# Gold (Counter)factuals 

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## Outline

- Motivation
- Model
- Data


## Questions

- there are various forms of money, used singly or jointly:
- coins (gold or silver)
- notes (backed or unbacked)
- deposits (narrow or not)
- we want to think about the policy choices governing these objects
- We are motivated by an array of questions:
- what if Bryan had won in 1896?
- switch from gold to silver
- a form of narrow banking
- was there enough gold for the gold standard?
- what happens when a country goes on/off gold/silver?
- we want to think of the price level implications
- we're structural guys
- we need a plausibly parametrized model of commodity money
- we will apply to the 1800-1913, but first we fit to 1500-1800 (Price Revolution)


## Model with one metal

- consumption good $x_{t}$, in exogenous supply
- metal stock $Q_{t}$, also exogenous
- metal can be held in nonmonetary form (jewelry) $d_{t+1}$ or in the form of money
- a firm transforms money into jewelry (melts) or jewelry into money (mints)
- representative agent's preferences:

$$
E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(x_{t}, d_{t+1}\right)
$$

- constraints:

$$
\begin{array}{r}
p_{t} x_{t}+q_{t} h_{t} \leq m_{t} \\
m_{t+1} \leq \Pi_{t}+m_{t}-p_{t} x_{t}-q_{t} h_{t} \\
d_{t+1}=d_{t}+h_{t}+Q_{t+1}-Q_{t}
\end{array}
$$

given initial stocks $m_{0}$ and $d_{0}$.

- firm's profits:

$$
\Pi_{t}=p_{t} x_{t}+n_{t}-\mu_{t}+q_{t} h_{t}
$$

- the firm's problem yields:

$$
q_{t} \geq 1, n_{t}=0 \text { and } \mu_{t}=m_{t} \text { if } q_{t}>1
$$

## Steady State

$$
\begin{aligned}
h & =0 \\
m & =p x \\
q & =1 \\
m & =Q-d \\
u_{d} & =\frac{1}{p}(1-\beta) u_{x}
\end{aligned}
$$

which can be written as an equation in $d$

$$
(Q-d)=(1-\beta) \frac{u_{x}}{u_{d}} x
$$

and the price level is

$$
p=(1-\beta) \frac{u_{x}}{u_{d}}
$$

- $u(x, d)=x^{1-\alpha} d^{\alpha}$

$$
\begin{aligned}
d & =a Q \\
p & =(1-a) \frac{Q}{x}
\end{aligned}
$$

price is linear in $Q / x$

- $u(x, d)=\left((1-\alpha) x^{\rho}+\alpha d^{\rho}\right)^{1 / \rho}$

$$
\begin{aligned}
\left(\frac{d}{x}\right)^{1-\rho} & =\frac{1-a}{a}\left(\frac{Q}{x}-\frac{d}{x}\right) \\
p+\tilde{a} p^{\sigma} & =\frac{Q}{x}
\end{aligned}
$$

with $\sigma=1 /(1-\rho), \tilde{a}=\left(\frac{a}{1-a}\right)^{\sigma} ; p$ is increasing but nonlinear function of $Q / x$

## Introducing token money

- cash-in-advance constraint is

$$
Q-d+e \bar{m}=p x
$$

where $e$ is quantity of silver per "dollar"

- bound on the silver value of token money:

$$
e \bar{m} \leq(1-\beta) \frac{u_{x}(x, Q)}{u_{d}(x, Q)} x
$$

- below the bound
- metal and token coexist at the rate e, $p \sim u_{x} / u_{d}$
- $d$ is endogenous: $d=a(Q+e \bar{m}), p=(1-a)(Q+e \bar{m}) / x$
- above the bound
- all metal is in jewelry (price of metal is at its minimum), the dollar price level is $p=\bar{m} / x$
- in an open economy, the metal is exported, the world price of metal may fall
- Hawtrey effect


## Bimetallism

- modify preferences:

$$
E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(x_{t}, d_{1, t+1}, d_{2, t+1}\right)
$$

(with 1:gold, 2:silver) and cash-in-advance constraint:

$$
e_{t} m_{1 t}+m_{2 t}=p_{t} x_{t}
$$

where $e_{t}$ is a free parameter (in oz silver / oz gold)

- steady-state equations are

$$
\begin{aligned}
\left(Q_{1}-d_{1}\right) \frac{v_{1 d}}{v_{2 d}}+Q_{2}-d_{2} & =(1-\beta) \frac{u_{x}}{v_{2 d}} \\
e & =\frac{v_{1 d}}{v_{2 d}}
\end{aligned}
$$

- two equations in three unknowns $\left(d_{1}, d_{2}, e\right)$
- government can set e subject to bounds derived by imposing $d_{1}=Q_{1}$ or $d_{2}=Q_{2}$
- solution method:
- pick $e$ and solve for $d_{1}, d_{2}$
- if one $d_{i} \geq Q_{i}$ then set $d_{i}=Q_{i}$ and solve for $d_{j}, e=v_{1 d} / v_{2 d}$
- these are "tight" bounds: they can be made looser by adding costs of converting metal $\leftrightarrow$ money (seigniorage, transport costs)
$-u(x, d)=x^{\gamma} d_{1}^{\alpha_{1}} d_{2}^{\alpha_{2}}$
- the bounds are

$$
\frac{\alpha_{1}}{\alpha_{2}+\gamma} \frac{Q_{2}}{Q_{1}} \leq e \leq \frac{\alpha_{1}+\gamma}{\alpha_{2}} \frac{Q_{2}}{Q_{1}}
$$

- within the bounds, $d_{i}=a_{i}\left(e Q_{1}+Q_{2}\right), p \sim\left(e Q_{1}+Q_{2}\right) / x$
- $u(x, d)=\left(\gamma x^{\rho}+\left(\alpha_{1} d_{1}{ }^{\epsilon}+\alpha_{2} d_{2}{ }^{\epsilon}\right)^{\rho / \epsilon}\right)^{1 / \rho}$

$$
\begin{aligned}
\left(\frac{d_{2}}{x}\right)^{1-\rho} & =b\left(\frac{Q}{x}-c \frac{d_{2}}{x}\right) \\
p & \sim \frac{d_{2}}{x}
\end{aligned}
$$

- we need data on $p, Q, x$ (per capita), e
- for this kind of history:
- quantities are heavily reconstructed/guestimated
- prices are relatively plentiful, but with lots of holes
- cheap way to aggregate unbalanced panel of price time series $\left\{y_{i t}\right\}$ :
- model $y_{i t}$ as an observation on a latent factor $z_{t}$

$$
\begin{align*}
y_{i t} & =\lambda_{i} z_{t}+\epsilon_{i t}  \tag{1}\\
z_{t} & =z_{t-1}+\eta_{t} \tag{2}
\end{align*}
$$

- estimate $\lambda_{i}, E\left(\epsilon^{2}\right), E\left(\eta^{2}\right)$ with MLE and Kalman filter


## Prices

- Robert C. Allen's CPI series from iisg.nl
- Antwerp, Amsterdam, London, Paris, Strasbourg, Florence, Naples, Valencia, Madrid, Augsburg, Leipzig, Munich, Vienna, Gdansk, Krakow, Warsaw, Lwow
- from 1264 to 1913 (not for all cities!)
- intended to be silver prices


## Prices



## Gold-silver ratio

- classic Soetbeer (1879) series:
- starts in 1687, derived from market value of gold coin in Hamburg
- before? Two approaches:
- average legal ratios across countries
- find market value of gold coin elsewhere
- Quarterly international netting, clearing and settlement meetings
- the Genoese merchants left Lyon in 1535, settled in Besançon (Bisenzone) and later Northern Italy
- records for exchange rates against mostly Spanish and Italian cities, but also Netherlands and Germany
- exchange rates expressed in terms of a gold-based unit per currency

Bisenzone


## Ratio



## Ratio



## Ratio



Dynamics of gold-silver ratio

- recall that, in the model, $e$ is arbitrarily chosen but must reside within bounds
- changes in metal stocks move the bounds
- the English and French ratios illustrate the dynamics


## Legal ratios and minting



## Legal ratios and minting



## Catch-up

- 17th c.: countries are trying to keep pace with rising relative price of gold
- why? To induce minting of gold, but why do they care? (our model is silent)
- by 1720, England gives up and stays on gold
- France continues and stabilizes the ratio for Europe until 1873 (14.5 until 1785, 15.5 after)


## Metal stocks

- the same Soetbeer (1879) estimated worldwide annual output rates of gold and silver from 1492
- since then, much research has tried to refine:
- Hamilton (1934) counted flows coming into Spain
- Morineau (1985) counted flows reported in European newspapers
- many historians have used New World archives (Bakewell 1971, Brading and Cross 1972...)
- new estimates for Brazil, Russia

Annual output



From flows to stocks

- we need estimates of stocks in 1492
- but we have reliable ones!
- we need estimates of stocks in 1492
- but we have reliable ones!
above the 297 tonnes reported by Velde and Webber. Given the research capabilities available to Velde and Webber in their positions in the Federal Reserve System, and given that other independent analyses have produced similar results, it is reasonable to rely upon their estimate that the total stock in 1492 was 297 tonnes.


## Stocks and the Ratio



## Population, GDP

- classic Maddison (2001) data
- can be updated with lots of recent work on England, Holland, Northern Italy, Germany, Spain, France
- De Vries (2003) for population and urbanization


## Population

|  | 1400 | 1450 | 1500 | 1550 | 1600 | 1650 | 1700 | 1750 | 1800 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1400 | 1450 | 1500 | 1550 | 1600 | 1650 | 1700 | 1750 | 1800 |
| England and Wales | 2.2 | 2.2 | 2.6 | 3.2 | 4.4 | 5.6 | 5.4 | 6.1 | 9.2 |
| Scotland | 0.7 | 0.7 | 0.8 | 0.9 | 1.0 | 1.0 | 1.0 | 1.3 | 1.6 |
| GB | 2.8 | 2.9 | 3.4 | 4.1 | 5.4 | 6.6 | 6.4 | 7.4 | 10.8 |
| lreland | 0.8 | 0.9 | 1.0 | 1.1 | 1.4 | 1.8 | 2.8 | 3.2 | 5.3 |
| UK | 3.7 | 3.8 | 4.4 | 5.2 | 6.8 | 8.4 | 9.2 | 10.6 | 16.1 |
| Holland | 0.2 | 0.2 | 0.3 | 0.3 | 0.5 | 0.8 | 0.9 | 0.8 | 0.8 |
| NL |  |  |  |  |  |  |  |  |  |
| Holland | 0.6 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.0 | 1.1 | 1.3 |
| Netherlands | 0.8 | 0.9 | 1.0 | 1.2 | 1.4 | 1.8 | 1.9 | 1.9 | 2.1 |
| ltaly (CN) | 5.3 | 6.0 | 6.6 | 7.2 | 8.3 | 7.0 | 8.5 | 9.6 | 10.8 |
| Italy (S) | 3.1 | 3.5 | 3.9 | 4.2 | 4.8 | 4.3 | 4.8 | 5.7 | 7.0 |
| Italy | 8.4 | 9.5 | 10.5 | 11.4 | 13.1 | 11.3 | 13.3 | 15.3 | 17.8 |
| Spain | 3.4 | 3.8 | 4.2 | 5.3 | 6.4 | 6.8 | 7.1 | 8.2 | 11.1 |
| Belgium | 1.1 | 1.3 | 1.4 | 1.7 | 1.6 | 2.0 | 2.0 | 2.2 | 2.9 |
| Germany | 5.8 | 7.3 | 8.7 | 12.0 | 15.4 | 9.5 | 13.4 | 17.4 | 21.6 |
| Scandinavia | 1.2 | 1.4 | 1.5 | 1.7 | 2.0 | 2.6 | 2.9 | 3.6 | 5.0 |
| France | 11.6 | 14.0 | 16.4 | 19.5 | 19.6 | 20.5 | 22.6 | 24.6 | 28.2 |
| Portugal | 0.7 | 0.8 | 0.9 | 1.4 | 1.9 | 2.0 | 2.3 | 2.4 | 2.9 |
| other Europe | 4.4 | 5.1 | 6.7 | 7.4 | 8.7 | 8.1 | 8.6 | 10.7 | 13.9 |
| Total Europe | 41.0 | 47.7 | 55.6 | 66.7 | 76.9 | 73.0 | 83.3 | 96.9 | 121.6 |

## GDP/capita

|  | 1400 | 1450 | 1500 | 1550 | 1570 | 1600 | 1650 | 1700 | 1750 | 1800 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| England | 1090 | 1055 | 1114 | 1135 | 1143 | 1123 | 1100 | 1630 |  |  |
| Scotland | 818 | 791 | 836 | 851 | 857 | 842 | 825 | 1201 |  |  |
| GB | 1059 | 993 | 1053 | 1072 | 1080 | 1071 | 1058 | 1563 | 1710 | 2080 |
| Ireland | 526 | 526 | 526 | 571 |  | 615 |  | 715 |  | 840 |
| UK | 885 | 887 | 909 | 909 | 909 | 862 | 792 | 1282 | 1421 | 1748 |
| Holland | 1245 | 1432 | 1483 | 1697 | 1783 | 2372 | 2171 | 2403 | 2440 | 2617 |
| NL |  |  |  |  |  |  |  |  |  |  |
| Holland | 934 | 1074 | 1112 | 1273 |  | 1779 |  | 1802 | 1510 | 1218 |
| NL |  | 1172 | 1217 | 1395 |  | 2002 | 2041 | 2080 | 1900 | 1752 |
| Italy (C+N) | 1761 | 1943 | 1537 | 1545 | 1511 | 1391 | 1566 | 1486 | 1614 | 1486 |
| Italy (S) | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 | 1100 |
| Italy | 1601 | 1668 | 1403 | 1381 | 1337 | 1244 | 1271 | 1350 | 1403 | 1244 |
| Spain | 885 | 889 | 889 | 940 | 990 | 944 | 820 | 880 | 910 | 962 |
| Belgium |  | 900 | 929 | 1043 | 1089 | 1073 | 1203 | 1264 | 1375 | 1497 |
| DE |  | 1100 | 1332 | 1113 |  | 894 | 1130 | 1068 | 1162 | 1140 |
| SE |  | 900 | 1000 | 1100 | 1368 | 1076 | 1303 | 1691 | 1255 | 1175 |
| France | 985 | 981 | 935 | 860 | 804 | 901 | 963 | 992 | 992 | 1045 |
| Portugal | 615 | 615 | 615 | 615 | 829 | 863 | 1033 | 914 | 1197 | 1022 |
| total |  | 1129 | 1101 | 1040 |  | 996 | 1057 | 1154 | 1191 | 1225 |


|  | 1450 | 1500 | 1550 | 1600 | 1650 | 1700 | 1750 | 1800 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| ratio | 10.90 | 10.55 | 11.05 | 11.70 | 14.30 | 15.10 | 14.77 | 15.54 |
| population | 47.7 | 55.6 | 66.7 | 76.9 | 73.0 | 83.3 | 96.9 | 121.6 |
| GDP | 53.8 | 61.2 | 69.3 | 76.7 | 77.2 | 96.1 | 115.4 | 148.9 |
| GDP/capita | 1.13 | 1.10 | 1.04 | 1.00 | 1.06 | 1.15 | 1.19 | 1.22 |
| prices | 0.58 | 0.52 | 0.64 | 1.13 | 1.28 | 1.12 | 1.04 | $1.67(1.47)$ |
| gold (t) | 280 | 356 | 653 | 890 | 1113 | 1341 | 2426 | 3624 |
| silver (kt) | 3.50 | 4.02 | 7.71 | 20.1 | 37.0 | 50.9 | 67.5 | 98.1 |

- Preference parameters $\theta$, exogenous variables: $Q, x, e$
- Endogenous variables: $d_{i}$ (we don't observe), $p$ (we do)
- Given $\{Q, x, e\}$ find $\theta$ to minimize $\sum_{t}\left(\hat{p}_{t}-p_{t}\right)^{2}$
- warning: we don't have that many observations


## Cobb-Douglas preferences



Cobb-Douglas preferences


Cobb-Douglas preferences


CD vs. CES preferences


CD vs. CES preferences


- the Price Revolution literature has suggested other factors
- urbanization $\rightarrow$ higher velocity
- financial changes (private and public banks, circulating liabilities)
- do we need these additional factors?
- the minting data might provide further tests of the bounds
- once we're happy with our parameters, we move on to the 19th c.!


## An early Hawtrey effect

- around 1600 Spain began to issue token money (copper)
- copper drove out silver by 1628
- the government started manipulating the currency and demand collapsed

An early Hawtrey effect


## An early Hawtrey effect



An early Hawtrey effect


