Estimating Fiscal Limits: The Case of Greece *

Huixin Bi[†] and Nora Traum[‡]

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Abstract

This paper uses Bayesian methods to estimate the 'fiscal limit' distribution for Greece implied by a rational expectations framework. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic 'fiscal limit,' and sovereign default risk. A fiscal limit measures the debt level beyond which the government is no longer willing to finance, causing a partial default to occur. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data until 2010Q4. We find that the probability of default on Greek debt remained close to zero from 2001 until 2009, when it began to rise sharply to the range of 5% to 10% by 2010Q4. The model also predicts a probability of default between 60-80% by 2011Q4, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the real interest rate in Greece in 2011 is within forecast bands of our rational expectations model. Finally, model comparisons based on Bayes factors strongly favor the nonlinear model specification with an endogenous probability of default over a linearized specification.

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[†]Bank of Canada, 234 Wellington Street, Ottawa, ON K1A0G9, Canada; hbi@bank-banque-canada.ca
[‡]North Carolina State University, Nelson Hall 4108, Box 8110 Raleigh, NC 27695-8110; nora_traum@ncsu.edu

1. Introduction

During the past five years, there has been growing concern over the fiscal positions of several Eurozone nations, as evidenced by the rapid increase in long term interest rate spreads of several countries' bonds against German bonds. The spread between Greek bonds and German bonds rose from 2.35 percentage points in 2009 to nearly 30 points by the end of 2011. Such marked spreads have lead to debate over whether financial markets have mispriced default risks before or after the start of the crisis [examples include Aizenman *et al.* (2012) and De Grauwe and Ji (2012)]. This paper contributes to the debate by exploring the extent to which a rational expectations model and macroeconomic fundamentals can explain the rising interest rates on sovereign debt in Greece.

This paper uses Bayesian methods to estimate and evaluate a real business cycle (RBC) model that allows for sovereign default. We estimate the model for Greece during the post-EMU period until the end of 2010. Two key findings emerge.

First, the rapid deterioration of confidence in Greek debt over 2011 is consistent with behavior from a simple rational expectations model. Using the estimated structural parameters, we compute model-implied default probabilities for Greece's debt-to-GDP ratios. We find that the probability of default on Greek debt remained close to zero from 2001 until 2009, when it began to rise sharply to the range of 5% to 10% by the fourth quarter of 2010. By the end of 2011, the model predicts a probability of default between 60-80%, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the quarterly real interest rate observed in Greece over 2011 is well within the forecast bands of our rational expectations model. These results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals.

Second, model comparisons based on Bayes factors strongly favor the nonlinear model specification with an endogenous probability of default over a log-linearized specification without default. In addition, some parameters are more precisely estimated with the nonlinear specification.

We consider a closed economy in which the government finances transfers and expenditures by collecting distortionary income taxes and issuing bonds. The bond contract is not enforceable and depends on the maximum level of debt that the government is politically able to service, a so-called 'fiscal limit.' At each period, if the level of government debt surpasses the effective fiscal limit, then the government reneges on a fraction of its debt. We assume that the effective fiscal limit is drawn from an underlying distribution.

We model the fiscal limit distribution with a logistical function and estimate parameters related to the distribution, so that fiscal limit is data-driven in our framework. Our approach abstracts from the government's strategic incentives to default, as developed in Eaton and Gersovitz (1981) and Arellano (2008). In those models, sovereign defaults are optimal decisions made by a benevolent social planner as responses to unlucky external shocks. Instead, our approach allows both bad policy and bad luck to contribute to a crisis, so that we can examine the general contribution of macroeconomic fundamentals to sovereign risks.

The economy switches between the default and no-default regimes endogenously, depending upon the level of government debt and the fiscal limit distribution. Thus, the model cannot be solved using a first-order approximation. Instead, it is solved using the monotone mapping method, and estimated using Bayesian inference methods and a sequential Monte Carlo approximation of the likelihood [similar estimation methods are used in Fernandez-Villaverde and Rubio-Ramirez (2007), Doh (2011), and Amisano and Tristani (2010)].

This paper is also related to a large empirical literature that studies the determinants of sovereign default risk premia through reduced-form panel regressions. Recent examples include Lonning (2000), Lemmen and Goodhart (1999), Codogno *et al.* (2003), Alesina *et al.* (1992), Bernoth *et al.* (2006), Haugh *et al.* (2009), Bernoth and Erdogan (2011), Abmann and Hogrefe (2009), and Maltritz (2011). This literature does not always fully agree on the relative importance of different factors in determining risk premia, suggesting the importance of country-specific macroeconomic fundamentals to explain sovereign risk premia. In addition, related work by Ostry *et al.* (2010) estimate historical fiscal responses to construct 'debt limits,' although these limits are backward-looking by construction. Our work builds upon the foundation Bi and Traum (2012), who provide a coherent framework for estimating forward-looking 'fiscal limits,' but do not examine the extent to which a rational expectations model can explain the interest rates surges on sovereign debt.

2. Model

Following Bi (2012), our model is a closed economy with linear production technology, whereby output depends on the level of productivity (A_t) and the labor supply (n_t).[1] Private consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint,

$$c_t + g_t = A_t n_t. \tag{1}$$

[1]Ceteris paribus, the assumption that the economy is open and all debt is held by foreigners raises the observed risk premium relative to the closed economy environment, as foreigners do not experience the negative wealth effects of debt and, in turn, have less incentive to hold debt. Thus, the estimates from our closed economy framework can be thought as the *lower* bound to estimates of the fiscal limit.

The deviation of technological productivity A_t from its steady state value A follows the $AR(1)$ process

$$A_t - A \;=\; \rho^A(A_{t-1} - A) + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2). \tag{2}$$

2.1 Government

The government finances lump-sum transfers to households (z_t) and exogenous, unproductive purchases by levying a tax (τ_t) on labor income and issuing one-period bonds (b_t). We denote q_t as the price of the bond in units of consumption at time t. For each unit of the bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At each period, there is an effective fiscal limit, specified in terms of the debt-to-GDP ratio (s_t^*), which represents the maximum debt burden the government is willing and able to repay. If current outstanding debt obligations, relative to GDP, are below this level, then the government fully repays its liabilities. However, if the current level is equal to or above this limit, then the government partially defaults on its obligations by a fixed δ share. Thus, the amount of unpaid bonds in any period (Δ_t), which we call default rate, is summarized by

$$\Delta_t = \begin{cases} 0 & \text{if } s_{t-1} < s_t^* \\ \delta & \text{if } s_{t-1} \geq s_t^* \end{cases}$$

We assume that the effective fiscal limit s_t^* is stochastic and drawn from an exogenous distribution, $s_t^* \sim \mathcal{S}^*$. Previous studies have shown that economic models give rise endogenously to a distribution for a fiscal limit through dynamic Laffer curves and the maximum amount of debt that the government is able to finance by tax revenue (examples include Bi (2012), Davig *et al.* (2010)). However, political considerations may make the government unwilling and unable to achieve this limit, due to unmodeled features such as the political inability to raise taxes. In the case of Greece, the protests against austerity measures in 2010 and 2011 suggest the relevance of such political considerations. Therefore, we simply take the fiscal limit's distribution as exogenously given. We model the cumulative density function of the fiscal limit distribution as a logistical function with parameters η_1 and η_2 dictating its shape.

$$p_{t-1} \equiv P(s_{t-1} \geq s_t^*) = \frac{\exp(\eta_1 + \eta_2 s_{t-1})}{1 + \exp(\eta_1 + \eta_2 s_{t-1})} \tag{3}$$

where s_t is defined as b_t/y_t, and p_{t-1} is the default probability associated with the debt-GDP ratio s_{t-1}. As shown in section 3.2.1, the estimated distribution reflects the economic and political fiscal limit distribution implied by the data.

The government's budget constraint is given by

$$\tau_t A_t n_t + b_t q_t = \underbrace{(1-\Delta_t)b_{t-1}}_{b_t^d} + g_t + z_t. \tag{4}$$

The tax rate and government spending evolve according to the rules,

$$\tau_t = \underbrace{(1-\rho^\tau)\tau + \rho^\tau \tau_{t-1} + \varepsilon_t^\tau}_{u_t^\tau} + \gamma^\tau \left(b_t^d - b\right) \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2) \tag{5}$$

$$g_t = \underbrace{(1-\rho^g)g + \rho^g g_{t-1} + \varepsilon_t^g}_{u_t^g} + \gamma^g \left(b_t^d - b\right) \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \tag{6}$$

with $AR(1)$ components being denoted as u_t^τ and u_t^g and x denoting the steady state level of any variables x_t. The non-distortionary transfers are modeled as a residual in the government budget constraint, exogenously determined by the AR(1) process,

$$z_t - z = \rho^z (z_{t-1} - z) + \varepsilon_t^z \qquad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2). \tag{7}$$

Since transfers are not included as an observable in our estimation, z_t can be thought of as a residual capturing all movements in government debt that are not explained by the model.

2.2 Household

With access to the sovereign bond market, a representative household chooses consumption (c_t), hours worked (n_t), and bond purchases (b_t) by solving,

$$\max \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t - h\bar{c}_{t-1}\right) + \phi \log(1-n_t)\right) \tag{8}$$

$$s.t. \qquad A_t n_t (1-\tau_t) + z_t - c_t = b_t q_t - (1-\Delta_t) b_{t-1} \tag{9}$$

The household's utility for consumption is relative to a habit stock given by a fraction of aggregate consumption from the previous period $h\bar{c}_{t-1}$ where $h \in [0,1]$. The household's first-order conditions are

$$\phi \frac{c_t - h\bar{c}_{t-1}}{1-n_t} = A_t(1-\tau_t) \tag{10}$$

$$q_t = \beta E_t \Big((1-\Delta_{t+1}) \frac{c_t - h\bar{c}_{t-1}}{c_{t+1} - h\bar{c}_t} \Big). \tag{11}$$

The bond price reflects the household's expectation about the probability and magnitude of sovereign default in the next period. The optimal solution to the household's maximization problem must also satisfy the transversality condition,

$$\lim_{j\to\infty} E_t \beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)} (1-\Delta_{t+j+1}) b_{t+j} = 0. \tag{12}$$

2.3 Model Solution

Other than the specifications for exogenous state variables, the core equilibrium conditions are

$$q_t = \frac{b_t^d + z_t + g_t - \tau_t A_t n_t}{b_t} \tag{13}$$

$$q_t = \beta(c_t - hc_{t-1}) E_t \frac{1-\Delta_{t+1}}{c_{t+1} - hc_t}. \tag{14}$$

The first equation is derived from the government budget constraint, while the second is from the household's first-order conditions. We use these conditions and the monotone mapping method (Coleman (1991), Davig (2004)) to solve for the decision rule of the bond price in terms of the state vector. At time t, the state vector is $(b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$, and the decision rule of the bond price can be written as $q_t = q(b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$. Appendix A discusses the solution procedure in detail.

3. Estimation

The model is estimated for Greece over the period 2001Q1-2010Q4. The estimation is over the post-EMU period, as interest rates during the pre-Euro period are susceptible to exchange rate risk from which our model abstracts. Five observables are used for the estimation: real output, the government spending-to-GDP ratio, the tax revenue-to-GDP ratio, the government debt-to-GDP ratio, and a 10-year sovereign real interest rate. For the estimation, we use percentage deviations of each observable from its steady state value. Figure 2 depicts the data used for estimation. Appendix B.1 provides a detailed description of the data.

3.1 Methodology

We estimate the model using Bayesian methods. The equilibrium system is written in the nonlinear state-space form, linking observables v_t to model variables x_t:

$$x_t = f(x_{t-1}, \epsilon_t, \theta) \tag{15}$$

$$v_t = Ax_t + \xi_t, \tag{16}$$

where θ denotes model parameters and ξ_t is a vector of measurement errors distributed $N(0, \Sigma)$. We assume that Σ is a diagonal matrix and calibrate the standard deviation of each measurement error to be 20% of the standard deviation of the corresponding observable variable.[2]

We use a particle filter to approximate the likelihood function. For a given sequence of observations up to time t, $v^t = [v_1, ..., v_t]$, the particle filter approximates the density $p(x_t|v^t, \theta)$ by applying a law of large numbers to a series of simulations using a swarm of particles x_t^i $(i = 1, ..., N)$ (see appendix B.2 for more details). The particle filter is applicable for nonlinear and non-Gaussian distributions, and it is increasingly used to estimate nonlinear DSGE models, to which class our model belongs. Recent examples include An and Schorfheide (2007), Fernandez-Villaverde and Rubio-Ramirez (2007), Amisano and Tristani (2010), Fernandez-Villaverde *et al.* (2011), and Doh (2011).

We combine the likelihood $L(\theta|v^T)$ with a prior density $p(\theta)$ to obtain the posterior density kernel, which is proportional to the posterior density. We assume that parameters are independent a priori. However, we discard any prior draws that do not deliver a unique rational expectations equilibrium, as we restrict the analysis to the determinacy parameter subspace.[3] We construct the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm (see appendix B.3 for more details). In each estimation, we sample 150,000 draws from the posterior distribution and discard the first 50,000 draws.[4] The likelihood is computed using 60,000 particles, and posterior analysis is conducted using every 25th draw from the posterior.

[2]Estimating measurement errors provides complications with nonlinear estimation techniques. See Doh (2011) for more discussion of the role of measurement error in nonlinear DSGE model estimation.

[3]A technical appendix of the authors provides more discussion on this point. Assuming a 30% annualized default rate, only 0.38% of the prior distribution falls outside the determinacy region.

[4]We use Fortran MPI code compiled in Intel Visual Fortran for the estimation. We use the computer server system at the Bank of Canada, each CPU of which uses Xeon CPU X5680 at 3.33GHz and has 23 processors with 64G RAM. One evaluation using the particle filter takes 5 seconds. These computational constraints limit the number of draws from the Metropolis-Hastings algorithm.

3.2 Prior Distributions

We impose dogmatic priors over some parameters, which are listed in table C. The discount rate is 0.99, so that the deterministic net interest rate is 1%.[5] We calibrate the household's leisure preference parameter ϕ such that a household spends 25% of its time working at the steady state. We calibrate the deterministic debt-to-GDP ratio, government spending-to-GDP ratio, and tax rate to the mean values of the data sample.

The priors for the remaining parameters are listed in table C. The prior for habit persistence h is similar to those in the linear DSGE estimation literature, e.g. Smets and Wouters (2007). For the remaining parameters, we first use ordinary least squares and estimate an AR(1) process for GDP and processes for government spending, the tax rate, and transfers given by equations (5)-(7).[6] The results are used as general guidance for the region of the parameter space for the ρ, σ, and γ parameters.

For the responses of government spending and taxes to debt, we form priors for the long run responses in terms of percentage deviations from steady state, that is

$$\gamma^{g,L} \equiv \frac{\bar{b}}{\bar{g}} \frac{\gamma^{g}}{(1-\rho^{g})}, \quad \gamma^{t,L} \equiv \frac{\bar{b}}{\bar{\tau}} \frac{\gamma^{\tau}}{(1-\rho^{\tau})}$$

These values are more comparable to estimates in the literature. Since determinacy is sensitive to the combination of the $\gamma^{\tau,L}$ and $\gamma^{g,L}$ parameters, we restrict the lower bound of the $\gamma^{\tau,L}$ ($\gamma^{g,L}$) prior to a value that ensures determinacy when only $\gamma^{\tau,L}$ ($\gamma^{g,L}$) finances debt.

For the standard deviations of shocks, we form priors for the standard deviations relative to relevant steady state variables: $\sigma_{k,p} \equiv \sigma_k/\bar{J}$ for $J = \{A, g, \tau, z\}$ and $k = \{a, g, \tau, z\}$. This gives standard deviations as percentage deviations, which provides more intuitive comparisons across values.

3.2.1 Fiscal Limit

We estimate one parameter from the fiscal limit distribution, given by equation (3). Given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters η_1 and η_2 can be uniquely determined by

$$\eta_2 = \frac{1}{\tilde{s}-\hat{s}} \log\left(\frac{\tilde{p}}{\hat{p}} \frac{1-\hat{p}}{1-\tilde{p}}\right), \quad \eta_1 \quad = \quad \log\frac{\tilde{p}}{1-\tilde{p}} - \eta_2 \tilde{s}. \tag{17}$$

[5] The mean of our data is 0.8% for Greece.

[6] We back out the model-consistent tax rate and transfers series implied by our observables for this exercise.

For a given default rate δ, $\tilde{s}$ and $\tilde{p}$ represent the default probabilities at the debt-GDP ratios of $\hat{s}$ and $\hat{p}$ respectively. Since they provide a more intuitive description about the fiscal limit distribution than η_1 and η_2, we fix $\tilde{p}$ and $\hat{p}$ at certain levels and estimate the corresponding debt-GDP ratios, $\tilde{s}$ and $\hat{s}$, instead of estimating η_1 and η_2 directly. We choose $\tilde{p} = 0.3$ and $\hat{p} = 0.999$. Unfortunately, given that defaults are not observed in our data sample, the data is unlikely to be informative about the upper bound of the distribution. Therefore, we estimate $\tilde{s}$ and fix the difference between $\tilde{s}$ and $\hat{s}$ to be 60% of steady-state output. This difference is chosen to capture the observation that once risk premia begin to rise, they do so rapidly.[7] Given the lack of guidance for the parameter $\tilde{s}$, we adopt a diffuse uniform prior over the interval 1.4 to 1.8, implying that the debt level associated with a 30% probability of default ranges from 140 to 180% of GDP.

3.2.2 δ Identification

To our knowledge, aside from Bi and Traum (2012), this paper is the only attempt to estimate a DSGE model of sovereign default. Thus, prior to estimating the model with real data, we performed several estimations with simulated data.[8] Unfortunately, the results revealed that we cannot jointly identify the default rate δ and the fiscal limit parameter $\tilde{s}$ when the data exclude observed defaults, since various combinations of δ and $\tilde{s}$ are consistent with the same risk premium. Given this limitation, we estimate our model for two different calibrations of δ: 0.05 and 0.075. These calibrations imply annualized rates of default δ^A of 20% and 30% respectively, which falls within the range of actual default rates in emerging market economies over the period 1983 to 2005, as documented by Bi (2012).

3.3 Posterior Estimates

Figure 1 plots the prior and posterior distributions for parameters when the annualized default rate is set to 30%. Table 2 compares the medians and 90% credible intervals of the posterior distributions estimated under both default rate specifications. For comparison, the means and 90% intervals from the priors are also listed. The data appears informative for all of the parameters, as the 90% credible intervals are smaller than those from the prior distributions.

The estimates of $\tilde{s}$ appear robust to the default rate calibration. If the annualized default rate δ^A is 30%, the debt-to-GDP ratio that is associated with a 30% probability of default $\tilde{s}$ is between 1.53-1.59, with the median being 1.56. If the annualized default rate is 20%, the

[7]The difference of 60% of output, albeit ad-hoc, should not change the key estimation results as the data are unlikely to be informative about the upper bound of distribution.

[8]The results are available in a technical appendix from the authors.

debt-to-GDP ratio that is associated with a 30% probability of default is between 1.51-1.58, with the median being 1.55. Holding $\tilde{s}$ constant, a higher default rate implies a larger risk premium in the model. For higher values of δ^A, agents expect to lose more face value of debt following a default. Thus, households demand a higher interest rate to compensate for this risk. However, the interest rate in the estimated model is the same across the default rate calibrations, as it comes from the data used in the estimation. As a result, the estimate of the response of government spending to debt $\gamma^{g,L}$ changes across the two default rate specifications. The posterior for $\gamma^{g,L}$ has more values concentrated at higher levels when $\delta^A = 0.3$ than the posterior when $\delta^A = 0.2$. Ceteris paribus, a larger $\gamma^{g,L}$ implies a stronger response of government spending to debt, which lowers the risk premium. Most other parameter estimates do not change across the default rate calibrations, although the posterior 90% intervals for habit h and the response of the tax rate to debt $\gamma^{\tau,L}$ are tighter when $\delta^A = 0.2$.

For comparison, we also list the estimates implied by a log-linearized version of our model without default. The system of equations for the log-linearized model is listed in appendix C.[9] Comparing the results to the nonlinear model, it appears that allowing default in the standard RBC model may help to identify the fiscal policy responses in Greece. The estimates from the linear model suggest that the data is not informative about $\gamma^{g,L}$ and σ^z, as the 90% credible interval from the posterior distribution mirrors that from the prior distribution, as shown in table (2). In addition, $\gamma^{\tau,L}$ is estimated less precisely than in the nonlinear specifications. More formal comparisons of the linear and nonlinear model specifications are provided in the next section.

4. Analysis

4.1 Model Fit

To examine how well the model fits the data, we compute smoothed estimates of model variables using the sequential monte carlo approximation of the forward-backward smoothing recursion.[10] Figure 2 compares the smoothed values from the nonlinear model with $\delta^A = 0.3$ and from the linearized model without default to the observable variables. For each specification, the fitted values are computed using the corresponding posterior median. The fit for most variables is quite accurate, with output and the real interest rate being the least precise.

[9] We use the Kalman filter to calculate the likelihood function and initialize the Metropolis-Hastings algorithm using the posterior mode and inverse Hessian at the posterior mode.

[10] See Doh (2011) for a detailed explanation of the procedure.

We also compute smoothed estimates of the measurement errors $E(\xi_t|v^T, \theta)$ and report their mean absolute values and relative standard deviations in table 3. For the estimation, the standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. For most observables, the actual estimated relative standard deviation is less than 20%, suggesting that the measurement error did not introduce many constraints for the model fit. The exception is the measurement error for the real interest rate in the nonlinear model and for output in the linear model. Table 3 also shows that mean absolute values of measurement error are close to zero.

Given that the different model specifications imply similar smoothed estimates, we perform posterior odds comparisons to determine which model is favored by the data. Bayes factors are used to evaluate the relative model fit of the linear and two nonlinear specifications. Table 4 presents the results. Bayes factors are based on log-marginal data densities calculated using Geweke's (1999) modified harmonic mean estimator with a truncation parameter of 0.5. The results demonstrate that the data strongly prefer the nonlinear model specifications over the linearized framework. In addition, it appears the data can only weakly distinguish the two nonlinear models with different calibrated default rate, as the log Bayes factor is 6.

4.2 Default Probability and Interest Rate Dynamics

In this section, we use the estimated structural estimates to evaluate the historical probability of default in Greece and explore the model's ability to forecast the fiscal deterioration in Greece since 2010, which is the end of our estimation period.

4.2.1 Default Risk

Figure 3 depicts model-implied historical sovereign default probabilities for Greece, based upon the estimated fiscal limit distribution when $\delta^A = 0.3$. Solid lines show the median and 90% posterior interval for the probability of default, calculated using the actual debt-to-GDP ratios from our debt and output observables for the estimated sample period.[11] In addition, the dashed lines provide the median and 90% posterior interval for default probabilities for the out-of-sample quarterly debt-to-GDP ratios observed in 2011.

Figure 3 shows that Greek debt had virtually zero probability of default from 2001 until 2009. Starting in 2009, the probability of default rose steadily, and ranged from 5-10% by the end of the estimated period 2010Q4. Estimates from the model's fiscal limit distribution thus

[11] For the estimation, we use data in terms of percentage deviations from the sample average. In contrast, we need level variables to back out model-implied probabilities of default. For model consistency, we convert the percentage deviations of the data to level variables using the steady state model variables.

reflect the deterioration in confidence in Greek debt in 2010. The model-implied probabilities, being consistent with the observed debt-to-GDP ratios in 2011, increase dramatically, ranging from 60-80% by 2011Q4. The striking build-up reflects the unsustainability of the Greek fiscal position and suggests imminent default. The predictions are consistent with the debt restructuring arrangements that took place at the beginning of 2012, which can be viewed as an effective default on Greek debt.

4.2.2 Out-of-Sample Interest Rate Forecasts

Over the course of 2011, the long term nominal interest rate in the secondary market for Greek government bonds rose from 9.1 percentage points in December 2010 to 21.2 in December 2011 based on BIS data, and to 31.2 based on Bloomberg data.[12] In this section, we examine the estimated model's ability to predict this sharp increase in the Greek interest rate.

To examine this issue, we use the posterior median estimates when $\delta^A = 0.3$ to simulate four quarters of time series 10,000 times starting from the fitted values for model variables in 2010Q4.[13] This gives a distribution for the forecasted path of the real interest rate in 2011. Figure 4 displays the median (blue, dotted line) and 90% interval (blue, dashed lines) of these model-implied interest rate forecasts for 2011. The figure also plots the path of the real interest rate implied from the data (black solid line for BIS data and dotted red line for Bloomberg data).[14]

Figure 4 shows that the surge in the real interest rate in Greece is well within forecast bands of our rational expectations model. Thus, it is possible for model forecasts to be consistent with the 2011 interest rate path, suggesting that the actual interest rate surge can be explained by macroeconomic fundamentals in a rational expectation framework.

The model-implied forecast band for 2011 ranges dramatically from approximately 6-35 percentage points. One might be tempted to conclude that model forecasts vary by bands too large to be of much use. To explore this issue, we repeat our forecasting exercise starting from the fitted values for model variables in 2006Q4. That is, we calculate the model-implied distribution for the interest rate path in 2007, assuming only information through 2006 was known. Figure 5 plots the median (blue, dotted line) and 90% interval (blue, dashed lines) of the model-implied interest rate forecasts, along with the path of the data (black solid line). Again, the actual interest rate path is well within the forecast band, but the band ranges a bit over 2 percentage points and is much tighter than the 2011 band. This is because

[12]Quarterly averages of Bloomberg and BIS data differ only in 2011Q3 and 2011Q4.

[13]See section 4.1 for details on the construction of fitted values.

[14]We constructed the real interest rate in 2011 using the same procedure documented in appendix B.1.

macroeconomic fundamentals were more stable in 2006Q4 than in 2010Q4, as reflected by the virtually zero probability of default in 2006Q4. Thus, while forecast bands appear tight in good times, large forecast bands may emerge at a turning point with deteriorating fiscal position, which is inherent in the nonlinear model.

The results suggest that the surge in the Greek interest rate premium and the rapid deterioration of confidence in Greek debt in 2011 are consistent with behavior from a simple rational expectations model, and that Greek debt was not mispriced in 2011.

5. Conclusion

This paper uses Bayesian methods to estimate the fiscal limit distribution and the associated sovereign default probability for Greece. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic fiscal limit, and sovereign default risk. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. We model the fiscal limit distribution with a logistical function, which illustrates the market's belief about the government's ability to service its debt at various debt levels.

Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data. We find that the probability of default on Greek debt remained close to zero from 2001 until 2009, when it began to rise sharply to the range of 5% to 10% by the fourth quarter of 2010. By the end of 2011, the model predicts a probability of default between 60-80%, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the Greek real interest rate in 2011 is within forecast bands of our rational expectations model. The results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals. Finally, model comparisons based on Bayes factors strongly favor our nonlinear model specification with an endogenous probability of default over a linearized specification without default.

In current ongoing research, we are estimating the model for other European countries, so as to allow cross-country comparisons of default probabilities. Although our nonlinear model allows interactions among fiscal policy instruments and the fiscal limit, it is only a first step to understand and estimate probabilities of default for developed countries. Exploration of other model features are worthy of attention, including the interaction of monetary and fiscal policies, the interaction of the financial sector and the government, and open economy considerations.

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A Solving the Nonlinear Model

Other than the end-of-period government debt, all other variables are either exogenous or can be computed in terms of the current state $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$.

$$\tau_t = u_t^\tau + \gamma^\tau\left(b_t^d - b\right) \tag{A.1}$$

$$g_t = u_t^g + \gamma^g\left(b_t^d - b\right) \tag{A.2}$$

$$z_t = (1-\rho^z)z + \rho^z z_{t-1} + \varepsilon_t^z \tag{A.3}$$

$$A_t = (1-\rho^A)A + \rho^A A_{t-1} + \varepsilon_t^A \tag{A.4}$$

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

Given the utility function, consumption is determined by,

$$c_t = \frac{\phi h c_{t-1} + (A_t - g_t)(1-\tau_t)}{1+\phi-\tau_t}. \tag{A.5}$$

We use the monotone mapping method to solve for the decision rule of the bond price in terms of the state vector. In terms of computation, the most time-consuming part is the loop iterations of the numerical integration in equation (14) of the text.

$$\begin{aligned} E_t\frac{1-\Delta_{t+1}}{c_{t+1}-hc_t} &= \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\int_{s_{t+1}^*}\frac{1-\Delta_{t+1}}{c_{t+1}-hc_t} \\ &= \left(1-\Phi(s_t \geq s_{t+1}^*)\right)\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1}{c_{t+1}-hc_t}\Big|_{\text{no default}} \\ &+ \Phi(s_t \geq s_{t+1}^*)\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1-\delta}{c_{t+1}-hc_t}\Big|_{\text{default}} \end{aligned} \tag{A.6}$$

Thus, the integration in Equation (A.6) can be re-written as

$$\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1}{c_{t+1}-hc_t} = \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\int_{\varepsilon_{t+1}^\tau}\frac{1+\phi-\tau_{t+1}}{(1-\tau_{t+1})(A_{t+1}-g_{t+1}-hc_t)} \tag{A.7}$$

$$= \int_{\varepsilon_{t+1}^\tau}\frac{1+\phi-\tau_{t+1}}{1-\tau_{t+1}}\int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g}\frac{1}{A_{t+1}-g_{t+1}-hc_t} \tag{A.8}$$

The logarithmal utility function helps to reduce the 4-dimension integration into 1- and 2-dimension integrations. The decision rule for government debt, $b_t = f^b(\psi_t)$, is solved in the following steps:

- Step 1: Discretize the state space ψ_t with grid points of $n_b = 25, n_c = 5, n_A = 5, n_g = 7, n_z = 12, n_\tau = 11$.[15] Make an initial guess of the decision rule f_0^b over the state space.

- Step 2: At each grid point, solve the following core equation and obtain the updated rule f_i^b using the given rule f_{i-1}^b. The integral in the right-hand side is evaluated as described above using numerical quadrature.

$$\frac{b_t^d + z_t + g_t - \tau_t A_t n(\psi_t)}{f_i^b(\psi_t)} = \beta(1-\Delta_{t+1})E_t \frac{c(\psi_t) - hc_{t-1}}{c(\psi_{t+1}) - hc(\psi_t)} \tag{A.9}$$

where $\psi_{t+1} = \left(\underbrace{(f_{i-1}^b(\psi_t), \Delta_{t+1})}_{b_t^d}, c_t, A_{t+1}, u_{t+1}^g, z_{t+1}, u_{t+1}^\tau\right)$.

- Step 3: Check the convergence of the decision rule. If $|f_i^b - f_{i-1}^b|$ is above the desired tolerance (set to $1e-5$), go back to step 2; other wise, f_i^b is the decision rule and used to evaluate the particle filter as described below.

B Estimation

B.1 Data Description

Five observables for Greece over the period 2001Q1-2010Q4 are used for the estimation: real output, the government spending to GDP ratio, the tax revenue to GDP ratio, the government debt to GDP ratio, and a 10-year real interest rate. This appendix provides documentation for the construction of these series. First, data for real GDP, government spending, tax revenue, government debt, and the 10-year interest rate are constructed as follows.

Real GDP. Constructed by dividing the nominal quarterly gross domestic product from the OECD quarterly National Accounts (using the expenditure approach, series B1_GE) by the gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Gov. Spending. Constructed using general government final consumption expenditure from the OECD quarterly National Accounts (series P3S13) divided by the gross domestic product deflator (constructed using the expenditure approach, series B13).

[15]The grid boundaries are fixed throughout the estimation to ensure the same solution precision for different parameter draws.

Real Tax Revenue. A quarterly tax revenue series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we construct a measure of total tax revenue by combining Eurostat quarterly series for tax receipts on income/wealth, production and imports, capital taxes, and social contributions. We seasonally adjust this series using Demetra+ and the tramo-seat RSA4 specification. Next, using an annual nominal tax revenue series from the OECD volume 90 (consisting of indirect and direct taxes and social security contributions, TIND + TY + SSRG), we interpolate a quarterly frequency series using the method of Chow and Lin (1971)[16] and the seasonally adjusted quarterly Eurostat tax revenue series for the interpolation. Finally, we construct a quarterly real tax revenue series by dividing the interpolated series by the OECD's gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Gov. Debt. A quarterly government debt series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we seasonally adjust using Demetra+ and the tramo-seat RSA4 specification the Eurostat quarterly series for nominal gross government consolidated debt. Next, using the annual nominal gross public debt series (under the Maastricht criterion) from the OECD volume 90, we interpolate a quarterly frequency series using the method of Chow and Lin (1971) and the seasonally adjusted quarterly Eurostat tax revenue series for the interpolation. Finally, we construct a quarterly real debt series by dividing the interpolated series by the OECD's gross domestic product deflator (constructed using the expenditure approach, series B13).

Real Interest Rate. To construct a 10-year real interest rate measure, we use data for the nominal interest rate, i_t (taken from the BIS) and the expected inflation rate, π_t^e. Our measure of expected inflation for Greece is the expected inflation series from the Survey of Professional Forecasts EU-area five year ahead expected inflation series. The gross real interest rate is constructed using the relation

$$R_t = \frac{1 + i_t}{1 + \pi_t^e}$$

Data for Estimation. We calculate the government spending to GDP ratio, tax revenue to GDP ratio, and government debt to GDP ratio by taking each real fiscal series described above and dividing by our real GDP series. The estimation uses these three series, along with the real GDP and real interest rate series described above. For each series, we transform the series into percentage deviations from its mean value over the period 2001Q1-2010Q4. In addition, the real GDP series is linearly detrended. The black sold lines of figure 2 graphs the observables.

[16]Forni *et al.* (2009) use a similar approach.

B.2 Particle Filter Algorithm

Let v^T denote $\{\hat{v}_t\}_{t=1}^T$, which evolves according to equations (15) and (16) in the text. To evaluate the likelihood function $L(\theta|v^T)$, we use a sequential Monte Carlo filter (specifically, the sequential importance resampling filter of Kitagawa (1996)). The algorithm is as follows:

- Step 1. Initialize the state variable x_0 by generating 60,000 values from the unconditional distribution $p(x_0|\theta)$. Denote these particles by x_0^i for $i = 1, ..., 60,000$. Draw $40,000$ values from standard normal distributions for each of the structural shocks (ϵ^A, ϵ^g, ϵ^t, ϵ^z) and 40,000 values from a standard uniform distribution for fiscal limit probabilities. Denote the vector of these particles by u^i. By induction, in period t these are particles $u^{t|t-1,i}$.

- Step 2. Construct $x^{t|t-1,i}$ using equation (15) in the text. Assign to each draw ($u^{t|t-1,i}$, $x^{t|t-1,i}$) a weight defined as:

$$w_t^i = \frac{1}{(2\pi)^{5/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(y_t - Ax^{t|t-1,i}\right)' \Sigma \left(y_t - Ax^{t|t-1,i}\right)\right] \tag{B.1}$$

- Step 3. Normalize the weights:

$$\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i}$$

  Update the values of $x^{t|t-1,i}$ by sampling with replacement 60,000 values of $x^{t|t-1,i}$ using the relative weights $\tilde{w}_t^i$ and the residual resampling algorithm.

- Repeat steps 2-3 for $t \leq T$.

The log-likelihood function is approximated by

$$L(\theta|v^T) \simeq \sum_{t=1}^T \ln\left(\frac{1}{60,000}\sum_{i=1}^{60,000} w_t^i\right) \tag{B.2}$$

B.3 MCMC Algorithm

The random walk Metropolis-Hastings algorithm used for estimation works as follows:

- Step 1. Compute the posterior log-likelihood for 500 draws from the priors. Call the draw with the highest posterior log-likelihood value θ^*.

- Step 2. Starting from θ^*, generate a MCMC chain using the following random-walk proposal density

$$\theta_{j+1}^{prop} = \theta_j^{prop} + c\mathcal{N}(0, \Lambda), \quad j = 1, ..., 100,000$$

where Λ is the covariance matrix of 500 draws from the priors and $c > 0$ is a tuning parameter set to determine the acceptance ratio.

- Step 3. Compute the acceptance ratio $\varphi = \min\left\{\frac{p(\theta_{j+1}^{prop}|v^T)}{p(\theta_j|v^T)}, 1\right\}$. Given a draw u from the standard uniform distribution. Then $\theta_{j+1} = \theta_{j+1}^{prop}$ if $u < \varphi$ and $\theta_{j+1} = \theta_j$ otherwise. Repeat for $j = 1, ..., 100,000$.

- Step 4. Update the random walk proposal density in the following way. Update Λ to be the covariance matrix from the previous draws $\{\theta_j\}_1^{100,000}$. Update θ^* to be the mean of previous draws $\{\theta_j\}_1^{100,000}$. Starting from the new θ^*, proceed through steps 2 and 3 for 150,000 draws from the new MCMC chain.

We burn the first 50,000 draws from the final MCMC chain and thin every 25 draws.

C Log-Linearized Model Equations

The log-linearized system of equations for the basic variant of the model without default are:

$$\hat{c}_t - \frac{1}{1+h} E_t \hat{c}_{t+1} + \frac{1-h}{1+h} \hat{R}_t = \frac{h}{1+h} \hat{c}_{t-1} \tag{C.1}$$

$$\frac{1}{1-h} \hat{c}_t + \frac{n}{1-n} \hat{n}_t - \hat{A}_t + \frac{\tau}{1-\tau} \hat{\tau}_t = \frac{h}{1-h} \hat{c}_{t-1} \tag{C.2}$$

$$\frac{c}{y} \hat{c}_t + \frac{g}{y} \hat{g}_t = \hat{A}_t + \hat{n}_t \tag{C.3}$$

$$\frac{b}{y} \hat{b}_t - \frac{g}{y} \hat{g}_t - \frac{z}{y} \hat{z}_t + \tau(\hat{\tau}_t + \hat{A}_t + \hat{n}_t) = R * \frac{b}{y} (\hat{R}_{t-1} + \hat{b}_{t-1}) \tag{C.4}$$

$$\hat{g}_t = (1-\rho^g) \hat{g}_{t-1} - \gamma^{g,L} (1-\rho^g) b_{t-1} + \sigma_{g,p} \epsilon_t^g, \;\; \epsilon_t^g \sim N(0,1) \tag{C.5}$$

$$\hat{\tau}_t = (1-\rho^\tau) \hat{\tau}_{t-1} + \gamma^{\tau,L} (1-\rho^\tau) b_{t-1} + \sigma_{\tau,p} \epsilon_t^\tau, \;\; \epsilon_t^\tau \sim N(0,1) \tag{C.6}$$

$$\hat{z}_t = (1-\rho^z) \hat{z}_{t-1} + \sigma_{z,p} \epsilon_t^z, \;\; \epsilon_t^z \sim N(0,1) \tag{C.7}$$

$$\hat{A}_t = (1-\rho^a) \hat{A}_{t-1} + \sigma_{a,p} \epsilon_t^a, \;\; \epsilon_t^z \sim N(0,1) \tag{C.8}$$

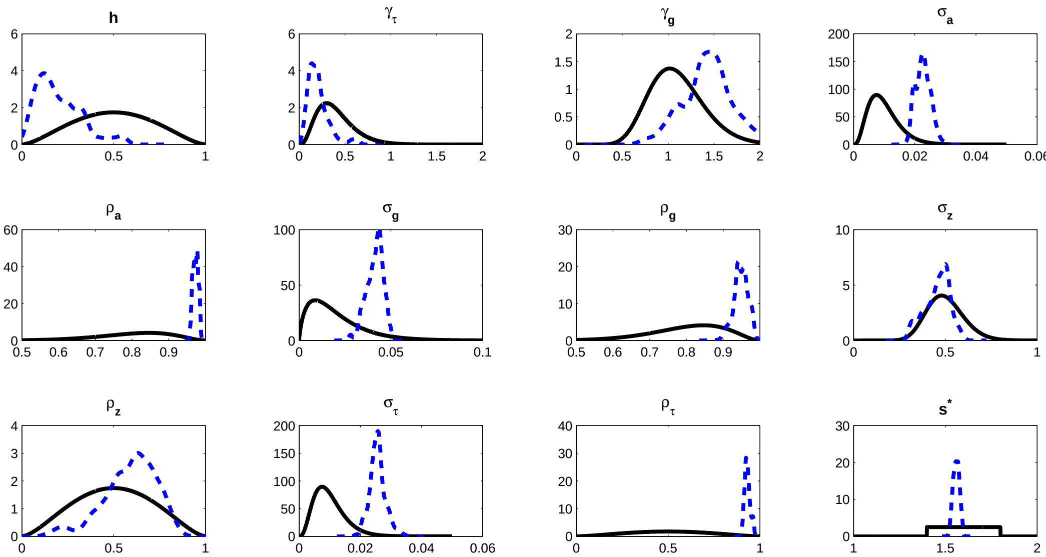

Figure 1: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with $\delta^A = 0.3$.

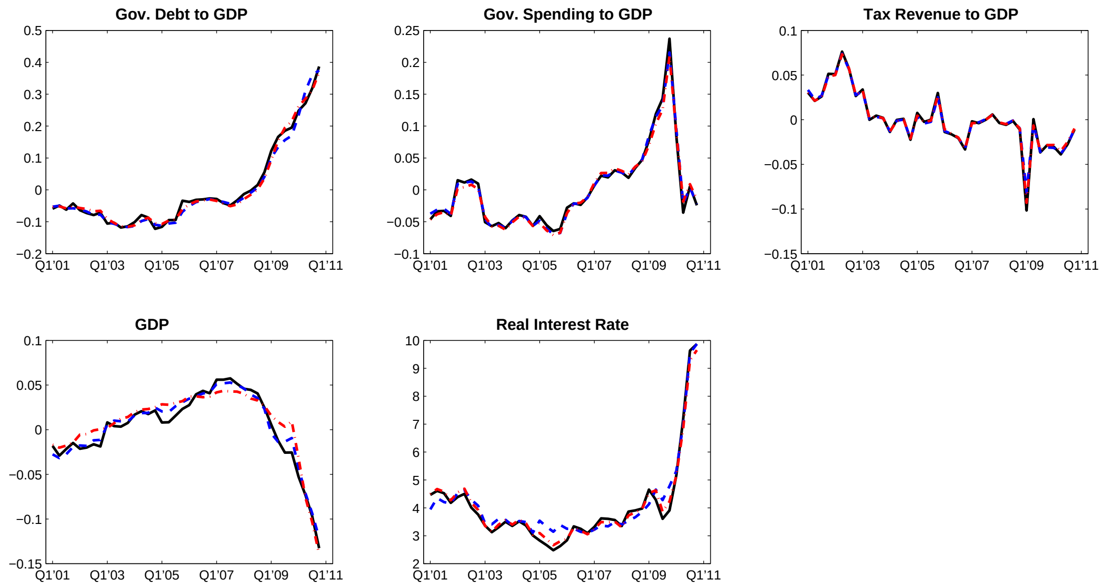

Figure 2: Fitted values for various estimations for Greece. Black, solid lines: data. Blue, dashed lines: Nonlinear model with $\delta^A = 0.3$. Red, dotted-dashed lines: Linear model.

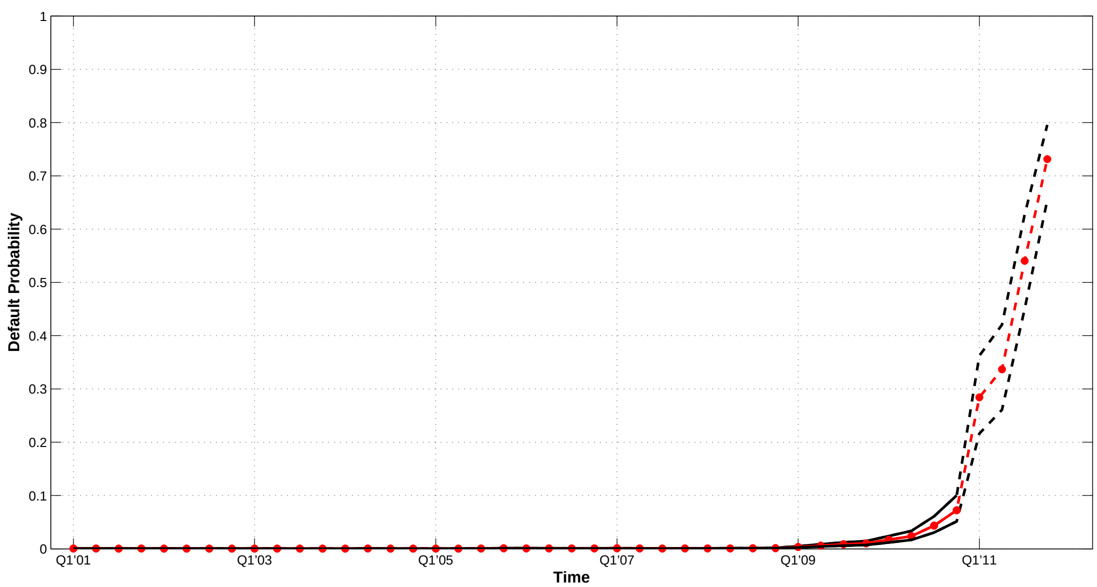

Figure 3: Model-implied sovereign default probabilities for Greece. Solid lines denote the median and 90% confidence interval probabilities for in-sample debt-to-GDP ratios. Dashed lines denote the median and 90% confidence interval probabilities for out-of-sample debt-to-GDP ratios.

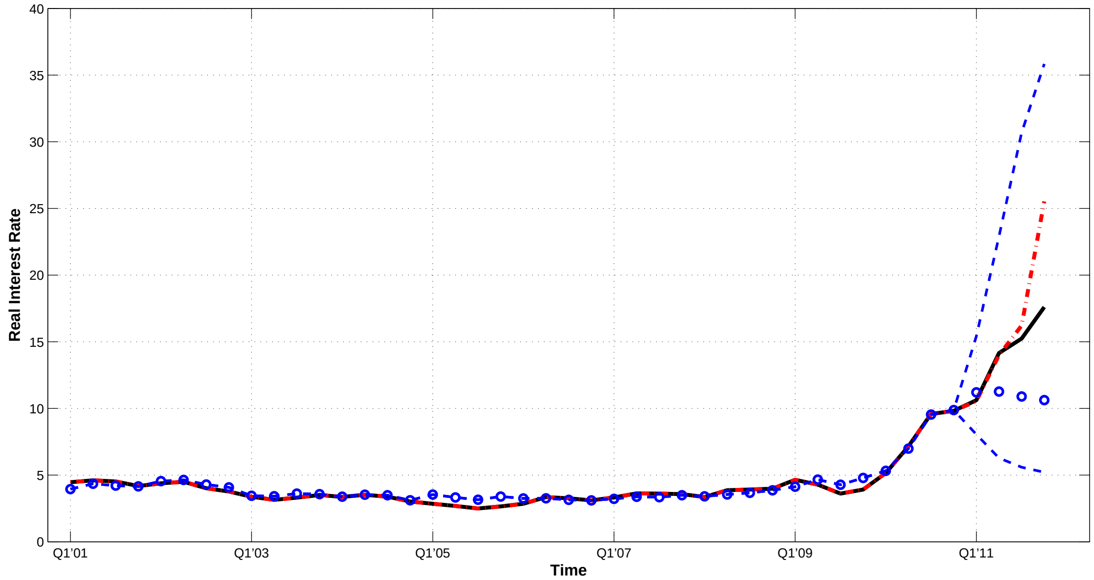

Figure 4: Data versus fitted and forecast values for the Greek interest rate. The median (blue, dotted line) and 90% interval (blue, dashed lines) of model-implied interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates. The black solid line shows BIS data, and red dotted line shows Bloomberg data.

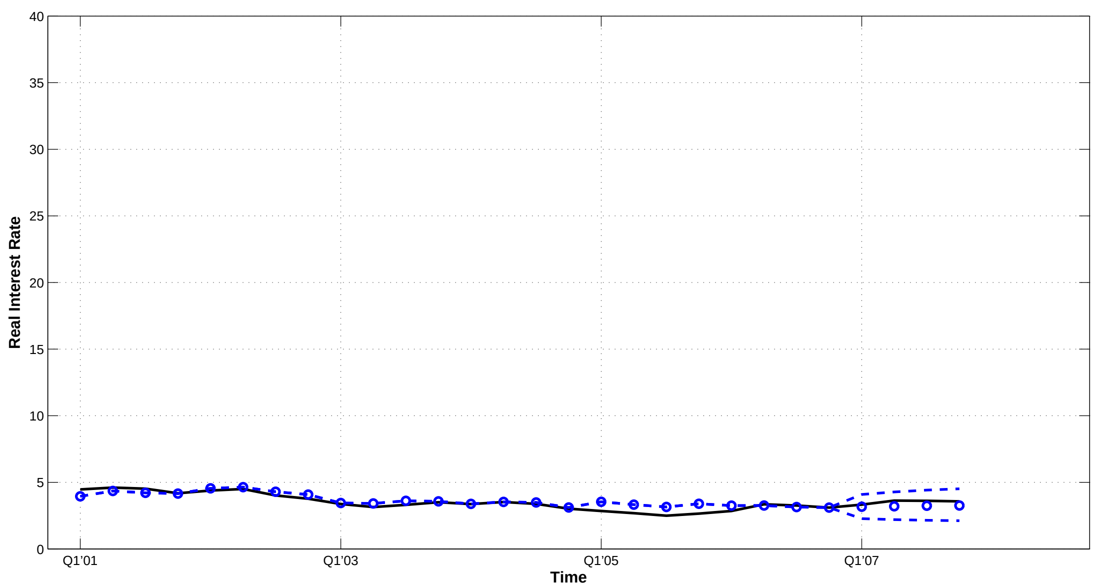

Figure 5: Data versus fitted and forecast values for the Greek interest rate. The median (blue, dotted line) and 90% interval (blue, dashed lines) of model-implied interest rate forecasts for 2007 are calculated based on the posterior median parameter estimates. The black solid line shows BIS data.

Table 1: Calibration and Priors for Greece

Calibration

β	0.99
$\bar{n}$	0.75
$\bar{g}/\bar{y}$	0.181
$\bar{b}/\bar{y}$	1.095*4
τ	0.333

Priors

	Function	Mean	St. Dev.
h	Beta	0.5	0.2
$\tilde{s}^*$	Uniform	1.6	0.013
$\gamma^{\tau,L}$	Gamma	0.4	0.2
$\gamma^{g,L}$	Gamma	1.1	0.3
ρ^a	Beta	0.8	0.1
ρ^g	Beta	0.8	0.1
ρ^τ	Beta	0.5	0.2
ρ^z	Beta	0.5	0.2
$\sigma_{a,p}$	Gamma	0.005	0.01
$\sigma_{g,p}$	Gamma	0.02	0.015
$\sigma_{\tau,p}$	Gamma	0.005	0.01
$\sigma_{z,p}$	Gamma	0.5	0.1

Table 2: Greece Estimates.

	Prior		**Posterior: $\delta^A = 0.3$**		**Posterior: $\delta^A = 0.2$**		**Posterior: Linear**	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	0.5	[0.17, 0.83]	0.17	[0.05, 0.47]	0.11	[0.03, 0.26]	0.73	[0.59, 0.83]
$\tilde{s}^*$	1.6	[1.42, 1.78]	1.56	[1.53, 1.59]	1.55	[1.51, 1.58]	-	-
$\gamma^{\tau,L}$	0.4	[0.14, 0.78]	0.19	[0.07, 0.46]	0.36	[0.27, 0.50]	0.61	[0.30, 1.04]
$\gamma^{g,L}$	1.1	[0.66, 1.64]	1.44	[0.96, 1.88]	1.08	[0.81, 1.22]	1.09	[0.66, 1.63]
ρ^a	0.8	[0.61, 0.94]	0.97	[0.96, 0.98]	0.97	[0.96, 0.98]	0.95	[0.94, 0.96]
ρ^g	0.8	[0.61, 0.94]	0.95	[0.91, 0.98]	0.93	[0.90, 0.96]	0.96	[0.91, 0.98]
ρ^z	0.5	[0.17, 0.83]	0.61	[0.32, 0.79]	0.64	[0.52, 0.87]	0.63	[0.39, 0.82]
ρ^τ	0.5	[0.17, 0.83]	0.93	[0.91, 0.96]	0.95	[0.93, 0.97]	0.93	[0.90, 0.95]
$\sigma_{a,p}$	0.01	[0.003, 0.02]	0.023	[0.019, 0.027]	0.019	[0.014, 0.025]	0.019	[0.015, 0.024]
$\sigma_{g,p}$	0.02	[0.003, 0.05]	0.042	[0.033, 0.048]	0.040	[0.031, 0.048]	0.04	[0.034, 0.052]
$\sigma_{z,p}$	0.5	[0.35, 0.68]	0.47	[0.32, 0.56]	0.38	[0.35, 0.43]	0.50	[0.38, 0.65]
$\sigma_{\tau,p}$	0.01	[0.003, 0.02]	0.026	[0.021, 0.030]	0.025	[0.023, 0.029]	0.022	[0.018, 0.027]

Table 3: Smoothed estimates of measurement error.

Greece		$\frac{b_t}{y_t}$	$\frac{g_t}{y_t}$	$\frac{T_t}{y_t}$	y_t	R_t
Nonlinear $\delta^A = 0.3$	mean absolute value	0.01	0.004	0.002	0.005	0.001
	relative standard deviation	0.12	0.10	0.11	0.16	0.22
Linear	mean absolute value	0.01	0.006	0.002	0.01	0.0003
	relative standard deviation	0.09	0.13	0.08	0.29	0.10

Table 4: Model Fit Comparisons

Model Specification	Bayes Factor Rel. to M1
M1: Nonlinear Model w/ $\delta^A = 0.3$	1
M2: Nonlinear Model w/ $\delta^A = 0.2$	exp[6]
M3: Linear	exp[53]