

Inflation and Monetary Regimes

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**Abstract:** Correlations of inflation with the growth rate of money increase when data are averaged over longer time periods. Correlations of inflation with the growth of money also are higher when high-inflation as well as low-inflation countries are included in the analysis. We show that serial correlation in the underlying inflation rate ties these two observations together and explains them. We present evidence that averaging increases the correlation of inflation and money growth more when the underlying inflation rate has higher serial correlation.

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## 1 Introduction

Is inflation related to money growth? Many have interpreted recent low correlations of money growth and inflation as evidence that inflation is not related to money growth under all circumstances, perhaps especially in low-inflation environments.

There is a large literature showing that money growth and inflation are related. The earliest papers in modern times were associated with the Money and Banking workshop at the University of Chicago (Friedman 1956). While Anderson and Jordan's (1968) paper using quarterly data for the United States was controversial, it clearly showed that money and inflation were related for 1952 to 1968. More recent papers suggest that such a relationship is not as close or as informative since the decline in inflation in the U.S. in the 1980s.

Kishor and Kochin (2007) show that part if not all of the explanation for the change in the importance of money growth in the United States is the change in the importance attached to inflation in monetary policy. When the monetary authority targets inflation using a control variable, the simple relationship between inflation and the control variable will decline because the control variable is changing to offset other influences on inflation. Kishor and Kochin show that the evidence for the United States is quite consistent with this analysis and an increasing emphasis on stabilizing inflation in U.S. monetary policy.

Empirical results across countries are not unequivocal either. Lucas (1980), Lothian (1985), Dwyer and Hafer (1988, 1999), McCandless and Weber (1995), Rolnick and Weber (1997) and others find substantial correlations of money growth and inflation across countries for different time periods. Moroney (2002) and De Grauwe and Polan (2005) examine a common criticism of such analyses, namely that the correlations are driven by the high inflation countries and there is little relationship between money growth and inflation for low inflation countries. Moroney (2002) selects countries based on money growth rates and finds a positive relationship between money growth and inflation in low-money-growth countries, but the relationship is stronger and more striking when countries with higher money growth are included in the analysis. De Grauwe and Polan present evidence that the correlations are close to zero or zero for low inflation countries. Frain (2004), responding to the 2001 working-paper version of De Grauwe and Polan's paper, removes countries with visible, documented data discontinuities or less than 25 years of data. He finds non-zero correlations of inflation and money growth relative to real income growth for low inflation countries as well as high inflation countries. He also finds regression coefficients for low inflation countries that are not different than

one at the five percent significance level.<sup>1</sup> Lothian and McCarthy (2009) proceed in a different way, comparing differences in growth rates across periods with evident differences in inflation. Even though the mean inflation rate in the high-inflation regime is less than ten percent, they find a close connection between the increase in inflation to these low levels and an increase in the the growth of money relative to real income.

The length of time over which growth rates are computed has an important influence on the analysis as well. Dwyer and Hafer (1988, 1999) show that the relationship across countries is not particularly obvious over periods as short as a year and is unambiguous over five-year periods. McCandless and Weber (1995) use data over 30 year periods to analyze the relationship and find clear relationships. De Grauwe and Polan (2005) find a relationship when using all countries over a 30-year period as does Frain (2004) for a 25-year period. While interesting and possibly informative, if the only reliable relationship between money growth and inflation is over a quarter of century, that certainly is very long run.

## 2 Some Evidence on Money Growth and Inflation

Before proceeding to our analysis, we document the results discussed above, including the importance of averaging over time and the implications of using only low money-growth countries. First, there is a noticeably closer relationship between inflation and money growth over longer periods than shorter periods. This is at least as strong a characteristic of the data as the other observation: countries with relatively high money growth show this relationship more clearly and make a substantial contribution to the apparent relationship across countries. We also summarize some empirical results about coefficients in regressions of inflation on money growth.

Throughout this paper, we measure the nominal quantity of money and its growth rate relative to real income. Adjusting the nominal quantity of money in this manner is useful if the income elasticity of the demand for money is unity or not too far from unity. While this is not a particularly important adjustment when there is substantial variation in inflation relative to real income growth, it is more important when inflation variation is on the order of magnitude of the variation in inflation.

[Insert Figure 1 about here]

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<sup>1</sup> The standard errors are larger for low-inflation countries, but confidence intervals include one and do not include zero.

The price level and money relative to real income are strikingly similar for both of the two high inflation countries shown in Figure 1, Brazil from 1912 to 2006 and Chile from 1940 to 2006. The price level is the Gross Domestic Product (GDP) deflator. The measure of money is the nominal quantity of money divided by real GDP. All of the series are set to have average values of 100 for the time periods covered. The vertical axis is a proportional scale, making it possible to read growth rates from the slopes of the lines. The closeness of the behavior of the price level and money in both graphs, including the decreases in inflation toward the end of the periods, is striking.

[Insert Figure 2 about here]

Figure 2 shows similar graphs for two countries with relatively low inflation, Japan and the United States for the parts of the postwar period with consistent data. The movements of the price level and money relative to real income are similar, but definitely not as close as in the graphs for Brazil and Chile. As earlier papers indicate, there is little obvious short-term relationship. In the United States, money relative to income is low compared to the price level in the 1990s. The fall in the price level in Japan from 1998 to 2006 is associated with lower growth of money relative to income for those years, but there is no corresponding fall in the level of money relative to income. Money relative to income increases 0.8 percent per year from 1998 to 2006 while prices fall 1.3 percent per year.

[Insert Figure 3 about here]

Figure 3 shows average inflation rates and growth rates of money relative to real income across countries. As does Frain (2004), we use the term “excess money growth” instead of the more cumbersome “growth of money relative to real income”. The upper left panel shows the relationship between the two for all 166 countries for which we have data for twelve or more consecutive years. We include data starting in 1985 or later through the end of the period. Not all of the countries have data for the whole period, for example Albania’s data begin in 1994. The upper right panel shows the relationship for countries with an excess money growth rate less than 50 percent, 159 of the countries. The lower left panel shows the relationship for countries with excess money growth less than 20 percent, and the lower right panel shows the relationship for countries with excess money growth less than 10 percent. We use the growth of money relative to real income instead of the inflation rate to pick countries with low inflation because regressions of inflation on money growth have biased coefficients if countries are picked on the basis of the dependent variable, the inflation rate.<sup>2</sup>

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<sup>2</sup> Suppose that inflation and the growth of money relative to real income are related with a coefficient of one and money growth is exogenous to inflation. If inflation is used to pick countries, then the dependent variable is being used to select the

For all of the countries, there is a positive relationship between inflation and excess money growth. We find a positive relationship for low inflation countries which we define as those with excess money growth less than ten percent per year. The correlation monotonically decrease with decreases in the cutoff growth of excess money but it does not go to zero. The correlation is 0.47 even for countries with the average growth of excess money less than ten percent.

Figure 3 also shows lines for regressions of inflation on excess money growth. The slopes in these regressions also are used by some as a criteria for evaluating the usefulness of money as a predictor of inflation, with coefficients close to one being considered more supportive (Moroney, 2002; Frain, 2004; DeGrauwe and Polan, 2005). As the data are cut off at lower growth rates of excess money, the regression coefficients decrease, with the regression coefficients in this figure decreasing from 1.01 for all the data, to 0.99 for countries with excess money growth less than 50 percent, to 0.88 for countries with excess money growth less than 20 percent and to 0.41 for countries with excess money growth less than 10 percent.

The regression coefficient is substantially less than unity for lower growth rates of excess money. Kisher and Kochin's (2007) analysis suggests why this is so. The correlation of inflation and excess money growth is zero if all deviations from a constant target inflation rate are unpredictable. The fall in the correlation and the regression coefficient is consistent with their analysis if low inflation countries have less variability of inflation targets and therefore less correlation of inflation and excess money growth.

[Insert Figure 4 about here]

Figure 4 shows the relationship between inflation and money growth when the data are averaged over successively shorter periods. The upper left panel shows the relationship over all the years for which we have data on each country, which is as much as 21 years and as few as twelve years. The upper right panel shows the relationship with data averaged over the last ten years for which we have data. The lower panels show the relationship with data averaged over five years and one year. It is clear that the relationship becomes weaker over shorter periods. This is consistent with averages presented over five years and less presented by Dwyer and Hafer (1988, 1999).

Figure 4 also shows regression coefficients of inflation on excess money growth. These coefficients also decrease as the data are averaged over shorter periods,

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observations. Countries with inflation greater than 10 percent and growth of money relative to real income less than 10 percent are excluded but countries with inflation less than 10 percent and growth of money relative to real income greater than 10 percent are included. This selection biases the regression coefficient downward from one.

from 1.01 for all the data to 0.46 for one year of data.

### 3 Money Growth and Inflation

Why are money growth and inflation more closely related when data are averaged over long time periods and when high inflation countries are included in an analysis of inflation and money growth? In this section, we provide an explanation based on variation of the underlying inflation rate relative to the demand for money. This analysis predicts that higher serial correlation of the underlying inflation rate is associated with a larger increase in the correlation between inflation and money growth as more years are averaged. We also show that the size of the slope coefficient in a regression of inflation on money growth is uninformative about whether the quantity theory holds. The quantity theory is consistent with a slope coefficient of unity in a regression of inflation on money growth and it is consistent with a slope coefficient less than unity.

Suppose that the demand for money has unit income elasticity and no other variables systematically affect demand. Then

$$\mu_t - y_t = \pi_t + \varepsilon_t \tag{1}$$

where  $\mu_t$  is the growth rate of the nominal quantity of money in period  $t$ ,  $\pi_t$  is the inflation rate,  $y_t$  is the growth rate of real income and  $\varepsilon_t$  is an error term in the demand for money.

#### 3.1 Inflation Targeting

Suppose that the monetary authority's actions target the inflation rate, whether this is intentional or not, and the target is  $\pi_t^*$  which varies over time. This relationship can be written

$$\pi_t = \pi_t^* + \eta_t, \tag{2}$$

where  $\eta_t$  is the error term in this equation. For simplicity, we suppress the subscript  $t$ . Combining (1) and (2) results in

$$\mu - y = \pi + \varepsilon = \pi^* + \eta + \varepsilon. \tag{3}$$

The correlation of the inflation rate (2) and excess money growth (3) is

$$\rho = \text{Corr}[\pi, \mu - y] = \frac{\text{Cov}[\pi^* + \eta, \pi^* + \eta + \varepsilon]}{\text{SD}[\pi^* + \eta] \text{SD}[\pi^* + \eta + \varepsilon]}, \tag{4}$$

which equals

$$\rho = \frac{\text{Var}[\pi^* + \eta] + \text{Cov}[\varepsilon, \pi^* + \eta]}{SD[\pi^* + \eta] (\text{Var}[\pi^* + \eta] + 2\text{Cov}[\varepsilon, \pi^* + \eta] + \text{Var}[\varepsilon])^{1/2}}. \quad (5)$$

At first glance, it is not obvious this is particularly helpful. Suppose, though, that the error term in the demand for money is orthogonal to the target price level and errors in hitting it, i.e.  $\text{Cov}[\varepsilon, \pi^* + \eta] = 0$ . Then the correlation of the inflation rate and excess money growth simplifies to

$$\rho = \frac{SD[\pi^* + \eta]}{(\text{Var}[\pi^* + \eta] + \text{Var}[\varepsilon])^{1/2}}. \quad (6)$$

This equation for the correlation can be interpreted in an informative way. First off, suppose that the variance of the inflation target and errors in hitting it are zero. Equation (6) states the obvious: the correlation of the inflation rate with excess money growth is zero. If there is substantial variance in inflation targets or errors in generating that inflation rate relative to the demand for money, then the correlation will be closer to one. This is related to the analysis by Kishor and Kochin (2007); it also provides a tentative explanation of Figure 3. For countries with similar inflation targets, i.e. little variance of inflation targets or errors in hitting them, the correlation across countries of inflation with excess money growth will be low if not zero. At the other end of the range between zero and one, zero variance of the error term in the growth of money demand implies

$$\text{Corr}[\pi, \mu - y] = 1. \quad (7)$$

The demand for money and the monetary authority's inflation target may well have different characteristics over time. Suppose that the inflation target varies gradually over time and the demand for money varies more over short periods of time. Then the relative variance of  $\pi^* + \eta$  and  $\varepsilon$  will change as data are averaged over different time periods. Over short periods, the variance of the demand for money will be larger relative to the variance in the supply; over longer periods, the variance of the demand for money decreases relative to the variance in the supply. In the limit, the variance in demand goes to zero and the correlation of the inflation rate with excess money growth goes to one.

Orthogonality of the error term in the demand for money and the error in the supply of money and changes in the target inflation rate is sufficient for this characterization of the correlations but is not necessary. The correlation can be written

$$\rho = \frac{1 + \text{Cov}[\varepsilon, \pi] / \text{Var}[\pi]}{(1 + 2\text{Cov}[\varepsilon, \pi] / \text{Var}[\pi] + \text{Var}[\varepsilon] / \text{Var}[\pi])^{1/2}}. \quad (8)$$

Even if  $\text{Cov}[\varepsilon, \pi] \neq 0$ , this correlation approaches one as  $\text{Var}[\pi]$  increases relative to  $\text{Cov}[\varepsilon, \pi]$  and  $\text{Var}[\varepsilon]$ . In short, a higher correlation of inflation and excess money growth is to be expected with a higher variance of the inflation target and errors hitting it if the covariance of errors in the growth of money demand with the inflation rate and the variance in errors in the demand for money do not increase proportionately.

The regression coefficient from a regression of inflation on excess money growth will not be unity even though the quantity theory holds in this setup. This regression coefficient is

$$\beta_{\pi|\mu-y} = \frac{\text{Cov}[\pi, \mu - y]}{\text{Var}[\mu - y]}, \quad (9)$$

which can be rewritten as

$$\beta_{\pi|\mu-y} = \frac{\text{Var}[\pi]}{\text{Var}[\pi] + 2\text{Cov}[\varepsilon, \pi] + \text{Var}[\varepsilon]} + \frac{\text{Cov}[\pi, \varepsilon]}{\text{Var}[\pi] + 2\text{Cov}[\varepsilon, \pi] + \text{Var}[\varepsilon]}. \quad (10)$$

This does not obviously equal one, and it does not equal one in general. Even if  $\text{Cov}[\varepsilon, \pi] = 0$ ,

$$\beta_{\pi|\mu-y} = \frac{\text{Var}[\pi]}{\text{Var}[\pi] + \text{Var}[\varepsilon]}, \quad (11)$$

which is less than one unless  $\text{Var}[\varepsilon]$  is zero.<sup>3</sup> Stated more positively,  $\beta_{\pi|\mu-y}$  approaches one as  $\text{Var}[\varepsilon]/\text{Var}[\pi]$  goes to zero but, with inflation targeting, the coefficient does not equal one even if the covariance of errors in the growth of money demand and the inflation rate is zero. This result does not hold under all circumstances.

### 3.2 Control of Money Supply

Instead of being determined by the demand for money as it would be under inflation targeting, suppose the supply of money is determined by

$$\mu = \pi^* + y + \zeta, \quad (12)$$

where  $\zeta$  is the error term and the demand for money is the same as equation (1). The money supply is determined with a target inflation rate as the goal but the growth rate of the nominal quantity of money is changed to effect the

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<sup>3</sup> The similarity of this formula and the one for regressions with errors in the right-hand-side variables is not an accident. With inflation targeting, shocks to the demand for money affect the growth of the nominal quantity of money but not the inflation rate. This is similar to measurement error in a right-hand-side variable that has no effect on a left-hand-side variable.

goal. The growth rate of real income is included in the equation for the supply of money with a coefficient of one to reflect the growth of demand due to real income. The central bank can achieve its target inflation rate by changing the growth rate of the nominal quantity of money with the growth rate of real income. This equation (12) can be rewritten

$$\mu - y = \pi^* + \zeta. \quad (13)$$

Equating the growth of the demand for the nominal quantity of money (1) and the supply of the nominal quantity of money (13) yields

$$\pi = \pi^* + \zeta - \varepsilon. \quad (14)$$

It follows that the correlation of the inflation rate and excess money growth  $\rho_m$  is

$$\rho_m = \frac{1 - \text{Cov}[\varepsilon, \pi^* + \zeta] / \text{Var}[\pi^* + \zeta]}{(1 - 2\text{Cov}[\pi^* + \zeta, \varepsilon] / \text{Var}[\pi^* + \zeta] + \text{Var}[\varepsilon] / \text{Var}[\pi^* + \zeta])^{1/2}}. \quad (15)$$

If  $\text{Cov}[\varepsilon, \pi^* + \zeta] = 0$ , then

$$\rho_m = \frac{\text{SD}[\pi^* + \zeta]}{(\text{Var}[\pi^* + \zeta] + \text{Var}[\varepsilon])^{1/2}} = \frac{1}{(1 + \text{Var}[\varepsilon] / \text{Var}[\pi^* + \zeta])^{1/2}} \quad (16)$$

which approaches one as  $\text{Var}[\varepsilon] / \text{Var}[\pi^* + \zeta]$  goes to one. This is the same conclusion as above under inflation targeting.

The conclusion concerning regression coefficients does change though. The coefficient from regressing the inflation rate on excess money growth is

$$\begin{aligned} \beta_{\pi|\mu-y}^* &= \frac{\text{Cov}[\pi, \mu - y]}{\text{Var}[\mu - y]} = \frac{\text{Var}[\pi^* + \zeta] - \text{Cov}[\varepsilon, \pi^* + \zeta]}{\text{Var}[\pi^* + \zeta]} \\ &= 1 - \frac{\text{Cov}[\varepsilon, \pi^* + \zeta]}{\text{Var}[\pi^* + \zeta]}. \end{aligned} \quad (17)$$

If  $\text{Cov}[\varepsilon, \pi^* + \zeta] = 0$ , then  $\beta_{\pi|\mu-y}^* = 1$ . This is not true if  $\text{Cov}[\varepsilon, \pi^* + \zeta] \neq 0$ , although  $\beta_{\pi|\mu-y}^*$  approaches one as  $\text{Cov}[\varepsilon, \pi^* + \zeta] / \text{Var}[\pi^* + \zeta]$  goes to zero.

In sum, if the covariance of the errors in the demand for money and supply of money is zero, the correlation of inflation and excess money growth increases to one as the variance in the demand for money goes to zero relative to the the variance in the supply for money. This conclusion concerning the correlation's value holds whether the monetary regime is one of inflation targeting or control of the money supply. The coefficient in a regression of inflation on the excess money growth rate depends on how the nominal quantity of money is determined.

#### 4 Persistence in the Underlying Inflation Rate

In this section, we derive testable predictions concerning the evolution of the underlying inflation rate and the correlation of excess money growth and inflation. We show that serial correlation of the underlying inflation rate is consistent with increases in the correlation of inflation and excess money growth as data are averaged over longer periods, and higher serial correlation is consistent with a greater increase in the correlation.

Consistent with the argument above, suppose that errors in the demand for money are serially uncorrelated but suppose that the underlying inflation rate, or inflation target, evolves over time according to

$$\pi^* = \beta\pi_{-1}^* + \nu, \quad (18)$$

where  $\nu$  is serially uncorrelated as well and  $0 \leq \beta < 1$ . Assume  $\text{Cov}[\pi_{-1}^*, \nu] = 0$ . If  $\text{Cov}[\varepsilon, \pi] = 0$ , the earlier analysis show that the one-period correlation of inflation and excess money growth is

$$\rho = \frac{\text{SD}[\pi]}{(\text{Var}[\pi] + \text{Var}[\varepsilon])^{1/2}}. \quad (19)$$

Let  $\pi_2 = (\pi + \pi_{-1})/2$  and  $\varepsilon_2 = (\varepsilon + \varepsilon_{-1})/2$ . Given these definitions, the correlation of two-period averages of inflation and excess money growth is

$$\rho_2 = \frac{\text{SD}[\pi_2]}{(\text{Var}[\pi_2] + \text{Var}[\varepsilon_2])^{1/2}}. \quad (20)$$

Is  $\rho_2 > \rho$ ? Because  $\text{Var}[\pi_2] = \frac{1}{2}\text{Var}[\pi] + \frac{1}{2}\text{Cov}[\pi, \pi_{-1}]$  and  $\text{Var}[\varepsilon_2] = \frac{1}{2}\text{Var}[\varepsilon]$ ,

$$\rho_2 = \left[ \frac{\text{Var}[\pi] + \text{Cov}[\pi, \pi_{-1}]}{\text{Var}[\pi] + \text{Cov}[\pi, \pi_{-1}] + \text{Var}[\varepsilon]} \right]^{1/2}. \quad (21)$$

Given serially uncorrelated errors that are mutually uncorrelated,  $\text{Cov}[\pi, \pi_{-1}] = \beta\text{Var}[\pi^*]$  and therefore

$$\rho_2 = \left[ \frac{\text{Var}[\pi] + \beta\text{Var}[\pi^*]}{\text{Var}[\pi] + \beta\text{Var}[\pi^*] + \text{Var}[\varepsilon]} \right]^{1/2}. \quad (22)$$

The issue is whether

$$\rho_2 \stackrel{\geq}{<} \rho. \quad (23)$$

This is equivalent to deciding whether

$$\beta \frac{\text{Var}[\pi^*]}{\text{Var}[\pi]} \frac{\text{Var}[\varepsilon]}{\text{Var}[\pi]} \stackrel{\geq}{<} 0. \quad (24)$$

Since this is positive by assumption, it follows that

$$\rho_2 > \rho_1. \quad (25)$$

This algebra makes it clear that the correlation goes up with averaging as  $\beta \frac{\text{Var}[\pi^*]}{\text{Var}[\pi]} \frac{\text{Var}[\varepsilon]}{\text{Var}[\pi]}$  increases. The parameter  $\beta$  represents the serial correlation in the underlying inflation rate. Increases in  $\beta$  increase the difference between the one-period and two-period correlations. If  $\beta = 0$ , then  $\rho_2 = \rho_1$ . If  $\beta > 0$ , then  $\rho_2 > \rho_1$ . The other terms represent the product of the variation in the inflation target relative to the inflation rate and the variation in the demand for money relative to the inflation rate, which affect the magnitude of the increase in the correlation.

## 5 The Generalized Local Level Model

We can examine the relationship between the correlations and the serial correlation of the underlying inflation rate  $\pi^*$  in a state-space model in which  $\pi^*$  is unobservable. We use a Bayesian analysis to derive the posterior distribution of the serial correlation parameter  $\beta$  and the relationship between  $\beta$  and the correlation between inflation and excess money growth as the data are averaged over longer periods.

The generalized local level model is a simple state-space model involving an observation (measurement) equation and a state (transition) equation.<sup>4</sup> We have  $T$  observations on inflation  $\pi = (\pi_1, \dots, \pi_T)$ . The state variable is the underlying inflation rate  $\pi^* = (\pi_1^*, \dots, \pi_T^*)$  which is not directly observed. The observation and state equations are (for  $t = 1, \dots, T$ )

$$\pi_t = \pi_t^* + \eta_t \quad (26)$$

$$\pi_t^* = \delta(1 - \beta) + \beta \pi_{t-1}^* + u_t, \quad (27)$$

where

$$\begin{bmatrix} \eta_t \\ u_t \end{bmatrix} \stackrel{\text{iid}}{\sim} \text{N}(0, \Sigma), \quad \text{where } \Sigma = h^{-1} \begin{bmatrix} 1 & 0 \\ 0 & \psi \end{bmatrix}. \quad (28)$$

We impose the restriction  $-1 < \beta \leq 1$ . The local level model itself as in Koop (2003) is characterized by  $\beta = 1$ .

It is important to deal with the unobserved observation in period 0 in a clean way. Let  $\pi_0^* = \delta + w$ , where  $w \sim \text{N}(0, h^{-1}\lambda)$  and  $w$  is independent of the other

<sup>4</sup> This model is a generalization of the local level model given in Koop (2003). The generalization allows the unobserved state variable to be stationary.

disturbances. Eliminating  $\pi_0^*$  from the state equation for  $\pi_1^*$  produces

$$\pi_1^* = \delta + \beta w + u_1. \quad (29)$$

Consequently, we see that  $\pi_1^* \sim N(\delta, h^{-1}(\beta^2\lambda + \psi))$ . (Note that if  $\beta = 0$ , then  $\lambda$  does not appear in the distribution for  $\pi_1^*$ .)

Given this setup, we now show that  $\pi$  follows a restricted ARMA(1,1) for  $t \geq 2$ . We can eliminate the unobserved state variable and obtain

$$\pi_t = \begin{cases} \delta + \beta w + u_1 + \eta_1 & t = 1 \\ \delta(1 - \beta) + \beta\pi_{t-1} + u_t + \eta_t - \beta\eta_{t-1} & t \geq 2. \end{cases} \quad (30)$$

Note  $E_0[w] = E_0[u_t] = E_0[\eta_t] = 0$ . Therefore,  $E_0[\pi_t] = \delta$  for all  $t \geq 1$ . Define  $\omega_t := u_t + \eta_t - \beta\eta_{t-1}$ . Let  $\gamma_\omega(\tau)$  denote the autocovariance function for  $\omega$ . Then  $\gamma_\omega(0) = \psi h^{-1} + (1 + \beta^2)h^{-1}$ ,  $\gamma_\omega(1) = -\beta h^{-1}$ , and  $\gamma_\omega(\tau) = 0$  for  $\tau \geq 2$ . This autocovariance function is characteristic of an MA(1). As such, we can reexpress  $\omega_t$  as  $\omega_t = v_t - \xi v_{t-1}$ , where  $|\xi| < 1$ .<sup>5</sup> We see that  $\pi$  is a restricted ARMA(1,1) for  $t \geq 2$ . If  $\beta = 1$ , then  $\pi$  is an IMA(1,1).

The posterior distribution for the unobservables conditional on the observable series  $\pi$  is

$$p(\pi^*, h, \phi|\pi) = p(\pi^*, h|\pi, \phi) p(\phi|\pi), \quad (31)$$

where

$$\phi := (\delta, \beta, \lambda, \psi). \quad (32)$$

The factorization on the right-hand side of (31) will prove convenient. The posterior distribution (31) can be obtained from the joint distribution as follows:

$$\begin{aligned} p(\pi^*, h, \phi|\pi) &\propto p(\pi, \pi^*, h, \phi) \\ &= p(\pi, \pi^*, h|\phi) p(\phi) \\ &= p(\pi|\pi^*, h) p(\pi^*|h, \phi) p(h|\phi) p(\phi). \end{aligned} \quad (33)$$

<sup>5</sup> According to this parameterization of  $\omega$ ,  $\gamma_\omega(0) = (1 + \xi^2)\sigma_v^2$  and  $\gamma_\omega(1) = -\xi\sigma_v^2$ . Therefore,

$$\begin{aligned} \xi &= \frac{1 + \beta^2 + \psi - \sqrt{(1 + \beta^2 + \psi)^2 - 4\beta^2}}{2\beta} \\ \sigma_v^2 &= \frac{1 + \beta^2 + \psi + \sqrt{(1 + \beta^2 + \psi)^2 - 4\beta^2}}{2h}. \end{aligned}$$

Thus the local level model imposes a restriction between  $\xi$  and  $\beta$ . Consequently not all ARMA(1,1) processes can be expressed as a generalized local level model. Moreover,  $\beta \neq \xi$  as long as  $\beta = 0$ . Therefore the problem of local non-identification due to cancelation of common factors is absent.

The observation and state equations provide  $p(\pi|\pi^*, h)$  and  $p(\pi^*|h, \phi)$  respectively. To complete the model we must specify the priors  $p(h|\phi)$  and  $p(\phi)$ . We defer consideration of  $p(\phi)$  until later. Let the prior for  $h$  be independent of  $\phi$ :  $p(h|\phi) = p(h)$ , with  $p(h)$  given by the Gamma distribution:<sup>6</sup>

$$h \sim \text{G}(\underline{s}^{-2}, \underline{\nu}). \quad (34)$$

This prior for  $h$  delivers analytical expressions for the conditional posterior  $p(\pi^*, h|\pi, \phi)$  and for the marginal likelihood  $p(\pi|\phi)$ .

In order to derive the aforementioned analytical expressions for the conditional posterior and marginal likelihood, it is convenient to change the parametrization. As a preliminary, stack the observation equations as follows:

$$\pi = \pi^* + \eta. \quad (35)$$

Using  $\pi^* = W\theta$ , we can write (35) as

$$\pi = W\theta + \eta \quad (36)$$

where

$$\theta := \begin{bmatrix} \pi_1^* \\ \pi_2^* - \beta \pi_1^* \\ \vdots \\ \pi_T^* - \beta \pi_{T-1}^* \end{bmatrix} \quad (37)$$

and  $W$  is the  $T \times T$  matrix such that

$$W_{ij} = \begin{cases} \beta^{i-j} & i \geq j \\ 0 & \text{otherwise.} \end{cases} \quad (38)$$

(Note  $|W| = 1$ .) As long as we condition on  $\phi$ , we can treat  $\beta$ —and hence  $W$ —as known. The advantage of the new parametrization appears in the state-

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<sup>6</sup> We adopt the parametrization of the Gamma distribution given by Koop (2003, p. 326): If  $z \sim \text{G}(\delta, \nu)$ , then the density of  $z$  is given by

$$f_{\text{G}}(z|\delta, \nu) = \begin{cases} c_{\text{G}}^{-1} z^{\frac{\nu-2}{2}} \exp\left(\frac{-\nu z}{2\delta}\right) & 0 < z < \infty \\ 0 & \text{otherwise,} \end{cases}$$

where

$$c_{\text{G}} = \left(\frac{2\delta}{\nu}\right)^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)$$

is independent of  $z$  and  $\Gamma(\cdot)$  is the Gamma function. Note  $E[z] = \delta$  and  $\text{Var}[z] = 2\delta^2/\nu$ .

dynamics:

$$\theta_t = \begin{cases} \delta + \beta w + u_1 & t = 1 \\ \delta(1 - \beta) + u_t & t > 1. \end{cases} \quad (39)$$

The distributions for  $\pi$  and  $\theta$  implied by (36) and (39) are

$$\pi \mid (\theta, h, \phi) \sim \text{N}(W\theta, h^{-1}I_T) \quad (40a)$$

$$\theta \mid (h, \phi) \sim \text{N}(\underline{\theta}, h^{-1}\underline{V}), \quad (40b)$$

where

$$\underline{\theta} = \begin{bmatrix} \delta \\ \delta(1 - \beta) \\ \vdots \\ \delta(1 - \beta) \end{bmatrix} \quad \text{and} \quad \underline{V} = \begin{bmatrix} \beta^2 \lambda + \psi & 0_{T-1}^\top \\ 0_{T-1} & \psi I_{T-1} \end{bmatrix}. \quad (41)$$

Equation (36) can be interpreted as a normal linear regression model for which (40b) and (34) form a natural conjugate prior:

$$(\theta, h) \mid \phi \sim \text{NG}(\underline{\theta}, \underline{V}, \underline{s}^{-2}, \underline{\nu}). \quad (42)$$

Given the conjugacy of the prior (42) with respect to the likelihood (40a), the posterior inherits the form of the prior; namely,

$$(\theta, h) \mid (\pi, \phi) \sim \text{NG}(\bar{\theta}, \bar{V}, \bar{s}^{-2}, \bar{\nu}), \quad (43)$$

where<sup>7</sup>

$$\bar{\theta} = \bar{V}(\underline{V}^{-1}\underline{\theta} + W^\top y) \quad (44)$$

$$\bar{V} = (\underline{V}^{-1} + W^\top W)^{-1} \quad (45)$$

$$\bar{\nu} = \underline{\nu} + T \quad (46)$$

$$\bar{\nu} \bar{s}^2 = \underline{\nu} \underline{s}^2 + (y - W\bar{\theta})^\top (y - W\bar{\theta}) + (\bar{\theta} - \underline{\theta})^\top \underline{V}^{-1} (\bar{\theta} - \underline{\theta}). \quad (47)$$

The conditional posterior for  $\pi^*$  is given by

$$\begin{aligned} p(\pi^* \mid \pi, h, \phi) &= p(\theta \mid \pi, h, \phi) \big|_{\theta=W^{-1}\pi^*} \\ &= \text{N}(W^{-1}\pi^* \mid \bar{\theta}, h^{-1}\bar{V}) = \text{N}(\pi^* \mid \bar{\pi}^*, h^{-1}\bar{V}_{\pi^*}), \end{aligned} \quad (48)$$

where  $\bar{\pi}^* = W\bar{\theta}$  and  $\bar{V}_{\pi^*} = W\bar{V}W^\top$ . Thus we have established the posterior distribution for  $\pi^*$  and  $h$  conditional on  $\phi$ :

$$(\pi^*, h) \mid (\pi, \phi) \sim \text{NG}(\bar{\pi}^*, \bar{V}_{\pi^*}, \bar{s}^{-2}, \bar{\nu}). \quad (49)$$

<sup>7</sup> See Koop (2003, p. 187).

We now turn to the posterior for  $\phi$ . The marginal posterior for  $\phi$  can be expressed as

$$p(\phi|\pi) \propto p(\pi|\phi) p(\phi). \quad (50)$$

The marginal likelihood of  $\phi$  is<sup>8</sup>

$$p(\pi|\phi) = \iint p(\pi, \theta, h|\phi) d\theta dh = c \left( \frac{|\overline{V}|}{|\underline{V}|} \right)^{1/2} (\overline{\nu} \underline{s}^2)^{-\overline{\nu}/2}, \quad (51)$$

where

$$c = \frac{\Gamma(\overline{\nu}/2) (\underline{\nu} \underline{s}^2)^{\underline{\nu}/2}}{\Gamma(\underline{\nu}/2) \pi^{T/2}}. \quad (52)$$

(Note  $c$  is free of  $\phi$ .) The first equality in (51) is identically true while the second equality follows from the specific functional forms given in (34) and (40).

The parameter space for  $\phi = (\delta, \beta, \lambda, \psi)$  is

$$\Phi = (-\infty, \infty) \times (-1, 1] \times (0, \infty) \times (0, \infty). \quad (53)$$

We adopt the following prior:

$$p(\phi) = (2\pi)^{-1} e^{-(\frac{1}{2}\delta^2 + \lambda + \psi)}. \quad (54)$$

In addition, values for  $(\underline{s}^2, \underline{\nu})$  must be specified. We adopt a noninformative prior, setting  $\underline{\nu} = 0$  and setting  $\underline{s}^2$  to an arbitrary value (since  $\underline{s}^2$  enters the posterior only via the product  $\underline{\nu} \underline{s}^2$ ).

We can make draws of  $\phi$  from  $p(\phi|\pi)$  via the symmetric random-walk Metropolis MCMC algorithm. The algorithm produces a sequence of draws  $\{\phi^{(r)}\}_{r=1}^R$ . Given  $\phi^{(r)}$  one computes  $\phi^{(r+1)}$  as follows: Draw  $\phi' \sim N(\phi^{(r)}, \Omega)$  and  $u \sim U(0, 1)$ , and then set

$$\phi^{(r+1)} = \begin{cases} \phi' & \frac{p(\phi'|\pi)}{p(\phi^{(r)}|\pi)} \geq u \\ \phi^{(r)} & \text{otherwise.} \end{cases} \quad (55)$$

Equation (55) shows that if the proposal  $\phi'$  is “uphill” from the current point  $\phi^{(r)}$  in the sense that  $p(\phi'|\pi) \geq p(\phi^{(r)}|\pi)$ , then it is always accepted (i.e., added to the output sequence); by contrast, if the proposal is “downhill” from the current point, then it is accepted with a probability that is proportional to the likelihood ratio. (If the proposal is out of bounds, i.e. if  $\phi' \notin \Phi$ , then  $p(\phi'|\pi) = 0$  and the proposal is never accepted.) Note that if the proposal is not accepted, then the current point  $\phi^{(r)}$  is placed again in the output sequence, which (among other things) induces serial correlation in the sequence of draws. To make the algorithm operational, one chooses a starting value  $\phi^{(0)} \in \Phi$  and the

<sup>8</sup> See Koop (2003, p. 189)

covariance matrix  $\Omega$ . (The covariance matrix provides a scale for the random-walk step size.) One must also specify the number of burn-in draws to discard and the amount of thinning to do (if any) after the burn-in period.<sup>9</sup>

## 6 Our Estimates

We have a dataset with 166 countries, each of which has continuous annual data for twelve or more years. The number of observations on inflation in each country ranges from twelve to 21. While it would be even better to compare across monetary regimes within a country, the paucity of observations implied by such a strategy leads us to examine the data across countries.

### 6.1 Estimates of the Serial Correlation

Let  $\phi_i$  denote the parameters for the  $i$ -th country. The parameter of interest is  $\{\beta_i\}$ . We make draws of  $\phi_i$  from  $p(\phi_i|\pi_i)$  via the symmetric random-walk Metropolis MCMC algorithm. For each country we make two runs, using the results of the first to calibrate the second. For the first run, we adopt a starting value of  $\phi_i^{(0)} = (0, .5, 1, 1)$  and we use  $\Omega = \text{diag}(10^{-4}, 10^{-2}, 10^{-2}, 10^{-2})$  as the covariance matrix for the scale of the step size. We make  $10^4$  burn-in draws and then make  $10^5$  draws, keeping every one in  $10^2$ . Next we compute the mean and covariance of the  $10^3$  draws produced by the first run. For the second run, we set  $\phi_i^{(0)}$  to the computed mean (from the first run) and we set  $\Omega$  to .2 times the computed covariance matrix. We make  $10^4$  burn-in draws for the second run and then make  $10^5$  draws, keeping every one in  $10^2$ . This produces a total of  $10^3$  draws for each country to approximate the posterior distribution of  $\phi_i$ . The draws are not independent and the average first-order autocorrelation across all parameters and all countries is .13. There are only four countries for which the maximum autocorrelation for any of the four parameters in  $\phi_i$  is above .5. An approximation for the effective number of independent draws is given by

$$\tilde{n} \approx R \frac{1 - \rho}{1 + \rho}, \quad (56)$$

where  $R$  is the number of draws and  $\rho$  is the first-order autocorrelation. In our case  $R = 10^3$ . For  $\beta$ ,  $\tilde{n}$  averages about 790 and is not less than 340.

The posterior means and the 90% highest posterior density regions for  $\beta_i$  are shown in Figure 5. The point estimates for all but 12 countries are positive and the regions do not include zero for 96 of the 166 countries.

<sup>9</sup> Koop (2003, Section 5.5) provides a detailed summary of the Metropolis MCMC.

[Insert Figure 5 about here]

## 6.2 Relation to Correlations

We are interested in the relation across countries between the posterior means of the autoregressive coefficients and the correlations of the inflation rate and the growth rate of excess money. Let  $\bar{\beta}_i$  denote the posterior mean of  $\beta_i$  and  $\bar{\rho}^{is}$  denote the sample correlation between the inflation rate and the excess money growth rate for country  $i$  over  $s$  periods. We summarize the relationship by the correlation between  $\bar{\beta} = (\bar{\beta}_1, \dots, \bar{\beta}_n)$  and  $\bar{\rho}^s = (\bar{\rho}^{1s}, \dots, \bar{\rho}^{ns})$  by the simple correlation  $r_s$ .

Figure 6 shows scatterplots for  $\bar{\rho}^{is}$  versus  $\bar{\beta}_i$  for  $s \in \{1, 3, 5\}$ . The top row shows plots for all 166 countries, while the bottom row shows plots for only those 119 countries for which we have at least 20 observations. Twenty observations would be considered a small sample for estimating a serial correlation coefficient in most contexts, but requiring 20 observations eliminates over a quarter of these countries.

[Insert Figure 6 about here]

To estimate the posterior distribution of  $r_s$ , we apply the Bayesian bootstrap to  $r_s$ , producing  $\{r_s^{(m)}\}_{m=1}^M$ . The Bayesian bootstrap works as follows. Make a draw  $w^{(m)}$  from the flat Dirichlet distribution and compute  $r_s^{(m)}$  using  $w^{(m)}$  as probabilities:

$$r_s^{(m)} = \frac{c^{(m)}}{\sqrt{v_\beta^{(m)} v_{\rho_s}^{(m)}}} \quad (57)$$

from

$$\begin{aligned} m_\beta^{(m)} &= \sum_{i=1}^n w_i^{(m)} \bar{\beta}_i & v_\beta^{(m)} &= \sum_{i=1}^n w_i^{(m)} (\bar{\beta}_i - m_\beta^{(m)})^2 \\ m_{\rho_s}^{(m)} &= \sum_{i=1}^n w_i^{(m)} \bar{\rho}^{is} & v_{\rho_s}^{(m)} &= \sum_{i=1}^n w_i^{(m)} (\bar{\rho}^{is} - m_{\rho_s}^{(m)})^2 \\ & & c^{(m)} &= \sum_{t=1}^n w_t^{(m)} (\bar{\beta}_t - m_\beta^{(m)}) (\bar{\rho}^{ts} - m_{\rho_s}^{(m)}). \end{aligned}$$

Figure 7 shows the estimated posterior distributions of the correlations between the serial correlation coefficients and the correlations of inflation and money growth different time spans. All of the  $r_s$  are positive, indicating that the posterior means of the serial correlation coefficients and the correlation of money growth and inflation clearly are positive.

[Insert Figure 7 about here]

Next we compare  $r_1$  with  $r_3$  and  $r_5$ . In particular, we compute  $d_3^{(m)} := r_3^{(m)} - r_1^{(m)}$  and  $d_5^{(m)} := r_5^{(m)} - r_1^{(m)}$ . The results are shown in Figure 8. We find the fraction of  $d_3^{(m)}$  that is positive is about .91 while the fraction of  $d_5^{(m)}$  that is positive is about .66. If we use only countries that have at least 20 observations, then the fractions increase to about .99 and .93, respectively.

These distributions are evidence in favor of the proposition that an increase in the serial correlation coefficient leads to an increase in the correlation of money growth and inflation.

[Insert Figure 8 about here]

## 7 Conclusion

The relationship between inflation and excess money growth still is controversial. We find a positive correlation across all countries. The correlation falls as countries with higher excess money growth are excluded, but the correlation is 0.47 across countries with excess money growth of ten percent or less. We show that the lower correlation for low inflation countries is not surprising if low inflation countries have lower variation in unpredictable changes in the supply of money relative to unpredictable changes in the demand for money.

We also show that the size of a regression coefficient of inflation on excess money growth is uninformative about the quantity theory. If errors in the supply of money are uncorrelated with errors in the demand for money, then a regression of inflation on the growth rate of money will have a slope coefficient of unity. On the other hand, if this correlation is not zero, as it is with explicit or implicit inflation targeting, the nominal quantity of money is endogenous and a regression of inflation on money growth will not deliver a coefficient of unity. This is perfectly consistent with the quantity theory holding. While regression coefficients equal to unity may seem like a plausible way to evaluate the quantity theory, the quantity theory is consistent with coefficients less than one as well as equal to one.

Higher correlations between money growth and inflation when data are averaged over time is consistent with this same analysis. We show that positive serial correlation of the underlying inflation rate is consistent with higher correlations of excess money growth with inflation as the growth rates are computed over longer time periods. We also show that monetary regimes with more sustained deviations of inflation from its mean will show greater increases in the correlation of excess money growth and inflation as the growth rates are computed over longer time periods.

We then test these implications. We find substantial variation in serial correlation of underlying inflation rates and we find this variation is positively related to the increase in correlations as data are averaged over longer periods.

Our results indicate that sustained excess money growth is positively correlated with inflation. The greater appearance of that relationship when data are averaged over time and when countries with sustained deviations of inflation from its mean inflation are quite consistent with the quantity theory holding.

## 8 Data Appendix

We analyze annual data for the United States and for 182 countries. The data across countries include available data for 1985 and subsequent years. These data are from the *World Development Indicators* website, the March 2008 CD for *International Financial Statistics*, and from Haver. Haver is the source of the data on Taiwan. The nominal and real Gross Domestic Product (GDP) data primarily are from *World Development Indicators*. These data are supplemented by data from *International Financial Statistics* when these *IFS* data are more complete or consistent.<sup>10</sup> We use the data on money plus quasimoney from *IFS* except when the *WDI* data cover a longer period or have more significant digits.<sup>11</sup> The price index is the Gross Domestic Product deflator and nominal income is GDP. Table A1 in the Appendix lists all the countries and the periods over which we have GDP and money data.

Data for some individual countries are from Haver or country-specific sources. Data for Taiwan are from Haver because these data are not available in either *WDI* or *IFS*.

Inspection of some series suggested discontinuities in the underlying data from *WDI* and *IFS*. As it turned out, all of the issues concerned the nominal quantity of money. When collecting data from an individual central bank's website, we collected the monetary series emphasized by the central bank. The nominal quantity of money for Belgium is M3 from the National Bank of Belgium's website. The quantity of money for Canada is M2 from Haver. The quantity of money for Japan is M2 including certificates of deposit from the Bank of Japan's website. Earlier and later series are spliced by the average monthly ratio of 0.995519 in the overlapping period April 1998 to March 1999. The quantity of money for New Zealand is M3 from the Bank of New Zealand. The growth rate of the nominal quantity of money for Macedonia is M2 from the Central Bank. The nominal quantity of money for the United Kingdom is M4 from the Bank of England. All data for the United States are from the Federal Reserve Bank of St. Louis's website and the nominal quantity of money is M2.

The long-term data for Brazil and Chile are from Rolnick and Weber (1997)

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<sup>10</sup> We use some or all *IFS* data for Anguilla, Aruba, Barbados, Cambodia, Cape Verde, Fiji, Kuwait, Libya, the Maldives, Montserrat, Qatar and San Marino.

<sup>11</sup> The *WDI* data available to us often contain more significant digits when there are large changes in the quantity of money. We use *WDI* data for Argentina, Bolivia, Brazil, Chile, the Democratic Republic of the Congo, Cyprus, Ethiopia, Ghana, Guinea, Lao People's Democratic Republic, Lebanon, Malta, Nicaragua, Peru, Romania, Samoa, San Marino, Slovenia, Sudan, Turkey, Turkmenistan, Uganda, Uruguay and Zimbabwe.

updated by *World Development Indicators*.<sup>12</sup> The monetary variable from Rolnick and Weber's data is their M2. The data are updated by spliced data from the *World Development Indicators* for 1986 through 2006.

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<sup>12</sup>Rolnick and Weber (1995) provide the data sources in their Data Appendix. We thank Warren Weber for providing the data.

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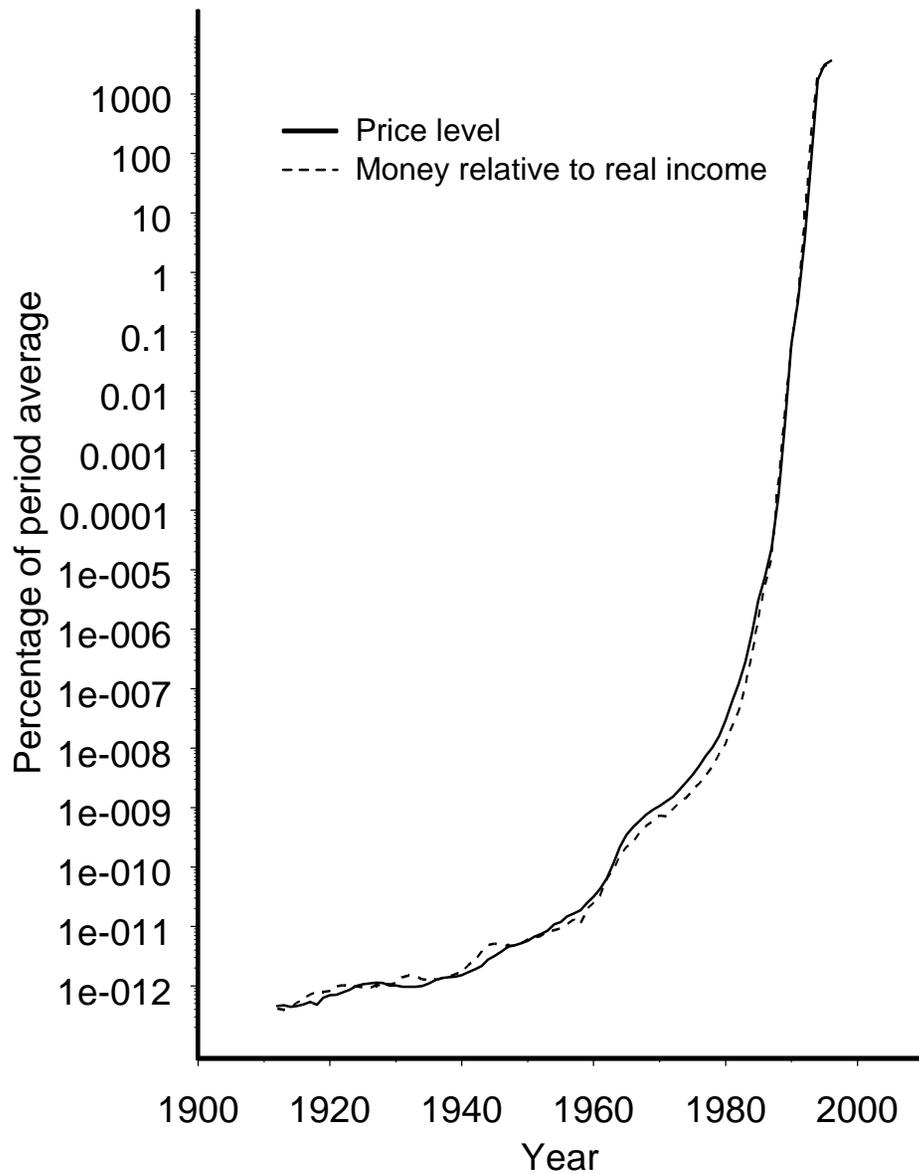
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Figure 1  
Money and Prices in Brazil and Chile

Brazil 1912 to 2006



Chile 1940 to 2006

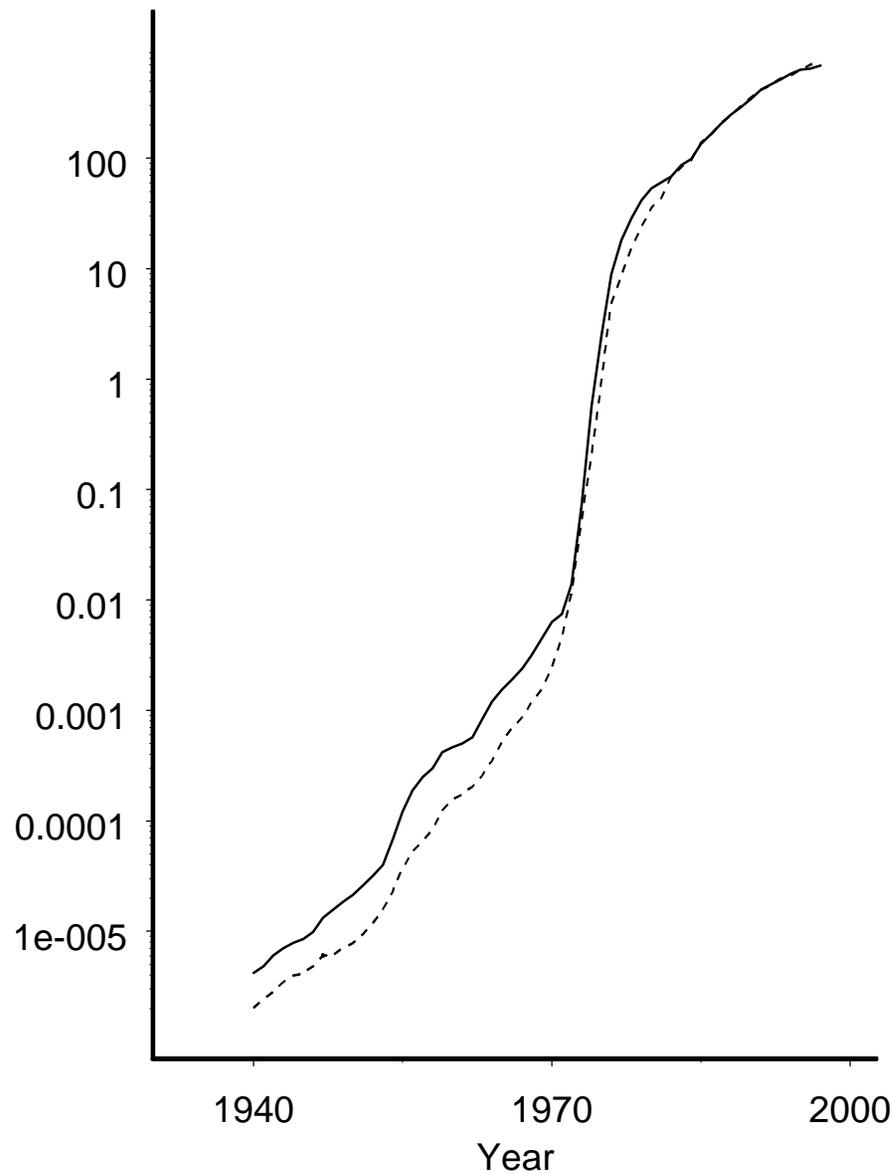
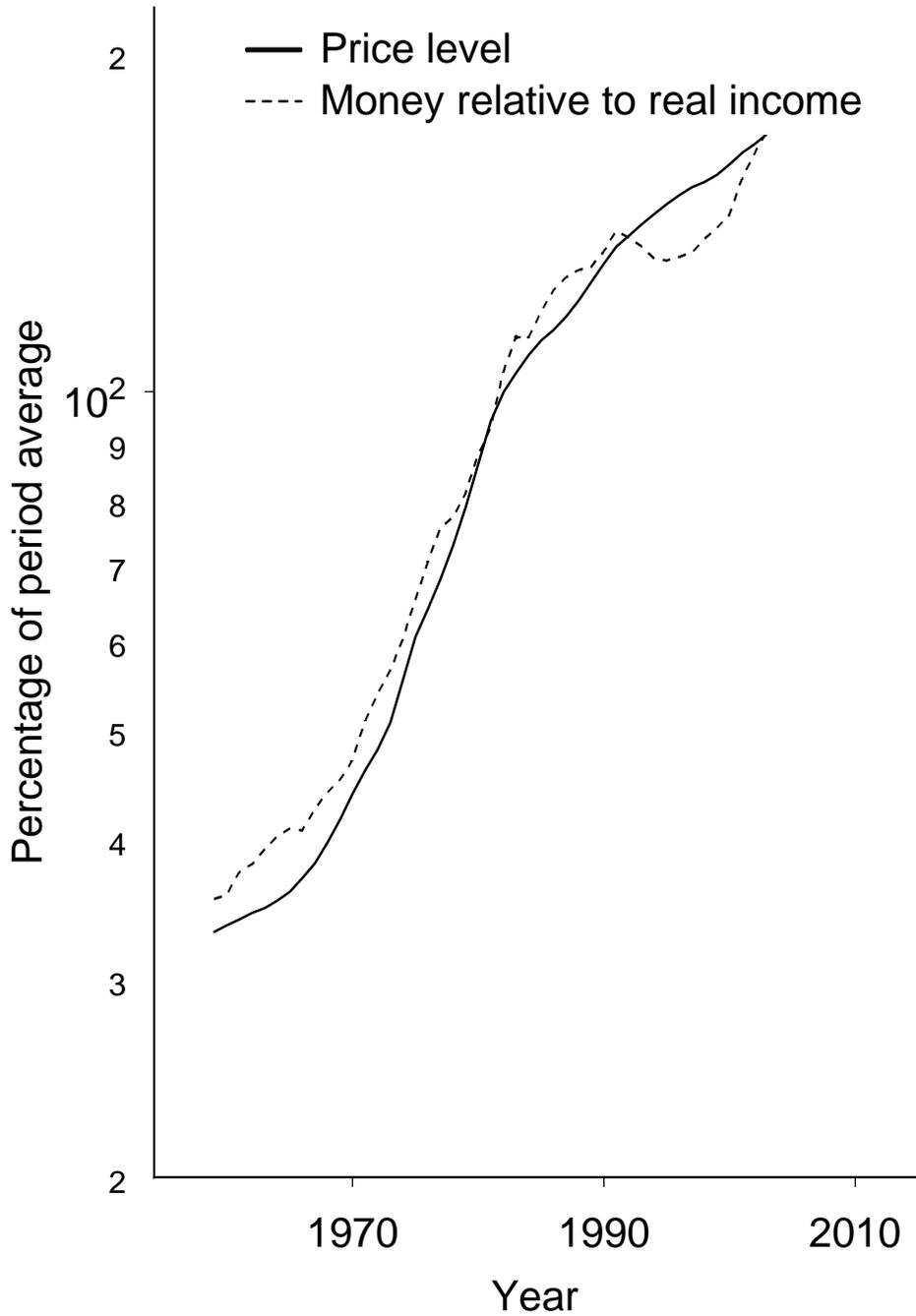


Figure 2  
Money and Prices in the U.S. and Japan

United States 1959 to 2007



Japan 1967 to 2007

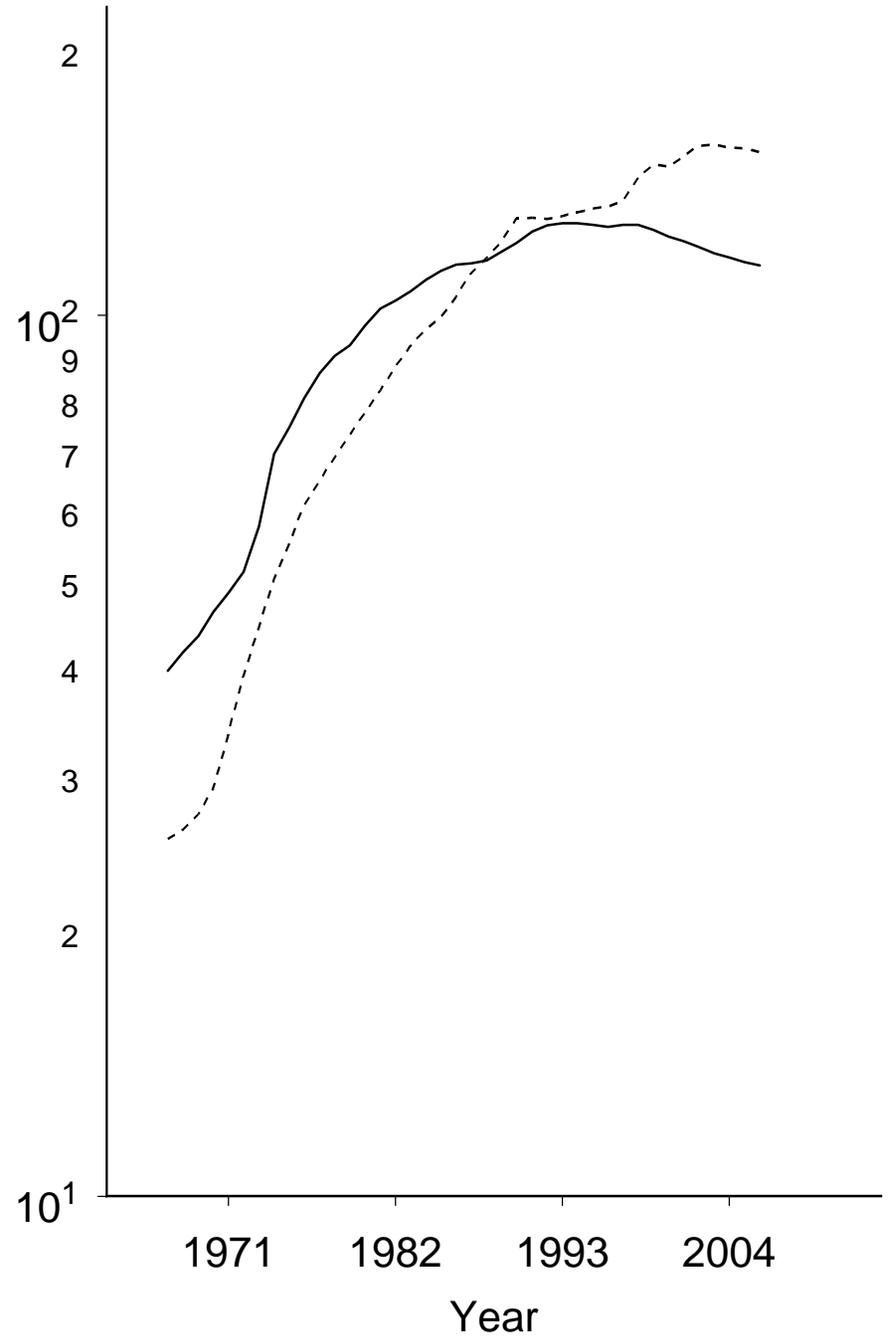


Figure 3

Inflation and Excess Money Growth for Lower Growth Rates of Excess Money

Note: The slope indicated in the figure is the slope of the regression line. The solid line in the figure is the regression line. The dotted line is a regression from the origin with a slope of one.

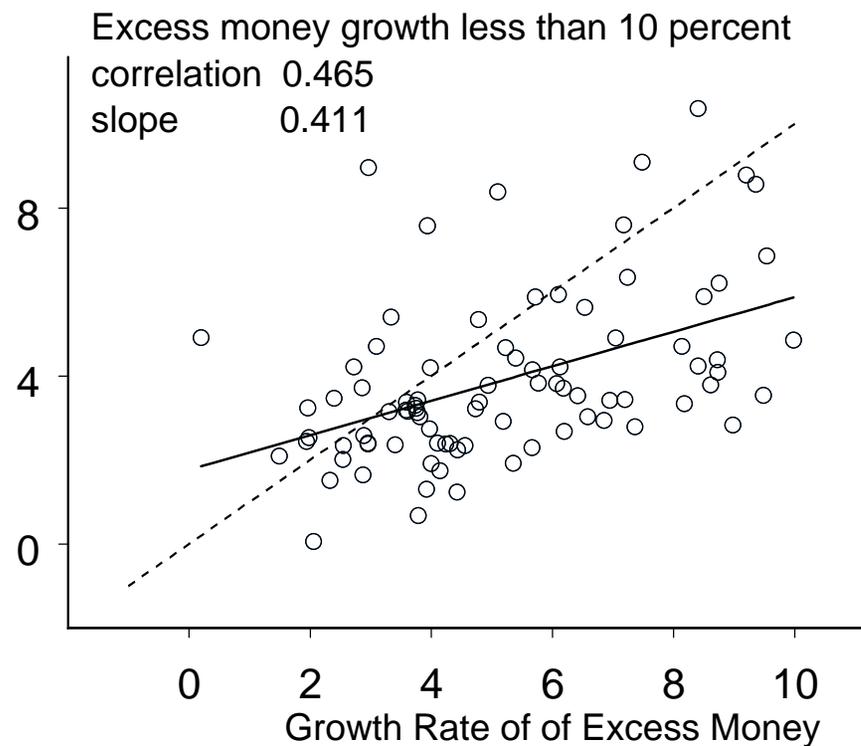
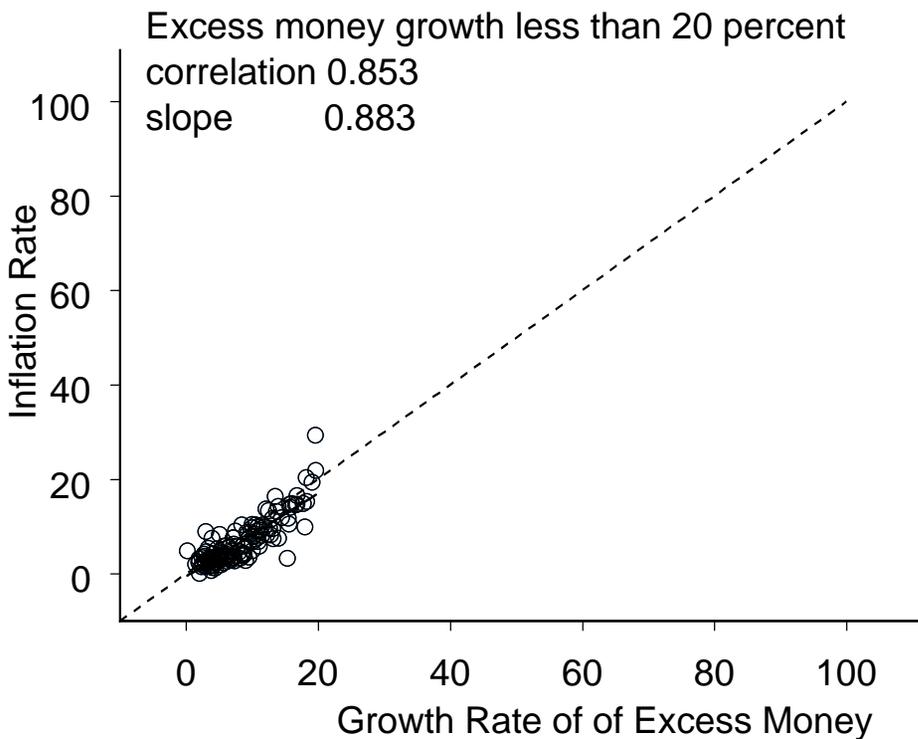
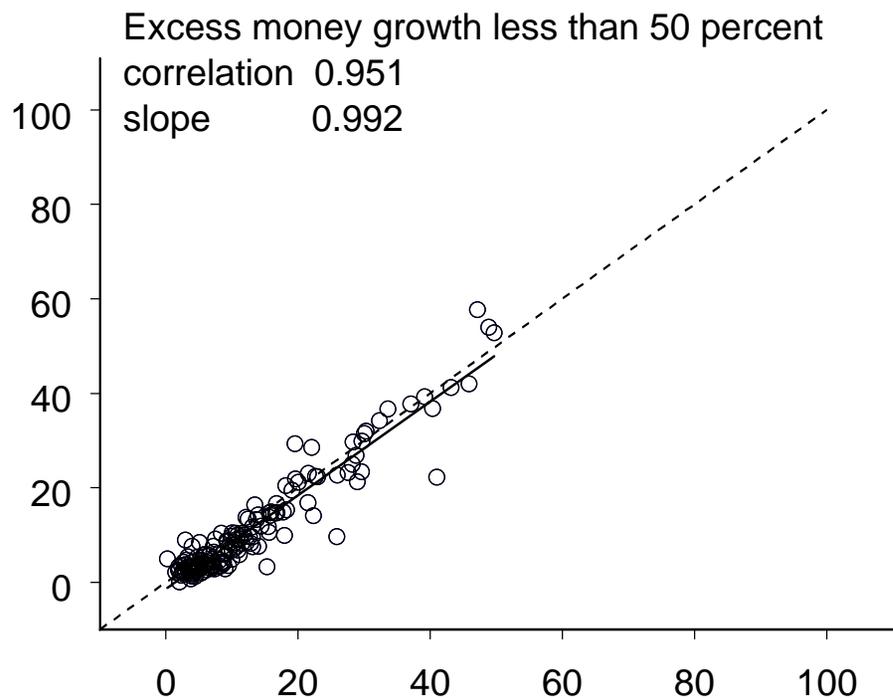
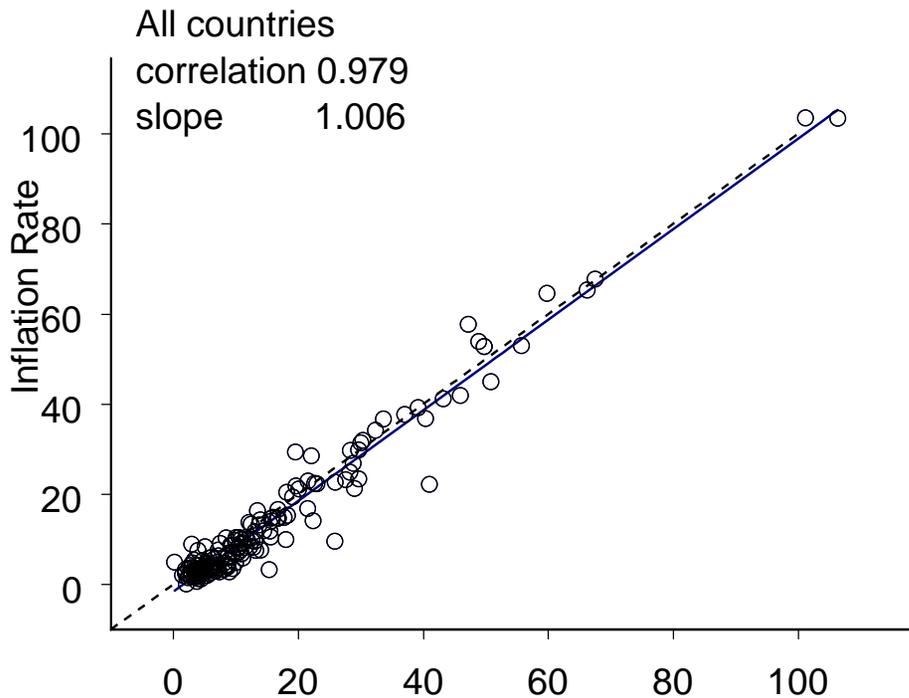


Figure 4

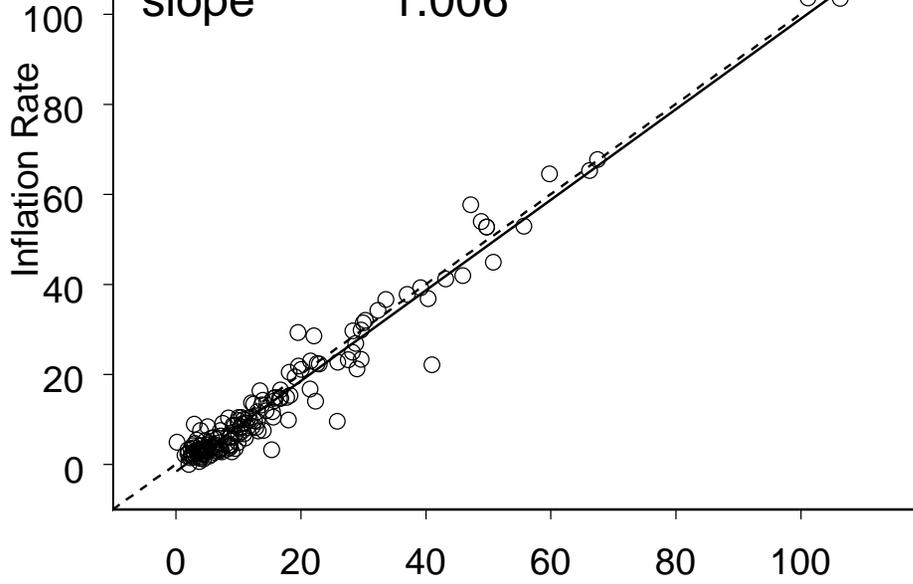
Inflation and Excess Money Growth over Shorter Time Periods

Note: The slope indicated in the figure is the slope of the regression line. The solid line in the figure is the regression line. The dotted line is a regression from the origin with a slope of one.

All years

correlation 0.979

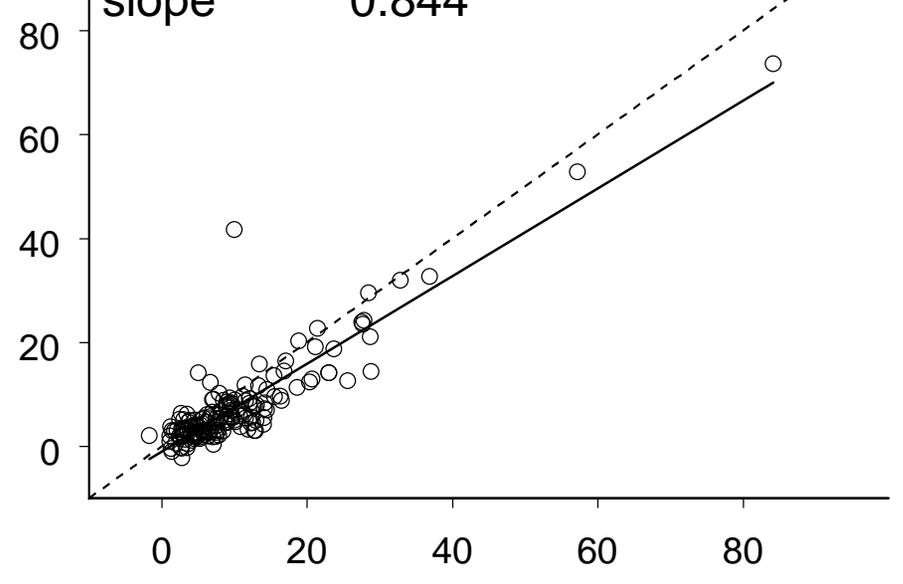
slope 1.006



Ten years

correlation 0.972

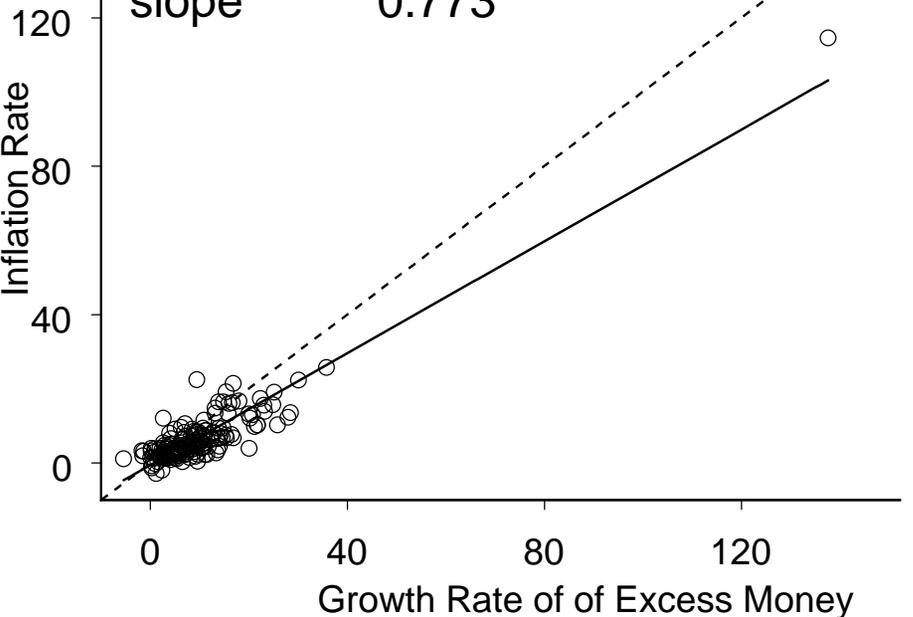
slope 0.844



Five years

correlation 0.916

slope 0.773



One year

correlation 0.807

slope 0.460

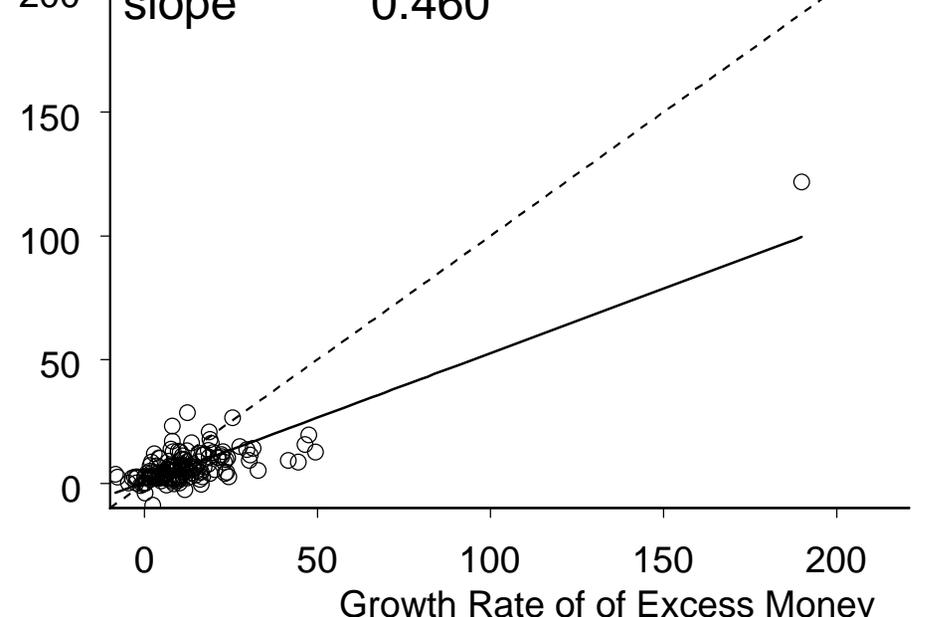


Figure 5. Posterior means and 90% highest posterior density regions for  $\beta_i$  for 166 countries (sorted by mean).

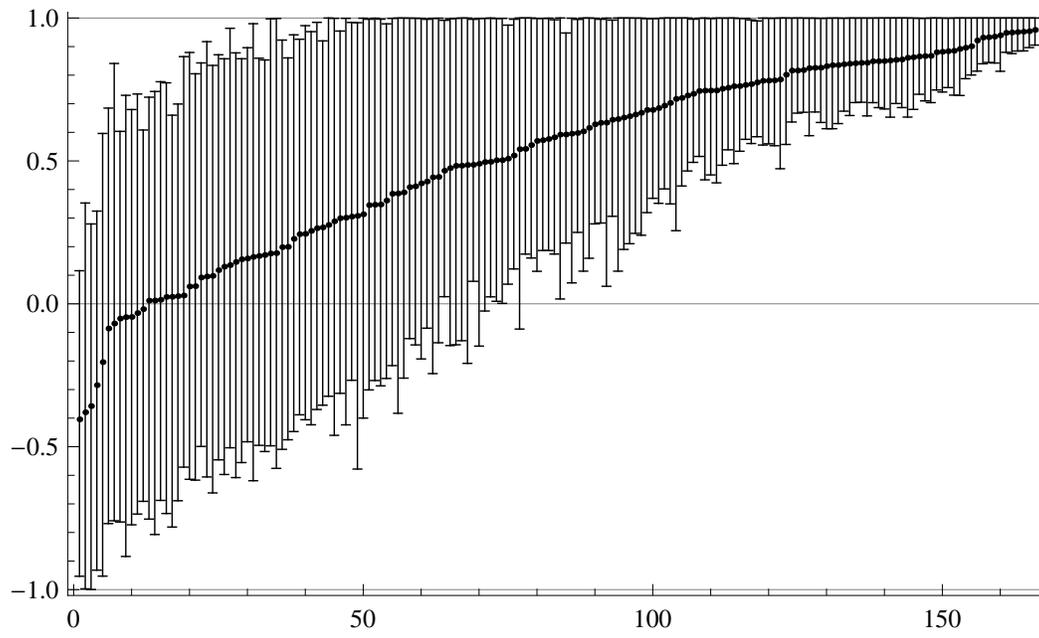
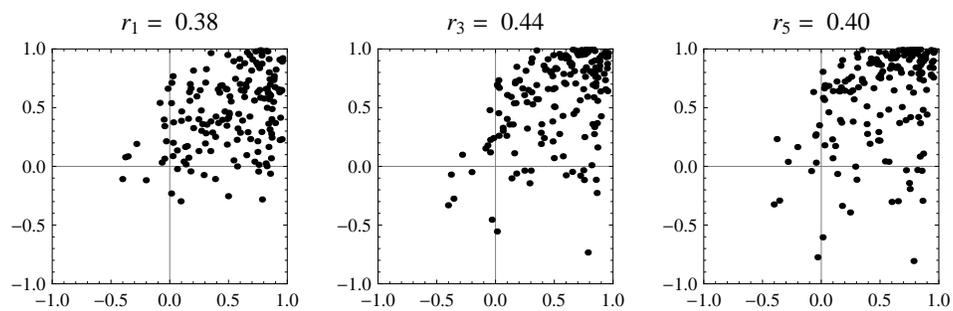


Figure 6. Scatterplots of  $\bar{\rho}^{is}$  versus  $\bar{\beta}_i$  for  $s \in \{1, 3, 5\}$ . The sample correlations  $r_s$  are shown.

All 166 Countries



119 countries with at least 20 observations

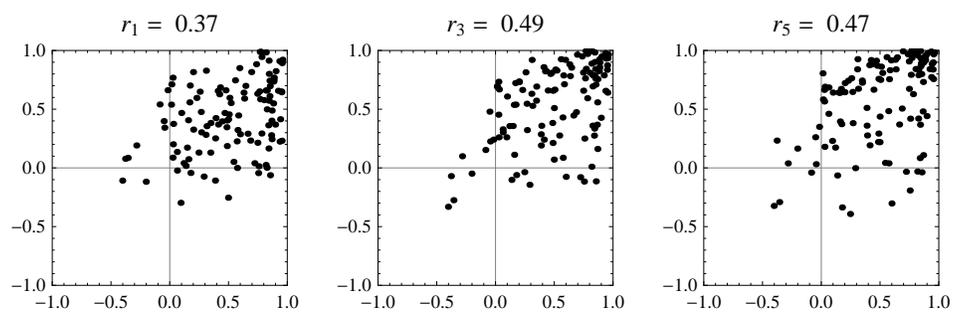


Figure 7. Bayesian bootstrap distributions of  $r_s$ .

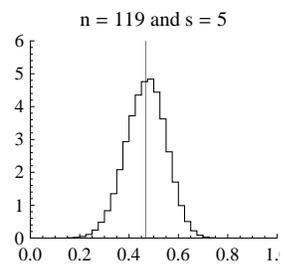
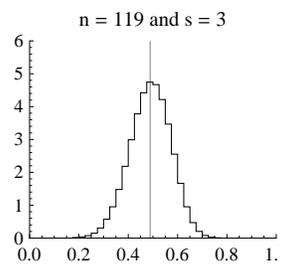
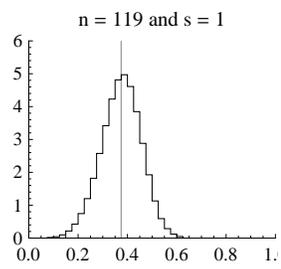
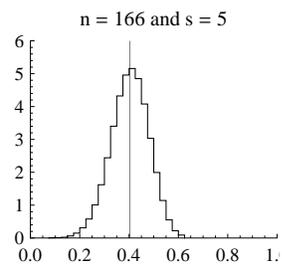
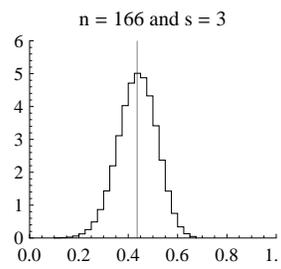
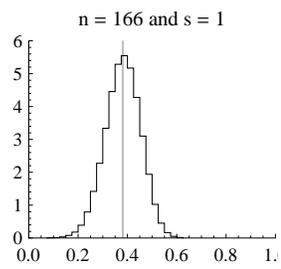


Figure 8. Differences  $d_3$  (thick) and  $d_5$ .

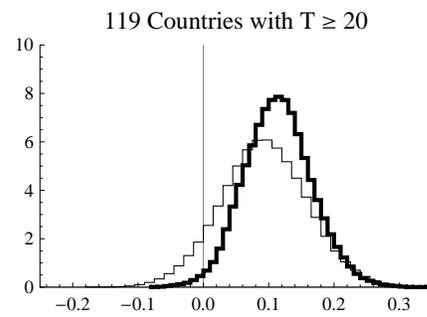
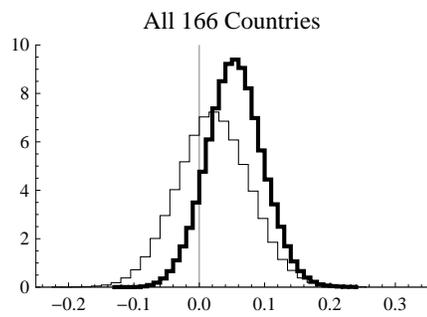


Table 1  
Countries and Years

Country	First Year	Last Year	Country	First Year	Last Year
Albania	1994	2006	Brazil	1985	2006
Algeria	1985	2006	Bulgaria	1991	2006
Anguilla	1990	2005	Burkina Faso	1985	2006
Antigua and Barbuda	1985	2006	Burundi	1985	2006
Argentina	1985	2006	Cambodia	1993	2006
Armenia	1992	2006	Cameroon	1985	2006
Aruba	1991	2004	Canada	1985	2006
Australia	1985	2006	Cape Verde	1985	2006
Austria	1985	1997	Central African Republic	1985	2006
Azerbaijan	1992	2006	Chad	1985	2006
Bahamas, The	1985	2002	Chile	1985	2006
Bahrain	1985	2005	China	1985	2006
Bangladesh	1985	2006	Colombia	1990	2006
Barbados	1985	2004	Comoros	1985	2006
Belarus	1994	2006	Costa Rica	1985	2006
Belgium	1985	1998	Cote d'Ivoire	1985	2006
Belize	1985	2006	Croatia	1993	2006
Benin	1985	2006	Cyprus	1985	2006
Bhutan	1985	2006	Czech Republic	1993	2006
Bolivia	1985	2006	Denmark	1985	2006
Botswana	1985	2006	Djibouti	1990	2006
Dominica	1985	2006	Haiti	1985	2006
Dominican Republic	1985	2006	Honduras	1985	2006
Ecuador	1985	2006	Hong Kong	1991	2006
Egypt	1985	2006	Hungary	1985	2006
El Salvador	1985	2006	Iceland	1985	2006
Equatorial Guinea	1985	2006	India	1985	2006
Estonia	1991	2006	Indonesia	1985	2006

Country	First Year	Last Year	Country	First Year	Last Year
Ethiopia	1985	2006	Iran	1986	2006
Euro Area	1995	2007	Israel	1985	2006
Fiji	1985	2006	Italy	1985	1997
Finland	1985	1997	Jamaica	1985	2006
France	1985	1997	Japan	1985	2006
Gabon	1985	2006	Jordan	1985	2006
Gambia, The	1985	2006	Kazakhstan	1993	2006
Germany	1985	1997	Kenya	1985	2006
Ghana	1985	2006	Kuwait	1992	2006
Grenada	1985	2006	Laos	1987	2006
Guatemala	1985	2006	Latvia	1993	2006
Guinea	1989	2005	Lebanon	1988	2006
Guinea-Bissau	1986	2006	Lesotho	1985	2006
Guyana	1985	2006	Liberia	1991	2006
Libya	1985	2006	Netherlands	1985	1997
Lithuania	1993	2006	New Zealand	1985	2006
Macao	1985	2006	Nicaragua	1985	2006
Macedonia	1992	2006	Niger	1985	2006
Madagascar	1985	2006	Nigeria	1985	2005
Malawi	1985	2006	Norway	1985	2003
Malaysia	1985	2006	Oman	1985	2005
Maldives	1985	2006	Pakistan	1985	2006
Mali	1985	2006	Panama	1985	2006
Malta	1985	2006	Papua New Guinea	1985	2006
Mauritania	1985	2003	Paraguay	1985	2006
Mauritius	1985	2006	Peru	1985	2006
Mexico	1985	2006	Philippines	1985	2006
Moldova	1991	2006	Poland	1990	2006
Mongolia	1991	2006	Portugal	1985	1997

Country	First Year	Last Year	Country	First Year	Last Year
Montserrat	1985	2005	Qatar	1985	2006
Morocco	1985	2006	Republic of the Congo	1985	2006
Mozambique	1988	2006	Romania	1985	2006
Myanmar	1985	2005	Russia	1993	2006
Namibia	1990	2006	Rwanda	1985	2005
Nepal	1985	2006	Saint Kitts and Nevis	1985	2006
Saint Lucia	1985	2006	Taiwan	1985	2007
Saint Vincent and the Grenadines	1985	2005	Tanzania	1988	2006
Samoa	1985	2006	Thailand	1985	2006
Saudi Arabia	1985	2006	Togo	1985	2006
Senegal	1985	2006	Tonga	1985	2006
Seychelles	1985	2006	Trinidad and Tobago	1985	2006
Sierra Leone	1985	2006	Tunisia	1985	2006
Singapore	1985	2006	Turkey	1985	2006
Slovakia	1993	2006	Uganda	1985	2006
Slovenia	1991	2006	Ukraine	1992	2006
Solomon Islands	1985	2006	United Arab Emirates	1985	2005
South Africa	1985	2006	United Kingdom	1985	2006
South Korea	1985	2006	United States	1985	2006
Spain	1985	1997	Uruguay	1985	2006
Sri Lanka	1985	2005	Vanuatu	1985	2006
Sudan	1985	2006	Venezuela	1985	2006
Suriname	1985	2006	Yemen	1990	2006
Swaziland	1985	2006	Zambia	1993	2006
Sweden	1985	2006	Zimbabwe	1985	2005
Switzerland	1985	2006			
Syria	1985	2006			