

Some Unpleasant Properties of Loglinearized Solutions When the Nominal Rate Is Zero

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Abstract: Does fiscal policy have large and qualitatively different effects on the economy when the nominal interest rate is zero? An emerging consensus in the New Keynesian (NK) literature is that the answer to this question is yes. Evidence presented here suggests that the NK model's implications for fiscal policy at the zero bound may not be all that different from its implications for policy away from it. For a range of empirically relevant parameterizations, employment increases when the labor tax rate is cut and the government purchase multiplier is less than 1.05.

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1. Introduction

The recent experiences of Japan, the United States and Europe with zero/near-zero nominal interest rates have raised new questions about the conduct of monetary and fiscal policy in a liquidity trap. A large and growing body of new research has emerged that provides answers using New Keynesian (NK) frameworks that explicitly model the zero bound on the nominal interest rate. Modeling the zero bound on the nominal interest rate is particularly important in the NK model because the interest rate policy of the monetary authority plays a central role in stabilizing the economy. Very low nominal interest rates constrain the ability of monetary policy to respond to shocks and this may result in macroeconomic instability.

Recent research has found that fiscal policy has very different effects on the economy when the nominal interest rate is zero. Eggertsson (2011) finds that employment falls in response to a cut in the labor tax rate, a property that he refers to as the “paradox of toil.” Christiano et al. (2011) and Woodford (2011) conclude that the size of the government purchase multiplier is close to two or even larger. These results have sharp implications for the conduct of fiscal policy in low interest rate environments. If supply-side stimulus is contractionary and demand-side fiscal policies are particularly potent then governments should rely exclusively on demand side fiscal stimulus when the central bank’s actions are constrained by the zero lower bound.

This paper proposes and solves a tractable stochastic nonlinear NK model that honors the zero bound on the nominal interest rate and that also reproduces the large output and small inflation declines observed during the U.S. Great Recession. We encounter some parameterizations of the model that are consistent with previous results. However, the novel contribution of our paper is that we find other empirically relevant parameterizations of the model where the government purchase multiplier is about one or less and the response of employment to a cut in the labor tax rate is positive.

These new findings are important because they raise the possibility that there might

also be a role for using supply side policies to stabilize the economy in low interest rate environments. On the one hand, the case for demand side measures is weaker since their efficacy is small. On the other hand, the case for supply-side measures is stronger because they are expansionary.

Why are the results presented here different from previous findings? One reason is the solution method. Previous results are based on a solution method that models the nonlinearity induced by the zero bound on the nominal interest rate but loglinearizes the other equilibrium conditions about a zero inflation steady-state. This solution method zeroes out the resource costs of price adjustment which affects the local dynamics of the model at the zero bound. A comparison of loglinear (LL) and nonlinear (NL) solutions reveals that the LL solution sometimes incorrectly predicts that supply side stimulus is contractionary when in fact it is expansionary.

A second and distinct reason for our findings is the parameterization of the model. The GR was associated with a 7% decline in GDP but only a 1% decline in the annualized inflation rate (see Christiano et al. 2011). We calibrate the model to these targets and this has implications for the size of the government purchase multiplier. Intuitively, government purchases are primarily a demand shifter and reproducing the GR targets results in a relatively flat aggregate supply schedule. At the zero bound, the government purchase multiplier can still be large in this situation. Indeed, Woodford (2011) has found that the government purchase multiplier can be arbitrarily large in the neighborhood of a point that can be indexed by the expected duration of zero interest rates.¹ This region of the parameter space is small under our calibration scheme. If the expected duration of zero interest rates exceeds 7 quarters or is less than 5 quarters, the government purchase multiplier is small using either the LL or the NL equilibrium conditions.

These points are made in a nonlinear stochastic NK model with Rotemberg (1982)

¹Intuitively, the government purchase multiplier is large because the aggregate demand and aggregate supply schedules are nearly parallel.

quadratic price adjustment costs. Rotemberg adjustment costs are widely used when studying the zero lower bound (Benhabib et al. 2001, Evans et al. 2008, Aruoba and Schorfheide 2013, Eggertsson et al. 2013) because the dimension of the state space is small. In our setup, output and inflation in the zero lower bound state solve a system of two nonlinear equations, which are the nonlinear analogues of what Eggertsson and Krugman (2012) refer to as “aggregate demand” and “aggregate supply” schedules. Some merits of this approach are that we can provide a graphical representation of the NL equilibrium conditions, an analytical characterization of the model’s key properties and an easy and accurate strategy for computing all equilibria. This final merit is important because we encounter multiple zero bound equilibria. LL solution methods, in contrast, have the property that aggregate supply and aggregate demand have a single crossing point at the zero bound.

Many NK models use Calvo price adjustment instead. The standard LL solution method also zeroes out the resource costs of price dispersion in Calvo models of price adjustment. Omitting this term can also create similar biases under Calvo price adjustment. Section 5.3 and the Online Appendix provide evidence that this is the case using a tractable (but stylized) model of Calvo price adjustment. In this sense, our findings are likely to apply to a large class of NK models.

Our research is closest to research by Christiano and Eichenbaum (2012) who consider related questions in a similar model. They show that imposing a particular form of E-learning rules out one of the two equilibria that occur in their model, and find that the qualitative properties of the remaining equilibrium are close to the LL solution. Our main conclusions about the size and sign of fiscal multipliers do not rely on multiplicity of zero bound equilibrium. In fact, some of our most interesting results occur in regions of the parameter space where equilibrium is unique and the aggregate demand and aggregate supply schedules have their conventional slopes. We also describe some problems with applying their E-learning equilibrium selection strategy. It does not omit all forms of multiplicity and sometimes selects equilibria that are not empirically relevant while ruling

out other equilibria that reproduce observations from the GR.

Our research is also related to recent work by Mertens and Ravn (2010) who consider zero bound sunspot equilibria. A major advantage of our setup is that it is straightforward to find all equilibria by finding the zeros of an equation. We encounter new cases of multiplicity, most significantly the possibility of multiple zero bound equilibria. Ascertaining the presence of multiple zero bound equilibria is a daunting task in richer NK models such as those considered by Gust et al. (2012), Aruoba and Schorfheide (2013) or Fernandez-Villaverde et al. (2012). Results presented here offer guidance about the regions of the parameter/shock space where multiplicity is most likely to arise in medium-scale NK models.

The remainder of our analysis proceeds in the following way. Section 2 describes the model and equilibrium concept. Section 3 explains how the model is parameterized. Section 4 characterizes equilibrium using the NL and the LL equilibrium restrictions. Section 5 documents that fiscal multipliers may be small and orthodox at the zero lower bound. Finally, Section 6 concludes.

2. Model and equilibrium

We consider a stochastic NK model with Rotemberg (1982) quadratic costs of price adjustment faced by intermediate goods producers. Monetary policy follows a Taylor rule when the nominal interest rate is positive but is restricted from falling below zero. The equilibrium analyzed here is the Markov equilibrium proposed by Eggertsson and Woodford (2003).

2.1. The model

Households. The representative household chooses consumption c_t , labor supply h_t , and bond holdings b_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^t d_j \right) \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\nu}}{1+\nu} \right\} \quad (1)$$

subject to the budget constraint

$$b_t + c_t = \frac{b_{t-1}(1 + R_{t-1})}{1 + \pi_t} + (1 - \tau_{w,t})w_t h_t + T_t. \quad (2)$$

where ν governs the elasticity of labor supply and σ is the curvature parameter for consumption.² R_t and π_t are the net nominal interest rate and the net inflation rate, respectively, and the after-tax real wage is $(1 - \tau_{w,t})w_t$. The preference discount factor from period t to $t + 1$ is βd_{t+1} , and d_t is a preference shock. We assume that the value of d_{t+1} is revealed at the beginning of period t . The variable T_t includes transfers from the government and profit distributions from the intermediate producers. The optimality conditions for consumption and labor supply choices are

$$c_t^\sigma h_t^\nu = w_t(1 - \tau_{w,t}), \quad (3)$$

and

$$1 = \beta d_{t+1} E_t \left\{ \frac{1 + R_t}{1 + \pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^\sigma \right\}. \quad (4)$$

Final good producers. Perfectly competitive final good firms use a continuum of intermediate goods $i \in [0, 1]$ to produce a single final good with the technology: $y_t = [\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$.

²Our preferences over leisure make no distinction between the number of days worked in a period and the number of hours worked per day. Formally, we are treating the two margins as perfect substitutes from the perspective of the representative household.

The profit maximizing input demands for final goods firms are

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} y_t, \quad (5)$$

where $P_t(i)$ denotes the price of the good produced by firm i and P_t the price of the final good. Thus $\pi_t = P_t/P_{t-1} - 1$. The price of the final good satisfies $P_t = [\int_0^1 P_t(i)^{1-\theta} di]^{1/(1-\theta)}$.

Intermediate goods producers. Intermediate good i is produced according to $y_t(i) = z_t h_t(i)$, where z_t , the state of technology, is common to all producers. Labor is homogeneous and thus real marginal cost for all firms is w_t/z_t . Producer i sets prices to maximize

$$E_0 \sum_{t=0}^{\infty} \lambda_{c,t} \left[(1 + \tau_s) \frac{P_t(i)}{P_t} y_t(i) - \frac{w_t}{z_t} y_t(i) - \frac{\gamma}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 y_t \right] \quad (6)$$

subject to the demand function (5). Producers take the stochastic discount factor, $\lambda_{c,t} \equiv \beta^t (\prod_{j=0}^t d_j) c_t^{-\sigma}$, as given. The sales subsidy τ_s satisfies $(1 + \tau_s)(\theta - 1) = \theta$, or that profits are zero in a steady-state with zero inflation. The final term in brackets is the cost of price adjustment. We assume it is proportional to aggregate production y_t , so that the share of price adjustment costs in the aggregate production depends only on inflation. The optimality condition for intermediate producers in a symmetric equilibrium with $(P_t(i), y_t(i), h_t(i)) = (P_t, y_t, h_t)$ for all i is

$$\pi_t(1 + \pi_t) = \frac{\theta}{\gamma} \left(\frac{w_t}{z_t} - 1 \right) + \beta d_{t+1} E_t \left\{ \left(\frac{c_t}{c_{t+1}} \right)^\sigma \frac{y_{t+1}}{y_t} \pi_{t+1} (1 + \pi_{t+1}) \right\}. \quad (7)$$

Monetary policy. Monetary policy follows a Taylor rule that respects the zero lower bound on the nominal interest rate:

$$R_t = \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \widehat{gdp}_t), \quad (8)$$

where $r_t^e \equiv 1/(\beta d_{t+1}) - 1$ and \widehat{gdp}_t is the log deviation of GDP from its steady-state value.³

The aggregate resource constraint is given by

$$c_t = (1 - \kappa_t - \eta_t)y_t, \quad (9)$$

where $\kappa_t \equiv (\gamma/2)\pi_t^2$ is the resource cost of price adjustment and where government purchases are $g_t = \eta_t y_t$. GDP in our economy, gdp_t , is

$$gdp_t \equiv (1 - \kappa_t)y_t = c_t + g_t. \quad (10)$$

This definition of GDP assumes that the resource costs of price adjustment are intermediate inputs and are consequently subtracted from gross output when calculating GDP.

The term κ_t plays a central role in the analysis that follows. Section 4 shows that loglinearizing equation (10) around a zero inflation steady-state can result in incorrect inferences about the local dynamics of this economy at the zero lower bound and relates this result to κ_t . Whenever the inflation rate changes, κ_t also changes and (10) implies that GDP and labor input h_t move differently, possibly even in opposite directions. However, if equation (10) is loglinearized about a zero inflation steady-state κ_t disappears and GDP and labor input are identical. A term like κ_t occurs in many NK models. For instance, the resource cost of price dispersion is an analogous term that appears in the resource constraint under Calvo pricing (see Yun 2005). Thus, loglinearizing the resource constraint about a zero inflation rate under Calvo pricing creates the same potential biases. We present results for a model with Calvo price setting in Section 5.3 that illustrate this point.

³The assumption that monetary policy responds directly to variations in d_t is made to facilitate comparison with other papers in the literature.

2.2. Markov equilibrium with zero interest rates

Following Eggertsson and Woodford (2003), we analyze the zero bound using a two state Markov equilibrium concept. Let s_t denote the state of the economy which is either low or high, $s_t \in \{L, H\}$. The initial state, s_0 , is L and s_t evolves according to a time-homogeneous Markov rule. The transition probability from state L to L is $p < 1$ and H is an absorbing state. All exogenous variables including the preference shock d_{t+1} , technology shock z_t , and fiscal policy $\{\tau_{w,t}, \eta_t\}$ change if and only if s_t changes: $\{d_{t+1}, z_t, \tau_{w,t}, \eta_t\}$ equals $\{d^L, z^L, \tau_w^L, \eta^L\}$ when $s_t = L$, and $\{1, z, \tau_w, \eta\}$ when $s_t = H$.

Under these assumptions, the equilibrium is characterized by two distinct values for prices and quantities. Prices and quantities in state L are denoted with the superscript L and prices and quantities in state H have no superscript. In state H the economy rests in a steady-state with a zero inflation rate and a positive nominal interest rate. More formally, $h = \{(1 - \tau_w)/(z^{\sigma-1}(1 - \eta)^\sigma)\}^{1/(\sigma+\nu)}$ and $\pi = 0$ if $s_t = H$.

Equilibria with a zero nominal interest rate in state L are subsequently referred to as *zero bound equilibria*.⁴

2.3. Employment and inflation in a zero bound equilibrium

An attractive feature of the model is that the equilibrium conditions for employment and inflation state L can be summarized by two equations in these two variables.⁵ These equations are nonlinear versions of what Eggertsson and Krugman (2012) refer to as “aggregate supply” (AS) and “aggregate demand” (AD) schedules. In what follows, we adopt the same shorthand when referring to these equations.

The AS schedule summarizes intermediate goods firms’ price setting decisions, the

⁴Other restrictions are that $p\beta d^L < 1$, to guarantee that utility is finite, and that prices and quantities are non-negative.

⁵In this model there is no meaningful distinction between hours and employment. Henceforth the term employment is used because employment is the focal point of policy discussions. Also we express the schedules in terms of employment rather than output because it is easier to ascertain the response of employment to a cut in the labor tax rate.

household's intratemporal first order condition, and the aggregate resource constraint. To obtain the AS schedule, start with (7) and substitute out the real wage using (3). Then use (9) to replace consumption with labor input. In a zero bound Markov equilibrium, the AS schedule in state L is

$$\pi^L(1 + \pi^L) = \frac{\theta}{\gamma(1 - p\beta d^L)} \left[\frac{(1 - \kappa^L - \eta^L)^\sigma (h^L)^{\sigma+\nu}}{(1 - \tau_w^L)(z^L)^{1-\sigma}} - 1 \right] \quad (11)$$

where $\kappa^L = (\gamma/2)(\pi^L)^2$.

The AD schedule summarizes the household's Euler equation and the resource constraint. It is obtained by substituting consumption out of the household's intertemporal Euler equation (4) using the resource constraint (9). The resulting AD schedule in a zero bound Markov equilibrium in state L is

$$\frac{h^L}{h} = \frac{z}{z^L} \frac{1 - \eta}{1 - \kappa^L - \eta^L} \left(\frac{1 - p\beta d^L / (1 + \pi^L)}{(1 - p)\beta d^L} \right)^{\frac{1}{\sigma}}. \quad (12)$$

3. Parameterization of the model

A principal claim of this paper is that LL solutions of the NK model can break down in empirically relevant situations. This section describes our strategy for producing empirically relevant parameterizations of the model.

3.1. Parameters that are not specific to the zero bound

An object of central interest is the slope of the conventional loglinear New Keynesian Phillips Curve which is given by $slope(NKPC) \equiv \theta(\sigma + \nu)/\gamma$. Its value influences the slope of the AS schedule using both the LL and the NL equilibrium conditions. There are many combinations of these parameters that can make $slope(NKPC)$ big or small. The baseline parameterization of the model fixes some of these parameters and estimates others using Bayesian methods.

Preferences over consumption are assumed to be logarithmic ($\sigma = 1$) because this is a common reference point in the DSGE literature. It is also well known that β is not well identified in DSGE models. Consequently, β is fixed at 0.997 which implies an annual rate of time preference of 1.2% (see also Denes et al. 2013). The parameter θ is set to 7.67, which implies a markup of 15%.⁶ This choice of θ lies midway between previous estimates from disaggregate and aggregate data. Broda and Weinstein (2004) find that the median value of this parameter ranges from 3 to 4.3 using 4-digit industry level data for alternative country pairs. Denes et al. (2013) estimate θ to be about 13 in a NK model that is similar to ours.

The government purchase share parameter η is fixed at 0.2 and the labor tax rate τ_w at 0.2. This leaves ν , the curvature parameter for leisure and γ , the adjustment cost parameter and the coefficients of the Taylor rule. These parameters are estimated using Bayesian methods on quarterly U.S. data on inflation, the output gap and the Federal Funds rate over a sample period that extends from 1985:I through 2007:IV. The estimated posterior mode of γ is 458.4 and 90% of its posterior mass lies between 315 and 714. This value is larger than Ireland (2003) who estimates a value of 162, and Gust et al. (2012) who estimate $\gamma = 94$. Experiments with a tighter prior on γ produces estimates of γ that lie in the range of 100 to 150 but most posterior mass lies outside of the prior. Instead of ruling out these other lower estimates of γ , we report results for values of γ that range from 100 to 600. The posterior mode for ν , which governs the curvature of the disutility of work, is 0.28 with 90% of its posterior mass lying in the interval 0.08 and 0.63. These estimates in conjunction $\theta = 7.67$ and $\sigma = 1$ imply that $slope(NKPC) = 0.0214$ which is close to the value of 0.024 estimated by Rotemberg and Woodford (1997).

Finally, the posterior modes of the Taylor rule parameters are $\phi_\pi = 3.46$ and $\phi_y = 1.63$. A complete set of estimation results can be found in Section F of the Online Appendix.

⁶We found that θ and γ are not individually identified by our estimation procedure. Given the central role played by γ in the NK model, we decided to fix θ and estimate γ .

3.2. Parameters and shocks that are specific to the zero bound

The remaining parameters are the shocks in the zero bound state, $\{d^L, z^L, \tau_w^L, \eta^L\}$, and the persistence parameter p which governs the expected duration of the zero bound. This final parameter is important for the local dynamics of the model at the zero bound and results will be reported for a large range of values of $p \in [0.05, 0.95]$. For each choice of p , d^L and z^L are chosen to hit output and inflation targets from the U.S. GR.⁷ The specific targets are taken from Christiano et al. (2011). They provide empirical evidence that the U.S. financial crisis that ensued after the collapse of Lehman Brothers in the third quarter of 2008 produced a decline in output of 7% and a decline in the inflation rate of 1%.

Using these targets the resource costs of price adjustment constitute 0.14% of gross output at the baseline value of γ . It is difficult to directly measure the overall magnitude of the resource cost of price adjustment but a rough idea of the potential magnitude of this cost is provided by Levy et al. (1997) who find that menu costs constitute 0.7% of revenues of supermarket chains.

4. Characterization of equilibrium

This section compares and contrasts the dynamics of the model in the zero bound state using the LL and the NL equilibrium conditions. The NK model has very rich dynamics at the zero bound and much of this richness is lost using the standard LL solution method. In particular, the AD and AS schedules can have conventional local slopes at the zero bound and multiple zero bound equilibria can occur.

4.1. Characterization of zero bound equilibria using the loglinearized equilibrium conditions

An important strand of the previous literature has worked with equilibrium conditions that are loglinearized around a perfect foresight steady-state with zero inflation and a

⁷Fiscal policy is fixed at its steady-state value, i.e. $\{\tau_w^L, \eta^L\} = \{\tau_w, \eta\}$ and Section G of the Online Appendix shows that there is a unique mapping from these two targets to $\{d^L, z^L\}$.

positive nominal interest rate. We start by briefly highlighting this and several other key properties of the LL equilibrium in the zero bound state.⁸

The most salient properties of the LL solution at the zero bound are that the slopes of both the AD schedule, $\text{slope}(AD^{LL}) \equiv (1 - p)\sigma/p$, and the AS schedule, $\text{slope}(AS^{LL}) \equiv \text{slope}(NKPC)/(1 - p\beta)$, are positive.⁹ Thus, only two types of zero bound equilibria can occur. In a *Case I* zero bound equilibrium *the AD schedule is steeper* whereas, in a *Case II* zero bound equilibrium *the AS schedule is steeper*.

An example of a Case I equilibrium is shown in Figure 1a. It reports the configuration of the AD^{LL} and AS^{LL} schedules in the low state with no shocks, L_{ns} , and in the low state with shocks to preferences and technology, L .¹⁰ Most of the recent literature on the zero bound has focused exclusively on Case I equilibria. Equilibrium is globally unique in Case I equilibria (see Proposition 4 in Braun et al. 2012). When the shocks are set to their steady-state levels, the AD and AS schedules cross at the steady-state ($R > 0$). However, the AD schedule has a kink that arises from imposing the zero bound restriction on the Taylor rule. If the shocks are instead set to hit the GR targets, the schedules cross in the region where $R = 0$ and $\text{slope}(AD^{LL})$ is larger than $\text{slope}(AS^{LL})$.

Case I equilibria are associated with relatively low values of p . As the expected duration of zero interest rates increases, AD^{LL} rotates to the right. A longer episode of zero interest rates means a longer expected duration of deflation and this in turn has a stronger contractionary effect on current demand. A larger value of p rotates AS^{LL} to the left. Firms recognizing that the expected duration of low demand is longer are willing to take on bigger price cuts (see also Eggertsson 2011). When p is sufficiently large $\text{slope}(AS^{LL})$ becomes larger than $\text{slope}(AD^{LL})$ and the zero bound equilibrium switches to Case II.

⁸A more complete discussion of the properties of LL solutions at the zero bound can be found in Woodford (2011) and Braun et al. (2012).

⁹These slope definitions are specific to the situation where $R = 0$.

¹⁰We thank a referee for suggesting that we use this type of figure. Similar figures are also used in Mertens and Ravn (2010).

Figure 1b provides an example of a Case II zero bound equilibrium. The AD schedule now crosses the AS schedule twice when shocks are set to their steady-state values. The upper crossing point occurs at the steady-state ($R > 0$) and a second lower crossing point occurs when $R = 0$. Bullard (2010), Mertens and Ravn (2010) and Aruoba and Schorfheide (2013) introduce sunspot variables into Case II equilibria that allow for switches from the steady-state equilibrium to the zero bond equilibrium with no shocks to fundamentals. We are interested in Case II equilibria that can reproduce the GR targets and for this reason we allow for shocks to preferences and technology. This induces shifts in the AD schedule as shown in Figure 1b. Under weak conditions described in Section C of the Online Appendix there is a single $R = 0$ crossing point of AD^{LL} and AS^{LL} and it follows that the zero bound equilibrium is always unique using the LL equilibrium conditions.

Case I and Case II equilibria have different local dynamics and thus very different implications for fiscal policy (Mertens and Ravn, 2010) in low interest rate environments. In a Case I equilibrium supply shocks have unorthodox effects on output. For instance, a cut in the labor tax rate shifts the equilibrium down and inflation and employment (and output) fall. However, in a Case II equilibrium employment and inflation increase in response to a labor tax rate cut. Demand shocks such as an increase in government purchases can have potent effects on output in Case I equilibria. The government purchase multiplier always exceeds one in Case I equilibria and is sometimes much larger than one. In Case II equilibria, the government purchase multiplier is always smaller than one and may even be negative. To understand these results, consider, for instance, how the size of the government purchase multiplier varies with p . When p is small $slope(AS^{LL})$ is also small and the government purchase multiplier, which is primarily a demand shifter, is small. The size of the government purchase multiplier increases with p up to the bifurcation point which occurs when $slope(AD^{LL}) = slope(AS^{LL})$. Beyond this point the equilibrium switches to a Case II equilibrium and it follows that the government purchase multiplier is less than one.

From this brief summary one can understand why Case I equilibria have received so much attention in the literature. In this type of equilibrium conventional supply side stimulus is contractionary and should be avoided at the zero bound. Policies that stimulate aggregate demand though are particularly effective.

4.2. Characterization of zero bound equilibria in the NK model

We now characterize equilibrium of the model using the NL equilibrium conditions. Two new results emerge. The AD schedule can be downward sloping at the zero bound and for some parameterizations of the model there are multiple zero bound equilibria.

4.2.1. Slopes of Aggregate Demand and Aggregate Supply

The slopes of the AD and AS schedules vary with the size of the shocks when one uses the NL equilibrium conditions. However, it is straightforward to derive analytical expressions for their *local* slopes in the neighborhood of the zero bound equilibrium by loglinearizing around (h^L, π^L) instead of the zero inflation steady-state.¹¹

At the zero bound, the slope of the AD schedule in the neighborhood of (h^L, π^L) is

$$\text{slope}(AD) \equiv \left[\frac{1}{\sigma} \frac{p\beta d^L / (1 + \pi^L)^2}{1 - p\beta d^L / (1 + \pi^L)} + \frac{(\kappa^L)'}{1 - \kappa^L - \eta^L} \right]^{-1}. \quad (13)$$

$\text{Slope}(AD)$ consists of the inverse of the two terms in brackets. Note that the first term captures the same tradeoffs between employment and inflation as $\text{slope}(AD^{LL})$. It is positive and simplifies to $\text{slope}(AD^{LL})^{-1}$ when $(d^L, \pi^L) = (1/\beta, 0)$. What is new is the second term in equation (13) which reflects the fact that price adjustment on the margin is costly and absorbs resources.¹² It is negative in a deflationary zero bound equilibrium and acts like a leaky bucket, driving a wedge between what is produced and what is available for consumption.

¹¹The restrictions $c^L > 0$ and $h^L > 0$ imply that attention can be restricted to $\{\pi^L : 1 - \kappa^L - \eta^L > 0, 1 - p\beta d^L / (1 + \pi^L) > 0, \text{ and } (1 - p\beta d^L)\gamma\pi^L(1 + \pi^L) + \theta > 0\}$.

¹²Note that the second term of (13) disappears when $\text{slope}(AD)$ is evaluated at $(d^L, \pi^L) = (1/\beta, 0)$.

The effect of the resource costs of price adjustment on AD is most pronounced when the expected duration of zero interest rates is short. Inspection of equation (13) reveals that at $p = 0$, $slope(AD)$ is unambiguously negative. Increasing p from zero rotates the AD schedule to the right for the reasons described in Section 4.1 and it becomes steeper until its slope eventually turns positive. Thus, $slope(AD^{LL})$ and $slope(AD)$ will have different signs when the expected duration of zero interest rates is sufficiently short and will both be positive when the expected duration is sufficiently long.

The parameters σ and γ are also important for the sign of $slope(AD)$. A larger value of σ reduces the size of the first term in equation (13), and a larger value of γ increases the resource cost of price adjustment and thus the magnitude of the second term. It follows that $slope(AD) < 0$ will be negative for a larger range of values of p when σ and/or γ are large.

The slope of the AS schedule can also be negative at the zero bound. Loglinearizing the nonlinear AS schedule at (h^L, π^L) yields

$$slope(AS) \equiv \left[\frac{1 - p\beta d^L}{slope(NKPC)} \frac{1 + 2\pi^L}{mc^L} + \frac{\sigma}{\sigma + \nu} \frac{(\kappa^L)'}{1 - \kappa^L - \eta^L} \right]^{-1} \quad (14)$$

where marginal cost in state L is given by $mc^L = \pi^L(1 + \pi^L)(1 - p\beta d^L)\gamma/\theta + 1$.

$Slope(AS)$ is given by the inverse of the two terms in brackets. The first term is unambiguously positive and reflects the same factors that determine $slope(AS^{LL})$. Note that this term simplifies to $slope(AS^{LL})^{-1}$ when evaluated at $(d^L, \pi^L) = (1, 0)$. The second term is negative in a deflationary zero bound equilibrium but disappears at $(d^L, \pi^L) = (1, 0)$. This term reflects how the resource cost of price adjustment affects the supply of goods. Less deflation reduces the amount of resources that are absorbed by costly price adjustment. This creates a positive wealth effect that puts downward pressure on labor supply. If this effect is strong enough employment can fall even though the wage has risen.

The value of p also plays an important role in determining $slope(AS)$. Suppose that

$(\kappa^L)' = \gamma\pi^L$ is sufficiently small so that $\text{slope}(AS) > 0$ when $p = 0$.¹³ Then as p is increased the AS schedule rotates to the left becoming steeper until its slope turns negative. Consequently, the signs of $\text{slope}(AS^{LL})$ and $\text{slope}(AS)$ are most likely to agree at smaller values of p .

The analysis so far is useful for understanding the slopes of the AD and AS schedules in the neighborhood of a crossing point but it is silent about the number of crossing points at the zero bound. The solution to the NL equilibrium conditions at the zero bound can be reduced to finding the zeros of a nonlinear function in the inflation rate. We show next that the NK model may have multiple zero bound equilibria with distinct local configurations of AD and AS.

4.2.2. *New types of zero bound equilibria in the NK model*

A conventional configuration of AD and AS. The results from the previous section suggest that the NK model has rich dynamics at the zero bound and that some of this richness is lost when one loglinearizes the equilibrium conditions at a steady-state with zero inflation. Perhaps the most important new case is that of a *downward sloping AD* and an *upward sloping AS schedule* at the zero bound. Figure 2c shows that this case occurs using the baseline parameterization of the model. The shocks are chosen to reproduce the Great Recession GDP and inflation targets and $p = 0.4$. This case, which is subsequently referred to as *Case III*, has the property that equilibrium is globally unique.

Multiple zero bound equilibria. The AD^{LL} and AS^{LL} schedules generically have a unique intersection at the zero bound. But in the true model, there are regions of the parameter space where multiple zero bound equilibria occur. Figure 2d provides an example of this situation using the baseline parameterization with $p = 0.88$. Under this configuration of parameters, there are *two zero bound equilibria* and one equilibrium with a positive nominal

¹³This condition is satisfied for all of the results we report below.

interest rate. We refer to this situation as a *Case MZB* zero bound equilibrium.¹⁴ The two zero bound equilibria have different local properties. In this example, the zero bound equilibrium that hits the GR targets exhibits $\text{slope}(AS) > \text{slope}(AD) > 0$. At the second zero bound equilibrium $0 > \text{slope}(AS) > \text{slope}(AD)$. The inflation rate in this second equilibrium is implausibly small at -16.8% per annum. However, the two zero bound equilibria are much closer for other choices of p . If, for instance, p is set to 0.86, the inflation rate in the nontargeted zero bound equilibrium is -0.8% and GDP declines by -6.7%. These magnitudes are similar to those in the targeted equilibrium which reproduces an inflation rate of -1% and a 7% decline in GDP. In this example, the targeted equilibrium has $\text{slope}(AD) > \text{slope}(AS) > 0$ and the second equilibrium has $\text{slope}(AS) > \text{slope}(AD) > 0$.

These two examples illustrate that the local properties of Case MZB zero bound equilibria are very sensitive to the particular choice of p in this region of the parameter space. Increasing p by 0.02 results in equilibria with very different local properties. The reason why the local dynamics are so complicated in this region is because it includes the bifurcation point where $\text{slope}(AD) = \text{slope}(AS)$.

Case I and II equilibria also occur using the NL equilibrium conditions (Figures 2a and 2b). In other words, there are regions of the parameter space in the NK model where $\text{slope}(AD) > \text{slope}(AS) > 0$ and equilibrium is globally unique (Case I) and there are other regions of the parameter space where there is one zero bound equilibrium with $\text{slope}(AS) > \text{slope}(AD) > 0$ and a second equilibrium with a positive interest rate equilibrium (Case II).¹⁵

¹⁴Case MZB can be divided into a number of sub cases that vary according to the slope of the targeted and the non-targeted zero bound equilibria. The number of distinct types of equilibrium is large and we thus choose to bundle them together in a single case.

¹⁵Cases I, II, III and MZB do not exhaust all of the possible configurations of $\text{slope}(AD)$ and $\text{slope}(AS)$. Most notably, none of the examples have $\text{slope}(AS) < 0$. This situation only occurs at very high values of $p > 0.98$ using the baseline parameterization of the model.

4.3. When and how do LL solutions fail?

Figure 3 reports the regions where each of the four cases occur using the NL equilibrium conditions (first row) and the regions where the two cases occur using the LL solution (second row). The figures in the first column report results for alternative values of $p \in [0, 0.945]$ ranging and for $\sigma \in [0.5, 2]$ and the results in the second column consider alternative values of p and $\gamma \in [100, 600]$. For each parameterization the shocks $\{d^L, z^L\}$ are adjusted to reproduce the GR inflation and GDP targets and in situations with multiple zero bound equilibria only the targeted equilibrium is reported.

Consider the baseline parameterization which is denoted with a solid black line. Sections 4.1 – 4.2 establish that $\text{slope}(AD) < 0$ for smaller values of p but that $\text{slope}(AD^{LL}) > 0$ for all p . A comparison of rows 1 and 2 of Figure 3 reveals that this breakdown in the LL solution occurs for all values of $p \leq 0.56$.

A second breakdown of the LL solution occurs in the interval $p \in [0.857, 0.890]$ using the baseline parameterization. For both solutions this interval contains a bifurcation point.¹⁶ However, the LL solution fails to register the fact that there are multiple zero bound equilibria. The qualitative properties of the targeted NL equilibrium and the LL equilibrium are generally the same for $p \in [0.857, 0.890]$. But the magnitudes of the slopes of the AD and AS schedules can be quite different.

A disturbing aspect of these results is that the bifurcation occurs at values of p that are close to values maintained in previous research. Denes et al. (2013) report an estimate of $p = 0.86$ and Christiano and Eichenbaum (2012) posit a value of $p = 0.775$. Our results indicate that small variations in p in this region can have a large effect on the nature of the zero bound equilibrium.

Section 4.2 shows that the size of region when $\text{slope}(AD) < 0$ is increasing in σ and γ . Recall that a higher value of σ reduces both $\text{slope}(AD)$ and $\text{slope}(AS)$. Figure 3 shows that

¹⁶Using the NL solution the bifurcation occurs at $p \approx 0.861$ and using the LL solution it occurs at $p \approx 0.866$.

increasing σ has a particularly large effect on the size of the Case III region. It increases from $[0, 0.56]$ when $\sigma = 1$ to $[0, 0.71]$ when $\sigma = 2$. Varying γ , which only reduces $slope(AS)$, has a smaller effect on the size of the Case III region but a larger effect on the location of the bifurcation point and thus the size of the Case II and Case MZB regions. It was pointed out above that our estimated baseline value of γ is higher than some other estimates. If a value of $\gamma = 100$ is used instead, the Case III region only includes $p \leq 0.22$. However, equilibrium is globally indeterminate (either Case MZB or Case II) for all $p \geq 0.77$.

5. Small and orthodox fiscal multipliers at the zero bound

This section establishes that the NK model can be used to build a case for the efficacy of supply side fiscal stimulus in a low interest rate environment. The argument is developed in two steps. We start by showing that a labor tax cut may increase employment using empirically relevant parameterizations of the NK model. Then we demonstrate that for these same parameterizations of the model, the government purchase multiplier is sometimes close to one. Taken together these two points strengthen the case for supply side policies and weaken the case for demand side policies in low interest rate environments.

5.1. Labor Tax Multiplier

Eggertsson (2011) has found that a reduction in the labor tax rate lowers employment in the NK model at the zero bound and refers to his result as a “paradox of toil.” Eggertsson and Krugman (2012) using a similar line of reasoning argue that labor tax rate cuts should be avoided when the economy is in a liquidity trap. These results are derived using LL solutions and attention is limited to equilibria that are globally unique. It follows from the results in Section 4.1 that *all* zero bound equilibria satisfy $slope(AD^{LL}) > slope(AS^{LL}) > 0$. In other words, all zero bound equilibria are Case I equilibria and the labor tax multiplier is positive. Cutting the labor tax rate shifts the AS schedule outward along a stable AD schedule and employment falls. However, the results in Section 4.2 show that if one uses the NL equilibrium conditions instead, limiting attention to equilibria that are globally unique

is not sufficient to rule out a negative labor tax multiplier. Equilibrium is also unique in Case III equilibria and they have the property that $\text{slope}(AS) > 0 > \text{slope}(AD)$ and thus that supply side stimulus is expansionary.

Figure 4 provides information that allows the reader to easily discern when the paradox of toil occurs in the NK model and when it does not by partitioning the parameter space according to the sign and magnitude of the labor tax multiplier. The upper panels report labor tax multipliers using the NL equilibrium conditions and configurations of the shocks that reproduce the GR inflation and GDP targets. Baseline values of σ and γ are denoted with a line and the targeted equilibrium is reported in situations where there are multiple zero bound equilibria. For purposes of comparison the lower panels report results based on the LL solution using the same shocks.

The labor tax multiplier is negative in the red region. In the lower panels (LL solution) this only occurs when $p > 0.86$. This region of the parameter space is where Case II zero bound equilibrium occur (see Figure 3). They have the property that $\text{slope}(AS^{LL}) > \text{slope}(AD^{LL}) > 0$ but as discussed above, the equilibrium is not globally unique.

The two upper panels of the figure, in contrast, have two disjoint red regions where the labor tax multiplier is negative. The leftmost red region in the first row of Figure 4 is of particular interest because equilibrium is globally unique. It corresponds to Case III (see Figure 3). The AD and AS schedules have their conventional slopes and it follows that the labor tax multiplier is negative. For the baseline parameterization this region obtains for $p \in [0, 0.57]$. When $\sigma = 2$, the labor tax multiplier is negative for p as high as 0.71. The leftmost red region is smallest when $\gamma = 100$. However, a reduction in the size of the left red region is offset by an increase in the size of the red region on the right, which consists of Case MZB equilibria and Case II equilibria.¹⁷ Lower values of γ can have a big impact on inference. For instance, Eggertsson (2011) produces a paradox of toil with $p = 0.77$. As

¹⁷For the Case MZB equilibria, the targeted equilibrium has a slope configuration with $\text{slope}(AS) > \text{slope}(AD) > 0$.

can be seen in Figure 4 there is no paradox of toil at this choice of p when $\gamma \leq 150$.

One of the reasons previous research on the zero bound has attracted so much attention is because the magnitude of the paradox of toil is large. Denes et al. (2013), for instance, report a median posterior labor tax multiplier of 0.1, i.e. employment *increases* by 0.1% when the labor tax is *raised* by one percentage point, in a loglinearized NK model that is calibrated to the GR. Figure 4 also reports information on the size of the paradox of toil in regions of the parameter space where it occurs. The paradox of toil is generally small using the NL equilibrium conditions. It only exceeds 0.1 in the blue region which is close but just to the left of the bifurcation point where the $slope(AD) = slope(AS)$. In this region both the size and the sign of the labor tax multiplier is very sensitive to small changes in the value of p .¹⁸

5.2. Government purchase multipliers

A number of papers including most notably Christiano et al. (2011) have found that the fiscal multiplier is large in the NK model at the zero bound. The government purchase multiplier is also large for some parameterizations of our model. However, it is close to one and sometimes is even less than one for a range of empirically relevant parameterizations of the model. Moreover, there is considerable overlap in the regions of the parameter space with small government purchase multipliers and negative labor tax multipliers.

Government purchase multipliers using the NL equilibrium conditions are reported in the upper panels of Figure 5. Government purchase multipliers that are less than one are colored red. They only occur to the right of the bifurcation ($p \geq 0.861$) and correspond to either Case II or targeted Case MZB equilibria. In either case the zero bound equilibrium satisfies $slope(AS) > slope(AD) > 0$ which immediately implies that the sign of the labor tax multiplier is negative. In principal, the government purchase multiplier could also be negative in this region. In practice though this only occurs in a very small neighborhood

¹⁸Section D of the Online Appendix shows that the labor tax multiplier is inversely related to $slope(AD) - slope(AS)$.

just to the right of the bifurcation point. This region of the parameter space has two other notable properties. An increase in government purchases is deflationary and equilibrium is globally indeterminate.

The remaining parameterizations of the model have government purchase multipliers that are larger than 1 but in some cases they are very close to 1. For instance, using the baseline values of σ and γ , the government purchase multiplier is less than 1.05 when $p \in [0, 0.73]$ (green) and it only exceeds 1.5 in the very small blue region that is just to the left of the bifurcation point. Combining these results with the previous findings on the labor tax multiplier implies that for all $p \in [0, 0.56]$, there is a unique zero bound equilibrium and it has the properties that the government purchase multiplier is small and the labor tax multiplier has an orthodox sign.

A comparison of the upper and lower panels of Figure 5 shows that the LL solution works reasonably well in predicting the size of the government purchase GDP multiplier. There are large green regions with small government purchase multipliers less than 1.05 using either solution and the region where the government purchase multiplier exceeds 1.5 is small in both the upper and lower panels.

Since the government purchase multiplier is primarily a demand shifter, one might be concerned that our finding stems from the fact that the slope of the AS schedule is very flat and that this is due to our setting of $\text{slope}(NKPC)$. Section A of the Online Appendix illustrates how the results change when $\text{slope}(NKPC)$ is increased from its baseline value of 0.0214 to 0.06. The size of the green region is smaller but the size of the blue region with multipliers in excess of 1.5 continues to be very small.

Finally, note that smaller values of γ reduce $\text{slope}(NKPC)$ and this increases the size of the region where the government purchase multiplier is less than one. Christiano and Eichenbaum (2012) set $\gamma = 100$ and choose $p = 0.775$. Panel B) of Figure 5 indicates that with these choices our model produces a government purchase multiplier of less than 1 for the GR.

5.3. Discussion

Expected duration of zero interest rates. We have found that the expected duration of zero interest rates plays a central role in determining the properties equilibrium at the zero bound. Given the importance of the magnitude of p , it is worthwhile to discuss grounds for entertaining large and small values of p . The NK model produces large positive fiscal multipliers when p is slightly less than 0.861 using the baseline parameterization. However, the local dynamics of the NK model change if p exceeds this value and it is hard to rule out larger values of p on empirical grounds. The actual duration of zero interest rates has been much longer than 7 quarters in the U.S. and Japan. Interest rates have been close to zero since the fourth quarter of 2008 in both countries.¹⁹ Case II is the most common type of equilibrium for $p > 0.861$ in Figure 3. And in this type of equilibrium the government purchase multiplier is less than 1 and employment increases in response to a cut in the labor tax rate.

The other region where supply side policies are likely to be most effective is when the expected duration of zero interest rates is short. Should one take seriously small values of p ? First, p does not have to be all that small to obtain a small government purchase multiplier or a negative labor tax multiplier. For instance, if p is set to 0.775 as in Christiano and Eichenbaum (2012), the government purchase GDP multiplier is 1.09 using the baseline parameterization of our model and it drops to 1.05 if σ is set to 2 instead. Similarly, when $\sigma = 2$ the labor tax multiplier is negative for p as large as 0.705.

Second, it is difficult to rule out even very short expected durations of zero interest rates on a priori grounds. In Aruoba and Schorfheide (2013), for instance, the expected duration of zero interest rates is often only one quarter. Their model requires a long sequence of negative monetary policy shocks to account for the fact that the U.S. policy rate has been about zero since 2008.²⁰

¹⁹Japan has had two other episodes of very low interest rates: March 1999 – July 2000 and March 2001 – June 2006 (see Hayashi and Koeda 2013).

²⁰A number of other recent papers consider medium-scale NK models with zero bound constraints and

A distinct reason for ruling out low values of p is that values around 0.4 or less require positive technology shocks and large values of d^L to reproduce the GR targets. A positive technology shock does not play a central role in our findings. Section A of the Online Appendix contains results that repeat our analysis, holding technology fixed and varying θ instead, and the size of the regions with small and orthodox fiscal multipliers increase.

However, a large value of the preference shock is essential if this simple model is to reproduce a 7% decline in GDP when the expected duration of state L is very short. For instance, at $p = 0.4$ a value of $d^L = 1.0445$ is required to reproduce the GR with the baseline parameterization. It is beyond the scope of this paper to determine what caused the GR. But we do feel that it is important to use observations from the GR to discipline the model.

Calvo price adjustment. The analysis has used Rotemberg price setting. We believe that our finding that the LL solution fails at the zero bound is not specific to the form of costly price adjustment. As described above price dispersion using Calvo price setting also reduces the resources that are available for private and public consumption. In particular, if Calvo price setting is used instead the term κ in the resource constraint becomes $\kappa_t \equiv (x_t - 1)/x_t$ where x_t summarizes the relative price dispersion described in Yun (2005).²¹

Unfortunately, x_t is an endogenous state variable and the zero bound equilibrium becomes much more complicated to compute. To give the reader an indication about what might happen under Calvo price adjustment Section B of the Online Appendix derives results for a stylized but tractable model with Calvo price adjustment. In this model x_t is only allowed to take on two distinct values: $x_t = x^L$ in state L and $x_t = 1$ in state H .

find it difficult to solve/estimate specifications that can reproduce the long periods of zero interest rates experienced by Japan and the U.S. (see e.g. Gust et al. (2012) or Fernandez-Villaverde et al. (2012)). Thus, it appears that the experiences of the U.S., Japan and other countries with long episodes of low interest rates are a puzzle for most of the recent NK literature.

²¹A complete description of the equilibrium condition in the Calvo model is provided in the Online Appendix.

This assumption is valid if the LL solution is accurate because x_t is constant at 1 when loglinearized around the zero inflation steady-state.

Figure 6 compares the AD and the AS schedules under this version of Calvo pricing with the baseline model using a value of $p = 0.4$. The figure shows that the two models of price adjustment are almost indistinguishable in the neighborhood of the equilibrium. In particular, the equilibrium using Calvo pricing also has conventionally sloped AS and AD schedules. Section B of the online Appendix illustrates that this version of the Calvo pricing also has very similar properties to Rotemberg at higher values of p . For instance, the Calvo model also has a Case I equilibrium when $p = 0.8$ and a Case II equilibrium when $p = 0.9$.²²

Other shocks. Our message that supply side stimulus can be expansionary at the zero bound has implications for other supply shocks. For instance, Christiano et al. (2011) find that the response of output to an improvement in technology is contractionary at the zero bound. This finding runs counter to empirical evidence in Wieland (2013) that suggests improvements in technology are also expansionary at the zero bound. In our model positive technology shocks are expansionary in Case III and Case II equilibria when $\sigma = 1$ (see Section D.2 of the Online Appendix).

Our finding that the LL solution works well when computing the government purchase multiplier depends on the size of the shocks. The LL solution exhibits much larger biases if the shocks are calibrated to observations from the Great Depression instead. The interested reader is referred to Braun and Körber (2011) for more details.

Other parameterizations. We have described how our results vary with p , σ and γ but it is possible that other values of the parameters that we have held fixed matter. In Section

²²To draw AS and AD under Calvo pricing, the probability that a firm is unable to change its price (α) is calibrated such that the loglinearized New Keynesian Phillips curve has the same slope as in the Rotemberg model with our baseline parameterization. The implied value is $\alpha = 0.88$. In the Rotemberg model, the slope of the New Keynesian Phillips curve is given by $\theta(\sigma + \nu)/\gamma$ while in the Calvo model it is $(1 - \alpha)(1 - \beta\alpha)(\sigma + \nu)/\alpha$.

A of the Online Appendix we report additional results that are designed to address this concern. We continue to find regions of the parameter space that have small government purchase multipliers and negative labor tax multipliers. Perhaps the most important new finding is that multiple zero bound equilibria occur in much larger regions of the parameter space and, in particular, in regions where p is very small (see Figures 8 and 11 of the Online Appendix).

Equilibrium selection. In situations with multiple equilibria, we have adopted the convention of reporting the targeted zero bound equilibrium that reproduces the GR calibration target. Is this a reasonable way to proceed? Christiano and Eichenbaum (2012) propose using an E-learning criterion to rule out multiple equilibria. Applying that criterion here does not always resolve the issue of multiple zero bound equilibria in our model. This is because in some cases both the targeted and the non-targeted zero bound equilibrium are E-learnable.²³ What is perhaps more troubling is that when the E-learning criterion is applied to Case II equilibria, it rules out the zero bound equilibrium and selects the positive interest rate equilibrium which fails to hit the GR targets. This selection works in the same way using either the NL or the LL solution. We are not convinced that it is reasonable to use an equilibrium selection criterion that rejects the zero bound equilibrium in Figure 2c and instead selects the positive interest rate equilibrium. We think it makes more sense to select the zero bound equilibrium because it reproduces the GR facts and to rule out the positive interest rate equilibrium instead.

6. Conclusion

In this paper we have documented the properties of a tractable nonlinear New Keynesian model that honors the zero lower bound on the nominal interest rate and also reproduces

²³For instance, when $p = 0.865$ the non-targeted zero bound equilibrium that produces a 13% decline in GDP and an annualized rate of deflation of 5% is also E-learnable. This example also has a positive interest rate equilibrium that is also E-learnable. It has the property that GDP falls by 0.6% and that employment is 5% above its steady-state level.

the large output and small inflation declines that occurred during the U.S. Great Recession. Some parameterizations of the model support the contention that supply side fiscal policies should be avoided in low interest rate environments and that demand side policies should be relied on instead. However, two of the principal arguments underlying this contention (labor tax cuts are contractionary and the government purchase multiplier is large) are not robust. Other empirically relevant parameterizations of the same NK model have much smaller government purchase multipliers and also provide a rationale for supply side measures such as a labor tax cut. In particular, one cannot dismiss the possibility that supply side policies are expansionary at the zero bound and solving the NK model using nonlinear methods plays an important role in reaching this conclusion.

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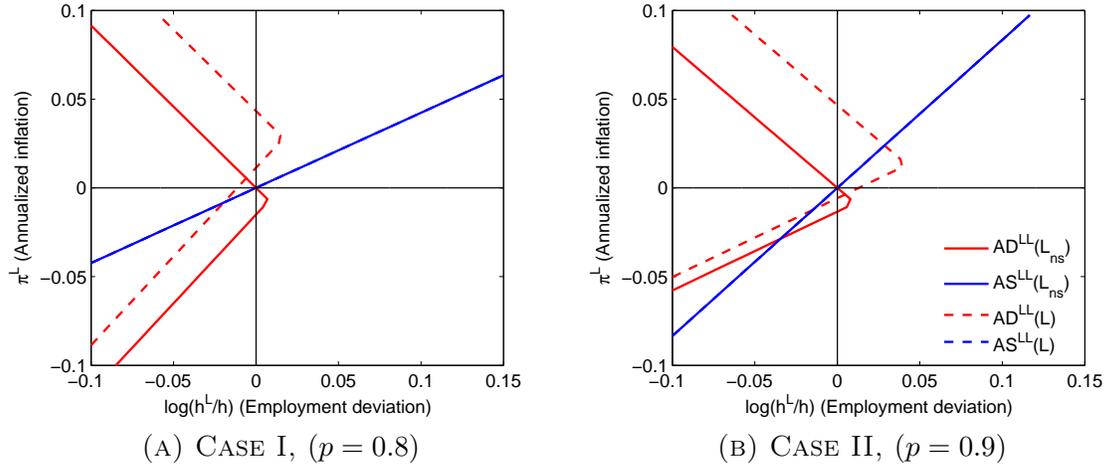
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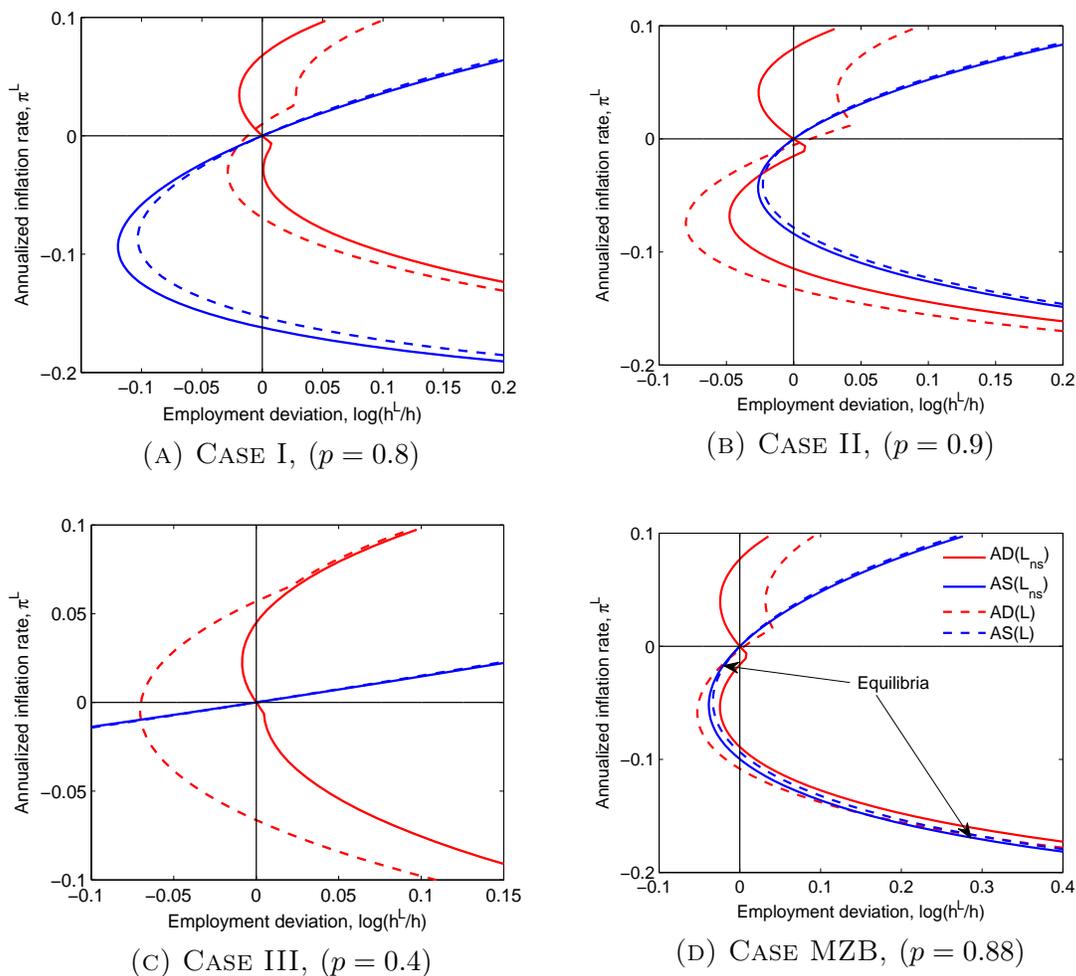
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FIGURE 1: ZERO-BOUND EQUILIBRIA USING LOGLINEARIZED EQUILIBRIUM CONDITIONS



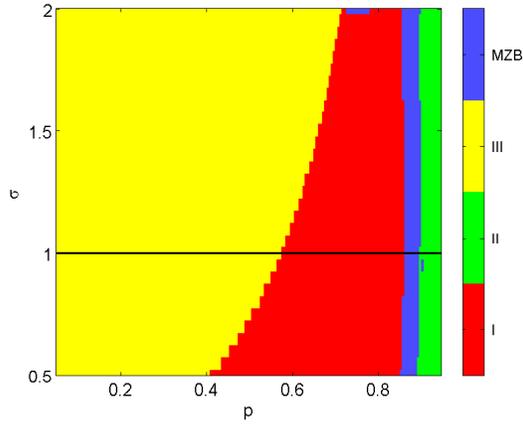
Notes: The plots use the baseline parameterization of the model. The schedules labeled L use values of \hat{d}^L and \hat{z}^L that reproduce the Great Recession targets using the nonlinear equilibrium conditions and the schedules labeled L_{ns} set the shocks to their steady-state values. The loglinearized AS schedule is the same in states L and L_{ns} because $\sigma = 1$.

FIGURE 2: ZERO-BOUND EQUILIBRIUM THAT CAN BE FOUND USING NONLINEAR SOLUTION METHODS.

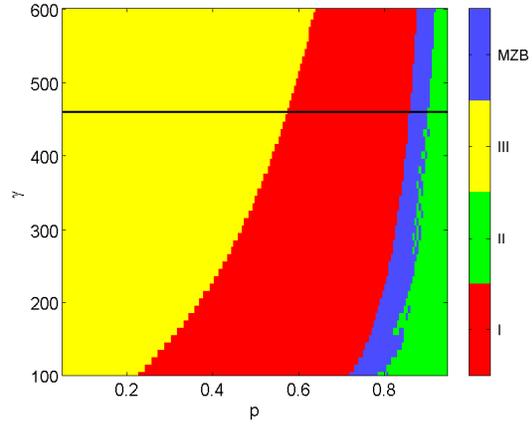


Notes: The plots are based on our baseline parameterization. The schedules labeled L_{ns} set all shocks to their steady-state variables. The schedules labeled L use shocks \hat{d}^L and \hat{z}^L that reproduce our Great Recession targets for GDP and inflation using the nonlinear equilibrium conditions.

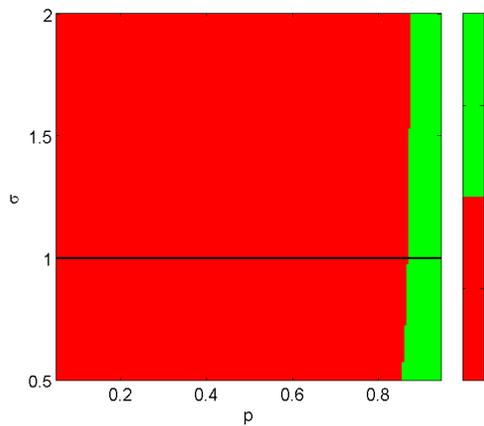
FIGURE 3: TYPES OF ZERO BOUND EQUILIBRIA FOR ALTERNATIVE VALUES OF PRICE ADJUSTMENT COSTS AND RISK AVERSION.



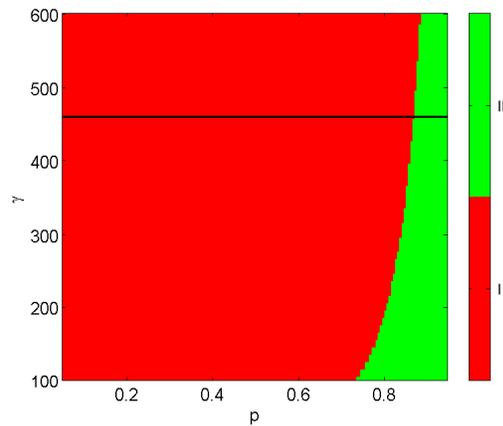
(A) NL EQUILIBRIUM: ALTERNATIVE COMBINATIONS OF p AND σ .



(B) NL EQUILIBRIUM: ALTERNATIVE COMBINATIONS OF p AND γ .



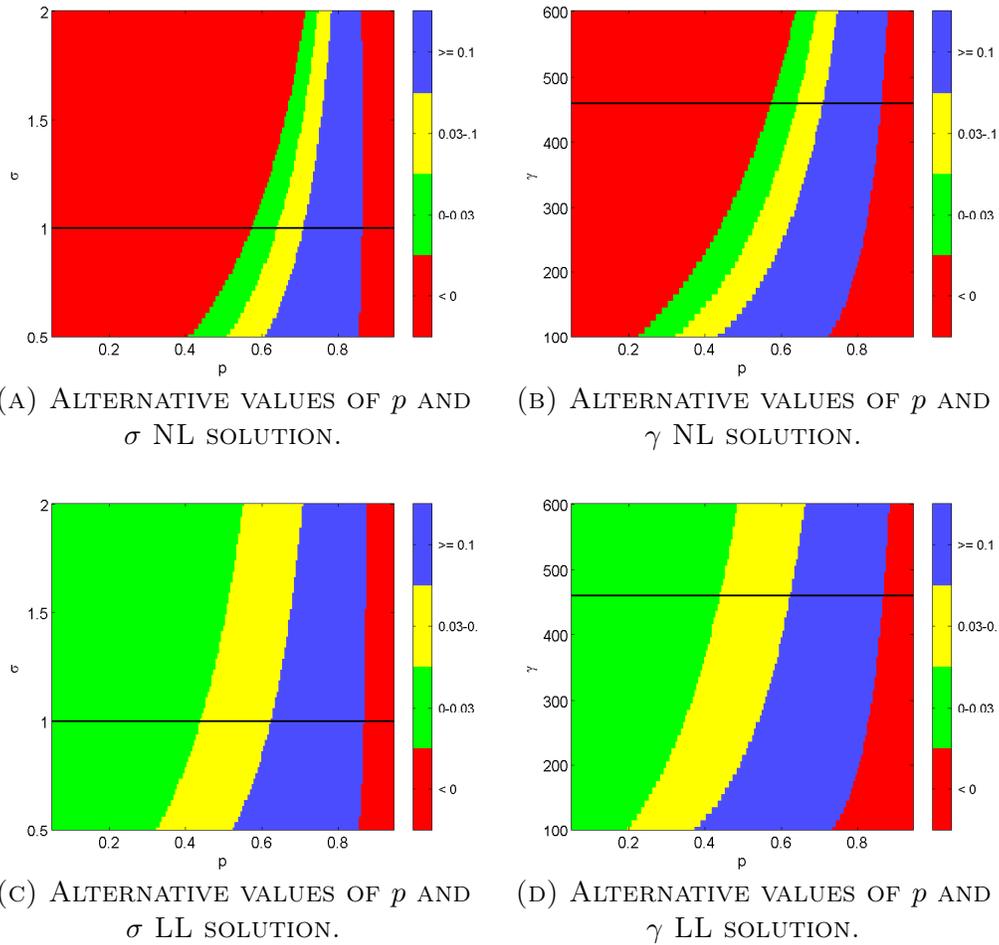
(C) LL EQUILIBRIUM: ALTERNATIVE COMBINATIONS OF p AND σ .



(D) LL EQUILIBRIUM: ALTERNATIVE COMBINATIONS OF p AND γ .

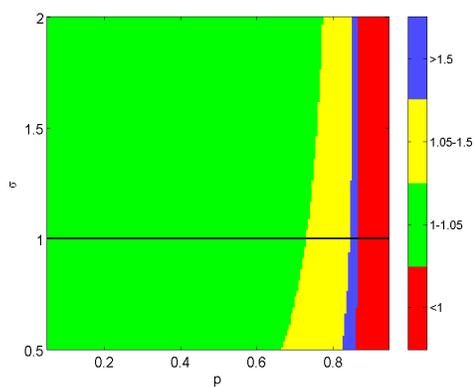
Notes: Red: Case I ($\text{slope}(AD) > 0 > \text{slope}(AS)$); Light Green: Case II ($\text{slope}(AS) > \text{slope}(AD) > 0$); Yellow: Case III ($\text{slope}(AS) > 0 > \text{slope}(AD)$); Blue: Case MZB (multiple zero bound equilibria); The baseline parameterization of the model is denoted with a black line.

FIGURE 4: THE LABOR TAX FISCAL MULTIPLIER AT ALTERNATIVE VALUES OF p , σ AND γ .

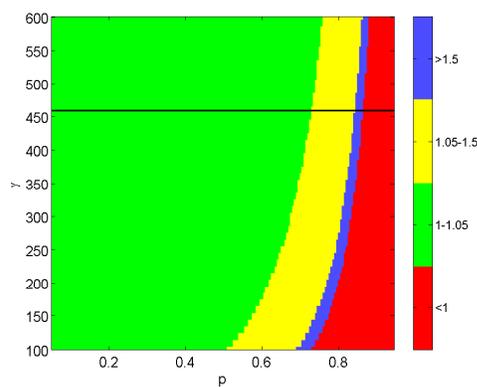


Notes: Red: Labor tax multiplier is negative (employment increases when the labor tax is cut); Green: Labor tax multiplier is in $[0, 0.03]$; Yellow: labor tax multiplier is in $(0.03, 0.1]$; Blue: labor tax multiplier exceeds 0.1. The black line denotes the baseline value of each parameter.

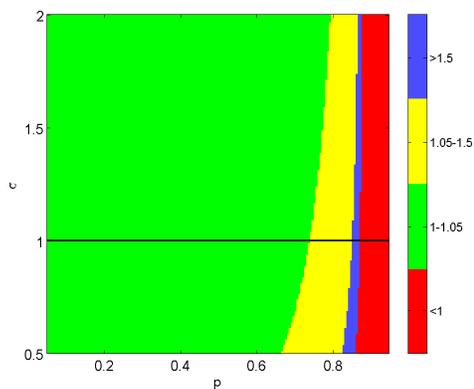
FIGURE 5: RESPONSE OF OUTPUT TO AN INCREASE IN GOVERNMENT PURCHASES AT ALTERNATIVE VALUES OF p , σ AND γ .



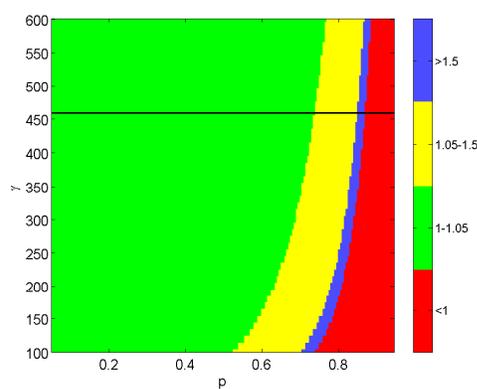
(A) ALTERNATIVE VALUES OF p AND σ NL SOLUTION.



(B) ALTERNATIVE VALUES OF p AND γ NL SOLUTION.



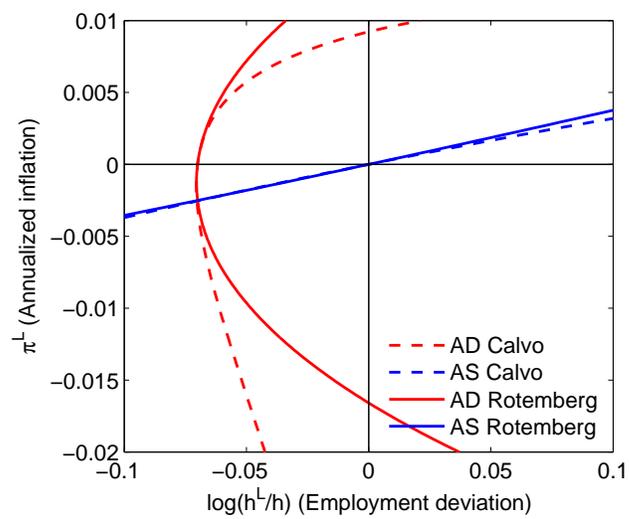
(C) ALTERNATIVE VALUES OF p AND σ LL SOLUTION.



(D) ALTERNATIVE VALUES OF p AND γ LL SOLUTION.

Notes: Red: the government-purchase-GDP-multiplier is less than 1; Green: the multiplier is in $[1, 1.05]$; Yellow: the multiplier is in $[1.05, 1.5]$, blue: the multiplier exceeds 1.5. The baseline parameterization is denoted with a line.

FIGURE 6: ZERO-BOUND EQUILIBRIA IN THE CALVO VS. ROTEMBERG MODEL
($p = 0.4$)



Online Appendix: Some unpleasant properties of loglinearized solutions when the nominal rate is zero.*

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*These views are our own and not those of the Federal Reserve System and the Bank of England.

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Appendix A Robustness Analysis

This section conducts several robustness checks. We report results for an alternative calibration strategy that holds the level of technology fixed and report results using parameterizations of our model that are based on [Christiano and Eichenbaum \(2012\)](#) and [Denes, Eggertsson, and Gilbukh \(2013\)](#). Our result that the NK model may exhibit orthodox and small fiscal multipliers at the zero bound continues to obtain when we use an alternative calibration scheme and consider other regions of the parameter space.

A.1 No-technology-shock calibration scheme

Most analyses of the zero bound that use the [Eggertsson and Woodford \(2003\)](#) Markov equilibrium also posit a single shock to demand. One advantage of that strategy is that employment is depressed in state L for the entire range of model parameters that we consider in the paper.

Our analysis focuses on parameterizations of the model that are empirically relevant in the sense that they all reproduce the GR declines in output and inflation. If we are to continue to reproduce these two facts with $z^L = z = 1$, we will need to adjust another parameter instead. We choose to adjust the Dixit-Stiglitz parameter, θ . Adjusting θ in conjunction with d^L has no effect on the AD schedule because θ is not an argument of the AD schedule. Thus, the local properties of the AD schedule are the same as before. It is downward sloping when p is sufficiently low and rotates to right as p is increased eventually turning positive. However, θ enters $slope(NKPC)$ and adjusting it in this way renders the AS schedule independent of p .¹ For our baseline parameterization of the model, $slope(AS) = 0.036$ using the no-technology-shock calibration scheme. This is about the same value of $slope(AS)$ that occurs using the baseline calibration scheme with technology shocks for $p \approx 0.415$.²

The fact that the AS curve no longer varies with p has two main consequences. The first consequence is that the locus of p 's where $slope(AD)$ and $slope(AS)$ become equal and then cross is shifted to the right. This modification enlarges the size of the Case I equilibrium region where the labor tax multiplier is unconventional. Comparing [Figure 1](#) with [Figure 4](#) in the paper we see that the Case I region starts at about the same value of p in both figures. However, using this alternative calibration scheme the size of the Case I region extends to about $p = 0.965$. This in turn shrinks the size of the two indeterminacy regions (Case II and MZB). The size of the Case III equilibrium regions is largely unaffected and it follows that there continues to be a large region where the LL solution yields the wrong slope of the AD

¹This calibration strategy implies that θ adjusts with $p\beta d^L$ in a way that keeps $\theta/(1 - p\beta d^L)$ constant.

²For this value of p , the calibrated z^L approximately equals $z = 1$.

schedule and thus produces an incorrect sign for the labor tax multiplier. This can be seen by comparing the upper panels of Figure 2 with the lower panels. In the upper panels that show the NL solution we see a large red region where employment increases when the labor tax is cut. The LL solution, in contrast, is green in this region indicating a paradox of toil.

The second consequence is that the AS curve is now flatter at higher values of p and the fiscal multipliers are correspondingly smaller. For instance, using our baseline parameterization the fiscal multipliers are smaller using the no-technology shock calibration scheme in comparison to our baseline calibration scheme when $0.415 < p \leq 0.863$. This effect can be readily discerned for the labor tax multiplier by comparing the upper panel of Figure 5 with the upper panel of Figure 2.³ It is even more pronounced for the government purchase multiplier. For instance the yellow region with government purchase multipliers between 1.05 and 1.5 begins at $p = 0.73$ in the upper panel of Figure 6 (in the paper) for our baseline parameterization. In Figure 3 the yellow region does not begin until p reaches a value of 0.86. In fact, the government purchase multiplier is less than 1.5 for all choices of $p \leq 0.95$ using the no-technology-shock calibration scheme. It is only in the immediate neighborhood of the asymptote, which occurs at $p \approx 0.965$, that the government purchase multiplier exceeds 1.5.

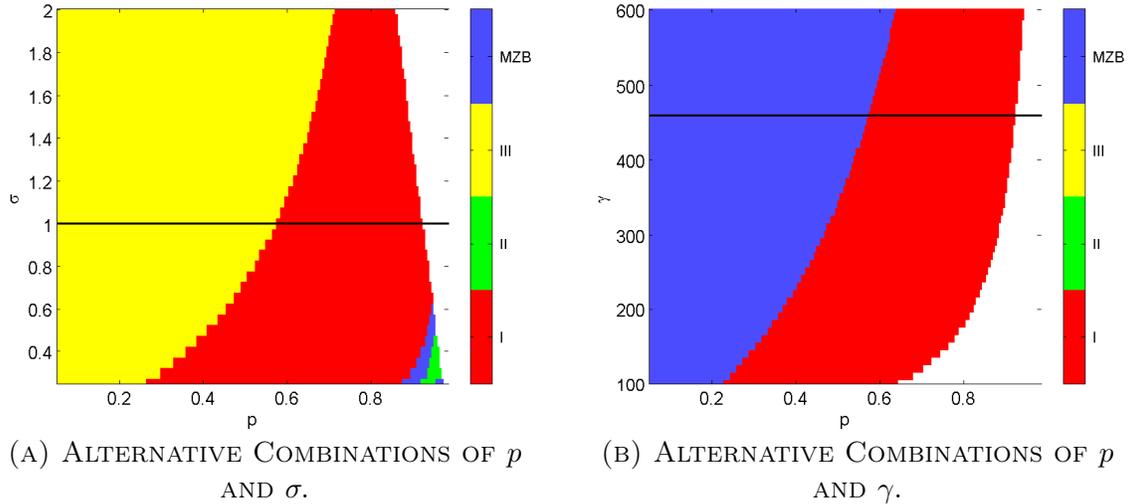
So far we have not said anything about the range of values taken on by θ . Higher values of p are associated with a smaller value of θ , and θ is declining in σ and γ . Some of the results reported in Figures 1–3 need $\theta < 1$ to hit the GR targets. These these regions are reported in white. Even though we can compute equilibria with values of $\theta < 1$ due to our subsidy scheme, θ in this range imply negative markups and are not of economic interest. To provide some indication about when this occurs suppose that γ is held fixed at its baseline value of 458.4 and $\sigma = 1$ then θ falls below 1 when p reaches 0.915. The associated values of the labor tax and government purchase multipliers are 0.56 and 1.1 respectively. But most estimates of θ are two or higher (see e.g. Broda and Weinstein (2004)). If we use our baseline parameterization of the model and limit attention to values of $\theta \geq 2$, then $p \geq 0.84$ are ruled out. Imposing this restriction implies that the government purchase GDP multiplier is 1.04 or less and that the labor tax multiplier is 0.17 or less.

In contrast to the calibration scheme with a technology shock, the no-technology-shock calibration scheme has the advantage that hours in state L are always below the steady-state. However, that calibration scheme imposes a restriction on θ instead. A common feature of both calibration schemes is that it is impossible to find empirically relevant parameter values for if $\sigma > 2$ and p is large.

Overall, the general pattern of results that emerges using this calibration scheme is consis-

³For instance, the blue region in Figure 5 (in the paper) with labor tax multipliers in excess of 1 begins at $p = 0.71$ for our baseline parameterization. The blue region in Figure 2, in contrast, begins at $p = 0.79$.

FIGURE 1: TYPES OF ZERO BOUND EQUILIBRIA FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: NO TECHNOLOGY SHOCK



Notes: Red: Case I ($\text{slope}(AD) > 0 > \text{slope}(AS)$); light Green: Case II ($\text{slope}(AS) > \text{slope}(AD) > 0$); yellow: Case III ($\text{slope}(AD) < 0 < \text{slope}(AS)$); blue: Case MZB (multiple zero bound equilibria); dark Green: Case IV ($\text{slope}(AD) > 0 > \text{slope}(AS)$); white: $\theta < 1$

tent with our previous results. We find large regions where LL solution produces an incorrect sign for the labor tax multiplier and the magnitudes of the multipliers are even smaller under this calibration scheme for many choices of p . Perhaps the biggest difference is that the size of the indeterminacy regions is much smaller now. This follows from the fact that a flatter AS schedule acts to push the asymptote as indexed by p to the right.

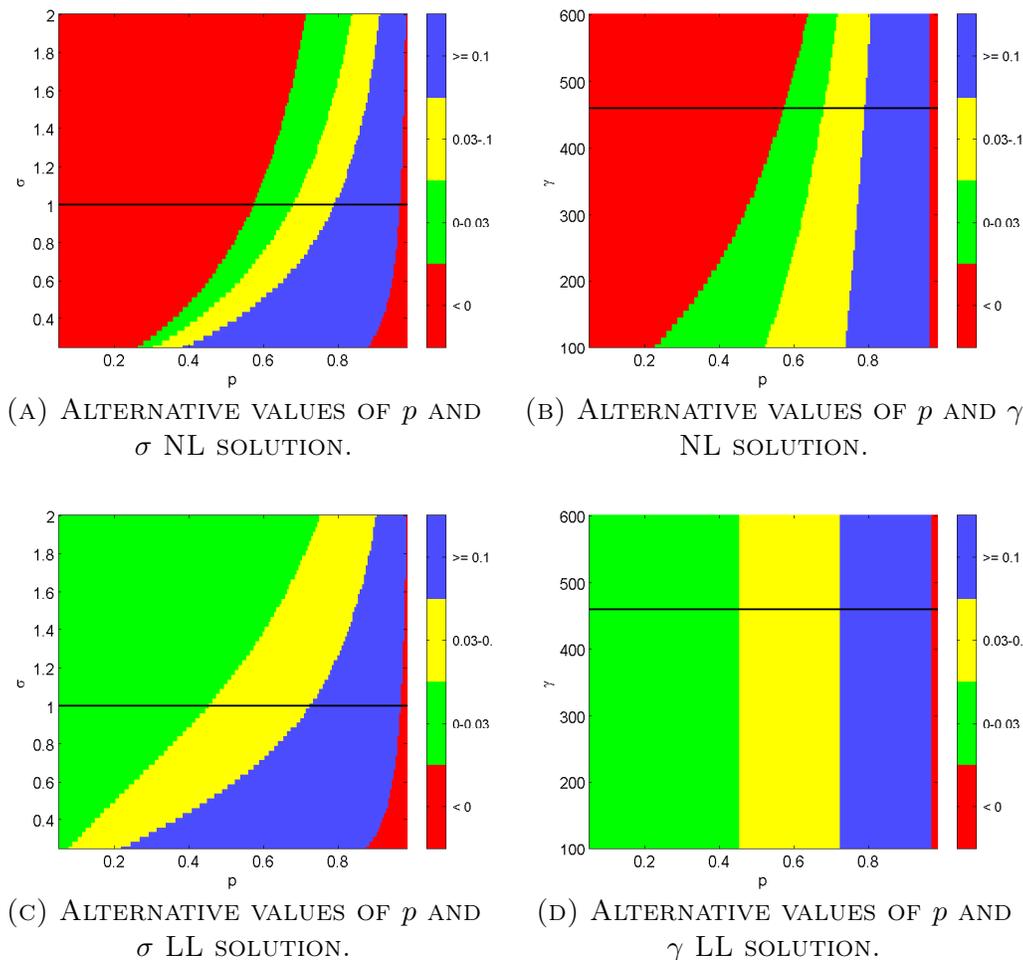
A.2 Accounting for the Great Recession with the parameterization of Christiano and Eichenbaum (2012)

Christiano and Eichenbaum (2012) find that the government purchase multiplier exceeds 2 in a nonlinear Rotemberg model that is very close to ours.⁴ In our model this can also occur but only in the neighborhood of the point where $\text{slope}(AD) = \text{slope}(AS)$. Moreover, in this neighborhood the sign and magnitudes of the fiscal multipliers are very sensitive to small perturbations of p and other structural parameters. It is thus interesting to investigate why their government purchase multipliers are so large.

Following their paper, we set the preference discount factor $\beta = 0.99$, the coefficient of relative risk aversion for consumption to $\sigma = 1$ and the curvature parameter for leisure to

⁴ Christiano, Eichenbaum, and Rebelo (2011) report similar results but it is easier for us to compare our results with the results of Christiano and Eichenbaum (2012) because they also posit Rotemberg adjustment costs.

FIGURE 2: TAX MULTIPLIER ON HOURS FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: NO TECHNOLOGY SHOCK



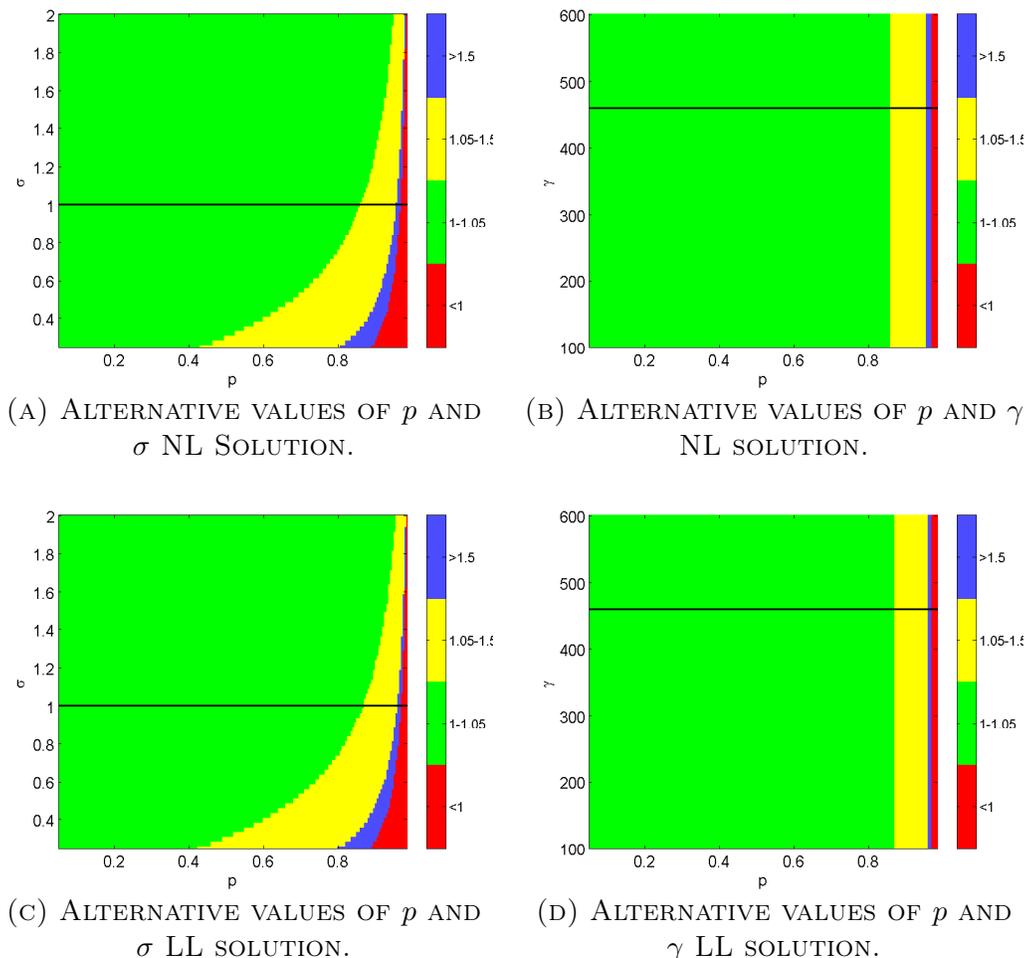
Notes: Red: labor tax multiplier is negative (employment increases when the labor tax is cut); green: labor tax multiplier is in $[0, 0.03]$; yellow: labor tax multiplier is in $(0.03, 1.0]$; blue: labor tax multiplier exceeds one; white: $\theta < 1$. The line denotes the baseline value of each parameter.

$\nu = 1$. The technology parameter θ that governs the substitutability of different types of goods is set to 3, the adjustment costs of price adjustment to $\gamma = 100$, and the conditional probability of exiting the low state to $p = 0.775$. The labor tax τ_w is set to 0.2, the government purchases share in output η to 0.2, and the subsidy to intermediate goods producers τ_s is set so that steady-state profits are zero. Finally, the coefficients on the Taylor rule are $\phi = 1.5$ and $\phi_y = 0$. With this parameterization our loglinearized system is identical to the one in [Christiano and Eichenbaum \(2012\)](#).⁵

We first examine whether some small differences in the nonlinear models are crucial for

⁵One difference between the models is that [Christiano and Eichenbaum \(2012\)](#) fix the level of government purchases as opposed to its share in output. However, the loglinearized systems are equivalent when one considers the same type of fiscal policy shock.

FIGURE 3: GOVERNMENT PURCHASE MULTIPLIER ON GDP FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: NO TECHNOLOGY SHOCK



Notes: Red: government-purchase-GDP-multiplier < 1 ; green: the multiplier is in $[1, 1.05]$; yellow: the multiplier is in $[1.05, 1.5]$; blue: the multiplier exceeds 1.5; white: $\theta < 1$. The baseline parameterization is denoted with a line.

the differences in results, by solving our model using their parameter values.⁶ It turns out the differences in the two models are inconsequential and we are able to reproduce the government purchase multipliers reported in [Christiano and Eichenbaum \(2012\)](#) by setting $d^L = 1.0118$. The resulting government purchase multiplier for GDP is 2.2 using the NL solution and 2.8 using the LL solution method.

The reason their government purchase multipliers are so large is because the [Christiano and Eichenbaum \(2012\)](#) parameterization has a very steep AS schedule. Their parameterization implies that $\text{slope}(NKPC)$ is about 0.06 which is about three times larger than our

⁶We assume that the resource costs of price adjustment apply to gross production ($\gamma\pi_t^2 y_t$) whereas they assume that the resource costs of price adjustment only apply to GDP ($\gamma\pi_t^2(c_t + g_t)$).

baseline value of 0.02.⁷ From Equation (19) we know that a larger value of $slope(NKPC)$ translates directly into a steeper AS curve. A steeper AS curve also results in a much larger inflation response to a d^L shock of a given size. In particular, their parameterization associates the 7% decline in output in the LL solution with a 7% decline in the annualized inflation rate. If we solve the model using the NL equilibrium conditions instead, output and the annualized inflation rate both fall by 5%. In fact, their value of $slope(NKPC)$ is so large that the model overstates the GR inflation targets for all values of p using either the LL or the NL equilibrium conditions.⁸

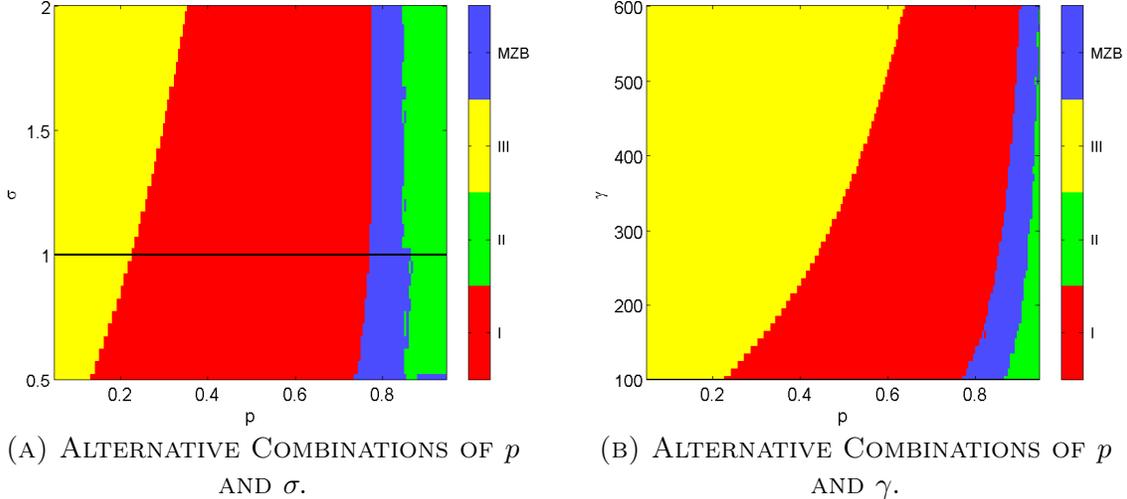
Given how different their results are from ours we would like to adjust their parameterization so that the model can hit the two GR targets using the NL solution. One way to do this is to hold fixed their choices of the structural parameters and thus $slope(NKPC)$ and to use the technology shock to hit the inflation rate. Results reported in Figures 4–7 implement this scheme. From Figure 4 we see that under this calibration strategy their model parameterization ($\sigma = 1, p = 0.775$, and $\gamma = 100$) falls in the indeterminacy region and is just to the left of the point as indexed by p where $slope(AS)$ becomes equal to $slope(AD)$ then crosses. The targeted equilibrium exhibits $slope(AD) > slope(AS) > 0$ and the resulting government purchase multiplier is 4.6. The non-targeted zero bound equilibrium exhibits $slope(AS) > slope(AD) > 0$ and there is a third equilibrium with a positive interest rate as shown in Figure 5. We have pointed out in the paper that the local properties of equilibrium and thus the sign and magnitude of the fiscal multipliers is very sensitive to the precise choice of parameters in this region of the parameter space. That result obtains here too. For instance, if p is increased from 0.775 to 0.79, the equilibrium switches to Case II and the government purchase multiplier is -10.0.

Christiano and Eichenbaum (2012) do not allow for technology shocks. So we also calibrate the model by adjusting θ and holding z fixed. This calibration scheme will also shift the asymptote to the right in the p dimension and it is possible that inferences about the size of the government purchase multiplier will be more robust. However, if we are to reproduce the GR facts using this calibration scheme we must also reduce $slope(NKPC)$ and in order to make the AS schedule flatter. We achieve this by lowering the value of θ to 1.24 and also increasing γ to 300. The reason why we have to adjust both of these parameters is because if we try to recalibrate their model by adjusting d^L and θ only, the resulting value of θ is less than 1 and thus not economically meaningful.

⁷Their parameterization implies $slope(NKPC) = 0.06$ when the share of government purchases in output is held fixed. If instead the level of government purchases is held fixed as they assume, $slope(NKPC)$ is 0.0675.

⁸This result also occurs if we set d^L to produce a 7% decline in output using the NL equilibrium conditions.

FIGURE 4: TYPES OF ZERO BOUND EQUILIBRIA FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITH TECHNOLOGY SHOCK



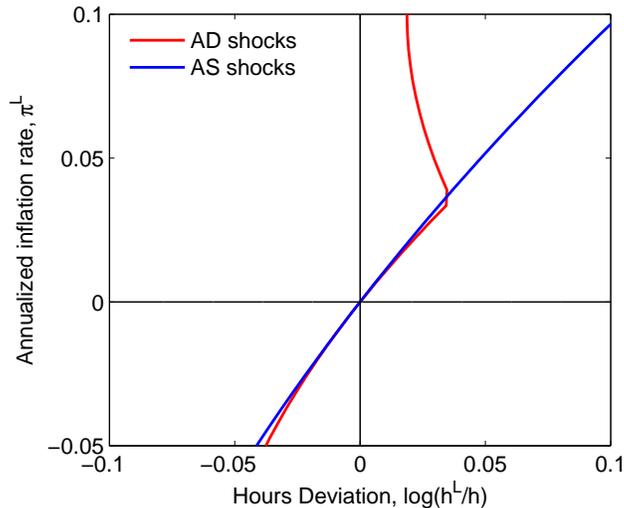
Notes: Red: Case I ($\text{slope}(AD) > 0 > \text{slope}(AS)$); light green: Case II ($\text{slope}(AS) > \text{slope}(AD) > 0$); yellow: Case III ($\text{slope}(AD) < 0 < \text{slope}(AS)$); blue; Case MZB (multiple zero bound equilibria); dark green: Case IV ($\text{slope}(AD) > 0 > \text{slope}(AS)$).

Figures 8-10 report results using the no-technology-shock calibration scheme. We saw above that the no-technology shock calibration scheme resulted in lower fiscal multipliers for larger values of p because $\text{slope}(AS)$ is independent of p .⁹ This is also true here. For instance, the equilibrium is now determinate and of type Case II and the government purchase multiplier is 1.06 when ($\sigma = 1, \theta = 1.24, \gamma = 300, p = 0.775$). In fact, the government purchase multiplier only exceeds 1.5 when $p \in [0.94, 0.965]$ but in that area, $\theta < 1$. (8).

The no-technology-shock results have several other noteworthy features. Now Case MZB equilibria occur at higher values of σ even when p is small and far away from the asymptote. The targeted zero bound equilibrium in this region continues to have $\text{slope}(AD) < 0$ and $\text{slope}(AS) > 0$ and it follows that the labor tax multiplier has an orthodox sign in this entire region (Figure 9). In the non-target equilibrium inflation and output exceed their steady-state levels but it is still a ZLB equilibrium because d^L is large. Note also that the overall size of the region with a downward-sloping AD schedule is smaller in Figure 8 as compared to our baseline calibration without technology shocks reported in the left panel of Figure 1. This is because the value of $\gamma = 300$ is lower than our baseline value of 458.4. The pattern of equilibria when γ is varied is qualitatively similar in the right panels of Figures 1 and 8. Most of the results with $\gamma < 300$ are not economically meaningful because the associated

⁹The value of $\text{slope}(AS) = 0.0342$ using our Christiano and Eichenbaum (2012) reference parameterization of the model: ($\sigma = 1, \theta = 1.24, \gamma = 300, p = 0.775$).

FIGURE 5: ZERO-LOWER BOUND EQUILIBRIUM AT $p = 0.775$: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITH TECHNOLOGY SHOCK



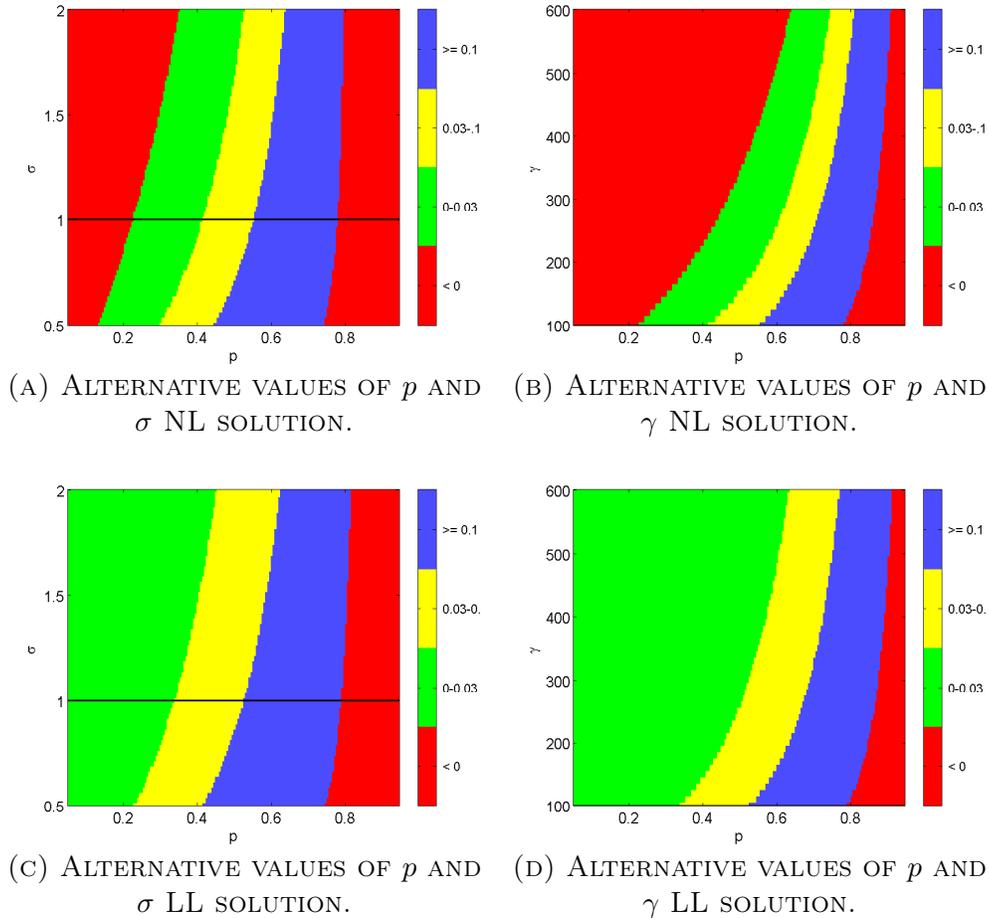
value of $\theta < 1$. Still, θ is positive and we can compute an equilibrium due to the tax subsidy. Thus, to facilitate comparison with our other results, the right panel of figures 8 - 10 also report values of γ that are less than 300. At higher levels of γ the LL solution significantly overstates the size of the region where the sign of the employment response to a tax cut is unorthodox. In regions where the NL solution indicates that the equilibrium is of Case I and thus unorthodox, the LL solution overstates the size of the labor tax.

A.3 Accounting for the Great Recession with the parameterization of Denes, Eggertsson and Gilbukh (2013)

We now consider the parameterization of Denes, Eggertsson, and Gilbukh (2013). Their estimated parameterization is interesting because their estimates imply a much smaller value of $\text{slope}(NKPC) = 0.0075$ than we have considered up to this point.

Denes, Eggertsson, and Gilbukh (2013) consider a NK framework with Calvo price adjustment and firm specific labor markets and a single shock to d^L . This is a different model from ours and the results that follow should not be interpreted as saying anything about their structural model. The common link between their model and ours is that they solve their model using the LL solution method we described in the paper and the loglinearized reduced form of their model and is identical to ours. They estimate their model parameters using an overidentified Quasi-Bayesian method of moments procedure with two moments that they associate with the GR: an output decline of 10% and an (annualized) decline in the inflation rate of -2%. The resulting estimates are: $(p, \theta, \beta, \sigma, \nu) = (0.857, 13.23, 0.997, 1.22, 1.69)$.

FIGURE 6: LABOR TAX MULTIPLIER ON EMPLOYMENT FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITH TECHNOLOGY SHOCK



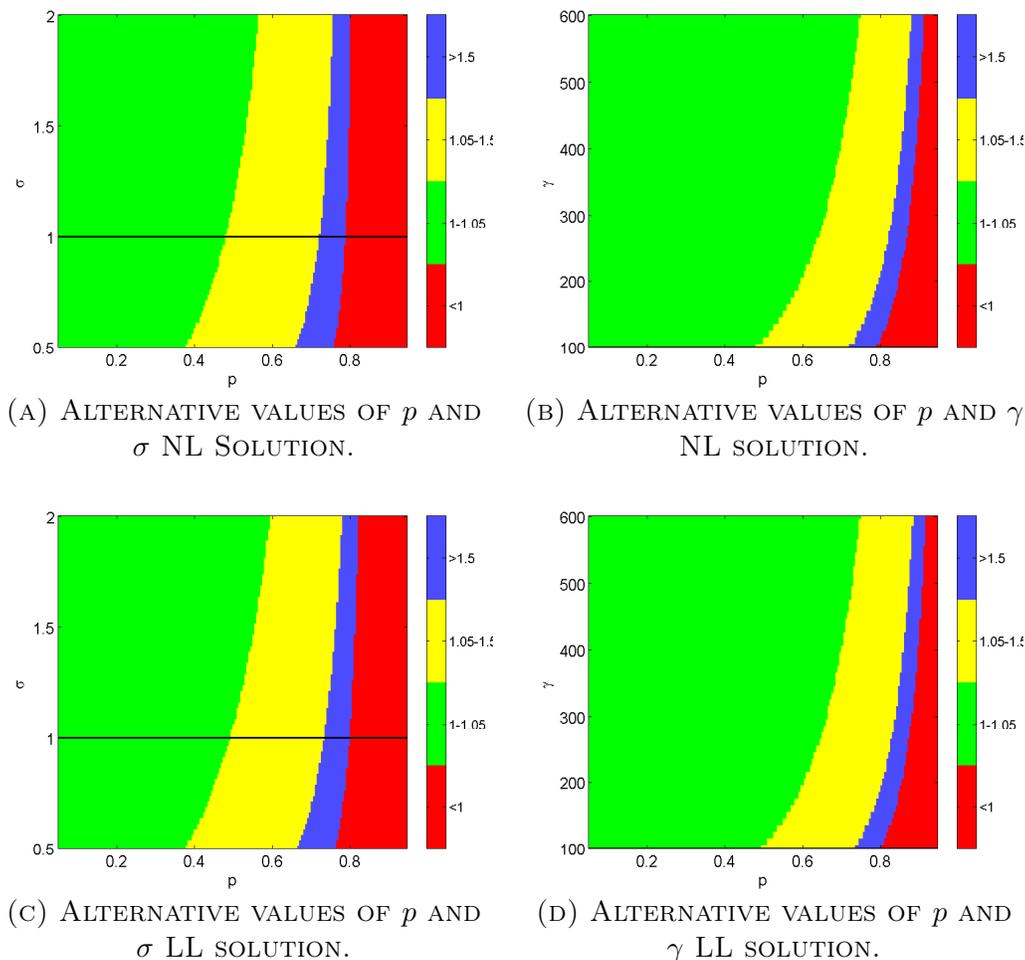
Notes: Red: labor tax multiplier is negative (employment increases when the labor tax is cut); green: labor tax multiplier is in $[0, 0.03]$; yellow: labor tax multiplier is in $(0.03, 1.0]$; blue: labor tax multiplier exceeds one. The baseline values of each parameter are denoted with a black line.

Finally, their fiscal parameters are set in the same way that we have assumed up to now.

We are interested in understanding the properties of our model in this region of the parameter space. However, in order to do that we must first make some small adjustments to their estimates. Our practice has been to calibrate the model using our *nonlinear* equilibrium conditions. Here we adjust d^L and γ to reproduce our GR inflation and output targets using our NL equilibrium conditions.¹⁰ The resulting value of $\gamma = 6341$. These adjustments have only a very small effect on $\text{slope}(NKPC)$. It rises from 0.0075 using the estimated parameterization of [Denes, Eggertsson, and Gilbukh \(2013\)](#) to 0.0086 using our calibrated

¹⁰The value of γ implied by their estimated reduced form is very large and calibrating our model in this way brings the value of γ down a bit and allows us to use their estimated value of θ as a reference point.

FIGURE 7: GOVERNMENT PURCHASE MULTIPLIER ON GDP FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITH TECHNOLOGY SHOCK

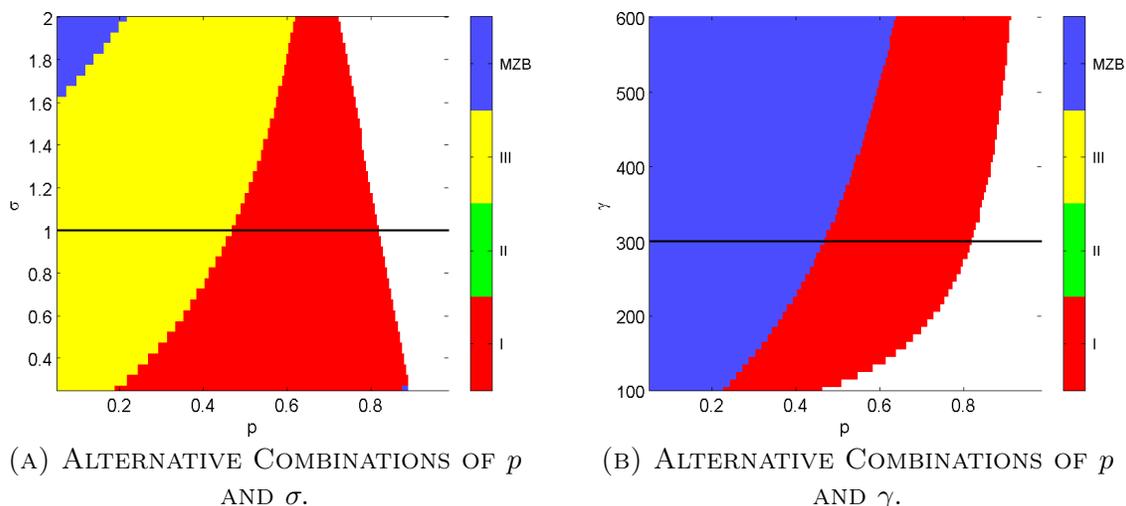


Notes: Red: government-purchase-GDP-multiplier < 1 ; green: the multiplier is in $[1, 1.05]$; yellow: the multiplier is in $[1.05, 1.5]$; blue: the multiplier exceeds 1.5. The baseline parameterization is denoted with a line.

values of d^L and γ .

Why is γ so large for this parameterization? Our discussion in Section 4.1 and 4.2 of the paper implies that $\text{slope}(NKPC)$ must be small to produce a Case 1 equilibrium when p is large. Indeed, the value of $\text{slope}(NKPC)$ here is less than half the size implied by our baseline calibration. To understand why γ is large observe that the [Denes, Eggertsson, and Gilbukh \(2013\)](#) estimates of the other parameters in $\text{slope}(NKPC)$ including θ , σ , and ν are all much higher than our estimates. The fact that these other parameters are so large implies that γ must also be very large if $\text{slope}(NKPC)$ is to be small enough to reproduce the GR targets at high values of p , see also Appendix .

FIGURE 8: TYPES OF ZERO BOUND EQUILIBRIA FOR ALTERNATIVE VALUES OF PRICE ADJUSTMENT COSTS AND RISK AVERSION: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITHOUT TECHNOLOGY SHOCK



Notes: Red: Case I ($\text{slope}(AD) > 0 > \text{slope}(AS)$); light green: Case II ($\text{slope}(AS) > \text{slope}(AD) > 0$); yellow: Case III ($\text{slope}(AD) < 0 < \text{slope}(AS)$); blue: Case MZB (multiple zero bound equilibria); dark green: Case IV ($\text{slope}(AD) > 0 > \text{slope}(AS)$); white: $\theta < 1$.

The most noteworthy new property of the model is shown in Figure 11.¹¹ The Case III region is now much smaller and instead there is a very large Case MZB region at low values of p . The size of this region increases with σ and γ . Throughout this region there are two zero bound equilibria, the targeted equilibrium has $\text{slope}(AD) < 0 < \text{slope}(AS)$ and the second has $0 < \text{slope}(AD) < \text{slope}(AS)$.

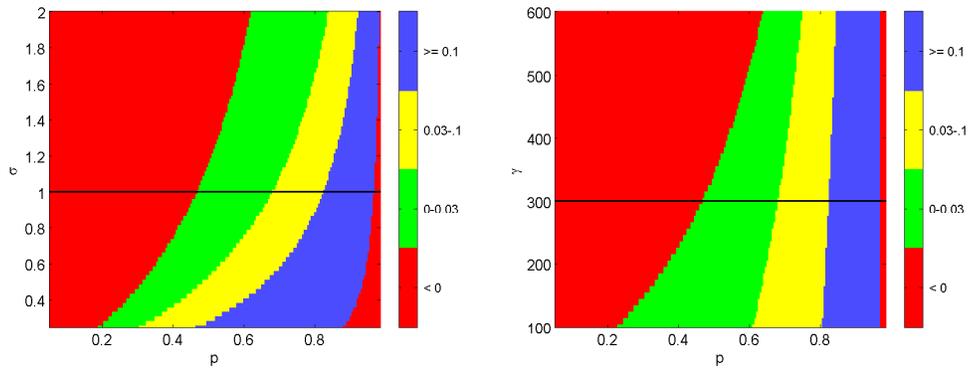
The model has a unique Case III equilibrium at the reference value of $p = 0.857$, estimated by Denes, Eggertsson, and Gilbukh (2013). This implies that employment increases in response to a cut in the labor tax (Figure 12). Using the LL solution one would conclude instead that there is a paradox of toil and that it is large (0.11).

The size of the government purchase multiplier using the NL equilibrium conditions is 1.08 at the reference parameterization and with marginally higher σ it would fall below 1.05 (Figure 13). There are some larger differences between the NL and LL government purchase multipliers here. The LL solution produces larger government purchase multipliers at lower values of p . However, once again we see that a government purchase multiplier in excess of 1.5 only occurs in a very small region of the parameter space as indexed by p , σ and γ .

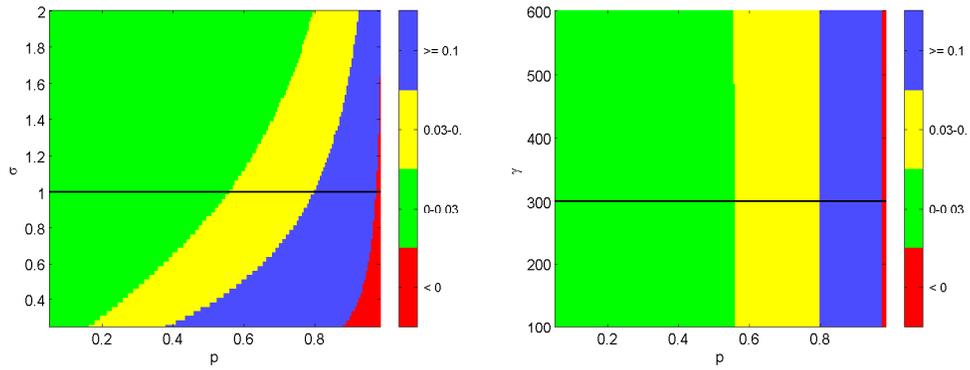
To summarize, the results we have reported here are consistent with the message of our paper. The NK model may also exhibit orthodox and small fiscal multipliers at the zero

¹¹To conserve on space we only report figures using the no-technology-shock calibration scheme for this parameterization of our model.

FIGURE 9: TAX MULTIPLIER ON EMPLOYMENT FOR ALTERNATIVE VALUES OF RISK AVERSION: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITHOUT TECHNOLOGY SHOCK



(A) ALTERNATIVE VALUES OF p AND σ NL SOLUTION. (B) ALTERNATIVE VALUES OF p AND γ NL SOLUTION.

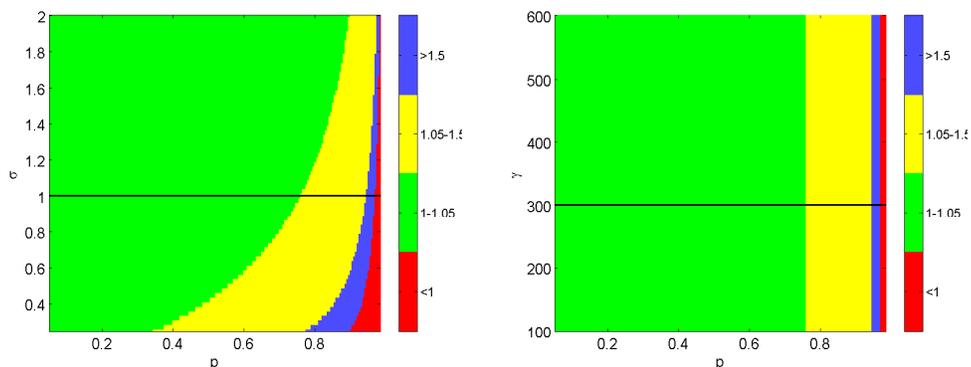


(C) ALTERNATIVE VALUES OF p AND σ LL SOLUTION. (D) ALTERNATIVE VALUES OF p AND γ LL SOLUTION.

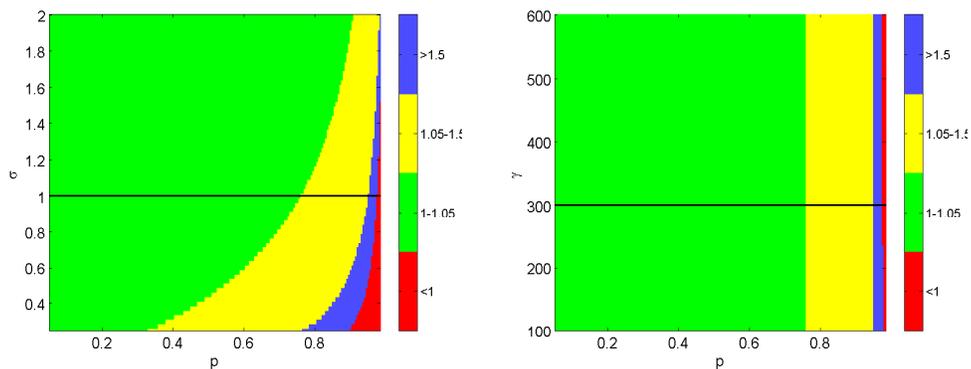
Notes: Red: labor tax multiplier is negative (employment increases when the labor tax is cut); green: labor tax multiplier is in $[0, 0.03]$; yellow: labor tax multiplier is in $(0.03, 1.0]$; blue: labor tax multiplier exceeds one; white: $\theta < 1$. The line denotes the baseline value of each parameter.

bound in these other regions of the parameter space.

FIGURE 10: GOVERNMENT PURCHASE MULTIPLIER ON GDP FOR ALTERNATIVE VALUES OF RISK AVERSION: CHRISTIANO-EICHENBAUM (2012) PARAMETERIZATION WITHOUT TECHNOLOGY SHOCK



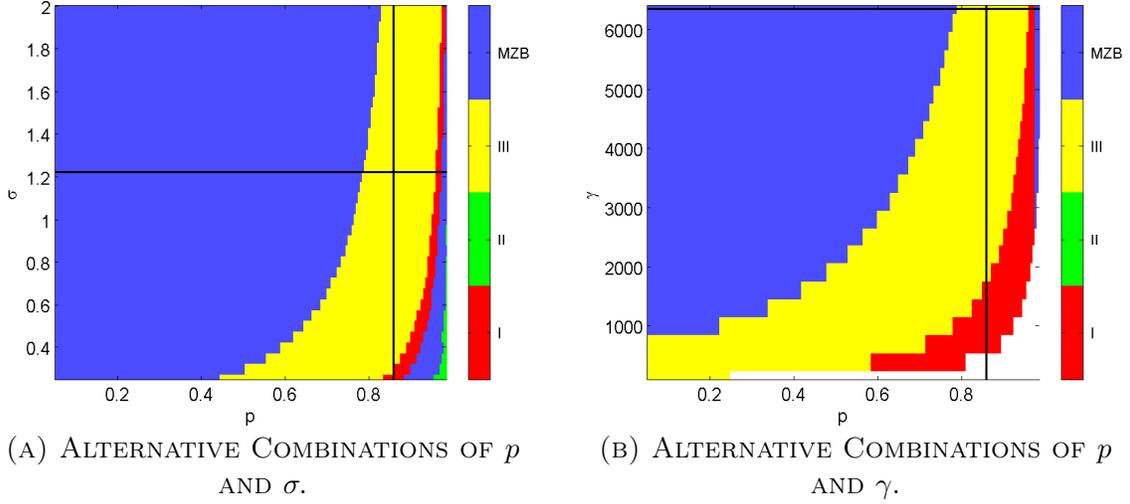
(A) ALTERNATIVE VALUES OF p AND σ NL SOLUTION. (B) ALTERNATIVE VALUES OF p AND γ NL SOLUTION.



(C) ALTERNATIVE VALUES OF p AND σ LL SOLUTION. (D) ALTERNATIVE VALUES OF p AND γ LL SOLUTION.

Notes: Red: government-purchase-GDP-multiplier < 1 ; green: the multiplier is in $[1, 1.05]$; yellow: the multiplier is in $[1.05, 1.5]$, blue: the multiplier exceeds 1.5; white: $\theta < 1$. The baseline parameterization is denoted with a line.

FIGURE 11: TYPES OF ZERO BOUND EQUILIBRIA FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: DENES ET AL. (2013).
PARAMETERIZATION WITH NO TECHNOLOGY SHOCKS.



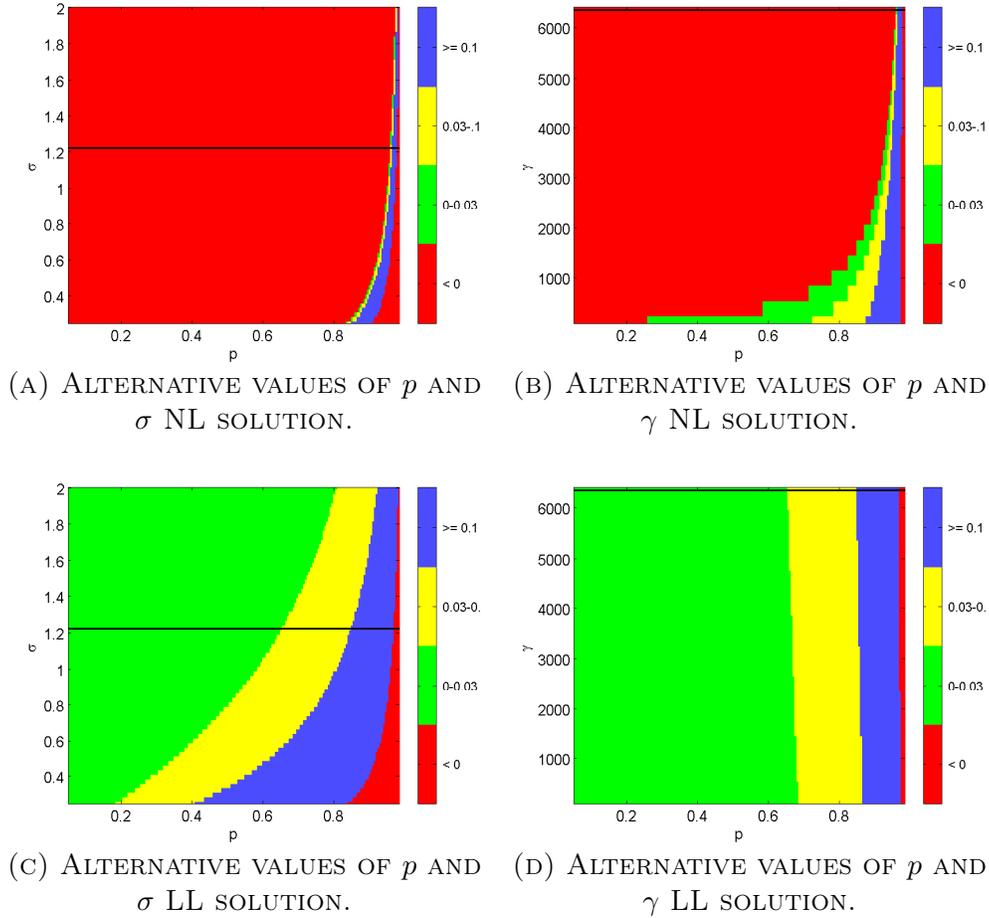
Notes: Red: Case I ($\text{slope(AD)} > 0 > \text{slope(AS)}$); light green: Case II ($\text{slope(AS)} > \text{slope(AD)} > 0$); yellow: Case III ($\text{slope(AD)} < 0 < \text{slope(AS)}$); blue: Case MZB (multiple zero bound equilibria); dark green: Case IV ($\text{slope(AD)} > 0 > \text{slope(AS)}$); white: $\theta < 1$.

Appendix B Calvo model with a single labor market

This section presents the equilibrium conditions in the Calvo model with a single labor market. They are given by

$$\begin{aligned}
 c_t^\sigma h_t^\nu &= w_t(1 - \tau_{w,t}), \\
 1 &= \beta d_{t+1} E_t \left\{ \frac{1 + R_t}{1 + \pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^\sigma \right\} \\
 gdp_t &= \frac{1}{x_t} z_t h_t, \\
 c_t &= \left(\frac{1}{x_t} - \eta_t \right) z_t h_t, \\
 as_{1,t} &= gdp_t + \beta \alpha d_{t+1} E_t \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (1 + \pi_{t+1})^{\theta-1} as_{1,t+1} \right], \\
 as_{2,t} &= \frac{c_t^\sigma h_t^\nu}{(1 - \tau_{w,t})} \frac{gdp_t}{z_t} + \beta \alpha d_{t+1} E_t \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (1 + \pi_{t+1})^\theta as_{2,t+1} \right], \\
 \tilde{P}_t &= \frac{as_{2,t}}{as_{1,t}}, \\
 x_t &= (1 - \alpha) \tilde{P}_t^{-\theta} + \alpha (1 + \pi_t)^\theta x_{t-1}, \\
 1 &= (1 - \alpha) \tilde{P}_t^{1-\theta} + \alpha (1 + \pi_t)^{\theta-1} \\
 R_t &= \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \widehat{gdp}_t)
 \end{aligned}$$

FIGURE 12: TAX MULTIPLIER ON EMPLOYMENT FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: DENES ET AL. (2013).
PARAMETERIZATION WITH NO TECHNOLOGY SHOCKS

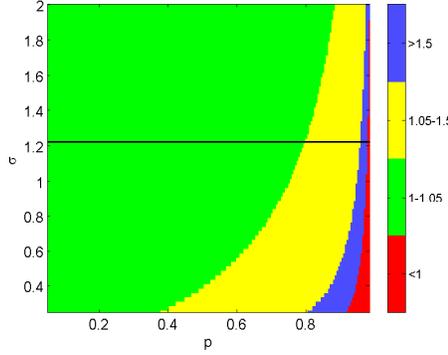


Notes: Red: labor tax multiplier is negative (employment increases when the labor tax is cut); green: labor tax multiplier is in $[0, 0.03]$; yellow: labor tax multiplier is in $(0.03, 1.0]$; blue: labor tax multiplier exceeds one; white: $\theta < 1$. The line denotes the baseline value of each parameter.

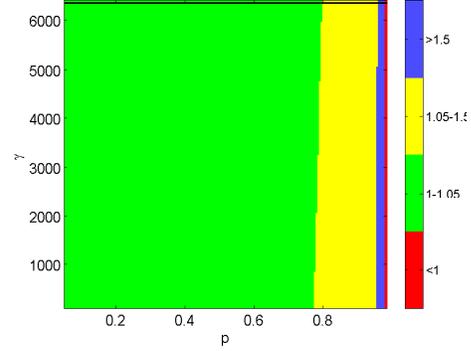
where \tilde{P}_t is the real price which is chosen by firms that can change their nominal prices at time t , x_t summarizes the relative price dispersion, and α is the probability that a firm is unable to change its price. The term $1/x_t$ introduces the wedge between GDP (y_t) and gross output ($z_t h_t$), and $1 - 1/x_t$ acts as κ_t in our baseline Rotemberg model. In NK models with Calvo price setting, there is a non-degenerate relative price distribution and as a result the allocation of the factors of production (labor in this model) is inefficient, i.e. higher price dispersion reduces GDP compared to the maximal production level that is possible with the same level of factor input.

Because x_t is a state variable, the equilibrium conditions cannot be summarized by the AD and the AS schedules without any additional simplifying assumptions. We make the following

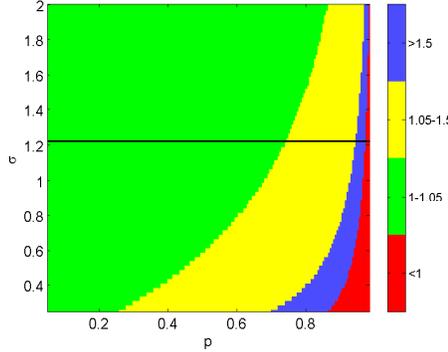
FIGURE 13: GOVERNMENT PURCHASE MULTIPLIER ON GDP FOR ALTERNATIVE VALUES OF RISK AVERSION AND PRICE ADJUSTMENT COSTS: DENES ET AL. (2013) PARAMETERIZATION WITH NO TECHNOLOGY SHOCKS, NONLINEAR (TOP) VS. LOGLINEAR (BOTTOM)



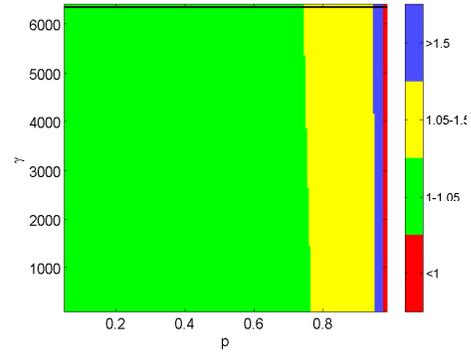
(A) ALTERNATIVE VALUES OF p AND σ NL SOLUTION.



(B) ALTERNATIVE VALUES OF p AND γ NL SOLUTION.



(C) ALTERNATIVE VALUES OF p AND σ LL SOLUTION.



(D) ALTERNATIVE VALUES OF p AND γ LL SOLUTION.

Notes: Red: government-purchase-GDP-multiplier < 1 ; green: the multiplier is in $[1, 1.05]$; yellow: the multiplier is in $[1.05, 1.5]$; blue: the multiplier exceeds 1.5; white: $\theta < 1$. The baseline parameterization is denoted with a line.

assumption: x is constant at x^L when the shocks are (d^L, z^L) and becomes 1 immediately after the shocks dissipate. This allows us to use the AD and the AS schedules to characterize a zero bound equilibrium.

Once the shocks dissipate, all variables jump to the zero inflation steady-state, where $x = 1$, $gdp = h = \{(1 - \tau_w)/(1 - \eta)^\sigma\}^{1/(\sigma+\nu)}$, $as_1 = h/(1 - \beta\alpha)$, $as_2 = (1 - \eta)^\sigma h^{1+\sigma+\nu}/\{(1 - \beta\alpha)(1 - \tau_w)\}$, and $\tilde{P} = 1$. In the L state, by assumption, x^L can be written as

$$x^L = \frac{(1 - \alpha)(\tilde{P}^L)^{-\theta}}{1 - \alpha(1 + \pi^L)^\theta} = \frac{1 - \alpha}{1 - \alpha(1 + \pi^L)^\theta} \left\{ \frac{1 - \alpha(1 + \pi^L)^{\theta-1}}{1 - \alpha} \right\}^{\frac{\theta}{\theta-1}}.$$

The AD schedule is identical to the AD schedule in the Rotemberg model, except that the

term κ^L is now equal to $(x^L - 1)/x^L$ (equation (13) in the main paper).

The AS schedule is a little bit more complicated. First observe

$$\tilde{P}^L = \frac{(c^L)^{-\sigma} a s_2^L}{(c^L)^{-\sigma} a s_1^L}.$$

Then using

$$(c^L)^{-\sigma} a s_1^L = (c^L)^{-\sigma} g d p^L + \beta \alpha d^L \{p(c^L)^{-\sigma} (1 + \pi^L)^{\theta-1} a s_1^L + (1-p)c^{-\sigma} a s_1\}$$

and

$$(c^L)^{-\sigma} a s_2^L = \frac{(c^L)^\sigma (h^L)^\nu}{(1 - \tau_w^L)} \frac{g d p^L}{z^L} + \beta \alpha d^L \{p(c^L)^{-\sigma} (1 + \pi^L)^\theta a s_2^L + (1-p)c^{-\sigma} a s_2\},$$

we obtain ¹²

$$\tilde{P}^L = G(h^L, \pi^L).$$

Because

$$\tilde{P}^L = \left\{ \frac{1 - \alpha(1 + \pi^L)^{\theta-1}}{1 - \alpha} \right\}^{\frac{1}{1-\theta}},$$

the AS schedule is characterized by

$$\left\{ \frac{1 - \alpha(1 + \pi^L)^{\theta-1}}{1 - \alpha} \right\}^{\frac{1}{1-\theta}} = G(h^L, \pi^L).$$

Figure 14 shows the AD-AS diagram for $p = 0.8$ and $p = 0.9$. For both cases the AD and the AS schedules are upward sloping, but as in the Rotemberg model the slope configurations switch: the AD schedule is steeper in the former case but is flatter in the latter case.

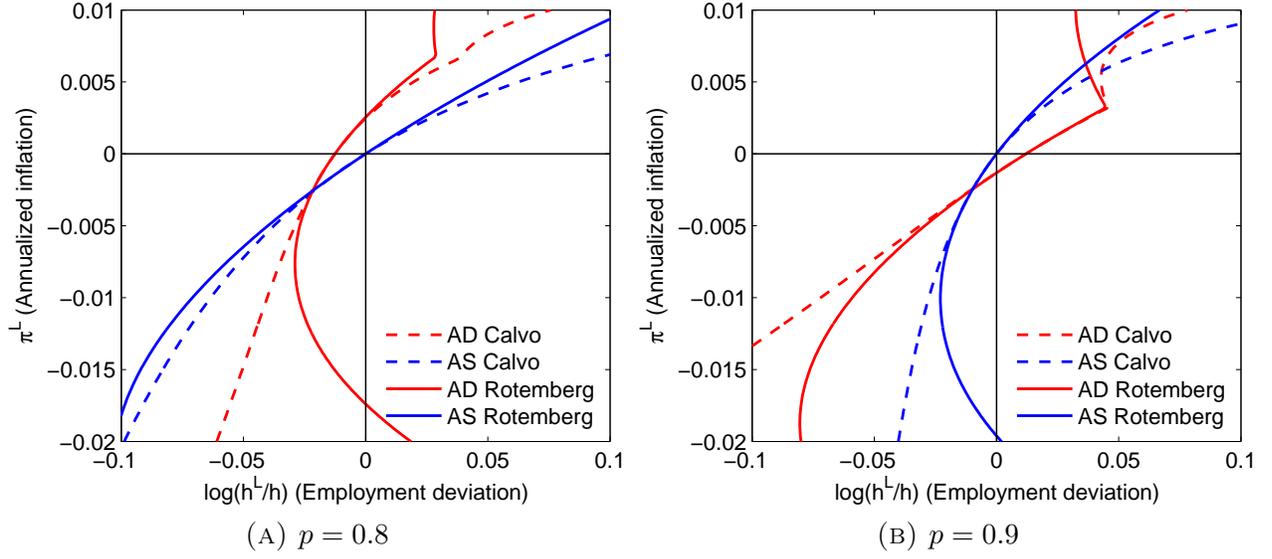
Appendix C Existence of a zero bound equilibrium in the LL model

To make the argument more transparent we assume that $\hat{\eta}^L = \hat{\tau}_w^L = 0$.

Proposition 1 Existence of a zero bound equilibrium in the LL model. *Suppose $\hat{\eta}^L = \hat{\tau}_w^L = 0$, $(\phi_\pi, \phi_y) \geq (p, 0)$, $\hat{d}^L \geq 0$, $\hat{z}^L \leq 0$, $0 < p \leq 1$, $\sigma \geq 1$, and that AD^{LL} and AS^{LL}*

¹²This is because both c^L and $g d p^L$ can be expressed by x^L , h^L , and exogenous variables and parameters, and because x^L is a function of π^L .

FIGURE 14: ZERO-BOUND EQUILIBRIA IN THE CALVO VS. ROTEMBERG MODEL



do not coincide in state L . Then there exists a unique zero bound equilibrium with deflation and depressed labor input, $(\pi^L, \hat{h}^L) < (0, 0)$, if

Case I

1a) $\text{slope}(AD^{LL}) > \text{slope}(AS^{LL})$ and

1b) $(\text{slope}(AD^{LL}) - \text{slope}(AS^{LL}) \frac{\sigma-1}{\sigma+\nu}) \hat{z}^L - \frac{\hat{r}_L^e}{p} > 0$,

or

Case II

2a) $\text{slope}(AD^{LL}) < \text{slope}(AS^{LL})$ and

2b) $(\text{slope}(AD^{LL}) - \text{slope}(AS^{LL}) \frac{\sigma-1}{\sigma+\nu}) \hat{z}^L - \frac{\hat{r}_L^e}{p} < 0$.

If the parameters do not satisfy either both 1a) and 1b) or alternatively both 2a) and 2b), then there is no zero bound equilibrium with depressed labor input $\hat{h}^L < 0$. ¹³

¹³The final statement leaves open the possibility that a zero bound equilibrium with $\hat{h}^L \geq 0$ exists for parameterizations that satisfy 1a) and 2b) (or 1b) and 2a)). This is possible when z^L is sufficiently low. If it is assumed that $\hat{z}^L = 0$, then the final clause can be removed and *any* ZLB equilibrium must satisfy both (1a) and (1b) or both (2a) and (2b). For further details see [Braun, Körber, and Waki \(2012\)](#).

Proof The AD and the AS schedules are

$$\pi^L = [\text{slope}(AD)\hat{z}^L - \frac{\hat{r}_L^e}{p}] + \text{slope}(AD)\hat{h}^L, \text{ and } \pi^L = \text{slope}(AS)\frac{\sigma-1}{\sigma+\nu}\hat{z}^L + \text{slope}(AS)\hat{h}^L.$$

They are upward-sloping, for both $\text{slope}(AD)$ and $\text{slope}(AS)$ are positive.

First, assume (1a) and (1b). They imply that the AD schedule is strictly steeper than the AS schedule, and that the intercept term is strictly higher for the AD schedule than for the AS schedule. It follows that at the intersection $\hat{h}^L < 0$. Solving for π^L , we obtain

$$\begin{aligned} \pi^L &= \frac{1}{\text{slope}(AS) - \text{slope}(AD)} \left[\text{slope}(AS)\left\{\text{slope}(AD)\hat{z}^L - \frac{\hat{r}_L^e}{p}\right\} - \text{slope}(AD)\text{slope}(AS)\frac{\sigma-1}{\sigma+\nu}\hat{z}^L \right] \\ &= \frac{\text{slope}(AS)}{\text{slope}(AS) - \text{slope}(AD)} \left[\text{slope}(AD)\hat{z}^L + \text{slope}(AD)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}_L^e}{p} \right]. \end{aligned}$$

Since $\text{slope}(AS) - \text{slope}(AD) < 0$, π^L is negative at the intersection if and only if the terms in the square brackets are positive.

$$\begin{aligned} &\text{slope}(AD)\hat{z}^L + \text{slope}(AD)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}_L^e}{p} \\ &\geq \text{slope}(AD)\hat{z}^L + \text{slope}(AS)\frac{\sigma-1}{\sigma+\nu}(-\hat{z}^L) - \frac{\hat{r}_L^e}{p} \\ &\quad (\text{By assumption, } (\sigma-1)(-\hat{z}^L) \geq 0 \text{ and } \text{slope}(AS) - \text{slope}(AD) < 0.) \\ &> 0. \quad (\text{By condition 1a).} \end{aligned}$$

Thus, at the intersection of the AD and the AS schedules, $(\pi^L, \hat{h}^L) < (0, 0)$.

What remains to show is that at the intersection, the Taylor rule implies zero nominal interest rate. The linear part of the Taylor rule prescribes

$$\begin{aligned} &\hat{r}^e + \phi_\pi \pi^L + \phi_y \hat{y}^L \\ &< p \left(\text{slope}(AD) - \text{slope}(AS)\frac{\sigma-1}{\sigma+\nu} \right) \hat{z}^L + \phi_\pi \pi^L + \phi_y \hat{y}^L. \quad (\text{Condition 1a).} \end{aligned}$$

Since $\hat{y}^L = \hat{z}^L + \hat{h}^L$, we know that $(\hat{z}^L, \pi^L, \hat{y}^L)$ are all negative. Condition 1b) implies that the coefficient on \hat{z}^L is strictly positive. Together with the assumption $(\phi_\pi, \phi_y) \geq 0$, it follows that the RHS of the above inequality is strictly negative, and the nominal interest

rate at the intersection is zero.

Next, assume (2a) and (2b). Proof is almost the same as that in the case with (1a) and (1b). The only difference is that ϕ_π needs to be sufficiently large to have $\hat{r}_L^e + \phi_\pi \pi^L + \phi_y \hat{y}^L < 0$. Since the AD schedule is upward sloping and $\hat{h}^L < 0$, π^L is smaller than the intercept of the AD schedule, $\text{slope}(AD)\hat{z}^L - \hat{r}_L^e/p$. Thus, the assumption $\phi_\pi \geq p$ implies

$$\hat{r}_L^e + \phi_\pi \pi^L + \phi_y \hat{y}^L \leq \hat{r}_L^e + p\pi^L \leq \hat{r}_L^e + p\{\text{slope}(AD)\hat{z}^L - \hat{r}_L^e/p\} \leq p \times \text{slope}(AD)\hat{z}^L \leq 0.$$

The nominal interest rate is thus zero.

Finally, suppose 1a) holds but 1b) doesn't. Then the AD is no steeper than the AS, and the intercept of the AD is larger than the intercept of the AS. When the AD and the AS are parallel but their intercepts differ, then there is no intersection and thus no equilibrium with $R = 0$. When the AS is strictly steeper than the AD, then their intersection satisfies $\hat{h}^L > 0$, and there is no ZLB equilibrium with $\hat{h}^L \leq 0$.

The same argument goes through for the case where 2a) holds but 2b) doesn't. \square

Appendix D Loglinearization of the AD and the AS schedules using the L state as a reference point and formulas for multipliers

D.1 Loglinearization of the AD and the AS schedules

When computing multipliers it is sometimes convenient to loglinearize the AD and AS schedules about state L instead. Let $\Delta\pi = \pi - \pi^L$, $\Delta h = \ln(h/h^L)$, $\Delta z = \ln(z/z^L)$, $\Delta\eta = \eta - \eta^L$, and $\Delta\tau_w = \tau_w - \tau_w^L$, then the loglinearized AD equation is

$$\begin{aligned} 1 &= (1-p)\beta d^L \frac{(1-\kappa^L - \eta^L)^\sigma (z^L)^\sigma (h^L)^\sigma}{(1-\eta)^\sigma z^\sigma h^\sigma} \left(1 + \sigma(\Delta h + \Delta z) - \frac{\sigma\Delta\eta}{1-\kappa^L - \eta^L} - \frac{\sigma\gamma\pi^L\Delta\pi}{1-\kappa^L - \eta^L}\right) \\ &\quad + \frac{p\beta d^L}{1+\pi^L} \left(1 - \frac{\Delta\pi}{1+\pi^L}\right) \\ &= \frac{p\beta d^L}{1+\pi^L} \left(1 - \frac{\Delta\pi}{1+\pi^L}\right) + \left(1 - \frac{p\beta d^L}{1+\pi^L}\right) \left(1 + \sigma(\Delta h + \Delta z) - \frac{\sigma\Delta\eta}{1-\kappa^L - \eta^L} - \frac{\sigma\gamma\pi^L\Delta\pi}{1-\kappa^L - \eta^L}\right). \end{aligned}$$

Thus

$$\begin{aligned}
slope(AD) &= \frac{(1 - \frac{p\beta d^L}{1+\pi^L})\sigma}{\frac{p\beta d^L}{(1+\pi^L)^2} + (1 - \frac{p\beta d^L}{1+\pi^L})\frac{\sigma\gamma\pi^L}{1-\kappa^L-\eta^L}} \\
icept(AD) &= -\frac{(1 - \frac{p\beta d^L}{1+\pi^L})\frac{\sigma}{1-\kappa^L-\eta^L}}{\frac{p\beta d^L}{(1+\pi^L)^2} + (1 - \frac{p\beta d^L}{1+\pi^L})\frac{\sigma\gamma\pi^L}{1-\kappa^L-\eta^L}} [\Delta\eta - (1 - \kappa^L - \eta^L)\Delta z] \\
&= slope(AD)\Delta z - slope(AD)\frac{\Delta\eta}{1 - \kappa^L - \eta^L}.
\end{aligned}$$

Loglinearizing the AS equation yields:

$$\begin{aligned}
0 &= \frac{(1 - \kappa^L - \eta^L)^\sigma h_L^{\sigma+\nu}}{(1 - \tau_w^L)(z^L)^{1-\sigma}} \left[1 + (\sigma + \nu)\Delta h - (1 - \sigma)\Delta z - \frac{\sigma\Delta\eta}{1 - \kappa^L - \eta^L} + \frac{\Delta\tau_w}{1 - \tau_w^L} - \frac{\sigma\gamma\pi^L\Delta\pi}{1 - \kappa^L - \eta^L} \right] \\
&\quad - 1 - (1 - p\beta d^L)\frac{\gamma}{\theta} [\pi^L(1 + \pi^L) + (1 + 2\pi^L)\Delta\pi] \\
&= -(1 - p\beta d^L)\frac{\gamma}{\theta}(1 + 2\pi^L)\Delta\pi + \left[(1 - p\beta d^L)\frac{\gamma}{\theta}\pi^L(1 + \pi^L) + 1 \right] \\
&\quad \times \left[(\sigma + \nu)\Delta h - (1 - \sigma)\Delta z - \frac{\sigma\Delta\eta}{1 - \kappa^L - \eta^L} + \frac{\Delta\tau}{1 - \tau_w^L} - \frac{\sigma\gamma\pi^L\Delta\pi}{1 - \kappa^L - \eta^L} \right].
\end{aligned}$$

Thus

$$\begin{aligned}
slope(AS) &= \frac{[(1 - p\beta d^L)\frac{\gamma}{\theta}\pi^L(1 + \pi^L) + 1](\sigma + \nu)}{(1 - p\beta d^L)\frac{\gamma}{\theta}(1 + 2\pi^L) + [(1 - p\beta d^L)\frac{\gamma}{\theta}\pi^L(1 + \pi^L) + 1]\frac{\sigma\gamma\pi^L}{1-\kappa^L-\eta^L}} \\
icept(AS) &= \frac{[(1 - p\beta d^L)\frac{\gamma}{\theta}\pi^L(1 + \pi^L) + 1]}{(1 - p\beta d^L)\frac{\gamma}{\theta}(1 + 2\pi^L) + [(1 - p\beta d^L)\frac{\gamma}{\theta}\pi^L(1 + \pi^L) + 1]\frac{\sigma\gamma\pi^L}{1-\kappa^L-\eta^L}} \\
&\quad \times \left[-(1 - \sigma)\Delta z - \frac{\sigma\Delta\eta}{1 - \kappa^L - \eta^L} + \frac{\Delta\tau}{1 - \tau_w^L} \right] \\
&= slope(AS)\frac{1}{\sigma + \nu} \left[-(1 - \sigma)\Delta z - \frac{\sigma\Delta\eta}{1 - \kappa^L - \eta^L} + \frac{\Delta\tau}{1 - \tau_w^L} \right].
\end{aligned}$$

Loglinearizing the aggregate resource constraint $c_L = (1 - \eta^L - \kappa^L)z^L h^L$ yields

$$\Delta c = \Delta h + \Delta z - \frac{\Delta\eta}{1 - \kappa^L - \eta^L} - \frac{\gamma\pi^L\Delta\pi}{1 - \kappa^L - \eta^L}.$$

Loglinearizing GDP $gdp^L = (1 - \kappa^L)z^L h^L$ yields

$$\Delta gdp = \Delta h + \Delta z - \frac{\gamma\pi^L\Delta\pi}{1 - \kappa^L}.$$

D.2 Multiplier formulas

Labor tax multiplier

Our multiplier measures are based on the above system that is loglinearized around a zero bound equilibrium. Note that

$$\Delta h = \frac{icept(AS) - icept(AD)}{slope(AD) - slope(AS)},$$

and

$$\Delta \pi = slope(AD)\Delta h + icept(AD) = slope(AS)\Delta h + icept(AS).$$

The labor tax multiplier on hours is thus

$$\frac{\partial \Delta h}{\partial \Delta \tau_w} = \frac{1}{slope(AD) - slope(AS)} \frac{\partial icept(AS)}{\partial \Delta \tau_w} = \frac{1}{\frac{slope(AD)}{slope(AS)} - 1} \frac{1}{\sigma + \nu} \frac{1}{1 - \tau_w}.$$

And the multiplier on inflation is:

$$\frac{\partial \Delta \pi}{\partial \Delta \tau_w} = slope(AD) \frac{\partial \Delta h}{\partial \Delta \tau_w} = \frac{slope(AD)}{\frac{slope(AD)}{slope(AS)} - 1} \frac{1}{\sigma + \nu} \frac{1}{1 - \tau_w}.$$

These multipliers are the *marginal* ones, for they are derived from elasticities.

Clearly, the slopes of the AD and the AS schedules are crucial for the multipliers. The sign of the multiplier on hours is positive when the relative slope, $slope(AD)/slope(AS)$, is bigger than one, and negative when it is less than one. Therefore, whenever the AD and the AS schedules have different signs, the multiplier on hours is negative. The multiplier is positive only when both schedules have the same signed slopes and the AD schedule is steeper. The absolute size of the multiplier explodes as the two schedules' slopes become closer.

Government purchase multiplier

To calculate the government purchases multiplier, it is convenient to start by deriving the multipliers associated with perturbations in the share of government purchases in output:

$$\frac{\partial \Delta h}{\partial \Delta \eta} = \frac{\frac{slope(AD)}{slope(AS)} - \frac{\sigma}{\sigma + \nu}}{\frac{slope(AD)}{slope(AS)} - 1} \frac{1}{1 - \kappa^L - \eta^L}$$

and

$$\begin{aligned}\frac{\partial \Delta \pi}{\partial \Delta \eta} &= \text{slope}(AD) \frac{\frac{\text{slope}(AD)}{\text{slope}(AS)} - \frac{\sigma}{\sigma + \nu}}{\frac{\text{slope}(AD)}{\text{slope}(AS)} - 1} \frac{1}{1 - \kappa^L - \eta^L} - \text{slope}(AD) \frac{1}{1 - \kappa^L - \eta^L} \\ &= \frac{\text{slope}(AD)}{\frac{\text{slope}(AD)}{\text{slope}(AS)} - 1} \frac{\nu}{\sigma + \nu} \frac{1}{1 - \kappa^L - \eta^L} \\ \frac{\partial \Delta gdp}{\partial \Delta \eta} &= \frac{\partial \Delta h}{\partial \Delta \eta} - \frac{\gamma \pi^L}{1 - \kappa^L} \frac{\partial \Delta \pi}{\partial \Delta \eta}.\end{aligned}$$

Since $\Delta g = \Delta \eta / \eta^L + \Delta h$,

$$\frac{\partial \Delta g}{\partial \Delta \eta} = \frac{1}{\eta^L} + \frac{\partial \Delta h}{\partial \Delta \eta}.$$

We can then calculate the multipliers associated with perturbations in the level of government purchases as follows

$$\begin{aligned}\text{Government purchases hours multiplier} &: \left(h^L \times \frac{\partial \Delta h}{\partial \Delta \eta} \right) / \left(g^L \times \frac{\partial \Delta g}{\partial \Delta \eta} \right) \\ \text{Government purchases GDP multiplier} &: \left(gdp^L \times \frac{\partial \Delta gdp}{\partial \Delta \eta} \right) / \left(g^L \times \frac{\partial \Delta g}{\partial \Delta \eta} \right) \\ \text{Government purchases inflation multiplier} &: \left(\frac{\partial \Delta \pi}{\partial \Delta \eta} \right) / \left(g^L \times \frac{\partial \Delta g}{\partial \Delta \eta} \right).\end{aligned}$$

Note that the government purchases increase with η when $\partial \Delta g / \partial \Delta \eta$ is positive. In such a case, the sign of the consumption response determines whether the government purchase multiplier on GDP is bigger than or less than one. Because the Euler equation implies that consumption and inflation are positively related when the nominal rate is constant, it suffices to know whether the inflation response is positive or not. What determines its sign and size is

$$\text{slope}(AD) / \left\{ \frac{\text{slope}(AD)}{\text{slope}(AS)} - 1 \right\}.$$

If the AD schedule is upward-sloping, the inflation response is positive when the AS schedule is also upward-sloping but flatter than the AD schedule. If both schedules are upward-sloping and the AS schedule is steeper, then the inflation response is negative. If the AD schedule is instead downward-sloping, the inflation response is positive either (i) when the AS schedule is upward-sloping, or (ii) when the AS schedule is downward-sloping and steeper than the AD schedule.

Effects of a technology shock

The responses of employment, output and inflation to a change in technology can be derived in the following way

$$\frac{\partial \Delta h}{\partial \Delta z} = \frac{\frac{\sigma-1}{\sigma+\nu} - \frac{\text{slope}(AD)}{\text{slope}(AS)}}{\frac{\text{slope}(AD)}{\text{slope}(AS)} - 1},$$

and it follows that

$$\frac{\partial \Delta y}{\partial \Delta z} = \frac{\partial \Delta h}{\partial \Delta z} + 1$$

and

$$\frac{\partial \Delta \pi}{\partial \Delta z} = \text{slope}(AS) \left[\frac{\partial \Delta h}{\partial \Delta z} + \frac{\sigma - 1}{\sigma + \nu} \right].$$

Observe, that output increases and hours and inflation fall in response to an improvement in technology when $\sigma = 1$ in a Case III equilibrium since $\text{slope}(AD) < 0 < \text{slope}(AS)$. In a Case II equilibrium we have ($0 < \text{slope}(AD) < \text{slope}(AS)$) and it follows that an improvement in technology increases employment, output and the inflation rate when $\sigma = 1$.

Appendix E Loglinear Slope of New Keynesian Phillips Curve

This section provides a more detailed analysis on the restrictions imposed by the calibration target.

Denote the slope coefficient (on output) in the loglinear New Keynesian Phillips curve by

$$\text{slope}(NKPC) := \frac{\theta(\sigma + \nu)}{\gamma},$$

. Then the AS schedule can be written as

$$\pi^L = \frac{\text{slope}(NKPC)}{(1 - p\beta)} \hat{h}^L + \frac{\text{slope}(NKPC)}{(1 - p\beta)(\sigma + \nu)} \left[-\sigma \frac{\hat{\eta}^L}{1 - \eta} + \frac{\hat{\tau}_w^L}{1 - \tau_w} - (1 - \sigma) \hat{z}^L \right]. \quad (1)$$

This relationship holds not only for our model but also for a wide range of NK models including those with Calvo price setting. Importantly, $\text{slope}(NKPC)$ is independent of

TABLE 1: VALUES OF p AND $slope(NKPC)$ THAT REPRODUCE THE GREAT RECESSION TARGETS

p	$slope(NKPC)$
0.16	0.03
0.44	0.02
0.58	0.015
0.66	0.012
0.72	0.01
0.86	0.005

(p, \hat{d}^L) .

First, we argue that **if $\hat{z}^L = \hat{\eta}^L = \hat{\tau}^L = 0$, the loglinearized model is unable to reproduce the Great Recession target with high p unless $slope(NKPC)$ is sufficiently low.**

Under the stated assumptions,

$$1 - p\beta = slope(NKPC) \frac{\hat{h}^L}{\pi^L} \Leftrightarrow p = \frac{1}{\beta} \left[1 - slope(NKPC) \frac{\hat{h}^L}{\pi^L} \right].$$

Our Great Recession target is $(\hat{h}^L, \pi^L) \approx (-0.07, -0.01/4)$, hence $\hat{h}^L/\pi^L \approx 28$. This implies the following:

- (A) For the model to reproduce the Great Recession target, it is necessary that $slope(NKPC) \leq 1/28 \approx 0.036$ (this is implied by the non-negativity of p)
- (B) When $\beta \approx 1$, the value of p with which the model reproduces the Great Recession targets is reported in Table 1:

To put these numbers in perspective, consider values of the Calvo parameter implied by these values of $slope(NKPC)$. In the NK model with Calvo price setting and a homogeneous labor market, $slope(NKPC)$ is given by the formula

$$slope(NKPC) = \frac{(1 - \alpha)(1 - \beta\alpha)}{\alpha}(\sigma + \nu),$$

where α is the Calvo parameter. If $(\sigma, \nu) = (1, 1)$ and $\beta \approx 1$, the right hand side equals 0.01 when the Calvo parameter α is as high as 0.93, and equals 0.02 when α is around 0.905. If $(\sigma, \nu) = (1, 0.28)$ and $\beta \approx 1$ as in our baseline specification, the right hand side equals 0.01 when α is around 0.916, and equals 0.02 when α is around 0.883.¹⁴ Intuitively, the GR

¹⁴In our model, $slope(NKPC)$ is about 0.021 and thus it corresponds to a Calvo parameter of $\alpha \approx 0.883$.

target inflation rate is so much lower than the target output decline that the New Keynesian Phillips Curve has to be very flat in order to be consistent with the target.

Note that the arguments so far are conditional on $\hat{z}^L = 0$. Deep recessions may bring about some production efficiency loss through e.g. resource misallocation, and/or lower utilization rates for production factors. When $\hat{z}^L < 0$ is allowed, the AS schedule is

$$\pi^L = \frac{\text{slope}(NKPC)}{(1-p\beta)} \hat{y}^L - \frac{\text{slope}(NKPC)}{(1-p\beta)} \frac{1+\nu}{\sigma+\nu} \hat{z}^L. \quad (2)$$

Note that we rewrite it with $\hat{y}^L = \hat{h}^L + \hat{z}^L$. This is because we are fixing the target values for inflation rate and GDP, and with a technology shock GDP and labor input are different. Restrictions imposed by the calibration targets are most transparently seen when labor input \hat{h}^L is replaced by $\hat{y}^L - \hat{z}^L$. This leads to the following expression

$$p = \frac{1}{\beta} \left[1 - \text{slope}(NKPC) \left\{ \frac{\hat{y}^L}{\pi^L} - \frac{1+\nu}{\sigma+\nu} \frac{\hat{z}^L}{\pi^L} \right\} \right]. \quad (3)$$

For pre-specified targets $(\hat{y}^L, \pi^L) < (0, 0)$, lowering $\hat{z}^L < 0$ increases the implied value for p for given preference parameters and $\text{slope}(NKPC)$. For instance, If $\sigma = 1$ and $\beta \approx 1$, then a value of p of 0.76 can be produced by $\text{slope}(NKPC)$ of about 0.015 together with $\hat{z}^L = -0.03$, or $\text{slope}(NKPC)$ of about 0.02 in conjunction with $\hat{z}^L = -0.04$. This is intuitive: for a given \hat{y}^L , negative technology shocks add inflationary pressure, and the NKPC does not need be so flat to produce a small amount of disinflation together with a relatively large decline in output.

It is worth mentioning that the above discussion does not use any information about the AD schedule, and hence these results also obtain in the true nonlinear model as long as loglinearization methods approximates the AS schedule well.

Next, we point out that **when p is close to one an asymptote or a Case 1 equilibrium is only possible if $\text{slope}(NKPC)$ is very small**. To understand this, observe that $\text{slope}(AD) \gtrless \text{slope}(AS)$ can be written as

$$\frac{(1-p)(1-p\beta)}{p} \gtrless \frac{\text{slope}(NKPC)}{\sigma}.$$

When the left hand side is larger (smaller) than the right hand side, the AD schedule is steeper (flatter) than the AS schedule.

This relationship has several implications. First, let \bar{p} be the value of p which satisfies the above with equality. When $p \rightarrow \bar{p}$, $\text{slope}(AD)/\text{slope}(AS) \rightarrow 1$ and the denominators in the

multiplier formulas go to zero as well. This results in an asymptote with very large positive or negative fiscal multipliers on each side of it. Second, since the left hand side of this inequality is decreasing in p , if we want to entertain very high p and $\text{slope}(AD) > \text{slope}(AS)$, then the right hand side $\text{slope}(NKPC)/\sigma$ must be sufficiently low. For example, when $\beta \approx 1$ and $p = 0.9$, the left hand side is approximately 0.0111. When $\sigma = 1$, then $\text{slope}(NKPC) < 0.0111$ must hold. This restriction is not very tight for modestly large p : e.g. for $p = 0.8$ and $\beta \approx 1$, the left hand side is around 0.05, and when $\sigma = 1$, the requirement is $\text{slope}(NKPC) < 0.05$.

Appendix F Estimation

Our Bayesian estimation procedure uses the log-linearized equilibrium conditions to solve the model and derive the likelihood function and assumes that agents assigned zero ex-ante probability to the possibility of $R = 0$. The estimated model has a more general shock structure than the two-state Markov model presented in Section 2 of the paper. In addition, to shocks to demand d_t and technology z_t , we allow for a shock to monetary policy ϵ_t . This makes it possible to estimate the model using three observables, the output gap, inflation and the nominal interest rate.¹⁵ The specification of the model that is estimated is given by the following equations. The nonlinear aggregate demand schedule is:

$$1 = \beta d_t E_t \frac{(1 + R_t)(gdp_{t+1}(1 - \eta))^{-\sigma}}{(1 + \pi_{t+1})(gdp_t(1 - \eta))^{-\sigma}} \quad (4)$$

and the aggregate supply schedule is:

$$\begin{aligned} \gamma \pi_t(1 + \pi_t) + (1 + \tau_s)(\theta - 1) &= \theta \frac{(gdp_t(1 - \eta))^\sigma gdp_t^\nu}{(1 - \tau_w)z_t^{1+\nu}(1 - \kappa_t)^\nu} \\ &+ \beta d_t E_t \frac{(gdp_t(1 - \eta))^\sigma gdp_{t+1}(1 - \kappa_t)}{(gdp_{t+1}(1 - \eta))^\sigma gdp_t(1 - \kappa_{t+1})} \gamma \pi_{t+1}(1 + \pi_{t+1}) \end{aligned} \quad (5)$$

The Taylor Rule is given by:

$$R_t = \rho R_{t-1} + (1 - \rho)(\phi_\pi \pi_t + \phi_y \hat{y}_t) + \epsilon_t \quad (6)$$

where \hat{y}_t , the log GDP gap, is given by:

$$\hat{y}_t = \ln((\exp(gdp_t)(1 - \eta))^{\sigma/(\sigma+\nu)} / (1 - \tau_w)^{1/(\sigma+\nu)}). \quad (7)$$

¹⁵Our measure of the output gap uses the Congressional Budget Office measure of potential GDP.

The shocks to demand and technology are assumed to follow AR 1 rules:

$$\log d_t = \rho_d \log d_{t-1} + u_{d,t} \quad (8)$$

$$\log z_t = \rho_z \log z_{t-1} + u_{z,t} \quad (9)$$

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + u_{\epsilon,t} \quad (10)$$

where the shocks are assumed to be Gaussian with zero means and variance-covariance matrix

$$\Omega \equiv \begin{bmatrix} \sigma_d & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_\epsilon \end{bmatrix} \quad (11)$$

We estimated the model using version 4.3.3 of Dynare. When computing the posteriors, we specified Metropolis Hastings chains of length 60,000 and used 10 parallel chains. After some experimentation we set the scale of the jumping distribution for the Metropolis-Hastings algorithm to 0.68 which produced an acceptance ratio that ranged from 0.2-0.3. The other DYNARE options for Metropolis Hastings were set at their default values.

Priors, posterior modes, posterior means and 5 and 95 percentiles for all estimated parameters are reported in Table 2.

Appendix G Calibration

For our baseline exercises, we fix $(\beta, \sigma, \nu, \theta, \gamma)$ at their estimated/calibrated values. For given p , we adjust (d^L, z^L) to hit the inflation and output targets (π^L, gdp^L) . The level of technology in the high state (H) normalized to 1, and thus the steady-state values of all prices and allocations are known. We also know the value of consumption in the L state $c^L = (1 - \kappa^L - \eta^L)gdp^L / (1 - \kappa^L)$, because $z^L h^L = gdp^L / (1 - \kappa^L)$.

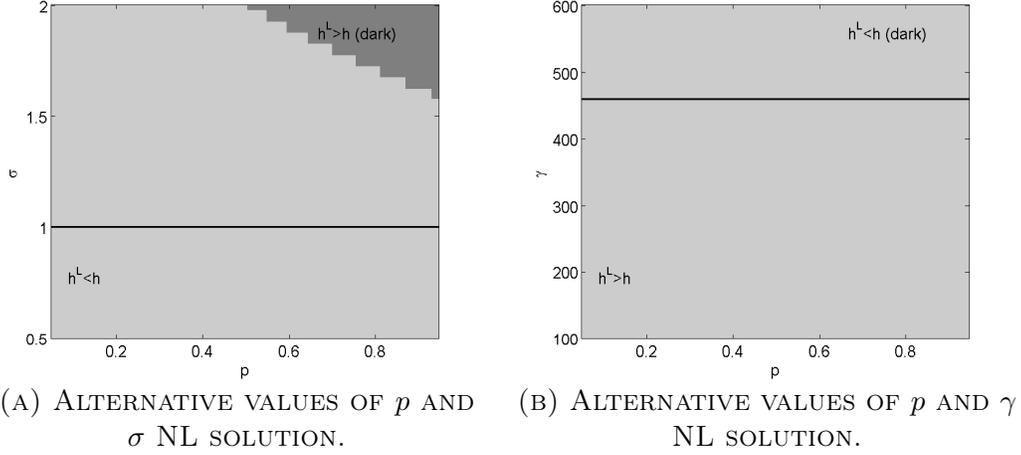
For a given p , we can solve the AD equation for d^L :

$$d^L = \left[\frac{p\beta}{1 + \pi^L} + (1 - p)\beta \left(\frac{c^L}{c} \right)^\sigma \right]^{-1}.$$

We then solve the AS equation for z^L :

$$\begin{aligned} \pi^L(1 + \pi^L) &= \frac{1}{1 - p\beta d^L} \frac{\theta}{\gamma} \left[\frac{(1 - \kappa^L - \eta^L)^\sigma (h^L)^{\sigma+\nu}}{(1 - \tau_w^L)(z^L)^{1-\sigma}} - 1 \right] \\ &= \frac{1}{1 - p\beta d^L} \frac{\theta}{\gamma} \left[\frac{(c^L)^{\sigma+\nu}}{(1 - \tau_w^L)(z^L)^{1+\nu}(1 - \kappa^L - \eta^L)^\nu} - 1 \right]. \end{aligned}$$

FIGURE 15: REGIONS WHERE EMPLOYMENT IS DEPRESSED AT THE ZERO LOWER BOUND FOR ALTERNATIVE VALUES OF p , σ AND γ .



Notes: Light gray: employment is below its steady-state value; dark gray: employment exceeds its steady-state value. The line denotes the baseline value of each parameter.

Note that all variables in this second equation are known except for z^L .

When considering the specification with constant technology, we restrict $z^L = 1$, we vary θ to hit the target. This proceeds in the following way. The preference shock d^L is calibrated first in the same way as before. This step does not require knowledge of θ . Then we use the second equation which is derived from the AS equation, to solve for θ . When calibrating our model to the parameterization of [Denes, Eggertsson, and Gilbukh \(2013\)](#), we restrict $z^L = 1$ and fix θ at their estimated value of this parameter, and then adjust d^L and γ instead to satisfy the above two equations.

Appendix H Employment at the zero bound

We have calibrated the model to produce a 7% decline in GDP. The resource costs of price adjustment, however, drive a wedge between GDP and employment and it is possible in some situations for employment in state L to exceed its steady-state level even though GDP is below its steady-state level. Figure 15 displays when this occurs. The reason this occurs is that technology in this situations is very depressed. If instead technology is held fixed as described in Section A, employment is always depressed at the zero bound.

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TABLE 2: PARAMETER ESTIMATES.

Parameter	Prior			Posterior			
	distribution	mean	std. dev.	mode	mean	5%	95%
ν	gamma	0.5	0.25	0.28	0.37	0.08	0.63
γ	normal	150	200	458.4	510.6	315.4	703.8
ϕ_y	normal	0	1	1.63	1.72	1.06	2.33
ϕ_π	normal	3	1	3.46	3.58	2.38	4.77
ρ_r	beta	0.75	0.1	0.86	0.85	0.81	0.90
ρ_ϵ	beta	0.75	0.1	0.86	0.86	0.81	0.91
ρ_z	beta	0.75	0.12	0.96	0.95	0.93	0.98
ρ_d	beta	0.75	0.1	0.88	0.88	0.83	0.92
σ_ϵ	inverse gamma	0.007	0.008	0.0014	0.0015	0.0012	0.0018
σ_z	inverse gamma	0.01	0.008	0.0052	0.0058	0.0038	0.0078
σ_d	inverse gamma	0.007	0.008	0.0027	0.0027	0.0023	0.0031

These estimates use U.S. quarterly data on the output gap, inflation rate and Federal Funds rate with a sample period of 1985:I-2007:IV.