Christiano's presentation on Monetary Policy, 1982 to the present:

I will devote about 1/3 of my time discussing the many things that have occurred from 1982 to 2007. After that, I will discuss the issues addressed in the attached manuscript, which pertain to the policies often referred to as 'QE1': the impact on a dysfunctional banking system of large scale government asset purchases and reductions in the cost of funds to banks.

Government Policy, Credit Markets and Economic Activity*
(preliminary)

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Abstract

The US government has recently conducted large scale purchases of assets and implemented policies that reduced the cost of funds to financial institutions. Arguably these policies have helped to correct credit market dysfunctions, allowing interest rate spreads to shrink and output to begin a (tentative) recovery. We study four models of financial frictions which provide different ways of understanding the channels by which policies might have had these effects.

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[1] This section is based on ongoing joint work with Tao Zha.

1. Introduction

1.1. Preliminary Observations

A recession began in late 2007. The initial slowdown in economic activity came on the heals of a substantial drop in housing prices and asset prices more generally. In addition, the slowdown in late 2007 was accompanied by a rise in financial market commercial paper spreads, suggesting a dysfunction in those markets (see Figure 1). The decline in activity accelerated in late 2008 and this was accompanied by a further sharp rise in spreads that now also affected non-financial firms. The Federal Reserve responded aggressively by driving its usual policy instrument, the Federal Funds rate, to its limit of zero and undertaking other actions that departed sharply from its traditional strategy. The Fed's traditional policy strategy sets short term interest rates by exchanging short term US Treasury securities for reserves at the Fed. The Fed deviated from this policy in two ways, both of which can be observed by examining selected items on its balance sheet (see Table 1). First, there was an enormous expansion in the size of the balance sheet, with assets and liabilities jumping from about \$0.85 trillion in late 2006, to \$2.3 trillion in late 2010. This jump primarily reflected an increase in the Fed's holdings of privately issued securities.[2] Second, the Fed altered the mix of short term versus long term US government securities in its balance sheet. In the first period, 36% of the Fed's holdings of US government securities was in the form of short term securities. Only a few years later, this percent had fallen to a mere 2.5%. Since the total holdings of securities rose relatively little over the same period, the Fed in effect acquired a large amount of long term debt in exchange for short term debt.

Still, there is casual evidence that suggests the Fed's unconventional monetary policy helped.[3] The Fed's large scale asset purchases began in late 2008 and risk spreads in commercial paper rates for financial firms dissipated quickly thereafter. In addition, the Fed undertook an enormous expansion in its asset purchase program in March, 2009 and after this, corporate bond spreads also began to come down. Soon, aggregate output began a (tentative) recovery and the NBER declared an end to the recession in June, 2009. Of course, it is difficult to say what part of the recovery (if any) was due to the Fed's unconventional monetary policy, what part was due to the tax and spending actions in the American Recovery and Reinvestment Act of 2009, and what part simply reflects the internal dynamics of the business cycle. Many observers suppose that the Fed policies had at least some effect.

These observations raise two key questions:

- What are the financial market dysfunctions that are manifested in a sharp rise in interest rate spreads?

[2] The lion's share of this increase was in the Fed's holdings of mortgage backed securities. To a much smaller extent, the Fed also increased its holdings of other privately issued securities, such as liabilities of American Insurance Group and Bear Sterns. The acquisition of the liabilities of American Insurance Group and Bear Sterns was accomplished indirectly, with the Fed investing in limited liability companies which acquired the assets. See the "Note on consolidation" after Table 10 in Federal Reserve Statistical Release H.4.1, October 14, 2010.

[3] See, for example, Gagnon, Raskin, Remache, and Sack (2010). For a less sanguine perspective on the effectiveness of the Fed's policy, see Krishnamurthy and Vissing-Jorgensen (2010) and Taylor and Williams (2009).

- How do sharp reductions in short term interest rates and large scale government asset purchases correct these dysfunctions?

The answers to these questions are important for determining which asset market program should be undertaken and at what scale. Traditional macroeconomic models used in policy analysis in central banks are silent on these questions.

We survey the answers to the above questions from the perspective of four standard models taken from the banking literature and inserted into a general equilibrium environment. In each case, we restrict the model environment to be simple enough to allow us to focus sharply on the main ideas. As a result, most of the details that are required to ensure that models fit the data well are left out. For example, the models have only two periods, most shocks are left out of the analysis and we abstract from such things as labor effort, capital utilization, habit persistence, nominal variables, money, price and wage-setting frictions, etc. In addition, we make assumptions that allow us to abstract from the effects of changes in the distribution of income in the population. For the reasons described in section 2 below, it is particularly important to relax this assumption in more general analyses of unconventional monetary policy. Ultimately, the questions raised above must be addressed in fully specified dynamic, stochastic general equilibrium models. Work is already well under way on this task.[4] Our hope is that this paper may give a bird's eye view of the properties of the different models, in terms of their implications for the questions raised above.

Our survey does not examine all models of financial frictions. For example, we do not review models in which these effects occur because they serve to satisfy a demand for liquidity (see, e.g., Moore (2009) and Kiyotaki and Moore (2008)).[5] Instead, we review models that are in the spirit of Mankiw (1986), Bernanke, Gertler and Gilchrist (1999) (BGG), Gertler and Karadi (2009) and Gertler and Kiyotaki (2010). We review four models. The first three focus on financial frictions in the banking sector. Among these, the first two feature moral hazard problems and the third features adverse selection. The fourth model features asymmetric information and monitoring costs along the lines stressed in BGG.

We adopt a very stylized characterization of what happened since 2007. We then investigate how the different models interpret this characterization and what they imply for policy. The characterization is that net worth of the financial system dropped and this led to a rise in interest rate spreads and a fall in intermediation. We ask how four different tax-financed government interventions help under these circumstances: (i) transfers of net worth to economic agents, (ii) reduction in the cost of funds to financial firms, (iii) equity injections into financial firms, (iv) loans to financial and nonfinancial firms. Regarding (iii), we define an equity injection as follows. It occurs when the government provides funds to a bank and requires that the government receives its share of the resulting profits. In the case

[4] There is now a large literature devoted to constructing quantitative dynamic, stochastic general equilibrium models for evaluating the consequences of government asset purchase policies. For a partial list of this work, see Ajello (2010), Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997), Christiano, Motto and Rostagno, (2003,2010), Curdia and Woodford (2009), del Negro, Eggertsson, Ferrero and Kiyotaki, (2010), Dib (2010), Gertler and Karadi (2009), Gertler and Kiyotaki (2010), Fisher (1999), Hirakata, Sudo and Ueda (2009a,2009b,2010), Meh and Moran (2010), Ueda (2009), Zeng (2010).

[5] The Moore and Kiyotaki and Moore ideas are pursued quantitatively in Ajello (2010), and del Negro, Eggertsson, Ferrero and Kiyotaki (2010).

of (iv), we define a government loan to a bank as a commitment of funds on the same terms as those received by ordinary depositors.

All the models suggest that (i) helps. This is perhaps not surprising, since (i) in effect undoes what we presume to be the cause of the trouble. All the models also suggest that (ii) helps, though the precise mechanisms through which this happens varies. There is less agreement among the models in the case of (iii) and (iv). Whether these policies work depend on the details of the financial frictions.

1.2. Overview of the Model Analysis

We first describe a simple version of the Gertler-Karadi and Gertler-Kiyotaki model. In this framework, there is a particular moral hazard problem in the financial sector.[6] This problem stems from the fact that bankers have the ability to abscond with a fraction of the assets they have under management. A repeated version of the one period model that we study provides a crude articulation of the post 2007 events. Before 2007, the financial system functioned smoothly, was characterized by normal interest rate spreads and the first-best allocations were supported in equilibrium. Then, with the collapse in banking net worth credit markets became dysfunctional, with the consequence that banking interest rate spreads jumped and the volume of intermediation and output fell. Specifically, banks responded to the decline in net worth by restricting the amount of deposits that they issued. They did so out of a fear that if they tried to maintain the level of deposits in the face of a decline in net worth, depositors would lose confidence and take their money elsewhere. Depositors would do this anticipating that the rise in bank leverage would trigger bankers to abscond with bank assets. From this perspective, a sharp cut in the cost of funds to banks calms the fears of depositors by raising bank profits and providing bankers an incentive to continue doing business normally.

In the case of direct equity injections and loans, we follow Gertler-Karadi and Gertler-Kiyotaki in assuming that the government has the power to prevent banks from absconding with government funds. Under these circumstances, it is perhaps not surprising that government equity injections and loans, (iii) and (iv), are effective. By redirecting intermediation through the government, the economy is able to sidestep the frictions in the financial system that are the source of the problem. Of course, if the nature of the financial market frictions are not something that can be avoided by using the government in this way, then one suspects (iii) and (iv) are less likely to be helpful. This is the message of our second model.

Out second model captures moral hazard in banking in a different way. We suppose that bankers must exert a privately observed and costly effort to identify good investment projects. The problem here is not that bankers may run off with funds. Instead, it is that bankers may exert too little effort to make sure that assets under management are invested wisely. To ensure that bankers exert an efficient amount of effort, they must have the incentive to do so. This can be accomplished in the model if bankers are allowed to keep the increment to bank profits that occur as a consequence of their efforts. To accomplish this, bank deposit rates must be independent of the performance of bank asset portfolios. In this way, any increase or decrease in the return on that portfolio accrues to the banker.

[6] For other analyses in this spirit, see Holmstrom and Tirole (1997), and Meh and Moran (2010).

In the model, bankers - even those that exert high effort - will occasionally experience low returns on their portfolios. In these states of the world, their revenues will be below their deposit costs. As a result, for non-state contingent deposit rates to be feasible, bankers must have sufficient net worth to cover their losses in case their assets perform badly.

In our hidden effort model, efficient allocations are supported when net worth is high enough, but financial markets become dysfunctional when net worth drops so far that bankers cannot cover their losses in the bad state. In this case, deposit rates must be state contingent. Rates must be low in the bad state. And, deposit rates on banks that enjoy good returns must be relatively high in order to compensate for low returns of banks with losses. This state contingent pattern in deposit rates cuts into bankers' incentives to exert effort. The result is a low overall return on bank portfolios, and therefore less deposits by households and less investment. Policy (ii), by reducing the cost of funds to banks is obviously helpful. This is because the policy reduces banks' liabilities in the bad state and so increases the likelihood that deposit rates can be state non-contingent.

Policies (iii) and (iv) have no effect in the model. Although the proof involves details, the result is perhaps not surprising. The government equity injections and bank loans that we consider do not involve side stepping any financial frictions in the way that our first model of moral hazard does. This is because we could not think of special advantages available to the government to ensure good effort by bankers, apart from the profit motive already in the market. This model illustrates the general principle that the sources of moral hazard matter for whether a particular asset purchase programs is effective.

Our third model focuses on adverse selection as a source of financial market frictions.[7] To many observers, adverse selection seems like a natural framework for thinking about the financial market turmoil of recent years. Before 2007 owners of mortgage backed securities (MBS) had little difficulty selling these assets because most thought they were very secure. When housing prices collapsed, a cloud of uncertainty settled over the whole market as people wondered which MBS had been most affected. Market participants suspected that institutions selling MBS did so because they knew their MBS to be of poor quality. As a result, the market assigned a low price to these assets. This ensured that the only MBS trading was in fact bad MBS, while holders of good MBS could not sell them at anything better than fire-sale prices.[8] These general considerations motivate us to include a model of adverse selection in our analysis.

In our model there are entrepreneurs with investment opportunities.[9] Although the entrepreneurs have their own net worth, they also require external finance if they are to operate their projects. The problem is that some projects are good risks and others are bad and this leads to a classic adverse selection problem in credit markets. Because bankers cannot differentiate between risky and safe projects, they must offer the same loan contracts on all of them. The interest rate banks charge entrepreneurs must be high relative to their

[7] There are several analyses of the recent credit crisis that focus on adverse selection in credit markets. See, for example, Chari, Shourideh and Zetlin-Jones (2010), Fishman and Parker (2010).

[8] This is a summary of the argument in Shimer, http://gregmankiw.blogspot.com/2008/09/case-against-paulson-plan.html. See Eisfeldt (2004) for a theoretical analysis that blends adverse selection and liquidity problems.

[9] Our model is adapted from the model in Mankiw (1986). We modify the model somewhat and also insert it into a general equilibrum environment.

deposit rates in order to compensate for their losses on the riskier loans. This positive interest rate spread is made even larger by adverse selection. With loan rates high, safer entrepreneurs choose not to borrow and so the loan pool is tilted towards riskier borrowers. With deposit rates low, the supply of saving is low. As a result, total intermediation and investment are low.

When entrepreneurial net worth drops, as in the period after 2007, then the adverse selection problem becomes more severe as entrepreneurs' dependence on external finance increases. The model predicts an increase in interest rate spreads and a drop in intermediation and investment. We investigate what policies can correct the resulting dysfunction in credit markets. In the model, deposit rates do not signal the full social marginal product of loans, and this is why intermediation is inefficiently low. Subsidies to banks' cost of funds improves the situation.

Our fourth model of financial frictions is a two-period version of the one proposed in Bernanke, Gertler and Gilchrist (1999) (BGG).[10] This is an important model of financial frictions, as there is a large literature analyzing the model in fully specified, dynamic general equilibrium frameworks. The model has proved useful in the analysis of macroeconomic data.[11] The model implies that intermediation is inefficiently low. In our version of the BGG model, it is the monitoring costs on borrowers that prevent the equilibrium from being efficient. Thus, any government policy that allows the economy to reduce monitoring costs is welfare improving. Thus a tax-financed transfer of net worth to entrepreneurs that is large enough for them to avoid the banking system altogether improves welfare by eliminating monitoring costs. However, it does not follow that more reasonably-sized local changes in net worth improve welfare, and this is a question that we are still studying. Turning to (ii), we find that a policy of subsidizing bank interest rate costs improves welfare. However, we find that the effect is quantitatively very small. In this respect, the BGG model's policy implications are different from the implications of the other three models considered. In those models, we show that interest rate subsidies have large effects, and can even move the economy to the first-best equilibrium allocations.

The rest of the paper is organized as follows. The second section below describes what we call the Ricardian irrelevance result, which sets out a basic challenge that any model of private asset purchases must address. The following two sections describe the two models of moral hazard. Section 5 studies the model of adverse selection. Second 6 describes the two-period version of the BGG model of financial frictions and its implications for policy. A final section presents concluding remarks.

[10] In this model the financial frictions primarily stem from problems in the non-financial sector. However, the basic asymmetric information and monitoring cost framework has been extended to to banks in the work of Hirakata, Sudo and Ueda (2009a,2009b,2010), Ueda (2009), Zeng (2010).

[11] See the Christiano, Motto and Rostagno (2003,2010) analysis of the US Great Depression, and of the past three decades' business cycles in the Euro Area and the US. An earlier DSGE model application of the costly state verification and monitoring cost idea can be found in the influential contribution by Carlstrom and Fuerst (1997). For another contribution of this idea in a DSGE model, see Jonas Fisher's doctoral dissertation, published as a 1996 Federal Reserve Bank of Chicago working paper and also in Fisher (1999).

2. Unconventional Policy and the Ricardian Irrelevance Proposition

Any analysis of unconventional policy must confront a basic question. If the government acquires privately issued assets by levying taxes (either in the present or the future), then the ownership of the asset passes from private agents to the government, which holds the asset in trust for the taxpayer. In the simplest economic model, it makes no difference. In such a model, when the government purchases private assets, households reduce their purchase of the asset by the same amount and nothing of consequence changes. The government purchases of private assets are irrelevant to allocations. We refer to this irrelevance result as the Ricardian-irrelevance result, because it is closely related to the Ricardian equivalence result emphasized by Barro (1974).[12] Any analysis in which government asset purchases have real effects must explain what assumptions have been made to defeat the irrelevance result. We present examples of such assumptions in this section.

One way to defeat the Ricardian irrelevance builds on heterogeneity in the population. For example, suppose that a subset of the population has a special desire to hold a certain asset (for example, 30 year Treasury bonds).[13] If the government engages in a tax financed purchase of that bond, then in effect the bond is transferred from the subset of the population that holds it initially, to all taxpayers. Such a redistribution of assets among heterogeneous agents can be expected to change prices and allocations. This type of logic may be useful for interpreting the evidence which suggesting that the change in the composition of the Fed's portfolio of Treasury bonds reported in Table 1 may have had real effects.[14] We do not investigate this line of analysis further here, since we abstract from changes in the distribution of income in the population.

There are other ways in which tax financed purchases of private securities may have real effects. In the examples we explore, this can happen by changing the market rate of interest.

3. Moral Hazard I: Gertler-Karadi, Gertler-Kiyotaki[15]

We construct a two-period model. In the first period, households make deposits in banks. Banks combine these deposits with their own net worth and provide funds to firms. In the second period, households purchase the goods produced by firms using income generated by bank profits and interest payments on deposits. The source of moral hazard in this section is that banks have an option to default by absconding with an exogenously fixed fraction of their total assets, leaving the rest to depositors. When a large fraction of a bank's assets are purchased with its own net worth, then banks hurt themselves by defaulting and they choose not to do so. In this case, we show that the equilibrium allocations correspond to the first-best efficient allocations. We refer to this scenario as 'normal times'. When banks' net worth is sufficiently low, banks restrict the supply of deposits. Banks do this because they know

[12] See also Wallace (1981), who stressed a similar result for open market operations.

[13] What I am calling the Ricardian irrelevance result is shown to apply to government purchases of long term debt in Eggertsson and Woodford (2003).

[14] The logic in the text may also provide the foundation for a theory of the effectiveness of sterilized interventions in the foreign exchange markets.

[15] This section is based on ongoing joint work with Tao Zha.

that if they planned a higher level of deposits, depositors would rationally lose confidence and take their deposits elsewhere. With the supply of deposits reduced in this way, and no change in demand, the interest rate on deposits is reduced. Because the return on bank assets is fixed by assumption, the result is an increase in banks' interest rate spreads.[16] We refer to the situation in which bank net worth is so low that the banking system is dysfunctional and conducts too little intermediation as 'crisis times'. Thus, the model articulates one view about what happened in the past few years: "a fall in housing prices and other assets caused a fall in bank net worth and initiated a crisis. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity was reduced." In contemplating such a scenario we imagine a version of our two-period model, repeated many times.

Government policy can push the economy from crisis to normal times by undoing the cause of economic dysfunction. One way it can do this is by purchasing bank assets. In the Gertler-Karadi and Gertler-Kiyotaki analysis it is assumed that the government has the ability to prevent banks from absconding with bank assets financed by equity or deposit liabilities to the government. We show that sufficiently large government purchases of bank assets can restore the banking system to normal times. In particular, government asset purchases cause interest rate spreads to disappear and total intermediation to return to its first best level. The logic of the Ricardian irrelevance result does not hold in crisis times because tax-financed government purchases of bank assets have an impact on the interest rate.

Another policy that can resolve a crisis is one in which the government provides tax-financed loans to firms. Under this policy the government returns the proceeds of its investment in firms to households in the form of lower taxes in second period. Households understand that this government policy is a substitute for their bank deposits and so they reduce the supply of deposits. If the government asset purchase program is large enough, deposits are reduced to a sufficiently low level that no one suspects banks will have the temptation to abscond. With the supply and demand for bank deposits both reduced, the deposit interest rate rises back up and the interest rate spread is wiped out. Total intermediation returns to its normal level because, though household deposits are relatively low, this is matched by a corresponding increase in government provision of funds. In this way, government purchases of the assets of nonfinancial business can resolve a crisis.

Finally, we show that a policy of subsidizing banks' cost of funds can push the economy out of a crisis. Such a policy works by increasing banks' profits during a crisis and so reducing their temptation to abscond with bank assets. Understanding that their depositors are aware of this, banks expand their deposits back to the first best level. That is, a sufficiently large subsidy to banks' cost of funds can push the economy out of a crisis and back into normal times.

We first describe the model. We then formally establish the properties of government policy just reviewed.

[16] So, profits per unit of bank deposits rises when banker net worth is low. However, total bank profits may be low because because of the lower net worth of the banks.

3.1. Model

There are many identical households, each with a unit measure of members. Some members are 'bankers' and others are 'workers'. There is perfect insurance inside households, so that all household members consume the same amount, c, in period 1 and C in period 2. In period 1, workers are endowed with y goods and the representative household makes a deposit, d, in a bank subject to its period 1 budget constraint:

$$c + d \leq y.$$

The representative household's period 2 budget constraint is:

$$C \leq Rd + \pi.$$

Here, R represents the gross return on deposits and π denotes the profits brought home by bankers. The household treats π as lump sum transfers. The intertemporal budget constraint is constructed using period 1 and period 2 budget constraints in the usual way:

$$c + \frac{C}{R} \leq y + \frac{\pi}{R}. \tag{3.1}$$

The representative household chooses c and C to maximize

$$u(c) + \beta u(C), \; u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \; \gamma > 0. \tag{3.2}$$

subject to R, π and (3.1). The solution to the household problem is:

$$c = \frac{y + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}, \; d = y - c, \; C = Rd + \pi. \tag{3.3}$$

We can see the basic logic of the Ricardian irrelevance proposition from (3.1). Suppose the government raises taxes, T, in period 1, uses the proceeds to purchase T deposits and gives households a tax cut, RT, in period 2. The periods 1 and 2 budget constraints are replaced by:

$$c + d \leq y - T, \; C \leq Rd + \pi + RT. \tag{3.4}$$

Using these two equations to substitute out for $d + T$ we obtain (4.22) and T is irrelevant for the determination of c and C. Deposits are determined residually by $d = y - T$. If the government increases T, then d drops by the same amount. Of course, if we change the environment in some way, then the Ricardian irrelevance proposition may no longer be true. This could happen, for example, if T affected R. To investigate this, we need to flesh out the rest of the model.

Bankers in period 1 are endowed with N goods. They accept deposits from households and purchase securities, s, from firms. Firms issue securities in order to finance the capital they use to produce consumption goods in period 2. Intermediation is crucial in this economy. Without it, production cannot occur and period 2 production is the only source of the period 2 consumption goods.

We first consider the benchmark case in which there are no financial frictions and the banking sector helps the economy to achieve the first best allocations. We suppose that the gross rate of return on privately issued securities is technologically fixed at R^k. Bankers combine their own net worth, N, with the deposits received, d, to purchase securities, s, from firms. Firms use the proceeds from selling these securities to purchase an equal quantity of goods which they turn into capital. The quantity of goods produced by firms in period 2 is sR^k. Goods producing firms make no profits, so sR^k is the revenue they pass back to the banks. Banks pay dR on household deposits in period 2. Bankers solve the following problem:

$$\pi = \max_{d} \left[sR^k - Rd \right], \tag{3.5}$$

where $s = N + d$ and N is the banker's state.

An equilibrium is defined as follows:

Benchmark Equilibrium: R, c, C, d, π such that
(i) the household and firm problems are solved
(ii) the bank problem, (3.5), is solved
(iii) markets for goods and deposits clear
(iii) $c, C > 0$

Condition (iii) indicates that we only consider interior equilibria, both here and elsewhere in the paper. A property of a benchmark equilibrium is $R = R^k$. To see this, suppose it were not so. If $R > R^k$ the bank would set $d = 0$ and if $R < R^k$ the bank would set $d = \infty$, neither of which is consistent with the equilibria that we study. Thus, in the benchmark case the interest rate faced by households in equilibrium coincides with the actual rate of return on capital. It is therefore not surprising that the first best allocations are achieved in this version of the model. That is, the allocations in the efficient, benchmark equilibrium coincide with the allocations that solve the following planning problem:[17]

$$\max_{c,C,k} u(c) + \beta u(C) \tag{3.6}$$

$$\text{subject to:} \qquad c + k \leq y + N, \; C \leq kR^k.$$

The interest rate spread in this economy is defined as $R^k - R$. In the benchmark equilibrium the interest rate spread is zero. This makes sense, since there are no costs associated with intermediation and there is no default. We summarize this result as follows:

Proposition 3.1. *A benchmark equilibrium has the properties:*
(i) the interest rate spread, $R^k - R$, is zero
(ii) d takes on its first-best value.

In this economy, the Ricardian irrelevance condition is satisfied. Tax financed government purchases of private assets have no impact.

We now introduce the moral hazard problem studied by Gertler-Karadi and Gertler-Kiyotaki. A bank has two options: 'default' and 'not default'. Not defaulting means that

[17] We assume the environment is such that $c < y$.

a bank simply does what it does in the benchmark version of the model. In this case, the bank earns profits

$$\pi = R^k (N + d) - Rd. \tag{3.7}$$

The option to default means that the banker can take a fraction, θ, of the assets and leave whatever is left for the depositors. A defaulting bank receives $\theta R^k (N + d)$ and its depositors receive $(1 - \theta) R^k (N + d)$. The bank chooses the no default option if, and only if doing so increases its profits:

$$(N + d) R^k - Rd \geq \theta (N + d) R^k. \tag{3.8}$$

By rearranging terms, we see that (3.8) is equivalent with

$$(1 - \theta) (N + d) R^k \geq Rd. \tag{3.9}$$

That is, a bank chooses the no default option if, and only if, doing so reduces what depositors receive.

Each bank takes the interest rate on deposits as given, and sets its own level of deposits, d.[18] Banks are required to post their intended values of d at the start of the period, so that households can assess whether or not a bank will default. We consider symmetric equilibria in which no bank chooses to default and the d posted by banks satisfy (3.8). In such an equilibrium an individual bank has no incentive to choose a level of deposits that violates (3.8) because depositors would in this case prefer to take their deposits to another bank, where they obtain a higher return (see (3.9)). In this setting, the banker solves the following problem:

$$\pi = \max_d \left[sR^k - Rd \right], \text{ subject to (3.8).} \tag{3.10}$$

The definition of equilibrium we use in the case that the banker has a default option is:

> **Financial Equilibrium**: R, c, C, d, π such that
> (i) the household and firm problems are solved
> (ii) the bank problem, (3.10), is solved
> (iii) markets for goods and deposits clear
> (iii) $c, C, d > 0$.

The difference between a financial equilibrium and a benchmark equilibrium lies in the definition of the banker problem.

When the bank's incentive constraint, (3.8), is non-binding, then $R^k = R$ and the no default condition reduces to:

$$NR^k \geq \theta (N + d) R^k.$$

If N is sufficiently large, (3.8) is non-binding and the equilibrium has the property that d is at its first-best level and the interest rate spread is zero.

Now suppose that N is sufficiently small (consider, for example, the case, $N = 0$) that (3.8) strictly binds.[19] In this case, the financial equilibrium would not be characterized

[18] I suspect that allowing banks to also choose their own deposit rate in addition to its level of deposits changes nothing.

[19] That is, the multiplier on (3.8) in the Lagrangian representation of (3.10) is non-zero.

by $R^k = R$. The only equilibrium is one in which R is below R^k. To see why, note that a reduction in R directly helps to restore (3.8) by increasing the term on the left of the inequality. In addition, the fall in R reduces d and this reduces both the left and right sides of (3.8).[20] We summarize this result in the following proposition:

Proposition 3.2. *When (3.8) is non-binding, the financial market equilibrium allocations are first-best and the interest rate spread is zero. When (3.8) binds, then the equilibrium values of d and R are below their first-best levels and the interest rate spread is positive.*

A sequentially repeated version of this model economy provides a rough characterization of events before and after 2007. Suppose that N was large in the early period, so that the economy was operating at its efficient level and no part of actual spreads was due to the type of default considerations addressed here. Then, in late 2007 the net worth of banks suddenly began to fall as a consequence of the collapse in housing prices. When the participation constraint began to bind, spreads opened up. The volume of intermediation - and the investment it supported - then collapsed.

3.2. Implications for Policy

We now consider the effects of four kinds of tax-financed unconventional monetary policies: injections of equity into banks, deposits in banks, direct loans to entrepreneurs and subsidies to banks' cost of funds. In each case, the policy is financed by lump sum taxes, T, in the first period. In the case of the asset purchase policies, the government transfers the proceeds back to households in the form of a second period tax reduction.

3.2.1. Equity Injections into Banks

In the case of an equity injection, the government transfers T to each bank. The government requires the banks to repay the earnings, R^kT, on the assets financed by the equity. The government transfers the R^kT back to households in period 2 in the form of a tax reduction.

We assume that unlike the household, the government has the power to prevent the bank from absconding with any part of the assets financed by T. Thus, for a bank that receives an equity injection of T, the incentive to default is still the object on the right of the inequality in (3.8). An equity injection also has no impact on a bank's profits:

$$(N + T + d)\, R^k - Rd - R^kT = (N + d)\, R^k - Rd.$$

Thus, for a given level of deposits, d, an equity injection has no effect on a bank's decision to default. However, the government's equity injection does affect the representative household's choice of d.

[20] Here, I use the fact that d is increasing in R. To see this, substitute out for π in (3.3) using (3.7) and solve for c to obtain:

$$c = \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k}\,(N + y)\,.$$

Evidently, equilibrium consumption is strictly decreasing in R, so that d is strictly increasing in R.

To understand how the representative household responds to the tax implications of an equity injection, a suitable adjustment of (3.3) implies:

$$c = \frac{y - T + \frac{\pi}{R} + \frac{R^k T}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Note that T does not directly cancel in the numerator because the rate of interest enjoyed by the government when it does an equity injection is different from the household's rate of return on deposits when (3.8) binds and $R^k \neq R$. To understand the general equilibrium impact of T on c it is necessary to substitute out for π (3.7):

$$c = \frac{y - T + \frac{R^k(N+d) - Rd}{R} + \frac{R^k T}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

The household's period 1 budget constraint implies $d = y - T - c$. Using this to substitute out for d in the above expression and rearranging:

$$\begin{aligned} c &= \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k}(N + y) \\ d &= y - c - T. \end{aligned} \tag{3.11}$$

Interestingly, the general equilibrium effect of T on consumption is nil, despite the difference between the government's and the household's interest rate. From the latter expression, we see that a rise in T has no impact on c and so has a one-for-one negative impact on d. If (3.8) is non-binding, then the equity injection is irrelevant. There is no impact on total intermediation, $d + T$, and the interest rate spread remains unchanged at zero.

Now suppose that (3.8) is binding. The fall in d with a rise in T increases the right side of (3.8) and reduces the left side, thus making the incentive constraint less binding. With T large enough, the incentive constraint ceases to bind altogether and an analogous argument to the one leading up to proposition 3.2 establishes that the interest rate spread is eliminated, $R = R^k$, while total intermediation, $T + d$, achieves its first best level.

To see what level of T achieves the first-best, let d^* denote the level of deposits in a benchmark equilibrium (d^* can be found by solving (3.6) and setting $d^* = k - N$). Our assumption that (3.8) is strictly binding implies that

$$NR^k < \theta (N + d^*),$$

so that d^* is not part of a financial equilibrium. Set T to the value, T^*, that solves

$$NR^k = \theta (N + d^* - T^*). \tag{3.12}$$

We summarize the preceding results in the form of a proposition:

Proposition 3.3. *When (3.8) is non-binding tax-financed equity injections have no impact on total intermediation, $d+T$, and on the interest rate spread, $R^k - R$. When (3.8) binds, tax financed equity injections reduce the interest rate spread and increase total intermediation. A sufficiently large injection restores spreads and total intermediation to their first-best level.*

A numerical example is reported in Table 2. The model parameter values are:

$$N = 1,\ y = 10,\ \gamma = 1,\ \beta = 0.97,\ \theta = 0.38,\ R^k = 1.10.$$

The value of θ was taken from Gertler-Kiyotaki. In the no-intervention equilibrium, $T = 0$. In the example, $T^* = 2.78$, after rounding. Profits are equal to the gains of deviating in each equilibrium, so that the incentive constraint, (3.8), is satisfied as an equality in both equilibria. But, the constraint is strictly binding in the first equilibrium and marginally non-binding in the second. Note that household deposits are lower in the second equilibrium than the first, although total intermediation is higher in the second equilibrium than the first.

We can express the equations of the model in words as follows. When N falls enough, the supply of deposits by banks decreases because the incentive constraint binds on the banks. This creates an interest rate spread by reducing the deposit rate (the return on assets is fixed in this example). A tax-financed government purchase of assets causes the demand for deposits by households to decrease, pushing the deposit rate back up and reducing the interest rate spread. The decrease in deposits is somewhat offset by the rise in the deposit rate and this is why $d + T$ increases with the government intervention. The intervention is welfare improving because it pushes the economy back up to the first best allocations.

3.2.2. Government Deposits in Banks and Loans to Firms

Suppose the government makes tax-financed deposits, T, in banks in period 1. In period 2 it returns the proceeds to households in the form of a tax cut in the amount, RT. It is easy to verify that c and d are determined according to (3.11) in this case. As a result, total deposits, $d + T$, are invariant to T, for a given R.

If we assume that banks can as easily default on the government as on households, then total deposits, $d + T$ enter the incentive constraint and the tax-financed deposits are irrelevant. However, suppose that the government can prevent banks from defaulting on any part of its deposits. In that case, the profits earned by banks on government deposits, $\left(R^k - R\right) T$, are not counted in the incentive constraint, (3.8). With only household deposits in the incentive constraint, the analysis is identical to the analysis of equity injections.

Now consider the case where the government makes tax-financed loans directly to entrepreneurs. This case is formally identical to the case of tax-financed equity injections. For a given R, $d + T$ is invariant to T. However, because only d enters the incentive constraint, (3.8), the reduction in d that occurs with a rise in T relaxes the incentive constraint in case it is binding. This results in an increase in R and, hence, a rise in total intermediation. We summarize these results in the following proposition:

Proposition 3.4. *If the government can prevent bank defaults on its own bank deposits, then the effects of tax-financed government deposits in banks resemble the effects of equity injections summarized in proposition 3.3. Direct government loans to firms have the same effects as those of equity injections.*

3.2.3. Interest Rate Subsidies and Net Worth Transfers to Banks

We now consider a policy in which the government subsidizes the interest rate that banks pay on deposits. Suppose that the equilibrium is such that the incentive constraint, (3.8), is binding. As in the previous subsection this implies that the first-best level of deposits (i.e., the one that solves (3.6) with 'deposits' identified with $k - N$) violates (3.8):

$$(N + d^*) R^k - Rd^* < \theta (N + d^*) R^k, \tag{3.13}$$

when the deposit rate, R, is at its efficient level, R^k. Let $\tau > 0$ be the solution to:

$$(N + d^*) R^k - R^k (1 - \tau) d^* = \theta (N + d^*) R^k. \tag{3.14}$$

Note that there exists a unique value of $\tau > 0$ that solves this equation because the left side is increasing in τ and the left exceeds the right when $\tau = 1$. To finance the transfer, $\tau R^k d^*$, to banks the government levies taxes, $T = \tau R^k d^*$, on households in the second period. We now verify that this policy, together with $d = d^*$, $R = R^k$ and c, C at their first-best levels, c^*, C^*, satisfies all the equilibrium conditions. Bank profits in the second period are:

$$\pi = (N + d^*) R^k - R^k (1 - \tau) d^* = (N + d^*) R^k - R^k d^* + R^k \tau d^*.$$

Total household income is

$$\begin{aligned} & Rd + \pi - T \\ = \; & (N + d^*) R^k. \end{aligned}$$

The latter result and the assumption that c^*, C^* solve (3.6) imply that the household problem is solved. The fact that the incentive constraint is satisfied implies that the bank problem, (3.10), is solved. We summarize these findings as follows:

Proposition 3.5. *Suppose (3.8) binds in equilibrium, so that deposits are strictly below their first-best level in a financial equilibrium. Then, a subsidy to bank deposit liabilities at the rate defined by (3.14) ensures that the first best allocations are supported as a financial equilibrium.*

Next, we consider the case in which taxes are levied on households in the first period and the proceeds are given to bankers as a supplement to their net worth. The net worth transfer is financed by taxes on households in period 1. Suppose the equilibrium is such that the incentive constraint, (3.8), is binding. This implies that the first-best level of deposits, d^*, violates (3.8) and that (3.13) is satisfied with R at its efficient level, R^k.

Let T denote the tax-financed transfer of net worth to bankers. The pre-tax level of banker net worth is N and after taxes it it $N + T$. We conjecture, and then verify, as for T sufficiently large, the financial equilibrium has the property that deposits equal $d^* - T$, the incentive constraint is non-binding, and c, C coincide with their first-best values. Let T be the solution to:

$$(N + d^*) R^k - R^k (d^* - T) = \theta (N + d^*) R^k. \tag{3.15}$$

Note that $N+d^*$ is unaffected under the tax policy and the conjecture about the equilibrium. A unique $T > 0$ that solves (3.15) is guaranteed to exist because the left side is monotonically increasing in T and the left side is assumed to be smaller than the right when $T = 0$.

To understand how the representative household responds to the tax financed equity injection, a suitable adjustment of (3.3) implies:

$$c = \frac{y - T + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Under our conjecture, $R = R^k$ and π is given by the expression on the left of the equality in (3.15). Substituting, we obtain (3.11), the level of consumption in the first-best equilibrium. This verifies our conjecture about the period 1 level of consumption. It is straightforward to verify that the first-best level of period 2 consumption satisfies the period 2 household budget constraint. We summarize our findings as follows:

Proposition 3.6. *Suppose (3.8) binds in equilibrium, so that deposits are strictly below their first-best level in a financial equilibrium. Then, a tax-financed transfer of net worth to bankers at a level defined in (3.15) ensures that the first best allocations are supported as a financial equilibrium.*

4. Moral Hazard II: Unobserved Effort

The basic framework of the model here is the same as in the previous subsection. The difference lies in the source of moral hazard. To make a high return for depositors, we assume that bankers must exert an unobserved and costly effort. The properties of the model are in some respects similar to those of the previous subsection. In particular, the model can be used to articulate the idea that the banking system was efficient prior to 2007, but then became dysfunctional with the fall in bank net worth. The decline in net worth leads to a rise in interest rate spreads and a fall in intermediation and the activities that it supports. The model differs from the one in the previous section in terms of some of its implications for policy, however. Following is an intuitive summary of the analysis. After that comes the formal presentation.

4.1. Overview

There are two periods. There is a large number of households. Each household has many bankers and workers. Bankers are endowed in the first period with their own net worth and they combine this with deposits to acquire securities from firms. There is a large number of firms, each having access to one investment project. Some firms have a good investment project that has a high (constant) gross rate of return. If these firms receive one unit of goods in period 1, they are able to produce R^g goods in period 2. Other firms are bad and we denote the gross rate of return on their investment projects by R^b, where $R^b < R^g$. The rates of return, R^g and R^b, are exogenous and technologically determined. The role of banks is to identify the investment projects that have a high return.

Empirically, we observe that some banks enjoy higher profits than others, and we interpret this as reflecting that banks cannot hold a fully diversified portfolio of assets. This could be because there are many different types of investment projects - differentiated according to industry, geographic location, etc. - and there are gains to specializing in the identification of good projects of a particular type. In the model, these observations are captured by the assumption that banks can invest in at most one firm. The task of a banker is to exert an unobserved and costly effort, e, to identify a firm with a good project. The ex post rate of return on the banker's investment is observed, but this does not reveal the banker's effort. This is because e only affects the probability, $p(e)$, that a banker will identify a good firm.

For bankers to have an incentive to exert effort they must receive a reward that is positively linked to the performance of their investment portfolio. We define a full information benchmark level of effort and of intermediation as those that occur in competitive markets in the special case that bankers' efforts are fully observable. This level of effort also occurs when e is not observed and the banker pays a rate of return on its deposit liabilities that is independent of the outcome of its investment project. In this way, the banker fully internalizes the marginal benefit of increased effort. However, for this arrangement to work, it is necessary that the banker have enough of its own net worth. Otherwise the banker would not have sufficient funds to pay depositors in the probability $1 - p(e)$ event that its project turns out to be bad.[21] When net worth is too low in this sense, the banking system becomes 'dysfunctional' in that it performs too little intermediation relative to the full information benchmark. As in the previous section, we refer to the situation in which net worth is too low and the system is dysfunctional as a 'crisis time'. We refer to a situation when net worth is adequate and the banking system produces the ideal outcomes in the sense defined above as a 'normal time'.

We can loosely think of the bad state as a bankruptcy state and in a calibration, this state would occur with low probability. For the purpose of defining the interest rate spread, we think of the 'interest rate' paid by a bank on its source of funds as the rate they pay, R_g^d, when the good state is realized. This notion of the interest rate is similar to that of the face value of a bond, which specifies what the holder receives as long as nothing goes wrong with the issuing firm. The ultimate source of funds for banks are the households and they receive an interest rate that is risk free. This is so, because the representative household is perfectly diversified across banks (it accomplishes this using a mutual fund) and so it receives the average rate of return across all banks. This is what we refer to as the risk free rate. The 'interest rate spread' is defined as the difference between R_g^d, the interest rate that banks pay on deposits in the high return state, and the risk free rate, R:

$$R_g^d - R. \tag{4.1}$$

In normal times, $R_g^d - R = 0$, so that the interest rate spread is zero. In crisis times, the rate of return on deposits is low in the bad state and thus higher in the good state. As a result, $R_g^d - R > 0$ in crisis times.

The state contingency in deposit rates in crisis times implies that bankers have too little incentive then to exert high effort. As a result, they identify relatively fewer good projects, with the consequence that the risk free rate is lower. This in turn implies that household

[21] Bankers do not have access to funds other than their own and those provided by depositors.

saving is lower. Thus, a fall in banker net worth triggers a scenario in which the interest rate spread is high and the level of intermediation is low, relative to the ideal outcome. In this respect, the model implications resemble those in the previous subsection. In the model considered here, the interest rate spread compensates for the low returns paid by banks with bad investments. In the model of the previous section the interest rate spread also reflects a concern about things going wrong. However, that concern is about something that is not observed in equilibrium (it is a fear that banks would abscond with a fraction of their assets if the level of deposits were higher).

In terms of their implications for unconventional policy, however, the two models differ. In the model of the previous section equity injections have no effect in normal times and they improve the efficiency of the economy in crisis times. In the model here, equity injections in normal times are counterproductive because they reduce bankers' incentives to exert effort. The intuition is simple. We treat an equity injection as a 'loan' from the government that must be repaid according to the actual return that the bank receives on the investment it makes with the loan. The direct impact of this sort of loan on the bank is nil since it generates zero net cash flows across states of nature. However, there is a general equilibrium effect that matters. From the point of view of the household, an equity injection corresponds to a tax hike in the first period, followed by a tax reduction in the second period. Because this pattern of taxes satisfies part of the household's desire to save, the household responds by reducing its own deposits. With fewer deposits, the banker has less incentive to exert effort. With less effort, the average quality of bank investments falls. This produces a fall in the risk free interest rate paid to households and causes them to save less. The net effect is that intermediation falls below its ideal level.

It turns out that in crisis times, an equity injection has *no* effect in the model. This is because in crisis times there is an additional positive effect from equity investments which cancels the negative effects in normal times that were discussed in the previous paragraph. Recall, the definition of a crisis time is that net worth is too low to permit a state-non contingent interest rate on deposits. When household deposits with banks are reduced in response to an equity injection, it becomes possible to reduce the degree of state contingency in deposit rates. This is because, with lower deposits the amount of money owed by banks in the bad state is smaller and more likely to be manageable with bank net worth. The reduced state contingency in deposit rates improves the incentive of banks to exert effort. This positive effect exactly cancels the negative effects that occur in normal times.

The following sections present the formal description of the model and the results, respectively.

4.2. Model

There are many identical households, each composed of many workers and bankers. The workers receive an endowment, y, in period 1 and the households allocate the endowment between period 1 consumption, c, and period 1 deposits in mutual funds, d. All quantity variables are expressed in per household member terms. The gross rate of return on deposits is risk free and is denoted by R. The preferences of the representative household are as in the previous example, in (3.2). Optimality of the deposit decision is associated with the usual intertemporal Euler equation. This Euler equation and the first period budget constraint

are given by:

$$c + d = y \tag{4.2}$$
$$u'(c) = \beta R u'(C). \tag{4.3}$$

In the second period, households receive Rd and profits from their bankers, π. In the interior equilibria that we study, the second period budget constraint is satisfied as a strict equality:

$$C = Rd + \pi.$$

We impose the following restriction on the curvature parameter in the utility function (see (3.2)):

$$0 < \gamma < 1. \tag{4.4}$$

The upper bound on γ ensures that the equilibrium response of d to R is positive, which we view as the interesting case.

Bankers receive an endowment, N, in the first period. They combine N with deposits received from mutual funds and buy securities that finance the investment of a firm. Firms are perfectly competitive and costless to operate, so the bank receives the entire return on its firm's investment project. The probability, $p(e)$, that the firm whose securities the bank buys are good is determined according to the following function:

$$p(e) = a + be, \tag{4.5}$$

so that $p'(e) = b$, $p''(e) = 0$. We only consider model parameter values that imply $0 < p(e) < 1$ in equilibrium.

Our primary interest is in the scenario with 'financial frictions', in which the mutual fund does not observe the effort, e, made by the banker. However, before discussing this scenario, it is useful to discuss, as a benchmark, the full information version of the model in which e is observed by the mutual fund. We always assume that e is observed by the banker's household. Without this assumption, the banker would not be able to receive perfect consumption insurance from the household.

4.3. Observable Effort Benchmark

A loan contract between a banker and a mutual fund is characterized by four numbers, $\left(d, e, R_g^d, R_b^d\right)$. Here, R_g^d, R_b^d denote the gross returns on d paid by bankers whose entrepreneurs turns out to be good and bad, respectively. All four elements of the contract are assumed to be directly verifiable to the mutual fund in the observable effort benchmark. Throughout, we assume that sufficient sanctions exist so that verifiable deviations from a contract never occur.

The representative competitive mutual fund itself takes deposits, d, from households and commits to paying households a gross rate of return, R. The mutual fund is competitive in that it treats R as exogenous. Because the representative mutual fund is perfectly diversified, its revenues from deposits, d, are $p(e) R_g^d d + [1 - p(e)] R_b^d d$. The mutual fund must repay

Rd to depositors, so that the profits of the mutual fund are $p(e) R_g^d d + [1 - p(e)] R_b^d d - Rd$. Because mutual funds are competitive, profits must be zero:[22]

$$p(e) R_g^d d + [1 - p(e)] R_b^d d = Rd. \tag{4.6}$$

We assume the banker's only source of funds for repaying the mutual fund is the earnings on its investments. In each state of nature the bankers must earn enough to pay its obligation to the mutual fund in that state of nature:

$$R^g (N + d) - R_g^d d \geq 0, \; R^b (N + d) - R_b^d d \geq 0.$$

In practice, these constraints will either never bind or they will only bind in the bad state of nature. Thus, an additional restriction on the menu of contracts, $\left(d, e, R_g^d, R_b^d\right)$, available to a bank is

$$R^b (N + d) - R_b^d d \geq 0. \tag{4.7}$$

The problem of the banker is to select a contract, $\left(d, e, R_g^d, R_b^d\right)$, from the menu defined by (4.6) and (4.7).

A banker's ex ante reward from a loan contract, is:

$$\lambda \left\{ p(e) \left[R^g (N + d) - R_g^d d \right] + (1 - p(e)) \left[R^b (N + d) - R_b^d d \right] \right\} - \frac{1}{2} e^2, \tag{4.8}$$

where $e^2/2$ is the banker's utility cost of expending effort and λ denotes the marginal value of consumption for the household of the banker. In addition, d denotes the deposits issued by the banker and is distinct from the deposit decision of the banker's household. As part of the terms of the banker's arrangement with its household, the banker is required to seek a contract that maximizes (4.8). Throughout the analysis we assume the banker's household observes all the variables in (4.8) and that the household has the means to compel the banker to do what the household requires of it.

The Lagrangian representation of the banker's problem is

$$\begin{aligned} &\max_{e, d, R_g^d, R_b^d} \lambda \{ p(e) \left[R^g (N + d) - R_g^d d \right] \\ &+ (1 - p(e)) \left[R^b (N + d) - R_b^d d \right] \} - \frac{1}{2} e^2 \\ &+ \mu \left[p(e) R_g^d d + (1 - p(e)) R_b^d d - Rd \right] + \nu \left[R_b^d d - R^b (N + d) \right] \end{aligned} \tag{4.9}$$

where μ is the Lagrange multiplier on (4.6) and $\nu \leq 0$ is the Lagrange multiplier on (4.7).

An interior equilibrium for this economy is:

Observable Effort Equilibrium: c, C, e, d, R, λ, R_g^d, R_b^d such that
(i) the household maximization problem is solved
(ii) mutual funds earn zero profits
(iii) the banker problem, (4.9), is solved
(iv) markets clear
(v) $c, C, d, e > 0$

[22] If instead profits were positive, bankers would set $d = \infty$, but this exceed the resources of households, who make the deposits. If a positive value of d produced negative profits, then profit maximizing bankers would earn zero profits by setting $d = 0$. But, this would be less than the positive amount of deposits supplied by households in the interior equilibria that we study.

We now study the properties of this equilibrium.

The first order conditions associated with the banker problem in equilibrium are:

$$\begin{aligned}
e &: \lambda p'(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right] - e + \mu p'(e)\left(R_g^d - R_b^d\right)d = 0 \\
d &: \lambda\left\{p(e)\left(R^g - R_g^d\right) + (1-p(e))\left(R^b - R_b^d\right)\right\} + \mu\left[p(e)R_g^d + (1-p(e))R_b^d - R\right] + \nu\left(R_b^d - R^b\right) = 0 \\
R_g^d &: -\lambda p(e)d + \mu p(e)d = 0 \\
R_b^d &: -\lambda(1-p(e))d + \mu(1-p(e))d + \nu d = 0 \\
\mu &: p(e)R_g^d d + (1-p(e))R_b^d d = Rd \\
\nu &: \nu\left[R_b^d d - R^b(N+d)\right] = 0,\ \nu \leq 0, R_b^d d - R^b(N+d) \leq 0,
\end{aligned}$$

where "$x:$" indicates the first order condition with respect to the variable, x. Adding the R_g^d and R_b^d equations, we obtain:

$$\mu = \lambda - \nu. \tag{4.10}$$

Substituting (4.10) back into the R_g^d equation, we find

$$\nu = 0,$$

so that the cash constraint is non-binding. Substituting the latter two results back into the system of equations, they reduce to:

$$e : e = \lambda p'(e)\left(R^g - R^b\right)(N+d) \tag{4.11}$$

$$d : R = p(e)R^g + (1-p(e))R^b \tag{4.12}$$

$$\mu : R = p(e)R_g^d + (1-p(e))R_b^d. \tag{4.13}$$

Note from (4.11) that in setting effort, e, the banker looks only at the sum, $N+d$, and not at how this sum breaks down into the component reflecting banker's own resources, N, and the component reflecting the resources, d, supplied by the mutual fund. By committing to care for d as if these were the banker's own funds, the banker is able to obtain better contract terms from the mutual fund. The banker is able to commit to the level of effort in (4.11) because e is observable to the mutual fund and throughout the analysis we assume that all actions which are verifiable are enforceable.

The profits, π, brought home by the bankers in the representative household in period 2 are:

$$\pi = p(e)\left[R^g(N+d) - R_g^d d\right] + (1-p(e))\left[R^b(N+d) - R_b^d d\right].$$

The large household assumption assures that the probability, $p(e)$, corresponds to the fraction of successful bankers and $1-p(e)$ corresponds to the fraction of unsuccessful bankers. Thus, the representative household's second period budget constraint is:

$$\begin{aligned}
C &= Rd + p(e)\left[R^g(N+d) - R_g^d d\right] + (1-p(e))\left[R^b(N+d) - R_b^d d\right] \\
&= R(N+d),
\end{aligned} \tag{4.14}$$

where the second equality uses the zero profit condition of mutual funds. The five equilibrium conditions, (4.11), (4.12), (4.2), (4.3) and (4.14), can be used to determine values for

$$c, C, e, d, R.$$

Also,

$$\lambda = \beta u'(C).$$

The observable effort version of the model does not uniquely determine the values of the state contingent return on deposits, R_g^d, R_b^d. These are restricted only by (4.13) and (4.7). For example, there is an equilibrium in which deposits have the following state contingent pattern: $R_g^d = R^g$, $R_b^d = R^b$. There may also be an equilibrium in which deposit rates are not state contingent, so that $R_g^d = R_b^d = R$. However, for the latter to be an equilibrium requires that N be sufficiently large. The equilibrium values of c, C, e, d, R, λ, are the same across all state contingent returns on deposits that are consistent with (4.13) and (4.7).

4.4. Unobservable Effort

We now suppose that the banker's effort, e, is not observed by the mutual fund. Thus, whatever d, R_g^d, R_b^d is specified by the contract, a banker will in any case choose e to maximize:

$$\lambda \left\{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1 - p(e)) \left[R^b (N+d) - R_b^d d \right] \right\} - \frac{1}{2} e^2.$$

The first order condition necessary for optimality is:

$$e : e = \lambda p'(e) \left[\left(R^g - R^b \right) (N+d) - \left(R_g^d - R_b^d \right) d \right]. \tag{4.15}$$

Note that $R_g^d > R_b^d$ reduces the banker's incentive to exert effort. This is because in this case the banker receives a smaller portion of the marginal increase in expected profits caused by a marginal increase in effort. Understanding that e will be selected according to (4.15), a mutual fund will only offer contracts, $\left(d, e, R_g^d, R_b^d\right)$, that satisfy not just (4.7), but also (4.15).

In light of the previous observations, the Lagrangian representation of the banker's problem is:

$$\begin{aligned} &\max_{\left(e, d, R_g^d, R_b^d\right)} \lambda \left\{ p(e) \left[R^g (N+d) - R_g^d d \right] + (1 - p(e)) \left[R^b (N+d) - R_b^d d \right] \right\} - \frac{1}{2} e^2 \\ &+ \mu \left[p(e) R_g^d d + (1 - p(e)) R_b^d d - R d \right] \\ &+ \eta \left(e - \lambda p'(e) \left[\left(R^g - R^b \right) (N+d) - \left(R_g^d - R_b^d \right) d \right] \right) \\ &+ \nu \left[R_b^d d - R^b (N+d) \right]. \end{aligned} \tag{4.16}$$

where η is the Lagrange multiplier on (4.15).

The equilibrium concept used here is:

Unobservable Effort Equilibrium: c, C, e, d, R, λ, R_g^d, R_b^d such that
(i) the household maximization problem is solved
(ii) mutual funds earn zero profits
(iii) the banker problem, (4.16), is solved
(iv) markets clear
(v) $c, C, d, e > 0$

To understand the properties of this equilibrium, consider the first order necessary conditions associated with the banker problem, (4.16):

$$\begin{aligned}
e &: \lambda p'(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right] - e + \mu p'(e)\left(R_g^d - R_b^d\right)d \\
&\quad +\eta\left(1 - \lambda p''(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right]\right) = 0 \\
d &: 0 = \lambda p(e)\left[R^g - R_g^d\right] + \lambda(1 - p(e))\left[R^b - R_b^d\right] + \mu\left[p(e)R_g^d + (1-p(e))R_b^d - R\right] \\
&\quad -\eta\lambda p'(e)\left[\left(R^g - R^b\right) - \left(R_g^d - R_b^d\right)\right] + \nu\left(R_b^d - R^b\right) \\
R_g^d &: -\lambda p(e) + \mu p(e) + \eta\lambda p'(e) = 0 \\
R_b^d &: -\lambda(1 - p(e)) + \mu(1 - p(e)) - \eta\lambda p'(e) + \nu = 0 \\
\mu &: R = p(e)R_g^d + (1 - p(e))R_b^d \\
\eta &: e = \lambda p'(e)\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right] \\
\nu &: \nu\left[R_b^d d - R^b(N+d)\right] = 0,\ \nu \leq 0, \left[R_b^d d - R^b(N+d)\right] \leq 0.
\end{aligned} \tag{4.17}$$

We refer to these equations - and their counterparts below - by their names to the left of the colon. Add the R_g^d and R_b^d equations, to obtain (4.10). After using (4.10) to substitute out for μ in (4.17), making use of (4.5) and rearranging:[23]

$$\begin{aligned}
e &: (\lambda - \nu)b\left(R_g^d - R_b^d\right)d + \eta = 0 \\
d &: R = p(e)R^g + (1 - p(e))R^b \\
R_g^d &: \nu p(e) = \eta\lambda b \\
\mu &: R = p(e)R_g^d + (1 - p(e))R_b^d \\
\eta &: e = \lambda b\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right)d\right] \\
\nu &: \nu\left[R_b^d d - R^b(N+d)\right] = 0,\ \nu \leq 0, \left[R_b^d d - R^b(N+d)\right] \leq 0.
\end{aligned} \tag{4.18}$$

We distinguish two cases. Equilibrium in 'normal times' corresponds to the case when N is sufficiently large that the cash constraint is nonbinding, so that $\nu = 0$. Equilibrium in 'crisis times' corresponds to the case when $\nu < 0$.

We first consider the properties of equilibrium in normal times. Substituting $\nu = 0$ into the R_g^d equation, we deduce that in an interior equilibrium with $d, \lambda > 0$, the multiplier on the incentive constraint, η, is zero. With $\eta = 0$ and the fact, $\lambda - \nu > 0$, the e and μ equations imply:

$$R_g^d = R_b^d = R. \tag{4.19}$$

It then follows from the η equation that:

$$e : e = \lambda b\left(R^g - R^b\right)(N+d). \tag{4.20}$$

[23] The d equation in (4.18) is a simplified version of the d equation in (4.17), obtained as follows. Substitute from (4.10) and the R_g^d and μ equations in (4.17) into the d equation in (4.17) yields, after some algebra:

$$0 = (\lambda - \nu)\left[p(e)R^g + (1 - p(e))R^b - R\right].$$

The result follows from the observation that $(\lambda - \nu)$ is strictly positive since $\nu \leq 0$ and $\lambda > 0$ in an interior equilibrium. The e equation in (4.18) is a simplified version of the e equation in (4.17), after making use of (4.10) and the ν equation in (4.17).

Equations (4.20) and the μ equation in (4.18), together with the three household equilibrium conditions, (4.2), (4.3) and (4.14), represent 5 conditions. These conditions can be used to determine the following 5 variables:

$$c, C, e, d, R.$$

A notable feature of the equilibrium in normal times is that the incentive constraint, (4.15), is non-binding and the allocations are efficient in the sense that they coincide with the allocations in the version of the model in which effort is observable. This may at first seem surprising, since effort is now not observable and d belongs to the mutual fund, not the banker. The level of effort exerted by the banker in the unobservable effort equilibrium coincides with what it is in the observable effort equilibrium because the loan contract transfers the full marginal product of effort to the banker. This is accomplished by making the rate of interest on banker deposits not state contingent (see (4.19)). The interest rate spread in this equilibrium (see (4.1)) is zero in normal times. We state these results as a proposition:

Proposition 4.1. *When the cash constraint, (4.7), does not bind (i.e., $\nu = 0$), then the allocations in the unobserved effort equilibrium coincide with those in the observed effort equilibrium and the interest rate spread is zero.*

We now turn to the case in which the cash constraint is binding, so that $\nu < 0$ and

$$\nu : R_b^d d = R^b (N + d).$$

In this case, the observed and unobserved effort equilibria diverge, since the cash constraint never binds in the observed effort equilibrium. The R_g^d equation in (4.18) implies $\eta < 0$, so that according to the e equation in (4.18),

$$R_g^d > R_b^d \tag{4.21}$$

in an interior equilibrium with $d > 0$. It then follows from the η equation in (4.18) that

$$e < \lambda b \left(R^g - R^b\right) (N + d).$$

That is, the banker in a crisis equilibrium exerts less effort, for given $N + d$, than it does in the observed effort equilibrium. The reason is that with (4.21), the banker does not capture the full marginal return from effort. With reduced effort, equation d in (4.18) shows that equilibrium R is smaller. Given (4.4), the household equilibrium conditions, (4.2), (4.3) and (4.14), imply a lower d, reinforcing the low e. We summarize these findings in the following proposition:

Proposition 4.2. *When the cash constraint, (4.7), binds (i.e., $\nu < 0$), then d and R in the unobserved effort equilibrium are lower than they are in the observed effort equilibrium, and the interest rate spread is positive.*

4.5. Implications for Policy

In this section, we consider the impact of government deposits and equity injections into banks, and show that thse are not helpful in crisis times. We then show that bank deposit rate subsidies and transfers of net worth to banks can solve the crisis completely by eliminating the interest rate spread and moving allocations to their efficient levels.

4.5.1. Government Deposits into Banks

Consider the case where the government raises taxes, T, and deposits the proceeds in the mutual fund. The household's period 1 budget constraint is given by:

$$c + \tilde{d} = y, \tag{4.22}$$

where $\tilde{d}$ denotes $d+T$, and d denotes deposits placed by households in the mutual fund. The intertemporal condition, (4.3) is unaffected by the change. The household's second period budget constraint is unaffected by the change, except that d is replaced by $\tilde{d}$. Similarly, the equilibrium conditions associated with the banker problem, (4.16), are unchanged, with the exception that d is replaced by $\tilde{d}$. In particular, if the government deposits taxpayer money into the mutual funds, taxpayers will reduce their deposits by the same amount and there will be no change. From the point of view of households in the economy, it is the same whether deposits are held in their capacity as taxpayers or directly in their own name. What fraction of deposits is allocated to them as taxpayers and what fraction is allocated in their own name is irrelevant to households.[24]

4.5.2. Equity Injections into Banks

In this section we adopt the same interpretation of equity injections as in section 3.2.1. That is, the government raises taxes, T, and hands these over to the banks. The government then requires that the banks repay the earnings they actually make on these funds. Under this policy, the expected profits of the bank are

$$p(e)\left[R^g(N+T+d) - R_g^d d - R^g T\right] + (1-p(e))\left[R^b(N+T+d) - R_b^d d - R^b T\right].$$

Note that taxes enter revenues symmetrically with deposits and the bank's own net worth. On the cost side, equity injections require that banks pay the government the actual rate of return on projects. Thus, equity injections have no direct impact on bank profits, because they enter revenues and costs in exactly the same way. For the same reason, equity injections also do not change the banker's cash requirement in the bad state. That is, the bank requirement that revenues be no smaller than costs is, in the presence of equity injections,

$$R^b(N+d+T) \geq R_b^d d + R^b T,$$

so that T cancels from both sides and thus coincides with (4.7). We conclude that the expression of the banker's problem, (4.16) is completely unaltered by the presence of equity injections.

[24] Of course, this requires that households not be at a corner of zero with d and that the interest rate received by the government coincide with the one they receive themselves.

Now consider the household problem. The period 1 budget constraint is:

$$c + d \leq y - T, \tag{4.23a}$$

reflecting that equity injections, T, are financed with taxes on households. The government transfers the revenues from equity injections back to households in period 2. In this way, the period 2 household budget constraint is:

$$\begin{aligned} C &= Rd + p(e)\left[R^g(N+d) - R_g^d d\right] + (1-p(e))\left[R^b(N+d) - R_b^d d\right] \\ &\quad + \left[p(e)R^g + (1-p(e))R^b\right]T \\ &= R(N+d+T). \end{aligned} \tag{4.24}$$

The last terms on the right reflects that the government's distribution of equity among banks is completely diversified. The intertemporal Euler equation, (4.3), is unchanged.

From the household problem we see that an increase in T induces an equal reduction in d, for a given value of R. Note that although T does not enter the banker's problem, d does. Thus, it is likely that T has an indirect effect.

Consider first the case when the cash constraint in the bad state is not binding, $\nu = 0$. In this case, the problem solved by the banker's contract is given by (4.16) with $\nu = 0$, so that (4.19) and (4.20) are satisfied. In this case, increased equity injections, for a given interest rate, R, reduce deposits and so reduce the banker's incentives to exert effort, e (see (4.20)). This in turn can be expected to produce a fall in R, so that d falls some more. Thus, $d + T$ falls with a tax-financed equity injection in normal times when the cash constraint is not binding. The intuition for this result is described in section 4.1. We summarize this result in the form of a proposition:

Proposition 4.3. *If the cash constraint, (4.7), is not binding, then an equity injection produces a fall total intermediation, $d + T$.*

In a crisis time when the cash constraint in the bad state is binding, the fall in $d+T$ that occurs with an equity injection is offset by a second effect. The two cancel, and so equity injections are irrelevant in a crisis, according to this model. The second effect occurs because a fall in deposits, d, loosens the cash constraint, (4.7), in the bad state. This relaxation of (4.7) requires an increase in the rate of return on deposits for banks in the bad state. The reduction in the state contingency of deposit rates enhances bankers' incentives to exert effort. As a result, the fraction of good projects that are identified is increased, so that the risk free rate rises, leading to a rise in deposits. Formally,

Proposition 4.4. *If the cash constraint is binding, then an equity injection has no impact on consumption, c, C or the interest rate, R, or the volume of intermediation, $d + T$.*

See the appendix for a proof of this proposition.

4.5.3. Interest Rate Subsidies and Net Worth Transfers to Banks

As we have emphasized, the heart of the problem in crisis times is that banks with bad investments do not have enough resources to cover their losses when bank deposit rates are state non-contingent. Equilibrium in this case requires that deposit rates covary positively with the return on the bank portfolio. This is where the problem lies. State non-contingency in the banks' deposit rate is crucial if they are to have enough incentive to exert the efficient level of effort. This reasoning suggests two policies that can solve the problem. First, by reducing the costs of their deposits, a tax-financed subsidy on banks' cost of funds makes it possible for banks to cover their losses in bad states and for bank deposit rates to be state non-contingent. Second, a tax-financed transfer of equity to banks also allows them to cover their losses in bad states with state non-contingent deposit rates.

Consider first case of interest rate subsidies. Suppose we have the allocations and returns in the observable effort equilibrium. The assumption that we are in a crisis time implies that if $R = R_b^d = R_g^d$, where R solves (4.12), then the cash constraint, (4.7), is violated:

$$Rd > R^b (N + d). \tag{4.25}$$

Let τ solve

$$(1 - \tau) Rd = R^b (N + d). \tag{4.26}$$

A value of $\tau > 0$ is guaranteed to exist because the left side of this expression is monotonically decreasing in τ and it is zero when $\tau = 1$. All the equilibrium conditions associated with the banker problem (see (4.18)) are satisfied, with $\nu = 0$. As a result, the banker exerts the level of effort that occurs in the observed effort equilibrium (see (4.20)). The key thing is that state non-contingency of deposit rates cause the banker to exert effort as though the deposits belonged to the banker. The fact that the level of deposit rates is lower across the realized outcomes of its investment is irrelevant to the effort exerted by the banker.

It remains only to verify that the household decisions in the observable effort equilibrium also solve their problem in the unobservable effort equilibrium with an interest rate subsidy. The households' period 1 budget constraint, (4.2), is unaffected. The household's intertemporal Euler equation, (4.3), is also not affected. The only household equilibrium condition that requires attention is its second period budget constraint, because the tax subsidy to banks is financed by period 2 taxes, $T = \tau Rd$,

$$C = Rd + \pi - T.$$

Bank profits, π, are higher under the interest rate subsidy than they are in the observable effort equilibrium. However, they are higher by exactly T. So, the assumption that the period 2 household budget constraint is satisfied in an observable effort equilibrium implies that the allocations in that equilibrium also satisfy the above budget constraint with taxes. We summarize these findings as follows:

Proposition 4.5. *Suppose the cash constraint in an unobservable effort equilibrium is binding. The interest rate subsidy, (4.26), financed by a period 2 tax on households causes the allocations in the unobservable effort equilibrium to coincide with those in the observable effort equilibrium.*

We now turn to tax financed transfers of net worth to banks. Suppose again that the cash constraint is binding in the unobservable effort equilibrium. The government raises taxes, T, in period 1 and transfers the proceeds to banks. If the transfer is sufficiently large, then the cash constraint in the unobservable effort equilibrium ceases to bind. To establish this result, suppose we have the allocations in the observable effort equilibrium in hand. The assumption that we are in a crisis time implies that if $R = R_b^d = R_g^d$, where R solves (4.12), then the cash constraint, (4.7), is violated, as in (4.25).

We first consider the response of the observable effort equilibrium to $T > 0$. Banks' pretax net worth is N and after taxes their net worth is $N + T$. We conjecture, and then verify, that with $T > 0$, deposits decline one-for-one in the observable effort equilibrium and period 1 and period 2 consumption allocations do not change. Suppose that T satisfies:

$$R\left(d - T\right) = R^b\left(N + d\right). \tag{4.27}$$

The value of T that satisfies this equation exists and is unique because the left side is monotonically decreasing and continuous in T and it is zero when $T = d$. According to (4.27) the cash constraint is (marginally) non-binding. It is easily verified that the household period 1 budget constraint and Euler equations in the observable effort equilibrium are satisfied (see (4.2) and (4.3)). It is also easily verified that households' second period income is invariant to T.[25] Finally, the bank equilibrium conditions, (4.11), (4.12), (4.13), easily seen to be satisfied. We conclude that we have an observable effort equilibrium. Because in addition the cash constraint is satisfied, it follows that we have an unobserved effort equilibrium too. We summarize our finding as follows:

Proposition 4.6. *Suppose the cash constraint in an unobservable effort equilibrium is binding. The net worth subsidy, (4.27), financed by a period 1 tax on households causes the allocations in the unobservable effort equilibrium to coincide with those in the observable effort equilibrium.*

5. Adverse Selection

In this section, we consider credit market frictions that occur when there is a lemons problem such as the one emphasized in Akerlof (1970). There is a group of entrepreneurs with access to investment projects. Because their own net worth is not sufficient to finance their investment project, they must rely on external finance from banks. Some of the entrepreneurs are relatively safe and others are riskier, but the bank cannot differentiate between them. To compensate for losses from the riskier entrepreneurs the interest rate spread - the difference between the rate charged by the bank to borrowers and the cost of funds to the bank - must be positive. The price distortions associated with the interest rate spread have the consequence that intermediation and investment are below their efficient levels. A drop in entrepreneurial net worth aggravates the distortions because entrepreneurs become more dependent on external finance.

We insert the entrepreneurs and bankers into the type of general equilibrium environment considered in the other sections of this paper. When entrepreneurial net worth falls and R

[25] This requires performing substitutions similar to those in (4.14).

falls, interest rate spreads jump and the supply of saving from the rest of the economy drops and investment drops. In this way, the environment rationalizes the type of observations that motivate this paper, about the behavior of interest rates spreads and intermediation in the recent market turmoil.

Consistent with the analysis of Mankiw (1986) and Bernanke and Gertler (1990), who consider a similar environment, we find that a subsidy to banks' cost of funds can ameliorate the problem.[26] Indeed, a suitable choice of the interest rate subsidy can make the market allocations coincide with the efficient allocations. This is so, even though the subsidy policy does not require observing borrowers' individual risks while our efficient allocations are defined for a benevolent planner who does observe those risks. A subsidy to entrepreneurs, by reducing their dependence on external finance, can also improve allocations.

5.1. The Model

We adopt the same basic environment as in the rest of the paper. The economy is populated by many large and identical households. The representative household has a unit measure of members composed of workers, bankers and entrepreneurs. The measure of entrepreneurs is $e < 1$. All these individuals receive perfect consumption insurance from households. Workers and entrepreneurs receive endowments of y and N, respectively in the first period. Here, $y > 0$ is measured in household per capita terms. We find it convenient to measure $N < 1$ in entrepreneur per capita terms. Thus, in household per capita terms the quantity of entrepreneurial net worth is eN. The equilibrium conditions associated with the household are as in the other parts of this paper, but are reproduced here for convenience:

$$c + d = y \tag{5.1}$$

$$c^{-\gamma} = \beta R C^{-\gamma}, \ \gamma > 0 \tag{5.2}$$

$$C \leq Rd + e\pi. \tag{5.3}$$

Here, c denotes first period household consumption, d denotes deposits, and C denotes second period consumption. These three variables are measured in household per capita terms. The object, π, denotes earnings, in entrepreneur per capita terms, brought home in period 2 by entrepreneurs. Finally, R denotes the gross rate of interest on deposits. The Euler equation in (5.2) indicates that the household's deposit decision is unchanged from what it was in previous sections. We now discuss the problems of the entrepreneurs and the bankers.[27]

Each entrepreneur can take one of two options. It can deposit its net worth in a bank and earn RN. Alternatively, it can undertake a particular investment project. The nature of the investment project available to an entrepreneur is determined by the values of θ and p that the entrepreneur draws from a joint cdf, $F(\theta, p)$. An entrepreneur's realized values of $\theta > 0$ and $p \in [0, 1]$ are known only to the entrepreneur and to the household from which it came. The distribution, F, is known to all. All projects are indivisible and require an investment of one good in period 1. In period 2, θ goods are produced with probability p

[26] Our model is most closely related to the one in Mankiw (1986).

[27] Our model is an adaptation on the model in Mankiw (1986), especially the example on page 463.

and zero goods are produced with probability $1-p$. The analysis is greatly simplified by a particular assumption we place on the joint distribution, F :

$$\theta p = \bar{\theta}.$$

Here, $\bar{\theta}$ is a non-random parameter known to all. Thus, each entrepreneur's investment project generates the same expected return, but differs in terms of riskiness. This restriction captures the spirit of the environment sketched in the introduction to this section, and at the same time allows us to obtain analytic results. With our restriction, we can characterize F by specifying a distribution for p and then setting $\theta = \bar{\theta}/p$. We assume that p is drawn from a uniform distribution with support, $[0,1]$.

Because $N < 1$, an entrepreneur must obtain a bank loan if it is to operate his investment technology. After the entrepreneur and banker leave the household, their location is randomized. Because there is a continuum of households, the probability that an entrepreneur meets a banker from its own household is zero. So, the fact that an entrepreneur's own household knows everything about the entrepreneur's investment project implies nothing for what the entrepreneur's banker knows. We suppose that the entrepreneur's banker can only observe whether the entrepreneur's project succeeds or fails. In case the project succeeds, the bank cannot tell ex post what that project's value of θ was. As a result, the interest payment by the entrepreneur to its bank can only be contingent on whether the project succeeds or fails. We denote the interest rate paid by the entrepreneur in the event that the project succeeds by r. Because the entrepreneur has no resources in the event that the project fails, the interest rate in that event must be zero.

Entrepreneurs which choose not to operate their project return RN to their household. Among the active entrepreneurs, p bring home $\theta - r(1-N)$ and $1-p$ bring home nothing. When choosing whether or not to activate its project, the entrepreneur behaves in the best interest of its household by maximizing the expected value of what it brings home. The entrepreneur does this in exchange for perfect consumption insurance from the household. The household is assumed to have the means to enforce correct behavior on the part of its entrepreneurs because of our assumption that an entrepreneur's own household observes the values of θ and p of each of its member entrepreneurs. Thus, the entrepreneur invests its net worth, N, in its project and borrows $1-N$ if and only if its realized value of p satisfies:

$$\bar{\theta} - pr(1-N) \geq RN.$$

The interval of p implicit in the previous expression can be written as follows:

$$0 \leq p \leq \bar{p}(r), \ \bar{p}(r) = \frac{\bar{\theta} - RN}{r(1-N)}. \tag{5.4}$$

The object, $\bar{p}(r)$, summarizes several interesting features of the equilibrium. Under our distributional assumption for p, $\bar{p}(r)$ corresponds to the fraction of entrepreneurs that invest:

$$\int_0^{\bar{p}(r)} dp = \bar{p}(r). \tag{5.5}$$

The object, $\bar{p}(r)$ also corresponds to the total quantity of entrepreneurial investment, in entrepreneurial per capita terms. The average value of p among the entrepreneurs that

invest is denoted $\Pi(r)$, where

$$\Pi(r) = \frac{\int_0^{\bar{p}(r)} p dp}{\bar{p}(r)} = \frac{1}{2}\bar{p}(r). \tag{5.6}$$

Finally, we show below that $\bar{p}(r)$ determines the interest rate spread, the difference between the interest rate, r, paid by entrepreneurs succeed and the risk free rate, R. We restrict our attention to model parameterizations that imply the efficient allocations (see the next subsection) and the equilibrium allocations are interior. This means the usual non-negativity constraints on quantities and also, $0 < \bar{p}(r) < 1$.

We now turn to the bankers, which we assume are competitive. Because each bank is fully diversified across entrepreneurs, its revenues are non-stochastic. That banks are competitive implies their profits must be zero. A bank's average earnings on each unit loan made is $\Pi(r) r$. The cost of a unit of deposits for a bank is R, so that the each bank's zero profit condition is:

$$\Pi(r) r = R. \tag{5.7}$$

Much of the economics of the model are summarized in (5.7). For example, multiplying (5.6) by r and using (5.7), we obtain an expression for the interest rate spread:

$$\text{'interest rate spread'} = \frac{r}{R} = \frac{2}{\bar{p}(r)}. \tag{5.8}$$

According to this expression, the interest rate spread is at least 2, and can be much higher. The reason is simple. Suppose all entrepreneurs borrowed, so that $\bar{p} = 1$. In this case, the average probability of success is 1/2 (see (5.6)). With half the entrepreneurs bankrupt and unable to pay, the ones that do pay must pay $2R$ if the bank is to be able to pay R to its depositors.[28]

Interestingly, (5.7) completely determines the rate of interest, R, in the model. To see this, substitute (5.4) into (5.6), to obtain:

$$\Pi(r) r = \frac{1}{2}\frac{\bar{\theta} - RN}{1 - N}.$$

According to this expression, a bank's revenues from a loan are independent of the interest rate on the loan. We now summarize the intuition for this result. A higher r means that banks earn more revenues from entrepreneurs who borrow and repay their loan. However, this positive impact on revenues is canceled by an adverse selection effect. When a bank raises r, entrepreneurs with a high probability of repaying their loan decide to become inactive. As a result, the average probability that an entrepreneur repays the loan falls (see (5.4) and (5.5)). In principle, this is not necessarily a problem because the lower probability entrepreneurs also enjoy a better outcome when they are successful. However, this is little comfort to a bank that must charge a state non-contingent interest rate, r. A fixed interest rate on loans prevents the bank from sharing in the huge payoffs experienced by low p entrepreneurs when they are successful. This is why a bank's revenues are independent of r.

[28] Clearly, an empirically plausible version of our model would require a density function for p that places greater mass on higher p.

Adverse selection also explains why the revenue function, $\Pi(r)\,r$, is decreasing in R. As R increases, high p entrepreneurs switch to being inactive and this reduces the average p among borrowers, reducing bank revenues per unit of loan extended.

These properties of $\Pi(r)\,r$ imply that the zero profit condition of entrepreneurs completely determines the equilibrium value of R. That is, R is determined exclusively by variables specific to the loan market and not by, for example, households' intertemporal preferences. In our model, the quantity of saving by households must adjust passively to the R that is set by (5.7).

Clearing in the loan market requires that the quantity of investment, $e\bar{p}(r)$, equals the quantity of household deposits, d, plus the quantity of net worth, eN, in the hands of entrepreneurs:

$$e\bar{p}(r) = d + eN. \tag{5.9}$$

We now obtain a simplified expression for period 2 household income. Averaging entrepreneurial earnings over all entrepreneurs, we obtain:

$$\pi = \int_0^{\bar{p}(r)} \left\{\bar{\theta} - pr(1-N)\right\} dp + \int_{\bar{p}(r)}^1 NR = \bar{p}(r)\left[\bar{\theta} - \Pi(r)\,r(1-N)\right] + (1-\bar{p}(r))\,NR. \tag{5.10}$$

Adding $e\pi$ to household earnings on deposits yields the equilibrium expression for total household income in the second period:

$$Rd + e\bar{p}(r)\left[\bar{\theta} - \Pi(r)\,r(1-N)\right] + e(1-\bar{p}(r))\,NR = e\bar{p}(r)\,\bar{\theta}.$$

The expression after the equality is obtained after substituting out for R and d using (5.7) and (5.9). The object, $e\bar{p}(r)\,\bar{\theta}$, represents the total period 2 output from investment projects, in household per capita terms. Replacing total household income with its equilibrium value of $e\bar{p}(r)\,\bar{\theta}$ and evaluating (5.3) at equality, we obtain the household's second period budget constraint in equilibrium:

$$C = e\bar{p}(r)\,\bar{\theta}. \tag{5.11}$$

Consistent with Walras' law, (5.11) is also the second period resource constraint.

We have the following definition of equilibrium:

> **Adverse Selection Equilibrium**: c, C, d, r, R, π, $\bar{p}(r)$ such that
> (i) c, C, d solve the household problem given R, π
> (ii) banks earn zero profits
> (iii) entrepreneurs maximize expected revenues

Equilibrium is straightforward to compute for this economy. The five equilibrium conditions, (5.1), (5.2), (5.7), (5.9), and (5.11), as well as the definitions of π and $\bar{p}(r)$ in (5.10) and (5.4), respectively, are sufficient to determine the seven equilibrium variables. Evaluate (5.7) using (5.4) and the definition in (5.6) and solve the resulting expression for R:

$$R = \frac{\bar{\theta}}{2-N}. \tag{5.12}$$

Combine (5.11) and (5.2) and use (5.4):

$$c = (\beta R)^{-\frac{1}{\gamma}} \frac{\bar{\theta} - RN}{r(1-N)} \bar{\theta} e. \tag{5.13}$$

Use the latter expression and (5.9) to substitute out for c and d in (5.1). Solving the resulting expression for r, and making use of (5.12), we obtain:

$$r = 2eR \frac{(\beta R)^{\frac{-1}{\gamma}} \bar{\theta} + 1}{y + eN}. \tag{5.14}$$

In this way, all the equilibrium variables can be computed uniquely as long as the model parameters are such that an interior equilibrium - $\bar{p}(r) < 1$ and $c, C > 0$ - exists.[29] We summarize these results as follows:

Proposition 5.1. *When an interior adverse selection equilibrium exists, it is unique and is characterized by (5.12), (5.13) and (5.14).*

5.2. Implications for Policy

We begin by discussing the planner problem for this economy. We then show that two types of subsidy schemes improve the equilibrium allocations: a tax-financed transfer of net worth to entrepreneurs and a tax subsidy to banks. Tax financed government deposits with the banks have no effect because they do not affect the bank interest rate on deposits. Households respond to the increase in taxes by reducing their deposits one-for-one with the increase in taxes and government deposits.

5.2.1. Efficient Allocations

We consider the allocations selected by a benevolent planner who observes an entrepreneur's p, though not the outcome of its project. We use these allocations as a benchmark from which to evaluate the adverse selection equilibrium and various policy interventions studied in the next section.

The planner faces the period 1 resource constraint,

$$c + d \leq y. \tag{5.15}$$

To describe the planner's decisions about which and how many projects to activate and to derive the planner's period 2 resource constraint, we find it useful to describe the model environment using a particular figure.

Figure 2 arranges all the agents in the economy in the unit square. Each point in the square corresponds to a particular household (vertical dimension) and member of household

[29] A sufficient condition is that, in addition to $N < 1$, the parameters satisfy

$$(y + eN)(\bar{\theta}\beta)^{\frac{1}{\gamma}} \leq e\left[(\bar{\theta}\beta)^{\frac{1}{\gamma}} + \bar{\theta}\right].$$

(horizontal dimension). There is a unit measure of households and a unit measure of members of any given household. We suppose that the box is constructed in period 1, just after each entrepreneur has drawn its value of p. A horizontal line inside the box highlights one particular household. The points on the line to the left of e correspond to the entrepreneurs. The points to the right of e correspond to the workers and bankers. The entrepreneurs are ordered according to their value of p, from $p = 0$ to $p = 1$ passing from left to right. For any particular $p \in [0, 1]$, the entrepreneur with that investment project is indicated by the point, pe, on the horizontal axis. Each point on a vertical line through pe corresponds to the entrepreneurs with the given p in the cross section of households. The θ's that all these entrepreneurs draw correspond to the population of θ's for the given p. The mean of that population is $\bar{\theta}$,independent of p.

The planner must decide how many entrepreneurs in the interval, 0 to e, to activate. If the planner elects to activate an entrepreneur with a particular p, it instructs all the entrepreneurs in the cross section of households with that p to activate their project. The planner does this because its decision is made after observing p, but before observing anyone's realized value of θ. The planner is indifferent about which projects to activate. Each project is the same to the planner because each has the same mean productivity, $\bar{\theta}$, and entrepreneurs suffer no cost to activate their project. As a result, there is no loss of generality in simply assuming that the planner selects entrepreneurs with p's extending from $p = 0$ to $p = \bar{p}$, for some $\bar{p} \leq 1$. This corresponds to the mass of entrepreneurs in the interval, 0 to $e\bar{p}$, in the figure.

Consider a mass of entrepreneurs on an arbitrary interval, Δ, inside $[0, e]$. The resource cost of activating these entrepreneurs in the cross section of households is the area of the rectangle with base Δ inside the unit square. The latter area is just Δ itself. This reflects the assumption that there is a unit mass of households and that each project costs one unit of resources to activate. The available net worth, N per entrepreneur, is sufficient to operate the entrepreneurs corresponding to the interval, 0 to eN. Since these resources have no alternative use, the planner applies them. Activating additional entrepreneurs is costly to the planner because this requires suppressing consumption in period 1. Suppose the planner considers activating an additional mass, d, of entrepreneurs. This corresponds to the entrepreneurs extending from the point, eN to the point, $eN+d$ in the figure. Activating these entrepreneurs requires d resources. So, if the planner wishes to activate a measure, $e\bar{p}$, of entrepreneurs, then $d + eN$ resources are needed, subject to:

$$e\bar{p} \leq d + eN. \tag{5.16}$$

When the planner activates entrepreneurs from 0 to $e\bar{p}$, the total amount of goods available in period 2 is $e\bar{p}\bar{\theta}$. Thus, the second period resource constraint for the planner is:

$$C \leq e\bar{p}\bar{\theta}. \tag{5.17}$$

The planner's problem is to solve:

$$\max_{c,C,\bar{p},d} u(c) + \beta u(C),$$

subject to $0 \leq \bar{p} \leq 1$, (5.15), (5.16) and (5.17) and $c, C \geq 0$. The unique interior solution is

characterized by the first order conditions evaluated at equality. Solving these, we obtain:

$$\begin{aligned} c &= \frac{y+eN}{\left(\beta\bar{\theta}\right)^{\frac{1}{\gamma}}+\bar{\theta}}\bar{\theta}, \\ e\bar{p} &= \frac{y+eN}{\left(\bar{\theta}\beta\right)^{\frac{1}{\gamma}}+\bar{\theta}}\left(\bar{\theta}\beta\right)^{\frac{1}{\gamma}}, \\ C &= c\left(\beta\bar{\theta}\right)^{\frac{1}{\gamma}}, \end{aligned} \tag{5.18}$$

with d given by solving (5.15) with equality. It is convenient to compare these allocations with the allocations in the adverse selection equilibrium. Substituting (5.14) into (5.13) and using (5.12), we obtain that first period consumption in the equilibrium is:

$$c = \frac{y+eN}{(\beta R)^{\frac{1}{\gamma}}+\bar{\theta}}\bar{\theta}.$$

Using (5.4) and making use of (5.12) and (5.14), we find that total resource use in the adverse selection equilibrium is:

$$e\bar{p}(r) = \frac{y+eN}{(\beta R)^{\frac{1}{\gamma}}+\bar{\theta}}(\beta R)^{\frac{1}{\gamma}}$$

According to (5.2), second period consumption in equilibrium is:

$$C = c(\beta R)^{\frac{1}{\gamma}}.$$

Evidently, the sole factor preventing the equilibrium from replicating the planner's allocations is that the interest rate, R, is too low. In the adverse selection equilibrium R is $\bar{\theta}/(2-N)$,but the actual rate of return on investment is $\bar{\theta}$. With the market sending the wrong signal to households about the return on investment, saving and investment are too low. The problem is aggravated if N falls.

5.2.2. Interest Rate Subsidies

In the previous subsection, we showed that the problem with the adverse selection equilibrium is that the deposit rate, R, is too low. This is because banks do not recover the returns that their loans make possible. This suggests two direct ways to repair the market mechanism: subsidize bank earnings or, equivalently, their cost of funds. We consider the latter here. We show that an appropriate interest rate subsidy can make the allocations in the adverse selection equilibrium coincide with the efficient allocations. Significantly, implementation of this policy does not require that the government observe any characteristics of the banks' borrowers. This stands in contrast with the planner in the previous subsection, which was assumed to observe each entrepreneur's p.

Denoting the pre-tax cost of funds to the bank by R, the after subsidy cost under our policy is $R/(1+\nu) < R$. We suppose that this subsidy is financed by a lump sum tax on households in the period 2, in the amount

$$T = (1-N)\,e\bar{p}(r)\left[R - \frac{R}{1+\nu}\right].$$

Here, the terms in front of the square bracket represent the total amount of loans made by the banks to the active entrepreneurs. The banks fund these loans with deposits taken from inactive entrepreneurs and households. The amount of the subsidy is $R - R/(1+\nu)$ per unit of loans made. The household's second period budget constraint, (5.3), is replaced by

$$C \leq Rd + e\pi - T.$$

Repeating the substitutions leading up to equation (5.11), taking account of the modified second period budget constraint of the household, we find that (5.11) continues to hold.

The impact of the tax subsidy on the equilibrium value of R is determined by studying the appropriately modified bank zero profit condition, (5.7):

$$\Pi(r)\, r = \frac{R}{1+\nu}. \tag{5.19}$$

Substituting (5.4) and (5.6) into the latter expression and solving for R :

$$R = \frac{\bar{\theta}(1+\nu)}{2 - N + N\nu}.$$

Evidently, to achieve $R = \bar{\theta}$ requires $\nu = 1$.

Recall that the seven conditions determining $c, C, d, r, R, \pi, \bar{p}(r)$ are (5.1), (5.2), (5.7), (5.9), (5.11), (5.10) and (5.4). We have verified that (5.11) continues to hold. Apart from (5.7), the other conditions are obviously unaffected by T. The only equilibrium condition that must be adjusted is the bank zero profit condition, (5.7), which we replace by (5.19). With $\nu = 1$, the interest rate that solves (5.19) is the efficient one, $R = \bar{\theta}$. Given that the other equations are unaffected, it follows from the discussion in the previous subsection that the efficient allocations are supported by the subsidy policy. We summarize this result in a proposition:

Proposition 5.2. *With an interest cost subsidy, $\nu = 1$, the allocations in the adverse selection equilibrium are efficient.*

5.2.3. Tax Financed Transfers to Entrepreneurs

Here, we consider a policy of raising lump sum taxes, T, on households in period 1 and transferring T/e to each entrepreneur. By setting

$$T = e(1 - N), \tag{5.20}$$

the transfer ensures that each entrepreneur has enough funds to fully finance its investment project. With this policy the banking system is circumvented and so there are no frictions. A problem with this policy is that it presses hard on a feature of the model that we have little confidence in. In particular, we assume that it is known how much net worth each entrepreneur has and how much they need for their investment project. In practice, these assumptions are not satisfied.

We now establish that the above tax-transfer scheme accomplishes what we claim it does. The value of R that satisfies the bank zero profit condition, (5.7), is still the one in (5.12),

except that N is replaced by the post-transfer level of entrepreneurial net worth. Since that is unity under the tax-transfer scheme, we have that $R = \bar{\theta}$, its value in the efficient equilibrium. It is also straightforward to verify that c, C and $\bar{p}$ satisfy (5.18). We have assumed that model parameters imply $\bar{p} \leq 1$. When $\bar{p} < 1$, then $d < 0$. That is, in this case some of the net worth transferred to entrepreneurs is recycled back to households through the loan market.

We summarize these results as follows:

Proposition 5.3. *Under the tax-transfer scheme in (5.20), the allocations in the adverse selection equilibrium are efficient.*

6. Asymmetric Information and Monitoring Costs

The following setup describes the simplest possible representation of an asymmetric information model along the lines proposed in BGG. In this model, the financial friction is not in the banks, as in the previous model. Instead, the financial friction reflects features of the problems of entrepreneurs. In terms of recent data, we can perhaps think of the model as pertaining to home buyers whose net worth has declined or the firms whose bond spreads rose so much during the recent financial market turmoil (see Figure 1). In principle, the model could also be interpreted as applying to actual banks. Real world banks resemble entrepreneurs in the BGG model. The entrepreneurs in the model have their own net worth, they accept loans (i.e., they 'take deposits') and they acquire assets.

It is natural to suppose that the model provides a rationalization for government purchases of the assets of non-financial institutions. This is because the BGG model appears to possess the pecuniary externality studied by Korinek (2010) and Lorenzoni (2008). In the BGG model, entrepreneurs are limited in their ability to acquire loans by a collateral constraint, where collateral corresponds to the market value of the capital that they own. The pecuniary externality idea suggests that if the central bank makes loans directly to a subset of entrepreneurs, the additional purchases of capital that these loans support will drive up the price of capital, thus loosening the collateral constraint of all the other entrepreneurs. The notion is that this will then allow economic activity to expand and move closer to the efficient level. We prove that tax-financed loans by the government to entrepreneurs in fact have no effect in the two-period version of the BGG model, contradicting the intuition just described. However, we assume that the government has no particular advantage over the private sector in handling the information problems in the BGG environment. Also, we show that a subsidy to banks' cost of funds helps, but only a quantitatively small amount. Finally, perhaps not so surprisingly, we show that a sufficiently large tax-financed transfer of net worth to entpreneurs allows the economy to support the first-best equilibrium outcomes.

We now describe the model. There are two periods. A large number of workers and entrepreneurs live in a representative family. Workers have an exogenous endowment, y, of income in the first period, while entrepreneurs possess k units of capital. Households allocate y to first period household consumption and to bank deposits. Entrepreneurs sell their capital to capital producers. The proceeds of this sale to capital producers constitutes entrepreneurs' net worth. Banks give a optimal standard debt contract to the entrepreneurs, who combine their net worth and loans to purchase new capital from capital producers.

Entrepreneurs use this capital (after it has been perturbed by an idiosyncratic shock) in the second period to produce output. They send their profits to households in the second period, and these are used to purchase consumption goods in that period for the members of the representative household.

We show that the Ricardian irrelevance result applies for loans made by the government to entrepreneurs. This is so, in spite of the fact that a key assumption underlying the Ricardian result is not satisfied, the assumption of frictionless financial markets. Tax-financed transfers of net worth to entrepreneurs and a subsidy to banks' cost of funds do help improve the efficiency of credit markets in the BGG model.

6.1. Capital Producers

Entrepreneurs sell their capital, k, to capital producers and receive $N = P_k k$, where P_k is the market price of capital. The capital producers operate the following production function:

$$K = k + f\left(\frac{I}{k}\right)k,$$

where

$$f\left(\frac{I}{k}\right) = \left(\frac{I}{k}\right)^{\gamma},\ 0 < \gamma < 1,$$

and I denotes investment goods. The profits of the capital producer are:

$$\begin{aligned} & P_K K - P_I I - P_k k \\ = \ & P_K k + P_K f\left(\frac{I}{k}\right)k - I - P_k k, \end{aligned}$$

where P_K denotes the market price of new capital. Also, I denotes investment, which can be converted one-for-one from first period output. Note that the profits are linear homogeneous in terms of k, K, I. As a result, they must be zero in equilibrium. We can see that the demand for k by capital producers will be infinite if $P_K > P_k$, since they can make infinite profits by making $k = \infty$ and $I = 0$. But, it's not so easy to rule out $P_K < P_k$. First order condition for k :

$$P_K\left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right] = P_k.$$

The first order condition for I :

$$P_K f'\left(\frac{I}{k}\right) = 1.$$

Note that the two first order conditions and the capital utilization condition imply zero profits.

6.2. Entrepreneurs

Entrepreneurs take their net worth and approach a bank for a loan, B. They use the proceeds to purchase

$$P_K K = N + B.$$

As soon as the entrepreneur purchases K, they experience a productivity shock,

$$\omega \tilde{} F, \quad \int_0^\infty dF(\omega) = 1,$$

where F is the cdf of a log-normal distribution. In period 2 the entrepreneur uses its capital, ωK, to produce

$$\omega K r^k$$

goods, where r^k is technologically determined. The entrepreneur receives a standard debt contract from the bank. In particular, the entrepreneur must pay the bank

$$ZB$$

in period 2 if it is able to do so. If it is not able to repay the loan, then the entrepreneur loses everything. The cutoff level of productivity, $\bar{\omega}$, such that the entrepreneur is just able to repay the loan is defined as:

$$\bar{\omega} K r^k = ZB. \tag{6.1}$$

The entrepreneur's expected payoff is given by:

$$\begin{aligned}
\int_{\bar{\omega}}^\infty \left[\omega K r^k - ZB\right] dF(\omega) &= \int_{\bar{\omega}}^\infty \left[\omega K r^k - \bar{\omega} K r^k\right] dF(\omega) \\
&= K r^k \int_{\bar{\omega}}^\infty \left[\omega - \bar{\omega}\right] dF(\omega) \\
&= N R^k L \left(\int_{\bar{\omega}}^\infty \left[\omega - \bar{\omega}\right] dF(\omega) \right),
\end{aligned}$$

where L denotes leverage and R^k denotes the rate of return on capital:

$$L \equiv \frac{N+B}{N}, \; R^k \equiv \frac{r^k}{P_K}.$$

From the definition of $\bar{\omega}$:

$$\bar{\omega} = \frac{ZB}{K r^k} = \frac{Z \frac{B}{N}}{\frac{P_K K}{N} R^k} = \frac{Z}{R^k} \frac{L-1}{L}.$$

From this we see that as $L \to \infty$, $\bar{\omega}$ converges to a constant. It follows that the ex ante objective of the entrepreneur is unbounded above in L, for any given Z and R^k. This is why the loan contract must specify both Z and B.

The entrepreneur's objective is to optimize the expected revenues of running a project:

$$\begin{aligned}
& N r^k L \left(\int_{\bar{\omega}}^\infty \left[\omega - \bar{\omega}\right] dF(\omega) \right) \qquad (6.2) \\
= \; & N L r^k \left(1 - G(\bar{\omega}) - \bar{\omega}\left[1 - F(\bar{\omega})\right]\right) \\
= \; & N L r^k \left(1 - \Gamma(\bar{\omega})\right),
\end{aligned}$$

say, where

$$\Gamma(\bar{\omega}) \equiv G(\bar{\omega}) + \bar{\omega}[1 - F(\bar{\omega})], \; G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega).$$

The entrepreneur's objective is dictated to it by the household, which provides the entrepreneur with perfect consumption insurance. The entrepreneur's relationship with its household is the same as the relationship between bankers and households in the previous sections.

Let R denote the gross interest rate on bank deposits. Then, the bank must pay RB to depositors. It receives

$$[1 - F(\bar{\omega})] ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) r^k K$$

from the entrepreneurs. So, the bank's zero profit condition is (using (6.1)):

$$[1 - F(\bar{\omega})] \bar{\omega} K r^k + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) r^k K = RB,$$

or,

$$[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] R^k \frac{P_K K}{B} = R,$$

so that the interest rate, R, has to equal the average return to the bank on a loan, B. The average gross return is $R^k P_K K$, but of this the bank only receives a fraction, $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$. Banks are indifferent over loan contracts, as long as they satisfy the previous equation. As a consequence, the zero profit condition of the banks is treated as a menu of contract by the entrepreneurs. The relationship between the entrepreneurs and the banks in this model is the same as the relationship between bankers and mutual funds in the previous section.

The contract selected by entrepreneurs maximize

$$NR^k (1 - \Gamma(\bar{\omega}))$$

with respect to $\bar{\omega}$ and L, subject to the zero profit condition:

$$L = \frac{1}{1 - \frac{R^k}{R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}.$$

Using the latter expression to substitute out for L in the objective, the problem is to choose $\bar{\omega}$ to maximize:

$$NR^k \frac{1 - \Gamma(\bar{\omega})}{1 - \frac{R^k}{R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]},$$

which implies the following first order optimality condition:

$$\frac{1 - F(\bar{\omega})}{1 - \Gamma(\bar{\omega})} = \frac{\frac{R^k}{R} [1 - F(\bar{\omega}) - \mu \bar{\omega} F'(\bar{\omega})]}{1 - \frac{R^k}{R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}.$$

This can be used to compute $\bar{\omega}$ for given R^k and R. Then the loan amount and interest rate are:

$$L = \frac{1}{1 - \frac{R^k}{R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}, \; Z = R^k \bar{\omega} \frac{L}{L - 1}.$$

6.3. Households and Government

In the first period, households place a deposit, B, with the banking system. Their budget constraint in the first period is

$$c + B \leq y.$$

In the second period, deposits generate a return,

$$(1-\tau) RB = (1-\tau) r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right],$$

where τ denotes the tax rate on saving. When entrepreneurs return home in the second period, they bring their profits with them. Because each household has many entrepreneurs the total amount of profits, π, is:

$$\pi = K r^k (1 - \Gamma(\bar{\omega})).$$

Bankers bring home zero profits. The second period budget constraint is:

$$\begin{aligned} C &\leq (1-\tau) RB + K r^k (1 - \Gamma(\bar{\omega})) \\ &= (1-\tau) r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + K r^k (1 - \Gamma(\bar{\omega})) + T, \end{aligned}$$

where T denotes lump sum transfers from the government.

The government's budget constraint is:

$$T = \tau RB = \tau r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right].$$

Note that if we combine the government's budget constraint with the household's second period budget constraint, we obtain the second period resource constraint:

$$\begin{aligned} C &\leq (1-\tau) r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + K r^k (1 - \Gamma(\bar{\omega})) + T \\ &= (1-\tau) r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + K r^k (1 - \Gamma(\bar{\omega})) + \tau r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] \\ &= r^k K \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + K r^k (1 - \Gamma(\bar{\omega})) = r^k K \left[1 - \mu G(\bar{\omega})\right]. \end{aligned}$$

The last equality corresonds to Walras' law. That is, consumption is no greater than total output, net of the output used up in monitoring by banks.

The household problem is to maximize

$$u(c) + \beta u(C)$$

subject to

$$c + b \leq y, \; C \leq (1-\tau) Rb + \pi + T$$

The first order necessary and sufficient conditions are:

$$\frac{u'(c)}{\beta u'(C)} = (1-\tau) R, \; c + \frac{C}{(1-\tau) R} = y + \frac{\pi + T}{(1-\tau) R}.$$

Suppose

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}$$

Then,

$$C = c\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}},\ c + \frac{c\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}}}{\left(1-\tau\right)R} = y + \frac{\pi + T}{\left(1-\tau\right)R},$$

or,

$$c = \frac{y + \frac{\pi+T}{(1-\tau)R}}{1 + \frac{(\beta[1-\tau]R)^{\frac{1}{\alpha}}}{(1-\tau)R}}$$

6.4. Equilibrium Conditions

The equilibrium conditions are divided into a household and government block, and a firm block. The first of these is:

Equation number	Household and Government conditions	Economic description
(1)	$C = c\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}}$	household first order condition
(2)	$C = r^k K\left[1-\mu G\left(\bar{\omega}\right)\right]$	resource constraint in the second period
(3)	$c + I = y$	resource constraint in the first period
(4)	$R^k = r^k / P_K$	definition of rate of return on capital

To understand the connection between B and I, recall

$$P_K K = N + B = P_k k + B.$$

But, zero profits for the capital producers implies:

$$I + P_k k = N + B.$$

The set of equilibrium conditions associated with the firm is:

Equation number	Efficiency conditions for firms	Economic description of condition
(5)	$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{R^k}{R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	contract efficiency condition
(6)	$\frac{P_K K}{P_k k} = \frac{1}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	leverage from bank zero profit condition
(7)	$P_k = P_K\left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right]$	efficiency condition of capital producers
(8)	$1 = P_K f'\left(\frac{I}{k}\right)$	optimality of I choice by capital producers
(9)	$K = \left[1 + f\left(\frac{I}{k}\right)\right]k$	capital accumulation technology

The above system represents 9 equations in 9 unknowns:

$$C, c, R, K, \bar{\omega}, B, P_k, P_K, I.$$

6.5. Implications for Policy

In the first subsection below, we show that an interest rate subsidy to the banks in this model improves social welfare. That this subsidy improves welfare reflects the monitoring costs. Absent monitoring costs, the equilibrium is efficient and subsidies are not warranted. After that, we show that government loans to entrepreneurs have no impact on the equilibrium. Finally, we show that transfers of net worth to entrepreneurs, if done at the rate, improves the allocations.

6.5.1. Interest Rate Subsidy to Banks

Denote the standard deviation that defines the log-normal distribution by σ. Let the exogenous parameters be as follows:

$$\mu = 0.2,\ \sigma = 0.34,\ \gamma = 0.55,\ \alpha = 0.9,\ y = 1527.44,\ r^k = 1.30,\ k = 1,\ \beta = 0.97,\ \tau = 0.$$

The endogenous variables are:

$$\begin{aligned} R &= 0.047,\ R^k = 0.0482,\ \bar{\omega} = 0.521,\ I = 400.62,\ K = 28.009,\ P_K = 26.969,\ P_k = 354.75,\ c = 11268.2 \\ C &= 36.278,\ G = 0.0183,\ F = 0.04,\ \Gamma = 0.519,\ L = 2.13,\ Z/R = 1.012,\ u = 34.083356439426971. \end{aligned}$$

For the calibration, we set $R^k/R = 1.03$, $r^k = 1.3$. Note how low the interest rates are. This is because γ is very small, so that the productivity of investment is very low. This could have been repaired by placing a multiplicative constant in front of the adjustment cost function.

The first best solution for this economy is:[30]

$$c = 1118.32,\ I = 409.11,\ K = 28.322,\ C = 36.82,\ u = 34.088653745007754$$

Note that the utility in the first best is superior to what it is in the equilibrium. To convert the utility difference into consumption units we ask how much the person in the equilibrium would pay to switch to the first best equilibrium. This is the decrease in consumption they are willing to suffer in the first best equilibrium so that they would be indifferent between the two equilibria.

$$u^1 \left(1 - \frac{\lambda}{100}\right)^{1-\alpha} = u^e,$$

where u^1 denotes utility in the first best equilibrium and u^e denotes utility in the equilibrium. Then,

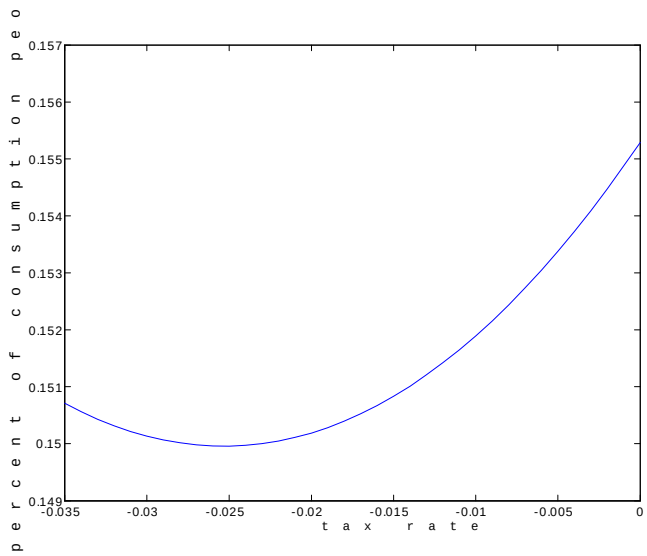
$$\lambda = 100 \left[1 - \left(\frac{u^e}{u^1}\right)^{\frac{1}{1-\alpha}}\right] = 0.155,$$

a very small number.

[30] The first-best allocations solve the following problem:

$$\begin{aligned} &\max_{c,C,I} u(c) + \beta u(C) \\ \text{subject to:} \quad &c + I \leq y,\ C \leq r^k K = r^k k \left[1 + f\left(\frac{I}{k}\right)\right]. \end{aligned}$$

We considered values of τ from 0 to -0.035. The results for utility are



The optimal tax rate is in the neighborhood of -2.5 percent. This establishes that the equilibrium for this economy is not efficient.

We now turn to the case in which there are no monitoring costs, $\mu = 0$. In this case, equation (5) can only be satisfied with $R = R^k$ and $\bar{\omega}$ disappears from that equation. Now consider equation (6):

$$\frac{1}{1-\Gamma(\bar{\omega})} = \frac{1+\left(\frac{I}{k}\right)^{\gamma}}{1+(1-\gamma)\left(\frac{I}{k}\right)^{\gamma}}.$$

We can think of this equation as defining $\bar{\omega}$ and, hence, Z (i.e., the spread). But, this variable does not enter the other equations, and so it plays no direct role in the system. The other equations are:

$$\begin{aligned}
&(1) \quad \tfrac{1}{\beta}\left(\tfrac{C}{c}\right)^{\alpha} = \tfrac{u'(c)}{u'(C)\beta} = (1-\tau)\,R^k = (1-\tau)\,\tfrac{r^k}{P_K} = (1-\tau)\,r^k\gamma\left(\tfrac{I}{k}\right)^{\gamma-1} \\
&(2) \quad C = r^k k\left[1+\left(\tfrac{I}{k}\right)^{\gamma}\right] \\
&(3) \quad c + I = y \ .
\end{aligned}$$

This represents three equations in three unknowns, c, C, I. Substitute out I using (3):

$$\begin{aligned}
&(1) \quad \tfrac{u'(c)}{u'(C)\beta} = (1-\tau)\,r^k\gamma\left(\tfrac{y-c}{k}\right)^{\gamma-1} \\
&(2) \quad C = r^k k\left[1+\left(\tfrac{y-c}{k}\right)^{\gamma}\right] \ .
\end{aligned}$$

The first-best problem for this economy is:

$$\begin{aligned}
&\max u(c) + \beta u(C) \\
C \quad &= \quad r^k k\left[1+\left(\frac{y-c}{k}\right)^{\gamma}\right].
\end{aligned}$$

Expressing this in Lagrangian form:

$$\max u(c) + \beta u(C) + \lambda\left(r^k k\left[1+\left(\frac{y-c}{k}\right)^{\gamma}\right] - C\right)$$

The first order conditions are:

$$\begin{aligned} u'(c) &= \lambda r^k \gamma \left(\frac{y-c}{k}\right)^{\gamma-1} \\ \beta u'(C) &= \lambda \end{aligned}$$

so that the necessary and sufficient conditions for optimization reduce to:

$$\begin{aligned} \frac{u'(c)}{\beta u'(C)} &= \overbrace{r^k \gamma \left(\frac{y-c}{k}\right)^{\gamma-1}}^{\text{marginal return on investment}} \\ C &= r^k k \left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right] \end{aligned}$$

Evidently, these equations coincide with (1)-(2) when $\tau = 0$. Thus, the equilibrium supports the first-best allocations. We summarize our findings as follows:

Proposition 6.1. *The allocations in the asymmetric information equilibrium with positive monitoring costs are inefficient, and efficiency is improved by providing a deposit subsidy to banks. When monitoring costs are zero, then the equilibrium is efficient and government intervention is not warranted.*

6.5.2. Government Loans to Entrepreneurs

The previous section established that the equilibrium of the system is inefficient. We now consider the impact on the equilibrium of two types of policy. In one, the government raises a lump sum tax, T, on households in the first period. It then lends the money to entrepreneurs using the same technology available to banks and transfers the proceeds to households in the second period in the form of a lump sum subsidy. We show here that this has no impact on equilibrium allocations, because it simply displaces, one-for-one, private saving. In effect, the Ricardian irrelevance result applies to this kind of policy.

In the first period, households place a deposit, b, with the banking system. Their budget constraint in the first period is

$$c + b \leq y - T.$$

The second period budget constraint is:

$$C \leq Rb + \text{government lump sum taxes+entrepreneurial earnings}$$

In the second period, deposits generate a return,

$$Rb = r^k K^h \left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right],$$

where K^h denotes the capital financed by the deposits, b :

$$K^h = k^h \left[1 + f\left(\frac{b}{k^h}\right)\right].$$

Here, k^h denotes the quantity of capital of the entrepreneurs financed by household deposits. These entrepreneurs return their profits to the households:

$$r^k K^h \left[1 - \Gamma\left(\bar{\omega}\right)\right].$$

As a result, household income from deposits plus the profits generated by entrepreneurs nourished by those deposits is:

$$\begin{aligned} Rb + r^k K^h \left[1 - \Gamma\left(\bar{\omega}\right)\right] &= r^k K^h \left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right] + r^k K^h \left[1 - \Gamma\left(\bar{\omega}\right)\right] \\ &= r^k K^h \left[1 - \mu G\left(\bar{\omega}\right)\right]. \end{aligned}$$

In addition, the households receive a tax rebate from the government plus profits from the entrepreneurs financed by the government, in the amount:

$$r^k K^g \left[1 - \mu G\left(\bar{\omega}\right)\right],$$

where K^g is the quantity of capital financed by tax dollars. Then, total household income in the second period is:

$$\begin{aligned} & r^k \left[K^h + K^g\right] \left[1 - \mu G\left(\bar{\omega}\right)\right] \\ = \; & r^k \left[k^h \left(1 + f\left(\frac{b}{k^h}\right)\right) + k^g \left(1 + f\left(\frac{T}{k^g}\right)\right)\right] \left[1 - \mu G\left(\bar{\omega}\right)\right] \end{aligned}$$

We suppose that

$$\frac{b}{k^h} = \frac{T}{k^g},$$

so that

$$\frac{b}{k^h} = \frac{T}{k^g} = \frac{b + T}{k^h + k^g} = \frac{I}{k},$$

where I denotes total investment and k denotes the total quantity of capital held by entrepreneurs. The second equality reflects:

$$\frac{b + T}{k^h + k^g} = \frac{\frac{k^h T}{k^g} + T}{\frac{b k^g}{T} + k^g} = \frac{\frac{k^h}{k^g} + 1}{\frac{b}{T} + 1} \frac{T}{k^g} = \frac{T}{k^g}.$$

So, total household income in the second period is:

$$C \leq r^k \left[k \left(1 + f\left(\frac{b + T}{k}\right)\right)\right] \left[1 - \mu G\left(\bar{\omega}\right)\right].$$

This (with the understanding, $b + T = I$) is the same as equation (2). Since the equilibrium conditions of this economy coincide with the equilibrium conditions of the economy without unconventional monetary policy, it follows that unconventional monetary policy has no effect. If the government extends loans to entrepreneurs, then private loans to them will be reduced by the same amount. This is so, even though the equilibrium involves an inefficiently low amount of lending.

6.5.3. Net Worth Transfers to Entrepreneurs

Here, we consider a policy in which the government raises taxes in the first period and then simply gives the banks the proceeds as a subsidy. With this policy, the government in effect can cause the economy to fully circumvent the financial frictions.

In this case, the government raises $T > 0$ in the first period and transfers the proceeds as a lump sum gift to entrepreneurs. The optimal policy in this case is to set $T = I^*$, the first best level of investment. In this case, the entrepreneurs have enough net worth that they do not need to borrow from banks. In this way there are no monitoring costs.

We first rework the equations associated with the banks and entrepreneurs. Entrepreneurial 'preferences' are:

$$r^k K \left(1 - \Gamma\left(\bar{\omega}\right)\right).$$

At the time that the loan contract is negotiated, N is a state variable. Divide the entrepreneur's preferences by N :

$$\frac{r^k P_K K}{N P_K}\left(1 - \Gamma\left(\bar{\omega}\right)\right) = R^k L \left(1 - \Gamma\left(\bar{\omega}\right)\right),$$

where $L = P_K K/N$.

The zero profit condition on banks is:

$$\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right] r^k K = RB.$$

To define leverage, note:

$$P_K K = N + B = P_k k + T + B,$$

where net worth is now $N = P_k k + T$. We require $B \geq 0$, so that

$$P_K K - P_k k - T \geq 0. \tag{6.3}$$

Leverage is:

$$L = \frac{N + B}{N} = \frac{P_k K}{P_k k + T}.$$

Now,

$$\frac{RB}{r^k K} = \frac{R}{R^k}\frac{B}{P_K K} = \frac{R}{R^k}\frac{B/N}{P_K K/N} = \frac{R}{R^k}\frac{L-1}{L},$$

so that the zero profit condition of the bank can be written

$$\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right) = \frac{R}{R^k}\frac{L-1}{L},$$

or,

$$\begin{aligned}
\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right] L &= \frac{R}{R^k}\left(L - 1\right) \\
\frac{R}{R^k} &= \left(\frac{R}{R^k} - \left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]\right) L \\
L &= \frac{\frac{R}{R^k}}{\frac{R}{R^k} - \left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]} = \frac{1}{1 - \frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right) - \mu G\left(\bar{\omega}\right)\right]}.
\end{aligned}$$

We conclude that in terms of the equilibrium conditions, (5) and (6), the only change is that the object on the left of (6) is replaced by

$$L = \frac{P_K K}{P_k k + T}.$$

Thus, the equilibrium conditions pertaining to the entrepreneur are:

Equation number	Efficiency conditions for firms	Economic description of condition
(5)	$\frac{1-F(\bar{\omega})}{1-\Gamma(\bar{\omega})} = \frac{\frac{R^k}{R}[1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})]}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	contract efficiency condition
(6)	$\frac{P_K K}{P_k k+T} = \frac{1}{1-\frac{R^k}{R}[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}$	leverage from bank zero profit condition
(7)	$P_k = P_K\left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right]$	efficiency condition of capital producers
(8)	$1 = P_K f'\left(\frac{I}{k}\right)$	optimality of I choice by capital producers
(9)	$K = \left[1 + f\left(\frac{I}{k}\right)\right]k$	capital accumulation technology

We now turn to the households. The households' first period budget constraint is:

$$c + B \leq y - T,$$

where c, $B \geq 0$. We can see from the above expression that this requires

$$T \leq y,$$

and from (6.3):

$$\overbrace{P_K\left[1 + f\left(\frac{I}{k}\right)\right]k - P_K\left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right]k}^{P_K K - P_k k} - T \geq 0,$$

or,

$$\overbrace{P_K f'\left(\frac{I}{k}\right)\frac{I}{k}k}^{P_K K - P_k k = I} \geq T,$$

or,

$$I \geq T.$$

In the second period, deposits generate a return,

$$RB = r^k K\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right],$$

Entrepreneurs return profits,

$$\pi = K r^k\left(1 - \Gamma(\bar{\omega})\right).$$

Profits returned by bankers are zero. The second period budget constraint is:

$$\begin{aligned} C &\leq RB + K r^k\left(1 - \Gamma(\bar{\omega})\right) \\ &= r^k K\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right] + K r^k\left(1 - \Gamma(\bar{\omega})\right) \\ &= r^k K\left[1 - \mu G(\bar{\omega})\right]. \end{aligned}$$

We conclude that equilibrium conditions (1)-(6) are unaffected by this policy.

The optimal choice of T is

$$T = I.$$

To verify this, note that in this case, $\bar{\omega} = 0$, $R^k = R$ solve (5) (note, $F(\bar{\omega}) = \Gamma(\bar{\omega}) = G(\bar{\omega}) = 0$). Also, with $T = I$ the left side of (6) is unity. Since the right side of (6) is also unity, that equation is satisfied. The remaining equations correspond to the equations for the first best equilibrium.

7. Concluding Remarks

In the past decade, dynamic stochastic general equilibrium (DSGE) models have been constructed that have proved useful for analyzing questions of interest to policy makers. For example, these models have been used to make the case that the Taylor principle is a valuable guide to monetary policy. In recent years, the Federal Reserve has undertaken large scale asset purchase programs and cut banks' cost of funds, with the objective of correcting dysfunctions in credit markets. The DSGE models developed to analyze the Taylor principle are silent on the rationale and appropriate scale of such programs. These models must integrate financial frictions in order to be useful given the new policy questions. This paper surveys four candidate models and summarizes some of their implications for government asset purchases.

Table 1: Selected Assets and Liabilities of the Federal Reserve (trillions of dollars)							
	10/18/2006				10/13/2010		
Total Assets	0.85				2.3		
US Treasury securities		0.77				0.82	
Bills			0.28				0.02
Bonds			0.49				0.80
Mortgage backed securities		0.				1.08	
Loans (including TAF)		0.				0.14	
Total Liabilities	0.85				2.3		
Federal Reserve Notes		0.76				0.92	
Deposits		0.16				1.3	
Depository institutions			0.10				1.1

Source: Federal Reserve tatisticalrelease, H.4.1, Statement of Condition of Each Federal Reserve Bank, 'total' column. Total assets and liabilities are correct and subtotals do not add because only selected items are reported

Table 2: Comparison of intervention and no intervention equilibrium		
	no-intervention	intervention
$R^k - R$	0.24	0.
d	3.76	1.63
c	6.24	5.58
C	5.23	5.96
$\pi = (N + d) R^k - Rd$	1.99	1.10
$\theta (N + d) R^k$	1.99	1.10
$u(c) + \beta u(C)$	-19696.564	-19696.549

Figure 1: Spreads, 3-month commercial paper (CP) over Tbills and BAA versus AAA corporate bonds

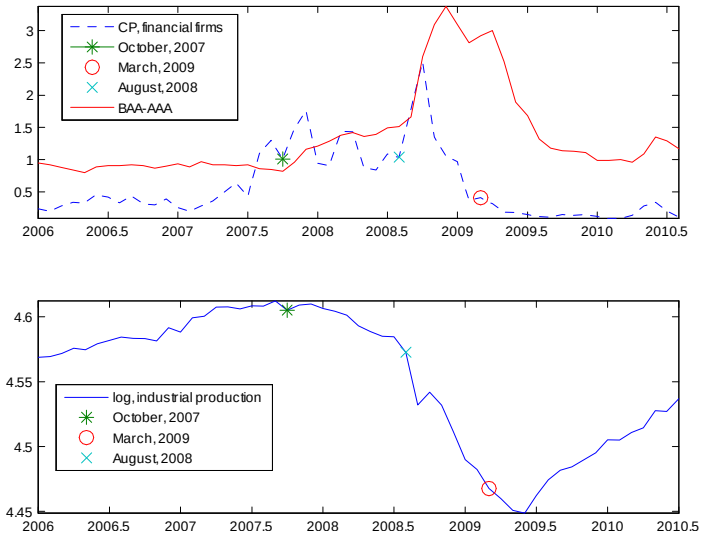
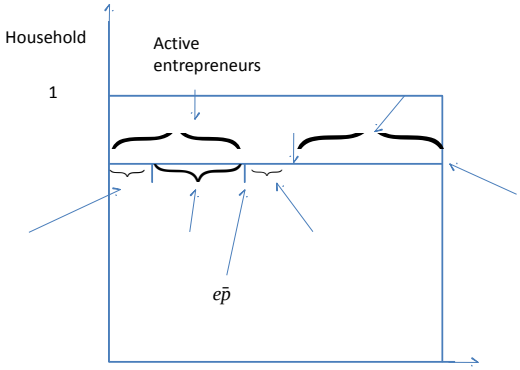


Figure 2: Agents in the Adver



References

[1] Ajello, Andrea, 2010, "Financial Intermediation, Investment Dynamics and Business Cycle Fluctuations," Northwestern Doctoral Dissertation, in process.

[2] Akerlof, George A., 1970, "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," The Quarterly Journal of Economics, Vol. 84, No. 3, August, pp. 488-500.

[3] Barro, Robert J., 1974, "Are government bonds net wealth?," Journal of Political Economy 82(6): 1095-1117.

[4] Bernanke, Ben, and Mark Gertler, 1990, "Financial Fragility and Economic Performance," The Quarterly Journal of Economics, Vol. 105, No. 1, February, pp. 87-114

[5] Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," Handbook of Macroeconomics, edited by John B. Taylor and Michael Woodford, pp. 1341-93, Amsterdam, New York and Oxford: Elsevier Science, North-Holland.

[6] Carlstrom, Charles T., and Timothy Fuerst, 1997, "Agency Costs, Net Worth and Business Fluctuations: A Computable General Equilibrium Analysis," American Economic Review 87, 893-910.

[7] Chari, V. V., Ali Shourideh and Ariel Zetlin-Jones, 2010, "Analyzing Policies to Repair Credit Markets," manuscript, Federal Reserve Bank of Minneapolis.

[8] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno, 2003, "The Great Depression and the Friedman-Schwartz Hypothesis," Journal of Money, Credit and Banking, Vol. 35, No. 6 (December, Part 2)

[9] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno, 2010, "Financial Factors in Business Cycles," manuscript, European Central Bank.

[10] Curdia, V. and Michael Woodford, 2009, "Credit Frictions and Optimal Monetary Policy," BIS Working Papers, 278.

[11] del Negro, Marco, Gauti Eggertsson, Andrea Ferrero and Nobuhiro Kiyotaki, 2010, "The Great Escape? A Quantitative Evaluation of the Fed's Non-Standard Policies," manuscript, Federal Reserve Bank of New York.

[12] Dib, Ali, 2010, "Capital Requirement and Financial Frictions in Banking: Macroeconomic Implication," manuscript, Bank of Canada, August 28.

[13] Eggertsson, Gauti and Michael Woodford, 2003, "The zero interest-rate bound and optimal monetary policy." *Brookings Panel on Economic Activity.*

[14] Eisfeldt, Andrea, 2004, "Endogenous Liquidity in Asset Markets," The Journal of Finance, 59(1), 1-30.

[15] Fisher, Jonas, 1999, "Credit Market Imperfections and the Heterogeneous Response of Firms to Monetary Shocks," Journal of Money, Credit and Banking, Vol. 31, No. 2, May, pp. 187-211.

[16] Fishman, Michael and Jonathan Parker, 2010, "Valuation and the Volatility of Financing and Investment," Northwestern University, manuscript, July.

[17] Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, 2010, "Large-Scale Asset Purchases by the Federal Reserve: Did They Work?," unpublished manuscript, Federal Reserve Bank of New York, March.

[18] Gertler, Mark and Peter Karadi, 2009,"A Model of Unconventional Monetary Policy," manuscript, New York University.

[19] Gertler, Mark, and Nobuhiro Kiyotaki, 2010, "Financial intermediation and credit policy in business cycle analysis," forthcoming chapter in Handbook of Monetary Economics, edited by Friedman and Woodford.

[20] Hirakata, Naohisa, Nao Sudo and Kozo Ueda, 2009a, "Chained Credit Contracts and Financial Accelerators," Discussion Paper Series E-30, Bank of Japan.

[21] Hirakata, Naohisa, Nao Sudo and Kozo Ueda, 2009b, "Capital Injection, Monetary Policy, and Financial Accelerators," manuscript, Bank of Japan, October 16.

[22] Hirakata, Naohisa, Nao Sudo and Kozo Ueda, 2010, "Do Banking Shocks Matter for the U.S. Economy?" manuscript, Bank of Japan.

[23] Holmstrom, Bengt and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and the Real Sector," The Quarterly Journal of Economics, Vol. 112, No. 3. (August), pp. 663-691.

[24] Kiyotaki, Nobuhiro and John Moore, 2008, "Liquidity, Business Cycles, and Monetary Policy," manuscript, Princeton University, April.

[25] Korinek, Anton, 2010, "Amplification Effects, Externalities, and Regulatory Responses," manuscript, University of Maryland.

[26] Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2010, "The Effect of Purchasing Long-term Securities on Interest Rates," unpublished manuscript, Kellogg School of Management, Northwestern University, October 22.

[27] Lorenzoni, Guido, 2008, "Inefficient Credit Booms," Review of Economic Studies, 75(3):809-833.

[28] Mankiw, N. Gregory, 1986, "The Allocation of Credit and Financial Collapse," The Quarterly Journal of Economics, Vol. 101, No. 3, August, pp. 455-470.

[29] Meh, Cesaire and Kevin Moran, 2010, "The Role of Bank Capital in the Propagation of Shocks," Journal of Economic Dynamics and Control, 34: 555-76.

[30] Moore, John, 2009, "Contagious Illiquidity," plenary lecture, joint conference, the XIV Annual Meeting of the Latin American and Caribbean Economic Association, and the XXV Latin American Meeting of the Econometric Society, October 1-3.

[31] Taylor, John B. and Williams, John C., 2009, "A Black Swan in the Money market," American Economic Journal: Macroeconomics 1, pp. 58-83.

[32] Ueda, Kozo, 2009, "Banking Globalization and International Business Cycles," manuscript, Bank of Japan, December 20.

[33] Wallace, Neil, 1981, "A Modigliani-Miller Theorem for Open-Market Operations," The American Economic Review, vol. 71, no. 3, June, pp. 267-274.

[34] Zeng, Zhixiong, 2010, "A Theory of the Non-Neutrality of Money with Banking Frictions and Bank Recapitalization," manuscript, Monash University, July.

A. Appendix A: Notes on the Unobsrved Effort Model

A.1. Computational Strategy for Solving the Model

For convenience, we repeat the equilibrium conditions associated with the banks here (see (4.18)):

$$\begin{aligned}
e &: (\lambda-\nu)\, b\left(R_g^d - R_b^d\right) d + \eta = 0 \\
d &: R = p(e)\, R^g + (1-p(e))\, R^b \\
R_g^d &: \nu p(e) = \eta\lambda b \\
\mu &: R = p(e)\, R_g^d + (1-p(e))\, R_b^d \\
\eta &: e = \lambda b\left[\left(R^g - R^b\right)(N+d) - \left(R_g^d - R_b^d\right) d\right] \\
\nu &: \nu\left[R_b^d d - R^b(N+d)\right] = 0,\ \nu \le 0,\ \left[R_b^d d - R^b(N+d)\right] \le 0.
\end{aligned} \tag{A.1}$$

Computing an equilibrium when the cash constraint, (4.7), is not binding, $\nu = 0$, is trivial. Here is an algorithm for computing an equilibrium when the cash constraint is binding, so that $\nu < 0$. Fix a value for R and solve for c, C, d, λ using the household equations, (4.2), (4.3) and (4.14), and the expression for the marginal utility of consumption. Compute R_b^d using the ν equation. The μ and η equations represent two equations in two unknowns, e and R_g^d. This can be solved non-linearly as follows. Fix a value for R_g^d. Solve for e using the η equation. Adjust R_g^d until the μ equation is satisfied. We checked for multiplicity of solutions by considering values of R_g^d on a grid and checking that the μ equation has only one solution. We now have R_b^d, R_g^d and e in hand. Adjust R until the d equation is satisfied. (Again, to investigate the possibility of multiple equilibria, we considered values of R on a fine grid and it appeared that there is only one value of R that satisfies the d equation.

It remains to verify that ν and η can be chosen so the e and the R_g^d equations are satisfied. With some algebra, we find

$$\nu = \frac{b^2\left(R_g^d - R_b^d\right) d\lambda^2}{b^2\left(R_g^d - R_b^d\right) d\lambda - p(e)}.$$

Once ν is computed using this last expression, η is found using the R_g^d equation. This completes the discussion of computing the equilibrium.

A.2. Proof of Proposition 4.4

Consider a baseline equilibrium for the unobserved effort model in which $\nu < 0$ and T is possibly non zero. Let $\bar{T} = T + \Delta T$, $\Delta T \neq 0$ denote the level of taxes and equity injections in a perturbed equilibrium. We conjecture (and then prove) that c, C, R, e take on the same values across the two equilibria. The values of the other endogenous variables may differ across the equilibria. We denote the values of the other endogenous variables in the baseline equilibrium by

$$d, \nu, \eta, R_b^d, R_g^d,$$

and we denote their values in the perturbed equilibrium by

$$\bar{d}, \bar{\nu}, \bar{\eta}, \bar{R}_b^d, \bar{R}_g^d, \tag{A.2}$$

where $\bar{d} = d - \Delta T$.

It is immediate that if the equations pertaining to the household, (4.3), (4.23a) at equality, and (4.24), are all satisfied in the baseline equilibrium, then they are also satisfied under the perturbed equilibrium under our conjecture. Now consider the equations pertaining to the banks, (A.1). Endogenous variables without hats are assumed to satisfy the equilibrium conditions in the baseline equilibrium. Our task is to verify that these equations are also satisfied under (A.2). [to be completed] This completes the discussion of proposition 4.4.

B. Appendix B: Computing the Equilibrium in the Asymmetric Information Model

We compute the equilibrium by finding $\tilde{R} \equiv RP_K$ such that $g\left(\tilde{R}\right) = 0$. To define the mapping from $\tilde{R}$ to g, fix a value for $\tilde{R}$. Solve for $\bar{\omega}$ using (5) and

$$\frac{R^k}{R} = \frac{r^k}{RP_K} = \frac{r^k}{\tilde{R}}.$$

Then, compute the right side of (6), which we call X. Rewrite the left side of (6) using (7) and (9), to obtain:

$$\begin{aligned}
\frac{P_K K}{P_k k} &= \frac{1 + f\left(\frac{I}{k}\right)}{1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}} = \frac{1 + \left(\frac{I}{k}\right)^\gamma}{1 + (1-\gamma)\left(\frac{I}{k}\right)^\gamma} = X \\
\left(\frac{I}{k}\right)^\gamma &= \frac{X - 1}{1 - X(1-\gamma)}
\end{aligned}$$

Solve this for I/k. Then, solve (9) for K/k, (8) for P_K and (7) for P_k. Finally,

$$R = \frac{\tilde{R}}{P_K}.$$

Combine (1), (2) and (3):

$$\begin{aligned}
c\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k K\left[1 - \mu G\left(\bar{\omega}\right)\right] \\
\left[y - I\right]\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k K\left[1 - \mu G\left(\bar{\omega}\right)\right] \\
\left[\frac{y}{k} - \frac{I}{k}\right]\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}} &= r^k \frac{K}{k}\left[1 - \mu G\left(\bar{\omega}\right)\right],
\end{aligned}$$

so that

$$\frac{y}{k} = \frac{\frac{I}{k}\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}} + r^k \frac{K}{k}\left[1 - \mu G\left(\bar{\omega}\right)\right]}{\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}}}.$$

Solve this for y/k. Solve (2) and (3) for c, C. In this way, we define mappings, $c\left(\tilde{R}\right), C\left(\tilde{R}\right)$. Then, let

$$g\left(\tilde{R}\right) \equiv C\left(\tilde{R}\right) - c\left(\tilde{R}\right)\left(\beta\left[1-\tau\right]R\right)^{\frac{1}{\alpha}}.$$

Adjust $\tilde{R}$ until $g\left(\tilde{R}\right) = 0$, i.e., (1) is satisfied.