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Asset-pricing Models and Economic Risk Premia: A Decomposition

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Asset-pricing Models and Economic Risk Premia: A Decomposition

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Abstract: The risk premia assigned to economic (nontraded) risk factors can be decomposed into three parts: (i) the risk premia on maximum-correlation portfolios mimicking the factors; (ii) (minus) the covariance between the nontraded components of the candidate pricing kernel of a given model and the factors; and (iii) (minus) the mispricing assigned by the candidate pricing kernel to the maximum-correlation mimicking portfolios. The first component is the same across asset-pricing models and is typically estimated with little (absolute) bias and high precision. The second component, on the other hand, is essentially arbitrary and can be estimated with large (absolute) biases and low precisions by multi-beta models with nontraded factors. This second component is also sensitive to the criterion minimized in estimation. The third component is estimated reasonably well, both for models with traded and nontraded factors. We conclude that the economic risk premia assigned by multi-beta models with nontraded factors can be very unreliable. Conversely, the risk premia on maximum-correlation portfolios provide more reliable indications of whether a nontraded risk factor is priced. These results hold for both the constant and the time-varying components of the factor risk premia.

JEL classification: G12

Key words: economic risk premium, asset-pricing models, mispricing, maximum-correlation mimicking portfolios, nontraded factors

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Introduction

The estimates of risk premia associated with economic (non-traded) risk variables are relevant for both practitioners and academics. Practitioners may be interested in knowing how to price new securities, which track economic risk factors. Academics are interested in knowing which economic risks are priced in the security markets. The issue of estimating economic risk premia has typically been addressed in the specific context of multi-beta models with non-traded factors, where all security risk premia are linear in the premia associated with the factors.

The articles that provide estimates of economic risk premia include Harvey (1989), Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), McElroy and Burmeister (1988), Ferson and Harvey (1991), and Jagannathan and Wang (1996). Unfortunately, estimates vary substantially in size, sign, and statistical significance from one study to the other. For example, the premium on inflation is negative and significant in Chen, Roll, and Ross (1986); positive and insignificant in McElroy and Burmeister (1988); negative and marginally significant in Ferson and Harvey (1991); and negative and insignificant in Jagannathan and Wang (1996). Another example is the premium on the slope of the term structure, which is negative and mostly insignificant in Chen, Roll, and Ross (1986); positive and significant in McElroy and Burmeister (1988); positive and marginally significant in Ferson and Harvey (1991); and negative and insignificant in Jagannathan and Wang (1996).

In this paper, we reconsider the evidence on economic risk premia in light of a novel decomposition. We define economic risk premia as the expected excess returns on theoretical exact mimicking portfolios. Hence, an economic risk premium is model dependent and equals the negative of the covariance between a normalized (mean-one) candidate pricing kernel and the risk factor. The economic risk premium can then be broken into three components: i) the expected excess return on the maximum-correlation portfolio tracking the factor;[1] ii) (minus)

[1]Maximum-correlation portfolios have special economic significance, since they are the hedging portfolios of Merton (1973). Fama (1996) shows that Merton's investors hold overall portfolios that minimize the return variance, for given expected return and covariances with the state variables. Other papers using maximum-correlation portfolios are Breeden (1979), Breeden, Gibbons, and Litzenberger (1989), Lamont (2001), Ferson, Siegel, and Xu (2005), and Van den Goorbergh, DeRoon, and Werker (2005).

the covariance between the non-traded components of the factor and of the candidate pricing kernel of a given model; and iii) (minus) the mis-pricing that a given model (pricing kernel) assigns to the maximum-correlation mimicking portfolio tracking the factor.

The first component is common to all models and the second and third components are model-dependent. The second component disappears if the kernel is traded; for example, if the kernel is linear in security (excess) returns. The second component is also somewhat arbitrary. Consider adding noise to both the factor and the kernel, with the noise uncorrelated with asset returns. This does not affect the pricing of securities, but it does affect the premium assigned to the factor. In other words, security-market data can only tell us whether the traded component of a factor is priced, while the conclusion of whether the non-traded component is also priced essentially depends on the model that one assumes. The third component disappears if the model exactly prices the maximum-correlation mimicking portfolio. This happens in the case of multi-beta models, when the weighting matrix used in the estimation of the coefficients of the pricing kernel is the inverse of the covariance matrix of returns.

We examine empirically the three components of the risk premia associated with the term structure, the dividend yield, consumption growth, the default premium, inflation, and the real rate of interest. We consider two multi-beta models with both traded and non-traded factors: the intertemporal capital asset pricing model (I-CAPM) and the consumption capital asset pricing model (C-CAPM). In addition, we consider two models with traded factors only: the Fama-French three-factor model and the capital asset pricing model (CAPM). We also examine the issue of time variation. Indeed, the I-CAPM postulates that certain economic variables should be priced risks *because* they affect the position of the investment-opportunity set.

We find that the economic risk premia assigned by the models with non-traded factors deviate substantially from the premia on maximum-correlation portfolios. In our setting, economic risk premia have the interpretation of Sharpe ratios because we standardize the factor by the corresponding standard deviation. Moreover, the economic risk premia estimates exhibit large (absolute) biases and standard errors, and are sensitive to the choice of weighting matrix. The premia on maximum-correlation portfolios themselves tend to be estimated precisely and with little bias. For example, in the case of the average conditional inflation risk premium in the context of the I-CAPM, we obtain estimates of -1.5290 and -1.0293, depending on the weighting matrix, with biases of 1.8546 and 1.2266, and standard errors of 0.9419 and 0.6843, respectively. These estimates can be compared with the average conditional risk premium on the maximum-correlation portfolio of 0.0036 with bias of 0.0006 and standard error of only 0.0163. We also document large discrepancies between the time variation of economic risk premia and the time variation of the premia on maximum-correlation portfolios. For example, consider again the inflation risk premium in the context of the I-CAPM. We estimate large negative impacts of the dividend yield on the conditional inflation premium: decreases of 1.2367 and 0.9349, depending on the weighting matrix, with standard errors of 0.7220 and 0.5193, respectively. The effect of the dividend yield on the conditional premium on the inflation maximum-correlation portfolio is much smaller and more precisely estimated: -0.0185 with a standard error of 0.0142.

We also show that the discrepancies between economic risk premia estimates and estimates of premia on maximum-correlation portfolios are mainly due to the non-traded common variability of factors and candidate pricing kernels. Hence, the non-traded component of an economic risk premium presents both conceptual and econometric challenges. From a conceptual standpoint, the non-traded component of an economic risk premium is largely arbitrary. As argued above, this non-traded component equals (minus) the covariance between the components of the pricing kernel and of the factor that are orthogonal to the span of asset returns; by adding noise unrelated to asset returns to the kernel and to the factor, one can make this non-traded component arbitrarily large, without affecting the pricing properties of the model. From an econometric standpoint, the non-traded component of an economic risk premium is difficult to estimate.

Importantly, the differences between estimates of economic risk premia and estimates of premia on maximum-correlation portfolios lead to differences in statistical inference. In the case of the I-CAPM, only two of the six average conditional risk premia on maximum-correlation portfolios are significant. In contrast, five of the six average conditional economic risk premia are significant when the weighting matrix is the identity matrix. For example, the p-value on a one-sided test that the average conditional premium on the portfolio tracking the inflation rate equals zero is 42.50%. The p-value for the average conditional risk premium on inflation (identity weighting matrix) is 0.30%.

We conclude that the estimates of economic risk premia based on multi-beta models with non-traded factors can be very unreliable. Hence, a better indicator of the risk premium on an economic risk factor is the expected excess return on the associated maximum-correlation portfolio.[2]

Related to the present paper is Balduzzi and Kallal (1997). Balduzzi and Kallal (1997) start from the same decomposition of economic risk premia, but focus on *admissible* kernels. Hence, they ignore the mis-pricing component of the economic risk-premium. In addition, the focus of Balduzzi and Kallal's (1997) analysis is to use the discrepancy between an economic risk-premium and the premium on a maximum-correlation portfolio to place a lower bound on the variability of any admissible pricing kernel. Indeed, they show how their volatility bounds are more stringent than the Hansen-Jagannathan (1991) bounds. Also related to the present paper are Kimmel (2004) and Balduzzi and Robotti (2005). Kimmel (2004) derives the asymptotic properties of estimates of the economic risk premia and of the premia on the maximum-correlation portfolios, for Gaussian i.i.d. returns. Balduzzi and Robotti (2005) compare the small-sample properties of tests of multi-beta models with non-traded factors, for two alternative formulations of the models: when the original factors are used; and when the factors are replaced by their projections onto the span of (excess) returns augmented with a constant, i.e. when the factors are replaced by the excess returns on the maximum-correlation portfolios.

This paper is organized as follows. Section I illustrates the decomposition of risk premia assigned by multi-beta models. Section II presents the orthogonality conditions imposed in estimation. Section III describes the data. Section IV presents the empirical results. Section V concludes.

[2]Note, though, that there is a special case where the two risk premia indicators are closely related. Assume that the multi-beta model has only one non-traded factor, and that the risk premium on the factor is estimated using a GLS-style cross-sectional regression. In this case, the unit-beta portfolio implicit in the cross-sectional regression has weights that are proportional to the weights of the maximum-correlation portfolio, and the Sharpe ratios of the unit-beta and of the maximum-correlation portfolios are the same. See Balduzzi and Robotti (2005) for further discussion.

I. Decomposing risk premia

A. Risk premia

We start by defining economic risk premia as expected excess returns on theoretical portfolios exactly mimicking the $K \times 1$ non-traded factors y_{t+1}. Since the factors are not traded, we cannot estimate their risk premia directly from security returns. Instead, we need a model to tell us what the risk premia are.[3] We denote with x_{t+1} the normalized (mean-one) candidate pricing kernel of a given asset-pricing model. We define the time-varying economic risk premia λ_t as

$$\lambda_t \equiv -\mathrm{Cov}_t(x_{t+1}, y_{t+1}), \tag{1}$$

where $\mathrm{Cov}_t(x_{t+1}, y_{t+1})$ denotes the conditional covariance between x_{t+1} and y_{t+1}. In the case where the asset-pricing model is a multi-beta model with non-traded factors, the corresponding pricing kernel is

$$x_{t+1} = 1 - [y_{t+1} - E_t(y_{t+1})]^\top b_t \equiv 1 - [y_{t+1} - E_t(y_{t+1})]^\top \Sigma_{yyt}^{-1} \lambda_t, \tag{2}$$

where b_t are the time-varying coefficients of the pricing kernel x_{t+1} and $E_t(y_{t+1})$ and Σ_{yyt} are the conditional factor means and covariances, respectively.

Hence, if the multi-beta model holds

$$E_t(x_{t+1} r_{t+1}) = 0, \tag{3}$$

where r_{t+1} is an $N \times 1$ vector of excess returns on a set of test assets. This means that

$$E_t(r_{t+1}) = \beta_t^\top \lambda_t, \tag{4}$$

where $\beta_t = \Sigma_{yyt}^{-1} \Sigma_{yrt}$ and Σ_{yrt} is the conditional cross-covariance matrix between y_{t+1} and r_{t+1}. In other words, the economic risk premia λ_t are defined based on a specific asset-pricing model. If the asset-pricing model is a multi-beta model, then all security risk premia are linear in the economic risk premia.

[3] Note that if the factors were traded, we could simply compute averages of their realizations in excess of the risk-free rate to obtain estimates of the economic risk premia. Yet, an asset-pricing model can still assign a premium to a traded factor that differs from its historical average excess return.

B. Decomposition

The main goal of this section is to provide an economic interpretation of the economic risk premia λ_t. For this purpose, consider the following decomposition of the pricing kernel x_{t+1}

$$x_{t+1} \equiv (x_{t+1} - x^\star_{t+1}) + (x^\star_{t+1} - q^\star_{t+1}) + q^\star_{t+1}, \tag{5}$$

where $x^\star_{t+1}$ is the projection of x_{t+1} onto a constant and the vector of asset excess returns r_{t+1}, and $q^\star_{t+1}$ is the minimum-variance (MV) admissible kernel, $1 - [r_{t+1} - E_t(r_{t+1})]^\top \Sigma_{rrt}^{-1} E_t(r_{t+1})$ (see Hansen and Jagannathan, 1991).[4]

Let $y^\star_{t+1} = (\gamma^\star_t)^\top r_{t+1}$ denote the variable part of the projection of y_{t+1} onto the augmented span of excess returns, i.e. on the span of excess returns augmented with a constant.[5] We have

$$\lambda_t \equiv -\mathrm{Cov}_t(q^\star_{t+1}, y_{t+1}) + \mathrm{Cov}_t[(x^\star_{t+1} - x_{t+1}), y_{t+1}] + \mathrm{Cov}_t[(q^\star_{t+1} - x^\star_{t+1}), y_{t+1}]. \tag{6}$$

It is easy to see that $-\mathrm{Cov}_t(q^\star_{t+1}, y_{t+1}) = (\gamma^\star_t)^\top E_t(r_{t+1}) \equiv \lambda^\star_t$, which are the expected excess returns on maximum-correlation portfolios.

Also, note that

$$\begin{aligned}
\mathrm{Cov}_t[(q^\star_{t+1} - x^\star_{t+1}), y_{t+1}] &= \mathrm{Cov}_t[(q^\star_{t+1} - x^\star_{t+1}), y^\star_{t+1}] \\
&= -\lambda^\star_t - (\gamma^\star_t)^\top \mathrm{Cov}_t(x^\star_{t+1}, r_{t+1}) \\
&= -(\gamma^\star_t)^\top E_t(r_{t+1}) - (\gamma^\star_t)^\top \mathrm{Cov}_t(x^\star_{t+1}, r_{t+1}) \\
&= -(\gamma^\star_t)^\top [E_t(r_{t+1}) + \mathrm{Cov}_t(x^\star_{t+1}, r_{t+1})] \\
&= -(\gamma^\star_t)^\top \alpha_t, \qquad (7)
\end{aligned}$$

where α_t is the mis-pricing of an asset-pricing model. Hence, we can write

$$\lambda_t = \lambda^\star_t - \mathrm{Cov}_t[(x_{t+1} - x^\star_{t+1}), (y_{t+1} - y^\star_{t+1})] - (\gamma^\star_t)^\top \alpha_t \equiv \lambda^\star_t + \delta_{nt} + \delta_{mt}. \tag{8}$$

In other words, the economic risk premia λ_t equal the premia on the maximum-correlation portfolios (λ^*_t), plus two components: the negative of the covariance between the non-traded components of x_{t+1} and of y_{t+1} (δ_{nt}), and the negative of the mis-pricing of the mimicking portfolios by the candidate kernel x_{t+1} (δ_{mt}).

[4] $E_t(r_{t+1})$ and Σ_{rrt} are the conditional means and covariances of asset excess returns, respectively.

[5] Note that γ^*_t is a $(N \times K)$ matrix.

C. Multi-beta models

Consider again the case where the pricing kernel $x_{t+1} = 1 - [y_{t+1} - E_t(y_{t+1})]^\top \Sigma_{yyt}^{-1} \lambda_t$. When risk premia are estimated by the standard generalized least squares (GLS) cross-sectional regressions, we have

$$\tilde{\lambda}_t = (\beta \Sigma_{rrt}^{-1} \beta^\top)^{-1} \beta \Sigma_{rrt}^{-1} E_t(r_{t+1}), \tag{9}$$

where the risk premia $\tilde{\lambda}_t$ coincide with the expected excess cash-flows on minimum-variance, unit-beta mimicking portfolios (see Balduzzi and Robotti, 2005). We also have

$$\tilde{\alpha}_t = [I - \beta^\top (\beta \Sigma_{rrt}^{-1} \beta^\top)^{-1} \beta \Sigma_{rrt}^{-1}] E_t(r_{t+1}). \tag{10}$$

It is easy to verify that

$$(\gamma_t^\star)^\top \tilde{\alpha}_t = 0. \tag{11}$$

Hence, in the special case where the asset-pricing model is a multi-beta model with non-traded factors, where the risk premia are estimated by a cross-sectional, GLS-style, regression, the risk premia on those factors can be written as

$$\tilde{\lambda}_t = \lambda_t^\star + \tilde{\delta}_{nt}. \tag{12}$$

D. "Noisy" factors

In this section we highlight how the premia assigned to non-traded risk factors are necessarily arbitrary. Indeed, by adding noise to a factor, where the noise is uncorrelated with factor and returns, one can make the risk premium arbitrarily large (in absolute value). This problem is not overcome by focusing on Sharpe ratios, which also increase (in absolute value) as the non-traded factors' volatility increases. Finally, it is also easy to show how the risk premia on the maximum-correlation portfolios are unaffected by non-traded volatility and, as the non-traded volatility of the factor increases, their contribution to the overall Sharpe ratio tends to zero.

Assume that we add to y_{t+1} mean-zero noise, e_{t+1}, uncorrelated with factors and asset returns. Consider now the premium assigned to the noisy factors $y_{t+1}^n = y_{t+1} + e_{t+1}$

$$\lambda_t^n \equiv -\mathrm{Cov}_t(x_{t+1}, y_{t+1}^n) = -\mathrm{Cov}_t(x_{t+1}, y_{t+1}) - \mathrm{Cov}_t(x_{t+1}, e_{t+1}) = \lambda_t - \mathrm{Cov}_t(x_{t+1}, e_{t+1}). \tag{13}$$

If x_{t+1} depends on e_{t+1} through the noisy factors y^n_{t+1}, then there is the potential for λ^n_t and λ_t to differ considerably, without any real underlying economic reason.

Indeed, for simplicity, consider the case where there is only one factor driving a linear kernel, where the kernel depends on y^n_{t+1}, rather than y_{t+1}: $x^n_{t+1} = 1 - [y^n_{t+1} - E_t(y^n_{t+1})]/\sigma^2_{yt}\lambda_t$.[6] We have

$$\lambda^n_t = \left(1 + \frac{\sigma^2_{et}}{\sigma^2_{yt}}\right)\lambda_t. \tag{14}$$

As σ^2_{et} increases, λ^n_t increases in absolute value, again without any change in the economic fundamentals. Moreover, even if we standardize the factor, so that λ^n_t has the dimension of a theoretical Sharpe ratio, we have

$$\frac{\lambda^n_t}{\sqrt{\sigma^2_{yt} + \sigma^2_{et}}} = \frac{1 + \sigma^2_{et}/\sigma^2_{yt}}{\sqrt{\sigma^2_{yt} + \sigma^2_{et}}}\lambda_t. \tag{15}$$

It is easy to see that the derivative of $(1 + \sigma^2_{et}/\sigma^2_{yt})/\sqrt{\sigma^2_{yt} + \sigma^2_{et}}$ w.r.t. σ^2_{et} is positive. Hence, even the theoretical Sharpe ratio on the noisy factors increases (in absolute value) as the noise in the factor increases.

On the contrary, the risk premium on the maximum-correlation portfolio is unaffected by non-traded volatility

$$-\mathrm{Cov}_t(q^\star_{t+1}, y^n_{t+1}) = -\mathrm{Cov}_t(q^\star_{t+1}, y_{t+1}) = \lambda^\star_t. \tag{16}$$

Moreover, going back to the original decomposition, if we use the model x^n_{t+1} to assign risk premia, we have

$$\lambda^n_t = \lambda^\star_t - \mathrm{Cov}_t[(x^n_{t+1} - x^\star_{t+1}), (y^n_{t+1} - y^\star_{t+1})] - (\gamma^\star_t)^\top \alpha_t. \tag{17}$$

While the first and third components are unaffected by non-traded volatility, the second component is affected. Obviously, if we standardize λ^n_t by the standard deviation of the noisy factor to obtain a Sharpe ratio, and we increase the noise, the first and third components of the Sharpe ratio converge to zero, while the second component diverges to plus or minus infinity.

[6] Since the noise in the factor is uncorrelated with returns, the coefficients of x_t and x^n_t are the same.

II. Further discussion

A. Traded and non-traded factors

In the general case, both traded and non-traded factors drive a candidate kernel, and the factors whose premia we want to estimate may not be the same non-traded factors driving the candidate pricing kernel. Hence, we can consider *three* sets of risk factors. The first set of K_1 factors, $y_{1,t+1}$, are excess security returns. The second set of K_2 factors, $y_{2,t+1}$, are economic, non-traded variables such as consumption growth. The third set of K_3 factors, $y_{3,t+1}$, are also non-traded variables whose risk premia we want to estimate such as unexpected inflation. The sets of factors K_1 and K_2 shape the candidate kernel.

B. Conditioning information and conditional variation

Denote with Z_t the $(J \times 1)$ vector of instruments $(1 \;\; z_t)^\top$, where the z_t are demeaned and standardized. Assume that expected returns and expected factors are linear functions of the instruments, while conditional variances and covariances are constant, $\Sigma_{rrt} = \Sigma_{rr}$, $\Sigma_{yyt} = \Sigma_{yy}$, and $\Sigma_{ryt} = \Sigma_{ry}$.[7] We redefine the factors $y_{2,t+1}$ and $y_{3,t+1}$ as the residuals of multivariate regressions of the original factors on the instruments Z_t. Moreover, the factors are also standardized.

C. Minimum-variance kernel

The MV kernel $q^\star_{t+1}$ has the following expression[8]

$$q^\star_{t+1} = 1 - [r_{t+1} - E_t(r_{t+1})]^\top \alpha_{rt}, \tag{18}$$

where $E_t(r_{t+1}) = \mu_r^\top Z_t$, $\alpha_{rt} = \alpha_r^\top Z_t$, and $\lambda^\star_t = (\lambda^\star)^\top Z_t$.

The closed-form solutions for the parameters of interest are

$$\hat{\mu}_r \;\; = \;\; \hat{E}(Z_t Z_t^\top)^{-1} \hat{E}(Z_t r_{t+1}^\top) \tag{19}$$

[7]This approach to incorporating conditioning information is used in Harvey (1989), as well as, for instance, in Campbell and Viceira (1996), and DeRoon et al. (1998, 2001).

[8]See Appendix A for the corresponding sets of moment conditions.

$$
\begin{aligned}
\hat{\alpha}_r &= \hat{E}(Z_t Z_t^\top)^{-1}\hat{E}(Z_t r_{t+1}^\top)\hat{\Sigma}_{rr}^{-1} && (20)\\
\hat{\lambda}^\star &= \hat{\alpha}_r \hat{\Sigma}_{ry_3}, && (21)
\end{aligned}
$$

where $\hat{\Sigma}_{rr} = \hat{E}[(r_{t+1} - \mu_r^\top Z_t)(r_{t+1} - \mu_r^\top Z_t)^\top]$ and $\hat{\Sigma}_{ry_3} = \hat{E}(r_{t+1} y_{3,t+1}^\top)$.

D. Candidate kernel

D.1. Non-traded factors only

The candidate kernel x_{t+1} is given by

$$
x_{t+1} = 1 - y_{2,t+1}^\top b_{2t}, \tag{22}
$$

where $b_{2t} = b_2^\top Z_t$. In all cases, we also assume $\lambda_t = \lambda^\top Z_t$. These are natural assumptions, since we have assumed that conditional expected excess returns are also linear in the instruments and since we have ruled out time variation in the second conditional moments.

This leads to

$$
\begin{aligned}
\hat{b}_2 &= \hat{E}(Z_t Z_t^\top)^{-1}\hat{E}(Z_t r_{t+1}^\top) W \hat{\Sigma}_{ry_2}(\hat{\Sigma}_{y_2 r} W \hat{\Sigma}_{ry_2})^{-1} && (23)\\
\hat{\lambda} &= \hat{b}_2 \hat{\Sigma}_{y_2 y_3}, && (24)
\end{aligned}
$$

where $\hat{\Sigma}_{ry_2} = \hat{E}(r_{t+1} y_{2,t+1}^\top)$ and $\hat{\Sigma}_{y_2 y_3} = \hat{E}(y_{2,t+1} y_{3,t+1}^\top)$. We consider two choices of weighting matrix: $W = I$ [ordinary least squares (OLS) case], and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case).

From the projection of x_{t+1} on the span of asset excess returns, we obtain

$$
x_{t+1}^\star = 1 - (y_{2,t+1}^*)^\top b_{2t}, \tag{25}
$$

where

$$
y_{2,t+1}^\star = (\gamma_2^\star)^\top [r_{t+1} - E_t(r_{t+1})]. \tag{26}
$$

This leads to[9]

$$
\hat{\gamma}_2^\star = \hat{\Sigma}_{rr}^{-1} \hat{\Sigma}_{ry_2}. \tag{27}
$$

[9]See Appendix B.1 for the corresponding sets of moment conditions.

D.2. Traded factors only

The candidate kernel x_{t+1} is given by

$$x_{t+1} = 1 - [y_{1,t+1} - E_t(y_{1,t+1})]^\top b_{1t}, \tag{28}$$

where $E_t(y_{1,t+1}) = \mu_{y_1}^\top Z_t$ and $b_{1,t} = b_1^\top Z_t$.

We have[10]

$$\hat{\mu}_{y_1} = \hat{E}(Z_t Z_t^\top)^{-1} \hat{E}(Z_t y_{1,t+1}^\top) \tag{29}$$

$$\hat{b}_1 = \hat{E}(Z_t Z_t^\top)^{-1} \hat{E}(Z_t y_{1,t+1}^\top) \hat{\Sigma}_{y_1 y_1}^{-1} \tag{30}$$

$$\hat{\lambda} = \hat{b}_1 \hat{\Sigma}_{y_1 y_3}, \tag{31}$$

where $\hat{\Sigma}_{y_1 y_1}^{-1} = \hat{E}[(y_{1,t+1} - \mu_{y_1}^\top Z_t)(y_{1,t+1} - \mu_{y_1}^\top Z_t)^\top]$.

From the projection of x_{t+1} on the span of asset excess returns, we obtain

$$x_{t+1}^\star = 1 - [y_{1,t+1}^* - E_t(y_{1,t+1}^\star)]^\top b_{1t}, \tag{32}$$

where

$$y_{1,t+1}^\star = (\gamma_1^\star)^\top r_{t+1}. \tag{33}$$

We have

$$\hat{\gamma}_1^\star = \hat{\Sigma}_{rr}^{-1} \hat{\Sigma}_{r y_1}, \tag{34}$$

where $\hat{\Sigma}_{r y_1} = \hat{E}[(r_{t+1} - \mu_r^\top Z_t)(y_1 - \mu_{y_1}^\top Z_t)^\top]$.

D.3. Traded and non-traded factors

In this case it is convenient to consider the residuals $\epsilon_{2,t+1}$ of a regression of the non-traded factors $y_{2,t+1}$ on the span of the traded factors $y_{1,t+1}$.[11] Hence, the candidate kernel x_{t+1} whose parameters we want to estimate is given by

$$x_{t+1} = 1 - [y_{1,t+1} - E_t(y_{1,t+1})]^\top b_{1t} - \epsilon_{2,t+1}^\top b_{2t}, \tag{35}$$

[10] See Appendix B.2 for the corresponding sets of moment conditions.

[11] In the empirical analysis, we verified that our findings were robust to the orthogonalization of the traded and non-traded factors driving the candidate kernel. Specifically, we ignored the pricing kernel restriction induced by the factor being traded, and we obtained results that were quantitatively very close to the ones reported in the tables.

where

$$\begin{aligned} b_{1t} &= b_1^\top Z_t \\ b_{2t} &= b_2^\top Z_t. \end{aligned}$$

We have[12]

$$\hat{b}_1 = \hat{E}(Z_t Z_t^\top)^{-1} \hat{E}(Z_t y_{1,t+1}^\top) \hat{\Sigma}_{y_1 y_1}^{-1} \quad (36)$$

$$\hat{b}_2 = \hat{E}(Z_t Z_t^\top)^{-1} \hat{E}\left[Z_t (r_{t+1} - \hat{\beta}_1^\top y_{1,t+1})^\top \right] W \hat{\Sigma}_{r\epsilon_2} (\hat{\Sigma}_{\epsilon_2 r} W \hat{\Sigma}_{r\epsilon_2})^{-1} \quad (37)$$

$$\hat{\lambda} = \hat{b}_1 \hat{\Sigma}_{y_1 y_3} + \hat{b}_2 \hat{\Sigma}_{\epsilon_2 y_3}, \quad (38)$$

where $\hat{\beta}_1 = \hat{\Sigma}_{y_1 y_1}^{-1} \hat{\Sigma}_{y_1 r}$, $\hat{\Sigma}_{r\epsilon_2} = \hat{E}[(r_{t+1} - \mu_r^\top Z_t)\epsilon_{2,t+1}^\top]$, and $\hat{\Sigma}_{\epsilon_2 y_3} = \hat{E}(\epsilon_{2,t+1} y_{3,t+1}^\top)$.

From the projection of x_{t+1} on the span of asset excess returns, we obtain

$$x_{t+1}^\star = 1 - [y_{1,t+1}^* - E_t(y_{1,t+1}^\star)]^\top b_{1t} - (\epsilon_{2,t+1}^*)^\top b_{2t}, \quad (39)$$

where

$$\epsilon_{2,t+1}^* = (\gamma_2^\star)^\top r_{t+1}. \quad (40)$$

We have $\hat{\gamma}_2^\star = \hat{\Sigma}_{rr}^{-1} \hat{\Sigma}_{r\epsilon_2}$.

E. Non-traded component and mis-pricing component

Recall that the non-traded and mis-pricing components of the risk premia, δ_{nt} and δ_{mt}, are the conditional expectations of $(x_{t+1}^\star - x_{t+1})y_{3,t+1}$ and $(q_{t+1}^\star - x_{t+1}^\star)y_{3,t+1}$, respectively. Hence, it is natural to assume that the δ_{nt} and δ_{mt} components are linear functions of the instruments Z_t

$$\delta_{nt} = \delta_n^\top Z_t \quad (41)$$

$$\delta_{mt} = \delta_m^\top Z_t. \quad (42)$$

Specifically, $\hat{\delta}_n$ and $\hat{\delta}_m$ are the parameters of regressions of $(x_{t+1}^\star - x_{t+1})y_{3,t+1}$ and $(q_{t+1}^\star - x_{t+1}^\star)y_{3,t+1}$ on Z_t.[13]

[12]See Appendix B.3 for the corresponding sets of moment conditions.

[13]See Appendix C for the corresponding sets of moment conditions.

III. Data

This section illustrates the data used in the empirical analysis. The period considered is March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables. The choice of the starting date is dictated by macroeconomic data availability.

A. Asset Returns

We use decile portfolio returns on NYSE, AMEX, and NASDAQ listed stocks. Ten *size* stock portfolios are formed according to size deciles on the basis of the market value of equity outstanding at the end of the previous year. If a market capitalization was not available for the previous year, the firm was ranked based on the capitalization on the date with the earliest available price in the current year. The returns are value-weighted averages of the firms's returns, adjusted for dividends. The securities with the smallest capitalizations are placed in portfolio one. The partitions on the CRSP file include all securities, excluding ADRs, which were active on NYSE-AMEX-NASDAQ for that year.[14]

All rates of return are in excess of the risk-free rate. The risk-free rate proxy is the 1–month Treasury Bill rate from Ibbotson Associates (SBBI module) and pertains to a bill with at least 1 month to maturity.

B. Economic Variables and Instruments

We concentrate on a set of six non-traded variables, which have been previously used in tests of multi-beta models and/or in studies of stock-return predictability. (See, for example, Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), McElroy and Burmeister (1988), Ferson and Harvey (1991, 1999), Downs and Snow (1994), and Kirby (1998)). These variables are statistically significant in multi-variate predictive regressions of means and volatilities or they have special economic significance.

[14]In addition to returns on size-sorted equity portfolios, we also use returns on the 25 size and book-to-market sorted portfolios, the "Fama-French" portfolios, available from Ken French's website. Results for this alternative choice of assets are briefly discussed in a series of footnotes.

INF is the monthly rate of inflation (Ibbotson Associates), percent per month.

CG denotes the logarithm of the monthly gross growth rate of per capita real consumption of nondurable goods and services, percent per month. The series used to construct consumption data are from CITIBASE. Monthly real consumption of nondurables and services are the GMCN and GMCS series deflated by the corresponding deflator series GMDCN and GMDCS. Per capita quantities are obtained by using data on resident population, series POPRES.

HB3 is the 1-month return of a 3-month Treasury bill less the 1-month return of a 1-month bill (CRSP, Fama Treasury Bill Term Structure Files), percent per month.

DIV denotes the monthly dividend yield on the Standard and Poor's 500 stock index (CITIBASE), percent per month.

REALTB denotes the real 1–month Treasury bill (SBBI), percent per month.

PREM represents the yield spread between Baa and Aaa rated bonds (Moody's Industrial from CITIBASE), percent per month.

All the variables are standardized by their standard deviation. Hence, the risk premia have the interpretation of Sharpe ratios on theoretical exact mimicking portfolios.[15] In addition, we consider three traded factors.

XVW represents the value-weighted NYSE-AMEX-NASDAQ index return (CRSP) in excess of the risk-free rate (SBBI), percent per month.

SMB (Small Minus Big) represents the average return on three small portfolios (small value, small neutral, and small growth) minus the average return on three big portfolios (big value, big neutral, and big growth), percent per month.

[15]Note that in obtaining standard errors and p-values by bootstrap, we bootstrap the original series, not the standardized series. Hence, we do account for the sampling variability in the estimates of the standard deviations of the factors. In all exercises other than the case of the "noisy factor," we assume that the factor is observed without measurement error.

HML (High Minus Low) represents the average return on two value portfolios (small value and big value) minus the average return on two growth portfolios (small growth and big growth),[16] percent per month.

We select a constant and the lagged values of XVW, DIV, and REALTB as instruments. The reason for using the lagged values of the economic variables as instruments is that, according to the I-CAPM intuition, the variables that drive asset returns should also be the variables affecting the risk-return trade-off, *i.e.* they should also be the variables predicting returns.[17,18]

IV. Empirical analysis

We consider four multi-beta models. The first model is the I-CAPM, with factors: market excess return, dividend yield, real T-bill rate, term structure, default premium, consumption growth, and inflation.[19] The second model is the C-CAPM, where the only factor driving the kernel is consumption growth. The third model is the Fama-French three-factor model. The fourth model is the CAPM, where the only factor driving the kernel is the excess market return.

We present results for our decomposition in three tables. Table I compares the estimates of the economic risk premia, λ, and of the premia on the maximum-correlation portfolios, $\lambda^\star$, for the four different models. Table II compares the "non-traded" components of the risk premia, δ_n, for the different models. Table III compares the mis-pricing components of the risk premia, δ_m, for the different models.

A. Bootstrap procedure

For all parameter estimates, we compute small-sample bias and empirical standard errors by parametric bootstrap. The bootstrap exercise is structured as follows:

[16] We thank Kenneth French for making the SMB and HML factors available.

[17] See, for example, Campbell (1996).

[18] While not reported in the tables, we also performed our analysis for the unconditional case, where the only instrument is the constant. Results for the unconditional case are briefly summarized in a few footnotes.

[19] This choice of factors for the I-CAPM is analogous to that of Ferson and Harvey (1991).

First, we estimate predictive regressions, by regressing excess returns on equity portfolios on the three instruments and a constant. Second, we estimate a first-order vector autoregression, VAR(1), for the nine non-traded and traded factors. Since the three instruments are lagged values of the factors, this gives us the law of motion of the factors and of the instruments predicting portfolio returns. Third, we jointly bootstrap the residuals in the predictive regressions and in the VAR(1). The VAR(1) residuals are fed into the estimated law of motion of the economic variables to generate bootstrap samples for the traded and non-traded factors and for the instruments. Using the bootstrap realizations of the instruments and of the predictive-regression residuals we generate bootstrap samples for excess returns.

The initial values of the factors and instruments are the beginning-of-sample values for the corresponding variables. The exercise is repeated 100,000 times.

B. Risk premia: λ and $\lambda^\star$

Table I reports the average conditional risk premia estimates (λ_0 and $\lambda_0^\star$) and the estimated coefficients relating the risk premium to the market excess return (λ_{XVW} and $\lambda_{XVW}^\star$), the dividend yield (λ_{DIV} and $\lambda_{DIV}^\star$), and the real T-bill rate (λ_{REALTB} and $\lambda_{REALTB}^\star$).

The first result emerging from Table I is that the pricing kernels with non-traded factors (I-CAPM and C-CAPM) lead to estimates of the λ parameters that deviate substantially from the corresponding $\lambda^\star$ values, and are much larger in absolute value.[20] Moreover, depending on the model, the estimates can also vary substantially. On the other hand, the

[20]The intuition for this result can be best seen in the case of a single factor and for a beta model that prices the portfolio mimicking the factor *exactly.* In this case, the result in (8) simplifies to

$$\lambda_t = \lambda_t^\star + \sigma_{\epsilon t}^2 b_t = \lambda_t^\star + \frac{\sigma_{\epsilon t}^2}{\sigma_{yt}^2}\lambda_t,$$

where $\sigma_{\epsilon t}^2$ is the residual variance from the projection of y_t onto the augmented span of excess returns. The expression above can be re-written as

$$\lambda_t = \lambda_t^\star \frac{\sigma_{yt}^2}{\sigma_{yt}^2 - \sigma_{\epsilon t}^2}.$$

Hence, λ_t is always larger than $\lambda_t^\star$ in absolute value, and the discrepancy between the two measures of the risk premium increases as the R-squared of the projection of the factor onto the augmented span of asset excess returns decreases.

pricing kernels with traded factors (Fama-French and CAPM) deliver λ estimates that are much closer to their $\lambda^\star$ counterparts. For example, consider the premium on consumption growth, which is of special economic significance. The $\lambda_0^\star$ estimate is 0.0103. The λ_0 estimate is -0.4977 for the I-CAPM/OLS and 0.7674 for the C-CAPM/OLS. Hence, not only can the discrepancies between the two sets of estimates be large, but the risk premium can even change sign depending on the model used. For a comparison, the Fama-French and CAPM models deliver λ_0 estimates that are much closer to $\lambda_0^\star$, 0.0257 and 0.0144, respectively.[21] A similar pattern holds for the inflation risk premium. The $\lambda_0^\star$ estimate is 0.0036. The λ_0 estimates are -1.5290 for the I-CAPM/OLS and -0.1355 for the C-CAPM/OLS; while the Fama-French and CAPM models deliver λ_0 estimates of -0.0198 and -0.0135, respectively.

The coefficients relating the conditional risk premia to the instruments also differ by orders of magnitude. For example, in the case of consumption growth, the $\lambda_{DIV}^\star$ estimate is 0.0093, while the λ_{DIV} estimates are -0.4733 for the I-CAPM/OLS and 0.5402 for the C-CAPM/OLS. On the other hand, the Fama-French and CAPM models deliver λ_{DIV} estimates of 0.0177 and 0.0120, respectively. In the case of inflation, the $\lambda_{DIV}^\star$ estimate is -0.0185, while the λ_{DIV} estimates are -1.2367 for the I-CAPM/OLS and -0.0954 for the C-CAPM/OLS; the Fama-French and CAPM models deliver λ_{DIV} estimates of -0.0185 and -0.0112, respectively.

The second result emerging from Table I is that the choice of weighting matrix for models with non-traded factors can make a substantial difference.[22] In the case of the I-CAPM consumption premium, for example, the estimate of the λ_0 parameter changes from -0.4977 to -0.1553 going from the OLS to the GLS specification; the estimate of the λ_{DIV} parameter changes from -0.4733 to -0.1540. For the I-CAPM inflation premium, the λ_0 estimate changes from -1.5290 to -1.0293, while the estimate of the λ_{DIV} changes from -1.2367 to -0.9349.[23]

The third result emerging from Table I is that the λ estimates assigned by models with non-traded factors can exhibit substantial small-sample biases and tend to be estimated

[21] Results for the unconditional versions of the models are quantitatively and qualitatively very similar. The $\lambda_0^\star$ estimate is 0.0092, while the λ_0 estimate is -0.4892 for the I-CAPM/OLS and 0.7674 for the C-CAPM/OLS. The Fama-French model and the CAPM deliver λ_0 estimates of 0.0249 and 0.0142, respectively.

[22] This is not surprising: the returns across the different size portfolios are substantially correlated and the covariance matrix of returns is far from being diagonal.

[23] In the unconditional case, the risk-premium estimate changes from -0.4892 to -0.1525.

imprecisely. Focusing again on the I-CAPM/OLS risk premium on consumption growth, the bias for the λ_0 estimates is 0.2851, with a standard error of 0.8003. On the other hand, biases for the models with traded factors are much smaller: -0.0035 and -0.0023 for the Fama-French model and the CAPM, respectively. Also modest are the empirical standard errors: 0.0130 and 0.0056. Similarly substantial are the biases and standard errors for the λ_{DIV} estimates: 0.2865 and 0.6118 for the I-CAPM/OLS. The corresponding biases and standard errors in the Fama-French and CAPM models are only 0.0012 and 0.0090, and 0.0016 and 0.0061, respectively.[24] For the case of inflation, the bias for the I-CAPM/OLS λ_0 estimate is 1.8546, with a standard error of 0.9419. Biases for the models with traded factors are 0.0046 (Fama-French) and 0.0021 (CAPM), with standard errors of 0.0139 and 0.0054. The bias for the I-CAPM/OLS λ_{DIV} estimate is 0.9322, with standard error of 0.7220. The corresponding biases and standard errors in the Fama-French and CAPM models are only -0.0006 and 0.0099, and -0.0015 and 0.0059, respectively.[25]

On the other hand, the $\lambda^\star$ estimates exhibit modest biases and standard errors. In the case of consumption growth, the bias in $\lambda_0^\star$ is basically non-existent and the empirical standard error is only 0.0163. The bias in $\lambda_{DIV}^\star$ is also small, 0.0025, and the empirical standard error is 0.0146.[26] In the case of inflation, the bias in the $\lambda_0^\star$ is 0.0006 and the empirical standard error is 0.0163. The bias in $\lambda_{DIV}^\star$ is -0.0033, and the empirical standard error is 0.0142.

We also illustrate the time-series properties of the two sets of premia, conditional on the realizations of the four instruments. For each premium, we report the conditional estimate, the median of the bootstrap distribution, and equi-tailed 90% confidence regions. Figure 1 presents the time-varying risk premia assigned by the I-CAPM/OLS to the six economic factors. Figure 2 presents the time-varying risk premia on the maximum-correlation portfolios.

[24] In the unconditional case, we have a bias of 0.2951, with a standard error of 0.7200. Biases for the Fama-French and CAPM are -0.0033 and -0.0023, respectively.

[25] Overall, the evidence using the Fama-French portfolios is similar. While the λ estimates tend to be smaller in absolute value and display somewhat smaller biases, it is still the case that the bias in the $\lambda^\star$ estimates is negligible.

[26] Again, similar results hold in the unconditional case: minimal bias and standard error of 0.0162.

The general message from these figures is that the λ_t estimates are on average much larger (in absolute value), more volatile, and less precisely estimated than the corresponding $\lambda_t^\star$ estimates. In addition, the λ_t estimates often fall outside of the 90% confidence bands, whereas the $\lambda_t^\star$ point estimates cannot be distinguished from the medians of the distribution.

C. Non-traded component: δ_n

Table II reports the average conditional non-traded component estimates ($\delta_{n,0}$) and the estimated coefficients relating the non-traded component to the market excess return ($\delta_{n,XVW}$), the dividend yield ($\delta_{n,DIV}$), and the real T-bill rate ($\delta_{n,REALTB}$).

It is immediately clear that for models with non-traded factors, this component is substantial, and its parameters are estimated with large (absolute) biases and imprecisely. Moreover, the estimates can be significantly affected by the choice of weighting matrix. For example, in the case of consumption growth, $\delta_{n,0}$ is -0.5047 for the I-CAPM/OLS and 0.7188 for the C-CAPM/OLS. The corresponding biases are 0.2857 and 0.0519, while the standard errors are 0.7924 and 0.4223. Going from the OLS to the GLS approach leads to estimates of -0.1650 and 0.1523 for the I-CAPM and C-CAPM, respectively.[27] We find a similar pattern for $\delta_{n,DIV}$. The estimate is 0.6170 for the I-CAPM/OLS and 0.5060 for the C-CAPM/OLS. The corresponding biases are -0.5840 and 0.1395, while the standard errors are 0.5159 and 0.4027. Going from the OLS to the GLS approach leads to estimates of 0.6402 and 0.1374 for the I-CAPM and C-CAPM, respectively. Similar patterns also hold for the parameters of the non-traded component of the inflation risk premium and for the other economic factors considered.[28]

D. Mis-pricing component: δ_m

Table III reports the average conditional mis-pricing component ($\delta_{m,0}$) and the coefficients relating the mis-pricing component to the market excess return ($\delta_{m,XVW}$), the dividend yield

[27] In the unconditional case, we have -0.4950 for the I-CAPM/OLS and 0.7193 for the C-CAPM/OLS, with biases of 0.2954 and 0.0506, and standard errors of 0.7120 and 0.4268. Going from the OLS to the GLS approach leads to estimates of -0.1611 and 0.1376 for the I-CAPM and C-CAPM, respectively.

[28] Results are qualitatively similar for the case of the Fama-French portfolios.

($\delta_{m,DIV}$), and the real T-bill rate ($\delta_{m,REALTB}$).

Unlike the non-traded component, the δ_m component tends to be fairly small for all models. Biases and standard errors are also modest. Moreover, for models with non-traded factors, the choice of weighting matrix is less relevant. The intuition for this is easily understood based on equation (11). For a multi-beta model with non-traded factors, where the parameters are estimated by cross-sectional GLS-style regressions, the mis-pricing of the portfolios mimicking the factors is identically zero. Hence, even when the estimation approach slightly departs from this paradigm, i.e., we employ the identity matrix as the weighting matrix, the mis-pricing of the mimicking portfolios is very modest.

For example, in the case of consumption growth, $\delta_{m,0}$ is -0.0034 for the I-CAPM/OLS and 0.0383 for the C-CAPM/OLS. The corresponding biases are -0.0007 and 0.0152, while the standard errors are 0.0128 and 0.0152. Going from the OLS to the GLS approach leads to estimates of -0.0006 and 0 for the I-CAPM and C-CAPM, respectively.[29]

A similar pattern holds for the $\delta_{m,DIV}$ coefficient: the estimates are 0.0013 for the I-CAPM/OLS and 0.0249 for the C-CAPM/OLS, with biases of -0.0068 and 0.0162, and standard errors of 0.0085 and 0.0284. Going from the OLS to the GLS approach leads to estimates of 0.0021 and 0 for the I-CAPM and C-CAPM, respectively.

E. Noisy factors

The results above highlight the fact that the source of problems in estimating economic risk premia, both conceptually and econometrically, is the non-traded variability shared by the factors and by the candidate pricing kernel assigning the risk premia. To further highlight this point, we consider an empirical example of the case of noisy factors.[30] As highlighted in Section I.D., when factors are measured with noise the non-traded component of the risk premium becomes more sizeable, and we know that this component tends to be estimated

[29]Similarly, in the unconditional case, we have -0.0034 for the I-CAPM/OLS and 0.0389 for the C-CAPM/OLS, with biases of -0.0003 and 0.0148, while the standard errors are 0.0116 and 0.0326. Going from the OLS to the GLS approach leads to estimates of -0.0005 and 0 for the I-CAPM and C-CAPM, respectively.

[30]Note that the seasonal adjustment in most macro-economic series is a natural source of measurement error. We thank Wayne Ferson for making this point.

with bias and imprecisely. At the same time, though, the estimates of the coefficients of the projection of the factor onto the span of asset excess returns will be less precise, affecting the precision of the estimates of $\lambda_t^\star$. Hence, it is interesting to see which one of the two effects dominates as the noise in the factor increases.

We consider the case of the consumption risk premium assigned by the C-CAPM, where the factor driving the kernel is now "noisy" consumption growth. We add to observed consumption growth mean-zero noise, orthogonal to asset excess returns, factor, and instruments, and we then estimate the consumption risk premium for different values of the volatility of the non-traded noise. Specifically, we generate a normal random variable with mean zero and variance equal to $c^2\sigma_y^2$, where c is a scalar ($c = 0, \sqrt{1/2}, 1, \sqrt{2}$). We regress the realizations of the $N(0, c^2\sigma_y^2)$ random variable on the augmented span of asset excess returns, factor, and instruments. Finally, we use the regression residuals e_{t+1} to form the noisy factor $y_{t+1}^n = y_{t+1} + e_{t+1}$. Results of the exercise are presented in Table IV.

Several patterns emerge from the table. First, as illustrated analytically in equation (15), the theoretical Sharpe ratio assigned by the C-CAPM increases monotonically with the volatility of non-traded noise. For example, the consumption risk premium in the base case (no noise) is 0.7674 (OLS). When $c = \sqrt{2}$, the estimate increases to 1.4023. Second, since the volatility of the factor increases with noise, the $\lambda^\star$ component decreases monotonically. For example, the $\lambda_t^\star$ estimate decreases from 0.0103 to 0.0056 as c increases from zero to $\sqrt{2}$. Third, the standard error of the λ_0 estimate also increases with the volatility of the noise. Consider again the OLS estimate. In the base case, the standard error is 0.4465. For $c = \sqrt{2}$, the standard error increases to 1.4636. Fourth, the standard error of the λ_0 estimate decreases slightly as the volatility of the noise increases. For example, going from $c = 0$ to $c = \sqrt{2}$, the standard error decreases from 0.0163 to 0.0134. As one would expect, these effects are mainly driven by the estimate of the non-traded component of the risk premium, δ_n. Hence, the addition of noise to a factor worsens substantially the properties of the estimates of λ_0, but has very little effect on the estimates of $\lambda_0^\star$.[31,32]

[31]As to biases, noise in the factor increases the absolute bias for the GLS estimates of λ_0, while the bias in the OLS estimates of λ_0 follows a non-monotonic pattern. Bias in the estimates of $\lambda^\star$ is very small and is essentially unaffected by noise in the factor.

[32]Results for the Fama-French portfolios are again qualitatively similar.

F. Statistical inference

Table V shows how the differences in λ and $\lambda^\star$ estimates for models with non-traded factors can translate into differences in statistical inference. For each coefficient, we report the p-value of a one sided significance test, based on the bootstrap distribution of the statistic.

The difference in inference between the $\lambda^\star$ and λ estimates can be striking. Of the 24 $\lambda^\star$ coefficients reported in table V, only three are significant at the 5% level. For the I-CAPM/OLS, on the other hand, there are 16 significant parameter estimates. Interestingly, the difference in inference persists when we consider a model with traded factors like the Fama-French three-factor model. In this case, there are still nine significant estimates out of 24.[33]

V. Conclusions

We define economic risk premia as the expected excess returns on theoretical portfolios exactly mimicking a non-traded risk factor. Since the factors are not traded, these premia depend on the specification of an asset-pricing model. If the model is a multi-beta model in the economic factors, then all security risk premia are linear in the economic risk premia.

We show how the risk premium assigned to a non-traded source of risk can be decomposed into three parts: i) the expected excess cash flow on the maximum-correlation portfolio; ii) (minus) the covariance between the non-traded components of the factor and of the candidate pricing kernel of a given model; and iii) (minus) the mis-pricing assigned by the candidate pricing kernel x_{t+1} to the maximum-correlation portfolio mimicking the factor.

We estimate the three components for four multi-beta models: the I-CAPM, the C-CAPM, the Fama-French model, and the CAPM. We show how models with non-traded factors (I-CAPM and C-CAPM) assign risk premia that deviate substantially from the premia on maximum-correlation portfolios. Moreover, the economic risk premia parameter estimates exhibit large (absolute) biases and standard errors, and are sensitive to the

[33] In the case of the Fama-French portfolios, we obtain essentially the same results for the estimates of the $\lambda^\star$ parameters. For the estimates of the λ parameters the results are similar, although the number of significant estimates decreases somewhat, especially in the OLS case.

choice of the weighting matrix. On the other hand, the premia on maximum-correlation portfolios tend to be estimated precisely and with little bias. We also show how the discrepancy between the economic risk premia parameter estimates and the parameter estimates of premia on maximum-correlation portfolios are mainly due to the common non-traded variability of factors and candidate pricing kernels. These patterns hold for both the constant and the time-varying component of the risk premia. Finally, we show that for models with non-traded factors, the differences in estimates of economic risk premia and premia on maximum-correlation portfolios translate into marked differences in statistical inference.

Hence, the parameter estimates of economic risk premia based on multi-beta models with non-traded factors can be very unreliable. Indeed, economic risk premia are intrinsically arbitrary, and even their theoretical reference values are affected by non-traded noise. We conclude that a better indicator of the market price of an economic risk factor is the expected excess return on the associated maximum-correlation portfolio.

Appendix

A. MV kernel

The following moment conditions identify the composition of the MV kernel and the parameters of risk premia $\lambda^\star$

$$
\begin{aligned}
E\left[(r_{t+1} - \mu_r^\top Z_t) Z_t^\top\right] &= 0 && (43)\\
E\left(r_{t+1} q^\star_{t+1} Z_t^\top\right) &= 0 && (44)\\
E\left\{\left[(1 - q^\star_{t+1}) y_{3,t+1} - \lambda^\star_t\right] Z_t^\top\right\} &= 0. && (45)
\end{aligned}
$$

B. Candidate kernel

B.1. Non-traded factors only

The following moment conditions identify the parameters of the candidate kernel

$$
\begin{aligned}
E\left\{[(1 - x_{t+1}) y_{3,t+1} - \lambda_t] Z_t^\top\right\} &= 0 && (46)\\
\Sigma_{y_2 r} W E(r_{t+1} x_{t+1} Z_t^\top) &= 0, && (47)
\end{aligned}
$$

for $W = I$ and $W = \hat{\Sigma}_{rr}^{-1}$.

B.2. Traded factors only

The following moment conditions identify the parameters of the candidate kernel

$$
\begin{aligned}
E[(y_{1,t+1} - \mu_{y_1}^\top Z_t) Z_t^\top] &= 0 && (48)\\
E\left\{[(1 - x_{t+1}) y_{3,t+1} - \lambda_t] Z_t^\top\right\} &= 0 && (49)\\
E(y_{1,t+1} x_{t+1} Z_t^\top) &= 0. && (50)
\end{aligned}
$$

B.3. Traded and non-traded factors

The following moment conditions identify the parameters of the candidate kernel

$$
\begin{aligned}
E(r_{t+1} - \beta_0 - \beta_1^\top y_{1,t+1}) &= 0 && (51)\\
E\left[(r_{t+1} - \beta_0 - \beta_1^\top y_{1,t+1}) y_{1,t+1}^\top\right] &= 0 && (52)
\end{aligned}
$$

$$E\left\{\left[y_{2,t+1} - \delta^\top y_{1,t+1}\right] y_{1,t+1}^\top\right\} = 0 \tag{53}$$

$$E[(y_{1,t+1} - \mu_{y_1}^\top Z_t) Z_t^\top] = 0 \tag{54}$$

$$E\left\{[(1 - x_{t+1}) y_{3,t+1} - \lambda_t]\, Z_t^\top\right\} = 0 \tag{55}$$

$$E(y_{1,t+1} x_{t+1} Z_t^\top) = 0 \tag{56}$$

$$\Sigma_{\epsilon_2 r} W E(r_{t+1} x_{t+1} Z_t^\top) = 0. \tag{57}$$

C. Non-traded component and mis-pricing component

The moment conditions identifying the parameters for δ_{nt} and δ_{mt} are

$$E\left\{\left[(x_{t+1}^\star - x_{t+1}) y_{3,t+1} - \delta_{nt}\right] Z_t^\top\right\} = 0 \tag{58}$$

$$E\left\{\left[(q_{t+1}^\star - x_{t+1}^\star) y_{3,t+1} - \delta_{mt}\right] Z_t^\top\right\} = 0. \tag{59}$$

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Table I: λ vs $\lambda^{\star}$

We use a sieve bootstrap exercise to investigate the statistical properties of λ_t and λ_t^* (see section IV for a description of the experiment). The set of instruments is common across asset-pricing models and includes a constant, the lagged value of the market excess return, the lagged value of the real TB rate, and the lagged value of dividend yield. We consider four multi-beta models. The first model is the I-CAPM (Panel A) with market excess return, dividend yield, real T-bill rate, term structure, default premium, consumption growth, and inflation as factors. The second model is the C-CAPM (Panel B), where the only factor driving the kernel is consumption growth. The third model is the Fama-French three-factor model (Panel C). The fourth model is the CAPM (Panel D), where the only factor driving the kernel is the excess market return. The exercise is repeated 100,000 times using a sample of 526 observations (March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables). In the first row, we report the average conditional risk premium estimate of λ_t^* (λ_0^*), and the conditional risk premium estimates of λ_t^* (λ_{XVW}^*, λ_{DIV}^*, λ_{REALTB}^*). In the second row, we report the average conditional risk premium estimate of λ_t (λ_0), and the conditional risk premium estimates of λ_t (λ_{XVW}, λ_{DIV}, λ_{REALTB}). When we investigate the properties of the λ estimates in presence of a pricing kernel driven by traded and non-traded factors (Panels A and B), we consider estimates based on the two weighting matrices $W = I$ (OLS case) and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case). In the third and fourth rows, we report the finite-sample (absolute) bias of λ_t^* and λ_t. In the fifth and sixth rows, we report the empirical standard errors of λ_t^* and λ_t.

Panel A: I-CAPM

	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	-0.0216	-0.0421	-0.0039	-0.0073
$\lambda(HB3)$	-2.9258(OLS) -2.1333(GLS)	-1.8399(OLS) -1.0244(GLS)	-1.2431(OLS) -0.6993(GLS)	-0.7050(OLS) -0.3363(GLS)
$Bias_{\lambda^\star}(HB3)$	-0.0008	-0.0008	-0.0008	-0.0005
$Bias_\lambda(HB3)$	2.2174(OLS) 1.5208(GLS)	0.7171(OLS) 0.2261(GLS)	1.0632(OLS) 0.5934(GLS)	0.5026(OLS) 0.2194(GLS)
$Emp.Std_{\lambda^\star}(HB3)$	0.0139	0.0189	0.0123	0.0123
$Emp.Std_\lambda(HB3)$	1.1849(OLS) 0.8467(GLS)	1.3262(OLS) 1.1787(GLS)	0.9060(OLS) 0.6337(GLS)	0.6449(OLS) 0.6019(GLS)
$\lambda^\star(DIV)$	-0.0629	-0.0412	-0.0446	-0.0672
$\lambda(DIV)$	0.4047(OLS) 0.3445(GLS)	0.7252(OLS) 0.4927(GLS)	0.5737(OLS) 0.5976(GLS)	-0.1872(OLS) -0.2475(GLS)
$Bias_{\lambda^\star}(DIV)$	-0.0004	0.0008	-0.0136	-0.0006
$Bias_\lambda(DIV)$	-0.6328(OLS) -0.3972(GLS)	-0.9597(OLS) -0.6248(GLS)	-0.6044(OLS) -0.3739(GLS)	0.0891(OLS) 0.1131(GLS)
$Emp.Std_{\lambda^\star}(DIV)$	0.0323	0.0328	0.0309	0.0314
$Emp.Std_\lambda(DIV)$	0.7266(OLS) 0.4863(GLS)	0.6694(OLS) 0.6727(GLS)	0.5319(OLS) 0.3643(GLS)	0.3465(OLS) 0.3516(GLS)
$\lambda^\star(CG)$	0.0103	0.0274	0.0093	-0.0042
$\lambda(CG)$	-0.4977(OLS) -0.1553(GLS)	0.2650(OLS) 0.3992(GLS)	-0.4733(OLS) -0.1540(GLS)	-0.6884(OLS) -0.4739(GLS)
$Bias_{\lambda^\star}(CG)$	0.0000	0.0001	0.0025	0.0003
$Bias_\lambda(CG)$	0.2851(OLS) 0.0838(GLS)	0.0335(OLS) -0.2115(GLS)	0.2865(OLS) 0.1045(GLS)	0.2758(OLS) 0.1134(GLS)
$Emp.Std_{\lambda^\star}(CG)$	0.0163	0.0209	0.0146	0.0145
$Emp.Std_\lambda(CG)$	0.8003(OLS) 0.5676(GLS)	0.8945(OLS) 0.7937(GLS)	0.6118(OLS) 0.4331(GLS)	0.4461(OLS) 0.4158(GLS)
$\lambda^\star(PREM)$	0.0045	-0.0166	0.0066	0.0053
$\lambda(PREM)$	-2.0030(OLS) -1.5788(GLS)	-5.9357(OLS) -4.4264(GLS)	-1.0638(OLS) -1.0387(GLS)	0.0163(OLS) -0.0227(GLS)
$Bias_{\lambda^\star}(PREM)$	0.0007	-0.0002	0.0013	-0.0004
$Bias_\lambda(PREM)$	2.0293(OLS) 1.5540(GLS)	5.0868(OLS) 3.5380(GLS)	1.1102(OLS) 0.9745(GLS)	-0.0631(OLS) -0.0323(GLS)
$Emp.Std_{\lambda^\star}(PREM)$	0.0124	0.0176	0.0115	0.0107
$Emp.Std_\lambda(PREM)$	1.5903(OLS) 1.1018(GLS)	1.7009(OLS) 1.5369(GLS)	1.2353(OLS) 0.8406(GLS)	0.8342(OLS) 0.7884(GLS)
$\lambda^\star(INF)$	0.0036	0.0247	-0.0185	-0.0011
$\lambda(INF)$	-1.5290(OLS) -1.0293(GLS)	-0.4731(OLS) 0.2274(GLS)	-1.2367(OLS) -0.9349(GLS)	-0.0454(OLS) 0.1329(GLS)
$Bias_{\lambda^\star}(INF)$	0.0006	0.0002	-0.0033	-0.0004
$Bias_\lambda(INF)$	1.8546(OLS) 1.2266(GLS)	1.2276(OLS) 0.3877(GLS)	0.9322(OLS) 0.6440(GLS)	0.2800(OLS) 0.0823(GLS)
$Emp.Std_{\lambda^\star}(INF)$	0.0163	0.0206	0.0142	0.0140
$Emp.Std_\lambda(INF)$	0.9419(OLS) 0.6843(GLS)	1.0754(OLS) 0.9634(GLS)	0.7220(OLS) 0.5193(GLS)	0.5259(OLS) 0.5011(GLS)
$\lambda^\star(REALTB)$	-0.0047	-0.0314	0.0139	-0.0009
$\lambda(REALTB)$	2.1160(OLS) 1.6540(GLS)	0.5644(OLS) -0.2079(GLS)	1.2322(OLS) 0.9903(GLS)	0.0981(OLS) -0.0480(GLS)
$Bias_{\lambda^\star}(REALTB)$	-0.0006	-0.0002	0.0024	0.0004
$Bias_\lambda(REALTB)$	-2.3240(OLS) -1.7028(GLS)	-1.4870(OLS) -0.5447(GLS)	-1.0231(OLS) -0.7464(GLS)	-0.3020(OLS) -0.1266(GLS)
$Emp.Std_{\lambda^\star}(REALTB)$	0.0150	0.0197	0.0133	0.0129
$Emp.Std_\lambda(REALTB)$	1.0721(OLS) 0.7754(GLS)	1.2174(OLS) 1.0948(GLS)	0.8253(OLS) 0.5856(GLS)	0.5873(OLS) 0.5578(GLS)

Panel B: C-CAPM

	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	-0.0216	-0.0421	-0.0039	-0.0073
$\lambda(HB3)$	0.0208(OLS) 0.0044(GLS)	0.0365(OLS) 0.0117(GLS)	0.0147(OLS) 0.0040(GLS)	0.0157(OLS) -0.0018(GLS)
$Bias_{\lambda^\star}(HB3)$	-0.0008	-0.0008	-0.0008	-0.0005
$Bias_\lambda(HB3)$	0.0007(OLS) -0.0028(GLS)	0.0041(OLS) -0.0028(GLS)	0.0052(OLS) -0.0005(GLS)	0.0026(OLS) 0.0005(GLS)
$Emp.Std_{\lambda^\star}(HB3)$	0.0139	0.0189	0.0123	0.0123
$Emp.Std_\lambda(HB3)$	0.0436(OLS) 0.0124(GLS)	0.0739(OLS) 0.0219(GLS)	0.0395(OLS) 0.0120(GLS)	0.0369(OLS) 0.0106(GLS)
$\lambda^\star(DIV)$	-0.0629	-0.0412	-0.0446	-0.0672
$\lambda(DIV)$	-0.1536(OLS) -0.0325(GLS)	-0.2688(OLS) -0.0864(GLS)	-0.1081(OLS) -0.0294(GLS)	-0.1156(OLS) 0.0132(GLS)
$Bias_{\lambda^\star}(DIV)$	-0.0004	0.0008	-0.0136	-0.0006
$Bias_\lambda(DIV)$	-0.0075(OLS) 0.0060(GLS)	-0.0083(OLS) 0.0158(GLS)	-0.0261(OLS) -0.0015(GLS)	-0.0079(OLS) -0.0044(GLS)
$Emp.Std_{\lambda^\star}(DIV)$	0.0323	0.0328	0.0309	0.0314
$Emp.Std_\lambda(DIV)$	0.0764(OLS) 0.0423(GLS)	0.0954(OLS) 0.0554(GLS)	0.0727(OLS) 0.0383(GLS)	0.0712(OLS) 0.0374(GLS)
$\lambda^\star(CG)$	0.0103	0.0274	0.0093	-0.0042
$\lambda(CG)$	0.7674(OLS) 0.1626(GLS)	1.3432(OLS) 0.4315(GLS)	0.5402(OLS) 0.1467(GLS)	0.5775(OLS) -0.0662(GLS)
$Bias_{\lambda^\star}(CG)$	0.0000	0.0001	0.0025	0.0003
$Bias_\lambda(CG)$	0.0672(OLS) -0.0335(GLS)	0.0989(OLS) -0.0724(GLS)	0.1582(OLS) 0.0043(GLS)	0.0646(OLS) 0.0141(GLS)
$Emp.Std_{\lambda^\star}(CG)$	0.0163	0.0209	0.0146	0.0145
$Emp.Std_\lambda(CG)$	0.4465(OLS) 0.2130(GLS)	0.6218(OLS) 0.2838(GLS)	0.4260(OLS) 0.1903(GLS)	0.5774(OLS) 0.1919(GLS)
$\lambda^\star(PREM)$	0.0045	-0.0166	0.0066	0.0053
$\lambda(PREM)$	0.0083(OLS) 0.0018(GLS)	0.0146(OLS) 0.0047(GLS)	0.0059(OLS) 0.0016(GLS)	0.0063(OLS) -0.0007(GLS)
$Bias_{\lambda^\star}(PREM)$	0.0007	-0.0002	0.0013	-0.0004
$Bias_\lambda(PREM)$	0.0022(OLS) -0.0009(GLS)	0.0017(OLS) -0.0023(GLS)	0.0023(OLS) 0.0001(GLS)	0.0012(OLS) -0.0000(GLS)
$Emp.Std_{\lambda^\star}(PREM)$	0.0124	0.0176	0.0115	0.0107
$Emp.Std_\lambda(PREM)$	0.0494(OLS) 0.0129(GLS)	0.0815(OLS) 0.0237(GLS)	0.0427(OLS) 0.0126(GLS)	0.0395(OLS) 0.0104(GLS)
$\lambda^\star(INF)$	0.0036	0.0247	-0.0185	-0.0011
$\lambda(INF)$	-0.1355(OLS) -0.0287(GLS)	-0.2372(OLS) -0.0762(GLS)	-0.0954(OLS) -0.0259(GLS)	-0.1019(OLS) 0.0117(GLS)
$Bias_{\lambda^\star}(INF)$	0.0006	0.0002	-0.0033	-0.0004
$Bias_\lambda(INF)$	-0.0072(OLS) 0.0073(GLS)	-0.0104(OLS) 0.0148(GLS)	-0.0247(OLS) -0.0002(GLS)	-0.0083(OLS) -0.0035(GLS)
$Emp.Std_{\lambda^\star}(INF)$	0.0163	0.0206	0.0142	0.0140
$Emp.Std_\lambda(INF)$	0.0814(OLS) 0.0382(GLS)	0.1179(OLS) 0.0523(GLS)	0.0778(OLS) 0.0349(GLS)	0.0751(OLS) 0.0344(GLS)
$\lambda^\star(REALTB)$	-0.0047	-0.0314	0.0139	-0.0009
$\lambda(REALTB)$	0.1340(OLS) 0.0284(GLS)	0.2346(OLS) 0.0754(GLS)	0.0944(OLS) 0.0256(GLS)	0.1009(OLS) -0.0116(GLS)
$Bias_{\lambda^\star}(REALTB)$	-0.0006	-0.0002	0.0024	0.0004
$Bias_\lambda(REALTB)$	0.0079(OLS) -0.0073(GLS)	0.0119(OLS) -0.0145(GLS)	0.0252(OLS) 0.0003(GLS)	0.0090(OLS) 0.0034(GLS)
$Emp.Std_{\lambda^\star}(REALTB)$	0.0150	0.0197	0.0133	0.0129
$Emp.Std_\lambda(REALTB)$	0.0825(OLS) 0.0381(GLS)	0.1207(OLS) 0.0523(GLS)	0.0790(OLS) 0.0348(GLS)	0.0761(OLS) 0.0344(GLS)

Panel C: Fama-French

	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	-0.0216	-0.0421	-0.0039	-0.0073
$\lambda(HB3)$	-0.0363	-0.0252	-0.0113	-0.0083
$Bias_{\lambda^\star}(HB3)$	-0.0008	-0.0008	-0.0008	-0.0005
$Bias_\lambda(HB3)$	0.0028	0.0006	0.0002	-0.0002
$Emp.Std_{\lambda^\star}(HB3)$	0.0139	0.0189	0.0123	0.0123
$Emp.Std_\lambda(HB3)$	0.0140	0.0151	0.0107	0.0097
$\lambda^\star(DIV)$	-0.0629	-0.0412	-0.0446	-0.0672
$\lambda(DIV)$	-0.0744	-0.0432	-0.0591	-0.0673
$Bias_{\lambda^\star}(DIV)$	-0.0004	0.0008	-0.0136	-0.0006
$Bias_\lambda(DIV)$	0.0118	0.0023	-0.0066	-0.0019
$Emp.Std_{\lambda^\star}(DIV)$	0.0323	0.0328	0.0309	0.0314
$Emp.Std_\lambda(DIV)$	0.0227	0.0321	0.0215	0.0314
$\lambda^\star(CG)$	0.0103	0.0274	0.0093	-0.0042
$\lambda(CG)$	0.0257	0.0222	0.0177	0.0155
$Bias_{\lambda^\star}(CG)$	0.0000	0.0001	0.0025	0.0003
$Bias_\lambda(CG)$	-0.0035	0.0002	0.0012	0.0004
$Emp.Std_{\lambda^\star}(CG)$	0.0163	0.0209	0.0146	0.0145
$Emp.Std_\lambda(CG)$	0.0130	0.0144	0.0090	0.0095
$\lambda^\star(PREM)$	0.0045	-0.0166	0.0066	0.0053
$\lambda(PREM)$	0.0171	-0.0080	-0.0011	0.0076
$Bias_{\lambda^\star}(PREM)$	0.0007	-0.0002	0.0013	-0.0004
$Bias_\lambda(PREM)$	0.0006	-0.0001	0.0013	-0.0001
$Emp.Std_{\lambda^\star}(PREM)$	0.0124	0.0176	0.0115	0.0107
$Emp.Std_\lambda(PREM)$	0.0140	0.0141	0.0089	0.0077
$\lambda^\star(INF)$	0.0036	0.0247	-0.0185	-0.0011
$\lambda(INF)$	-0.0198	-0.0238	-0.0185	-0.0142
$Bias_{\lambda^\star}(INF)$	0.0006	0.0002	-0.0033	-0.0004
$Bias_\lambda(INF)$	0.0046	0.0010	-0.0006	-0.0011
$Emp.Std_{\lambda^\star}(INF)$	0.0163	0.0206	0.0142	0.0140
$Emp.Std_\lambda(INF)$	0.0139	0.0161	0.0099	0.0093
$\lambda^\star(REALTB)$	-0.0047	-0.0314	0.0139	-0.0009
$\lambda(REALTB)$	0.0132	0.0130	0.0112	0.0097
$Bias_{\lambda^\star}(REALTB)$	-0.0006	-0.0002	0.0024	0.0004
$Bias_\lambda(REALTB)$	-0.0031	-0.0008	0.0006	0.0011
$Emp.Std_{\lambda^\star}(REALTB)$	0.0150	0.0197	0.0133	0.0129
$Emp.Std_\lambda(REALTB)$	0.0124	0.0151	0.0090	0.0077

Panel D: CAPM

	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	-0.0216	-0.0421	-0.0039	-0.0073
$\lambda(HB3)$	-0.0058	-0.0027	-0.0048	-0.0059
$Bias_{\lambda^\star}(HB3)$	-0.0008	-0.0008	-0.0008	-0.0005
$Bias_{\lambda}(HB3)$	0.0009	0.0004	-0.0007	-0.0002
$Emp.Std_{\lambda^\star}(HB3)$	0.0139	0.0189	0.0123	0.0123
$Emp.Std_{\lambda}(HB3)$	0.0049	0.0041	0.0055	0.0065
$\lambda^\star(DIV)$	-0.0629	-0.0412	-0.0446	-0.0672
$\lambda(DIV)$	-0.0653	-0.0304	-0.0542	-0.0666
$Bias_{\lambda^\star}(DIV)$	-0.0004	0.0008	-0.0136	-0.0006
$Bias_{\lambda}(DIV)$	0.0102	0.0022	-0.0071	-0.0018
$Emp.Std_{\lambda^\star}(DIV)$	0.0323	0.0328	0.0309	0.0314
$Emp.Std_{\lambda}(DIV)$	0.0199	0.0306	0.0213	0.0312
$\lambda^\star(CG)$	0.0103	0.0274	0.0093	-0.0042
$\lambda(CG)$	0.0144	0.0067	0.0120	0.0147
$Bias_{\lambda^\star}(CG)$	0.0000	0.0001	0.0025	0.0003
$Bias_{\lambda}(CG)$	-0.0023	-0.0005	0.0016	0.0004
$Emp.Std_{\lambda^\star}(CG)$	0.0163	0.0209	0.0146	0.0145
$Emp.Std_{\lambda}(CG)$	0.0056	0.0072	0.0061	0.0083
$\lambda^\star(PREM)$	0.0045	-0.0166	0.0066	0.0053
$\lambda(PREM)$	0.0065	0.0030	0.0054	0.0067
$Bias_{\lambda^\star}(PREM)$	0.0007	-0.0002	0.0013	-0.0004
$Bias_{\lambda}(PREM)$	-0.0010	-0.0001	0.0006	0.0001
$Emp.Std_{\lambda^\star}(PREM)$	0.0124	0.0176	0.0115	0.0107
$Emp.Std_{\lambda}(PREM)$	0.0047	0.0043	0.0051	0.0061
$\lambda^\star(INF)$	0.0036	0.0247	-0.0185	-0.0011
$\lambda(INF)$	-0.0135	-0.0063	-0.0112	-0.0137
$Bias_{\lambda^\star}(INF)$	0.0006	0.0002	-0.0033	-0.0004
$Bias_{\lambda}(INF)$	0.0021	0.0004	-0.0015	-0.0004
$Emp.Std_{\lambda^\star}(INF)$	0.0163	0.0206	0.0142	0.0140
$Emp.Std_{\lambda}(INF)$	0.0054	0.0069	0.0059	0.0080
$\lambda^\star(REALTB)$	-0.0047	-0.0314	0.0139	-0.0009
$\lambda(REALTB)$	0.0092	0.0043	0.0076	0.0094
$Bias_{\lambda^\star}(REALTB)$	-0.0006	-0.0002	0.0024	0.0004
$Bias_{\lambda}(REALTB)$	-0.0014	-0.0003	0.0010	0.0003
$Emp.Std_{\lambda^\star}(REALTB)$	0.0150	0.0197	0.0133	0.0129
$Emp.Std_{\lambda}(REALTB)$	0.0047	0.0051	0.0052	0.0067

Table II: Non-traded component

We use a sieve bootstrap exercise to investigate the statistical properties of $\delta_{n,t}$ (see section IV for a description of the experiment). The set of instruments is common across asset-pricing models and includes a constant, the lagged value of the market excess return, the lagged value of the real TB rate, and the lagged value of dividend yield. We consider four multi-beta models. The first model is the I-CAPM (Panel A) with market excess return, dividend yield, real T-bill rate, term structure, default premium, consumption growth, and inflation as factors. The second model is the C-CAPM (Panel B), where the only factor driving the kernel is consumption growth. The third model is the Fama-French three-factor model (Panel C). The fourth model is the CAPM (Panel D), where the only factor driving the kernel is the excess market return. The exercise is repeated 100,000 times using a sample of 526 observations (March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables). In the first row, we report the average conditional non-traded component estimate of $\delta_{n,t}$ ($\delta_{n,0}$), and the conditional non-traded component estimates of $\delta_{n,t}$ ($\delta_{n,XVW}$, $\delta_{n,DIV}$, $\delta_{n,REALTB}$). When we investigate the properties of the $\delta_{n,t}$ estimates in presence of a pricing kernel driven by traded and non-traded factors (Panels A and B), we consider estimates based on the two weighting matrices $W = I$ (OLS case) and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case). In the second row, we report the finite-sample (absolute) bias of $\delta_{n,t}$. In the third row, we report the empirical standard error of $\delta_{n,t}$.

Panel A: I-CAPM

	$\delta_{n,0}$	$\delta_{n,XVW}$	$\delta_{n,DIV}$	$\delta_{n,REALTB}$
$\delta_n(HB3)$	−2.8983(OLS) −2.1117(GLS)	−1.7924(OLS) −0.9823(GLS)	0.7665(OLS) 0.5325(GLS)	0.2356(OLS) 0.3722(GLS)
$Bias_{\delta_n}(HB3)$	2.2167(OLS) 1.5212(GLS)	0.7278(OLS) 0.2272(GLS)	−0.9531(OLS) −0.6234(GLS)	0.0242(OLS) −0.2121(GLS)
$Emp.Std_{\delta_n}(HB3)$	1.1780(OLS) 0.8418(GLS)	1.3185(OLS) 1.1730(GLS)	0.6641(OLS) 0.6585(GLS)	0.8840(OLS) 0.7875(GLS)
$\delta_n(DIV)$	0.4660(OLS) 0.4051(GLS)	−5.9152(OLS) −4.4096(GLS)	−0.4962(OLS) 0.2023(GLS)	0.5959(OLS) −0.1761(GLS)
$Bias_{\delta_n}(DIV)$	−0.6272(OLS) −0.4028(GLS)	5.0857(OLS) 3.5378(GLS)	1.2185(OLS) 0.3879(GLS)	−1.4774(OLS) −0.5445(GLS)
$Emp.Std_{\delta_n}(DIV)$	0.7066(OLS) 0.4727(GLS)	1.6926(OLS) 1.5310(GLS)	1.0676(OLS) 0.9573(GLS)	1.2097(OLS) 1.0889(GLS)
$\delta_n(CG)$	−0.5047(OLS) −0.1650(GLS)	−1.2353(OLS) −0.6954(GLS)	0.6170(OLS) 0.6402(GLS)	−0.4781(OLS) −0.1627(GLS)
$Bias_{\delta_n}(CG)$	0.2857(OLS) 0.0851(GLS)	1.0623(OLS) 0.5935(GLS)	−0.5840(OLS) −0.3700(GLS)	0.2809(OLS) 0.1042(GLS)
$Emp.Std_{\delta_n}(CG)$	0.7924(OLS) 0.5623(GLS)	0.8998(OLS) 0.6291(GLS)	0.5159(OLS) 0.3539(GLS)	0.6048(OLS) 0.4280(GLS)
$\delta_n(PREM)$	−2.0056(OLS) −1.5829(GLS)	−1.0694(OLS) −1.0450(GLS)	−1.2185(OLS) −0.9169(GLS)	1.2198(OLS) 0.9770(GLS)
$Bias_{\delta_n}(PREM)$	2.0274(OLS) 1.5540(GLS)	1.1076(OLS) 0.9742(GLS)	0.9379(OLS) 0.6452(GLS)	−1.0280(OLS) −0.7474(GLS)
$Emp.Std_{\delta_n}(PREM)$	1.5818(OLS) 1.0964(GLS)	1.2275(OLS) 0.8356(GLS)	0.7160(OLS) 0.5147(GLS)	0.8192(OLS) 0.5811(GLS)
$\delta_n(INF)$	−1.5327(OLS) −1.0337(GLS)	−0.6943(OLS) −0.3285(GLS)	−0.1201(OLS) −0.1784(GLS)	−0.6806(OLS) −0.4700(GLS)
$Bias_{\delta_n}(INF)$	1.8490(OLS) 1.2248(GLS)	0.5028(OLS) 0.2200(GLS)	0.0898(OLS) 0.1140(GLS)	0.2741(OLS) 0.1130(GLS)
$Emp.Std_{\delta_n}(INF)$	0.9349(OLS) 0.6788(GLS)	0.6396(OLS) 0.5977(GLS)	0.3398(OLS) 0.3416(GLS)	0.4397(OLS) 0.4109(GLS)
$\delta_n(REALTB)$	2.1224(OLS) 1.6592(GLS)	0.0110(OLS) −0.0281(GLS)	−0.0453(OLS) 0.1342(GLS)	0.1009(OLS) −0.0470(GLS)
$Bias_{\delta_n}(REALTB)$	−2.3199(OLS) −1.7014(GLS)	−0.0626(OLS) −0.0319(GLS)	0.2794(OLS) 0.0828(GLS)	−0.3018(OLS) −0.1270(GLS)
$Emp.Std_{\delta_n}(REALTB)$	1.0650(OLS) 0.7702(GLS)	0.8291(OLS) 0.7841(GLS)	0.5199(OLS) 0.4962(GLS)	0.5818(OLS) 0.5533(GLS)

Panel B: C-CAPM

	$\delta_{n,0}$	$\delta_{n,XVW}$	$\delta_{n,DIV}$	$\delta_{n,REALTB}$
$\delta_n(HB3)$	0.0005(OLS) 0.0001(GLS)	0.0009(OLS) 0.0003(GLS)	0.0004(OLS) 0.0001(GLS)	0.0004(OLS) −0.0000(GLS)
$Bias_{\delta_n}(HB3)$	−0.0174(OLS) −0.0042(GLS)	−0.0269(OLS) −0.0076(GLS)	−0.0128(OLS) −0.0033(GLS)	−0.0117(OLS) 0.0014(GLS)
$Emp.Std_{\delta_n}(HB3)$	0.0451(OLS) 0.0344(GLS)	0.0733(OLS) 0.0221(GLS)	0.0383(OLS) 0.0117(GLS)	0.0358(OLS) 0.0097(GLS)
$\delta_n(DIV)$	−0.5212(OLS) −0.1104(GLS)	−0.9123(OLS) −0.2930(GLS)	−0.3669(OLS) −0.0996(GLS)	−0.3922(OLS) 0.0450(GLS)
$Bias_{\delta_n}(DIV)$	−0.0415(OLS) 0.0220(GLS)	−0.0583(OLS) 0.0499(GLS)	−0.1032(OLS) −0.0038(GLS)	−0.0400(OLS) −0.0109(GLS)
$Emp.Std_{\delta_n}(DIV)$	0.2848(OLS) 0.1442(GLS)	0.3816(OLS) 0.1900(GLS)	0.2726(OLS) 0.1293(GLS)	0.2648(OLS) 0.1294(GLS)
$\delta_n(CG)$	0.7188(OLS) 0.1523(GLS)	1.2581(OLS) 0.4041(GLS)	0.5060(OLS) 0.1374(GLS)	0.5408(OLS) −0.0620(GLS)
$Bias_{\delta_n}(CG)$	0.0519(OLS) −0.0336(GLS)	0.0746(OLS) −0.0725(GLS)	0.1395(OLS) 0.0018(GLS)	0.0525(OLS) 0.0138(GLS)
$Emp.Std_{\delta_n}(CG)$	0.4223(OLS) 0.1977(GLS)	0.5967(OLS) 0.2655(GLS)	0.4027(OLS) 0.1769(GLS)	0.3883(OLS) 0.1782(GLS)
$\delta_n(PREM)$	0.0017(OLS) 0.0004(GLS)	0.0029(OLS) 0.0009(GLS)	0.0012(OLS) 0.0003(GLS)	0.0013(OLS) −0.0001(GLS)
$Bias_{\delta_n}(PREM)$	−0.0147(OLS) −0.0031(GLS)	−0.0271(OLS) −0.0084(GLS)	−0.0126(OLS) −0.0029(GLS)	−0.0118(OLS) 0.0009(GLS)
$Emp.Std_{\delta_n}(PREM)$	0.0504(OLS) 0.0135(GLS)	0.0840(OLS) 0.0250(GLS)	0.0437(OLS) 0.0131(GLS)	0.0406(OLS) 0.0110(GLS)
$\delta_n(INF)$	−0.1746(OLS) −0.0370(GLS)	−0.3056(OLS) −0.0982(GLS)	−0.1229(OLS) −0.0334(GLS)	−0.1314(OLS) 0.0151(GLS)
$Bias_{\delta_n}(INF)$	−0.0263(OLS) 0.0065(GLS)	−0.0419(OLS) 0.0118(GLS)	−0.0457(OLS) −0.0036(GLS)	−0.0236(OLS) −0.0033(GLS)
$Emp.Std_{\delta_n}(INF)$	0.1087(OLS) 0.0526(GLS)	0.1515(OLS) 0.0706(GLS)	0.1039(OLS) 0.0478(GLS)	0.1005(OLS) 0.0475(GLS)
$\delta_n(REALTB)$	0.1060(OLS) 0.0224(GLS)	0.1855(OLS) 0.0596(GLS)	0.0746(OLS) 0.0203(GLS)	0.0797(OLS) −0.0091(GLS)
$Bias_{\delta_n}(REALTB)$	−0.0109(OLS) −0.0087(GLS)	−0.0190(OLS) −0.0186(GLS)	0.0061(OLS) −0.0030(GLS)	−0.0056(OLS) 0.0039(GLS)
$Emp.Std_{\delta_n}(REALTB)$	0.0667(OLS) 0.0276(GLS)	0.1040(OLS) 0.0403(GLS)	0.0629(OLS) 0.0256(GLS)	0.0602(OLS) 0.0248(GLS)

Panel C: Fama-French

	$\delta_{n,0}$	$\delta_{n,XVW}$	$\delta_{n,DIV}$	$\delta_{n,REALTB}$
$\delta_n(HB3)$	-0.0215	-0.0022	0.0020	-0.0015
$Bias_{\delta_n}(HB3)$	0.0001	0.0004	-0.0004	-0.0003
$Emp.Std_{\delta_n}(HB3)$	0.0107	0.0089	0.0067	0.0053
$\delta_n(DIV)$	-0.0058	0.0003	0.0017	0.0004
$Bias_{\delta_n}(DIV)$	0.0006	-0.0017	-0.0018	-0.0009
$Emp.Std_{\delta_n}(DIV)$	0.0081	0.0053	0.0036	0.0026
$\delta_n(CG)$	0.0063	0.0007	-0.0004	0.0006
$Bias_{\delta_n}(CG)$	-0.0005	0.0017	0.0013	0.0005
$Emp.Std_{\delta_n}(CG)$	0.0091	0.0056	0.0036	0.0026
$\delta_n(PREM)$	0.0126	-0.0014	-0.0022	0.0012
$Bias_{\delta_n}(PREM)$	0.0001	0.0006	0.0005	-0.0000
$Emp.Std_{\delta_n}(PREM)$	0.0122	0.0063	0.0049	0.0041
$\delta_n(INF)$	-0.0092	-0.0165	-0.0067	-0.0009
$Bias_{\delta_n}(INF)$	0.0021	0.0017	0.0013	-0.0007
$Emp.Std_{\delta_n}(INF)$	0.0109	0.0086	0.0057	0.0041
$\delta_n(REALTB)$	0.0088	0.0156	0.0064	0.0010
$Bias_{\delta_n}(REALTB)$	-0.0017	-0.0015	-0.0012	0.0006
$Emp.Std_{\delta_n}(REALTB)$	0.0105	0.0079	0.0054	0.0040

Panel D: CAPM

	$\delta_{n,0}$	$\delta_{n,XVW}$	$\delta_{n,DIV}$	$\delta_{n,REALTB}$
$\delta_n(HB3)$	0.0004	0.0002	0.0003	0.0004
$Bias_{\delta_n}(HB3)$	−0.0004	−0.0001	−0.0003	−0.0004
$Emp.Std_{\delta_n}(HB3)$	0.0010	0.0007	0.0010	0.0012
$\delta_n(DIV)$	0.0010	0.0005	0.0008	0.0010
$Bias_{\delta_n}(DIV)$	−0.0014	−0.0005	−0.0012	−0.0013
$Emp.Std_{\delta_n}(DIV)$	0.0011	0.0008	0.0012	0.0015
$\delta_n(CG)$	0.0000	0.0000	0.0000	0.0000
$Bias_{\delta_n}(CG)$	0.0005	0.0002	0.0006	0.0006
$Emp.Std_{\delta_n}(CG)$	0.0008	0.0006	0.0008	0.0010
$\delta_n(PREM)$	0.0001	0.0000	0.0001	0.0001
$Bias_{\delta_n}(PREM)$	0.0001	0.0001	0.0001	0.0001
$Emp.Std_{\delta_n}(PREM)$	0.0011	0.0008	0.0012	0.0014
$\delta_n(INF)$	−0.0003	−0.0001	−0.0002	−0.0003
$Bias_{\delta_n}(INF)$	−0.0001	−0.0001	−0.0002	−0.0002
$Emp.Std_{\delta_n}(INF)$	0.0008	0.0006	0.0009	0.0010
$\delta_n(REALTB)$	0.0003	0.0002	0.0003	0.0003
$Bias_{\delta_n}(REALTB)$	−0.0000	0.0001	0.0001	0.0001
$Emp.Std_{\delta_n}(REALTB)$	0.0008	0.0006	0.0009	0.0010

Table III: Mis-pricing component

We use a sieve bootstrap exercise to investigate the statistical properties of $\delta_{m,t}$ (see section IV for a description of the experiment). The set of instruments is common across asset-pricing models and includes a constant, the lagged value of the market excess return, the lagged value of the real T-bill rate, and the lagged value of dividend yield. We consider four multi-beta models. The first model is the I-CAPM (Panel A) with market excess return, dividend yield, real T-bill rate, term structure, default premium, consumption growth, and inflation as factors. The second model is the C-CAPM (Panel B), where the only factor driving the kernel is consumption growth. The third model is the Fama-French three-factor model (Panel C). The fourth model is the CAPM (Panel D), where the only factor driving the kernel is the excess market return. The exercise is repeated 100,000 times using a sample of 526 observations (March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables). In the first row, we report the average conditional non-traded component estimate of $\delta_{m,t}$ ($\delta_{m,0}$), and the conditional non-traded component estimates of $\delta_{m,t}$ ($\delta_{m,XVW}$, $\delta_{m,DIV}$, $\delta_{m,REALTB}$). When we investigate the properties of the $\delta_{m,t}$ estimates in presence of a pricing kernel driven by traded and non-traded factors (Panels A and B), we consider estimates based on the two weighting matrices $W = I$ (OLS case) and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case). In the second row, we report the finite-sample (absolute) bias of $\delta_{m,t}$. In the third row, we report the empirical standard error of $\delta_{m,t}$.

Panel A: I-CAPM

	$\delta_{m,0}$	$\delta_{m,XVW}$	$\delta_{m,DIV}$	$\delta_{m,REALTB}$
$\delta_m(HB3)$	−0.0059(OLS) −0.0000(GLS)	−0.0054(OLS) −0.0000(GLS)	−0.0000(OLS) 0.0015(GLS)	0.0020(OLS) −0.0004(GLS)
$Bias_{\delta_m}(HB3)$	0.0015(OLS) 0.0005(GLS)	−0.0099(OLS) −0.0002(GLS)	−0.0074(OLS) −0.0022(GLS)	0.0092(OLS) 0.0005(GLS)
$Emp.Std_{\delta_m}(HB3)$	0.0110(OLS) 0.0037(GLS)	0.0144(OLS) 0.0031(GLS)	0.0116(OLS) 0.0258(GLS)	0.0150(OLS) 0.0059(GLS)
$\delta_m(DIV)$	0.0016(OLS) 0.0023(GLS)	−0.0038(OLS) −0.0002(GLS)	−0.0016(OLS) 0.0004(GLS)	−0.0001(OLS) −0.0003(GLS)
$Bias_{\delta_m}(DIV)$	−0.0052(OLS) 0.0060(GLS)	0.0011(OLS) 0.0002(GLS)	0.0090(OLS) −0.0003(GLS)	−0.0093(OLS) 0.0001(GLS)
$Emp.Std_{\delta_m}(DIV)$	0.0105(OLS) 0.0301(GLS)	0.0112(OLS) 0.0031(GLS)	0.0134(OLS) 0.0055(GLS)	0.0133(OLS) 0.0040(GLS)
$\delta_m(CG)$	−0.0034(OLS) −0.0006(GLS)	−0.0039(OLS) 0.0000(GLS)	0.0013(OLS) 0.0021(GLS)	−0.0045(OLS) −0.0005(GLS)
$Bias_{\delta_m}(CG)$	−0.0007(OLS) −0.0014(GLS)	0.0017(OLS) 0.0008(GLS)	−0.0068(OLS) 0.0098(GLS)	0.0031(OLS) −0.0022(GLS)
$Emp.Std_{\delta_m}(CG)$	0.0128(OLS) 0.0069(GLS)	0.0087(OLS) 0.0026(GLS)	0.0085(OLS) 0.0208(GLS)	0.0104(OLS) 0.0048(GLS)
$\delta_m(PREM)$	−0.0019(OLS) −0.0003(GLS)	−0.0010(OLS) −0.0003(GLS)	0.0003(OLS) 0.0006(GLS)	−0.0015(OLS) −0.0005(GLS)
$Bias_{\delta_m}(PREM)$	0.0012(OLS) −0.0007(GLS)	0.0014(OLS) −0.0009(GLS)	−0.0025(OLS) 0.0020(GLS)	0.0024(OLS) −0.0014(GLS)
$Emp.Std_{\delta_m}(PREM)$	0.0097(OLS) 0.0038(GLS)	0.0080(OLS) 0.0027(GLS)	0.0090(OLS) 0.0045(GLS)	0.0088(OLS) 0.0033(GLS)
$\delta_m(INF)$	0.0001(OLS) 0.0007(GLS)	−0.0033(OLS) −0.0004(GLS)	0.0001(OLS) −0.0019(GLS)	−0.0036(OLS) 0.0003(GLS)
$Bias_{\delta_m}(INF)$	0.0050(OLS) 0.0012(GLS)	0.0004(OLS) −0.0001(GLS)	−0.0000(OLS) −0.0003(GLS)	0.0014(OLS) 0.0000(GLS)
$Emp.Std_{\delta_m}(INF)$	0.0112(OLS) 0.0065(GLS)	0.0065(OLS) 0.0018(GLS)	0.0057(OLS) 0.0151(GLS)	0.0072(OLS) 0.0034(GLS)
$\delta_m(REALTB)$	−0.0017(OLS) −0.0006(GLS)	0.0000(OLS) 0.0001(GLS)	0.0011(OLS) −0.0002(GLS)	−0.0018(OLS) 0.0000(GLS)
$Bias_{\delta_m}(REALTB)$	−0.0035(OLS) −0.0008(GLS)	−0.0001(OLS) 0.0000(GLS)	0.0010(OLS) −0.0000(GLS)	−0.0006(OLS) 0.0000(GLS)
$Emp.Std_{\delta_m}(REALTB)$	0.0110(OLS) 0.0047(GLS)	0.0052(OLS) 0.0018(GLS)	0.0064(OLS) 0.0033(GLS)	0.0062(OLS) 0.0024(GLS)

Panel B: C-CAPM

	$\delta_{m,0}$	$\delta_{m,XVW}$	$\delta_{m,DIV}$	$\delta_{m,REALTB}$
$\delta_m(HB3)$	0.0419(OLS) 0.0259(GLS)	0.0777(OLS) 0.0536(GLS)	0.0182(OLS) 0.0078(GLS)	0.0226(OLS) 0.0056(GLS)
$Bias_{\delta_m}(HB3)$	0.0190(OLS) 0.0022(GLS)	0.0318(OLS) 0.0056(GLS)	0.0188(OLS) 0.0037(GLS)	0.0148(OLS) −0.0004(GLS)
$Emp.Std_{\delta_m}(HB3)$	0.0344(OLS) 0.0188(GLS)	0.0524(OLS) 0.0270(GLS)	0.0315(OLS) 0.0170(GLS)	0.0309(OLS) 0.0164(GLS)
$\delta_m(DIV)$	0.4305(OLS) 0.1408(GLS)	0.6847(OLS) 0.2479(GLS)	0.3034(OLS) 0.1149(GLS)	0.3438(OLS) 0.0355(GLS)
$Bias_{\delta_m}(DIV)$	0.0343(OLS) −0.0156(GLS)	0.0492(OLS) −0.0350(GLS)	0.0907(OLS) 0.0160(GLS)	0.0327(OLS) 0.0071(GLS)
$Emp.Std_{\delta_m}(DIV)$	0.2358(OLS) 0.1195(GLS)	0.3151(OLS) 0.1520(GLS)	0.2259(OLS) 0.1100(GLS)	0.2202(OLS) 0.1110(GLS)
$\delta_m(CG)$	0.0383(OLS) 0(GLS)	0.0578(OLS) 0(GLS)	0.0249(OLS) 0(GLS)	0.0408(OLS) 0(GLS)
$Bias_{\delta_m}(CG)$	0.0152(OLS) 0(GLS)	0.0242(OLS) 0(GLS)	0.0162(OLS) 0(GLS)	0.0117(OLS) 0(GLS)
$Emp.Std_{\delta_m}(CG)$	0.0323(OLS) 0(GLS)	0.0421(OLS) 0(GLS)	0.0284(OLS) 0(GLS)	0.0282(OLS) 0(GLS)
$\delta_m(PREM)$	0.0022(OLS) −0.0031(GLS)	0.0283(OLS) 0.0204(GLS)	−0.0019(OLS) −0.0054(GLS)	−0.0003(OLS) −0.0059(GLS)
$Bias_{\delta_m}(PREM)$	0.0163(OLS) 0.0015(GLS)	0.0290(OLS) 0.0063(GLS)	0.0136(OLS) 0.0016(GLS)	0.0133(OLS) −0.0005(GLS)
$Emp.Std_{\delta_m}(PREM)$	0.0187(OLS) 0.0134(GLS)	0.0335(OLS) 0.0203(GLS)	0.0171(OLS) 0.0120(GLS)	0.0163(OLS) 0.0119(GLS)
$\delta_m(INF)$	0.0355(OLS) 0.0047(GLS)	0.0437(OLS) −0.0027(GLS)	0.0461(OLS) 0.0260(GLS)	0.0306(OLS) −0.0022(GLS)
$Bias_{\delta_m}(INF)$	0.0186(OLS) 0.0003(GLS)	0.0313(OLS) 0.0028(GLS)	0.0243(OLS) 0.0066(GLS)	0.0157(OLS) 0.0002(GLS)
$Emp.Std_{\delta_m}(INF)$	0.0399(OLS) 0.0258(GLS)	0.0516(OLS) 0.0308(GLS)	0.0384(OLS) 0.0236(GLS)	0.0363(OLS) 0.0232(GLS)
$\delta_m(REALTB)$	0.0327(OLS) 0.0106(GLS)	0.0806(OLS) 0.0472(GLS)	0.0059(OLS) −0.0085(GLS)	0.0221(OLS) −0.0015(GLS)
$Bias_{\delta_m}(REALTB)$	0.0193(OLS) 0.0021(GLS)	0.0312(OLS) 0.0044(GLS)	0.0167(OLS) 0.0009(GLS)	0.0142(OLS) −0.0009(GLS)
$Emp.Std_{\delta_m}(REALTB)$	0.0331(OLS) 0.0170(GLS)	0.0489(OLS) 0.0259(GLS)	0.0281(OLS) 0.0144(GLS)	0.0280(OLS) 0.0140(GLS)

Panel C: Fama-French

	$\delta_{m,0}$	$\delta_{m,XVW}$	$\delta_{m,DIV}$	$\delta_{m,REALTB}$
$\delta_m(HB3)$	0.0068	0.0192	-0.0094	0.0005
$Bias_{\delta_m}(HB3)$	0.0034	0.0011	0.0014	0.0006
$Emp.Std_{\delta_m}(HB3)$	0.0143	0.0138	0.0113	0.0097
$\delta_m(DIV)$	-0.0056	-0.0024	-0.0161	-0.0006
$Bias_{\delta_m}(DIV)$	0.0115	0.0032	0.0089	-0.0004
$Emp.Std_{\delta_m}(DIV)$	0.0329	0.0137	0.0220	0.0133
$\delta_m(CG)$	0.0091	-0.0059	0.0089	0.0192
$Bias_{\delta_m}(CG)$	-0.0031	-0.0015	-0.0026	-0.0004
$Emp.Std_{\delta_m}(CG)$	0.0172	0.0148	0.0132	0.0120
$\delta_m(PREM)$	0.0001	0.0100	-0.0055	0.0012
$Bias_{\delta_m}(PREM)$	-0.0002	-0.0007	-0.0005	0.0003
$Emp.Std_{\delta_m}(PREM)$	0.0109	0.0123	0.0089	0.0084
$\delta_m(INF)$	-0.0143	-0.0320	0.0067	-0.0121
$Bias_{\delta_m}(INF)$	0.0020	-0.0009	0.0014	-0.0000
$Emp.Std_{\delta_m}(INF)$	0.0162	0.0158	0.0125	0.0114
$\delta_m(REALTB)$	0.0091	0.0288	-0.0091	0.0096
$Bias_{\delta_m}(REALTB)$	-0.0008	0.0009	-0.0006	0.0001
$Emp.Std_{\delta_m}(REALTB)$	0.0144	0.0155	0.0116	0.0108

Panel D: CAPM

	$\delta_{m,0}$	$\delta_{m,XVW}$	$\delta_{m,DIV}$	$\delta_{m,REALTB}$
$\delta_m(HB3)$	0.0154	0.0393	−0.0012	0.0011
$Bias_{\delta_m}(HB3)$	0.0021	0.0014	0.0005	0.0007
$Emp.Std_{\delta_m}(HB3)$	0.0130	0.0187	0.0113	0.0102
$\delta_m(DIV)$	−0.0034	0.0104	−0.0105	−0.0004
$Bias_{\delta_m}(DIV)$	0.0120	0.0018	0.0077	0.0001
$Emp.Std_{\delta_m}(DIV)$	0.0308	0.0156	0.0214	0.0133
$\delta_m(CG)$	0.0041	−0.0207	0.0026	0.0188
$Bias_{\delta_m}(CG)$	−0.0029	−0.0008	−0.0015	−0.0005
$Emp.Std_{\delta_m}(CG)$	0.0158	0.0197	0.0129	0.0124
$\delta_m(PREM)$	0.0019	0.0196	−0.0013	0.0013
$Bias_{\delta_m}(PREM)$	−0.0017	−0.0002	−0.0007	0.0004
$Emp.Std_{\delta_m}(PREM)$	0.0112	0.0171	0.0099	0.0088
$\delta_m(INF)$	−0.0168	−0.0309	0.0076	−0.0123
$Bias_{\delta_m}(INF)$	0.0017	0.0003	0.0020	0.0003
$Emp.Std_{\delta_m}(INF)$	0.0155	0.0194	0.0124	0.0115
$\delta_m(REALTB)$	0.0135	0.0356	−0.0065	0.0100
$Bias_{\delta_m}(REALTB)$	−0.0008	−0.0001	−0.0015	−0.0001
$Emp.Std_{\delta_m}(REALTB)$	0.0142	0.0189	0.0117	0.0110

Table IV: Noisy Factor (C-CAPM)

We use a sieve bootstrap exercise to investigate the statistical properties of λ_t, λ_t^*, δ_{nt}, and δ_{mt} in presence of noise in the factor (see section IV for a description of the experiment). We focus on the unconditional estimates of the conditional C-CAPM, where the only factor driving the kernel is consumption growth. We introduce mean-zero noise orthogonal to asset excess returns, factor and instruments as follows: i) We generate a normal random variable with mean zero and variance equal to $c^2\sigma_y^2$, where c is a scalar ($c = 0, \sqrt{1/2}, 1, \sqrt{2}$); ii) we regress the $N(0, c^2\sigma_y^2)$ random variable on the augmented span of asset excess returns, factor and instruments; and iii) we use the regression residuals e_{t+1} to form the noisy factor $y_{t+1}^n = y_{t+1} + e_{t+1}$. The exercise is repeated 100,000 times using a sample of 526 observations (March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables). We report estimates of λ_0, λ_0^*, δ_{n0}, and δ_{m0} as well as small-sample (absolute) biases and empirical standard errors. We consider estimates of λ_0, λ_0^*, δ_{n0}, and δ_{m0} based on the two weighting matrices $W = I$ (OLS case) and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case).

	$c=0$ (no noise)	$c=\sqrt{1/2}$	$c=1$	$c=\sqrt{2}$
λ_0	0.7674(OLS) 0.1626(GLS)	0.9661(OLS) 0.2046(GLS)	1.1304(OLS) 0.2394(GLS)	1.4023(OLS) 0.2970(GLS)
$Bias_{\lambda_0}$	0.0672(OLS) -0.0335(GLS)	0.1066(OLS) -0.0580(GLS)	0.1123(OLS) -0.0857(GLS)	0.0293(OLS) -0.1411(GLS)
$Emp.Std_{\lambda_0}$	0.4465(OLS) 0.2130(GLS)	0.7120(OLS) 0.2812(GLS)	0.9768(OLS) 0.3342(GLS)	1.4636(OLS) 0.4128(GLS)
λ_0^*	0.0103	0.0082	0.0070	0.0056
$Bias_{\lambda_0^*}$	0.0000	-0.0002	-0.0002	-0.0003
$Emp.Std_{\lambda_0^*}$	0.0163	0.0148	0.0141	0.0134
δ_{n0}	0.7188(OLS) 0.1523(GLS)	0.9274(OLS) 0.1965(GLS)	1.0973(OLS) 0.2324(GLS)	1.3757(OLS) 0.2914(GLS)
$Bias_{\delta_{n0}}$	0.0519(OLS) -0.0336(GLS)	0.0882(OLS) -0.0578(GLS)	0.0920(OLS) -0.0854(GLS)	0.0078(OLS) -0.1408(GLS)
$Emp.Std_{\delta_{n0}}$	0.4223(OLS) 0.1977(GLS)	0.6857(OLS) 0.2675(GLS)	0.9471(OLS) 0.3213(GLS)	1.4279(OLS) 0.4006(GLS)
δ_{m0}	0.0383(OLS) 0(GLS)	0.0305(OLS) 0(GLS)	0.0260(OLS) 0(GLS)	0.0210(OLS) 0(GLS)
$Bias_{\delta_{m0}}$	0.0152(OLS) 0(GLS)	0.0187(OLS) 0(GLS)	0.0206(OLS) 0(GLS)	0.0218(OLS) 0(GLS)
$Emp.Std_{\delta_{m0}}$	0.0323(OLS) 0(GLS)	0.0353(OLS) 0(GLS)	0.0384(OLS) 0(GLS)	0.0439(OLS) 0(GLS)

Table V: P-values (λ vs $\lambda^{\star}$)

We use a sieve bootstrap exercise to investigate the statistical properties of λ_t and λ_t^* (see section IV for a description of the experiment). The set of instruments is common across asset-pricing models and includes a constant, the lagged value of the market excess return, the lagged value of the real T-bill rate, and the lagged value of dividend yield. We consider four multi-beta models. The first model is the I-CAPM (Panel A) with market excess return, dividend yield, real T-bill rate, term structure, default premium, consumption growth, and inflation as factors. The second model is the C-CAPM (Panel B), where the only factor driving the kernel is consumption growth. The third model is the Fama-French three-factor model (Panel C). The fourth model is the CAPM (Panel D), where the only factor driving the kernel is the excess market return. The exercise is repeated 100,000 times using a sample of 526 observations (March 1959-December 2002 for test assets and economic variables, and February 1959-November 2002 for the conditioning variables). In the first row, we report the bootstrap p-value of the average conditional risk premium estimate of λ_t^* (λ_0^*), and the bootstrap p-value of the conditional risk premium estimates of λ_t^* (λ_{XVW}^*, λ_{DIV}^*, λ_{REALTB}^*). In the second row, we report the bootstrap p-value of the average conditional risk premium estimate of λ_t (λ_0), and the bootstrap p-value of the conditional risk premium estimates of λ_t (λ_{XVW}, λ_{DIV}, λ_{REALTB}). When we investigate the properties of the λ estimates in presence of a pricing kernel driven by traded and non-traded factors (Panels A and B), we consider estimates based on the two weighting matrices $W = I$ (OLS case) and $W = \hat{\Sigma}_{rr}^{-1}$ (GLS case).

Panel A: I-CAPM				
	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	0.067	0.019	0.392	0.280
$\lambda(HB3)$	0.001(OLS) 0.001(GLS)	0.027(OLS) 0.111(GLS)	0.011(OLS) 0.022(GLS)	0.031(OLS) 0.139(GLS)
$\lambda^\star(DIV)$	0.028	0.100	0.158	0.018
$\lambda(DIV)$	0.060(OLS) 0.053(GLS)	0.011(OLS) 0.043(GLS)	0.013(OLS) 0.008(GLS)	0.176(OLS) 0.121(GLS)
$\lambda^\star(CG)$	0.262	0.096	0.313	0.375
$\lambda(CG)$	0.126(OLS) 0.301(GLS)	0.377(OLS) 0.177(GLS)	0.081(OLS) 0.234(GLS)	0.022(OLS) 0.066(GLS)
$\lambda^\star(PREM)$	0.364	0.168	0.311	0.285
$\lambda(PREM)$	0.012(OLS) 0.008(GLS)	0.000(OLS) 0.000(GLS)	0.035(OLS) 0.013(GLS)	0.458(OLS) 0.501(GLS)
$\lambda^\star(INF)$	0.425	0.116	0.139	0.478
$\lambda(INF)$	0.003(OLS) 0.004(GLS)	0.050(OLS) 0.596(GLS)	0.007(OLS) 0.006(GLS)	0.228(OLS) 0.445(GLS)
$\lambda^\star(REALTB)$	0.391	0.059	0.188	0.460
$\lambda(REALTB)$	0.002(OLS) 0.001(GLS)	0.042(OLS) 0.654(GLS)	0.009(OLS) 0.006(GLS)	0.207(OLS) 0.568(GLS)
Panel B: C-CAPM				
	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^\star(HB3)$	0.067	0.019	0.392	0.280
$\lambda(HB3)$	0.254(OLS) 0.198(GLS)	0.273(OLS) 0.190(GLS)	0.313(OLS) 0.256(GLS)	0.267(OLS) 0.294(GLS)
$\lambda^\star(DIV)$	0.028	0.100	0.158	0.018
$\lambda(DIV)$	0.040(OLS) 0.175(GLS)	0.016(OLS) 0.039(GLS)	0.116(OLS) 0.221(GLS)	0.068(OLS) 0.317(GLS)
$\lambda^\star(CG)$	0.262	0.096	0.313	0.375
$\lambda(CG)$	0.063(OLS) 0.172(GLS)	0.037(OLS) 0.044(GLS)	0.141(OLS) 0.219(GLS)	0.090(OLS) 0.329(GLS)
$\lambda^\star(PREM)$	0.364	0.168	0.311	0.285
$\lambda(PREM)$	0.401(OLS) 0.320(GLS)	0.415(OLS) 0.307(GLS)	0.414(OLS) 0.353(GLS)	0.384(OLS) 0.384(GLS)
$\lambda^\star(INF)$	0.425	0.116	0.139	0.478
$\lambda(INF)$	0.067(OLS) 0.156(GLS)	0.043(OLS) 0.053(GLS)	0.146(OLS) 0.207(GLS)	0.098(OLS) 0.305(GLS)
$\lambda^\star(REALTB)$	0.391	0.059	0.188	0.460
$\lambda(REALTB)$	0.070(OLS) 0.157(GLS)	0.047(OLS) 0.055(GLS)	0.150(OLS) 0.208(GLS)	0.101(OLS) 0.305(GLS)

Panel C: Fama-French				
	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^*(HB3)$	0.067	0.019	0.392	0.280
$\lambda(HB3)$	0.006	0.049	0.135	0.190
$\lambda^*(DIV)$	0.028	0.100	0.158	0.018
$\lambda(DIV)$	0.000	0.079	0.008	0.020
$\lambda^*(CG)$	0.262	0.096	0.313	0.375
$\lambda(CG)$	0.017	0.067	0.043	0.066
$\lambda^*(PREM)$	0.364	0.168	0.311	0.285
$\lambda(PREM)$	0.116	0.276	0.370	0.150
$\lambda^*(INF)$	0.425	0.116	0.139	0.478
$\lambda(INF)$	0.034	0.064	0.047	0.087
$\lambda^*(REALTB)$	0.391	0.059	0.188	0.460
$\lambda(REALTB)$	0.090	0.175	0.117	0.130
Panel D: CAPM				
	λ_0^* (λ_0)	λ_{XVW}^* (λ_{XVW})	λ_{DIV}^* (λ_{DIV})	λ_{REALTB}^*(λ_{REALTB})
$\lambda^*(HB3)$	0.067	0.019	0.392	0.280
$\lambda(HB3)$	0.091	0.181	0.203	0.167
$\lambda^*(DIV)$	0.028	0.100	0.158	0.018
$\lambda(DIV)$	0.000	0.142	0.014	0.020
$\lambda^*(CG)$	0.262	0.096	0.313	0.375
$\lambda(CG)$	0.006	0.151	0.058	0.057
$\lambda^*(PREM)$	0.364	0.168	0.311	0.285
$\lambda(PREM)$	0.073	0.191	0.165	0.138
$\lambda^*(INF)$	0.425	0.116	0.139	0.478
$\lambda(INF)$	0.008	0.157	0.063	0.060
$\lambda^*(REALTB)$	0.391	0.059	0.188	0.460
$\lambda(REALTB)$	0.027	0.167	0.110	0.097

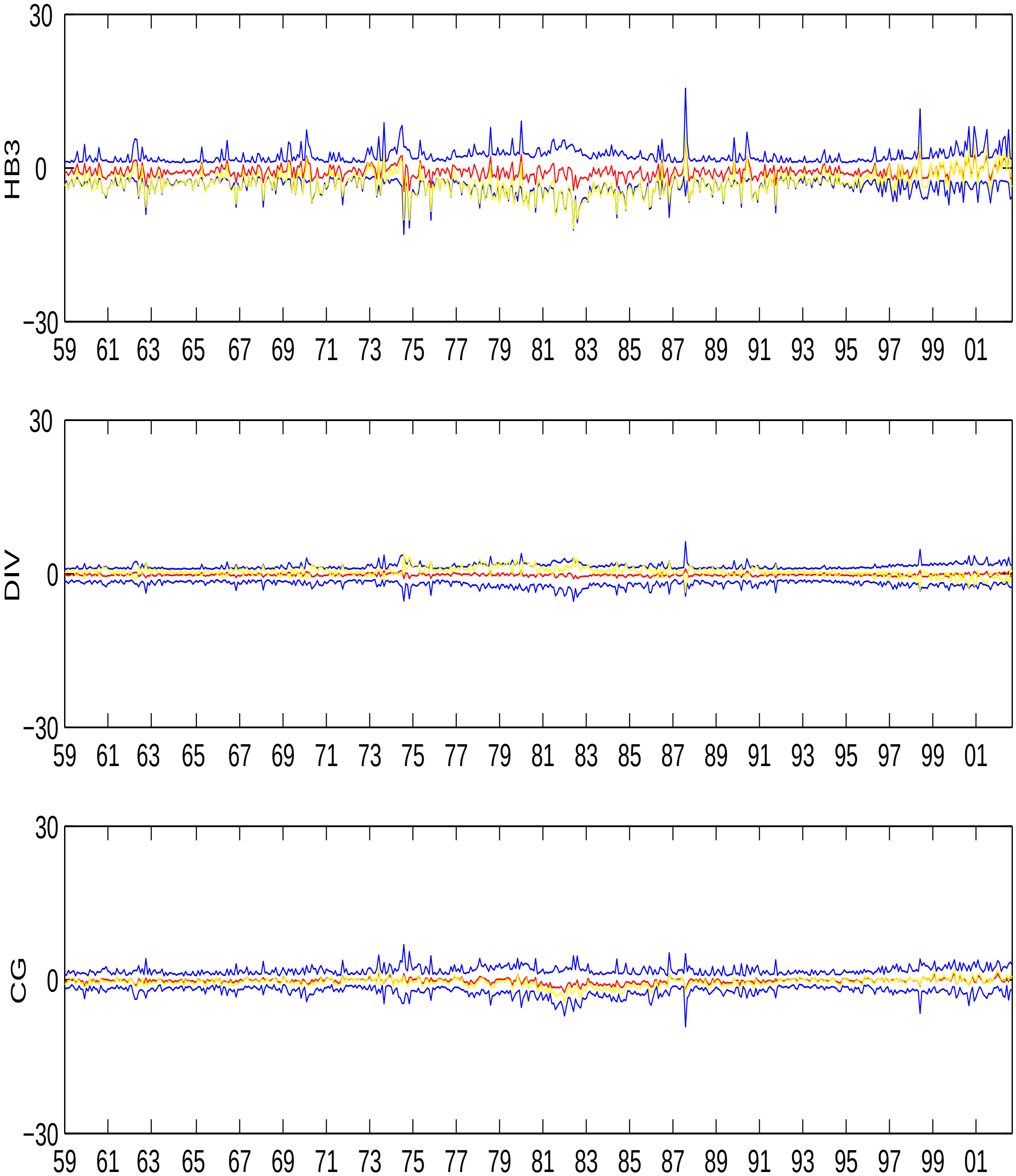

Figure 1. (A-C) Time-varying Risk Premia, λ_t. The graph illustrates the sample values of λ_t (yellow line) as well as the median (red line) and the 5^{th} and 95^{th} percentiles (blue lines) for HB3, DIV, CG, PREM, INF, and REALTB (I-CAPM/OLS case).

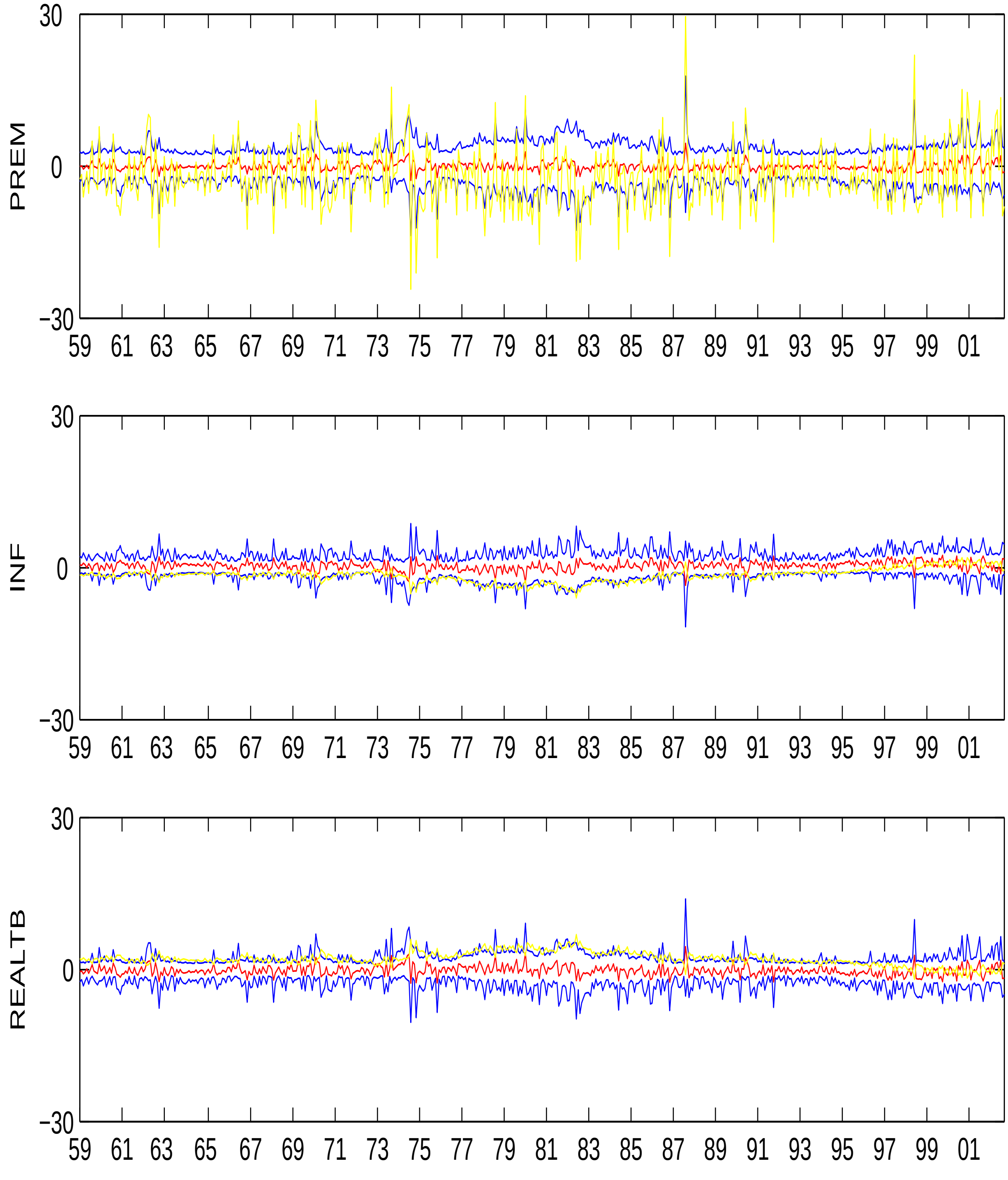

Figure 1. (D-F) Time-varying Risk Premia, λ_t.

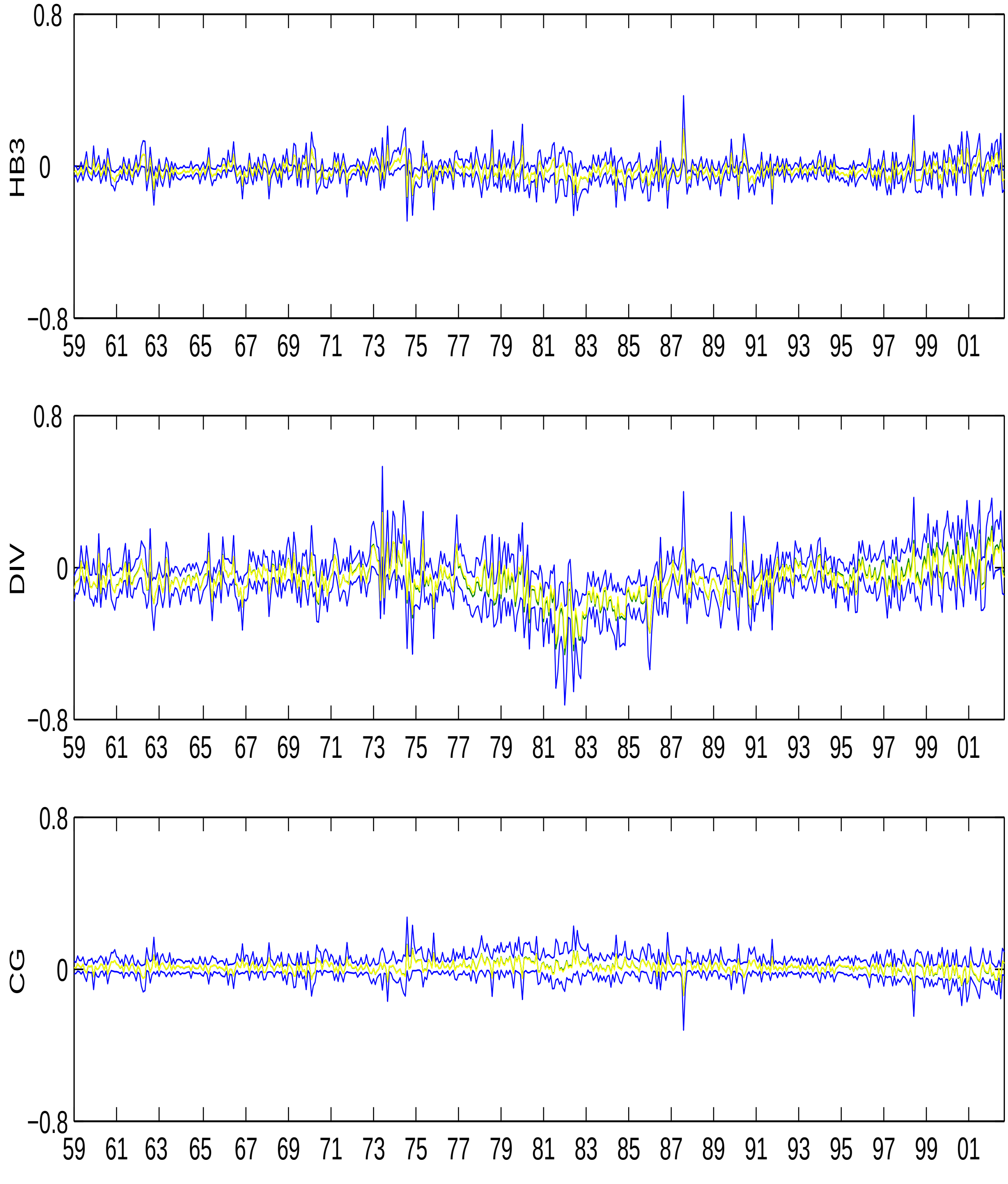

Figure 2. (A-C) Time-varying Risk Premia, $\lambda_t^\star$. The graph illustrates the sample values of λ_t^* (yellow line) as well as the 5^{th} and 95^{th} percentiles (blue lines) for HB3, DIV, CG, PREM, INF, and REALTB.

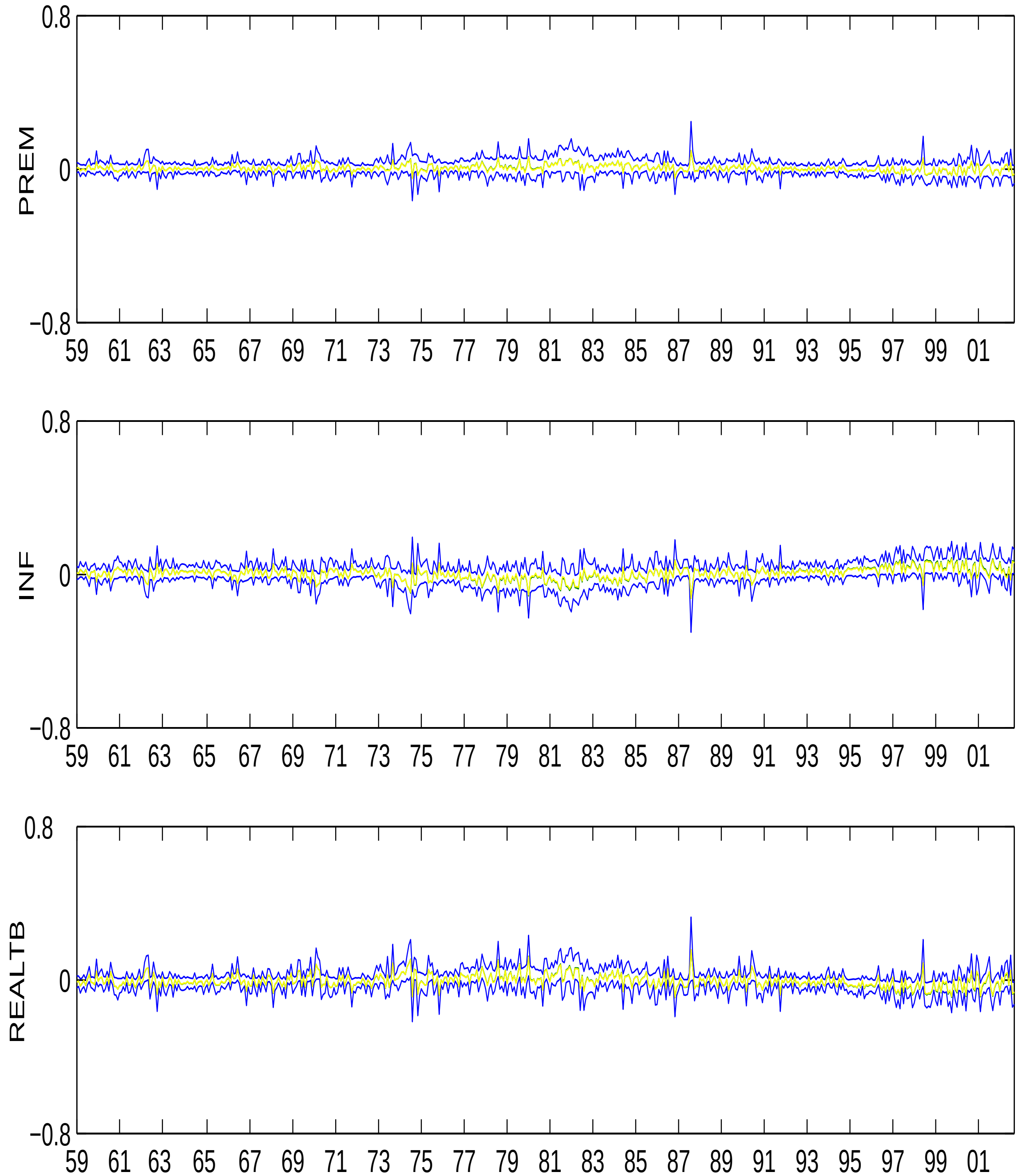

Figure 2. (D-F) Time-varying Risk Premia, $\lambda_t^{\star}$.