Cyclical Lending Standards: A Structural Analysis

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Abstract: Lending standards are a direct measure of credit conditions. We use the micro data merged from three separate sources to construct this measure and document that an uncertain macroeconomic outlook, rather than banks' balance sheet positions, was an important reason that a majority of banks tightened bank lending standards during the Great Recession. Our extensive data analysis disciplines how we introduce credit frictions in the banking sector into a macroeconomic model. The model estimation reveals that an exogenous shock to credit supply drives cyclical lending standards and accounts for a significant portion of fluctuations in bank loans and aggregate output.

JEL classification: E32, E44, G21, C51, C81, C82

Key words: asymmetric credit allocation, endogenous regime switching, debt-to-GDP ratio, heavy GDP, heavy loans, real estate, land prices, GDP growth target, nonlinear effects

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I. Introduction

Credit shocks to the banking sector have long been recognized as an important driving force behind the output fluctuation (Bernanke, 1983). The observed series of lending standards set by bank loan officers, perhaps the best direct gauge of credit conditions across countries, has been used as evidence to motivate a building of macroeconomic models with exogenous credit shocks for understanding the Great Recession (Perri and Quadrini, 2018). In this paper, we provide an extensive analysis of the cyclical fluctuation of lending standards using the micro data merged from three separate sources: the quarterly Consolidated Report of Condition and Income (Call Report) for banks and the analogous FR Y-9C report for bank holding companies, the Federal Reserve Board’s Senior Loan Officer Opinion Survey (SLOOS) on Bank Lending Practices, and the Center for Research in Security Prices (CRSP). We connect our data analysis to an introduction of informational frictions in the banking sector into a dynamic stochastic general equilibrium (DSGE) model.

Our data analysis consists of two parts. First, we exploit the micro data to construct the aggregate series of lending standards, as a measure of the weighted fraction of banks that tighten their lending standards. This constructed series of lending standards allows for potential influences of various aggregate shocks (supply and demand) on the loan market. Second, we explore the possible reasons why a bank tightens or loosens the standards at both the bank level and the aggregate level. By combining the Call Report, SLOOS, and CRSP, we document that balance sheet positions of banks measured by either their book or market values, while relevant for an individual bank to tighten its credit supply, were not a primary reason for a majority of banks to tighten their lending standards during the Great Recession. Instead, a more uncertain macroeconomic outlook was the main reason.

We avail ourselves of this data analysis to discipline our DSGE model by abstracting from considerations of the bank’s capital position while emphasizing informational frictions in the banking sector. This abstraction is based on the fact that only a very small fraction of banks viewed their current or expected capital positions as an important economic reason for tightening their credit supply during the Great Recession and even for these banks both book and market values of their capitalizations were as healthy as other banks. The model builds on Townsend (1979), Williamson (1987), and Greenwood, Sanchez, and Wang (2010) but with an intermediation process in which the degree of informational asymmetry between lender and borrower determines how many banks would engage in costly state verification. In our model, a shock to bank credit supply is equivalent to a productivity shock to the monitoring technology, which affects the bank’s probability of discovering misreports by borrowers. The informational frictions in our model manifest the moral hazard problem present in the lending market. More severe informational frictions reduce the bank’s probability of detecting misreports by borrowers and force an individual bank to conduct more frequent verification, which results in an increase of the fraction of banks that engage in costly state verification.
A fraction of banks engaging in state verification in the model is linked to a fraction of banks changing their lending standards in the data. Negative shocks to bank credit supply, originated from moral hazard in the lending market, exacerbate informational frictions in the model. A shock to credit supply in our model generates a countercyclical movement of the supply of bank loans as observed in the data. Other economic shocks, such as a technology shock to production and a risk shock to the borrower’s delinquency, shift the demand for bank credit, which moves the frequency of state verification and bank loans (output) in the same direction. Our estimated model reveals that credit supply shocks drive the cyclical fluctuation of bank lending standards and explain over 40% of the short-run fluctuations in bank loans and aggregate output. And these impacts are persistent over the four-year horizon. The persistent effects on aggregate conditions such as bank loans and output imply that negative shocks to credit supply produce a more uncertain economic outlook, which is the main reason for the tightening of lending standards as observed in the data.

Our paper relates to the literature on the role of financial factors in the business cycle. The bulk of the recent literature focuses on the role of borrowers’ net worth or corporate bond spreads in propagating shocks originating in nonfinancial sectors of the economy (Bernanke, Gertler, and Gilchrist, 1999; Christiano, Motto, and Rostagno, 2014; Gilchrist and Zakravšek, 2012, for example). A notable exception is Jermann and Quadrini (2012), who shift attention back to the role of shocks that originate directly in the financial sector (the so-called “financial shocks”) as a source of the business cycle fluctuation. In Jermann and Quadrini (2012), however, financial shocks stem from disruptions in the pledgeability of firms’ assets. Several other papers have studied sources of banks’ problems and the role of credit supply shocks with three approaches. The first approach focuses on banks’ incentive problems and the effect of changes in banks’ net worth or their ability to absorb disruptions hitting their liabilities (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Christiano and Ikeda, 2013; Quadrini, 2017). The second approach examines the impact of banks’ relaxation of credit constraints on the boom of house prices (Justiniano, Primiceri, and Tambalotti, 2017). The third approach emphasizes how banks’ liquidity mismatch opens up the possibility of bank runs (Diamond and Dybvig, 1983; Gertler and Kiyotaki, 2015, for example).

Our paper is motivated by different sources of the micro data showing that the decision made by a majority of banks to tighten their credit supply during the Great Recession was due to an uncertain or less favorable macroeconomic outlook, rather than the book or market values of banks’ balance sheet positions. This finding is consistent with the empirical evidence provided by Begenau, Bigio, Majerovitz, and Vieyra (2019) (BBMV hereafter) on the role of commercial banks’ capitalization in the business cycle. In another related paper, Bassett, Chosak, Driscoll, and Zakrašek (2014) (BCDZ hereafter) construct a diffusion index of “exogenous” lending standards. They apply this exogenous series to their vector autoregression (VAR) analysis. As various economic shocks

\[\text{1Since capital positions of investment banks could play a potentially important role in the propagation of the Great Recession, our paper focuses on commercial banks only for which we have relevant micro data.}\]
influence changes in bank lending standards, however, it is conceptually difficult, if not impossible, to construct purely exogenous lending standards via reduced-form econometric procedures. It would also be conceptually difficult to connect such an exogenous series to any shock in a general equilibrium model without an explicit linkage between observed lending standards and the theoretical model.

For these reasons, we depart from BCDZ’s approach and construct an aggregate series of bank lending standards that are not exogenous but affected by variations in macroeconomic conditions. Our DSGE model allows us to answer which structural shock drives the movement of lending standards. We place a special emphasis on how model and data should be synthesized in the context of credit conditions reflected in bank lending standards. We use the rich micro data information to discipline how certain credit frictions should be introduced into a structural model for the purpose of explaining the cyclical fluctuation of bank lending standards.

The rest of the paper is organized as follows. Section II discusses how the micro and macro data are constructed and analyzed. Section III presents a DSGE model with informational frictions in the banking sector and builds the linkage between the model and the observed series of lending standards. Section IV discusses the estimation strategy and analyzes the empirical results. Section V offers concluding remarks.

II. Credit supply tightening and cyclical lending standards

In this section, we first construct a series of bank lending standards. As this is an involved construction process, we provide a detailed description so that researchers can replicate or modify our data for macroeconomic analysis. We then analyze main economic reasons for banks to tighten their lending standards by merging the SLOOS, Call Report, and CRSP data.

II.1. Methodology. Our sample from 1990Q1 to 2017Q4 covers a longer period than BCDZ’s sample, which ends in 2012Q3. A longer length of the sample is not the main reason for us to construct our own series. Because the goal of this paper is to use the micro data to discipline our theoretical model, the data must be constructed by selecting categories of the survey as consistent as possible with the scope of the theory. This construction requirement applies to the other time series discussed in Section IV.1 as well.

From April 1990 until now, SLOOS has asked banks about changes in their lending standards. We focus on two loan categories relevant to the model: commercial and industrial loans (C&I loans) and consumer loans (CS loans). CS loans do not include credit cards and most loans are related to consumer durables. Participating banks are asked about whether and how they have tightened or eased their lending standards in the following wording: “Over the past three months, how have your bank’s credit standards for approving loans of type k
changed? where type k refers to C&I loans or CS loans. Banks are requested to respond to this questionnaire with a scale ranging from 1 to 5, where 1 means “eased considerably”, 2 “eased somewhat”, 3 “about the same or unchanged”, 4 “tightened somewhat”, and 5 “tightened considerably”. Since banks rarely report either easing or tightening standards considerably, we consolidate these responses on the following three point scale and create the categorical variable $I_{i,k,t}$ as

$$
I_{i,k,t} = \begin{cases} 
-1 & \text{if bank } i \text{ reports easing standards on loan category } k \text{ in quarter } t \\
0 & \text{if bank } i \text{ reports no changes in standards on loan category } k \text{ in quarter } t \\
1 & \text{if bank } i \text{ reports tightening standards on loan category } k \text{ in quarter } t 
\end{cases}
$$

The original responses to the survey are reported separately for large and middle-market firms with annual sales of $50$ million or more and for small firms with annual sales of less than $50$ million. We label the consolidated responses by $I_{i,k,t}$ where the superscript $j = S$ indicates small firms and $j = ML$ middle-market and large firms. In most cases, banks provide responses for both $j = S$ and $j = ML$. For cases in which banks provide a response for only one of $j = S$ and $j = ML$, we set $I_{i,k,t}^S = I_{i,k,t}^{ML}$ when the response for $j = S$ is missing and $I_{i,k,t}^{ML} = I_{i,k,t}^S$ when the response for $j = ML$ is missing. We define bank $i$ standards in quarter $t$ since August 1997 as

$$I_{i,k,t} = \frac{1}{2}[I_{i,k,t}^S + I_{i,k,t}^{ML}].$$

Prior to August 1997, loan officers in SLOOS were queried about lending standards for each of large (annual sales more than $250$ million), middle-market (annual sales between $50$ million and $250$ million), and small firms. To maximize comparability with equation (1), we define lending standards prior to the August 1997 survey as

$$I_{i,k,t} = \frac{1}{2} \left[ I_{i,k,t}^S + \frac{1}{2} \left[ I_{i,k,t}^{ML} + I_{i,k,t}^L \right] \right],$$

where $M$ denotes middle-market firms, $L$ large firms, and $S$ small firms. When we construct lending standards for C&I loans and consumer loans, the lending standard questions in the SLOOS after 2010 were split into standards for “auto loans to individuals or households” and consumer loans other than credit card and auto loans. For the SLOOS administered prior to 2011, only a single question was asked on standards for consumer loans other than credit cards.

Each SLOOS generally takes place in between the second (and final) scheduled Federal Open Market Committee (FOMC) meeting of each quarter and the first scheduled FOMC meeting of the subsequent quarter. A rare exception to this timing convention was the additional mid-September 1998 SLOOS used to assess the impact financial turbulence associated with the effect of the Long Term Capital Management crisis on...
We define a composite index of lending standards on C&I and CS loans for bank $i$ in quarter $t$ using the SLOOS administered early in quarter $t + 1$ as

$$S_{i,t} = \sum_{k} s_{i,k,t} \times I_{i,k,t}$$  \hspace{1cm} (3)$$

where $s_{i,k,t}$ is bank $i$’s outstanding type-$k$ loans in the quarter $t$ Call Report as a share of bank $i$’s outstanding C&I and CS loans.\(^7\) If we use lending standards on C&I loans only, $s_{i,k,t} = 1$ where $k$ refers to C&I loans only.

To construct lending standards that are consistent with our macroeconomic model, we estimate the fixed effects panel regression

$$S_{i,t} = \eta_i + \theta' Z_{i,t} + \zeta_{i,t}$$  \hspace{1cm} (4)$$

where $\eta_i$ is a bank-level fixed effect and $Z_{i,t}$ is a vector of bank-level controls primarily constructed from the Call Report in quarter $t$. The bank-level control variables included in $Z_{i,t}$ are listed in Table 1 and described in Appendix A. Most of these control regressors are self-explanatory. The stock market return and Tobin’s Q regressors in Table 1 are calculated with data from the CRSP; Tobin’s Q also utilizes data from quarterly FR Y-9 financial holding company microdata available from the Federal Reserve Bank of Chicago.\(^8\) The bank-level control variables constructed with first and higher-order lagged values from the Call Report are adjusted for mergers as in BCDZ.\(^9\)

Because bank $i$’s stock market returns are included in the panel regression, banks that are not held by a publicly traded holding company are excluded from the regression. Following BCDZ, we also remove banks with less than 20 quarters from the regression sample. Thus, our unbalanced sample in our combined 1990-2017 SLOOS dataset consists of 92 distinct banks (cf. 68 banks in BCDZ’s sample from mid-1991 to mid-2012). In all but one quarter

\(^6\)These data are downloaded from the Federal Reserve Bank of Chicago’s website (https://www.chicagofed.org/banking/financial-institution-reports/commercial-bank-data) prior to 2011 and from the Federal Financial Institutions Examination Council (FFIEC)’s website (https://cdr.ffiec.gov/public/) after 2010.

\(^7\)Type $k$ loans are not included in the sum whenever $I_{i,k,t}$ is missing.


banks in our regression sample account for 80% to 97% of total C&I loans in the SLOOS sample.

Controlling for the bank-level fixed effect and other bank-level variables is consistent with most macroeconomic models (our model included) from which bank-level variables are abstracted. The residual $\zeta_{i,t}$ is defined as lending standards for bank $i$ in quarter $t$. Unlike BCDZ, however, we do not control for $S_{i,t-1}$ and other macroeconomic demand factors because our theoretical model allows these factors to influence $\zeta_{i,t}$. One of the objectives of using the theoretical model is to determine how various economic shocks, demand and supply, influence the lending market as well as aggregate output.

The aggregate series of lending standards on C&I and CS loans is defined as

$$s_t = \sum_i w_{i,t} \times \zeta_{i,t},$$

where $w_{i,t}$ is bank $i$’s share of C&I and CS loans in total C&I and CS loans made by all banks in the fixed effects regression sample participating in the SLOOS administered early in quarter $t + 1$. C&I and CS loans in quarter $t$ are obtained from the Call Report and bank $i$’s weight only includes loan categories for which the corresponding SLOOS response $I_{i,k,t}$ is non-missing. A similar definition applies to lending standards on C&I loans alone.

Our constructed aggregate series of lending standards abstract from the effects of banks’ current or expected balance-sheet position on their lending standards. In the next section, we justify our methodology by showing that balance-sheet positions of banks measured by either their book or market values, while relevant for an individual bank to tighten its credit supply, were not a primary reason for a majority of banks to tighten their lending standards during the Great Recession.

II.2. What do the micro and macro data say about reasons of tightening lending standards? Figure 1 plots the constructed aggregate diffusion index for bank lending standards. The series indicates that bank lending standards began to tighten in 2007 and reached its peak in the middle of the Great Recession. Understanding the potential sources that drive the countercyclical fluctuation of such lending standards is the key to disciplining the theoretical model we build later in this paper. Senior loan officers in the SLOOS are

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$^{10}$We thank an anonymous referee for making this point.

$^{11}$Up to 60 domestically chartered U.S. commercial banks participated in each survey from 1990 to April 2012. Beginning in July 2012, the survey was expanded to as many as 80 banks. Although the list of banks participating in the survey panel is confidential, each of the 12 Federal Reserve Districts contains between 2 and 12 surveyed domestic banks headquartered in its geographic region and the panel “selection criteria are heavily weighted toward inclusion of the largest banks in each district that have a minimum ratio of commercial and industrial (C&I) loans to total assets and that are not specialty banks concentrated in one specific area of lending such as credit cards” (BCDZ). According to the online documentation for the survey (https://www.federalreserve.gov/data/sloos/about.htm), the SLOOS panel as of March 31, 2017 included 80 domestic banks, 47 of which had assets of $20 billion or more; the aggregate assets of the panel accounted for about 69% of total assets of all domestically chartered institutions.
asked of the possible reasons why the bank tightened or eased the standards on C&I loans. The exact wording of the questions has changed somewhat over time. For example, “uncertainty” about the economic outlook was not a part of the questionnaire prior to the 1998Q4 survey. The overall design of the questionnaire, however, has been consistent over time. In the following two sections, we analyze the reasons for tightening lending standards from the perspectives of both micro and macro data by linking the Call Report to the SLOOS.

II.2.1. Micro evidence. Table 1 reports the estimated values of $\theta$ in regression (4) for lending standards on both C&I loans and C&I and CS loans. As one can see, the estimates of most explanatory variables are statistically significant at the 1% level. In particular, the estimated effect of the Tier 1 capital ratio on the average bank’s lending standards is highly significant, implying that the balance-sheet position (the book value) is important for the average bank. This result is consistent with the micro literature on significant effects of financially constrained banks on firms that borrow from these banks (Jiménez, Ongena, Peydró, and Saurina, 2012, for example).

In addition to our finding of the significance of the bank’s capital position, we also find that the market value of the average bank’s assets, which is not captured by the book value (the Tier 1 capital ratio), explains the bank’s changes in lending standards. Table 2 reports the estimated values of $\theta$ in regression (4), similar to Table 1 but replacing Tobin’s Q with the bank’s market leverage as proposed by BBMV. Clearly, the estimate effect of the market leverage on the average bank’s lending standards is statistically significant. When there was a decline in the market value of bank’s (illiquid) assets during the Great Recession, for example, the bank’s book value might be healthy but lending standards would nonetheless tighten. The additional statistical power of market leverages in the regression results, as reported in Table 2, implies that the value of a bank’s capitalization may not be fully captured by its book value.

II.2.2. Macro evidence. The significant micro-level effects reported in the previous subsection do not necessarily translate into macro effects on aggregate bank lending and aggregate lending standards. In this subsection, we provide new evidence on this issue by studying two crucial factors (reasons) that influenced C&I lending standards: banks’ capital positions and macroeconomic outlooks (or aggregate uncertainty). Figure 2 displays the SLOOS response results for banks’ current or expected capital positions as a reason for tightening or easing lending standards. Respondents who stated that they tightened standards are represented by bars above zero on the y-axis (e.g., if 50% of the respondents stated that they tightened

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12The questionnaire concerns C&I loans only.

13Prior to 1995Q2, the wording of survey questions was slightly different about the reasons for tightening or easing lending standards. Instead of responding to whether the reasons were “very”, “somewhat”, or “not” important, the respondents were asked to state whether or not economic outlook or capital position is the main reason for tightening or easing the standards. The survey results, however, are very similar: banks’ capital position is not the main reason but economic outlook is more likely to be the main reason for a change.
standards, the positive portion of the bar adds up to 50). Those who stated that they eased standards are represented by bars below zero on the y-axis.\textsuperscript{14} As revealed in Figure 2, most respondents stated that their banks’ capital positions were not an important factor in changing their lending standards, even during the Great Recession period.\textsuperscript{15}

We link the SLOOS to the Call Report and examine how the book value of the bank’s assets is related to the SLOOS responses to current or expected capital position at the aggregate level. The top panel of Figure 3 displays the distribution of Tier 1 capital ratios for banks that responded how important their current or expected capital position was for tightening C&I lending standards. We focus on the Great Recession period marked by the second NBER recession bar in the figure. The interquartile range for the “Important” group of banks overlaps with the “Not important” group, indicating that how important the bank’s capital position was for tightening lending standards was not influenced by the bank’s book value at the aggregate level. For the “Important” group of banks, one may have expected that the book value would be lower, but it turns out that the book value was as healthy as the “Not important” group of banks (i.e., the capital ratio was above 6%).\textsuperscript{16} According to Basel II (prior to the introduction of Basel III in 2013), the minimum Tier 1 capital ratio was 4% and thus banks with the Tier 1 capital ratio above 6% were well-capitalized by the Basel II criterion. This aggregate result is consistent with BBMV’s finding and conclusion.

While the book value of a bank’s assets had little connection with the SLOOS responses to banks’ capital positions, the top chart of Figure 4 reveals that the market value is consistent with the SLOOS responses. In 2008-2009, the interquartile range of market leverages for the “Not important” group of banks was mostly below the interquartile range for the “Important” group. Most banks in this period reported that their capital positions were not an important reason for tightening their lending standards (Figure 2), and as confirmed by the top chart of Figure 4 this “Not important” group of banks was less likely to suffer market-value loss than the “Important” group of banks during the Great Recession. Both our micro data and BBMV’s empirical work reveal that neither book nor market leverage constraints strictly bound for most banks. In contrast to the micro-level finding reported in Table 2, therefore, the unimportance of banks’ capital positions as revealed in the SLOOS truly reflected the unimportance of both their book and market values at the aggregate level when lending standards were tightened by banks.

The most important factor that influenced banks’ tightening of their lending standards is aggregate uncertainty or macroeconomic outlook as shown in Figure 5. This factor is especially conspicuous during the Great Recession when nearly all banks regarded uncertainty or

\textsuperscript{14}There are some banks that did not respond to the survey questions.

\textsuperscript{15}Bassett and Covas (2013) show that these results are not biased, partly because respondents’ answers were confidential.

\textsuperscript{16}For some years, there are very few banks in the “Important” group. The extreme case is the year 2014 in which there is only one bank responding that capital position was an important reason for tightening lending standards. The wide interquartile range is simply an artificial interpolation of the Matlab graphics software. We keep such a wide range in our reporting so that the anonymity of this bank is protected.
economic outlook as an important reason for tightening their lending standards. To examine whether the book or market values of banks’ assets, possibly affected by borrowers’ real estate collateral values during the Great Recession, might influence banks’ responses to economic uncertainty or outlook, the bottom charts of Figures 3 and 4 report the distributions of book and market values of banks’ assets corresponding to their responses to uncertainty or economic outlook. For a precise comparison with the top chart in each figure, the sample includes only the banks selected for the top chart. The dashed lines extend from the minimum to the maximum of the distributions conditional on the banks used in the top chart. The diamond symbol represents the median of this conditional distribution. The boundaries of each interquartile range in the top chart may not match exactly the corresponding minimum and maximum values in the bottom chart because the Matlab graphics software interpolates end points of an interquartile range. Moreover, some banks stating that capital position was a “Not important” reason for tightening lending standards may state that uncertainty or economic outlook was an “Important” reason for tightening lending standards. Thus, banks in the “Not important” group and those in the “Important” group may shuffle around, depending on whether the question of capital position or economic uncertainty (outlook) was asked.

For banks indicating uncertainty or economic outlook as an important reason for tightening their lending standards, as revealed in the bottom charts of Figures 3 and 4, both the book and market values of their assets were healthy, especially during the Great Recession. In 2009, for example, the median Tier 1 capital ratio for this group of banks was around 8% and the median market leverage ratio was below 15. At the aggregate level, therefore, the uncertainty or economic outlook reason given by banks would capture macroeconomic conditions affected by shocks other than those to banks’ capital positions. In the theoretical model we build in the following section, we abstract the model from banks’ capital positions and study various economic shocks that underlie more uncertain or less favorable economic outlook, such as shocks to bank credit supply, production technology, firms’ asset values, and loan delinquency. In particular, we introduce the frequency of state verification to the banking sector with a stochastic monitoring technology to be linked to the fraction of banks that tighten lending standards reported in the SLOOS.\textsuperscript{17}

\section{III. The model}

The economy is inhabited by an infinitely-lived representative worker, a continuum of entrepreneurs with unit mass, a continuum of banks with unit mass, a representative final goods producer, and a representative capital good producer.

In each period, entrepreneurs rent capital and labor to produce intermediate goods, as an input into final goods production. Competitive banks provide loans to entrepreneurs

\textsuperscript{17}The other parts of the model are standard and the costly-state-verification feature has been widely adopted in the literature on financial frictions, from the seminal works of Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) to the more recent work of Christiano, Motto, and Rostagno (2014).
to finance the factor payment. The representative worker supplies capital and labor for production of intermediate goods and is entitled to the profit of the capital producer. All entrepreneurs are subject to idiosyncratic shocks to the production technology. Realizations of these shocks are not observed and can only be verified with costs.

III.1. **Technology.** We specify the technology processes in different production sectors: the intermediate goods sector, the final goods sector, and the capital sector.

III.1.1. **Intermediate goods.** There are a continuum of entrepreneurs indexed by $j \in [0, 1]$, each endowed with the technology for producing intermediate goods $j$:

$$Y_t(j) = A_t(j)z_tk_t^\alpha (j) h_t^{1-\alpha} (j),$$

where $k_t(j)$ and $h_t(j)$ represent capital and labor used to produce intermediate goods $j$, and $Y_t(j)$ represents intermediate goods $j$. Since entrepreneurs producing different intermediate goods are symmetric, we drop $j$ for notational brevity whenever there is no confusion. An aggregate technology shock, $z_t$, has the following stochastic process:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \epsilon_{z,t},$$

where $\epsilon_{z,t}$ is a standard normal random variable. The idiosyncratic productivity $A_t$ takes the form $A_t = \overline{A}_t \xi_t$, where $\xi_t$ is a shock to idiosyncratic productivity in the form of

$$\xi_t = \begin{cases} 
1 - \sigma \sqrt{\frac{1-\pi_t}{\pi_t}} & \text{with probability } \pi_t \\
1 + \sigma \sqrt{\frac{\pi_t}{1-\pi_t}} & \text{with probability } 1 - \pi_t
\end{cases}$$

such that the unconditional mean of this shock process is 1 and the unconditional variance is $\sigma^2$. Let

$$\pi_t = \frac{\vartheta_t}{1 + \vartheta_t},$$

where the risk shock $\vartheta_t$ follows a stochastic process specified as

$$\log \vartheta_t = (1 - \rho_\vartheta) \log \vartheta + \rho_\vartheta \log \vartheta_{t-1} + \sigma_\vartheta \epsilon_{\vartheta,t}$$

with $\epsilon_{\vartheta,t}$ being a normal random variable. Denote

$$A_{1,t} = \overline{A}_t \left[1 - \sigma \sqrt{\frac{1-\pi_t}{\pi_t}} \right],$$

$$A_{2,t} = \overline{A}_t \left[1 + \sigma \sqrt{\frac{\pi_t}{1-\pi_t}} \right].$$

The subscript 1 denotes a bad state and 2 a good state for entrepreneurs. We have $\overline{A}_t = E_t (A_t) = \pi_t A_{1,t} + (1 - \pi_t) A_{2,t}$ and $\text{Var}_t (A_t) = \pi_t (1 - \pi_t) (A_{2,t} - A_{1,t})^2$. Without loss of generality, we normalize $\overline{A}_t = 1$ so that for a typical entrepreneur, the expected output of intermediate goods, denoted as $y_t$, is $z_t k_t^\alpha h_t^{1-\alpha}$. Accordingly, variations in $\pi_t$ drive the variance of $A_t$ while the mean of $A_t$ is independent of $\pi_t$. Hence, the risk shock $\vartheta_t$ is essentially a shock to delinquency.
III.1.2. Final goods. There is a constant-elasticity-of-substitution (CES) technology for producing final goods by combining the differentiated intermediated goods:

\[ Y_t = \left[ \int_0^1 (Y_t(j))^{\mu} \, dj \right]^{\frac{1}{\mu}}, \]

where the elasticity of substitution is \( 0 < \mu < 1 \). Given the specification of idiosyncratic productivity, the above expression can be rewritten as

\[ Y_t = \left\{ \pi_t \left( A_{1,t} z t k^{\alpha}_t h^{1-\alpha}_t \right)^\mu + (1 - \pi_t) \left( A_{2,t} z t k^{\alpha}_t h^{1-\alpha}_t \right)^\mu \right\}^{\frac{1}{\mu}} = \left[ A^\mu_t \right]^{\frac{1}{\mu}} z t k^{\alpha}_t h^{1-\alpha}_t, \]

where

\[ A^\mu_t \equiv \pi_t \left( A_{1,t} \right)^\mu + (1 - \pi_t) \left( A_{2,t} \right)^\mu. \]

As in the standard model with monopolistic competitions, the final goods producer chooses varieties of intermediate goods to maximize its profit. The first-order condition for this maximization delivers the demand function for each intermediate good:

\[ P_{i,t} \equiv \frac{Y_t}{Y_{i,t}} \left( Y_{i,t} \right)^{1-\mu}, i \in \{1, 2\}, \quad (6) \]

where \( P_{i,t} \) is the price of intermediate goods at state \( i \), and \( Y_{i,t} \equiv A_{i,t} z t k^{\alpha}_t h^{1-\alpha}_t \) is the output of intermediate goods at state \( i \).

III.1.3. Installed capital. In each period, after the final goods production takes place, the capital producer purchases investment goods \( I_t \) (in units of consumption goods) from the final-goods producer (and \( (1 - \delta) K_t \) units of physical capital from households and entrepreneurs), and produces the new capital stock to be sold to households and entrepreneurs at the end of the period.

The technology to transform new investment into the installed capital involves installation costs, \( S (I_t/I_{t-1}) \), which increase with the rate of investment growth. Since the marginal rate of transformation from the previously installed capital stock (after it has depreciated) to new capital is unity, the price of new and used capital is the same. The capital producer’s period-\( t \) profit can be expressed as

\[ \Pi^k_t = q_t [(1 - \delta) K_t + \chi_t (1 - S (I_t/I_{t-1})) I_t] - q_t (1 - \delta) K_t - I_t, \]

where \( \chi_t \) is a marginal efficiency shock to investment (MEI shock) as in Justiniano, Primiceri, and Tambalotti (2011), which has the stochastic process

\[ \log \chi_t = \rho_{\chi} \log \chi_{t-1} + \sigma_{\chi} \epsilon_{\chi,t}, \]

where \( \epsilon_{\chi,t} \) is a standard normal random variable. The capital producer solves the following dynamic optimization problem:

\[ \max_{t, \Pi^k_t, \epsilon_{\chi,t}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \Pi^k_{t+j} \right\}, \]
where $\lambda_t$ is the Lagrangian multiplier for the household’s budget constraint. We assume $S(1) = S'(1) = 0$ and $S''(1) > 0$. It is straightforward to show that $\Pi^k_t = 0$ at the steady state.

We introduce a collateral shock to the economy as an exogenous source of variation in the value of capital, which is called a “capital quality shock” by Gertler, Kiyotaki, and Queralto (2012). Let $\widetilde{K}_{t+1}$ be the aggregate capital stock “in process” for period $t + 1$. Capital in process for period $t + 1$ is the sum of installed capital investment and the undepreciated capital:

$$\widetilde{K}_{t+1} = (1 - \delta) K_t + \chi_t (1 - S (I_t/I_{t-1})) I_t. \quad (7)$$

The capital in process for period $t + 1$ is transformed into capital for production after the realization of a multiplicative collateral shock, $\gamma_{t+1}$, where $\gamma_{t+1}$ is realized at the end of period $t$ after all decisions by economic agents at period $t$ are made. Hence, the aggregate capital for production at period $t + 1$ is

$$K_{t+1} = \gamma_{t+1} \widetilde{K}_{t+1}. \quad (8)$$

The collateral shock as an exogenous trigger of asset price dynamics has the stochastic process

$$\log \gamma_t = \rho_\gamma \log \gamma_{t-1} + \sigma_\gamma \epsilon_{\gamma,t},$$

where $\epsilon_{\gamma,t}$ is a standard normal random variable.

III.2. Workers. The representative worker has no access to the production technology, but provides physical capital and labor to intermediate-goods producers in each period. The household is entitled to the profit of the capital-goods producer. After the production takes place, the household makes optimal decisions on consumption, hours to work, and investment in physical capital. The representative worker solves the problem

$$E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[ \log \left( c^h_t - \omega_h c^h_{t-1} \right) - \phi_t \frac{H^{1+\nu}_t}{1+\nu} \right]$$

with $0 < \beta < 1$, subject to

$$c^h_t + q_t a^h_{t+1} = [q_t (1 - \delta) + r_t] \gamma_t a^h_t + w_t H_t + \Pi^k_t,$$

where $a^h_{t+1}$ is the physical capital purchased at the end of the period $t$ by the household, $c^h_t$ and $H_t$ denote the household’s consumption and total hours supplied, and $\Pi^k_t$ is the profit of the representative capital producer. Let $\theta_t = \Theta_t / \Theta_{t-1}$, which follows a stochastic process as

$$\log \theta_t = \rho_\theta \log \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t},$$

where $\epsilon_{\theta,t}$ is a normal random variable. The labor supply shock $\phi_t$ also follows an AR(1) process as

$$\log \phi_t = (1 - \rho_\phi) \phi + \rho_\phi \log \phi_{t-1} + \sigma_\phi \epsilon_{\phi,t},$$

where $\epsilon_{\phi,t}$ is a normal random variable.
III.3. **Entrepreneurs’ consumption-saving problem.** Consider a large entrepreneur family with a continuum of members. Each member faces an idiosyncratic productivity shock $\xi$. Assuming complete insurance against consumption risks, all members of the family enjoy the same consumption $c_t^e$.

The resource available to the entrepreneur family at the end of period $t$ includes the gross return for physical capital investment and the expected net project value by all members. The household uses these resources to finance family consumption spending $c_t^e$, as well as transfer $q_t a_{t+1}^e$ to be equally dispersed among family members as capital investment, as well as internal funds for intermediate good production. Accordingly, the consumption-saving problem of the entrepreneur family is to maximize

$$E_0 \sum_{t=0}^{\infty} (\beta^e)^t \log (c_t^e - \omega c_{t-1}^e)$$

with $0 < \beta^e < 1$

subject to

$$c_t^e + q_t a_{t+1}^e = [q_t (1 - \delta) + r_t] \gamma_t a_t^e + \left( \int_0^1 v_t(j) dj - q_t \gamma_t a_t^e \right),$$

where $v_t(j)$ is the contract value for entrepreneur $j$, which will be defined in Section III.4.3. We assume that $\beta^e < \beta$, where $\beta$ is the representative worker’s discount factor.

III.4. **Financial contract between entrepreneur and bank.** This section specifies the financial contract between an entrepreneur and a bank and builds a micro foundation for the observed aggregate index of lending standards.

III.4.1. **Loan contract.** Each bank is indexed by $j \in [0, 1]$. Before the final goods production takes place, entrepreneurs need to rent capital and labor. Here, the total cost of production for the entrepreneur $j$ is $r_t k_t(j) + w_t h_t(j)$. Entrepreneurs can use both their net worth and external borrowing to finance the factor payments. Entrepreneur $j$’s savings at the end of the last period, $a_t^e(j)$, is in the form of physical capital and entrepreneurs’ net worth is $q_t \gamma_t a_t^e(j)$, where $q_t$ is the price of capital. The gap between the cost of production and entrepreneurs’ net worth is financed by the bank, where the bank receives deposits from households.

In each period, an entrepreneur enters into a financial contract with a bank before idiosyncratic productivity shocks, $\xi_t$, are realized. Debt is repaid at the end of the period. Each bank provides loans to entrepreneurs and is subject to asymmetric information regarding the realized cash flow. Entrepreneurs self-report their productivity after the production takes place. Since information is asymmetric, an entrepreneur who receives $A_{2,t}$ has incentive to misreport the true productivity. On the other hand, an entrepreneur has no incentive to misreport $A_{1,t}$ when the actual realized idiosyncratic technology is $A_{1,t}$. Since the realized idiosyncratic shock is not publicly observable, payments to the bank at the end of the period are made according to the report submitted by the entrepreneur.

The key assumption is that banks can choose the frequency of state verification. Each time a bank $j$ verifies the state, it will involve a fixed verification cost equal to the project size, $y_t(j)$. The symbol $m_t(j)$ denotes the total monitoring input of the project committed by the
bank \( j \), and is measured in consumption goods. The frequency for state verification, denoted as \( \Pi_t(j) \), is therefore \( m_t(j)/y_t(j) \). Given the optimal choice of \( \Pi_t \), after the production takes place and entrepreneurs report their cash flow, a lottery chooses which banks will engage in verification. Given the nature of costly state verification, the bank will choose the frequency of verification only when a bad state is reported.

Once a bank verifies, it can detect misreporting with probability \( p_t \), which captures the monitoring technology. To capture the exogenous variations in the efficiency of such lending services, we assume \( p_t \) is a function of efficiency in lending technology, denoted as \( \varepsilon_t \) (i.e., \( p_t = p(\varepsilon_t), p(\varepsilon_t) \in [0, 1] \)). The stochastic process for \( \varepsilon_t \) takes the form

\[
\log \varepsilon_t = (1 - \rho_\varepsilon) \log \varepsilon + \rho_\varepsilon \log \varepsilon_{t-1} + \sigma_\varepsilon \varepsilon_{t,t},
\]

where \( \varepsilon_{t,t} \) is a standard normal random variable. We refer to \( \varepsilon_{t,t} \) as a credit supply shock as it affects lending efficiency directly.

Hence, the overall probability for a bank to find misreporting by an entrepreneur who receives \( A_{2,t} \) but reports \( A_{1,t} \) among all incidences of verification is an increasing function of both the frequency of bank verification and the probability of detecting misreporting upon verification.\(^{18}\) Denote such probability by \( P(\varepsilon_t, m_t(j)/y_t(j)) \), where both \( P_1 \) and \( P_2 \) are strictly positive and \( P_x \) is the partial derivative of \( P \) with respect to the \( x \)-th argument. Note that a higher frequency for verification increases the probability of finding misreporting, but at the same time, requires more monitoring input, and thus a higher cost of credit intermediation. Therefore, a bank’s optimal frequency of verification is affected by the probability of identifying misreporting upon verification, which, in turn, depends on the exogenous shocks to the lending efficiency. Without loss of generality, we specify the overall probability that the entrepreneur is found cheating as

\[
P(\varepsilon_t, m_t(j)/y_t(j)) = \begin{cases} 
1 - \frac{1}{(\varepsilon_t m_t(j)/y_t(j))^\psi} & < 1, \ \psi > 0 \\
0, & \text{for a report } \xi_t \neq \xi_{2t}, \\
0, & \text{for a report } \xi_t = \xi_{2t},
\end{cases}
\]

The timing of events for a period is as follows: at the beginning of the period, the lending efficiency shock \( \varepsilon_t \) is realized. Then, each bank engages in a loan contract with some entrepreneur by choosing the loan amount and overall monitoring input, which in turn determines \( \Pi_t \), the frequency of state verification. After the production takes place, entrepreneurs report their idiosyncratic productivity. Given that an entrepreneur reports \( A_{1,t} \), a lottery decides which banks will engage in verification.

III.4.2. Discussion of the credit supply shock. We provide an analysis that connects the credit supply shock in our model to reasons for changes in lending standards in the SLOOS. We assume that once a bank chooses to verify the state, it will observe only a noisy signal about

\(^{18}\)To be specific, with frequency of verification \( m/y \), the overall probability of detecting the misreport is \( P = 1 - (1 - p(\varepsilon))^{m/y} \).
the realized idiosyncratic technology. The signal $\hat{A}_t$ follows the random process

$$\hat{A}_t = A_t - \epsilon_t, \quad (9)$$

where $A_t = \overline{A}_t \xi_t$ is the realized cash flow, $\epsilon_t$ is i.i.d distributed and independent of $A_t$. The noise $\epsilon_t$ follows a binary distribution:

$$\epsilon_t = \begin{cases} 0 \text{ with probability } p(\epsilon_t) \\ A_{2,t} - A_{1,t} \text{ with probability } 1 - p(\epsilon_t) \end{cases}.$$ 

Recall that $p(\epsilon_t)$ is an increasing function of the credit supply shock $\varepsilon_t$. Note that when $\epsilon_t = 0$, the observed signal equals the realized productivity. Thus, $p(\epsilon_t)$—the probability of $\epsilon_t = 0$—measures the precision of the signal.

The probability that an entrepreneur receives $A_t = A_{2,t}$ but is found cheating is equal to

$$\Pr(\hat{A}_t = A_{2,t} | A_t = A_{2,t}) = \Pr(\epsilon_t = 0) = p(\epsilon_t),$$

where the first equality follows from the assumption that $\epsilon_t$ and $A_t$ are independent of each other. The probability that the entrepreneur is not caught of falsely defaulting is $1 - p(\epsilon_t)$. As $p(\epsilon_t)$ is reduced from 1 to 0.5, for example, $\text{var}(\hat{A}_t) = p(\epsilon_t) (1 - p(\epsilon_t)) (A_{2,t} - A_{1,t})^2$—the variance of the signal—increases, implying that the signal becomes noisier.

The role a credit supply shock plays in affecting the probability of detecting misreporting is equivalent to the fluctuation of $\text{var}(\hat{A}_t)$. During a recession, signals for entrepreneurs’ cash flows become noisier as the asymmetric information problem in the lending market worsens. Therefore, the probability for a bank to receive a strong signal for the realized cash flow of a healthy entrepreneur becomes lower, reflecting a more uncertain or less favorable economic outlook in the SLOOS. Banks respond by exerting more frequent verifications of entrepreneurs’ cash flows, which shows up as an observed increase of the fraction of banks that tighten lending standards.

In an extended cash-in-advance model that includes credit as an alternative to money as a medium for exchange, Benk, Gillman, and Kejak (2005) construct credit shocks to the payment technology using quarterly U.S. data on key variables. Such a credit shock is related to changes in the banking legislation during the U.S. financial deregulation era. In contrast, our credit supply shock originated from moral hazard in lending activities exacerbates informational frictions in the model, which can be thought as reflecting an uncertain economic outlook that leads to the cyclical fluctuation in lending standards observed in the data.

III.4.3. Optimal financial contract. Without loss of generality, we pair each individual entrepreneur to a specific bank. Denote the payoff to bank $j$ at state $i$ by $b_{it}(j)$ for $i \in \{1, 2\}$. Given the entrepreneur’s value of the loan contract with her and bank $j$, denoted by $v_t(j)$, and her net worth, the optimal contract problem for bank $j \in [0, 1]$ is to choose the quintuple \{$b_{it}(j), b_{2t}(j), k_t(j), h_t(j), m_t(j)$\} to maximize the bank’s expected profit:

$$\max_{b_{1t}(j), b_{2t}(j), k_t(j), h_t(j), m_t(j)} \{\pi_t b_{1t}(j) + (1 - \pi_t) b_{2t}(j) - (r_t k_t(j) + w_t h_t(j) - q_t \gamma_t a_t^e(j)) - \pi_t m_t(j)\}$$

(10)
subject to
\[ b_{1t}(j) \leq P_{1,t}A_{1,t}z_t k_t(j) h_t(j)^{1-\alpha}, \]  
\[ b_{2t}(j) \leq P_{2,t}A_{2,t}z_t k_t(j) h_t(j)^{1-\alpha}, \]  
\[ [1 - P(m_t(j)/(z_t k_t(j) h_t(j)^{1-\alpha}))][P_{2,t}A_{2,t}z_t k_t(j) h_t(j)^{1-\alpha} - b_{1t}(j)] \leq P_{2,t}A_{2,t}z_t k_t(j) h_t(j)^{1-\alpha} - b_{2t}(j), \]
\[ \pi_1(P_{1,t}A_{1,t}z_t k_t(j) h_t(j)^{1-\alpha} - b_{1t}(j)) + \pi_2(P_{2,t}A_{2,t}z_t k_t(j) h_t(j)^{1-\alpha} - b_{2t}(j)) = v_t(j), \]  
and the demand schedule represented by (6). The loan made by bank \( j \) is
\[ B_t(j) = r_t k_t(j) + w_t h_t(j) - \gamma_t a_t^i(j). \]

Constraints (11) and (12) are the limited liability constraints for both states. The incentive compatibility constraint (13) dictates that when the entrepreneur’s cash flow is in a good state, the expected benefit for her to misreport, represented by the left-hand-side term of (13), is less than or equal to the benefit of reporting the truth. The participation constraint (14) represents the contract value for the entrepreneur. The bank’s non-negative profit requires the entrepreneur’s contract value \( v_t(j) \) to satisfy
\[ v_t(j) \leq Y_t^{1-\mu} A_t^\mu (z_t k_t(j) h_t(j)^{1-\alpha}) - (r_t k_t(j) + w_t h_t(j) - \gamma_t a_t^i(j)) - \pi_t m_t(j). \]

The banking sector is competitive so that in equilibrium, each bank earns zero profit and equation (15) holds with equality.

**Lemma 1.** The optimal frequency of verification is the same for any two banks, i.e.,
\[ m_t(i)/y_t(i) = m_t(j)/y_t(j) = m_t/y_t, \forall i, j \in [0, 1], \ i \neq j. \]  

**Proof.** Since all entrepreneurs begin in period \( t \) with the same net worth \( q_t \gamma_t a_t^i(i) = q_t \gamma_t a_t^i(j), \forall i, j \in [0, 1], i \neq j, \) all banks are ex-ante homogeneous in the loan contract problem. The optimal contract problem for any two banks, \( i \neq j, \) will have the same solution. That is
\[ x_t(i) = x_t(j), \forall i, j \in [0, 1], \ i \neq j, \]
where \( x_t(j) = \{b_{1t}(j), b_{2t}(j), k_t(j), h_t(j), m_t(j)\} \).

Given Lemma 1, we establish the following proposition in regard to the fraction of banks engaging in verification.

**Proposition 1.** In equilibrium, the fraction of banks that verify the state ex-post is \( \pi_t \Pi_t \).

**Proof.** Since \( \Pi_t = \Pi_{jt} = \Pi_t \), the law of large numbers holds. Hence, the fraction of banks that ex-post verify the state when \( A_{1,t} \) is realized equals \( \Pi_t \). Also banks will monitor the state only when \( A_{1,t} \) is realized, which has probability \( \pi_t \). In each period, therefore, the fraction of banks that verify the state ex-post is \( \pi_t \Pi_t \).
Proposition 1 maps the aggregate index of lending standards in the SLOOS, \( s_t \), into our model. In the data, \( s_t \) is a (net) fraction of banks that tightened lending standards and \( \Delta s_t \) measures a change in the (net) fraction of banks in tightening lending standards in period \( t \). In our model economy, the fraction of banks that verify the state is \( \Pi_t \cdot \pi_t \). In a recession, the fraction of banks tightening lending standards increase (\( \Delta s_t > 0 \)), which corresponds to an increase in the fraction of banks that engage in state verification in the model. That is,

\[
\Delta s_t = \Delta [\Pi_t \cdot \pi_t] \equiv \Delta \frac{\pi_t m_t}{y_t},
\]

where \( m_t/y_t \) equals the frequency of verification for an individual bank by the law of large numbers.

### III.5. How various economic shocks drive the frequency of verification?

The bank’s frequency of verification is affected by both demand and supply shocks. The technology, risk, and collateral shocks, for example, influence the credit market from the demand side. In addition to the credit supply shock, these shocks potentially change the bank’s frequency of verification. In this section, we focus on how various shocks drive the comovement between bank loans and verification frequency. For the rest of the paper, we remove the index \( j \) for the bank-entrepreneur problem to keep the notation tractable.

#### III.5.1. Joint determination of bank loans and verification frequency.

Two equations characterize the relationship between \( m_t/y_t \) and \( y_t \), where \( y_t \) determines the amount of bank loans.\(^\text{19}\)

The first equation derives from a combination of the incentive compatibility constraint and the participation constraint in our model:

\[
y_t = \left[ \frac{v_t}{(1 - \pi_t)Y_t^{1-\mu}((A_{2,t})^{\mu} - (A_{1,t})^{\mu})(\varepsilon_t m_t/y_t)^\psi} \right]^{\frac{1}{\mu}}.
\]

The second equation derives from the bank’s first-order condition in respect to \( y_t \):

\[
\mu Y_t^{1-\mu} A_t^{\mu} (y_t)^{\mu-1} - p_t^y = \pi_t m_t/y_t \left( 1 + \frac{\mu}{\psi} \right),
\]

where \( p_t^y \equiv (r_t k_t + w_t h_t)/y_t \) is the unit cost of producing intermediate goods.

For each loan amount, equation (18) reveals the cost of the frequency of verification each bank needs to incur in order for that particular loan amount to satisfy the incentive compatibility constraint. Given the informational asymmetry, a larger amount of bank loans and thus a larger production scale tends to increase the entrepreneur’s gain of misreporting when the outcome is in the good state. This requires the bank to increase the frequency of verification to discourage the entrepreneur from misreporting, which implies a positive relationship between bank loans and verification frequency.

Equation (19) characterizes the optimal choice of intermediate goods \( y_t \) that maximizes the contract value. The presence of intermediation costs—unlike in the frictionless economy—drives a wedge between the marginal revenue product of \( y_t \) and the unit production cost \( p_t^y \).

\(^\text{19}\)See Supplemental Appendix S1 for details of solving the optimal contract.
A more frequent verification involves a higher monitoring cost, which eventually passes onto the entrepreneur. This leads to a higher overall marginal cost of intermediate goods (as the sum of the production cost and the intermediation cost). As a result, the size of the project and thus the amount of intermediate inputs have to be reduced, which leads to a negative relationship between \( y_t \) and \( m_t/y_t \).

Equations (18) and (19) together, therefore, determine the equilibrium relationship between bank loans and verification frequency. We show, in the next section, that this joint determination allows us to identify the credit supply shock from demand shocks.

III.5.2. Identifying the credit supply shock from demand shocks. We first explore how various shocks affect the comovement between bank loans and verification frequency. Figures 6 and 7 plot the relationship between \( y_t \) and \( m_t/y_t \) via equations (18) and (19): one positively sloped curve labeled as the SS curve and one negatively sloped curve labeled the DD curve. The intersection point A represents the initial equilibrium. In our model, various shocks affect the equilibrium outcome by shifting either the SS curve or the DD curve or both.

Figure 6 illustrates the impact of a negative shock to credit supply. This shock shifts the SS curve to the left as shown by the dotted line. In equilibrium, intermediate goods \( y_t \) and therefore bank loans decline while the verification frequency (or the fraction of banks engaging in verification) increases. Intuitively, this negative shock tightens the incentive compatibility constraint, which leads to a simultaneous increase in verification frequency and a decline in bank loans to enforce the true reporting mechanism. The equilibrium moves from point A to point B, implying countercyclical lending standards.

By contrast, other shocks shift either the DD curve alone or both SS and DD curves simultaneously. For example, a positive risk shock (i.e., a positive shock to the delinquency rate of bank loans measured by \( \pi_t \)) increases the marginal monitoring cost (the right-hand side of equation (19)) and thus shifts the DD curve to the left. At the same time, this risk shock shifts the SS curve to the right as well, further reducing the frequency of verification. A negative technology shock or a negative collateral shock, on the other hand, shifts both DD and SS curves (via a decrease in \( v_t \)) to the left, thus muting the effects on verification frequency. All these shocks imply either procyclical or acyclical movements of bank lending standards.

In the next section, we estimate the theoretical model against several macroeconomic series, including bank loans and changes in the fraction of banks that tighten or ease their lending standards. The model provides an economically interpretable relationship between bank loans and lending standards (i.e., the fraction of banks engaging in verification in the model); its estimation quantifies the different roles various shocks play in driving this relationship. One chief finding is that the credit supply shock accounts for most of the variation in the frequency of verification and thus plays a dominant role in explaining countercyclical lending standards observed in the data.
IV. Empirical analysis

We use our constructed series of lending standards, together with other standard macroeconomic variables, to estimate the theoretical model and provide an empirical analysis.

IV.1. Measurement. We fit the model to six quarterly U.S. time series: changes in bank lending standards on C&I and CS loans ($\Delta s_t^{Data}$), real per capita C&I and CS loans to the nonfinancial sector ($b_t^{Data}$), commercial banks’ charge-off rates on bank loans ($d_t^{Data}$), real per capita nondurable goods consumption ($c_t^{Data}$), real per capita investment in the nonfinancial sector ($i_t^{Data}$), and per capita hours worked in the nonfinancial sector ($h_t^{Data}$).

From the Call Report microdata, we construct C&I and CS loans to the nonfinancial sector to be usable for our theoretical model as well as other structural models. This task proves challenging. We first follow the FFIEC’s instruction book (http://www.ffiec.gov/pdf/FFIEC_forms/FFIEC031_FFIEC041_201503_i.pdf), which includes the detailed instructions to banks about how to exclude loans to financial institutions from loans secured by real estate (page 156 of the book). We then obtain C&I loans to nonfinancial institutions excluding those secured by residential real estate. In a third step, we obtain automobile loans and other consumer loans excluding loans outstanding on credit cards. We sum all these components to obtain the series of C&I and CS loans to the nonfinancial sector. We include automobile loans and consumer loans for other consumer durables to follow the convention in the DSGE literature by treating consumer durables as a part of investment goods in the theoretical model.

To be also consistent with our model, we construct the nominal consumption series as the sum of nominal nondurable goods consumption and nominal services consumption excluding housing services, the nominal investment series as the sum of nominal equipment investment, intellectual property products investment, and durable goods consumption, and the labor hours series as aggregate hours in the business sector excluding those in the finance and insurance industries. More involved effort is devoted to construction of the aggregate price index for consumption, which is computed as the Tornqvist price index for nondurable goods and services consumption excluding housing, and the annual capital stock consistent with the definition of our investment series. All the nominal variables are divided by the aggregate price index and then by the civilian noninstitutional population with ages between 25 to 64 years, so that these nominal variables are transformed to real variables per capita. The population series is smoothed with Christiano and Fitzgerald (2003)’s band-pass filter to eliminate seasonal fluctuations and breaks due to the Census’s population controls.

In addition to the series of bank lending standards displayed in Figure 1, Figure 8 displays log values of real per capita bank lending, real per capita consumption, real per capita investment, and per capita labor hours. It is evident from the top right panel of Figure 8 that bank loans experience a sharp decline at the beginning of the Great Recession and its recovery has been slow. Consumption, investment, and labor hours, displayed in the rest of the panels, show similar patterns but with different magnitudes. In particular, the
fall of investment during the Great Recession is more severe and persistent than the fall of consumption.

Equilibrium conditions, a list of endogenous variables, and the log-linearized system of equations are reported in Supplemental Appendices S2-S5. The variable \( \hat{x}_t \) is defined as \( \log x_t - \log x \), where \( x_t \) represents a variable of interest and \( x \) is the steady state value of \( x_t \). Since our theoretical model is trend-stationary, we follow the DSGE literature and remove from all (log) trend series the balanced linear trend defined by output growth. Changes in the fraction of banks engaging in verification is measured by equation (17), which can be rewritten as

\[
\Delta s_t^{\text{Data}} = b_s [\Delta \hat{m}_t - \Delta \hat{y}_t + \Delta \hat{n}_t],
\]

where \( b_s \) is a scale parameter for log-linearizing the term \( \Delta \hat{m}_t/\hat{y}_t \) in equation (17). The measurement equation for bank loans is

\[
\log b_t^{\text{Data}} = b_b \hat{b}_t.
\]

Because the series of bank loans includes long-term debts, the relationship between \( \log b_t^{\text{Data}} \) and \( \hat{b}_t \) would not be exact. We estimate the parameters \( b_b \) and \( b_s \) jointly with the other parameters in the model. The measurement equations for the other variables are

\[
\begin{align*}
\log c_t^{\text{Data}} &= \hat{c}_t, \\
\log i_t^{\text{Data}} &= \hat{i}_t, \\
\log h_t^{\text{Data}} &= \hat{h}_t, \\
\log d_t^{\text{Data}} &= \hat{d}_t,
\end{align*}
\]

where \( \log h_t^{\text{Data}} \) and \( \log d_t^{\text{Data}} \) are demeaned log variables.

IV.2. Estimation method. We apply the Bayesian methodology to estimation of this log-linearized system, using our own C/C++ code. The advantage of using our own code instead of using the Dynare software is the flexibility and accuracy we have for finding the posterior mode. Our Dynare code fails to converge with any of its optimization options. The failure is partly due to the difficulty of solving the steady state and partly due to the complexity of the model the Dynare software package has yet to confront with.\(^{20}\)

We use the log-linearized equilibrium conditions, reported in Supplemental Appendix S4, to form the likelihood function fit to the six quarterly U.S. time series from 1990Q2 to 2017Q4. We categorize the model’s parameters in three groups. The first group consists of those fixed at values commonly used or calibrated by the average patterns of the data: the capital share is set to 0.35; the subjective discount factor \( \beta \) is set to 0.995; the elasticity-of-substitution parameter \( \mu \) is set to 0.85, which implies a markup of 17.6%, consistent with the empirical evidence provided by Morrison (1992); the steady state hours is \( H = 0.3 \); the capital adjustment cost parameter is \( S'' = 2.5 \) as used in much of the DSGE literature;

\(^{20}\)We are in the process of collaborating with Dynare developers to make our estimation procedure available through the Dynare interface.
the parameter for the inverse of the Frisch elasticity of labor supply is set to $\nu = 1.0$; the delinquency parameter is $\bar{\delta} = 0.8\%$ so that the model’s average delinquency rate is the same as the data average 0.78%; the leverage ratio $B/\bar{Y}$ is 0.7 calculated from our quarterly data and consistent with the literature on financial frictions; the intermediation-cost ratio $\bar{\pi}m^c/\bar{Y}$ is 0.1194 at a quarterly frequency, equal to the ratio of financially intermediated services in the banking system to aggregate output, where financially intermediated services are constructed using the NIPA data by following Mehra, Piguillem, and Prescott (2011); the ratio of output to capital is 0.125 at quarterly frequency, calculated from our quarterly data; and the ratio of investment to capital is 0.0377 at quarterly frequency, which is also calculated from our quarterly data.

The second group of parameters are to be estimated. Table 3 reports the prior distribution of each of these parameters, where “Inv-Gamma” stands for an inverse Gamma probability density. Most of these prior settings, agnostic in nature, are used in the literature (see Liu, Wang, and Zha (2013) for detailed discussions). We discuss the few prior settings that are specific to our model. The unit cost of producing intermediate goods, $p^y$, is not a parameter but an implicit function of model parameters implied by the steady state. Solving for the steady state value $p^y$ and other steady state variables requires solving a system of nonlinear equations, which would be costly during the estimation phase. A nonlinear system may not have a solution or its solution can be difficult to find, which is the case for our model. This difficulty is one of the main reasons that the Dynare software package has difficulty in finding the posterior mode of this model. By finding the value of $p^y$ first, however, we can reverse-engineer the value of the parameter $\bar{\varepsilon}$. The steady state can then be solved recursively. This advancement makes the estimation feasible, even though the steady state for our model is complicated. The prior mean we set for $p^y$ is around 1.0 with the .90 probability interval between 0.5 and 1.5. We experiment with a much looser prior and our results are robust to different prior settings. The prior means for $b_s$ and $b_b$ are centered at 1. We also change the variance of the prior and the posterior results are not materially affected since the posterior modes are similar and the impulse responses do not change much.

The posterior modes, alongside the .90 posterior probability intervals, for the second group of parameters are reported in Tables 4 and 5. The standard Metropolis Markov chain Monte Carlo (MCMC) algorithm is used to simulate the posterior probability distribution. The price of intermediate goods is estimated to be 0.63, with the posterior distribution skewed to higher values than the posterior mode. The estimated slope parameters in the measurement equations for lending standards and bank loans are statistically significant. The slope parameter for lending standards is tightly estimated around one, implying that the dynamics generated from the model capture those of cyclical lending standards observed in the data. The estimated habit parameter for the household’s consumption is extremely small, even judged by the .90 posterior probability interval. The estimated habit parameter for the entrepreneur, however, is much larger.

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21 Some more sophisticated MCMC algorithms are discussed by Waggoner, Wu, and Zha (2016).
The persistence parameter for the credit supply shock is estimated above 0.8 according to the .90 posterior probability interval. Its posterior mode is smaller than those of the technology and risk shocks. The standard deviation of the credit supply shock is similar to that of the risk shock with both estimates above 0.20. The estimated collateral shock process has little persistence with a small standard deviation. In the section below, we discuss how credit supply shocks are transmitted to the fluctuations of bank loans and aggregate output.

The third group collects the remaining parameters. These parameters are obtained by solving the steady state given the parameter values in the first two groups. Since the steady state can be solved recursively, these parameter values can be calculated with little computing time for each MCMC simulation of the second group of parameters.

IV.3. Findings. Figure 9 reports the estimated impulse responses to a negative shock to bank credit supply. The frequency of verification rises on impact (top panel of the figure) so that the fraction of banks choosing to tighten lending standards (engaging in verification in our model) increases by almost 30% initially. A negative shock to bank credit supply causes a deterioration of lending efficiency and an increase of monitoring costs in the banking sector. These effects result in an increase of the fraction of banks tightening lending standards. Accordingly, both bank loans and output fall by 3% on impact (bottom two panels of the figure) and remain negative for years. The response of bank loans, which in turn affects output, is propagated through the so-called financial accelerator.\footnote{For the implications of our model on the risk premium, see Appendix B and also Gomes, Yaron, and Zhang (2003).} An initial fall of bank loans reduces the production scale for intermediate goods and the expected profit of the production project. After these initial effects, both the end-of-period net worth of the entrepreneur and the contract value of the next-period loan decline. The incentive problem is exacerbated, which contracts bank loans further and results in the hump-shape response of bank loans (bottom panel of Figure 9).

Key variance decompositions attributable to a credit supply shock (relative to all other shocks) are reported in Table 6. The credit supply shock explains over 40% of the output fluctuation on impact and the importance of this effect on output is persistent with a 20% contribution to the output fluctuation in four years (16 quarters). The contribution to the loan fluctuation is over 43% on impact and this effect is also persistent with 30% of the loan fluctuation over the four-year horizon that is explained by the shock to bank credit supply.

As can be seen from Table 6, the credit supply shock accounts for essentially all the fluctuation of the verification frequency (the fraction of banks that tighten lending standards or engage in verification). This result indicates the importance of our data analysis on the fraction of banks tightening lending standards in disciplining the kind of credit frictions needed to be embedded in the model. Various economic shocks, demand or supply, drive the fluctuation of bank lending standards in the data as well as the model’s corresponding fraction of banks that engage in verification. As discussed in Section III.5 and revealed in
Table 6, however, only a shock to bank credit supply in the model can generate countercyclical movements of the verification frequency, while shocks outside the banking sector shift the credit demand curve and tend to produce procyclical or acyclical movements in the fraction of banks that engage in verification.

V. Conclusion

As argued by Perri and Quadrini (2018), credit tightening in the banking sector plays an important role in propagating the depth and duration of the Great Recession across countries. In this paper, we not only construct a direct measure of credit tightening via changes in bank lending standards but, more importantly, provide an extensive analysis of the cyclical fluctuation in lending standards by using the micro and macro data merged from the SLOOS, Call Report, and CRSP. The detailed data analysis is usable for macroeconomic research on financial frictions in general and helps discipline the kind of credit frictions needed to be embedded in the model in particular. By estimating the model against standard macro variables and the series of the fraction of banks that change lending standards over the business cycle, we are able to quantify the importance of the credit supply shock in driving the countercyclical movements of bank lending standards as well as in generating the significant fluctuations of bank loans and aggregate output.
### Table 1. Panel regressions on lending standards

<table>
<thead>
<tr>
<th>Lending standards:</th>
<th>C&amp;I loans</th>
<th></th>
<th>C&amp;I and CS loans</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>S.E.</td>
<td>Est.</td>
<td>S.E.</td>
</tr>
<tr>
<td>Change in C&amp;I NIM(^#)</td>
<td>-2.756</td>
<td>1.157**</td>
<td>1.6602</td>
<td>2.657</td>
</tr>
<tr>
<td>Change in total NIM(^#)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in C&amp;I LLP(^#)</td>
<td>2.4600</td>
<td>0.523***</td>
<td>7.4044</td>
<td>0.998***</td>
</tr>
<tr>
<td>Change in total LLP(^#)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core loan share</td>
<td>0.6742</td>
<td>0.077***</td>
<td>0.6879</td>
<td>0.069***</td>
</tr>
<tr>
<td>Core deposit share</td>
<td>-0.659</td>
<td>0.060***</td>
<td>-0.654</td>
<td>0.053***</td>
</tr>
<tr>
<td>Log real assets</td>
<td>0.0206</td>
<td>0.009**</td>
<td>0.0361</td>
<td>0.008***</td>
</tr>
<tr>
<td>Stock returns</td>
<td>-0.041</td>
<td>0.007***</td>
<td>-0.040</td>
<td>0.006***</td>
</tr>
<tr>
<td>Tobin’s Q</td>
<td>-0.311</td>
<td>0.088***</td>
<td>-0.127</td>
<td>0.079</td>
</tr>
<tr>
<td>Tier 1 capital ratio</td>
<td>-2.423</td>
<td>0.417***</td>
<td>-1.748</td>
<td>0.373***</td>
</tr>
<tr>
<td>R(^2)</td>
<td>0.1143</td>
<td>0.1337</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* The superscripts *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The superscript \(^\#\) denotes the series adjusted for mergers. Lending standards for quarter \(t\) are reported early in quarter \(t + 1\). Control variables from the Call Report, bank holding company financial reports, and CRSP are variables at the end of quarter \(t\).
Table 2. Panel regressions on lending standards

<table>
<thead>
<tr>
<th></th>
<th>C&amp;I loans</th>
<th></th>
<th>C&amp;I + CS loans</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>S.E.</td>
<td>Est.</td>
<td>S.E.</td>
</tr>
<tr>
<td>Change in C&amp;I NIM#</td>
<td>-2.608</td>
<td>1.149**</td>
<td>1.1769</td>
<td>2.641</td>
</tr>
<tr>
<td>Change in total NIM#</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in C&amp;I LLP#</td>
<td>2.3660</td>
<td>0.519***</td>
<td>7.0202</td>
<td>0.993***</td>
</tr>
<tr>
<td>Change in total LLP#</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core loan share</td>
<td>0.6493</td>
<td>0.076***</td>
<td>0.6762</td>
<td>0.068***</td>
</tr>
<tr>
<td>Core deposit share</td>
<td>-0.673</td>
<td>0.058***</td>
<td>-0.689</td>
<td>0.052***</td>
</tr>
<tr>
<td>Log real assets</td>
<td>0.0305</td>
<td>0.009***</td>
<td>0.0431</td>
<td>0.008***</td>
</tr>
<tr>
<td>Stock returns</td>
<td>-0.028</td>
<td>0.007***</td>
<td>-0.029</td>
<td>0.006***</td>
</tr>
<tr>
<td>Market leverage</td>
<td>0.0078</td>
<td>0.001***</td>
<td>0.0060</td>
<td>0.001***</td>
</tr>
<tr>
<td>Tier 1 capital ratio</td>
<td>-1.699</td>
<td>0.423***</td>
<td>-1.158</td>
<td>0.379***</td>
</tr>
<tr>
<td>R²</td>
<td>0.1261</td>
<td>0.1444</td>
<td></td>
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</tr>
</tbody>
</table>

Note. The superscripts *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The superscript # denotes the series adjusted for mergers. Lending standards for quarter $t$ are reported early in quarter $t + 1$. Control variables from the Call Report, bank holding company financial reports, and CRSP are variables at the end of quarter $t$. 
### Table 3. Prior distributions of structural and shock parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
<th>a</th>
<th>b</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^y$</td>
<td>Price of intermediate goods</td>
<td>Gamma(a,b)</td>
<td>9.387</td>
<td>9.952</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega^h$</td>
<td>Household habit</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\omega^c$</td>
<td>Entrepreneur habit</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$b_s$</td>
<td>Scale for lending standards</td>
<td>Gamma(a,b)</td>
<td>9.387</td>
<td>9.952</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$b_b$</td>
<td>Scale for bank loans</td>
<td>Gamma(a,b)</td>
<td>9.387</td>
<td>9.952</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Technology</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>MEI</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_{\delta}$</td>
<td>Risk</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_{\gamma}$</td>
<td>Preference</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_{\epsilon}$</td>
<td>Credit supply</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_{\theta}$</td>
<td>Labor supply</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\rho_{\eta}$</td>
<td>Collateral</td>
<td>Beta(a,b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.025</td>
<td>0.776</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Technology</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>MEI</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_{\delta}$</td>
<td>Risk</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_{\gamma}$</td>
<td>Preference</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>Credit supply</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_{\theta}$</td>
<td>Labor supply</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma_{\eta}$</td>
<td>Collateral</td>
<td>Inv-Gamma(a,b)</td>
<td>3.26e-01</td>
<td>1.45e-04</td>
<td>1.0e-04</td>
<td>2.0</td>
</tr>
</tbody>
</table>

*Note.* “Low” and “high” denotes the bounds of the 90% probability interval for each parameter.

### Table 4. Posterior distributions of structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Posterior estimates</th>
<th>Mode</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^y$</td>
<td>Price of intermediate goods</td>
<td></td>
<td>0.63</td>
<td>0.54</td>
<td>1.57</td>
</tr>
<tr>
<td>$\omega^h$</td>
<td>Household habit</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
</tr>
<tr>
<td>$\omega^c$</td>
<td>Entrepreneur habit</td>
<td></td>
<td>0.68</td>
<td>0.45</td>
<td>0.67</td>
</tr>
<tr>
<td>$b_s$</td>
<td>Slope for lending standards</td>
<td></td>
<td>1.18</td>
<td>0.85</td>
<td>1.34</td>
</tr>
<tr>
<td>$b_b$</td>
<td>Slope for bank loans</td>
<td></td>
<td>1.68</td>
<td>1.19</td>
<td>1.93</td>
</tr>
</tbody>
</table>

*Note.* “Low” and “High” denote the bounds of the 90% probability interval for each parameter.
Table 5. Posterior distributions of shock parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Posterior estimates</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_z$</td>
<td>Technology</td>
<td>0.9982</td>
<td>0.9927</td>
<td>0.9995</td>
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<tr>
<td>$\rho_x$</td>
<td>MEI</td>
<td>0.2216</td>
<td>0.0062</td>
<td>0.5663</td>
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</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Risk</td>
<td>0.9356</td>
<td>0.8887</td>
<td>0.9688</td>
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</tr>
<tr>
<td>$\rho_\phi$</td>
<td>Preference</td>
<td>0.8545</td>
<td>0.8300</td>
<td>0.8993</td>
<td></td>
</tr>
<tr>
<td>$\rho_\epsilon$</td>
<td>Credit supply</td>
<td>0.8779</td>
<td>0.8161</td>
<td>0.9135</td>
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</tr>
<tr>
<td>$\rho_\phi$</td>
<td>Labor supply</td>
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<td>0.8747</td>
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<tr>
<td>$\rho_\gamma$</td>
<td>Collateral</td>
<td>0.0090</td>
<td>0.0041</td>
<td>0.2022</td>
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<tr>
<td>$\sigma_z$</td>
<td>Technology</td>
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<td>0.0198</td>
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<tr>
<td>$\sigma_x$</td>
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<td>0.0002</td>
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<td>$\sigma_\theta$</td>
<td>Risk</td>
<td>0.2168</td>
<td>0.2172</td>
<td>0.2762</td>
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<tr>
<td>$\sigma_\phi$</td>
<td>Preference</td>
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<td>0.0049</td>
<td>0.0080</td>
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<tr>
<td>$\sigma_\epsilon$</td>
<td>Credit supply</td>
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<td>0.2279</td>
<td>0.3008</td>
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<td>$\sigma_\phi$</td>
<td>Labor supply</td>
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<tr>
<td>$\sigma_\gamma$</td>
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<td>0.0052</td>
<td>0.0048</td>
<td>0.0073</td>
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</table>

Note. “Low” and “High” denote the bounds of the 90% probability interval for each parameter.

Table 6. Contributions of a credit supply shock to variance decompositions (%)

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Verification frequency</th>
<th>Bank loans</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.99</td>
<td>43.97</td>
<td>41.06</td>
</tr>
<tr>
<td>4</td>
<td>99.98</td>
<td>41.33</td>
<td>35.13</td>
</tr>
<tr>
<td>8</td>
<td>99.97</td>
<td>37.10</td>
<td>29.10</td>
</tr>
<tr>
<td>16</td>
<td>99.96</td>
<td>30.75</td>
<td>21.08</td>
</tr>
</tbody>
</table>
Figure 1. The constructed aggregate series of bank lending standards. The shaded bars represent NBER-dated recessions.
Figure 2. The bank’s current or expected capital position as a reason for tightening or easing lending standards. The longest shaded bars represent NBER-dated recessions. Answer “Important” includes the “Somewhat important” and “Very important” categories.
Figure 3. Top chart: the interquartile range of Tier 1 capital ratios for banks responding how important their current or expected capital positions were for tightening C&I lending standards. The sample only includes publicly-listed banks in CRSP with a holding company that files the FR Y-9C report. Quarterly responses are grouped into annual frequency such that a bank can appear up to four times in each box-plot. The red boxes represent the interquartile response range for the “not important” reason for tightening lending standards, and the blue boxes for the “important” reason. The diamond symbol inside each box represents the median of the response distribution. Bottom chart: the minimum, median, and maximum values of Tier 1 capital ratios for banks responding how important a more uncertain or less favorable economic outlook was for tightening C&I lending standards. For comparison, the selected banks correspond to those in the top chart exactly.
Figure 4. Top chart: the interquartile range of market leverages for banks responding how important their current or expected capital positions were for tightening C&I lending standards. The sample only includes publicly-listed banks in CRSP with a holding company that files the FR Y-9C report. Quarterly responses are grouped into annual frequency such that a bank can appear up to four times in each box-plot. The red boxes represent the interquartile response range for the “not important” reason for tightening lending standards, and the blue boxes for the “important” reason. The diamond symbol inside each box represents the median of the response distribution. Annual distributions with no observations are left blank.

Bottom chart: the minimum, median, and maximum values of market leverages for banks responding how important a more uncertain or less favorable economic outlook was for tightening C&I lending standards. For comparison, the selected banks correspond to those in the top chart exactly.
Figure 5. Aggregate uncertainty or macroeconomic outlook as a reason for tightening or easing lending standards. The longest shaded bars represent NBER-dated recessions. Answer “Important” includes the “Somewhat important” and “Very important” categories. The survey on “Uncertainty” as a reason for changing lending standards began in 1998Q4.
Figure 6. The impact of a negative shock to bank credit supply.
Figure 7. The impact of shocks outside the banking sector.
Figure 8. The U.S. time series constructed for estimation of the structural model. All variables are per capita. Loans, consumption, and investment are real variables (deflated by prices of consumption goods). The shaded bars represent NBER-dated recessions.
Figure 9. The impulse responses to a contractionary credit supply shock of one standard deviation. The star line represents the posterior model estimate. The dashed lines around the star lines represent the .90 posterior probability bands.
Appendix A. Data description

The variables in Tables 1 and 2 are defined below.

- **C&I loans**: industrial and commercial loans from banks.
- **CS loans**: bank loans to consumers (CS) excluding credit card loans, most of which are used for consumer durables.
- **Change in C&I NIM**: change in the ratio of net interest and fee income on C&I loans to the quarterly average of C&I loans.
- **Change in total NIM**: total net interest income divided by the quarterly average of total assets.
- **Change in C&I LLP**: quarterly change in the share of total outstanding C&I loans that are in nonaccrual status, where LLP is an abbreviation for “loan loss provisions.”
- **Change in total LLP**: quarterly change in the share of total outstanding loans and leases that are in nonaccrual status.
- **Core loan share**: ratio of the sum of C&I loans, real estate (excluding HELOCs) loans, credit card loans, and consumer loans to total assets.
- **Core deposit share**: ratio of the sum of total transaction accounts, money market deposit accounts, other non-transaction savings deposits, and total time deposits of less than $100,000 to total liabilities and minority interest.
- **Log real assets**: end-of-quarter total assets divided by the GDP Deflator (converted to 2005 dollars).
- **Tobin’s Q**: sum of market capitalization and total liabilities and minority interest for the bank’s holding company, divided by the holding company’s total assets.
- **Market leverage**: sum of market capitalization and total liabilities and minority interest for the bank’s holding company, divided by market capitalization of the bank’s holding company (Begenau, Bigio, Majerovitz, and Vieyra, 2019). All variables are at the end of the quarter.
- **Tier 1 capital ratio**: ratio of Tier 1 capital to average total consolidated assets excluding some deductions for capital ratio calculation purposes. The ratio is reported in the Call Report from the beginning of 2001 onward. Prior to 2001, we proxy this ratio as the difference between total equity capital and goodwill divided by the quarterly average of total assets.

Appendix B. Additional results

The series of bank lending standards used for our model estimation is constructed to be consistent with the model ingredients. In particular, investment in the model, as well as in most macroeconomic models, includes spendings on consumer durables, and bank loans are restricted to the nonfinancial sector. We construct bank lending standards on C&I and CS loans and on C&I loans alone. Figure B.1 display both series of bank lending standards and their dynamics are very similar. The Federal Reserve Board (FRB) also reports a series
of lending standards that are constructed from the micro data with a simple average. This series applies to standards on C&I loans only and differs in some respects from our loan weighted average series (top panel of Figure B.1).

Our model also has implications on the risk premium, defined as the spread between the average lending rate (the ratio of the average payoff to the bank in both states to the amount of bank loans) and the borrowing rate. Under the zero profit condition, the risk premium can be expressed as

\[
\pi_t b_{1t} + (1 - \pi_t) b_{2t} - 1 = \frac{\pi_t m_t}{p_t^y y_t - q_t c r a_t^2},
\]

where \(p_t^y \equiv (r_t k_t + w_t h_t) / y_t\) is the unit production cost of intermediate goods. According to equation (B.1), the risk premium increases with total verification/monitoring costs. Since verification is costly, banks charge a higher lending rate (in the good state) when more verification resources are needed to enforce the truth telling.

In addition to the cyclical fluctuation of lending standards, credit supply shocks contribute to the countercyclical fluctuation of the risk premium due to the countercyclicality of endogenous verification costs \(m_t\). By contrast, Gomes, Yaron, and Zhang (2003) show that the typical costly state verification model generates the procyclical risk premium, which is counterfactual. Their analysis focuses only on credit demand shocks, which, similar to demand shocks in our model, tend to produce the counterfactual procyclical intermediation premium. For example, in response to a negative collateral shock in our model, the demand for capital and thus the price of capital decrease. The decrease in the demand for capital decreases the demand for bank loans and thus the overall verification/monitoring cost. As a result, the risk premium increases in our model as well as in Gomes, Yaron, and Zhang (2003).

The credit supply shock in our model, on the other hand, generates the countercyclicality of the frequency of state verification (or the verification cost). Figure B.2 reports the estimated impulse responses to a negative shock to credit supply. The increase in the frequency of verification (credit supply tightening) is associated with an increase in the risk premium (top row of Figure B.2). Intuitively, an increase in the frequency of verification forces banks to put in more monitoring resources, which drives up the spread between lending and borrowing rates. Accordingly, bank loans decline on impact and continue to decline until the fifth quarter, while output falls on impact and the fall is persistent.
Figure B.1. The constructed diffusion series for bank lending standards. The loan weighted series (solid lines) is constructed by linking the Call Report to the SLOOS. The dashed line represents the unweighted series published by the FRB. The FRB does not publish a lending standards series related to the sum of C&I loans and CS loans. The shaded bars represent NBER-dated recessions.
Figure B.2. The impulse responses to a contractionary credit supply shock of one standard deviation. The star line represents the posterior mode estimate. The dashed lines around the star lines represent the .90 probability bands.
References


In the following appendices, all labels for equations, figures, tables, definitions, and propositions begin with S, standing for supplement to the main text.

**Appendix S1. Solving the Optimal Contract**

Since capital and labor enter symmetrically into the optimal contracting problem (10), to illustrate the intuition, we can solve this optimal contract problem in two steps: first we solve for the expected output of intermediate goods, together with the monitoring input and the payment to banks in each state. In the second step, we solve for the capital and labor input. With the definition of $p^y_t$ and $y_t$, the optimal contract problem (10) can be rewritten as

$$
\max_{b_1t, b_2t, y_t, m_t} \{ \pi_t b_{1t} + (1 - \pi_t) b_{2t} - (p^y_t y_t - q_t \gamma_t \alpha^*_t) - \pi_t m_t \}
$$

subject to

$$
b_{1t} \leq P_{1,t} A_{1,t} y_t,
$$

$$
b_{2t} \leq P_{2,t} A_{2,t} y_t,
$$

$$
[1 - P(m_t/y_t)] [P_{2,t} A_{2,t} y_t - b_{1t}] \leq P_{2,t} A_{2,t} y_t - b_{2t},
$$

$$
\pi_t (P_{1,t} A_{1,t} y_t - b_{1t}) + (1 - \pi_t) (P_{2,t} A_{2,t} y_t - b_{2t}) = v_t,
$$

**Proposition S1.** There is no monitoring if and only if

$$
Y^{1-\mu} \left( A_{1,t} y^f_t \right)^\mu \geq p^y_t y_t - q_t \gamma_t \alpha^*_t, \quad \text{where}
$$

$$
y^f_t \equiv \arg \max_{y_t} Y^{1-\mu} \left( \pi_t (A_{1,t})^\mu + (1 - \pi_t) (A_{2,t})^\mu \right) (y_t)^\mu - p^y_t y_t.
$$

**Proof.** See Appendix S1.1.

The intuition behind Proposition S1 is straightforward. As the production scale increases, if the payoff in the bad state becomes larger than the amount of bank loans, then there is no incentive for the bank to engage in costly monitoring. Proposition 1 implies that as the firm becomes large, it relies less on bank loans to finance the input costs. Accordingly, it is optimal for the bank to have zero monitoring if the bank loan advanced to the entrepreneur is less than what the bank can seize in the bad state. In the analysis below, we therefore assume that all entrepreneurs’ net worth is sufficiently small so that the financial constraint is always binding.

As in the standard costly-state-verification model, the following proposition characterizes the optimal contract when the incentive compatibility constraint is binding.

**Proposition S2.** Given that

$$
Y^{1-\mu} \left( A_{1,t} y^f_t \right)^\mu < p^y_t y_t - q_t \alpha_t,
$$

the limited liability constraint for state 1 is binding, the limited liability constraint for state 2 is nonbinding, and the incentive compatibility constraint is always binding.

**Proof.** See Appendix S1.2.
With Proposition S2, we can derive two equations to characterize the relationship between \( m_{t}/y_{t} \) and \( y_{t} \). Equation (18) is obtained by plugging equation (S2) and (S5) into the incentive compatibility constraint (S4). To obtain equation (19), we plug equations (6) and (S5) into (S1) and rewrite the optimal contract problem as

\[
\max_{y_{t}, m_{t}/y_{t}} \{ Y_{t}^{1-\alpha} A_{t}^{\alpha} y_{t}^{\alpha} - (p_{t}^{\beta} y_{t} - q_{t} a_{t}^{e}) - \pi_{t} y_{t} m_{t}/y_{t} - v_{t} \},
\]

subject to equation (18). The first-order conditions then give equation (19).

In the second stage, given the entrepreneur’s expected output of intermediate goods \( y_{t} \), the optimal inputs in capital and labor are obtained by solving the following cost minimization problem

\[
\min_{k_{t}, h_{t}} \{ r_{t} k_{t} + w_{t} h_{t} \}
\]

subject to

\[
z_{t} (k_{t})^{\alpha} (h_{t})^{1-\alpha} \geq y_{t},
\]

This gives

\[
k_{t} = \frac{y_{t}}{z_{t}} \left( \frac{w_{t}}{1-\alpha} \right)^{1-\alpha} \left( \frac{r_{t}}{\alpha} \right)^{\alpha},
\]

\[
h_{t} = \frac{y_{t}}{z_{t}} \left( \frac{r_{t}}{\alpha} \right)^{\alpha} \left( \frac{w_{t}}{1-\alpha} \right)^{-\alpha}.
\]

Finally, the competitiveness of the factor markets implies that the unit production cost is:

\[
p_{t}^{y} = \frac{1}{z_{t}} \left( \frac{r_{t}}{\alpha} \right)^{\alpha} \left( \frac{w_{t}}{1-\alpha} \right)^{1-\alpha}.
\]

S1.1. **Proof of Proposition S1.** We take two steps to prove Proposition S1. We first derive the necessary condition for the monitoring cost \( m_{t} = 0 \). We then derive its sufficient condition.

With \( m_{t} = 0 \), from the incentive compatibility constraint (13), we have

\[
P_{2,t} A_{2,t} y_{t} - b_{2,t} \geq P_{2,t} A_{2,t} y_{t} - b_{1,t}.
\]

Also, combining (14) with (15), we get

\[
[\pi_{t} P_{1,t} A_{1,t} + (1 - \pi_{t}) P_{2,t} A_{2,t}] y_{t} - [\pi_{t} (P_{1,t} A_{1,t} y_{t} - b_{1,t}) + (1 - \pi_{t}) (P_{2,t} A_{2,t} y_{t} - b_{2,t})] - \pi_{t} m_{t} \geq p_{t}^{y} y_{t} - q_{t} \gamma_{t} a_{t}^{e}.
\]

Plugging (S8) (with equality) into (S9), we obtain the necessary condition for \( m_{t} = 0 \).

\[
[\pi_{t} P_{1,t} A_{1,t} + (1 - \pi_{t}) P_{2,t} A_{2,t}] y_{t} - (1 - \pi_{t}) (P_{2,t} A_{2,t} y_{t} - b_{1,t}) - \pi_{t} (P_{1,t} A_{1,t} y_{t} - b_{1,t}) \geq p_{t}^{y} y_{t} - q_{t} \gamma_{t} a_{t}^{e}
\]

or

\[
P_{1,t} A_{1,t} y_{t} \geq P_{1,t} A_{1,t} y_{t} - b_{1,t} + p_{t}^{y} y_{t} - q_{t} \gamma_{t} a_{t}^{e}
\]

\[
\geq p_{t}^{y} y_{t} - q_{t} \gamma_{t} a_{t}^{e}
\]
where the second inequality is obtained from the limited liability condition (11). Plugging the demand function for intermediate goods into the above inequality, we obtain the necessary condition for \( m_t = 0 \) as in Proposition S1.

To prove the sufficiency, the financial contract can be simply designed as

\[
\begin{align*}
  b_{1,t} &= b_{2,t} = p_t^y y_t f^b - q_t \gamma_t a_t^e \\
  \text{(S10)}
\end{align*}
\]

Note that the payoff at the low state, \( P_{1,t} A_{1,t} y_t - (p_t^y y_t - q_t \gamma_t a_t^e) \), is non-negative by assumption and is thus feasible. Plugging (S10) into the incentive compatibility constraint (13), we have

\[
[1 - P(m_t/y_t)] \left[ P_{2,t} A_{2,t} y_t f^b - (p_t^y y_t f^b - q_t \gamma_t a_t^e) \right] \leq P_{2,t} A_{2,t} y_t f^b - (p_t^y y_t f^b - q_t \gamma_t a_t^e).
\]

Obviously, the above incentive compatibility constraint is always satisfied, even if no monitoring resource is used such that the probability of identifying misreporting is zero (i.e., \( P = 0 \)). Thus, the incentive compatibility constraint can be dropped from the bank problem (10). Also, the non-negative profit condition of the bank is satisfied. Hence, it is optimal to set \( m_t = 0 \) and \( P = 0 \). Intuitively, since the bank does not monitor in either state and the entrepreneur has an incentive to misreport, it is optimal to set the payoff at both states at the value equal to the bank’s finance cost.

**S1.2. Proof of Proposition S2.** Using the demand function for intermediate goods (6) to replace \( P_{1,t} \) in the optimal contract problem (10), we can write the Lagrangian as

\[
L = \pi_t b_{1,t} + (1 - \pi_t) b_{2,t} - (p_t^y y_t - q_t \gamma_t a_t^e) - \pi_t m_t
\]

\[
+ \lambda_1 [\pi_t A_t (y_t)^{\mu - 1} (A_{1,t} y_t)^{\mu - 1} - b_{1,t}] + (1 - \pi_t) (Y_t^{1 - \mu} (A_{2,t} y_t)^{\mu - 1} - b_{2,t}) - v_t
\]

\[
+ \lambda_2 [Y_t^{1 - \mu} (A_{2,t} y_t)^{\mu - 1} - b_{2,t} - (1 - P(m_t/y_t)) (Y_t^{1 - \mu} (A_{2,t} y_t)^{\mu - 1} - b_{1,t})]
\]

\[
+ \lambda_3_1 [Y_t^{1 - \mu} (A_{1,t} y_t)^{\mu - 1} - b_{1,t}] + \lambda_3_2 [Y_t^{1 - \mu} (A_{2,t} y_t)^{\mu - 1} - b_{2,t}],
\]

where \( \lambda_1, \lambda_2, \lambda_3_1, \) and \( \lambda_3_2 \) denote the Lagrange multipliers for the entrepreneur’s participation constraint, the incentive compatibility constraint, and limited liability constraints in state 1 and 2, respectively.

The first-order conditions are:

\[
\frac{\partial L}{\partial b_{1,t}} = \pi_t (1 - \lambda_1) + \lambda_2 (1 - P(m_t/y_t)) - \lambda_3_1 = 0, \quad \text{(S11)}
\]

\[
\frac{\partial L}{\partial b_{2,t}} = (1 - \pi_t) (1 - \lambda_1) - \lambda_2 - \lambda_3_2 = 0, \quad \text{(S12)}
\]

\[
\frac{\partial L}{\partial y_t} = -p_t^y + \lambda_1 \mu Y_t^{\mu - 1} \left[ \pi_t A_{1,t}^\mu + (1 - \pi_t) A_{2,t}^\mu \right] (y_t)^{\mu - 1}
\]

\[
+ \lambda_2 [Y_t^{1 - \mu} A_{2,t}^\mu (y_t)^{\mu - 1} + \frac{\partial P(m_t/y_t)}{\partial y_t} (Y_t^{1 - \mu} (A_{2,t} y_t)^{\mu} - b_{1,t}) - (1 - P(m_t/y_t)) Y_t^{1 - \mu} A_{2,t}^\mu (y_t)^{\mu - 1}]
\]

\[
+ (\lambda_3_1 A_{1,t}^\mu + \lambda_3_2 A_{2,t}^\mu) Y_t^{1 - \mu} (y_t)^{\mu - 1},
\]

\[
= 0 \quad \text{(S13)}
\]
\[
\frac{\partial L}{\partial m_t} = -\pi_t + \lambda_2 \frac{\partial P(m_t/y_t)}{\partial m_t} (Y_t^{1-\mu} (A_{2,t}y_t)^\mu - b_{1,t}) = 0. \tag{S14}
\]

**Proof.** From the first-order conditions, we have the following results:

**Result 1:** \( \lambda_1 \in (0, 1) \).

**Proof:** From (S12), we have \( \lambda_1 \in [0, 1] \). Since the participation constraint is binding, \( \lambda_1 \in (0, 1) \).

Now we turn to prove \( \lambda_1 \neq 1 \).

Suppose \( \lambda_1 = 1 \). Then from (S11) and (S12), we have \( \lambda_2 = \lambda_3 = \lambda_3 = 0 \). Therefore, (S14) implies that \( \pi_t = 0, \forall t \). This leads to a contradiction.

**Result 2:** \( \lambda_3 > 0 \); that is, the limited liability constraint for state 1 is binding.

**Proof:** A combination of Result 1 and (S11) gives this result immediately.

**Result 3:** \( \lambda_2 > 0, \lambda_3 = 0 \); that is, the incentive compatibility constraint is binding and the limited liability constraint for state 2 is not binding.

**Proof:** Suppose \( \lambda_3 > 0 \). Then, the limited liability constraint at state 2 is binding, \( b_{2,t} = Y_t^{1-\mu} (A_{2,t}y_t)^\mu \). We thus have two cases:

Case 1: \( \lambda_2 = 0 \).

This implies that the incentive compatibility constraint is not binding. Therefore, a combination of Result 2 and the incentive compatibility constraint implies

\[
b_{2,t} < Y_t^{1-\mu} (A_{2,t}y_t)^\mu - (1 - P(m_t/y_t))(Y_t^{1-\mu} (A_{2,t}y_t)^\mu - Y_t^{1-\mu} (A_{1,t}y_t)^\mu) < Y_t^{1-\mu} (A_{2,t}y_t)^\mu,
\]

which contradicts \( b_{2,t} = Y_t^{1-\mu} (A_{2,t}y_t)^\mu \).

Case 2: \( \lambda_2 > 0 \).

This implies that the incentive compatibility constraint is binding. Therefore, a combination of Result 2 and the incentive compatibility constraint implies

\[
Y_t^{1-\mu} (A_{2,t}y_t)^\mu - b_{2,t} = (1 - P(m_t/y_t))(Y_t^{1-\mu} (A_{2,t}y_t)^\mu - Y_t^{1-\mu} (A_{1,t}y_t)^\mu) > 0,
\]

which implies \( Y_t^{1-\mu} (A_{2,t}y_t)^\mu > b_{2,t} \), which again contradicts \( b_{2,t} = Y_t^{1-\mu} (A_{2,t}y_t)^\mu \).

Therefore, \( \lambda_3 = 0 \). With this result, a combination of Result 1 and equation (S12) gives us \( \lambda_2 > 0 \). \( \square \)

**Appendix S2. Equilibrium conditions**

The equilibrium for the model described in Section III is characterized by the following system of equations:

(E1) The household’s budget constraint (\( c_t^h \)):

\[
c_t^h + q_t a_{t+1}^h = [q_t (1 - \delta) + r_t] \gamma_t a_t^h + w_t H_t + \Pi_t^k. \tag{S1}
\]

(E2) Intertemporal Euler equation for the household (\( r_t \)):

\[
q_t MU_t^h = \beta E_t [\theta_{t+1}MU_{t+1}^h (q_{t+1} (1 - \delta) + r_{t+1}) \gamma_{t+1}]. \tag{S2}
\]
(E3) Marginal utility of consumption for the household \((MU^h_t)\):
\[
MU^h_t = \frac{1}{c^h_t - \omega_h c^h_{t-1}} - \beta \omega_h E_t \frac{1}{c^h_{t+1} - \omega_h c^h_t}.
\] (S3)

(E4) Optimal labor decision \((H_t)\):
\[
\phi_t H_t = w_t MU^h_t.
\] (S4)

(E5) Final goods \((Y_t)\):
\[
Y_t = [A^\mu_t]^{\frac{1}{\mu}} y_t.
\] (S5)

(E6) Bank lending \((B_t)\):
\[
B_t = p^y_t y_t - q_t \gamma_t a^e_t.
\] (S6)

(E7) Euler equation for the supply of credit \((v_t)\):
\[
y_t = \left[ \frac{v_t}{(1 - \pi_t) Y_t^{1-\mu} ((A_{2,t})^\mu - (A_{1,t})^\mu)} (\varepsilon_t m_t/y_t)^{\psi} \right]^{\frac{1}{\mu}}.
\] (S7)

(E8) Euler equation for the demand of credit \((m_t/y_t)\):
\[
\mu Y_t^{1-\mu} A^\mu_t (y_t)^{\mu-1} - p^y_t = \pi_t m_t/y_t \left( 1 + \frac{\mu}{\psi} \right),
\] (S8)

(E9) The production of intermediate goods by entrepreneurs \((w_t)\):
\[
y_t = z_t (k_t)^{\alpha} (h_t)^{(1-\alpha)}.
\] (S9)

(E10) Demand for capital (and labor) by entrepreneurs \((k_t)\):
\[
k_t = h_t \frac{\alpha w_t}{(1-\alpha) r_t}.
\] (S10)

(E11) Competitiveness in the intermediate goods market makes the price equal to the marginal cost \((p^y_t)\):
\[
p^y_t = \frac{1}{z_t} \left( \frac{r_t}{\alpha} \right)^{\alpha} \left( \frac{w_t}{(1-\alpha)} \right)^{(1-\alpha)}.
\] (S11)

(E12) Intertemporal Euler equation for entrepreneurs \((a^e_t)\):
\[
q_t MU^e_t = \beta^e E_t \left[ MU^e_{t+1} \left( r_{t+1} - q_{t+1} \delta + v'_{t+1} (a^e_{t+1}) \right) \gamma_{t+1} \right].
\] (S12)

(E13) Definition of \(MU^e_t\) \((MU^e_t)\):
\[
MU^e_t = \frac{1}{c^e_t - \omega_e c^e_{t-1}} - \beta^e \omega_e E_t \frac{1}{c^e_{t+1} - \omega_e c^e_t}.
\] (S13)

(E14) Entrepreneurs’ budget constraint \((c^e_t)\):
\[
c^e_t + q_t a^e_{t+1} = (r_t - \delta q_t) \gamma_t a^e_t + v_t.
\] (S14)

(E15) The capital producer’s period-\(t\) profit \((\Pi^k_t)\):
\[
\Pi^k_t = q_t \left[ (1 - \delta) K_t + \chi_t (1 - S (I_t/I_{t-1})) I_t \right] - q_t (1 - \delta) K_t - I_t.
\] (S15)
(E16) Optimality condition for the capital producer ($q_t$):

$$q_t = \frac{1 - E_t \beta_t q_{t+1} \frac{MU_{h_{t+1}}}{MU_t} \left[ q_{t+1} \chi_{t+1} S' (I_{t+1}/I_t) \left( \frac{h_{t+1}}{h_t} \right)^2 \right]}{\chi_t \left[ 1 - S' (I_t/I_{t-1}) \frac{h_t}{h_{t-1}} - S (I_t/I_{t-1}) \right]}.$$

(S16)

(E17) Labor market ($h_t$):

$$H_t = h_t.$$

(S17)

(E18) Capital market ($K_{t+1}$):

$$K_{t+1} = k_{t+1}.$$

(S18)

(E19) Asset market ($a^h_{t+1}$):

$$\bar{K}_{t+1} = \gamma_{t+1} (a^c_{t+1} + a^h_{t+1}).$$

(S19)

(E20) Aggregate capital accumulation ($I_t$):

$$K_{t+1} = \gamma_{t+1} K_{t+1} = \gamma_{t+1} [(1 - \delta) K_t + \chi_t (1 - S (I_t/I_{t-1})) I_t].$$

(S20)

(E21) Aggregate goods market ($y_t$):

$$Y_t = \bar{Y}_t + \pi_t m_t.$$

(S21)

(E22) Aggregate consumption ($C_t$):

$$C_t = c^h_t + c^e_t.$$

(S22)

(E23) Aggregate output ($\bar{Y}_t$):

$$\bar{Y}_t = C_t + I_t.$$

(S23)

(E24) Costs in the bad state ($b_{1t}$):

$$b_{1t} = Y_t^{1-\mu} (A_{1t} y_t)^\mu.$$

(S24)

(E25) Charge-off rate ($d_t$):

$$d_t = \frac{\pi_t - b_{1t}}{B_t}.$$

(S25)

(E26) Definition of the external finance premium ($S_t$):

$$S_t = \frac{\pi_t b_{1t} + (1 - \pi_t) b_{2t}}{p^y_t y_t - q_t \gamma_t a^e_t} - 1 = \pi_t m_t \frac{p^y_t y_t - q_t \gamma_t a^e_t}{p^y_t y_t - q_t \gamma_t a^e_t}.$$

(S26)

**APPENDIX S3. ENDOGENOUS VARIABLES**

(V1) $c^h_t$: consumption for the household.

(V2) $r_t$: the rent of capital.

(V3) $MU^h_t$: marginal utility of consumption for the household.

(V4) $H_t$: labor hours.

(V5) $Y_t$: final goods.

(V6) $B_t$: bank lending.

(V7) $v_t$: contract value for the firms.

(V8) $m_t$: state verification cost.
(V9) \( w_t \): wages.
(V10) \( k_t \): capital used by the firms.
(V11) \( p_t^w \): the cost of intermediate goods (working capital).
(V12) \( a_t^c \): (physical) capital held by the entrepreneur at the end of \( t - 1 \).
(V13) \( MU^e_t \): marginal utility of the entrepreneur’s consumption.
(V14) \( c^e_t \): consumption of the entrepreneur.
(V15) \( \Pi^k_t \): profit of capital producers.
(V16) \( q_t \): the price of capital.
(V17) \( h_t \): labor hired by the firms.
(V18) \( K_t+1 \): aggregate capital at the end of \( t \).
(V19) \( a_{t+1}^h \): physical capital owned by the household at the end of the period \( t \).
(V20) \( I_t \): aggregate investment.
(V21) \( y_t \): intermediate goods (working capital) for the firms.
(V22) \( C_t \): aggregate consumption.
(V23) \( \tilde{Y}_t \): aggregate output.
(V24) \( b_{1t} \): cost for the bankrupt firm.
(V25) \( d_t \): the charge-off rate.
(V26) \( S_t \): external finance premium.

The number of variables listed here must match the number of equations. Some equations in the system are simply definitions.

**APPENDIX S4. LOG-LINEARIZATION**

(LL1) From (S1)
\[
a^h a_{t+1}^h - [(1 - \delta) + r]a^h (\hat{\gamma}_t + \hat{a}_t^h) + c^h \hat{c}_t + wH \hat{H}_t - \hat{\Pi}_{tke} + \delta a^h \hat{q}_t - r a^h \hat{\gamma}_t - wH \hat{w}_t = 0,
\]
where \( \hat{\Pi}_{tke} = e^{\hat{\mu}_t} \).

(LL2) From (S2)
\[
E_t \hat{MU}_{t+1}^h + \beta (1 - \delta)E_t \hat{q}_{t+1} + \beta r E_t \hat{\gamma}_{t+1} + E_t \hat{\theta}_{t+1} + \gamma_{t+1} - \hat{MU}_{t}^h - \hat{q}_t = 0.
\]

(LL3) From (S3)
\[
\beta \omega_h \hat{c}_{t+1}^h - (1 + \beta \omega_h) \hat{c}_t^h - (1 - \beta \omega_h)(1 - \omega_h) \hat{MU}_t^h + \omega_h \hat{c}_{t-1}^h = 0.
\]

(LL4) From (S4)
\[
\nu \hat{H}_t - \hat{MU}_t^h + \hat{\phi}_t - \hat{w}_t = 0.
\]

(LL5) From (S5)
\[
\frac{1}{\mu} \hat{A}_t^\mu - \hat{Y}_t + \hat{\gamma}_t = 0.
\]
Note

\[ \overline{A}^\mu \hat{A}^\mu_{t} + (\overline{A})^\mu \left[ \pi \left( 1 + \sigma \sqrt{\frac{\pi}{1 - \pi}} \right)^\mu - \pi \left( 1 - \sigma \sqrt{\frac{1 - \pi}{\pi}} \right)^\mu \right] - \frac{\mu \sigma}{2} \sqrt{\frac{\pi}{1 - \pi}} \left[ \left( 1 + \sigma \sqrt{\frac{\pi}{1 - \pi}} \right)^{\mu - 1} + \left( 1 - \sigma \sqrt{\frac{1 - \pi}{\pi}} \right)^{\mu - 1} \right] \hat{\pi}_t = 0. \]

(LL6) From (S6)

\[ \mathcal{B} \hat{b}_t - \rho^y \hat{y}^y_t + a^e (\hat{q}_t + \hat{a}^e_t + \hat{\gamma}_t) - \rho^y \hat{y}_t = 0. \]

(LL7) From (S7)

\[ \frac{\mu (\overline{A}_1)^\mu}{(\overline{A}_2)^\mu - (\overline{A}_1)^\mu} \hat{A}_{1,t} - \frac{\mu (\overline{A}_2)^\mu}{(\overline{A}_2)^\mu - (\overline{A}_1)^\mu} \hat{A}_{2,t} + \psi \hat{\xi}_t + \psi \hat{\eta}_t + \frac{\pi}{1 - \pi} \hat{\pi}_t + \hat{v}_t - (1 - \mu) \hat{Y}_t - (\mu + \psi) \hat{y}_t = 0. \]

Note

\[ \hat{A}_{1,t} = \frac{\sigma}{2 \pi} \sqrt{\frac{\pi}{1 - \pi}} \hat{\pi}_t; \quad \hat{A}_{2,t} = \frac{1}{1 + \psi} \hat{\phi}_t. \]

(LL8) From (S8)

\[ \frac{\mu Y^{1 - \mu}}{\mu Y^{1 - \mu}} \overline{A}_1^\mu (y)^{\mu - 1} - p^y \hat{\pi}_t - \hat{m}_t - \hat{\pi}_t - \frac{p^y}{\mu Y^{1 - \mu}} \overline{A}_2^\mu (y)^{\mu - 1} - p^y \hat{\pi}_t \]
\[ + \left( 1 - \mu \right) \mu Y^{1 - \mu} \overline{A}_1^\mu (y)^{\mu - 1} - p^y \hat{Y}_t + \mu^2 Y^{1 - \mu} \overline{A}_1^\mu (y)^{\mu - 1} - p^y \hat{y}_t = 0. \]

(LL9) From (S9)

\[ (1 - \alpha) \hat{h}_t + \alpha \hat{k}_t - \hat{\gamma}_t + \hat{z}_t = 0. \]

(LL10) From (S10)

\[ \hat{h}_t - \hat{k}_t - \hat{r}_t + \hat{w}_t = 0. \]

(LL11) From (S11)

\[ \hat{p}^y_t - \alpha \hat{r}_t - (1 - \alpha) \hat{w}_t + \hat{z}_t = 0. \]

(LL12) From (S12)

\[ \hat{\gamma}_{t+1} + E_t \hat{M}U^e_{t+1} - \beta^e \delta E_t \hat{q}_{t+1} + \beta^r E_t \hat{r}_{t+1} + \beta^e v^e E_t \hat{v}_{t+1} - M \hat{U}^e_t - \hat{q}_t = 0. \]

(LL13) From (S13)

\[ \beta^e \omega^e E_t \hat{c}^e_{t+1} - (1 + \beta^e \omega^e) \hat{c}^e_{t} - (1 - \beta \omega^e) \hat{M}U^e_t + \hat{\omega}^e \hat{c}^e_{t-1} = 0. \]

(LL14) From (S14)

\[ a^e \hat{a}^e_{t+1} - (r - \delta) a^e (\hat{a}^e_t + \hat{\gamma}_t) + c^e \hat{c}^e_t + (1 + \delta) a^e \hat{q}_t - ra^e \hat{r}_t - \hat{v}_t = 0. \]

(LL15) From (S15)

\[ I \hat{\chi}_t - \hat{\Pi}^k + I \hat{q}_t = 0. \]
From (S16)
\[ \beta S'' \dot{I}_{t+1} + \dot{\tilde{I}}_t - (1 + \beta) S'' \dot{I}_t + \dot{q}_t + S'' \dot{I}_{t-1} = 0. \]

From (S17)
\[ \hat{H}_t - \hat{h}_t = 0. \]

From (S18)
\[ \hat{K}_{t+1} - \dot{k}_{t+1} = 0. \]

From (S19)
\[ \dot{a}^e (\hat{a}^c_{t+1} + \hat{\gamma}_{t+1}) + \dot{a}^h (\hat{a}^h_{t+1} + \hat{\gamma}_{t+1}) - K \hat{K}_{t+1} = 0. \]

From (S20)
\[ K \left( \hat{K}_{t+1} - \hat{\gamma}_{t+1} \right) - I \dot{\tilde{X}}_t - I \dot{\tilde{T}}_t - (1 - \delta) K \hat{K}_t = 0. \]

From (S21)
\[ \pi \mu \dot{m}_t + \pi \mu \dot{\tilde{m}}_t - Y \dot{\tilde{Y}}_t + \dot{Y} \dot{\tilde{Y}}_t = 0. \]

From (S22)
\[ C \dot{\tilde{C}}_t - c^e \dot{c}^e_t - c^h \dot{c}^h_t = 0. \]

From (S23)
\[ C \dot{\tilde{C}}_t + I \dot{\tilde{T}}_t - \dot{Y} \dot{\tilde{Y}}_t = 0. \]

From (S24)
\[ \dot{b}_{1t} = (1 - \mu) \dot{Y}_t + \mu \left( \dot{A}_{1t} + \dot{y}_t \right). \]

From (S25)
\[ \dot{d}_t = \dot{\pi}_t - \dot{b}_t + \frac{\mathcal{B}}{\mathcal{B} - \dot{b}_1} \dot{b}_t - \frac{\dot{b}_1}{\mathcal{B} - \dot{b}_1} \dot{b}_{1t} = \dot{\pi}_t + \frac{\dot{b}_1}{\mathcal{B} - \dot{b}_1} \left( \dot{b}_t - \dot{b}_{1t} \right). \]

From (S26)
\[ (p^r y - q^r a^e) S \dot{\tilde{s}}_t + p^r y \mathcal{S} (\dot{p}^r_t + \dot{y}_t) - q^r a^e \mathcal{S} (\dot{q}_t + \dot{\gamma}_t + \dot{\alpha}_t) = m \pi (\dot{m}_t + \dot{\pi}_t). \]

**Appendix S5. Log-linearized variables**

LV1 \( \dot{c}^h_t \): consumption for the household.

LV2 \( \dot{r}_t \): the rent of capital.

LV3 \( M U^h_t \): marginal utility of consumption for the household.

LV4 \( \dot{H}_t \): labor hours.

LV5 \( \dot{Y}_t \): final goods.

LV6 \( \dot{b}_t \): bank lending.

LV7 \( \dot{v}_t \): firm’s contract value.

LV8 \( \dot{m}_t \): state verification cost.

LV9 \( \dot{w}_t \): wages.

LV10 \( \dot{k}_t \): capital used by the firms.

LV11 \( \dot{p}^f_t \): the cost of intermediate goods (working capital).
(LV12) \( \hat{a}_t^c \): (physical) capital held by the entrepreneur at the end of \( t - 1 \).
(LV13) \( MU_t^e \): marginal utility of the entrepreneur’s consumption.
(LV14) \( \hat{c}_t^e \): consumption of the entrepreneur.
(LV15) \( \hat{\Pi}_t^k \): profit of capital producers.
(LV16) \( \hat{q}_t \): the price of capital.
(LV17) \( \hat{h}_t \): labor hired by the firms.
(LV18) \( \hat{K}_{t+1} \): aggregate capital at the end of \( t \).
(LV19) \( \hat{a}_{t+1}^h \): physical capital owned by the household at the end of the period \( t \).
(LV20) \( \hat{I}_t \): aggregate investment.
(LV21) \( \hat{y}_t \): intermediate goods (working capital) for the firms.
(LV22) \( \hat{C}_t \): aggregate consumption.
(LV23) \( \hat{Y}_t \): aggregate output.
(LV24) \( \hat{b}_t \): cost for the bankrupt firm.
(LV25) \( \hat{d}_t \): the charge-off rate.
(LV26) \( \hat{s}_t \): external finance premium.
(LV27) \( \hat{\pi}_t \): a new defined variable.
(LV28) \( \hat{A}_t^f \): a new defined variable.
(LV29) \( \hat{A}_{1,t} \): a new defined variable.
(LV30) \( \hat{A}_{2,t} \): a new defined variable.
(LV31) \( \hat{m}_t^c - \hat{y}_t^c \): a new defined variable.

The number of variables listed here must match the number of equations. Variables (LV27)-(LV31) are newly defined and thus do not belong to any original equation.

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