Evidence on the Efficiency of Index Options Markets

LUCY F. ACKERT AND YISONG S. TIAN

Ackert is a senior economist in the financial section of the Atlanta Fed's research department. Tian is an associate professor of finance at the Schulich School of Business, York University. *The authors thank Jerry Dwyer, Mark Fisher, Saikat Nandi, and Steve Smith for helpful comments.*

SINCE THE CHICAGO BOARD OPTIONS EXCHANGE INTRODUCED THE FIRST INDEX OPTION CONTRACT IN 1983, INDEX OPTIONS MARKETS HAVE HAD A SIGNIFICANT ROLE IN FINANCIAL MARKETS. INDEX OPTIONS HAVE BEEN ONE OF THE MOST SUCCESSFUL OF THE MANY INNOVATIVE FINANCIAL INSTRUMENTS INTRODUCED OVER THE LAST FEW DECADES, AS THEIR HIGH TRADING volume indicates. Index options give market participants the ability to participate in anticipated market movements without having to buy or sell a large number of securities, and they permit portfolio managers to limit downside risk. Given their prominence and functions, the pricing efficiency of these markets is of great importance to academics, practitioners, and regulators.

Well-functioning financial markets are vital to a thriving economy because these markets facilitate price discovery, risk hedging, and allocating capital to its most productive uses. Inefficient pricing of index options indicates that their market (and, possibly, other financial markets) is not doing the best possible job at these important functions. To detect inefficient pricing (often called mispricing) requires computing a theoretically efficient price or price range and comparing it with prices of options traded in financial markets. But valuing an index option in theory is complicated and challenging.

One popular approach to deriving option pricing relationships is based on a principal called no-arbitrage. This approach is a very powerful tool in the valuation of financial assets because it does not make strong assumptions about traders' behavior or market price dynamics. The principle simply assumes that arbitrageurs enter the market and quickly eliminate mispricing if a riskless profit opportunity exists. An arbitrageur is an individual who takes advantage of a situation in which securities are mispriced relative to each other. The arbitrageur buys the underpriced asset and sells the overpriced asset, locking in a riskless profit. In doing so, the arbitrageur drives the price of the underpriced asset up and the price of the overpriced asset down, thus eliminating mispricing. However, in a well-functioning economy—where there are no free lunches—there is no portfolio of assets that has zero cost today and a certain, positive payoff in the future. Similarly, there is no portfolio of assets that pays a positive amount with certainty today and requires no payment in the future. Arbitrage is critical for ensuring market efficiency because it forces asset prices to return to their implied, no-arbitrage values.

Many earlier studies report evidence of mispricing of index options, though arbitrage might have been limited. In some situations, market frictions restrict arbitrage so that investors simply cannot take advantage of available profit opportunities. For example, if arbitrageurs are subject to capital constraints and they cannot raise the capital necessary to form the riskless hedge, they will be unable to undertake trades that would move the market toward an efficient state (Shleifer and Vishny 1997). Similarly, the activity of arbitrageurs may be limited because the stock index underlying the option is often relatively difficult and costly to reproduce. To arbitrage based on a mispriced index option, investors may need to replicate the index by buying or selling a large basket or set of stocks that is perfectly correlated with the index. Doing so may be relatively difficult and costly, even for large investors (Ackert and Tian 1998b, 1999).[1]

The evidence of index option mispricing has been taken to indicate that options markets are inefficient and casts doubt on their contributions to price discovery, hedging, and efficient capital allocation. This article is a discussion of index option pricing aimed at analyzing earlier evidence of mispricing and presenting new evidence on index option pricing and its evolution. It first presents theoretical pricing relationships implied by no-arbitrage conditions. These conditions place bounds on possible efficient call and put option prices and imply relative pricing relationships between call and put option prices. A call (put) option is the option to buy (sell) an asset. Empirical tests of the conditions presented provide powerful insight into how options market efficiency has evolved over time. In contrast to many previous studies of options market efficiency, the arbitrage strategies examined here do not involve trading a stock index, and the relationships hold for any given value of the underlying asset. This approach avoids some of the difficulties that arise from impediments to arbitrage when, for example, an investor might have to short sell a large stock basket—that is, sell shares he or she does not own by borrowing them from another investor.

The article also reviews earlier studies of the pricing efficiency of index options markets and provides an empirical examination of the efficiency of the market for the popular Standard and Poor's (S&P) 500 index options. The results indicate some substantial deviations of market prices from theoretical pricing relationships. Importantly, S&P 500 index options are frequently mispriced, and the mispricing does not appear to have abated over time. The mispricing may not, however, indicate market inefficiency because there are limits to arbitrage.

Index options give market participants the ability to participate in anticipated market movements without having to buy or sell a large number of securities, and they permit portfolio managers to limit downside risk.

Arbitrage Pricing Relationships

In evaluating the efficiency of option pricing, a theoretical optimal price derived from a model frequently provides the basis for comparison. Such theoretical models often assume specific dynamics for the underlying asset in order to derive more well-defined restrictions on the efficient price. In contrast, tests of pricing efficiency based solely on no-arbitrage arguments may be more informative if the relationships are independent of the models, though restrictions they place on price may not be very demanding.

Arbitrage pricing relationships are based on the simple assumption that investors prefer more to less. If these pricing relationships are violated by actual prices after adjustment for the bid-ask spread and transaction costs, arbitrage profits may be possible by buying the underpriced asset(s) and short-selling the overpriced asset(s). As discussed previously, rational pricing of options imposes explicit restrictions on the relative prices of call and put options. If these restrictions are violated, arbitrage opportunities exist. Some arbitrage pricing relationships jointly test options and stock market efficiency and allow examination of the information exchange between these markets whereas others test options market efficiency alone and allow examination of how pricing has evolved over time.[2] The relationships and

1. Note that traders can use the very liquid S&P 500 futures contract to replicate the index.
2. Billingsley and Chance (1985) and Ronn and Ronn (1989) note that some tests are joint tests of options and stock market efficiency while others consider only options market efficiency. See Ackert and Tian (1998a, 1999) for examples of both types of relationships.

empirical tests reported in this article are of the latter type. Because stock market transactions are not involved, examining these relationships may provide a superior test of pricing across index options. Another advantage of these types of relationships is that they are unaffected by the different closing times in stock and options markets.

The arbitrage pricing relationships presented below allow examining whether options market efficiency improved over the sample period. Options on the S&P 500 index are European, and the discussion below applies to European options only. A European option may be exercised only at maturity whereas an American option may be exercised prior to maturity. All relationships account for the bid-ask spread because bid-ask spreads result in significant transaction costs for participants in options markets (Phillips and Smith 1980; Baesel, Shows, and Thorp 1983). Define:

- C^b: bid price of European call option;
- C^a: ask price of European call option;
- P^b: bid price of a European put option;
- P^a: ask price of a European put option;
- S: price of underlying asset;
- X: strike price;
- T: maturity of the option;
- r: risk-free rate of interest or Treasury bill rate;[3]
- t_i: transaction costs (other than those arising from the bid-ask spread) of buying or selling calls, puts, or Treasury bills, $i = c$, p, or r.

Three sets of arbitrage pricing relationships are presented: the box spread, call and put spreads, and call and put convexity. The box spread is a combination of call and put spreads that matches two pairs of call and put options.[4] This strategy requires that an investor purchase and sell calls (bullish call spread) with strike prices X_1 and X_2, respectively, while simultaneously selling and purchasing puts (bearish put spread) with strike prices X_1 and X_2, respectively. The box spread is a riskless strategy because the future payoff is always positive: the difference between two strike prices, $X_2 - X_1$, where $X_1 < X_2$. The payoff is illustrated in Chart 1. If bid-ask spreads and transaction costs are taken into account, the box spread is expressed by the following two inequalities:

$$(C_1^a - C_2^b) - (P_1^b - P_2^a) + (X_1 - X_2)e^{-rT} + t_1 \geq 0 \quad \text{(1a)}$$

and

$$(C_2^a - C_1^b) - (P_2^b - P_1^a) + (X_2 - X_1)e^{-rT} + t_1 \geq 0, \quad \text{(1b)}$$

where $t_1 = 2t_c + 2t_p + t_r$. In the absence of arbitrage, inequalities (1a) and (1b) hold.

In contrast to the box spread, the call (put) spread combines two call (put) options with identical maturity. The call spread strategy requires purchase of call option 1 and sale of call option 2, where $X_1 < X_2$, as illustrated in Chart 2. The call spread is expressed as

$$(C_2^a - C_1^b) + (X_2 - X_1)e^{-rT} + t_{2a} \geq 0, \quad \text{(2a)}$$

where $t_{2a} = 2t_c + t_r$. Similarly, the put spread involves the sale of put option 1 and purchase of put option 2 and is expressed as

$$(P_1^a - P_2^b) + (X_2 - X_1)e^{-rT} + t_{2b} \geq 0, \quad \text{(2b)}$$

where $t_{2b} = 2t_p + t_r$.

Finally, call (put) convexity creates a riskless position by combining three call (put) options where $X_1 < X_2 < X_3$. The call (put) convexity strategy requires purchase of call (put) options 1 and 3 and sale of call (put) option 2. Call convexity is expressed as

$$wC_1^a + (1 - w)C_3^a - C_2^b + 2t_c \geq 0 \quad \text{(3a)}$$

and put convexity is

$$wP_1^a + (1 - w)P_3^a - P_2^b + 2t_p \geq 0, \quad \text{(3b)}$$

where $w = (X_3 - X_2)/(X_3 - X_1)$. So, for example, if $w = ½$, call convexity involves the purchase of one call option 1 and one call option 3 for every two call option 2s sold. The payoff from this strategy, commonly referred to as a butterfly spread, is illustrated in Chart 3.

If the box spread, call spread, put spread, call convexity, or put convexity is violated, arbitrage profits are possible by taking appropriate option positions. For example, if the call spread (2a) is violated, index call option 1 is overvalued relative to call option 2. The arbitrageur would sell call 1 and buy call 2, investing the balance in a Treasury bill earning the risk-free rate. In the case of exercise of both call options at maturity, the arbitrageur closes the index position and earns a risk-free profit at maturity (time T) of $(C_1^b - C_2^a) + (X_1 - X_2)e^{-rT} - t_{2a} \geq 0$, where $t_{2a} = 2t_c + t_r$ (see the box for the details of this arbitrage).

Although a violation of any of the inequalities above indicates the presence of an arbitrage opportunity, the box spread inequalities (1a) and (1b) place more demanding restrictions on the pricing of options. In the absence of transaction costs, the box spread requires an equality among four option prices. In contrast, even ignoring trans-

CHART 1 Box Spread Payoff

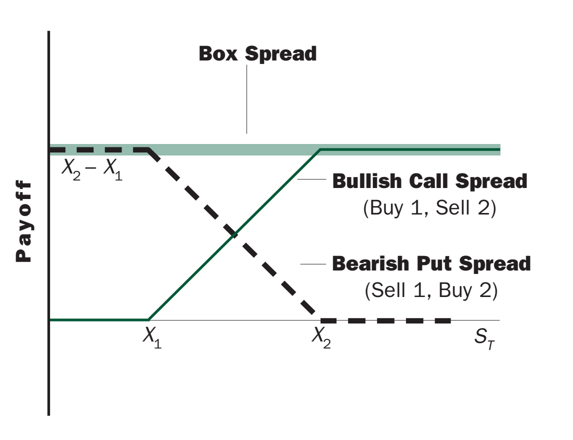


CHART 2 Call Spread Payoff

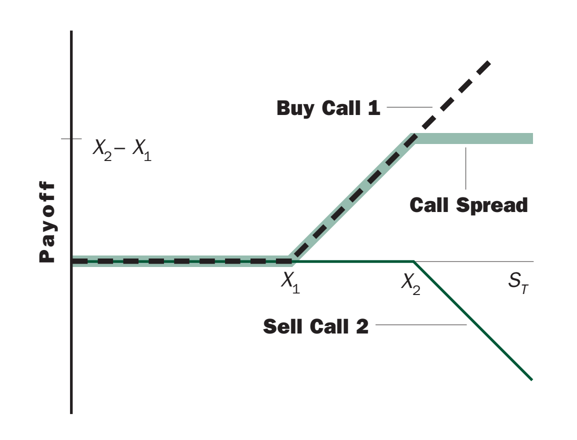
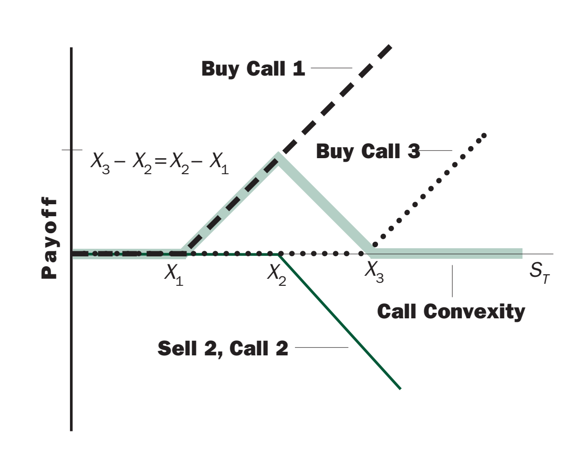


CHART 3 Call Convexity ($w = ½$)



action costs, call and put spreads and convexity are minimum-maximum (inequality) restrictions, and a wide range of prices is consistent with the boundaries they place on option prices, as is apparent in Charts 1–3.

Efficiency of Index Options Markets

Many empirical studies have tested pricing relations between put and call options, particularly for options on individual stocks. See, for example, Stoll (1969), Gould and Galai (1974), and Klemkosky and Resnick (1979). Some of these tests are based on theoretical option pricing models, such as the Black-Scholes (1973) or the Cox, Ross, and Rubinstein (1979) binomial option pricing models. Other tests are based simply on arbitrage arguments and are model-independent, including, for example, the box spread.

Although the empirical evidence generally supports some pricing relationships like put-call parity for individual stock options, significant mispricing has been reported in stock index options markets. For example, Evnine and Rudd (1985) use intraday data for a two-month period in 1984 and find frequent violations of boundary conditions and put-call parity for S&P 100 and Major Market index options, both of which are American options.[5] Evnine and Rudd further conclude that these options are significantly mispriced relative to theoretical values based on the binomial option pricing model. Chance (1987) also finds that put-call parity and the box spread are violated frequently for S&P 100 index options and that the violations are significant in size.[6] However, these results may not indicate market inefficiency for several reasons.

3. The assumption is that borrowing and lending rates are equal. Regarding the impact of this assumption on the results, see note 14.
4. The box spread is also a simple algebraic combination of the put-call parity relationship for each option. Put-call parity relates the put price, call price, exercise price, risk-free interest rate, and underlying asset price for options on the same asset with identical exercise price and expiration date. According to put-call parity, a pair of call and put options with identical maturity and strike price should be priced such that $C + Xe^{-rT} = P + S$, ignoring transaction costs and the bid-ask spread.
5. A boundary condition specifies a maximum or minimum price for an option. For example, an upper bound on the price of a call option is the value of the underlying asset because, no matter what happens, the option can never be worth more than the asset, that is, $C \leq S$. See note 4 above on put-call parity. For derivations of the various pricing relationships, see Merton (1973), Cox and Rubinstein (1985), Chance (1987), and Hull (1997).
6. In another study, Chance (1986) examines whether S&P 100 option prices are consistent with the Black-Scholes model and concludes that the model cannot be used to generate abnormal returns.

Arbitrage Opportunity When the Call Spread Does Not Hold

Ignoring the bid-ask spread and transaction costs, the call spread is expressed as

$$(C_2 - C_1) + (X_2 - X_1)e^{-rT} \geq 0.$$

Without loss of generality, it is always possible to rearrange the pair of calls so that $X_1 < X_2$. Given this arrangement, the price of the first call should always be greater than that of the second call; that is, $C_1 > C_2$. If the call spread is violated, that is, if

$$(C_2 - C_1) + (X_2 - X_1)e^{-rT} < 0,$$

then risk-free profit opportunities are present. Arbitrage profits are possible by taking appropriate positions in the options market. In this case, index call option 1 is overvalued relative to call option 2. The arbitrageur would sell call 1, buy call 2, and invest the balance in a Treasury bill earning the risk-free rate. At maturity, a call option is exercised if the stock price exceeds its exercise price. The cash flows from the strategy are as shown in the table.

At the inception of this strategy there is no initial investment, and at maturity there are three possible cash flows, all of which are positive. When $S < X_1$, neither option is exercised upon maturity and the arbitrageur accrues the entire amount invested in the Treasury bill as profit; that is, $(C_1 - C_2)e^{rT} > 0$. When $X_1 < S \leq X_2$, option 1 is exercised but not option 2 and the investment in the Treasury bill more than offsets the loss on the first option so that

$$(C_1 - C_2)e^{rT} - (S - X_1) > (C_1 - C_2)e^{rT} - (X_2 - X_1) > 0.$$

The first inequality holds because $(X_1 - S) > (X_1 - X_2)$ when $X_1 < S \leq X_2$, and the second, because the call spread is violated. Finally, if both options are exercised at maturity, positive profit also accrues, again because the call spread is violated. Therefore, profit is made by the arbitrageur in all three possible outcomes. After commission fees and the bid-ask spread are recognized, the violation of (2a) (see Tables 4 and 5) must be large enough to compensate for transaction costs; that is,

$$(C_2^a - C_1^b) + (X_2 - X_1)e^{-rT} + t_{2a} < 0.$$

Such an opportunity cannot persist as arbitrageurs will take advantage of the mispricing until (2a) holds.

Arbitrage When the Call Spread Is Violated

	Cash Flow			
		At Options' Maturity		
Strategy	Today	$S < X_1$	$X_1 < S \leq X_2$	$S > X_2$
Sell call 1	$+C_1$	—	$-(S - X_1)$	$-(S - X_1)$
Sell call 2	$-C_2$	—	—	$S - X_2$
Buy Treasury bill	$-(C_1 - C_2)$	$(C_1 - C_2)e^{rT}$	$(C_1 - C_2)e^{rT}$	$(C_1 - C_2)e^{rT}$
Total	0	$(C_1 - C_2)e^{rT}$	$(C_1 - C_2)e^{rT} - (S - X_1)$	$(C_1 - C_2)e^{rT} - (X_2 - X_1)$

These tests of market efficiency may be misleading because they use American options and the arbitrage conditions are for European options. Kamara and Miller (1995) point out that prior to their examination all tests of put-call parity used American options. In addition, tests of other arbitrage pricing relationships such as the box spread used data for American options (for example, Billingsley and Chance 1985). Because of the possibility of early exercise, these relationships may not be expected to hold for American options, and similar conditions for American options are frequently intractable. In their tests using S&P 500 index options that are European, Kamara and Miller find fewer and smaller violations.

Tests of put-call parity may also fail to indicate market inefficiency if arbitrage at low cost is not possible. Introducing index options helps to reduce arbitrage costs. In Canada, a stock basket, Toronto Index Participation Units (TIPS 35), has been traded since 1990. Ackert and Tian (1998a) examine the efficiency of Canadian index and options markets by comparing the number and size of violations in theoretical pricing relationships before and after the introduction of TIPs. They conclude that, although options market efficiency improved over their test period, the connection between options markets and stock markets did not. Ackert and Tian (1999) also examine the impact of a traded stock basket, Standard and Poor's Depositary Receipts (SPDRs), on the link between U.S. index options markets. They conclude that the introduction of a stock basket can enhance market efficiency because it removes one limit to arbitrage.

In summary, the results reported in earlier studies suggest that put-call parity is frequently violated in index options markets and that these options are often mispriced relative to prices predicted by theoretical models. To overcome problems in earlier studies, this study tests theoretical pricing relationships based on no-arbitrage conditions for European stock index options. It focuses on tests of options market efficiency independent of the stock market and includes the effects of transaction costs and bid-ask spreads. It also examines whether deviations from pricing relations declined over the 1986–96 sample period.

Empirical Results

All three arbitrage pricing relationships presented earlier are investigated for S&P 500 index options on each trading day in the sample, as described subsequently. The number and size of violations are recorded and analyzed. This approach allows examining the evolution of the index options market and provides insight into whether market efficiency increased over the sample period. Arbitrage based on violations of the relationships considered does not require a position in the underlying asset. In addition, all of the pricing relationships are independent of an option pricing model so that no assumption concerning the process underlying the stock price is required. Thus, the empirical tests are true tests of market efficiency instead of joint tests of market efficiency and model specification.[7] Finally, the analysis recognizes the limits that transaction costs and bid-ask spreads place on arbitrage.

The empirical investigation analyzes the efficiency of the S&P 500 index options market using daily data for the S&P 500 index and index options from January 1, 1986, through December 31, 1996. Daily closing prices, trading volume, and open interest for S&P 500 index call and put options are from the Chicago Board Options Exchange.[8] The three-month Treasury bill rate (a proxy for the risk-free interest rate) is from the *Federal Reserve Bulletin*. Bid-ask spreads and commissions are included so that the analysis recognizes the effect of transaction costs on pricing efficiency. The approach is conservative in that it uses closing bid and ask prices, rather than closing prices, in testing the pricing relationships.[9] Following Harris, Sofianos, and Shapiro (1994) and Kamara and Miller (1995), this research constructs bid and ask prices, based on the usual spread in option prices, from closing prices. The option bid-ask spread is estimated by adding or subtracting 1⁄32 (1⁄16) of a point if the price is less than (greater than or equal to) \$3.[10] Following Kamara and Miller (1995), commission costs (t_i) are \$30 for Treasury bills and \$2 (\$4) per option contract for 100 shares if the price is less than (greater than or equal to) \$1.

On each trading day during the test period, the three pricing relationships discussed above are tested: the box spread (1a) and (1b), call and put spreads (2a) and (2b), and call and put convexity (3a) and (3b). For each maturity month, two pairs of put and call options are used to examine the box spread. The put and call within each pair are matched

7. So, for example, there is no test of whether prices are consistent with those predicted by a particular model such as the Black-Scholes option pricing model.
8. All relationships tested require synchronous option prices. Inferences are limited by the fact that closing prices may be nonsynchronous. However, Evnine and Rudd (1985) and Kamara and Miller (1995) find very similar results using intraday and closing price data for S&P 100 and S&P 500 index options, respectively.
9. See Ronn and Ronn (1989), who demonstrate that the use of bid-ask prices is conservative. They note that the market maker commits to transacting at least one contract at the bid-ask quotes, but the effective spread may be narrower. Traders are sometimes able to bargain to obtain better prices so that trades occur inside the quoted spread.
10. Some traders may have access to better price quotes. The assumption in this article concerning the constructed spread appears to be reasonable based on the results reported by others, though the results may be affected by the assumption to the extent that the spread is over- or underestimated.

with an identical strike price, but two different strike prices are used for the two pairs. In contrast, the call (put) spread combines two call (put) options with identical maturity and different strike prices. Finally, call (put) convexity combines three call (put) options with identical maturity and different strike prices.

The frequency and severity of violations are tabulated for the full sample period as well as for each year in the sample.[11] Examining violations in the pricing relationships for each sample year provides insight into how the efficiency of the options market has changed as the market has developed over time. Tables 1 through 9 report the percentage of violations as well as the mean violation in dollars. Significant dollar violations are tested for by testing the null hypothesis that the mean dollar violation is zero. All reported *t* statistics use standard errors corrected for autocorrelation using a maximum likelihood procedure estimated by a Gauss-Marquardt algorithm (Judge and others 1985).[12]

To further investigate the persistence of violations in pricing relationships, the study examines whether arbitrage opportunities are evident the day following observed violations. Doing so provides an ex ante test, which, as Galai (1977) argues, a true test of market efficiency must be. Ex ante tests are executed from the trader's point of view and reflect the trader's ability to actually form the required, profitable portfolio. In an ex ante approach, current prices reveal arbitrage opportunities but execution is at prices that are yet to be revealed. Conducting ex ante tests involves identifying each day on which a particular violation occurs and tracking whether the violation persisted on the following trading day. Existence on the following day implies that traders did not fully eliminate arbitrage opportunities.

Table 1 reports the frequency and severity of violations and ex ante violations of the box spread, inequalities (1a) and (1b). For the two inequalities, the percentage and dollar amount of violations are similar (21.02 percent and $1.07 versus 23.78 percent and $1.08). For each relationship, the percentage of violations is substantial and the mean dollar violation is significantly different from zero.[13] The ex ante tests indicate that significant abnormal profit opportunities existed even on the day following the violation of a pricing relationship. For example, 28,292 violations of (1a) occurred, and of these violations 2,785 or 9.84 percent persisted on the following day with a significant mean violation of $1.02.[14]

Tables 2 and 3 report the percentage and dollar size of violations and ex ante violations of (1a) and (1b), respectively, for each year in the 1986–96 sample period. All mean dollar violations are significantly different from zero at the 1 percent significance level. Although some variation is observed in the extent to which the pricing relationships are violated across years, the results provide no evidence that options market efficiency improved over the sample

TABLE 1 Violations and Ex Ante Violations of the Box Spread (1a) and (1b)

	Box Spread (1a)		Box Spread (1b)	
	Total Violations	Ex Ante Violations	Total Violations	Ex Ante Violations
Frequency of Violations				
Number of Observations	134,606	28,292	134,606	32,014
Number of Violations	28,292	2,785	32,014	3,210
Percentage of Violations	21.02	9.84	23.78	10.03
Violations, in Dollars				
Mean	1.07	1.02	1.08	1.11
Standard Deviation	1.05	1.00	1.07	1.09
t statistic for nonzero mean	170.00***	54.19***	180.36***	57.74***

Note: This table reports the frequency and dollar size of violations of the box spread (1a) and (1b) using daily data for the S&P 500 index and index options from January 1, 1986, through December 31, 1996. An ex ante violation occurs when a particular violation persists into the following trading day. Asterisks *,**, or *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively, in a two-tailed test.

TABLE 2 Violations and Ex Ante Violations of the Box Spread (1a) by Year

Sample Year	Total Violations: Percentage of Violations	Total Violations: Mean Dollar Violation	Ex Ante Violations: Percentage of Violations	Ex Ante Violations: Mean Dollar Violation
1986	18.72	0.83	4.35	0.76
1987	22.30	1.24	8.03	1.13
1988	20.34	0.86	5.83	0.85
1989	13.94	0.91	7.54	0.81
1990	24.06	1.06	12.18	1.01
1991	21.62	0.99	9.11	0.90
1992	17.03	0.78	9.21	0.92
1993	18.29	0.86	9.42	0.76
1994	17.69	0.92	8.99	0.87
1995	20.76	1.02	10.75	1.05
1996	25.96	1.35	11.14	1.23
Overall	21.02	1.07	9.84	1.02

Note: This table reports the percentage and dollar size of violations of the box spread (1a) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

period. The frequency of violations remains high at approximately 20 percent of observations, even after taking into account trading costs, including the bid-ask spread and commission fees.

Next, violations of call and put spreads (2a) and (2b) and call and put convexity (3a) and (3b) are examined. As reported in Tables 4 and 7, significant mean dollar violations and ex ante dollar violations were observed for all four relationships. However, for all four the frequency of violations is quite low. The maximum percentage of violations (ex ante violations) across the four inequalities for the full sample is only 3.08 percent (8.04 percent). When the percentage and dollar violations by year reported in Tables 5 and 6 (8 and 9) for call and put spreads (convexity) are considered, there is no apparent trend. Although market efficiency does not appear to have improved over the sample period, the results suggest that options market valuations were generally consistent with these theoretical predictions.

A numerical example for the call spread provides perspective on the size of the violations reported in this article. On January 4, 1996, call options expiring on March 16, 1996, with strike prices 610 (X_1) and 615 (X_2) were priced at \$23.25 (C_1) and \$15.50 (C_2). The maturity date translates into a time to maturity of 0.1973 years (T), and the continuously compounded Treasury bill rate is 5.29 percent (r). Using inequality (2a) and ignoring transaction costs results in $15.50 - 23.25 + (615 - 610)e^{(-0.0529 \times 0.1973)} = -2.8019$ so that the size of the violation is \$2.80. Transaction costs are the sum of commission fees and the bid-ask spread and are (4 + 4 + 30)/100 + 1/8 = 0.505, which gives a net violation of \$2.30 (–2.8019 + 0.505).

11. In some cases, a few extreme outliers were detected. After checking and rechecking the original data sources, these outliers remained. However, removing these outliers does not change statistical inferences.
12. Autocorrelation in the dollar violations might be expected because the time to maturity for sample options may overlap. Diagnostic tests confirm the presence of significant positive autocorrelation. However, inferences are unchanged if ordinary least squares standard errors are used.
13. Note that inequality (1a) involves lending whereas inequality (1b) requires borrowing. Because similar frequency and magnitude of violations are observed across the two inequalities, the results suggest that the assumption of equal borrowing and lending rates does not explain the extent of profit opportunities.
14. Abnormal profit opportunities are not expected to persist and, thus, the mean ex post violation is expected to be zero. However, no directional relationship in the percentage of violations over the two-day time period is posited.

TABLE 3 Violations and Ex Ante Violations of the Box Spread (1b) by Year

	Total Violations		Ex Ante Violations	
Sample Year	Percentage of Violations	Mean Dollar Violation	Percentage of Violations	Mean Dollar Violation
1986	21.32	0.83	7.07	0.60
1987	26.23	1.27	11.16	1.26
1988	21.56	0.95	5.65	0.67
1989	15.90	0.90	6.36	0.86
1990	25.11	1.03	8.99	1.12
1991	24.16	0.99	9.28	1.03
1992	20.25	0.88	8.45	0.84
1993	19.19	0.81	6.77	0.67
1994	19.73	0.91	8.89	1.04
1995	23.87	0.95	10.61	0.83
1996	30.54	1.41	13.17	1.42
Overall	23.78	1.08	10.03	1.11

Note: This table reports the percentage and dollar size of violations of the box spread (1b) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

TABLE 4 Violations and Ex Ante Violations of the Call Spread (2a) and Put Spread (2b)

	Call Spread (2a)		Put Spread (2b)	
	Total Violations	Ex Ante Violations	Total Violations	Ex Ante Violations
Frequency of Violations				
Number of Observations	283,345	5,806	537,701	2,159
Number of Violations	5,806	467	2,159	145
Percentage of Violations	2.05	8.04	0.40	6.72
Violations, in Dollars				
Mean	1.05	1.09	1.30	1.08
Standard Deviation	1.04	1.06	1.22	1.06
t statistic for nonzero mean	77.24***	22.13***	49.47***	12.27***

Note: This table reports the frequency and dollar size of violations of the call spread (2a) and put spread (2b) using daily data for the S&P 500 index and index options from January 1, 1986, through December 31, 1996. An ex ante violation occurs when a particular violation persists into the following trading day. Asterisks *,**, or *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively, in a two-tailed test.

TABLE 5 Violations and Ex Ante Violations of the Call Spread (2a) by Year

Sample Year	Total Violations: Percentage of Violations	Total Violations: Mean Dollar Violation	Ex Ante Violations: Percentage of Violations	Ex Ante Violations: Mean Dollar Violation
1986	1.08	0.81	7.69	1.72
1987	2.10	0.92	8.70	0.70
1988	1.14	0.85	2.69	1.95
1989	1.71	0.91	6.54	0.86
1990	0.80	0.85	7.07	1.11
1991	3.14	1.17	8.36	0.90
1992	1.64	0.98	7.51	1.21
1993	1.48	0.73	6.65	0.52
1994	0.70	0.88	2.46	0.91
1995	3.92	0.99	11.16	1.14
1996	2.21	1.36	6.18	1.47
Overall	2.05	1.05	8.04	1.09

Note: This table reports the percentage and dollar size of violations of the call spread (2a) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

TABLE 6 Violations and Ex Ante Violations of the Put Spread (2b) by Year

Sample Year	Total Violations: Percentage of Violations	Total Violations: Mean Dollar Violation	Ex Ante Violations: Percentage of Violations	Ex Ante Violations: Mean Dollar Violation
1986	0.31	0.97	0	0
1987	1.83	1.75	4.75	1.66
1988	0.48	0.81	3.91	0.37
1989	0.19	0.90	3.70	0.25
1990	0.87	1.13	8.00	0.96
1991	0.25	0.97	4.42	0.43
1992	0.24	0.79	7.29	0.74
1993	0.17	0.96	2.78	0.19
1994	0.45	1.05	9.51	0.88
1995	0.12	1.00	9.80	1.06
1996	0.29	1.65	8.06	1.34
Overall	0.40	1.30	6.72	1.08

Note: This table reports the percentage and dollar size of violations of the put spread (2b) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

TABLE 7 Violations and Ex Ante Violations of Call Convexity (3a) and Put Convexity (3b)

	Call Convexity (3a)		Put Convexity (3b)	
	Total Violations	Ex Ante Violations	Total Violations	Ex Ante Violations
Frequency of Violations				
Number of Observations	882,954	27,206	2,244,467	20,439
Number of Violations	27,206	1,659	20,439	844
Percentage of Violations	3.08	6.10	0.91	4.13
Violations, in Dollars				
Mean	0.91	1.13	0.95	1.21
Standard Deviation	0.94	1.07	1.04	1.14
t statistic for nonzero mean	159.98***	43.01***	131.12***	30.81***

Note: This table reports the frequency and dollar size of violations of call convexity (3a) and put convexity (3b) using daily data for the S&P 500 index and index options from January 1, 1986, through December 31, 1996. An ex ante violation occurs when a particular violation persists into the following trading day. Asterisks *,**, or *** denote significance at the 10 percent, 5 percent, and 1 percent levels, respectively, in a two-tailed test.

TABLE 8 Violations and Ex Ante Violations of Call Convexity (3a) by Year

	Total Violations		Ex Ante Violations	
Sample Year	Percentage of Violations	Mean Dollar Violation	Percentage of Violations	Mean Dollar Violation
1986	2.11	0.72	0.61	0.02
1987	3.83	0.99	9.82	1.29
1988	1.37	0.79	1.12	0.20
1989	2.23	0.76	2.29	0.80
1990	1.76	0.64	4.07	0.72
1991	4.32	1.00	8.41	1.09
1992	1.82	0.75	3.17	1.02
1993	1.66	0.54	4.00	0.36
1994	0.85	0.55	4.09	0.82
1995	4.98	0.85	6.65	1.05
1996	3.45	1.07	5.12	1.33
Overall	3.08	0.91	6.10	1.13

Note: This table reports the percentage and dollar size of violations of call convexity (3a) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

TABLE 9 Violations and Ex Ante Violations of Put Convexity (3b) by Year

Sample Year	Total Violations: Percentage of Violations	Total Violations: Mean Dollar Violation	Ex Ante Violations: Percentage of Violations	Ex Ante Violations: Mean Dollar Violation
1986	1.06	0.58	0	0
1987	4.21	1.53	6.31	1.76
1988	0.77	0.71	2.06	0.49
1989	0.40	0.64	1.03	0.13
1990	1.82	0.82	5.59	0.93
1991	0.56	0.76	2.00	0.66
1992	0.45	0.70	2.91	0.70
1993	0.20	0.56	0.36	0.33
1994	0.76	0.78	3.94	1.13
1995	0.29	0.52	2.05	0.36
1996	1.01	0.89	3.51	1.06
Overall	0.91	0.95	4.13	1.21

Note: This table reports the percentage and dollar size of violations of put convexity (3b) using daily data for the S&P 500 index and index options for each year in the January 1, 1986, through December 31, 1996, sample period. An ex ante violation occurs when a particular violation persists into the following trading day. All mean dollar violations are significantly different from zero at the 1 percent significance level.

Taken together, significant violations of arbitrage pricing relationships are observed, even using ex ante tests, particularly for the box spread relationship. The differing results across the relationships tested are not surprising because the box spread is a more demanding test of market efficiency as compared with call and put spreads or convexity. The overall finding is that S&P 500 index options are frequently mispriced to a significant extent and that options market efficiency has not changed markedly over time.

Conclusion

This article examines the efficiency of the S&P 500 index options market using theoretical pricing relationships derived from stock index option no-arbitrage principles. It reports frequent and substantial violations of the box spread relationship in particular, even though the analysis reflects transaction costs. The results do not provide support for the argument that options market efficiency improved over time. However, at the same time, there were few violations of call and put spreads and convexity, which are less demanding tests of pricing efficiency than the box spread.

Market frictions appear to have a significant effect on arbitrageurs' abilities to take advantage of violations of no-arbitrage pricing relationships. Although the analysis reflects the market frictions imposed by the bid-ask spread and commission costs, other frictions may be significant. One such friction may be insufficient liquidity, which increases option traders' risk and may prevent them from eliminating arbitrage opportunities. In a liquid market a transaction can be quickly completed with little impact on prices.

REFERENCES

ACKERT, LUCY F., AND YISONG S. TIAN. 1998a. "The Introduction of Toronto Index Participation Units and Arbitrage Opportunities in the Toronto 35 Index Option Market." *Journal of Derivatives* 5, no. 4:44–53.

———. 1998b. "Investor Sentiment and Mispricing in Traded Stock Portfolios." Unpublished paper, Federal Reserve Bank of Atlanta.

———. 1999. "Efficiency in Index Options Markets and Trading in Stock Baskets." Federal Reserve Bank of Atlanta Working Paper 99-5, June.

BAESEL, JEROME B., GEORGE SHOWS, AND EDWARD THORP. 1983. "The Cost of Liquidity Services in Listed Options." *Journal of Finance* 38 (June): 989–95.

BILLINGSLEY, RANDALL S., AND DON M. CHANCE. 1985. "Options Market Efficiency and the Box Spread Strategy." *Financial Review* 20 (November): 287–301.

BLACK, FISCHER, AND MYRON S. SCHOLES. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (May/June): 637–54.

CHANCE, DON M. 1986. "Empirical Tests of the Pricing of Index Call Options." In *Advances in Futures and Options Research*. Vol. 1. Greenwich, Conn.: JAI Press.

———. 1987. "Parity Tests of Index Options." In *Advances in Futures and Options Research*. Vol. 2. Greenwich, Conn.: JAI Press.

COX, JOHN C., STEPHEN A. ROSS, AND MARK RUBINSTEIN. 1979. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7 (September): 229–63.

COX, JOHN C., AND MARK RUBINSTEIN. 1985. *Options Markets*. Englewood Cliffs, N.J.: Prentice-Hall.

EVNINE, JEREMY, AND ANDREW RUDD. 1985. "Index Options: The Early Evidence." *Journal of Finance* 40 (July): 743–56.

GALAI, DAN. 1977. "Tests of Market Efficiency of the Chicago Board Options Exchange." *Journal of Business* 50 (April): 167–97.

GOULD, JOHN P., AND DAN GALAI. 1974. "Transactions Costs and the Relationship between Put and Call Prices." *Journal of Financial Economics* 1 (July): 105–29.

HARRIS, LAWRENCE, GEORGE SOFIANOS, AND JAMES E. SHAPIRO. 1994. "Program Trading and Intraday Volatility." *Review of Financial Studies* 7 (Winter): 653–85.

HULL, JOHN C. 1997. *Options, Futures, and Other Derivative Securities*. 3d ed. Upper Saddle River, N.J.: Prentice-Hall.

JUDGE, GEORGE G., W.E. GRIFFITHS, R. CARTER HILL, HELMUT LUKTEPOHL, AND TSOUNG-CHAO LEE. 1985. *The Theory and Practice of Econometrics*. 2d ed. New York: John Wiley and Sons.

KAMARA, AVRAHAM, AND THOMAS W. MILLER JR. 1995. "Daily and Intradaily Tests of European Put-Call Parity." *Journal of Financial and Quantitative Analysis* 30 (December): 519–39.

KLEMKOSKY, ROBERT C., AND BRUCE G. RESNICK. 1979. "Put-Call Parity and Market Efficiency." *Journal of Finance* 34 (December): 1141–55.

MERTON, ROBERT C. 1973. "The Theory of Rational Option Pricing." *Bell Journal of Economics* 4 (Spring): 141–83.

PHILLIPS, SUSAN M., AND CLIFFORD W. SMITH JR. 1980. "Trading Costs for Listed Options: The Implications for Market Efficiency." *Journal of Financial Economics* 8 (June): 179–201.

RONN, AIMEE GERBARG, AND EHUD I. RONN. 1989. "The Box Spread Arbitrage Conditions: Theory, Tests, and Investment Strategies." *Review of Financial Studies* 2, no. 1:91–108.

SHLEIFER, ANDREI, AND ROBERT W. VISHNY. 1997. "The Limits of Arbitrage." *Journal of Finance* 52 (March): 35–55.

STOLL, HANS R. 1969. "The Relationship between Put and Call Option Prices." *Journal of Finance* 24 (December): 801–24.