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# A Staggered Pricing Approach to Modeling Speculative Storage: Implications for Commodity Price Dynamics

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**Abstract:** This paper embeds a staggered price feature into the standard speculative storage model of Deaton and Laroque (1996). Intermediate goods inventory speculators are added as an additional source of intertemporal linkage, which helps us to replicate the stylized facts of the observed commodity price dynamics. Incorporating this type of friction into the model is motivated by its ability to increase price stickiness which, gives rise to a higher degree of persistence in the first two conditional moments of commodity prices. The structural parameters of our model are estimated by the simulated method of moments using actual prices for four agricultural commodities. Simulated data are then employed to assess the effects of our staggered price approach on the time series properties of commodity prices. Our results lend empirical support to the possibility of staggered prices.

JEL classification: Q11, C15, E21, O13, G12

Key words: commodity price determination, staggered pricing, high persistence, conditional heteroskedasticity, simulated method of moments



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# 1 Introduction

The last decade has witnessed a surge in commodity prices and a widespread financialization of commodity products. The upward movements and the increased volatility of the commodity prices have been largely attributed to strong demand by China and other emerging markets as well as massive capital flows into the commodity markets by institutional investors, portfolio managers and speculators. While the importance of commodity price movements for the economic policy and investors' sentiment has generated a substantial research interest, the behavior and the determination of commodity prices is not yet fully understood. The main objective of this paper is to develop a structural model of commodity price determination that reflects the empirical properties (high persistence and conditional heteroskedasticity) of commodity prices. In order to achieve this goal and to gain further understanding into the fundamental factors that drive the observed behavior of commodity prices, we modify the structure of the speculative storage model from one where prices adjust almost instantaneously to harvest shocks to a setup where they change slowly and infrequently. More specifically, we depart from the assumption that market prices are determined in a perfectly competitive environment and extend the basic speculative storage model by explicitly introducing intermediate goods speculators with a staggered pricing rule. One appealing aspect of this approach is its ability to mimic some important characteristics of the actual commodity prices such as high persistence and conditional heteroskedasticity, which can be generated even in the absence of correlated harvest shocks.

The speculative storage model for commodity prices can be dated back to Gustafson (1958) who defines a set of optimal storage rules that state how much grain should be carried over into the next period given the current year supply. Moreover, by introducing intertemporal storage arbitrage and supply shocks, Gustafson (1958) incorporates rational expectations. This line of research is further elaborated in Muth (1961). Samuelson (1971) develops a model for commodities which determines the behavior of the prices as the solution to a stochastic dynamic programming problem. Furthermore, Beck (1993) builds upon the work by Muth (1961) and provides a theoretical basis for treating the variance of storable commodities as serially correlated which suggests that commodity prices may exhibit conditional heteroscedasticity. The presence of storage is instrumental in ensuring that the price variance in one period directly affects inventory variance which in turn is transmitted to next period's price variation. Williams and Wright (1991) provide a comprehensive discussion of the basic storage model and its extensions, and summarize the time series properties of storable commodities. Williams and Wright (1991) put an emphasis on the complex non-linear storage behavior resulting from the fact that aggregate storage cannot be negative.

Deaton and Laroque (1992, 1995, 1996) develop a partial equilibrium structural model of commodity price determination and apply numerical methods to test and estimate the model parameters, confronting for the first time the storage model with the documented behavior of actual prices. Their analysis suggests that the introduction of speculative inventories and serially correlated supply shocks do not appear to generate sufficient persistence in commodity prices although they prove to be successful in replicating the substantial volatility observed in the actual data.

More recently, numerous studies have focused on modifying the storage model in order to accommodate the persistence of commodity prices. Chambers and Bailey (1996) relax the *iid* assumption on harvest shocks, and study the price fluctuations of storable commodities, assuming that shocks are either time dependent or that the model exhibits periodic disturbances. Ng and Ruge-Murcia (2000) incorporate additional features into the storage model in an attempt to generate a higher degree of persistence in commodity prices. In particular, Ng and Ruge-Murcia (2000) allow for serially correlated shocks assuming that harvest follows a first-order moving average (MA(1)) process. They also examine the ability of production lags and heteroskedastic supply shocks, multi-period forward contracts and convenience yields to generate an increased persistence in commodity prices. Cafiero, Bobenrieth, Bobenrieth, and Wright (2011) demonstrate that the competitive storage model can give rise to high levels of serial correlation observed in commodity prices if more precise numerical methods are employed. Moreover, estimates for seven commodities supported the specification of the speculative storage model with positive constant marginal costs and no deterioration, which is in line with Gustafson (1958).

Furthermore, Cafiero, Bobenrieth, Bobenrieth, and Wright (2011) use a maximum likelihood framework to estimate the storage model with stock-outs, which is extended to include unbounded harvests and free disposal. Their results produce more accurate small sample estimates of the structural parameters of the model compared to the previous studies based on the pseudo-maximum likelihood procedure. Miao and Funke (2011) add shocks to the trends of output and demand. Evans and Guthrie (2007) include transaction cost frictions into the speculative storage model. One important finding that emerges from their analysis is that these frictions tend to have explanatory power for the dynamic behavior of spot and futures commodity prices. In a competitive equilibrium framework, the model of Evans and Guthrie (2007) is able to capture the serial correlation and GARCH characteristics of commodity prices. Finally, Arseneau and Leduc (2012) embed the speculative storage model into a general equilibrium framework. Their main result is that the interaction between storage and interest rates in general equilibrium increases the impact of competitive storage on commodity prices and leads to higher persistence than the one observed in the storage model with fixed interest rate.

In spite of this extensive literature for understanding the determinants and the dynamic patterns of commodity prices, reproducing the documented high persistence and conditional heteroskedasticity of actual prices within a well-articulated structural model proved to be a challenging task. In this paper, we address the issues regarding the commodity price dynamics in a unified fashion by embedding a staggered pricing mechanism into the speculative storage model. While Arseneau and Leduc (2012) also suggest to "introduce staggered price setting on the part of the final goods producing firm" in a general equilibrium framework as a possible extension for future research, our paper is the first to implement this approach and assess the properties of the model-generated commodity prices against the observed data.

In an attempt to depart from the assumption of perfect competition at both the production and storage activity, Newbery (1984), Williams and Wright (1991), and McLaren (1999) investigate the effects of market power on the storage behavior. Our model differs from their work along the dimension that the final bundler does not store the good and the storage is only done by intermediate risk neutral speculators. The final bundler only bundles intermediate prices in order to set the final price. Finally, Mitraille and Thille (2009) examine the market power in production with competitive storage by analyzing the effects that competitive storage has on the behavior of a monopolist. Using his market power, the monopolist can influence speculative activity by manipulating prices and consequently affect the distribution of prices. One of the findings of Mitraille and Thille (2009) is that stockouts occur less frequently under monopoly.

The focus of this paper is on the improved ability of the storage model with staggered prices to account for the empirical features of commodity prices. The main impact of staggered prices in our model is to dampen the movements in prices as well as the market power of intermediate speculators to affect prices. This leads to gradual adjustments and persistent responses of prices following a harvest shock. In addition to generating sufficient persistence in commodity prices, the staggered pricing approach allows us to match other important moments in the unconditional and conditional distributions of the commodity prices.

Nominal price rigidity is often incorporated in dynamic general equilibrium models with two widely used nominal price rigidity specifications in the literature. On one hand, the partial adjustment model developed by Calvo (1983), Rotemberg (1987), and Rotemberg (1996) allows for only a randomly chosen fraction of firms to adjust their prices according to some constant hazard rate in any given period. On the other hand, the staggered price setting rule adopted by Taylor (1980) and Blanchard and Fisher (1989) permits all firms to optimize their prices after a fixed number of periods.

In this paper, we assume that the pricing decisions are staggered as in Calvo (1983) and use a similar modeling framework as the one developed in McCandless (2008). Our results confirm the importance of staggered prices for commodity price dynamics and suggest that the staggered pricing mechanism appears to be consistent with the behavior of the actual data. Moreover, we show how our model can be used to analyze the response of commodity prices to harvest shocks which provides a framework for economic and policy evaluation.

The remainder of the paper is organized as follows. The competitive storage model with staggered prices as well as the statistical characterizations of this model are presented in Section 2. Section 3 studies the practical implications of our staggered price speculative storage model using simulated data. Section 4 contains a brief description of the data and the estimation method used in the paper, and presents the main empirical results. Section 5 concludes.

# 2 Competitive Storage Model with Staggered Prices

This section introduces the model setup and characterizes the equilibrium and statistical behavior of the model-generated commodity prices.

## 2.1 Model and Equilibrium Price Behavior

The rational expectations model determines the optimal inventory decisions by risk- neutral speculators. The basic version of the model developed by Deaton and Laroque (1992, 1995, 1996)[1] incorporates competitive storage into the consumer demand and supply dynamics and establishes the concept of stationary rational expectations equilibrium (SREE). The model with serial correla-

[1]For brevity, we denote hereafter the basic speculative storage model of Deaton and Laroque by DL.

tion in harvest shocks is tested by Ng and Ruge-Murcia (2000). In their paper, Ng and Ruge-Murcia (2000) consider an MA(1) specification for the model harvest shocks. Our model complements and extends the original DL model by embedding a staggered price setting into the speculative storage model. Regarding the harvest shock specification, we consider both (i) *iid* harvest shocks and (ii) MA(1) harvests shocks.

Our modified model has three types of commodity market participants: final consumers, intermediate risk neutral speculators and a bundler[2] who bundles the commodities in order to set the final price. In the absence of storage, the behavior of final consumers is characterized by a linear inverse demand function

$$p_t = P(z_t) = a + bz_t,$$

where $a$ and $b < 0$ are parameters to be estimated and $z_t$ denotes the harvest in period $t$.

Let the harvest $z_t$ be given by

$$z_t = \bar{z} + u_t,$$

where $\bar{z}$ is constant (perfectly inelastic) and $u_t$ is a random disturbance term which is assumed either to be *iid* or to follow an MA(1) process

$$u_t = e_t + \rho e_{t-1},$$

where $e_t$ is $iid(0, \sigma^2)$. If $\rho = 0$, we have the case of *iid* shocks as in DL, and when $\rho > 0$, we have MA(1) shocks as in Ng and Ruge-Murcia (2000). In this paper, we investigate both cases and show that when we add staggered prices, the case for $\rho = 0$ gives better results compared to the case of non-staggered prices and $\rho > 0$.

Intermediate risk neutral speculators or inventory holders know the current year harvest and demand the commodity to transfer to the next period. They will do so whenever they expect to make a profit above the storage and interest cost. The depreciation rate of storage is denoted by $\delta$. A simple form of proportional deterioration is considered which means that if in period $t$ the speculators store $I$ units of the commodity, they have at their disposal $(1 - \delta)I$ units at the beginning of the next period. Moreover, speculators have to pay the real interest rate on the value

[2]In the literature, it is common to use the term "monopolist" instead of the term "bundler" that we use in this paper. The reason that we prefer the latter is the following: in the staggered pricing literature, the final goods producer maximizes his profit by setting the price. In this paper, we do not consider any profit maximization and any type of price setting for the final goods producer. Instead, we use the final goods prices set as in (2.6).

of their storage. Let $r$ denotes the constant exogenous real interest rate. The sum of harvest and inherited inventories, denoted by $x_t$, is referred to as the amount on hand and is given by

$$x_t = (1-\delta)I_{t-1} + z_t.$$

The relationship between the amount of storage and its net profit can be summarized as

$$\begin{cases} I_t > 0 \text{ if } (1-\delta)/(1+r)\mathbb{E}_t[p_{t+1}] = p_t, \\ I_t = 0 \text{ otherwise,} \end{cases}$$

where $\mathbb{E}_t$ denotes the expectation given the information at time $t$.

The condition for non-negative inventories is the crucial source of non-linearity in the model. This specification does not allow the market participants to borrow commodities that have not yet been grown. In addition, intermediate speculators benefit from market power that reflects their ability to affect the price. In this framework, we assume that there is a continuum of intermediate speculators (of unit mass indexed by $k \in [0,1]$) and final big players in the market. Final players collect all the commodities from intermediate speculators and bundle intermediate speculators' prices into the final price in order to sell the commodity to consumers. Since we only have a few big players in the market, they are best described by an oligopolistic environment. In reality, the price level of many commodities is influenced either through the formation of cartels by producers or through government intervention by imposing export control agreements or keeping strategic stock reserves. Although some of those cartels brake up in the long run, as discussed in Gilbert (1987), all of them have a strong influence on commodity prices, at least in the short-run. Hence, the introduction of these final big players who bundle prices tends to generate persistence in commodity prices over consecutive periods.

For simplicity, we assume that there exists a bundler who bundles all intermediate speculators' prices into a single one. Each period $t$, a fraction $1-\gamma$ $(0 < 1-\gamma < 1)$ of the speculators are able to exploit their market power and get to reset the prices of their commodities $P_t^*(k)$. The rest, who did not benefit from their market power to affect prices, keep their prices at the same level as the last period: $P_t^*(k) = P_{t-1}^*(k)$. Given this staggered pricing rule, along with the assumptions that speculators are risk neutral and have rational expectations, intermediate speculators' current and expected future prices must satisfy

$$P_t^*(k) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t[P_{t+1}^*(k)] + \gamma P_t^*(k)\right\}. \tag{2.1}$$

The first term in the brackets represents the price if the harvest is sold to consumers in period $t$ and no inventories are carried over to the next period. The second term is known as the intertemporal Euler equation. This is the value of one unit stored if $1-\gamma$ of the speculators benefit form their market power to affect the price. This, in turn, happens if the speculators expect to cover their costs (after depreciation) from buying the commodity at time $t$. Since the current period bundler prices are not yet determined, it is important to stress that speculators, who do not reset their prices, use their own current prices and not the market ones in order to determine $P_t^*(k)$ in (2.1).

Finally, the bundler will bundle all intermediate prices together according to the following pricing rule (see McCandless (2008))

$$P_t^{1-\psi} = \gamma P_{t-1}^{1-\psi} + (1-\gamma) P_t^*(k)^{1-\psi},$$

where $P_t$ denotes the bundler final price of the good, the parameter $\psi$ is the gross markup of the intermediate goods speculators and $P_t^*(k)$ represents the price for intermediate goods speculators who can set their prices. Since all intermediate goods speculators who can fix their prices are assumed to have the same markup over the same marginal costs, $P_t^*(k)$ is the same for all intermediate risk neutral speculators who adjust their prices. Prices for intermediate speculators who cannot set their prices are the same as the previous period prices denoted by $P_{t-1}$.

In order to simplify the bundler's pricing rule, we use the log-linearized version of this equation so that the final price becomes

$$\tilde{p}_t = \gamma \tilde{p}_{t-1} + (1-\gamma) \tilde{p}_t^*(k), \tag{2.2}$$

where $\tilde{p}_t$ and $\tilde{p}_t^*$ denote the logarithm of $P_t$ and $P_t^*$, respectively.

After completing the description of our model, we elaborate on some important implications of equation (2.1). As implied by this equation, the intermediate risk neutral speculators' price follows a non-linear first-order Markov process with a kink at the price above which we do not have inventories. In the case of *iid* shocks, the kink is determined by

$$\hat{p} = (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}p(z) + \gamma \hat{p}.$$

This implies that

$$\hat{p} = \frac{1-\delta}{1+r}\mathbb{E}p(z) \tag{2.3}$$

which coincides with the kink given in DL.

However, as in Chambers and Bailey (1996), the price kink $\hat{p}$ in the case of correlated harvests shocks is no longer constant and varies with the current harvest. This is due to the fact that with serially correlated harvest shocks, speculators form their price forecasts using all the information contained in the current shock.

Under some regularity conditions, most notably $r + \delta > 0$ and that $z$ has a compact support, DL establish the existence of a solution to (2.1) when $\gamma = 0$ and shocks are independent. Indeed, to show the existence of the demand function for non-independent shocks, it is enough to prove the independent case conditioning on time $t$. In our case, we proceed by following a similar approach to proving that such an equilibrium exists. Assume that the demand $x_t$ always lies in a subset $\mathbb{X} = [\underline{z}, +\infty)$ of the real numbers and that the harvest shock $z_t$ belongs to a compact set $\mathbb{Z} = [\underline{z}, \bar{z}]$.

**Definition 2.1** *Assume that $\gamma \in [0, 1)$. A staggered stationary rational expectation equilibrium (SSREE) is a price function $f : \mathbb{X} \times \mathbb{Z} \to \mathbb{R}$ which satisfies the following equation*

$$p_t = f(x_t, z_t) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t f(z_{t+1} + (1-\delta)I_t, z_{t+1}) + \gamma f(x_t, z_t)\right\}$$

*where*

$$I_t = x_t - p^{-1}(p_t) = x_t - p^{-1}(f(x_t, z_t)). \tag{2.4}$$

This defines the price function

$$P_t^*(k) = f(x_t, z_t),$$

where $f(x_t, z_t)$ is the unique, monotone decreasing in its first argument, solution to the functional equation. Since this price function is non-linear, numerical techniques similar to the ones adopted by DL and Michaelides and Ng (2000) are used to solve for $f(x_t, z_t)$

$$f(x_t, z_t) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t f((z_{t+1} + (1-\delta)I_t), z_{t+1}) + \gamma f(x_t, z_t)\right\}.$$

In the case of independent shocks, we can remove the time subscript and the shocks in $f$.

When $\gamma = 0$ and the shocks are *iid*, we have the same model as considered by DL. Hence, the equilibrium is simply called SREE. In the following theorem we show that the staggered stationary rational expectation equilibrium (SSREE) coincides with the stationary rational expectation equilibrium (SREE) derived from the basic DL speculative storage model.

**Theorem 2.1** *If shocks are iid, then SSREE=SREE.*

**Proof** See Appendix A. ■

**Remark 2.1** *Theorem 2.1 shows that $p_t = P_t^*$. This allows us to use all of the results for the process $p_t$, that are available in the literature, for the process $P_t^*$.*

We next show that the final demand for the bundler in our staggered speculative model is different form the one in DL. It proves useful to compare the price processes in the speculative storage model with and without staggered prices for the market participants who can reset their prices. In the basic speculative storage model of DL, the market participants cannot hold negative inventories. If prices are expected to increase or decrease by less than the cost of carrying the commodity from one period to another, inventories are zero. If inventories are positive, the expected price next period is equal to the current price plus the storage costs. The final price of the commodity in the basic speculative storage model satisfies

$$p_t = \max\left\{p(x_t), \frac{1-\delta}{1+r}\mathbb{E}_t p_{t+1}\right\}.$$

Hence,

$$\begin{cases} p_t = \frac{1-\delta}{1+r}\mathbb{E}_t p_{t+1} & \text{if } I_t > 0; \\ p_t = p(x_t) & \text{if } I_t = 0. \end{cases}$$

However, as stated in the description of our speculative storage model with staggered prices, the intermediate risk neutral speculators price function satisfies

$$P_t^* = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t P_{t+1}^* + \gamma P_t^*\right\}.$$

In this case,

$$\begin{cases} P_t^* = (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t P_{t+1}^* + \gamma P_t^*, & \text{if } I_t > 0; \\ P_t^* = p(x_t) & \text{if } I_t = 0. \end{cases} \tag{2.5}$$

It can be easily seen from (2.5) that the prices for intermediate risk neutral speculators who can adjust them satisfy the same equation as the one that speculators face in the basic storage model of DL.

Since the final price process in the speculative storage model with staggered prices is given by

$$\tilde{p}_t = \gamma\tilde{p}_{t-1} + (1-\gamma)\tilde{p}_t^*(k), \tag{2.6}$$

one can infer that the demand of the bundler (the final demand) will be different from the demand presented by DL in the basic speculative storage model. We expect the final demand for speculative storage model with staggered prices to be in between the DL demand and the regular market demand. Moreover, we expect this demand to be more inelastic than the one derived from the basic speculative storage model. This is more consistent with the commodity elasticities estimated from actual data.

## 2.2 Statistical Characterization

Under the assumption of *iid* harvests shocks, the final log-price process satisfies equation (2.6). The bundler price can then be written as

$$P_t = {P_{t-1}}^{\gamma}\ P_t^{*1-\gamma}. \tag{2.7}$$

The persistence of commodity prices is then simply an outcome of the staggered prices which is extensively discussed in the literature on staggered pricing. Here, we provide an alternative explanation. From the logarithmic form of the relation (2.7), we have by induction that

$$\tilde{p}_{t+1} = (1-\gamma)\sum_{i=0}^{t} \gamma^i \tilde{p}^*_{t+1-i}$$

which in turn yields

$$P_{t+1} = \left(\prod_{i=0}^{t} {P^*_{t+1-i}}^{\gamma^i}\right)^{1-\gamma}.$$

This shows that $P_{t+1}$ shares overlapping terms prices in previous periods which gives rise to high persistence.

Next, we show that the final prices of the bundler exhibit conditional heteroskedasticity which is another salient characteristic of the observed commodity prices. Note that from (2.7), we have

$$\mathbb{E}_{t-1}(P_t^2) = {P_{t-1}}^{2\gamma}\mathbb{E}_{t-1}(P_t^{*2(1-\gamma)}) \tag{2.8}$$

and

$$(\mathbb{E}_{t-1}P_t)^2 = {P_{t-1}}^{2\gamma}(\mathbb{E}_{t-1}(P_t^{*1-\gamma}))^2. \tag{2.9}$$

Combining (2.8) and (2.9) and assuming that the shocks are *iid*, the conditional variance of the final prices is given by

$$\mathrm{Var}_{t-1}(P_t) = {P_{t-1}}^{2\gamma}\left[\mathbb{E}\left(f(z+(1-\delta)I_{t-1})^{2(1-\gamma)}\right) - (\mathbb{E}(f(z+(1-\delta)I_{t-1})^{1-\gamma})^2\right]. \tag{2.10}$$

In the absence of inventories in the previous period, $I_{t-1} = 0$, the variance reduces to

$$\mathrm{Var}_{t-1}(P_t) = {P_{t-1}}^{2\gamma}\mathrm{Var}\left(f(z)^{1-\gamma}\right). \tag{2.11}$$

From (2.10) and (2.11), we can see that the variance is time-varying and, as a result, the final commodity prices derived from our model exhibit conditional heteroskedasticity. In addition, it is worth noting that the variance also depends on the value of $\gamma$.

It is interesting to point out that the form of the conditional variance in (2.11) bears strong resemblance to modeling the conditional heteroskedaticity in interest rate models (see, for instance, Brenner, Harjes, and Kroner (1996)). In these models, there is a parameter that allows the volatility of interest rates to depend on the level of the process. Similarly, higher values of the parameter $\gamma$ in equation (2.11) indicate that the volatility of commodity prices is more sensitive to their past level which generates volatility clustering.

# 3 Model Comparisons Using Simulated Data

In this section we examine the statistical properties of the simulated data from our commodity price model with staggered pricing. In order to assess the qualitative and quantitative implications of our model, we compare it to the basic speculative storage model of DL and the modified version of the speculative model of Ng and Ruge-Murcia (2000). The model of Ng and Ruge-Murcia (2000) extends the DL model by adding serially correlated harvest shocks that follow an MA(1) process, as well as gestation lags, heteroskedastic supply shocks, multi-period forward contracts and convenience yields.

In our simulations, we calibrate the models using the parameter values estimated by Deaton and Laroque (1996) for a set of 12 commodities. These parameters $(a, b, \delta)$, presented in Table 1, are the same as the parameters used by Ng and Ruge-Murcia (2000). The data are simulated using *iid* harvest shocks or MA(1) harvest shocks with an MA parameter $\rho = 0.8$. We denote our speculative storage model with staggered prices by ADG.

Table 2 presents the results for the first-order autocorrelation of the simulated prices from the different models. The first column of Table 2 reports the autocorrelations from the actual data used in Deaton and Laroque (1996), the second column shows the results from the basic DL model ($\rho = 0$) and the third column contains the results obtained using DL model with MA(1) shocks ($\rho = 0.8$). The highest autocorrelation for the simulated prices from the DL model is for Maize (0.413 for the basic DL model and 0.644 for the specification with MA(1) harvest shocks). For all other commodities, the serial correlation in the simulated prices is well below the persistence in the actual prices.

The last two columns of Table 2 report the results from our model. For all commodities, the autocorrelation coefficients of the simulated prices based on the ADG model are much higher than those of the DL model specifications and are very close to the autocorrelations obtained from actual data. Once we account for staggered pricing, the additional effect of serially correlated harvest shocks is minimal.

Furthermore, Table 3 lends additional support to our ADG model with staggered prices. In this table, we compare the autocorrelation coefficients for the model by Ng and Ruge-Murcia (2000) with gestation lags, overlapping contracts and convenience yields to those computed from our ADG model in columns 4 and 5 of Table 2.

Ng and Ruge-Murcia (2000) add gestations lags to the DL basic specification in an attempt to reduce the number of periods where the intertemporal price link between periods with and without production is severed. Consequently, this increases the serial correlation in prices. For this purpose, Ng and Ruge-Murcia (2000) assume that there are odd and even periods and that harvest takes place in the even periods. Hence, the random disturbance term of the harvest process has a variance that could differ if the period is odd ($\sigma_1$) or even ($\sigma_2$). The highest autocorrelations are reached for a value of $\frac{\sigma_2}{\sigma_1} = 1.8$. This model is denoted by GS. The results from the GS specification are reported in column 2 of Table 3.

Ng and Ruge-Murcia (2000) also show, in contrast to the earlier literature on storage where contracts are absent and stockholders are free to roll-over their inventories, that a model with overlapping contracts can partially explain the high serial correlation in prices. Column 3, denoted by OV in Table 3 reports the corresponding autocorrelation coefficients.

Finally, Ng and Ruge-Murcia (2000) add a convenience yield to the DL model. Since inventory holders might derive convenience from holding inventories, Ng and Ruge-Murcia (2000) introduce both a speculative and a convenience motive for inventory holding. Hence, since the convenience yield partially compensates inventory holders for the expected loss when the basis is below carrying charges, their model with convenience yield generates a smaller number of stock-outs and, as a result, the demand for inventories for convenience purposes strengthens the intertemporal link resulting in a higher persistence of prices. Results for $c = 50$ are reported in column 4 of Table 3. The model is denoted by CY.

Overall, the results in Table 3 suggest that the different specifications of Ng and Ruge-Murcia (2000) cannot generate autocorrelation coefficients greater than 0.640 and they are below the autocorrelation coefficients from our ADG model and the actual data across all commodities.

# 4 Empirical Application

This section presents new empirical results from estimating the structural parameters of our proposed model using monthly data for four agricultural commodities..

## 4.1 Data

The data set employed in this empirical application consists of prices for four agricultural commodities: sugar, soybeans, soybean oil, and wheat. The commodity prices are obtained from the Commodity Research Bureau and are available at daily frequency for the period March 1983 – July 2008. The trading characteristics of these commodities are summarized in Table 4.

The spot price is approximated by the price of the nearest futures contract. Monthly commodity price series are constructed from daily data by averaging the daily prices in the corresponding month. The monthly frequency is convenient for studying the persistence and conditional heterosekdasticity in commodity prices. The real commodity prices are obtained by deflating the nominal spot prices by the CPI (seasonally adjusted) index obtained from the Bureau of Labor Statistics (BLS). Each deflated price series is then further normalized by dividing by the sample average. By performing this additional normalization, each series has a historical mean of one which allows us to conduct easier comparisons of the estimated parameters across various price series.

## 4.2 Estimation Method: Simulated Method of Moments

This section provides a brief description of the simulated method of moments (SMM) which is used for estimating the model parameters. The main advantage of SMM lies in its flexibility of the choice of moment conditions that allow us to identify the staggered pricing parameter $\gamma$. See Pakes and Pollard (1989), Lee and Ingram (1991) and Duffie and Singleton (1993) for a detailed description of the method and its asymptotic properties, and Michaelides and Ng (2000) for an investigation of its finite-sample properties in the context of the speculative storage model.

The SMM estimator requires repeatedly solving the model for given values of the structural parameters. For this reason, we present some computational details regarding the solution of the model. The function $f(x)$ is approximated using cubic splines and 100 grid of points for $x$. This function is calculated using an iterative procedure, starting with an initial value $f_0(x) = \max[p(x_t), 0]$. As in DL, the interest rate $r$ is not estimated but it is fixed at 5 percent per annum or 0.41 percent ($r = 1.05^{\frac{1}{12}} - 1 = 0.0041$) per month. In addition, we calibrate the depreciation rate $\delta$ and set it equal to 0.04 per month. One reason to calibrate $\delta$ is that the SMM estimator tends to over-estimate $\delta$ as indicated by Michaelides and Ng (2000). Finally, the harvest shocks $z$ are discretized using a discrete approximation of a standard normal random variable with $z$ taking one of the following 10 values: $(\pm 1.755, \pm 1.045, \pm 0.677, \pm 0.386, \pm 0.126)$, with equal probability of 0.1.

It is worth noting that the prices used for estimation of ADG model parameters represent the prices of intermediate risk neutral speculators, not the final prices that are given by the data set described above. Hence, we first retrieve the prices of intermediate risk neutral speculators from the final prices given by the time series of commodity prices using the equation

$$P_t{}^* = \left( \frac{P_t}{P_{t-1}{}^{\gamma}} \right)^{\frac{1}{1-\gamma}}. \qquad (4.1)$$

Let $\theta = (a, b, \gamma)'$ denote the vector of structural parameters of the model. Sample paths of commodity prices can be simulated from the assumed structural model for a candidate value of $\theta$. In what follows, we simulate one sample path of prices $\tilde{P}_t(\theta)$ of length $TH$, where $H = 20$ and $T$ is the sample size of the observed prices $P_t$. The SMM estimator of $\theta$ is then obtained by minimizing the weighted distance (using an optimal weighting matrix) between the moments of the observed data $P_t$ (empirical moments) and simulated data $\tilde{P}_t(\theta)$ (theoretical moments). Let

$m(P_t)$ and $m(\tilde{P}_t(\theta))$ denote the set of moments from the observed and simulated data. Then, the SMM estimator $\hat{\theta}$ is defined as

$$\hat{\theta} = \text{Argmin}_\theta D_T(\theta) V_T^{-1} D_T(\theta), \tag{4.2}$$

where

$$D_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} m(P_t) - \frac{1}{TH}\sum_{t=1}^{TH} m(\tilde{P}_t(\theta)),$$

and $V_T$ denotes a consistent estimator of

$$V = \lim_{T\to\infty} \text{Var}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} m(P_t)\right).$$

The vector of moments

$$m(P_t) = [P_t, (P_t - \bar{P})^i, (P_t - \bar{P})(P_{t-1} - \bar{P})]', \text{ for } i = 2, 3, 4, \tag{4.3}$$

is chosen to capture the dynamics and the higher-order unconditional moments of actual commodity prices. The long-run variance $V$ is estimated using the Parzen window

$$w(x) = \begin{cases} 1 - 6x^2 + 6|x|^3 & \text{if } |x| \le 1/2, \\ 2(1 - |x|^3) & \text{if } 1/2 \le |x| \le 1 \end{cases} \tag{4.4}$$

with four lags.

Under some regularity conditions, Lee and Ingram (1991) and Duffie and Singleton (1993) show that the SMM estimator is asymptotically normally distributed

$$\sqrt{T}(\hat{\theta} - \theta_0) \to N(0, \Omega_H), \tag{4.5}$$

where $\Omega_H = \left(1 + \frac{1}{H}\right)\left(\mathbb{E}\left[\frac{\partial m(\tilde{P}_t(\theta_0))}{\partial\theta}\right]' V^{-1} \mathbb{E}\left[\frac{\partial m(\tilde{P}_t(\theta_0))}{\partial\theta}\right]\right)^{-1}$. The derivatives $\partial m/\partial\theta$ are computed numerically and $\Omega_H$ is replaced by a consistent estimator in constructing the standard errors of the parameter estimates.

## 4.3 Empirical Results

The estimation results for the ADG model parameters are presented in Table 5. The standard errors of the estimated parameters, based on the asymptotic approximation described above, are reported in parentheses below the parameter estimates. The standard errors for the staggered price parameter $\gamma$ are low for all of the four commodities indicating that $\gamma$ is well identified and significantly different from zero. The mean of $\gamma$ for the four commodities is equal to 0.85. The parameter estimates for $b$ satisfy the constraint $b < 0$. For most of the cases, the standard errors of the estimated parameters $a$ and $b$ are relatively low.

In this paper, we argue that the high persistence and the conditional heteroskedasticity in commodity prices appear to be primarily driven by the staggered price parameter $\gamma$. To illustrate this, we simulate 200 price series, each of length of 300 observations. The set of parameters used to conduct the simulations is $(a, b, \delta) = (.7, -3, .04)$ and $r = .0041$. We compute the first-order autocorrelation for each series and then calculate the average over the Monte Carlo replications. We repeat the same exercise for four different values of $\gamma$, $\gamma = (0, 0.3, 0.6, 0.9)$. In the first three columns of Table 7 we report the first-order autocorrelation for the actual data, ADG and DL models, respectively. Table 6 shows that incorporating staggered prices into the speculative storage model does increase the first-order autocorrelation of the prices and makes it comparable to the sample autocorrelation of the actual data. More specifically, as $\gamma$ increases from $\gamma = 0$ (which represents the case for the DL model) to $\gamma = 0.9$, the first-order autocorrelation increases from 0.6 to 0.9.

To visualize the differences between the two models, Figure 1 plots the actual price of soybean, the simulated prices generated by our ADG model with *iid* harvest shocks and estimated parameters $(a, b, \gamma) = (0.352, -4.787, 0.909)$, and the simulated prices generated by DL model with estimated parameters $(a, b, \delta) = (0.723, -0.394, 0.130)$. It is clear from the graph that our staggered price model generates more persistent data with volatility clustering which is closer to the actual price dynamics of soybean prices presented in Figure 1. Also, in Figure 2 we trace the dynamic responses of the simulated commodity prices following a negative harvest shock. The gradual adjustment of the commodity prices from the ADG model stands in sharp contrast with the stronger but short-lived impact of the harvest shock on commodity prices in the DL model.

Next, in order to reveal the advantages of our ADG model in matching the dynamics in the first two conditional moments of the data, we simulate 200 series of prices, each of length of 300 observations, using the parameters estimated from ADG model (reported in Table 5). We repeat the same exercise, using the same values for the parameters $a$ and $b$ but setting $\gamma = 0$, which represents the case for the DL model. We filter the simulated prices from both the DL and ADG models using an AR(1) model and then fit a GARCH(1,1) model to each of the pre-filtered series using the following equations:

$$\begin{aligned} P_t &= a_0 + a_1 P_{t-1} + \epsilon_t \\ \epsilon_t &= \sigma_t z_t \\ \sigma_t^2 &= \kappa + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \end{aligned}$$

Figures 3 and 4 plot the distribution of the parameter estimates $\hat{\beta}$ and $\hat{\alpha}$ for the ADG and DL models. The figures clearly suggest that the ADG model provides an improvement over DL model by better capturing the conditional heteroskedasticity. In fact, the medians for $\hat{\beta}$ and $\hat{\alpha}$, generated by ADG model, are much closer to the parameters (denoted by bullets) estimated from actual data. Table 7 summarizes the results by reporting the means of the autocorrelations and the GARCH parameters for the ADG and DL models against the statistics from the actual data. Overall, the results lend strong support to the staggered pricing feature of the modified speculative storage model of commodity price determination.

# 5 Conclusion

The main objective of this paper is to propose a model which is able to reproduce the statistical characteristics of the actual commodity prices. Our modified speculative storage model embeds a staggered price feature into the DL storage model. The staggered pricing rule is incorporated by introducing intermediate good speculators and a final goods bundler. We examine the empirical relevance of the structural modification by comparing our model performance with several models in the literature, namely DL and the extended DL version of Ng and Ruge-Murcia (2000). Our analysis suggests that the proposed model outperforms the existing models along several dimensions such as matching the serial correlation and GARCH dynamics of the observed commodity prices. We also estimate the vector of structural parameters for the ADG model with uncorrelated harvest shocks using monthly data for four agricultural commodity prices. The results tend to suggest that the staggered price parameter is large and it proves to be instrumental in generating the documented persistence and conditional heteroskedasticity of commodity prices.

# A Appendix: Proof of Theorem 2.1

First, we state the assumptions for the theorem.

**Assumptions**: Assume that

A.1 $r + \delta > 0$.

A.2 The harvest shocks $z$ belong to a compact set $\mathbb{Z} = [\underline{z}, \bar{z}]$;

A.3 The function $p^{-1} : (q_0, q_1) \to \mathbb{R}$ is continuous and strictly decreasing such that

$$\lim_{q \to q_0} p^{-1}(q) = +\infty.$$

Furthermore, we have that $\underline{z} \in p^{-1}(p_0, p_1)$ and $p(\underline{z}) \in \mathbb{R}_+ \setminus \{0\}$.

Following Deaton and Laroque (1992), for any function $g$ on the set $\mathbb{X} = [\underline{z}, +\infty)$ we introduce a function $G$ on $\mathbb{Y} = \{(q, x) | x \in \mathbb{X}, \, p(x) \leq q < q_1\}$ which has the form

$$G(q, x) = (1 - \gamma)\frac{1 - \delta}{1 + r}\mathbb{E}g(z + (1 - \delta)(x - p^{-1}(q))) + \gamma q. \tag{A.1}$$

If $\gamma = 0$, then $G$ is the same as in Deaton and Laroque (1992). Let $G^{DL}$ denote the function when $\gamma = 0$:

$$G^{DL}(q, x) = \frac{1 - \delta}{1 + r}\mathbb{E}g(z + (1 - \delta)(x - p^{-1}(q))).$$

It can be seen that $G = (1 - \gamma)G^{DL} + \gamma p$.

Theorem 2.1 aims to find a function $f$ such that

$$f(x) = \max\{G(f(x), x), p(x)\}, \ \forall x \in \mathbb{X}, \tag{A.2}$$

where we also have $f = g$. To prove the theorem, we use the following lemma.

**Lemma A.1** *For a given $g$, the unique solution $f : \mathbb{X} \to \mathbb{R}$ to* (A.2) *equals $f^{DL}$, where $f^{DL}$ is the unique solution to the same problem when $\gamma = 0$.*

**Proof** For each $x$, $f(x)$ is the solution to the following equation for $q$

$$\max\{G(q,x) - q, p(x) - q\} = 0. \tag{A.3}$$

It can be seen that

$$G(q,x) - q = (1-\gamma)G^{DL}(q,x) + \gamma q - q = (1-\gamma)(G^{DL}(q,x) - q).$$

Thus, the solution $q$ is a solution to

$$\max\{(1-\gamma)(G^{DL}(q,x) - q), p(x) - q\} = 0. \tag{A.4}$$

But this is equivalent to solving[3]

$$\max\{G^{DL}(q,x) - q, p(x) - q\} = 0, \tag{A.5}$$

which gives the desired result. ■

This lemma shows that for any $g$, there is a unique $f$ which is the solution to (A.2). Therefore, we can introduce an operator $\mathbb{T}$ and denote $f$ with $\mathbb{T}g$.

**Proof of Theorem 2.1** From Lemma A.1 it follows that $\mathbb{T}$ is the same as the operator introduced in Deaton and Laroque (1992). It is shown in Deaton and Laroque (1992) that $\mathbb{T}$ is an operator from the set of non-increasing and continuous functions on $\mathbb{X}$ to itself and has a unique fixed point $f$, i.e., $f = \mathbb{T}f$. It then follows that this unique fixed point is the unique SSREE or SREE. This completes the proof of Theorem 2.1. ■

[3] For a positive number $\theta$ and two real numbers $a, b$, we have that $\max\{a,b\} = 0 \Leftrightarrow \max\{\theta a, b\} = 0$.

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Table 1: Parameter estimates from the DL model.

| Commodity | $a$ | $b$ | $\delta$ |
|---|---|---|---|
| Cocoa | 0.162 | -0.221 | 0.116 |
| Coffee | 0.263 | -0.158 | 0.139 |
| Copper | 0.545 | -0.326 | 0.069 |
| Cotton | 0.642 | -0.312 | 0.169 |
| Jute | 0.572 | -0.356 | 0.096 |
| Maize | 0.635 | -0.636 | 0.059 |
| Palm oil | 0.461 | -0.429 | 0.058 |
| Rice | 0.598 | -0.336 | 0.147 |
| Sugar | 0.643 | -0.626 | 0.177 |
| Tea | 0.479 | -0.211 | 0.123 |
| Tin | 0.256 | -0.170 | 0.148 |
| Wheat | 0.723 | -0.394 | 0.130 |

Table 2: First-order autocorrelations for the DL and ADG models.

| Commodity | Actual | DL | DL | ADG | ADG |
|---|---|---|---|---|---|
| | | $\rho = 0$ | $\rho = 0.8$ | $\rho = 0$ | $\rho = 0.8$ |
| | | $\gamma = 0$ | $\gamma = 0$ | $\gamma = 0.8$ | $\gamma = 0.8$ |
| Cocoa | 0.834 | 0.352 | 0.609 | 0.7715 | 0.8446 |
| Coffee | 0.804 | 0.219 | 0.576 | 0.7811 | 0.8501 |
| Copper | 0.838 | 0.335 | 0.619 | 0.8918 | 0.9074 |
| Cotton | 0.884 | 0.173 | 0.564 | 0.8626 | 0.9053 |
| Jute | 0.713 | 0.289 | 0.589 | 0.8817 | 0.9072 |
| Maize | 0.756 | 0.413 | 0.644 | 0.9246 | 0.9180 |
| Palm oil | 0.730 | 0.397 | 0.637 | 0.9079 | 0.9050 |
| Rice | 0.829 | 0.237 | 0.579 | 0.8700 | 0.9078 |
| Sugar | 0.621 | 0.266 | 0.583 | 0.8860 | 0.9184 |
| Tea | 0.778 | 0.213 | 0.571 | 0.8332 | 0.8893 |
| Tin | 0.895 | 0.238 | 0.567 | 0.7547 | 0.8462 |
| Wheat | 0.863 | 0.250 | 0.602 | 0.8834 | 0.9198 |

Table 3: First-order autocorrelations for the Ng and Ruge-Murcia (2000) and ADG models.

| Commodity | Actual | GL | OV | CY | ADG $\rho = 0$ $\gamma = 0.8$ | ADG $\rho = 0.8$ $\gamma = 0.8$ |
|---|---|---|---|---|---|---|
| Cocoa | 0.834 | 0.511 | 0.462 | 0.522 | 0.7715 | 0.8446 |
| Coffee | 0.804 | 0.433 | 0.385 | 0.530 | 0.7811 | 0.8501 |
| Copper | 0.838 | 0.526 | 0.394 | 0.608 | 0.8918 | 0.9074 |
| Cotton | 0.884 | 0.365 | 0.337 | 0.473 | 0.8626 | 0.9053 |
| Jute | 0.713 | 0.486 | 0.365 | 0.545 | 0.8817 | 0.9072 |
| Maize | 0.756 | 0.620 | 0.418 | 0.623 | 0.9246 | 0.9180 |
| Palm oil | 0.730 | 0.640 | 0.438 | 0.625 | 0.9079 | 0.9050 |
| Rice | 0.829 | 0.398 | 0.334 | 0.475 | 0.8700 | 0.9078 |
| Sugar | 0.621 | 0.427 | 0.370 | 0.424 | 0.8860 | 0.9184 |
| Tea | 0.778 | 0.428 | 0.302 | 0.509 | 0.8332 | 0.8893 |
| Tin | 0.895 | 0.428 | 0.355 | 0.472 | 0.7547 | 0.8462 |
| Wheat | 0.863 | 0.411 | 0.368 | 0.505 | 0.8834 | 0.9198 |

Table 4: Description of commodity price data.

| Description | Exchange | Contract size | Contract month |
|---|---|---|---|
| Foodstuffs | | | |
| SB : Sugar No.11/World raw | NYBOT | 112,000 lbs. | H,K,N,V |
| Grains and Oilseeds | | | |
| S : Soybean/No.1 Yellow | CBOT | 5,000 bu. | F,H,K,N,Q,U,X |
| BO : Soybean Oil/Crude | CBOT | 60,000 lb. | F,H,K,N,Q,U,V,Z |
| W : Wheat/No.2 Soft red | CBOT | 5,000 bu. | H,K,N,U,Z |

Notes: This table provides a brief description about each commodity. The first column presents the symbol description and the second one lists the futures exchange where the commodity is traded. In this table, CBOT refers to Chicago Board of Trade, NYBOT: New York Board of Trade. The third column states the contract size and the last column provides the contract months denoted by: F = January, G = February, H = March, J = April, K = May, M = June, N= July, Q = August, U = September, V = October, X = November and Z = December.

Table 5: Parameter estimation of the ADG model using SMM.

| Commodity | a | b | $\gamma$ |
|---|---|---|---|
| W | 0.4227 | -4.6606 | 0.9476 |
| | (0.0102) | (0.2929) | (0.0086) |
| BO | 0.7860 | -2.1265 | 0.7621 |
| | (0.0177) | (0.1354) | (0.0237) |
| S | 0.7209 | -2.7562 | 0.8524 |
| | (0.0454) | (0.3256) | (0.0343) |
| SB | 0.2264 | -5.6592 | 0.9474 |
| | (0.0195) | (0.4351) | (0.0099) |

Table 6: First-order autocorrelations for simulated price series.

| | $\gamma = 0$ | $\gamma = 0.3$ | $\gamma = 0.6$ | $\gamma = 0.9$ |
|---|---|---|---|---|
| Auto. corr. | 0.6122 | 0.7899 | 0.9172 | 0.9838 |

Table 7: First-order autocorrelation, and $\beta$ and $\alpha$ parameters from a GARCH(1,1) model.

| | Auto. corr. | | | $\beta$ | | | $\alpha$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Com. | Actual | ADG | DL | Actual | ADG | DL | Actual | ADG | DL |
| W | 0.9648 | 0.9899 | 0.6387 | 0.6977 | 0.6719 | 0.4834 | 0.2283 | 0.3006 | 0.5138 |
| BO | 0.9679 | 0.9550 | 0.5989 | 0.7903 | 0.5160 | 0.4089 | 0.1473 | 0.4709 | 0.5804 |
| S | 0.9697 | 0.9765 | 0.6180 | 0.3413 | 0.5674 | 0.4476 | 0.3410 | 0.4194 | 0.5483 |
| SB | 0.9620 | 0.9902 | 0.6680 | 0.9018 | 0.6781 | 0.4852 | 0.0798 | 0.2977 | 0.5126 |

Figure 1: Actual and simulated data for soybean prices. The simulated data is from models with staggered pricing (ADG) and without staggered pricing (DL).

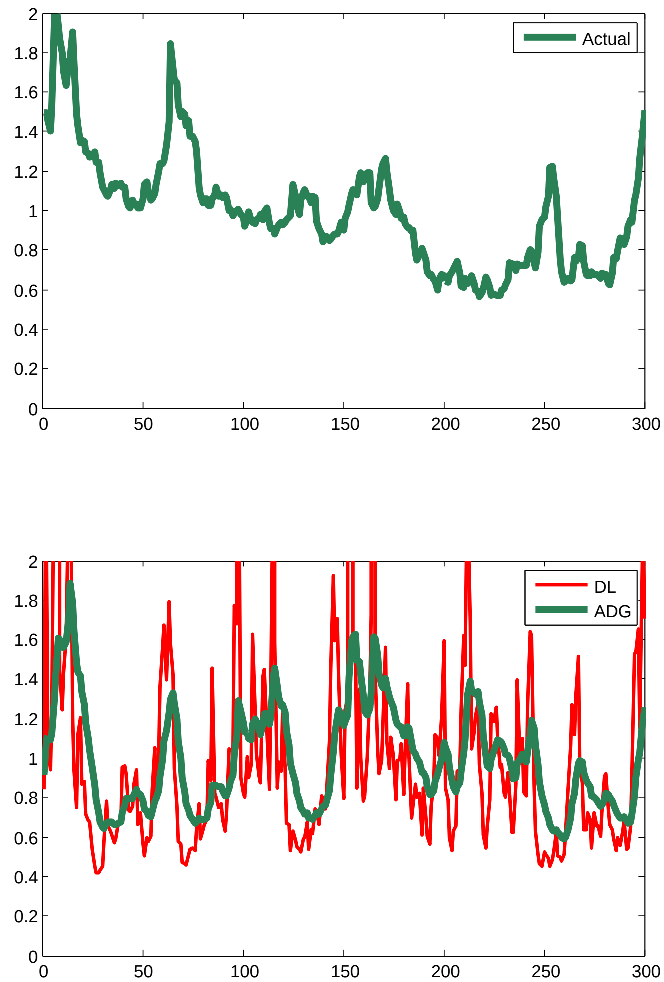

Figure 2: Impulse response function based on simulated data for soybean prices.

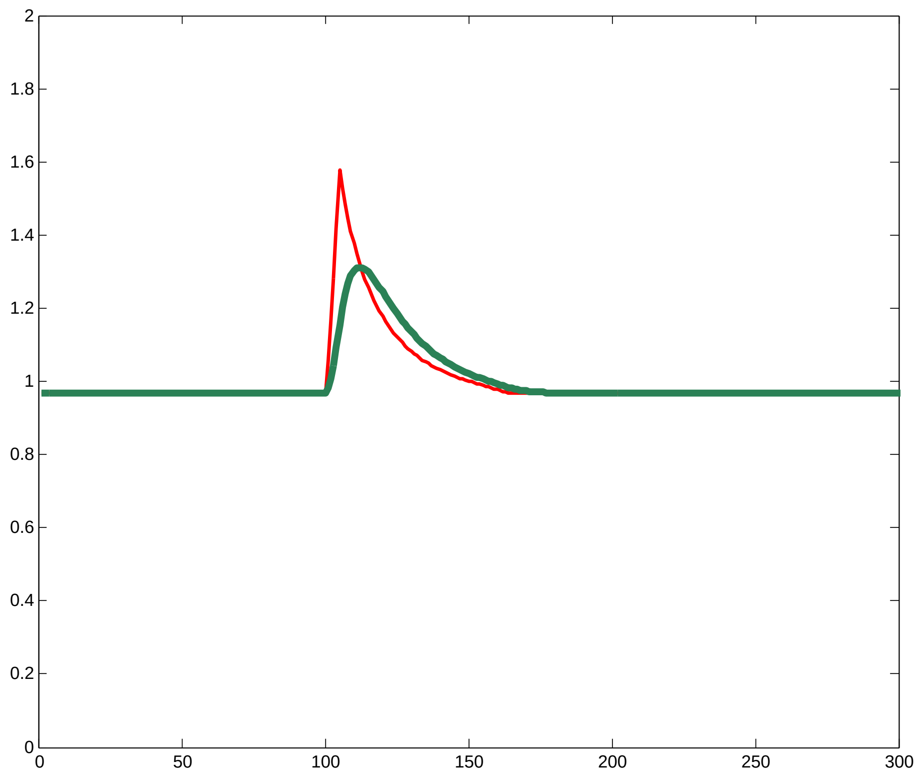

Figure 3: Distribution of $\hat{\beta}$ for simulated data from models with and without staggered pricing. The dashed line is based on data from the ADG model and the solid line is based on data from the DL model. The estimate of $\beta$ from actual data is denoted by a circle on the horizontal axis.

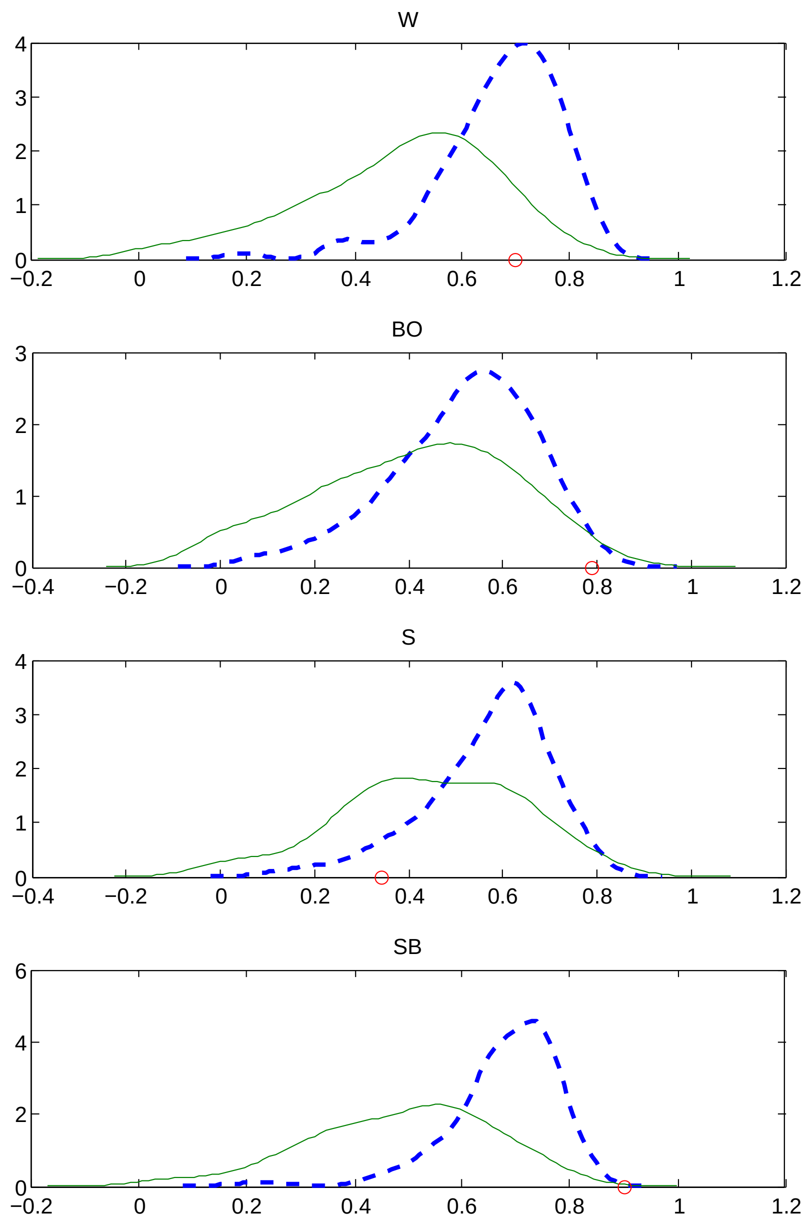

Figure 4: Distribution of $\hat{\alpha}$ for simulated data from models with and without staggered pricing. The dashed line is based on data from the ADG model and the solid line is based on data from the DL model. The estimate of $\alpha$ from actual data is denoted by a circle on the horizontal axis.