

Credit Ratings and the Securitization of Subprime Mortgages

John Hull

Joseph L. Rotman School of Management
University of Toronto

Presented at the Federal Reserve Bank of Atlanta
2010 Financial Markets Conference
Up from the Ashes: The Financial System after the Crisis
Atlanta, Georgia
May 11, 2010

Credit Ratings and the Securitization of Subprime Mortgages

John Hull

Abstract

This paper considers the lessons that can be learned from the securitization of subprime mortgages during the 1999 to 2007 period and the subsequent large losses of financial institutions. It critically examines the complexity of the products that were created and the AAA-ratings that were assigned to tranches. It reaches conclusions on how the subprime experience should influence the way securitizations are structured in the future.

Credit Ratings and the Securitization of Subprime Mortgages

During the 1999 to 2007 period securitization became increasingly popular in the United States. Many different types of assets were securitized: corporate loans, credit card receivables, auto loans, mortgages, etc. There are a number of reasons why securitization was so attractive. One is regulatory arbitrage. When banks securitized the assets on their balance sheets and bought the products that were created, they reduced their regulatory capital. Another is spread arbitrage. A bank may be able to package the cash inflows on the assets on its balance sheet so that the average interest rate paid on the products created is less than the average interest rate earned on the assets. A third reason involves what is known as the “originate-to-distribute” model. Securitization can be used to transfer to an outside investor the credit risks in the assets on its balance sheet so that a bank is able to originate more assets without increasing its capital. A fourth reason is that there were not enough AAA-rated securities to satisfy the demands of investors and so securitizations that manufactured AAA-rated securities were both attractive to investors and profitable for issuers.

The credit crisis led to securitization almost disappearing from the financial landscape in August 2007, but by the end of 2009 there were signs that it was returning. It is clearly important for regulators and market participants to understand the mistakes that were made and the way they can be avoided in the future.

Some of the objectives of securitizers are potentially socially desirable while others are not. Regulatory arbitrage is clearly in the second category. It is in essence a game between banks and their regulators. By securitizing mortgages and buying products that were created, banks moved their exposures from the banking book to the trading book, thereby reducing regulatory capital. Regulators have taken steps to change capital requirements to combat this. Regulatory arbitrage will of course never be eliminated but it is to be hoped that it will not be the main driver of securitization in the future.

Using securitization as way of transferring risks from the originator of the risks to investors (the originate-to-distribute model) is potentially much more socially desirable than regulatory arbitrage. Indeed, the transfer of risks in the economy is a major role of financial markets and essential to a well functioning market economy. There were some serious problems with the

way securitization was done in the first decade of the 21st century, but we should not throw out the baby with the bath water. It is important to understand what went wrong and how securitization can be improved going forward. This is the main goal of this paper.

The paper will focus on the securitization of subprime mortgages. This type of securitization played a key role in the credit crisis. Most of the losses experienced by financial institutions were as a result of their exposures to tranches created from subprime mortgages. This paper explains the nature of the products that were created and discusses their risks. It examines the criteria used by rating agencies when they gave AAA ratings to securitized products and whether these criteria were reasonably applied.

1. Securitized Products

The nature of the products that were created from subprime mortgages is explained in Gorton (2007). One of the objectives of designers of the products appears to have been to maximize the volume of AAA-rated securities created from a given pool of mortgages. This maximized the regulatory-arbitrage and spread-arbitrage benefits obtained from the securitization. Also, as already mentioned, AAA-rated securities were relatively easy to sell in the market.

Often close to \$90 of AAA-rated securities were ultimately created from each \$100 of subprime mortgages. By definition, subprime borrowers are high risk with poor credit scores. As a stand-alone credit it seems likely that a subprime mortgage would at best be rated BBB. Is it reasonable that \$90 of AAA-rated instruments can be created from \$100 of BBB-rated instruments? Some hedge fund managers thought not and made billions betting against the subprime market. In retrospect it is surprising that more investors did not agree with them.

1.1. ABSs

The tranches created from mortgages are known as asset-backed securities (ABSs). A typical structure is illustrated in Figure 1. The subprime mortgage portfolio might consist of a total of 1000 mortgages. This underlying collateral is tranching into one or more senior tranches rated AAA, a number of tranches rated AA, A, and BBB, and subordinated tranches which are either rated BB or unrated. A principal is allocated to each tranche. Sometimes, the total principal of the mortgages equals the total principal of the tranches. In other cases, there is some overcollateralization where the total principal of the mortgage portfolio exceeds the total principal of the tranches.

Typically 75% to 85% of the mortgage principal was allocated to AAA tranches. The principals allocated to other rating categories were much smaller. For example, the BBB tranches taken together typically accounted for 3% or less of the mortgage principal. As mentioned, one of the goals of the creator of an ABS was to maximize the volume of AAA-rated securities that were created. More generally, they wanted to maximize the average credit quality of the tranches that

were created. First, the AAA tranches were made as wide as possible; then the AA tranches were made as wide as possible; and so on.

The rules for allocating cash flows from the mortgages to tranches are defined by what is known as a “waterfall.” The interest payments promised to tranches typically decrease with seniority. Interest payments from mortgages are typically allocated to tranches in order of seniority so that the AAA-rated tranches get promised interest payments on their outstanding principal first; after that, the AA-rated tranches get their promised interest payments on their outstanding principal; and so on. The interest payments on subprime mortgages are sometimes fixed for the first two or three years and then become floating. The interest payments on tranches may be fixed or floating. When they are floating they are sometimes capped so that promised interest payments on the tranches do not exceed scheduled interest payments on the mortgages.

The principal payments (both scheduled and prepayments) are handled separately from interest payments and the rules are relatively complicated. There is typically a lockout period during which principal payments are sequential. This means that all principal payments go first to the most senior tranche. When that tranche has been completely amortized, they go to the next-most-senior tranche, and so on. After the lock out period, if certain performance targets are met, principal payments are allocated to tranches in proportion to their outstanding principals. However, if there is a “cumulative loss trigger event” (where cumulative losses on the mortgages are higher than certain levels) or a “delinquency event” (where the rate of delinquency over a three-month period is above a certain level), principal payments become sequential again.

As an approximation, it can be assumed that the repayment of principal is entirely sequential in stressed market conditions. The effect of this is that tranches bear losses in order of reverse seniority. The unrated tranches absorb losses first. Once their principal has been lost, the BB-rated tranches bear losses, and so on.

There were often several tranches corresponding to each rating category. For example, in the Structured Asset Investment Loan Trust (SAIL), issued in 2006, there were four AAA tranches (with equal seniority) accounting for 83.25% of the collateral; two AA-rated tranches (with unequal seniority) accounting for 8.2% of the collateral; three A-rated tranches (with unequal

seniority) accounting for 4.1% of the principal; and three BBB tranches (with unequal seniority) accounting for 2.8% of the principal.

The BBB tranches were often very thin. Although the total of all BBB tranches might account for about 3% of the total underlying principal, each individual BBB tranche was often only about 1% wide. If the macroeconomic environment is relatively benign so that there are large repayments of principal, the AAA tranche can be expected to shrink and the proportion of the remaining mortgage principal accounted for by the BBB tranches can be expected to increase. But, if default rates are high, a thin BBB tranche can easily lose its entire principal. In the SAIL structure just mentioned, the most senior BBB tranche (rated Baa1/BBB+) was 1.1% wide with 3.25% subordination; the next BBB tranche (rated Baa2/BBB) was 0.85% wide with 2.40% subordination, and the most junior BBB tranche (rated Baa3/BBB-) tranche was 0.8% wide with 1.60% subordination. Assuming principal payments are sequential, the three tranches will lose their entire principal if losses on the subprime mortgage portfolio are greater than 4.35%, 3.25%, and 2.40%, respectively.

1.2. ABS CDOs

Often there was a second level of securitization where tranches were created from tranches. The products created are known as ABS CDOs. Two types of ABS CDOs were common. These were a “High Grade ABS CDO” created from the AAA, AA, and A tranches of ABSs and a “Mezz ABS CDO” created from the BBB tranches of ABSs.

We will focus on the Mezz ABS CDO. Its creation is illustrated in Figure 2. Typically there were about 100 underlying BBB-rated tranches. The AAA-rated tranche in Figure 2 is typically less wide than that in Figure 1. However, it still usually accounted for more than 50% of the underlying principal of the ABSs.

Many ABS CDOs are managed. This means that the tranches forming the collateral do not remain fixed over time. A portfolio manager is allowed to trade a certain percentage of the underlying collateral each year. However there are restrictions relating to measures involving the ratings of the collateral, correlation, and the weighted average life of the underlying assets.

ABS CDOs are like ABSs in that the waterfall is complex. Losses tend to be allocated to the most junior tranches first. There are coverage tests and triggers which cause amortizations to be sequential and divert cash flows from junior to senior tranches. In certain circumstances, the senior tranche holders may be able to liquidate the assets. As in the case of ABSs it is reasonable to assume that principal payments are sequential in stressed market conditions.

1.3. CDO Squareds

The securitization did not always end with ABS CDOs. Sometimes portfolios were created from tranches of ABS CDOs and new tranches, some rated AAA, were created from the portfolios. These were known as CDO squareds. However, they might be more accurately termed “ABS cubeds” because they are asset-backed securities which are created from asset backed securities, which are themselves created from asset-backed securities. Figure 3, which is reproduced from Gorton (2008), and originally appeared in a UBS publication, illustrates the types of structures that were created.

1.4. Additional Complications

The structures that were created were not always as straightforward as has been described. Sometimes there were mixtures of ABSs and ABS CDOs in the same portfolio and sometimes other assets totally unrelated to residential mortgages were included. The result is that the many interrelationships between the securities that were created were very difficult to unravel. This lack of transparency made it very difficult for market participants to calculate how ABS CDOs would perform in different market conditions. As discussed in Smithson (2009), the lack of transparency made the valuation of tranches very difficult. As a result, market participants were more likely to rely on ratings.¹ However, in many instances the assumptions underlying the ratings were not critically examined.

¹ To examine the extent to which market participants relied on ratings, Adelino (2009) tested whether yield spreads on ABSs created from subprime mortgages at issuance contained information, in addition to that in their ratings, that would be useful in predicting subsequent performance. His conclusion was that they did not for AAA-rated securities. However, he did find some evidence that that information other than ratings were used to price lower-rated tranches

2. How Ratings Were Assigned to Tranches

The traditional business of rating agencies is the rating of corporate and sovereign bonds. This is based on a combination of analysis and judgment. The rating of structured products was a departure from this traditional business. Instead of analysis and judgment, it involved the application of a model. The rating agencies were quite open about the models they used. Standard and Poor's (S&P) and Fitch based their ratings on the probability of loss given by their models. If the calculated probability of loss on a structured product corresponded to the probability of loss on a AAA-rated bond, the structured product was rated AAA. Moody's, by contrast, based its ratings on the expected loss of principal. If the expected loss of principal on a structured product corresponded to the expected loss of principal from default on a Aaa-rated bond, the structured product was rated Aaa. When a structured product was designed, creators wanted to achieve their target ratings for tranches by meeting the model requirements of rating agencies.² Their objective was usually to make the total principal of the AAA tranches that they created as large as possible. Usually, they got advance rulings on ratings before finalizing product design.

Hull and White (2009) examine whether, given the criteria used by rating agencies, their ratings were reasonable. They assume that principal losses are allocated to tranches in order of reverse seniority. (As discussed, this is not an unreasonable assumption in the stressed market conditions that are of most concern to rating agencies.) The probability of loss and expected loss that were assumed for AAA- and BBB-rated bonds were taken from Moody's statistics and are shown in Table 1.

2.1. CDO Ratings

Suppose that Q is the fraction of mortgages in the pool that are expected to default within T years. If all the mortgages are of similar risk then Q is the probability of default for any individual mortgage. Hull and White consider a one-factor copula model. In these models, there is a factor that is common to all mortgages, which we will denote by M , and a factor specific to mortgage i which we will denote by Z_i . The factors M and Z_i are assumed to have independent zero mean unit variance distributions. In the model, mortgage i defaults within T years if

² See Brennan, Hein, and Poon (2008) for a discussion of this.

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i < K$$

where ρ is the correlation between the transformed times to default of any two mortgages. If F is the cumulative probability distribution of

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i$$

the unconditional probability of default is $F(K)$. The i th mortgage therefore defaults if

$$\sqrt{\rho}M + \sqrt{1-\rho}Z_i < F^{-1}(Q)$$

The realized default rate, conditional on the factor M , is

$$P = H\left(\frac{F^{-1}(Q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right) \quad (1)$$

where H is the cumulative probability distribution of Z_i .

Hull and White (2009) consider two assumptions about the M and the Z_i . The first is that they have standard normal distributions so that both F and H are cumulative standard normal distribution functions. This is the Gaussian copula model which was first suggested by Li (2000). As a result of work by Gregory and Laurent (2005) and others, it has become the standard market model for valuing synthetic CDOs. In the second assumption, M and Z_i have t -distributions with four degrees of freedom. Hull and White (2004) find that this model, which they refer to as the double t copula fits CDO market prices better than the Gaussian copula model. In equation (1), H is the cumulative distribution for a student t with four degrees of freedom and F is the cumulative distribution for the weighted sum of two student t distributions and must be calculated numerically.

Hull and White (2009) also consider two assumptions about the recovery rate. In the first assumption the recovery rate is fixed at 75%. In the second, it reduces from 100% to 50% as the default rate increases in such a way that it is 75% when the expected default rate is realized.

This leads to four models for ABSs:

1. Gaussian copula with constant recovery rate
2. Double t copula with a constant recovery rate
3. Gaussian copula with a stochastic recovery rate
4. Double t copula with a stochastic recovery rate.

The minimum attachment points calculated for a AAA-rated tranche using the four models are shown in Table 2. As mentioned, AAA-tranched tranches typically accounted for 75 to 85% of the underlying principal. This means that tranches with attachment points between 15% and 25% were rated AAA. Table 2 suggests that the AAA ratings were (ex ante) not totally unreasonable. We do not know what expected default rates and correlations were assumed by the rating agencies. Default experience on subprime mortgages during the 1999 to 2006 period was generally good. Cumulative default rates on subprime mortgages had been about 5% or less and the Basel II committee's research led to it using a value of ρ equal to 0.15 in the Gaussian copula model.³

Of course, there were a number of warning signals in 2006. The S&P Case-Shiller Composite 10 house price index, which was set at 100 in January 2000, reached over 225 in mid-2006, but had started to decline by the beginning of 2007. Although few people anticipated the full extent of the fall in house prices that took place in over the next two years, there was general agreement that some decline would take place. For obvious reasons, home owners are much more likely to default when house prices are falling than when they are rising. Mortgage default experience during the 1999 to 2006 period should therefore have been treated with caution. However a wide range of assumptions would have led to the conclusion that the AAA-ratings assigned to ABSs were reasonable.

Hull and White (2009) show that the expected loss assumption of Moody's tends to lead to lower minimum attachment points reinforcing the conclusion that, given their published criteria, the ratings were not totally unreasonable.

³ See Bank for International Settlements (2006, p77) and Hull (2009). Basel II uses essentially the same copula model that we do with M and the Z_i normally distributed.

2.2. ABS CDO Ratings

When ABS CDOs are considered it is necessary to define a “between-pool” factor, M_{bp} , and “within-pool” factors, $M_{wp,j}$. The factor M_{bp} affects the probability of default for all mortgages while $M_{wp,j}$ affects the probability of default only for mortgages in pool j . In this model, when distributions are assumed to be normal, the i th mortgage in the j th pool defaults if

$$\sqrt{\alpha\rho}M_{bp} + \sqrt{(1-\alpha)\rho}M_{wp,j} + \sqrt{1-\rho}Z_{ij} < \Psi^{-1}(Q)$$

where Z_{ij} is a variable affecting only the i th mortgage in the j th pool and Ψ is the cumulative probability distribution of

$$\sqrt{\alpha\rho}M_{bp} + \sqrt{(1-\alpha)\rho}M_{wp,j} + \sqrt{1-\rho}Z_{ij}$$

The factors and the variables Z_{ij} are independent of each other.

As before, the parameter ρ is the total within pool correlation. The parameter α indicates the proportion of the default correlation that comes from a factor common to all pools. When $\alpha = 0$ the default rates of different pools are independent of each other.⁴ At the other extreme, when $\alpha=1$, there is a single factor affecting all mortgage defaults and the default rates in all mortgage pools are the same.

A two-factor model is important when ABS CDOs are considered. One of the advantages cited for ABS CDOs over ABSs is that investors benefit from the across pool diversification as well as within pool diversification. Suppose that half of the underlying pools of an ABS CDO consist entirely of mortgages on Florida homes while the other half consist entirely of mortgages on California homes. If the default rate in California is less than perfectly correlated with the default rate in Florida, there is a diversification benefit to investors. The parameter α measures this diversification benefit. If α is low this extra diversification is valuable to investors, but if α is high it has very little value.

⁴ This is the assumption made by Coval (2008)

Research suggests that correlations increase in stressed market conditions. For example, Servigny and Renault (2002), who look at historical data on defaults and ratings transitions to estimate default correlations, find that the correlations are higher in recessions than in expansion periods. Das, Freed, Geng and Kapadia (2004) employ a reduced form approach and compute the correlation between default intensities. They conclude that default correlations increase when default rates are high. Ang and Chen (2002) find that the correlation between equity returns is higher during a market downturn. Given that they are most interested in what happens during stressed market conditions, this research suggests that rating agencies should have used a relatively high value of α . It should also be noted that if ABS mortgage pools are already well diversified across the United States, so that there is very little extra diversification benefit from forming an ABS CDO, then α should be close to 1.

The realized default rate for pool j conditional on M_{bp} and $M_{wp,j}$ is

$$\Phi \left(\frac{\Psi^{-1}(Q) - \sqrt{\alpha\rho}M_{bp} - \sqrt{(1-\alpha)\rho}M_{wp,j}}{\sqrt{1-\rho}} \right) \quad (3)$$

where Φ the cumulative probability distribution of Z_{ij} . The simplest version of the model is the case in which the M 's and Z 's have standard normal distributions. Hull and White (2009) also consider the case where they all have t distribution with four degrees of freedom (scaled so that the variance is one). They refer to this as the “triple t copula model.”

Table 3 shows results for two extreme models:

1. The two-factor Gaussian copula model a constant recovery rate of 75% on the underlying mortgages
2. The two-factor triple t copula model with the stochastic recovery rate

The ABS CDO is created from 100 BBB tranches of CDSs, each tranche being responsible for losses in the range 4% to 5% of the underlying portfolio. Results produced by Stanton and Wallace (2008) indicate that these choices of the attachment and detachment point are reasonable.

The pattern of results in Table 3 is different from that in Table 2. It is clear that the attachment point must be quite high for a wide range of assumptions. In some cases the attachment point is so high that a AAA-rating for even a very thin senior tranche is not warranted (i.e., the minimum attachment point is 100%).

Other results produced by Hull and White (2009) explore the impact of increasing the width of the underlying BBB tranches and introducing heterogeneity into the tranche attachment and detachment points. This does not alter the conclusion that, except in the case of a low- α Gaussian copula model, an attachment point below 50% is difficult to justify.

It should be noted that a CDO created from the triple BBB tranches of ABSs is quite different from a CDO created from BBB bonds. This is true even when the BBB tranches have been chosen so that their probabilities of default and expected losses are consistent with their BBB rating. The reason is that the probability distribution of the loss from a BBB tranche is quite different from the probability distribution of the loss from a BBB bond.

As in the case of ABSs, Moody's expected loss criterion tends to lead to a lower attachment point than the probability of loss criterion favored by S&P and Fitch. However, results show that for most assumptions the minimum attachment points are high.

An insight into the characteristics of the loss distribution of BBB-rated tranches can be obtained by considering an extreme case. Suppose tranches are infinitesimally thin and $\alpha=1$ so that the losses on tranches are perfectly correlated with each other. It is then the case that either a) the BBB tranches lose none of their principal or b) each BBB tranche loses its entire principal. An ABS CDO consisting of a portfolio of these tranches suffers either zero loss or 100% loss. It follows that every tranche of the ABS CDO are also in the situation where they either lose everything or nothing. There means that there should be no differences between the ratings of the tranches. (Indeed, they should all be rated BBB.)

3. Lessons for Securitization from the Crisis

One clear conclusion from the research in Section 2 is that resecuritization was badly flawed idea. In the future only one level of securitization is likely to be tolerated by investors. There is clearly something wrong when \$90 of AAA-rated securities cannot be created directly from a \$100 portfolio of mortgages, but financial engineers find a way to create it indirectly.

It might be argued that two levels of securitization can be justified if there are extra diversification benefits. However, two levels of securitization are not necessary to realize the full benefits of diversification. There is no reason why the portfolios used for the first level of mortgage securitization cannot be fully geographically diversified. We can go further than this. When forming portfolios to create ABSs we do not need to restrict ourselves to one asset class. A portfolio that is 25% mortgages, 25% auto loans, 25% credit card receivables, and 25% corporate debt can be used.

Using a portfolio such as this for a simple “pass-through” (where there is in effect one tranche created) would have no added value. Investors could create the return for themselves by creating a portfolio that was equally divided between a mortgage pass-through, an auto loan pass-through, a credit card receivables pass-through, and a corporate debt pass-through. However, creating the structure in Figure 1 from a portfolio that is diversified across asset classes investors would have added value because it would be doing something for investors that they could not do for themselves.

The huge losses during the credit crisis were as a result of the steep decline in US house prices. This suggests that it is dangerous to tie a securitization to one asset class (particularly one dependent on the price of a single good) and reinforces the arguments in favor of using well-diversified pools of assets. If investors insisted on diversified pools, it would not be possible for any one asset class to dominate the securitization market as it did prior to the credit crisis. This could be an added advantage.

Moving forward, transparency is an important goal. When the way in which a product works is clear to the market, it is likely to retain its liquidity in stressed market conditions. (Single-name

credit default swaps continued to trade actively throughout the crisis because all market participants understood how they worked.) Restricting securitizations to one stage should improve transparency. Another suggestion here is that issuers should be encouraged to provide software for their waterfalls. This software could be structured to make it easy for purchasers of tranches to quickly compute their returns in different scenarios.

Conclusions

The structured products created from mortgages were complex and often involved tranches being created from other tranches so that the interdependencies were very difficult to unravel and the exposures of investors could not easily be ascertained. Contrary to many of the opinions that have been expressed, the AAA ratings for the senior tranches of ABSs were not totally unreasonable given the published criteria used by rating agencies. However, the AAA ratings for Mezz ABS CDOs are much less defensible. Scenarios where all the underlying BBB tranches lose virtually all their principal are sufficiently probable that it is not reasonable to assign a AAA rating to even a quite thin senior tranche.

How should securitizations change in the future to reduce the risk of another credit crisis. This paper has made three key suggestions. First resecuritizations should be shunned by the market; second, the market should insist that underlying portfolios be well diversified both geographically and across asset classes; third, the market should require issuers to make the waterfalls as transparent as possible.

References

- Adelino, Manuel, 2009, "Do Investors Rely Only on Ratings? The Case of Mortgage-Backed Securities," Working paper, MIT Sloan School of Management.
- Ang, Andrew and Joseph Chen, 2002, "Asymmetric Correlations of Equity Portfolios," *Journal of Financial Economics*, 63, 443-494.
- Bank for International Settlements, 2006, "International Convergence of Capital Measurement and Capital Standards."
- Brennan, Michael J., Julia Hein, and Ser-Huang Poon, 2008, "Tranching and Rating," Working paper, Anderson School, UCLA.
- Coval, Joshua D., Jakub Jurek, and Erik Stafford, Working Paper, Harvard Business School, 2008.
- Das, Sanjiv R., Laurence Freed, Gary Geng and Nikunj Kapadia, 2006, "Correlated Default Risk," *Journal of Fixed Income*, 16, 2, 7-32.
- Gorton, Gary. 2008. "The Panic of 2007," Working Paper, Yale School of Management.
- Gregory, Jon and Jean-Paul Laurent, 2005, "Basket Default Swaps, CDO's and Factor Copulas," *Journal of Risk*, 7, 4, 103-22.
- Hull, John. 2009. *Risk Management and Financial Institutions*. 2nd edition, Upper Saddle River, NJ: Pearson.
- Hull, John, and Alan White, "Valuation of a CDO and n th to Default CDS without Monte Carlo Simulation," 2004, *Journal of Derivatives*, 12, 2, pp 8-23.
- Hull, John and Alan White, "The Risk of Tranches Created from Residential Mortgages," Working Paper, University of Toronto.
- Li, David X., 2000, "On Default Correlation: A Copula Approach," *Journal of Fixed Income*, 9, p. 43-54.
- Servigny, Arnaud de and Olivier Renault, 2002, "Default Correlation: Empirical Evidence," Working Paper, Standard and Poors.
- Smithson, Charles, 2009, "Valuing Hard-to-Value Assets and Liabilities: Notes on Valuing Structures Credit Products," *Journal of Applied Finance*, 1&2, pp 1-12.
- Stanton, Richard and Nancy Wallace, 2008, "ABX.HE Indexed Credit default Swaps and the valuation of Subprime MBS," Working Paper, Haas School of Business, University of Berkeley, California.

Figure 1: Creation of Tranches from a Portfolio of Subprime Mortgages

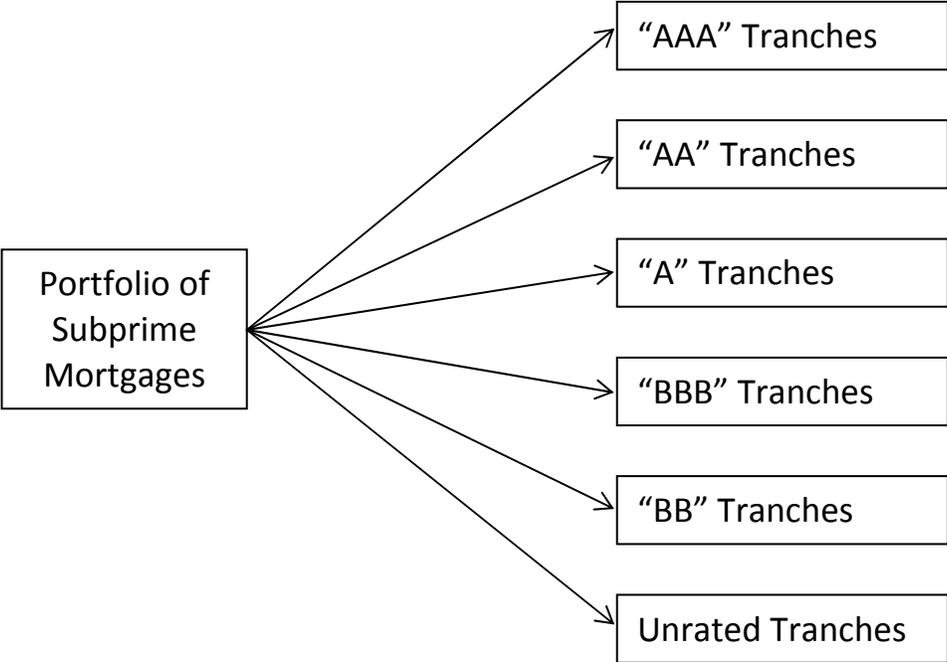


Figure 2: A “Mezz” ABS CDO Created from the BBB tranches of an ABS

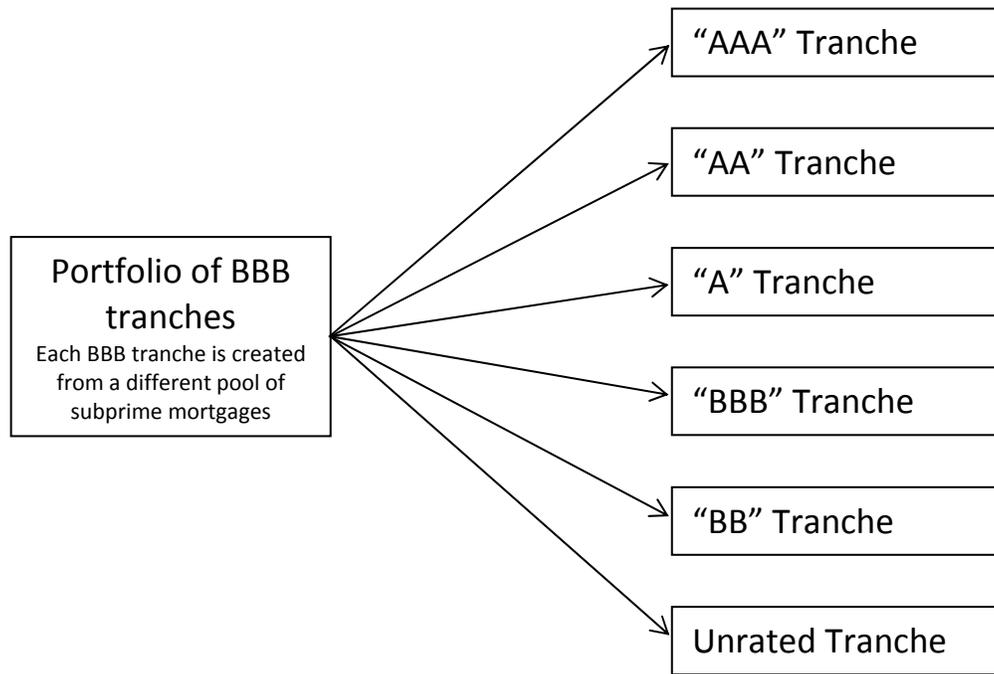


Figure 3: Illustration of the Pattern of Securitizations

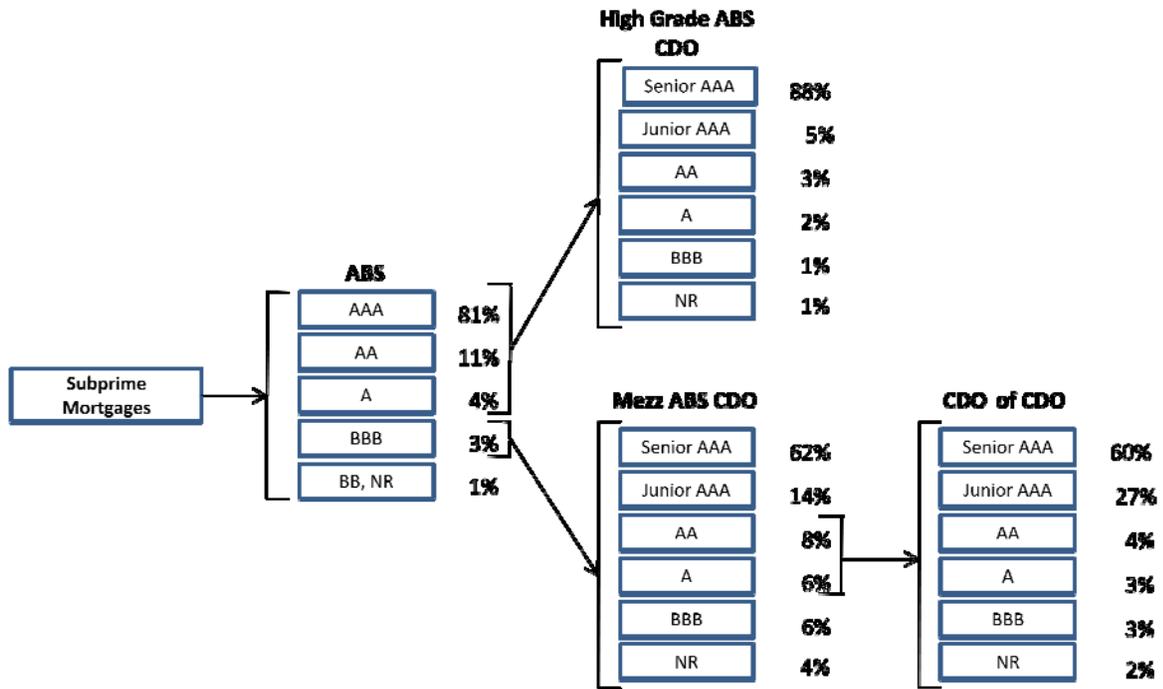


Table 1

Cumulative probability of default over 5 years for bonds initially rated AAA and BBB taken from Moody's statistics for the 1970 to 2007 period. Expected losses are calculated by assuming a recovery rate of 40%

	Probability of Loss	Expected Loss
AAA	0.1%	0.06%
BBB	1.8%	1.08%

Table 2

Minimum attachment point for the AAA rated tranche of an ABS to achieve a probability of loss less than 0.1%. In the constant recovery rate model the recovery rate is 75%. In the stochastic recovery rate model the recovery rate depends on the default rate and ranges from a high of 100% to a low of 50%.

		Expected Default Rate		
		5%	10%	20%
Gaussian Copula	$\rho = 0.05$	4.1%	6.8%	11.0%
Constant Recovery	$\rho = 0.10$	6.0%	9.4%	13.9%
	$\rho = 0.20$	9.6%	13.6%	18.2%
	$\rho = 0.30$	13.1%	17.2%	21.1%
Double t Copula	$\rho = 0.05$	7.6%	13.0%	18.2%
Constant Recovery	$\rho = 0.10$	13.6%	18.7%	21.9%
	$\rho = 0.20$	21.1%	23.2%	24.1%
	$\rho = 0.30$	23.7%	24.4%	24.7%
Gaussian Copula	$\rho = 0.05$	7.3%	11.6%	17.1%
Stochastic Recovery	$\rho = 0.10$	11.6%	17.3%	23.8%
	$\rho = 0.20$	19.1%	26.6%	33.4%
	$\rho = 0.30$	26.1%	34.1%	40.0%
Double t Copula	$\rho = 0.05$	15.0%	25.3%	33.4%
Stochastic Recovery	$\rho = 0.10$	27.2%	37.2%	41.8%
	$\rho = 0.20$	42.2%	46.3%	46.6%
	$\rho = 0.30$	47.4%	48.7%	47.8%

Table 3

Minimum attachment points for the AAA senior tranche of an ABS CDO. The ABS CDO is created from 100 BBB tranches of ABS tranches. The attachment point for each BBB tranche is 4% and the detachment point is 5%. The model determining the actual default rate is given in Section 2. The parameters α and ρ are defined so that the between pool copula correlation is $\alpha\rho$ and the within pool correlation is ρ . EDR is the expected default rate.

		$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.95$
Gaussian Copula	$\rho = 0.05$	17.1%	42.7%	73.5%	96.2%	99.9%
Constant Recovery	$\rho = 0.10$	29.7%	62.3%	89.7%	99.8%	99.9%
EDR=10%	$\rho = 0.20$	39.7%	73.6%	95.4%	99.9%	99.9%
	$\rho = 0.30$	43.5%	77.2%	96.7%	99.9%	99.9%
Gaussian Copula	$\rho = 0.05$	0.9%	2.6%	5.9%	10.1%	10.4%
Constant Recovery	$\rho = 0.10$	5.3%	16.1%	36.2%	66.3%	98.3%
EDR=5%	$\rho = 0.20$	14.5%	37.9%	69.1%	95.2%	99.9%
	$\rho = 0.30$	20.5%	48.8%	80.2%	98.7%	99.9%
Triple t copula	$\rho = 0.05$	95.9%	100.0%	100.0%	100.0%	100.0%
Stochastic Recovery	$\rho = 0.10$	93.8%	100.0%	100.0%	100.0%	100.0%
EDR=10%	$\rho = 0.20$	92.0%	100.0%	100.0%	100.0%	100.0%
	$\rho = 0.30$	90.3%	100.0%	100.0%	100.0%	100.0%
Triple t Copula	$\rho = 0.05$	82.9%	99.0%	100.0%	100.0%	100.0%
Stochastic Recovery	$\rho = 0.10$	84.1%	99.0%	100.0%	100.0%	100.0%
EDR=5%	$\rho = 0.20$	85.0%	99.0%	100.0%	100.0%	100.0%
	$\rho = 0.30$	80.0%	99.0%	100.0%	100.0%	100.0%