Smoothing the Shocks of a Dynamic Stochastic General Equilibrium Model

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The 2001 recession displayed unique characteristics in comparison to other recessions. Although moderate in terms of the decline in output, the recession was unique in that the contraction in measured output was driven almost entirely by retrenchment in business capital spending. Consumer spending growth remained positive, and residential investment maintained a very high level. Real gross domestic product (GDP) declined just 0.2 percent from its peak in the fourth quarter of 2000 to its trough in the third quarter of 2001. During this period, business investment in equipment and software fell 8.0 percent while consumer spending and residential investment grew 1.1 and 2.4 percent, respectively. Certain aspects of the recession, however, were fairly typical. The drop in payroll employment over the course of the recession was comparable to that in previous recessions—1.67 million, or 1.3 percent of total employment. In addition, as in every post–World War II recession, inflation rose just prior to the onset of the recession.

Just as the recession itself was in some ways unique, so too was the recovery that followed. Most recessions are followed by strong recoveries with above-trend real GDP growth and a turnaround in the labor market. The recovery following the 2001 recession, however, was characterized by moderate, uneven growth in output and employment losses that continued well after the end of the recession. Real GDP growth was just 2.3 percent in 2002, and employment declines continued into May 2003, totaling another 1 million jobs. On the other hand, core inflation, low by historical standards as the downturn began, moderated during this recovery period, consistent with previous episodes. The characteristics of the 2001 recession and subsequent recovery have raised many questions about the conventional wisdom for post–World War II U.S. business cycles.

If we believe that recessions are caused by external shocks to the economy, what type of perturbations or shocks affected U.S. output and inflation during the 2001 recession? Did these shocks persist after the end of the recession and into the recovery period in 2002? Given the unique characteristics of the 2001 recession, would we expect that the shocks to the economy were different from those that affected output in previous recessions? And, by the same token, would we expect that these shocks were the same as those that affected the path of inflation in previous recessions?

Finding an internally consistent and reasonable answer to those questions is important for several reasons. First, depending on the type of shocks affecting the economy at any particular time, the fiscal or monetary authorities may decide to respond. Second, if they do respond, their policy may depend on what forces drove the economy to that particular state. Hence, isolating and measuring such factors are crucial. Unfortunately, because of their nature, these perturbations cannot be observed, so they must be estimated. Clearly, these estimations are conditional on the framework used.

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In this article, we use a general equilibrium model as the framework to recover, or “back out,” these shocks. In this general equilibrium economy, four shocks affect the economy: a productivity shock, a demand shock, a markup shock, and a monetary policy shock. The parameters of the model are estimated so that the measurable variables of the model fit the observed data as closely as possible. Finally, we back out the realizations of the shocks, which are not observable, and match the model’s measurable variables to the observed data.

The next section describes the methodology used in this article and compares it with the more traditional approaches used in the literature. The article then describes a state-space representation, demonstrates how any model written in this form can be estimated using a Kalman filter, and explains how any unobservable variables of the model can be recovered from the data using the smoothing algorithm built into the Kalman filter. Next, we describe the sticky-price and sticky-wage model written in state-space form so that the Kalman filter can be used and the structural shocks can be backed out. Finally, we report and comment on our results.

The Methodology

Historically, the standard methodology for estimating and measuring economic shocks has used a structural vector autoregression (SVAR), which is a statistical relationship between a set of measured economic variables. SVAR models make explicit assumptions about the relationship between these variables. Each one of the measured variables is affected by actual observations of the rest of the variables, past observations of the whole set of variables, and unobservable economic structural shocks. A main supposition in these models is that the structural shocks affect only one of the variables contemporaneously—that is, the shocks are uncorrelated. Unfortunately, such models cannot be estimated directly. Instead, the researcher must estimate what is called a reduced-form vector autoregression.

In reduced-form VARs, each of the measured variables is exclusively driven by past observations of the whole set of measured variables and innovations that can be correlated (those that may affect more than one of the variables contemporaneously). It is important to highlight the difference between structural shocks in a structural VAR and innovations in a reduced-form VAR. While structural shocks are uncorrelated to each other (that is, they affect only one of the measured variables contemporaneously), innovations may affect more than one contemporaneous measured variable.

The next step is to map these estimated innovations onto the uncorrelated structural shocks. Unfortunately, there is more than one way to do this. To pick one of the possible maps, the researcher must make "identification assumptions," which are normally based on "out-of-the-model" economics. Therefore, although this technique is very easy to apply and analyze, it relies on model identification assumptions outside the model to estimate any structural shocks. We find the identification assumptions approach unsatisfactory because different assumptions may generate notably different structural shock estimates.

To avoid this problem, in this article we back out the shocks directly from a general equilibrium model. Because a general equilibrium model has the structural shocks built in, it is already a structural model, so we do not need to rely on any outside-the-model assumption to recover the structural shocks. Instead, we show how a general equilibrium model can be written in a state-space representation. Thus we can estimate its structural parameters, those defining preferences and technology, using a Kalman filter. An interesting feature of Kalman filtering is that it allows us to back out any unobservable variable of the model through a smoothing algorithm.

Many general equilibrium models can be used for estimating structural shocks. In this case, we choose a sticky-price and sticky-wage model like the one described by Erceg, Henderson, and Levin (2000) and extended by Rabanal and Rubio-Ramírez (2003a) to incorporate more structural shocks. We choose this particular version of the model for two reasons: First, this model has four structural shocks—productivity, demand, markup, and monetary policy—an uncommonly large number for a standard general equilibrium model. Using such a large number of structural shocks allows us to perform a deeper analysis of the perturbations that affected the U.S. economy in past recessions. Second, as Rabanal and Rubio-Ramírez (2003a) show, this particular general equilibrium model fits the U.S. data from 1964 up to today.

Most of the technical details of the Kalman filter and the smoothing algorithm are well known in the literature. In this article, we highlight only the main steps.[1]

The State-Space Representation

To recover the structural shocks from a general equilibrium model, we need two main ingredients. First, we need a general equilibrium model in which those shocks are specified. (We introduce the model and the structural shocks in the next section.) Second, we need to recover those shocks. We use a Kalman filter to estimate the shocks.[2] A Kalman filter allows us to estimate the parameters of the model so that the observed variables fit the data as closely as possible. Then, given those parameter estimates and the data, we back out a realization of the unobservable shocks that make the model's observed variables match the observable data.

To begin, we write the general equilibrium model in such a form that a Kalman filter can be implemented. This form is called the state-space representation, which is defined in the following way. Let η_t be the $(n \times 1)$ vector of the observed variables at date t, and let ξ_t be the $(r \times 1)$ vector of unobserved variables at date t. (This vector is also called the state vector.) The observed variables are those that can be measured

1. For a more detailed presentation of the estimation and smoothing algorithm, see Hamilton (1994). See Bauer, Haltom, and Rubio-Ramírez (2003) for a description of how to implement them in the case of a general equilibrium model.
2. A Kalman filter is a set of equations that provides an efficient computational means to estimate the state of a process in a way that minimizes the mean of the squared error.

while the unobserved variables cannot be measured but are hypothesized to be important. For example, imagine we want to explain consumer demand for a certain item. If we model consumer demand as a function of the price of the item and the willingness to buy it, the price will be an observable variable of the model while the willingness to buy will be unobservable.

In general, the state-space representation of a system is

(1) $\xi_{t+1} = F\xi_t + v_{t+1}$, and

(2) $\eta_t = H'\xi_t + w_t$,

where F and H' are matrices of the needed dimensions.[3] Equation (1) is called the state equation, and equation (2) is the observed equation. The variables v_t and w_t are uncorrelated normally distributed white noise vectors.[4] Therefore,

$$E(v_t v_\tau') = Q \text{ for } \tau = t, 0 \text{ otherwise,}$$

$$E(w_t w_\tau') = R \text{ for } \tau = t, 0 \text{ otherwise, and}$$

$$E(w_t v_\tau') = 0 \text{ for all } t, \tau.$$

The assumption that v_t and w_t are normally distributed is very important. Given that equations (1) and (2) are linear, the normality of v_t and w_t implies the normality of all the other variables in the system. This normality assumption underlies the main Kalman filter algorithm steps.[5]

Assume we want to explain the observed data $\eta^T = \{\eta_1, \eta_2, ..., \eta_t\}$. Once the model has been written in this form, Bauer, Haltom, and Rubio-Ramírez (2003) show how to write the likelihood function of η^T:

(3) $\ell(\eta^T | F, H', Q, R) = \Pi_{t=1}^{T} \ell(\eta_t | \eta^{t-1} F, H', Q, R)$.

We can either maximize this function and report the maximum likelihood estimates or, after specifying priors, report the posterior distributions. In the model, the structural shocks are not observable, so they will be part of ξ_t. Bauer, Haltom, and Rubio-Ramírez (2003) show how the Kalman filter can be used to obtain $(\xi_{t|T})_{t=1}{}^{T}$, where $\xi_{t|T} = E(\xi_t | \eta^T)$ is the linear projection of ξ_t on η^T and a constant. This projection is very important because it reveals which estimate of ξ_t is best, given all the observations. Therefore, since the structural shocks will be part of ξ_t, this projection is the main object of this article. Because the model is linear, the linear projection of variable ξ_t on η^T is the best estimate of ξ_t given η^T. Hence, this object will allow us to obtain the best estimate of any component of ξ_t at any moment in time given all the observed data. Since the structural shocks belong to ξ_t, we will be able to obtain the best estimate of the shocks.

The General Equilibrium Model

In this section we describe a model with sticky prices and sticky wages that we use to recover the structural shocks that affected the U.S. economy from 1954 to today. The model provides context for our main concern: to measure the influence of the respective perturbations in explaining inflation and output and to provide reference to the economic perturbations. At the same time, we want to understand how monetary policy responds to changes in the economic outlook and how much of an impact those policies have had. Therefore, we need a model in which monetary policy has real effects.

Until recently most macroeconomic models in which monetary policy has real effects were based on the assumption that agents were not rational. A new class of New Keynesian models combines Keynesian elements, such as rigidities and monopolistic power, with the rational-agents framework. The simplest version of this class of models considers only "sticky" prices (prices that adjust only slowly to market shortages or surpluses), but such models do not seem to be able to reproduce the persistence of inflation observed in the data. As Rabanal and Rubio-Ramírez (2003b) show, a model with both sticky prices and sticky wages can more closely replicate the observed inflation persistence.

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Another important feature of our model is that it explicitly defines four structural shocks: a productivity shock that affects labor productivity, a demand shock that affects demand for consumption, a monetary policy shock that affects interest rates, and a markup shock that affects contemporaneous inflation. Four perturbations is a large number for a general equilibrium model, a number that allows us to distinguish between different causes of the business cycle. Therefore, in this section, we present a dynamic stochastic general equilibrium model with monopolistic competition in the goods and labor markets, as the one described in Erceg, Henderson, and Levin (2000), and with both sticky prices and sticky wages.

The model consists of (1) a continuum of infinitely lived households, each selling a type of labor that is an imperfect substitute for other types; (2) a continuum of intermediate-good producers, each producing a specific good that is an imperfect substitute for other goods; and (3) a continuum of competitive final-good producers. The model assumes that intermediate-good producers and households face restrictions in the price- and wage-setting processes, respectively, as in Calvo (1983). Four types of exogenous shocks are considered: a technology shock, a monetary policy shock, a price markup shock, and a preference or demand shock. Households have access to complete markets so that we can abstract from distributional issues between those households that reset wages optimally and those that do not.[6] In the following discussion, the lowercase variables denote logarithmic deviations from the steady-state value.[7]

First, the Euler equation relates consumption, c_t, with the nominal rate of interest, r_t, inflation, Δp_t, and demand shock, g_t, in the following way:

$$c_t = E_t c_{t+1} - \sigma(r_t - E_t \Delta p_{t+1} + E_t g_{t+1} - g_t),$$

where $\sigma > 0$ is the degree of risk aversion and E_t is the expectation operator. Therefore, the higher the nominal interest rate, the lower tomorrow's expected inflation, or the

3. For examples of state-space representations of linear models, see Hamilton (1994, chap. 13).
4. This assumption can be relaxed.
5. For more details, see Bauer, Haltom, and Rubio-Ramírez (2003).
6. The model is not described fully here. We consider only the log-linear approximated equations that describe the symmetric equilibrium. For a full description, see Rabanal and Rubio-Ramírez (2003a).
7. For every variable X_t, we define the log-linear approximation as $x_t = \log X_t - \log(X^{SS})$, where X^{SS} is the variable's steady-state value.

lower the demand shock, the lower the consumption today. The intuition for these relations should be clear because (1) if the nominal interest rate is high, the household wants to save more (consume less), (2) if it expects low inflation, tomorrow's consumption will be cheaper than today's consumption, and (3) a negative demand shock reduces the household's desire to consume today.

Second, the production function relates output, y_t, with a productivity shock, a_t, and hours worked, n_t:

$$y_t = a_t + (1 - \delta)n_t,$$

where δ belongs to the open interval (0, 1).[8]

The marginal cost, mc_t, is related to wages, w_t, prices, p_t, hours worked, and output as

$$mc_t = w_t - p_t + n_t - y_t.$$

Therefore, the lower the output per hour or the higher the wage, the higher the marginal cost. On the other hand, the lower the price, the higher the marginal cost.

The marginal rate of substitution, mrs_t, between consumption and hours worked takes the form

$$mrs_t = (1/\sigma)c_t + \gamma n_t - g_t,$$

where $\gamma > 0$ determines the elasticity of the labor supply.

The pricing decision of the firm under the Calvo-type restriction delivers the following forward-looking equation for inflation:

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \kappa_p(mc_t + \lambda_t),$$

where $\kappa_p = (1 - \delta)(1 - \theta_p\beta)(1 - \theta_p)/\{\theta_p[1 + \delta(\varepsilon - 1)]\}$; λ_t is a markup shock; $\varepsilon = \lambda/(\lambda - 1)$, where λ is the steady-state value of the markup shock; θ_p is the probability that firms are allowed to change their prices under the Calvo-type restriction; and β is the discount factor of households. This equation, known as the New Keynesian Phillips curve, relates inflation today with expected inflation tomorrow, marginal cost, and the markup shock. If firms expect a large increase in prices tomorrow, if they face a large marginal cost today, or if they have a great deal of price power (reflected in a high markup shock), they will demand a large increase in today's prices.

The nominal wage growth equation, Δw_t, is

$$\Delta w_t = \beta E_t \Delta w_{t+1} + \kappa_w[mrs_t - (w_t - p_t)],$$

where $\kappa_w = (1 - \theta_w)(1 - \beta\theta_w)/[\theta_w(1 + \varphi\gamma)]$, θ_w is the probability that the households are allowed to change their wages under the Calvo-type restriction, and $\varphi > 0$ is the elasticity of substitution between different types of labor.

This equation relates nominal wage growth today with tomorrow's expected nominal wage growth, the marginal rate of substitution, and real wages. Therefore, if households expect a large increase in wages tomorrow or if they face a large difference between marginal utility and real wages, they will demand a large increase in today's wages.

This equation is very similar to the New Keynesian Phillips curve. Producers and households face similar Calvo-type restrictions when trying to change prices or wages.

The only difference is that producers are allowed to change prices with probability θ_p while households can change wages with probability θ_w.

One of the most important ingredients of the model is the specification of the monetary policy rule. We follow the tradition in the sticky-prices and sticky-wages literature and specify a Taylor rule for the nominal interest rate. Therefore today's nominal interest rate relates last period's rate, inflation, and the output gap in the following way:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\gamma_\pi \Delta p_t + \gamma_y y_t) + ms_t,$$

where ρ_r, which belongs to the open interval (0, 1), is an interest rate–smoothing parameter, γ_π is the elasticity of the nominal interest rate to the inflation rate, and γ_y is the elasticity of the nominal interest rate to the output gap.[9] Hence, the higher inflation or the output gap, the higher the nominal interest rate today. Also note that we include a monetary policy shock, ms_t, which will explain any departure of the actual interest rate from the specified rule. Finally, we note that negative monetary policy shocks imply a loosening of monetary policy with respect to the Taylor rule.

We also specify the law of motion for real wages:

$$w_t - p_t = w_{t-1} - p_{t-1} + \Delta w_t - \Delta p_t.$$

Finally, since there is no capital, output should equal consumption: $c_t = y_t$.

The last, but very important, step is to specify the law of motion of the four structural shocks as follows:

$$a_t = \rho_a a_{t-1} + \varepsilon^a_t,$$

$$g_t = \rho_g g_{t-1} + \varepsilon^g_t,$$

$$ms_t = \varepsilon^m_t, \text{ and}$$

$$\lambda_t = \varepsilon^\lambda_t,$$

where each innovation, ε^i_t, follows an $N(0, \sigma^2_i)$ distribution. Therefore, we allow both productivity and demand shocks to have persistence while monetary policy and markup shocks are independent and identically distributed.

Let $\varepsilon_t = (\varepsilon^a_t, \varepsilon^{ms}_t, \varepsilon^\lambda_t, \varepsilon^g_t)'$ and $\Theta = (\theta_p, \theta_w, \sigma_a, \sigma_{ms}, \sigma_\lambda, \sigma_g, \rho_r, \rho_a, \rho_g, \gamma_\pi, \gamma_y, \gamma, \sigma)'$ be the vector of structural parameters we are going to estimate.

The model does not have capital, and it is very difficult to estimate both β and δ. Second, it is not possible to simultaneously estimate φ and θ_{w}. Third, the same problem appears between λ and θ_{p}. Because of these difficulties, we set the following parameters: $\beta = 0.99$, $\varphi = 6$, $\lambda = 1.2$, and $\delta = 0.36$. Since we use quarterly data, $\beta = 0.99$ implies an annualized interest rate of 4.1 percent, $\varphi = 6$ implies a steady-state wage markup of 20 percent, $\lambda = 1.2$ implies a steady-state price markup of 20 percent over marginal cost, and $\delta = 0.36$ matches the calibrated share of labor in output.

8. As the production function makes clear, we do not consider capital in this model. We recognize this simplification should be relaxed, and we plan to do that in future research.
9. An interesting question is why the monetary authority needs to smooth the rate. Chugh (2003) shows how consumption externalities can justify this smoothing parameter.

The State-Space Representation of the Model

Having outlined the general equilibrium model to be used to recover the structural shocks, we now need to write the solution of the model in state-space form. We are then able to use a Kalman filter to estimate the parameters of the model (that is, Θ) and later back out the shocks. This process is not difficult, but it is rather cumbersome, so we highlight only the main step here, leaving a more detailed description for the appendix.

The model attributes declines in output to negative demand shocks and increases in output to positive productivity shocks.

To solve the general equilibrium model, we first use the Uhlig (1999) algorithm to eliminate the expectation operator and allow us to write a law of motion for the main variables of the model. The second step—writing that law of motion in state-space form to be able to use the Kalman filter—is described in Rabanal and Rubio-Ramírez (2003a) and Bauer, Haltom, and Rubio-Ramírez (2003).

The result of these two steps is the following state-space representation of the model's solution. If

$$x_t = (w_t - p_t, r_t, \Delta p_t, \Delta w_t, y_t)',$$

$$\mu_t = (n_t, mc_t, mrs_t, c_t)', \text{ and}$$

$$z_t = (a_t, ms_t, \lambda_t, g_t)',$$

the model's state-space representation is

$$\xi_t = F\xi_{t-1} + v_t, \text{ and}$$

$$\eta_t = H'\xi_t + w_t.$$

where $\xi_t = (x_t', z_t')'$ and $\eta_t = (w_t - p_t, r_t, \Delta p_t, y_t)'$. Hence, our observables will be real wages, the nominal interest rate, price inflation, and output.

Bauer, Haltom, and Rubio-Ramírez (2003) show that $w_t = 0$ and that F, H', and Q depend on the structural parameters of the model, Θ.

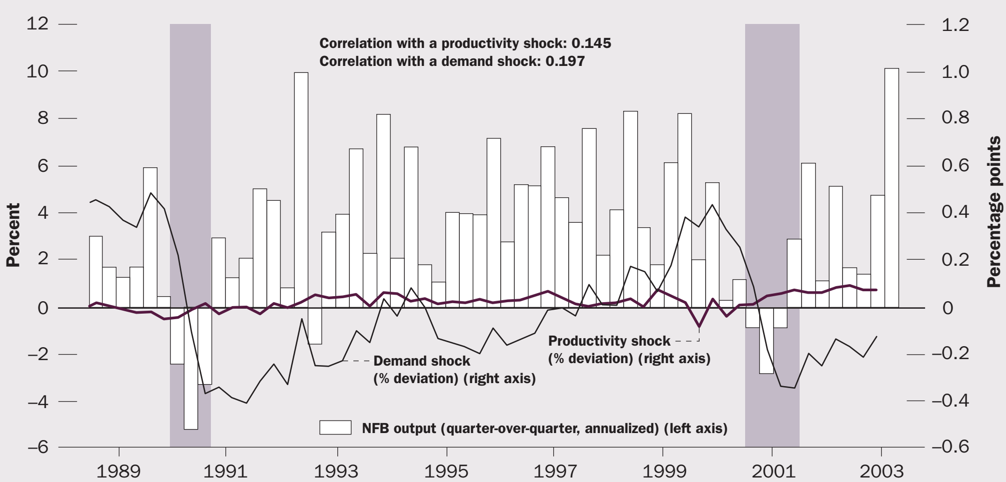
The main objective of this model is to back out the structural shocks of z_t. Given the definition of ξ_t, recovering z_t requires that we first back out ξ_t. To do so, we use the smoothing algorithm that allows us to obtain $(\xi_{t|T})_{t=1}{}^{T}$, where

$$\xi_{t|T} = E(\xi_t | \eta^T).$$

Data Description and the Estimation Procedure

For the output measure, we use nonfarm business sector output detrended using a Hodrick-Prescott (HP) filter, and for inflation we use the HP-detrended nonfarm business sector output deflator. We use federal funds rates as the nominal interest rate measure and HP detrended compensation per hour in the nonfarm business sector as a nominal wage measure.[10] The data are from three different periods: 1954:1 to 2003:2, 1960:1 to 2003:2, and 1982:1 to 2003:2. The 1954:1–2003:2 vintage is used because it is the longest sample available. The 1982:1–2003:2 vintage is used because many authors (see, for example, Galí and Gertler 1999) have argued that the characterization of monetary policy using a Taylor rule is valid only since 1982. The 1960:1–2003:2 vintage is used merely for robustness analysis.

Figure 1
The Correlation of Nonfarm Business Output Growth with Productivity and Demand Shocks



Source: U.S. Bureau of Labor Statistics and authors' calculations

Although we use a Bayesian estimation procedure, we do not want to use informative priors that reflect our subjective beliefs about parameter values. Instead, we use flat priors for all the structural parameters, combining them with the likelihood function as defined in equation (3) and using the Metropolis-Hasting algorithm to draw from the posterior distribution of the structural parameters.[11] Finally, we back out the shocks at the mean of the posterior distribution. The results are reported in the next section.

Results

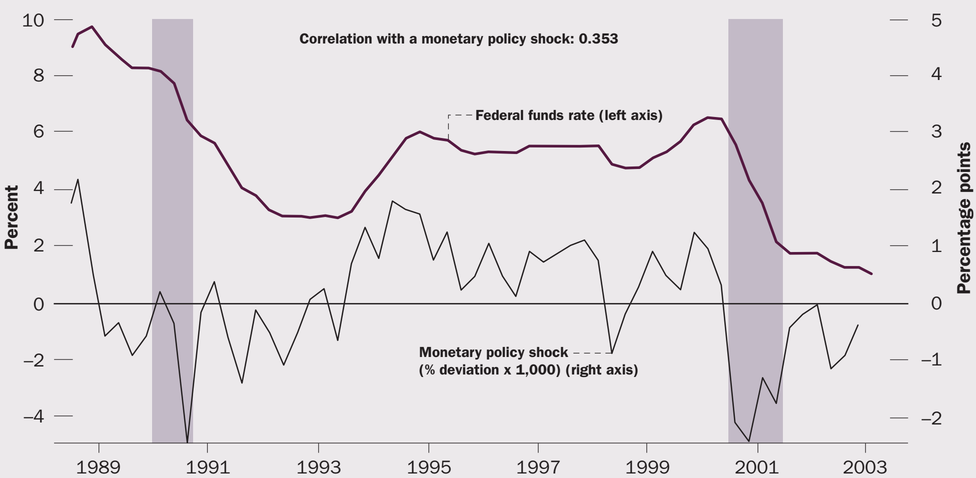
We now analyze the estimated shocks and relate them to movements in output and inflation. Estimates are reported only for the 1982:1–2003:2 period because the estimates for the other two vintages are very similar. The model appears to perform well along several dimensions but could be improved in some areas.

Figure 1 shows the correlation of the productivity and demand shocks with the growth in nonfarm business output for the 1989:1–2003:2 period. The model interprets large changes in output during recessions to be the result of negative demand shocks. In both the 1990–91 and the 2001 recessions, the model attributed the growth in output just prior to the recessions, in part, to large positive demand shocks (on the order of 40 to 50 percent deviation from its steady-state value). As output growth weakened and turned negative, these positive demand shocks lessened and turned sharply negative (on the order of 30 to 40 percent deviation from its steady-state value).

At the same time, the model does not attribute large declines in output to shocks from productivity. Just before the 1990–91 recession, a negative productivity shock

10. We use detrended data because the general equilibrium model used is stationary and does not include any trend.
11. See Rabanal and Rubio-Ramírez (2003a) for a description of the algorithm.

Figure 2
The Correlation of the Federal Funds Rate with a Monetary Policy Shock



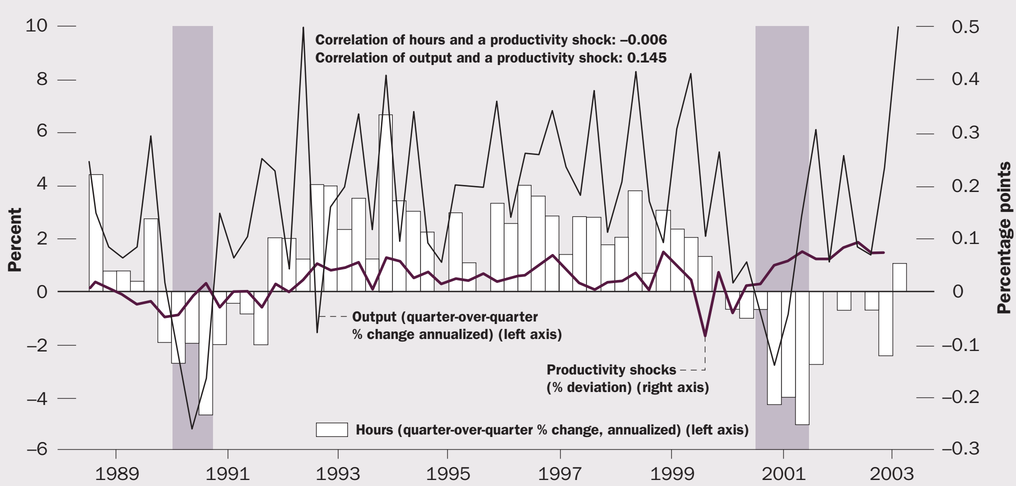
Source: Board of Governors of the Federal Reserve System and authors' calculations

occurred, but the shock was relatively small in magnitude and exhibited little persistence. During the 2001 recession and recovery, however, the model attributes the relatively small decline and subsequent strong growth in output in the face of a large demand shock to positive productivity shocks. This finding is consistent with recent data that show that measured productivity grew at an unusually strong pace during the 2001–03 period. Notably, the model also interprets the strong economic expansion of the 1990s to be the result of persistent positive shocks to productivity. Overall, the model generally attributes declines in output to negative demand shocks and increases in output to positive productivity shocks.

Figure 2 compares the nominal federal funds rate and the monetary policy shock for the same period. According to the Taylor rule, negative monetary policy shocks imply a loosening of monetary policy. The rule fits the data well because the monetary policy shocks are very small in magnitude, and the model generates monetary policy shocks that are consistent with recent events. However, both recessions are characterized by a sudden loosening of policy greater than what would be predicted by a Taylor rule. Also, the model captures the negative policy shocks of the early nineties, a period in which the Federal Reserve kept rates low in order to facilitate the rebalancing of bank balance sheets. The model also picks up the 1998 policy shock caused by the Asian crisis.

Despite its strengths, highlighted above, the model could be improved in several ways. The large swings of the demand shock in Figure 1 may indicate a model misspecification. The problem may lie in the Euler equation, which relates output today with output tomorrow. Thus, the model cannot explain big shifts in output and attributes them to g_t, the demand shock. In addition, the model is unable to distinguish between consumption and investment demand. The swings in the demand shock at each recession are similar in magnitude even though the underlying demand conditions were different. Consumption declined significantly during the 1990–91 recession but remained positive during the 2001 recession; the downturn in 2001 was

Figure 3
The Correlation of Nonfarm Business Output and Hours with a Productivity Shock



Source: U.S. Bureau of Labor Statistics and authors' calculations

driven largely by a decline in private domestic investment. The inability to distinguish among demand components stems from the lack of capital in the model.

Since capital is not specified in the model, productivity reflects labor productivity and not total factor productivity. Figure 3 graphs the productivity shock against nonfarm business output and hours worked. In both recessions, productivity deviated from its steady-state value. In the 1990–91 recession, output fell more than hours, and the model interpreted this difference as a (small) negative productivity shock. In the 2001 recession, hours fell more than output, resulting in a (large) positive productivity shock. While measured productivity is a closely followed economic variable, a more important gauge is total factor productivity, and a model with capital would provide a better estimate of this variable.

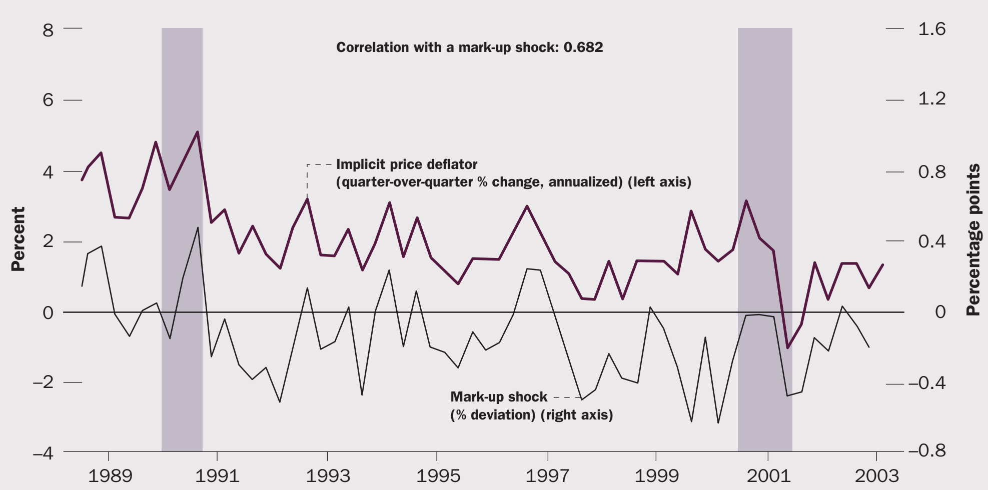
Another drawback to the model is that it does not fit the inflation data very well, but neither does any other model. Figure 4 plots the markup shock versus the nonfarm business implicit price deflator. The deviations from its steady-state value are frequent and sizable in magnitude. In addition, the correlation between the two series is very high—0.68—indicating that the model is not capturing the inflation dynamics and that it attributes frequent changes in inflation to λ_t, the markup shock.

Conclusions

In this article, we estimate a dynamic stochastic general equilibrium model with sticky prices and sticky wages, as in Erceg, Henderson, and Levin (2000). By recovering the structural shocks using a Kalman filter, we can map out the structural shocks from the model across time and compare the impact of these shocks on observable data.

The model interprets steep drops in output during recessions to be the result of negative demand shocks. In both the 1990–91 and the 2001 recessions, as output growth weakened and turned negative, the demand shocks lessened and turned sharply negative. The model attributes any strong growth in output to be mainly a

Figure 4
The Correlation of the Nonfarm Business Implicit Price Deflator with a Mark-Up Shock



Source: U.S. Bureau of Labor Statistics and authors' calculations

result of positive productivity shocks. This finding is consistent with recent data that show that measured productivity grew at an unusually strong pace during the 2001–03 period. Notably, the model also attributes the strong economic expansion of the 1990s to persistent positive shocks to productivity. Both recessions are characterized by a sudden loosening of policy greater than what would be predicted by a Taylor rule. Finally, the model does not capture the inflation dynamics and attributes frequent changes in inflation to the markup shock.

Future versions of the model will include capital and a broader set of structural shocks to better characterize the sudden drops in output and the inflation dynamics.

Appendix
The Solution of the Model in State-Space Form

Let

$$x_t = (w_t - p_t, r_t, \Delta p_t, \Delta w_t, y_t)',$$

$$\mu_t = (n_t, mc_t, mrs_t, c_t)', \text{ and}$$

$$z_t = (a_t, ms_t, \lambda_t, g_t)'.$$

Rabanal and Rubio-Ramírez (2003a) show how to write the model in the following way:

$$0 = Ax_t + Bx_{t+1} + C\mu_t + Dz_t,$$

$$0 = E_t(Fx_{t+1} + Gx_t + Hx_{t-1} + J\mu_{t+1} + K\mu_t + Lz_{t+1} + Mz_t), \text{ and}$$

$$z_t = Nz_{t-1} + \varepsilon_t.$$

Once the model is in this form, we can use the Uhlig (1999) algorithm to write its solution as

$$x_t = Px_{t-1} + Zz_t,$$

$$\mu_t = Rx_{t-1} + Sz_t, \text{ and}$$

$$z_t = Nz_{t-1} + \varepsilon_t.$$

In our model, the observable vector is $\eta_t = (w_t - p_t, r_t, \Delta p_t, y_t)'$. Bauer, Haltom, and Rubio-Ramírez (2003) demonstrate how to write these equations in state-space form:

$$\xi_t = F\xi_{t-1} + v_t, \text{ and}$$

$$\eta_t = H'\xi_t + w_t.$$

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