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Rational Exuberance: The Fundamentals of Pricing Firms, from Blue Chip to "Dot Com"

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Abstract: The author establishes that classic firm-valuation methods based on dividends (or equivalently free cash flows or residual income) can be modified to be based on any financial variable (V), such as sales, given V is cointegrated with the fundamental value (P) of the firm. The variable V (or a fraction of V) replaces dividends in the valuation formula, through a share liquidation scheme tied to V/P. The author shows that this modified valuation formula is equivalent to the classic fundamental valuation formula based on dividends, provided the share liquidation implicit in this scheme is accounted for. The use of nondividend information V permits an estimate of the fundamental value of a firm which should be more reliable than an estimate based on dividends alone, as dividends are well-known to be smoothed and can provide a poor indicator of future cash payments to investors. This approach is shown to complement existing valuation approaches that use dividends, permitting the fundamental valuation of firms which may or may not pay out dividends, have negative earnings, negative free cash flows, or even a negative book value (of shareholder equity). This extension of the classic fundamental valuation formula also provides a new methodology for calculating the fundamental asset price of any firm, including "dot-com" firms and privately held firms, utilizing nondividend information, such as sales, explicitly. Using dividends augmented with a cash flow from share liquidation, the author restates popular valuation methods, including the Gordon growth model, the residual income model, and the free cash flow model.

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It is shown here that *fundamental* valuation of a firm can be based on any variable that forms a stable long-run relationship with the fundamental value of the firm. In the language of econometrics, any variable that is cointegrated with price will do.[1] Such variables include, obviously, dividends paid by the firm. Virtually all existing/available valuation methods based on fundamentals make explicit or implicit use of dividends. This study outlines how other variables, including sales, revenues, total assets of the firm, or even possibly (in the case of dot-com firms) click-throughs[2] to a web page, could be used to *fundamentally* value a firm. This paper provides re-statements of popular valuation methods such as the Gordon Growth model (Gordon [1961]), the residual income model, the free cash flow model and the Donaldson and Kamstra (D&K) [1996] discounted dividend growth simulation technique. Note that the approach outlined here can be applied to firms with possibly negative earnings, negative book value of shareholder equity, and negative free-cash flows. This method can also be applied to the limiting case of a zero-dividend[3] firm.

The Miller and Modigliani [1961] (M&M) dividend irrelevance result established (under perfect certainty, perfect markets, and rational behavior) that dividend payments are arbitrary, given a fixed investment policy. Virtually any payment flow could be constructed with no impact on firm value, including the zero dividend case. The intuition for this result is simply that an investor can sell stock to generate income flows, or buy back stock with dividends issued by the company, to create the cash payments desired. This payment stream can be used to calculate the fundamental value of the firm, and no matter what feasible stream they choose, it will yield the same fundamental valuation, at least under the conditions of M&M. The pricing of a firm is straightforward conceptually. Implementation of pricing schemes is another matter.

One approach to pricing a firm is to use historical dividend payments and discount rate

[1]The notion that the price of a firm is itself integrated and thus possibly cointegrated with another variable is not controversial. Financial theory states clearly and under very general conditions that a function of prices and dividends will follow a martingale process, and this can also be seen to imply both are integrated and hence cointegrated with each other. See, for instance, Campbell Lo and MacKinlay [p. 257, 1997].

[2]Trueman, Wong and Zhang [2000] demonstrate that the market value of dot-com firms is correlated with internet usage.

[3]The zero dividend case has no cash payments made to the stockholders of the firm, including no stock re-purchases from the company. The zero dividend case may or may not have the firm re-investing all cash (i.e. a plowback ratio of one). The treatment here does not rely on a plowback ratio less than one.

data[4] to forecast future payments and discount rates. Restrictions on the dividend and discount rate processes are typically imposed to produce an analytic solution to the fundamental valuation equation (an equation that involves calculating the expectation of an infinite sum of discounted dividends). Discrete time approaches utilizing dividends include Gordon [1962], Hawkins [1977], Michaud and Davis [1982], Farrell [1985], Sorensen and Williamson [1985], Rappaport [1986], Barsky and DeLong [1993], Hurley and Johnson [1994,1998], D&K [1996], and Yao [1997]. Bakshi and Chen [1998] and Dong [2000] provide solutions by assuming dividends are proportional to earnings and model earnings. Campbell and Kyle [1993], Chiang et al. [1997], Bakshi and Chen [1998] and Dong [2000] all make use of continuous time tools to evaluate the fundamental present value equation. There are also approaches to valuation that are based on book value of equity, abnormal earnings and free cash flows,[5] linked to dividends and hence formal fundamental valuation by well-established accounting relationships. These approaches, the most popular of which include the residual income and free cash flows methods, address pricing by using the valuation of firm assets and income streams. See Ohlson [1995], Feltham and Ohlson [1995], and Penman and Sougiannis [1998] for instance. All these valuation methods implicitly or explicitly take the present value of the stream of firm-issued dividends to the investor.

In this paper, I formalize a method of generating a flow of cash payments through share liquidation to augment dividends, which complements existing valuation methods. I also explore the implications of using augmented dividends in place of firm-issued dividends for the basic Gordon model, the D&K model, the residual income and free cash flows methods. If the share liquidation of an investor's holdings of a firm is designed to deliver a cash return equal to, for instance, the earnings-to-price ratio (the earnings yield) or to a fraction of the sales-to-price ratio, a non-trivial fundamental price estimate can be obtained based on forecasts of future earnings or sales and their respective yield ratios. Pricing a firm based on its sales record (when it is possibly losing money on each and every transaction) is highly speculative, but *any* fundamentals estimation problem is inherently speculative. As will be

[4]Typically the discount rate equals a risk-free rate like the three month US t-bill rates plus an equity premium.

[5]Free cash flows are cash flows that could be withdrawn from a firm without lowering the current rate of growth. For a discussion of free cash flows and equity valuation see Hackel and Livnat [1996] or Penman and Sougiannis [1998]. Free cash flows are substantially different from accounting earnings and even accounting measures of the cash flow of a firm.

made explicit below, an underlying assumption is that a firm being priced will be profitable eventually. This is little different in character than the typical assumption that profitable firms will remain profitable.

In Section II the fundamental valuation equation is derived for the limiting case of a zero dividend firm. In Section III the issue of constructing cash payments for the zero dividend case is explored. In Section IV the classic fundamental valuation equation for dividend-paying firms is extended to incorporate shareholder-augmented cash payments, an approach which does not produce a different *expected* fundamental price, but which may produce a more reliable (i.e. lower variance) fundamental price *estimate*. In Section V some popular valuation approaches are modified for the augmented (through shareholder liquidation) dividend case, including the Gordon model, the D&K model, and the residual income and free cash flow valuation methods. The Appendix provides a detailed description of the extension of the D&K method. Section VI concludes.

II. Fundamental Valuation

Investor rationality requires that the current market price P_t of a stock which will pay a per share dividend (cash payment) D_{t+1} one period from now and then sell for P_{t+1}, discounting payments received during period t (i.e., from the beginning of period t to the beginning of period $t+1$) at rate r_t, must satisfy Equation 1:

$$P_t = \mathcal{E}_t \left\{ \frac{P_{t+1} + D_{t+1}}{1 + r_t} \right\} \tag{1}$$

where $\mathcal{E}_t$ is the expectations operator conditional on information available up to the end of period t. Solving Equation 1 forward under the transversality condition that the expected present value of P_{t+k} goes to zero as k goes to infinity (a "no-bubble" assumption) produces the familiar result that the market price equals the expected present value of future dividends (cash payments); i.e.,

$$P_t = \sum_{k=0}^{\infty} \mathcal{E}_t \left\{ \left(\Pi_{i=0}^{k} \left[\frac{1}{1 + r_{t+i}} \right] \right) D_{t+k+1} \right\}. \tag{2}$$

Assuming a flat term structure (constant discount rates $r_t = r$ for all t) for simplicity allows this expression to be re-written as

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{D_{t+k}}{(1+r)^k} \right\}. \tag{3}$$

This is just the fundamental valuation equation, which is not controversial and can be derived, as in Rubinstein [1976] and others, under the law of one price and non-satiation alone. Notice that the cash payments D_{t+k} include all cash disbursements from the firm, including cash dividends and share re-purchases. Fundamental valuation methods based directly on Equation 3 are typically called dividend discount models.

II.A. The Dividend Discount Model under the Limiting Case of Zero Dividends

I now turn to the special issues for the limiting case of a zero dividend firm. The zero dividend case has no cash payments made to the stockholders of the firm, including no stock re-purchases from the company. The zero dividend case may or may not have the firm re-investing all cash (i.e. a plowback ratio of one). Although the treatment here does not require the plowback ratio to be less than one, it does assume a fixed investment policy. Given an investment policy potentially as extreme as having all cash flows re-invested in the firm, no free-cash flows, no dividends, possibly even negative free-cash flows, negative earnings, and negative book value of shareholder equity, how might an investor value[6] the firm?

Consider a shareholder presented with the firm retaining all cash flows for the foreseeable future, that is, $D_t = 0$ for all t in the foreseeable future. The shareholder can re-construct the

[6] There is a widespread belief that some popular valuation approaches, such as the free cash flow method, can be directly applied to zero dividend firms. Indeed, the applicability to zero dividend firms is argued as an advantage of these methods over dividend discount valuation methods – see for instance Hackel and Livnat [p.10, 1996]. Ohlson [1990], however, makes clear that accounting valuation formulas *do* require dividends to be paid out, and emphasized this point by stating that every valuation method must have a precise link to dividends to avoid the status of a tautology.

dividend-paid case by selling a fraction f_t of their stock holdings[7] to receive a cash payment of $f_t \cdot P_t$. Suppose for the moment that at some point in the distant future, time period $t+K$, the firm is expected to liquidate all assets and pay out a terminal dividend per share equal to P_{t+K}.[8] The M&M irrelevance result establishes that any (feasible) schedule of payments will yield the same fundamental value P_t (at least under the M&M assumptions on markets). Suppose the shareholder holds N_t shares at the beginning of period t, each with value P_t.

At the beginning of period $t+1$, a "dividend" per share is generated on the N_t shares by selling $N_t f_{t+1}$ shares at price P_{t+1}, generating a total cash payment of $N_t f_{t+1} P_{t+1}$. Similarly, cash payments are generated into the future, $N_{t+k} f_{t+k+1} P_{t+k+1}$; and in the final period when the firm liquidates, a terminal cash payment of P_{t+K} is paid out on the remaining N_{t+K-1} shares at the beginning of period $t+K$. Assume again for simplicity a flat term structure of interest rates with $r_t = r$ for all t.

The fundamental valuation equation that price P_t equals the discounted present value of cash payments (i.e. dividends) reveals that the value of these cash payments on the shareholder's N_t shares as of the beginning of period t is simply the sum of the present value of the generated cash payments $N_{t+k} f_{t+k+1} P_{t+k+1}$ and the terminal liquidation cash payment $N_{t+K-1} P_{t+K}$,

$$N_t P_t = \sum_{k=1}^{K-1} \mathcal{E}_t \left\{ \frac{N_{t+k-1} f_{t+k} P_{t+k}}{(1+r)^k} \right\} \quad + \quad \mathcal{E}_t \left\{ \frac{N_{t+K-1} P_{t+K}}{(1+r)^K} \right\}. \tag{4}$$

Notice that a fraction f_{t+k} of total share holdings are sold each period $t+k$, so that the total share holdings is declining,

$$N_{t+k-1} = (1 - f_{t+k-1})\, N_{t+k-2}; \quad k = 2, 3, ..., K. \tag{5}$$

[7] Assume for simplicity that fractional shares may be bought or sold. Note that if the agent purchasing the shares is the firm, the firm is not retaining all cash flows, it is distributing cash by stock re-purchases. Also note that implicitly I am assuming that $0 \leq f_t \leq 1$.

[8] There are other approaches available to develop the case of zero cash payments. M&M, for instance, considered this case by noting a single investor could purchase all the shares from other investors at time $t+K$ at price P_{t+K} and then determine her own dividend payments as she pleases. This is equivalent to the treatment here, although M&M did not focus on producing a fundamental value estimate.

Recursively substituting out for N_{t+k-1} in Equation 5 produces

$$N_{t+k-1} = \left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right) N_t; \quad k = 2, 3, ..., K \tag{6}$$

where $f_t \equiv 0$, $0 < f_{t+i} < 1$ for all $i > 0$. Substituting Equation 6 into Equation 4 yields

$$N_t P_t = \sum_{k=1}^{K-1} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right) N_t f_{t+k} P_{t+k}}{(1+r)^k} \right\} + \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{K-1}\left[1 - f_{t+i}\right]\right) N_t P_{t+K}}{(1+r)^K} \right\}.$$

N_t is in the time t information set and can be thus extracted from the expectation. The per share value (as of the beginning of period t) can be written as:

$$P_t = \sum_{k=1}^{K-1} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right) f_{t+k} P_{t+k}}{(1+r)^k} \right\} + \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{K-1}\left[1 - f_{t+i}\right]\right) P_{t+K}}{(1+r)^K} \right\}. \tag{7}$$

Under the conditions of M&M, the irrelevance of cash payments means that the expected value of Equation 7 does not change with choice of K, the timing of the cash payments. Notice that as K increases, the first term on the right-hand side of Equation 7 is strictly increasing, which means the second term must be strictly declining to zero[9] as K approaches ∞. Provided K is large enough, $\mathcal{E}_t \left\{ \left(\Pi_{i=0}^{K-1}\left[1 - f_{t+i}\right]\right) P_{t+K}/(1+r)^K \right\}$ vanishes.

The M&M dividend irrelevance result allows K to be set to ∞ without any loss of generality, so that the fundamental valuation of this constructed series of dividends is

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right) f_{t+k} P_{t+k}}{(1+r)^k} \right\}. \tag{8}$$

[9]This holds if f_{t+k}, P_{t+k} and r are positive, provided additionally that f_{t+k} does not converge to zero as k increases. In the trivial case of $P_{t+k} = 0$ for some $k \leq K$, the first term is not increasing, but the second term is 0. Recall that I am also not considering the trivial case of $f_{t+k} \geq 1$. If $f_{t+k} \geq 1$ the shareholder liquidates her entire holdings in period $t+k$ and this is equivalent to moving the terminal payment of the firm up to period $t+k$ from the period $t+K$.

Notice that the Equation 8 price holds regardless of the shareholder's choice of cash payments – the M&M result. That is, Equation 8 gives a general solution to the problem of pricing a firm which has never paid out a dividend, and this solution does not depend on the shareholder's choice for cash payments into the future. This solution merely generates a price using *one possible choice* of cash payments, and given the M&M result this price is the price of the firm for *any other choice* of cash payments, at least under the conditions of the M&M result. Of course, Equation 8 states a general result without indicating how f_{t+i} could be chosen, and this solution indicates we must know future prices to solve for today's price, which is not very satisfying. It is to these issues I now turn.

III. Generating Streams of Income (Dividends) under the Zero Dividend Case

There are many strategies available to generate streams of cash payments under the zero dividend case, but not all lead to interesting price estimates. What is needed is to have future cash payments and hence price depend on quantities that can be forecasted based on what is known today. A tautological price forecast should also be avoided. One strategy is to pick a fixed payout yield on stock holdings, $f_t = f$. Another strategy is to exploit a yield ratio like earnings-to-price, book-to-price, or sales-to-price and have a payout f_t equal to the yield, or a fraction of the yield.

III.A. Pricing with a Constant Payout Yield

The choice of the fraction f_{t+i} to be a fixed f (say a yield of 7%) is a natural starting point. Substituting f for f_{t+i} in Equation 8 and carrying through expectations yields

$$P_t = f \cdot \mathcal{E}_t \left\{ \frac{P_{t+1}}{1+r} \right\} + \sum_{k=2}^{\infty} \frac{(1-f)^{k-1} f \mathcal{E}_t \left\{ P_{t+k} \right\}}{(1+r)^k}. \tag{9}$$

In the case of zero-dividend firms, the price is expected to grow at the discount rate[10] r so that $\mathcal{E}_t \left\{ P_{t+k} \right\} / ((1+r)^k)$ equals P_t. Equation 9 simplifies to

[10] See, for instance Ohlson [1991] for a discussion in the context of the growth of earnings when firms pay out less than 100% of earnings.

$$P_t = f \cdot P_t + \sum_{k=2}^{\infty}(1-f)^{k-1} f P_t = P_t \cdot \left(f + f \cdot \sum_{k=2}^{\infty}(1-f)^{k-1} \right) = P_t.$$

This follows from results on sums of geometric series – for values $0 < f < 1$ the expression $\left(f + f \cdot \sum_{k=2}^{\infty}(1-f)^{k-1}\right)$ equals one.

It is thus transparent that any rule which liquidates a constant proportion of total holdings in order to derive a dividend stream and calculate the present value produces a tautological price estimate, not an interesting price estimate.

III.B. Pricing using Financial Yield Ratios

The question of how to choose a non-constant yield ratio f_{t+k} is related to the derivation of the central equation of this paper, Equation 8. This derivation requires that the term $\mathcal{E}_t \left\{ \left(\Pi_{i=0}^{K-1} \left[1 - f_{t+i} \right] \right) P_{t+K}/(1+r)^K \right\}$ vanish as K increases. This will hold provided f_{t+k} does not converge to zero as k increases, given that f_{t+k}, P_{t+k} and r are all positive.[11] The requirement that f_{t+k} does not converge to zero as k increases provides an identifying restriction on the yield ratio f_{t+k}. For instance, there are constant payouts $\$\Delta$ per share ($f_{t+k} = \Delta/P_{t+k}$ for all k) that lead to f_{t+k} collapsing to 0 as k increases, and Equation 8 will not obtain.[12]

To identify an appropriate yield ratio, first consider a financial variable that is fundamentally related to firm value, like earnings, sales, or book value of shareholder equity – a variable whose per share quantity cannot vary far from the per share price of the firm. In the language of econometrics, such a variable would be cointegrated with price.[13] Call this variable V. The yield ratio[14] using V is simply V_{t+k}/P_{t+k}. If V_{t+k} and P_{t+k} are cointegrated,

[11] I am not considering the trivial case of $f_{t+k} \geq 1$. If $f_{t+k} \geq 1$ the shareholder liquidates her entire holdings in period $t+k$ instead of period $t+K$. Also recall I am considering the zero dividend case here, under which $\mathcal{E}_t \left\{ P_{t+k}/(1+r)^k \right\} = P_t$

[12] Equation 8 will hold if Δ is chosen so large as to liquidate the entire portfolio.

[13] Financial theory states clearly and under very general conditions that a function of prices and dividends will follow a martingale process, and this can also be seen to imply both are integrated and hence cointegrated with each other. See, for instance, Campbell Lo and MacKinlay [p. 257, 1997]. Empirical evidence also strongly supports the notion that price has a stable relationship – is cointegrated with – a variety of firm-specific variables, such as earnings, book value and sales. See, for instance, Beaver and Morse [1978], Wilcox [1984], Estep [1985], Peters [1991], Bauman and Miller [1997], Leibowitz [1997] and Leibowitz [1999].

[14] In the case of some financial variables like total assets or sales, it will be necessary to choose V equal to

then with no loss in generality V_{t+k} can be rewritten as $(\alpha_0 + \alpha_1 P_{t+k} + \epsilon_{t+k})$ where ϵ_{t+k} is I(0) (stationary) with bounded variance, uncorrelated with P_{t+k}, and $\alpha_1 > 0$. Rule out the degenerate case of $P_{t+k} = 0$. Then $f_{t+k} = V_{t+k}/P_{t+k}$ is converging to $\alpha_1 (> 0)$ as k increases, satisfying the identifying restriction on f_{t+k}. Finding a V that is cointegrated with price is a necessary, though not sufficient condition to form an admissible yield ratio. V_{t+k} must also be non-negative for all $k > 0$ and the ratio V_{t+k}/P_{t+k} must be less than or equal to 1 for all $k > 0$.[15]

Set $f_{t+i} = V_{t+i}/P_{t+i}$; $i > 0$ ensuring that V is both greater than zero and chosen so that f_{t+i} lies below 1. Substitute this into Equation 8 to yield

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1} \left[1 - f_{t+i}\right]\right) V_{t+k}}{(1+r)^k} \right\}. \tag{10}$$

This produces the basic dividend discount model of Equation 3 if $f_{t+i} = 0$ for all $i > 0$ and $V_{t+k} = D_{t+k}$.

The issue in estimating Equation 10 is one of forecasting V_{t+k} and the yield ratio f_{t+i}. Just as forecasts of dividends are based on historical patterns of dividend payments and extrapolations based on similar firms, forecasts of V_{t+k} and f_{t+i} can be based on past values of these variables and/or knowledge of these variables from similar firms. For instance, choice of V to equal firm earnings would be expected to lead to a long run average of roughly 6% for f, based on the S&P 500 average earnings yield over the past twenty years.

Among the candidates for V are earnings, sales, revenues, shareholder equity, and total assets. If the variable V has been negative historically, as earnings can be even for well-established companies, the most straightforward solution is to make use of some other financial variable that has a long-run stable relationship with firm value, say sales. If a firm has no sales yet, the search may have to be further widened, to possibly include variables like total assets.

The derivations leading to Equation 10 demonstrate that the problem of pricing a negative earnings (even zero-sales) "dot-com" firm is *not* a problem of a different character than

a fraction of the variable, to ensure the yield ratio V_{t+k}/P_{t+k} lies between 0 and 1.

[15]In order to avoid complicating the notation I will not explicitly consider generalizing these results to a V_{t+k} that is occasionally less than zero, or a V_{t+k}/P_{t+k} occasionally greater than 1.

pricing blue chip dividend-yielding firms. Pricing a "dot-com" firm simply requires a bigger leap of faith – the assumption that a zero-dividend, negative earnings firm with little or no track record will eventually be able to earn positive earnings and in the long run look like other firms, say the typical S&P 500 firm.

This approach can be extended to unpriced firms, as forecasting f_{t+i} does not require market prices. Instead an iterative technique may be used. For instance, the yield ratio can be calibrated to the S&P 500 firms' yields for the first iteration, fundamental prices estimated, yield ratios constructed with these estimated prices, and then fundamental prices re-estimated with these new yield ratio estimates, and so on until fundamental prices and yield ratios do not change from one iteration to another. For a detailed description of this iterative process, see Steps A-C of the appendix.

IV. Extension to Dividend-Paying Firms

One concern practitioners have with basing valuation of dividend-paying firms on the dividend record of these firms is that dividends are typically smoothed and are set low enough so that the dividend payments can be maintained through economic downturns. Authors such as Hackel and Livnat [p.9, 1996] argue that these sorts of considerations imply that historical records of dividend payments may thus be poor indicators of future cash payments to investors. Consider re-writing the fundamental valuation equation to incorporate cash payments issued from the firm (i.e. dividends) augmented by liquidating a fraction of the shareholder's holdings. To do this, Equation 8 must be augmented to include dividend payments, making use of Equation 3. Simple algebra, together with application of the M&M dividend irrelevance result, yields

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right)\left(f_{t+k}P_{t+k} + D_{t+k}\right)}{(1+r)^k} \right\} \tag{11}$$

which can also be written as

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right)\left(V_{t+k} + D_{t+k}\right)}{(1+r)^k} \right\} \tag{12}$$

where $f_{t+i} = V_{t+i}/P_{t+i}$, $0 < f_{t+i} < 1$ for all $i > 0$ and where $f_t \equiv 0$.

Under M&M the fundamental price calculated by Equation 12 and that calculated by Equation 3 will have the same expectation for dividend-paying firms – the true fundamental value of the firm. The decision to use one or the other would be based on the properties of the resulting fundamental price *estimate*. An estimate based on dividends alone may be less reliable (have larger variance) than one based on a richer information set.

V. Some Popular Valuation Approaches Modified for the Augmented Dividend Case

There are a variety of fundamental valuation methods that have become popular, largely distinguished by the assumptions imposed on dividend growth rates and discount rates.

V.A. The Gordon Growth Model

Perhaps the most widely used valuation method is the Gordon Growth model. In order to derive the classic Gordon Growth model, first return to the classic valuation formula, Equation 2, which does not impose the flat term rate assumption. Define the growth rate of dividends from the beginning of period t to the beginning of period $t+1$ as $g_t^d \equiv (D_{t+1} - D_t)/D_t$, and write

$$P_t = D_t \sum_{k=0}^{\infty} \mathcal{E}_t \left\{ \Pi_{i=0}^{k} \left[\frac{1 + g_{t+i}^d}{1 + r_{t+i}} \right] \right\}. \tag{13}$$

It is straightforward to derive from Equation 13 the Gordon fundamental price estimate:

$$P_t^G = D_t \left[\frac{1 + g^d}{r - g^d} \right], \tag{14}$$

where r is the constant discount rate value and g^d is the (conditionally) constant growth rate of dividends.[16]

V.A.1. The Gordon Growth Model Under Dividend Augmentation

In order to investigate the Gordon price with dividend augmentation, re-write Equation 12, relaxing the flat interest rate term structure assumption, as

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \left(\Pi_{i=0}^{k-1} \left[\frac{1 - f_{t+i}}{1 + r_{t+i}} \right] \right) (V_{t+k} + D_{t+k}) \right\}. \tag{15}$$

Define $A_t = D_t + V_t$, and A's growth rate as $g_t^a \equiv (D_{t+1} + V_{t+1} - (D_t + V_t))/(D_t + V_t)$, so that A_{t+k} may be written as follows:

$$A_{t+k} = \left(\Pi_{i=0}^{k-1} \left[1 + g_{t+i}^a \right] \right) A_t; \quad k = 1, 2, 3, ..., \infty \tag{16}$$

Substituting Equation 16 into Equation 15 and noticing that A_t is in the time t information set yields

$$P_t = A_t \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \Pi_{i=0}^{k-1} \left[\frac{(1 - f_{t+i})(1 + g_{t+i}^a)}{1 + r_{t+i}} \right] \right\}.$$

or equivalently

$$P_t = A_t \sum_{k=0}^{\infty} \mathcal{E}_t \left\{ \Pi_{i=0}^{k} \left[\frac{(1 - f_{t+i})(1 + g_{t+i}^a)}{1 + r_{t+i}} \right] \right\}. \tag{17}$$

[16]The derivation of the Gordon fundamental price estimate requires constant discount rates $r_{t+i} = r$ because Jensen's Inequality will not allow us to extract the denominator of Equation 13 from the expectations operator if interest rates are not constant. The derivation does not require constant growth rates of dividends, however, but does require *conditionally* constant growth rates of dividends: $\mathcal{E}_t \left\{ g_{t+i}^d \right\} = g^d$ for all $i > 0$, and $\mathcal{E}_t \left\{ g_{t+i}^d g_{t+j}^d \right\} = \mathcal{E}_t \left\{ g_{t+i}^d \right\} \mathcal{E}_t \left\{ g_{t+j}^d \right\} = (g^d)^2$ for all $i > 0$, $j > 0$. Finally, $g^d < r$ is required as well.

Notice the similarity between Equation 13 and Equation 17. Equation 13 has the fundamental price equal to an infinite sum of discounted dividend growth rates times the most recent dividend. Equation 17 has the fundamental price equal to an infinite sum of growth rates of the dividend augmented payout, A, times the most recent value of A, where now the "discount" rate is $(1 - f_{t+i})/(1 + r_{t+i})$ rather than $1/(1 + r_{t+i})$.

Adjusting Equation 17 to calculate a Gordon Growth price requires the assumption of a conditionally constant yield ratio A_t/P_t with an expectation of f, a conditionally constant cash payment growth rate with expectation g^a, as well as a constant discount rate r.[17] Recall that $f_t = 0$. Into Equation 17 substitute f for f_t, $t \geq 1$, g^a for g_t, $t \geq 0$, and r for r_t, $t \geq 0$, and carry expectations through to yield:

$$P_t = A_t \left(\frac{1 + g^a}{1 + r} + \left(\frac{1 + g^a}{1 + r} \right) \sum_{k=1}^{\infty} \Pi_{i=1}^{k} \left[\frac{(1 - f)(1 + g^a)}{1 + r} \right] \right)$$

or

$$P_t = A_t \left(\frac{1 + g^a}{1 + r} \right) \left(1 + \sum_{k=1}^{\infty} \left[\frac{(1 - f)(1 + g^a)}{1 + r} \right]^k \right).$$

Results from sums of geometric series (for f, g^a, r each greater than 0 and less than 1, $g^a \leq r$) deliver Equation 18.

$$P_t^{G,v} = A_t \left(\frac{1 + g^a}{r - g^a + f(1 + g^a)} \right). \tag{18}$$

This is the Gordon Growth model's price for the augmented dividend case, which could be based on a financial variable V equal to earnings, if earnings are positive, or sales, for instance.

For the zero-dividend case, $D = 0$, $r = g^a$, and Equation 18 simplifies to $P_t^{G,v} = A_t/f$. In words, the Gordon price equals the cash payment divided by the expected cash yield ratio. The use of yield ratios to price firms, such as suggested by $P_t^{G,v} = A_t/f$, is often referred

[17] Also required is $\mathcal{E}_t \left\{ g^a_{t+i} g^a_{t+j} \right\} = \mathcal{E}_t \left\{ g^a_{t+i} \right\} \mathcal{E}_t \left\{ g^a_{t+j} \right\} = (g^a)^2$, $\mathcal{E}_t \left\{ f_{t+i} f_{t+j} \right\} = \mathcal{E}_t \left\{ f_{t+i} \right\} \mathcal{E}_t \left\{ f_{t+j} \right\} = f^2$, and $\mathcal{E}_t \left\{ g^a_{t+i} f_{t+j} \right\} = \mathcal{E}_t \left\{ g^a_{t+i} \right\} \mathcal{E}_t \left\{ f_{t+j} \right\} = g^a * f$ for all $i > 0$, $j > 0$.

to as the relative value method or the constant P/E model (when the yield is based on earnings). References to this sort of approach can be found in textbooks like Brealey et al. [1992], and journal articles such as Peters [1991] and Penman [1998]. Equation 18 under the zero-dividend case, $D = 0$, can be viewed as a formal justification for the use of yield ratios to price firms with no dividend record. This result also points out the (strong) assumptions underlying the use of yield ratios to price firms.

For the dividend payout case with no cash flow augmentation ($A_t = D_t$, $f = 0$, $g^a = g^d$), Equation 18 simplifies to Equation 14, the classic Gordon Growth model.

V.B. The Residual Income Valuation and Free Cash Flow Valuation Models

There are a number of manipulations of the dividend discount model of Equation 3 that have become popular, based on readily available accounting data of operating or financial activities. By far the most popular and well-laid out approaches are the residual income valuation model and the free cash flow valuation method. See for instance Feltham and Ohlson [1995], Penman and Sougiannis [1998], and Lee et al. [1999].

In order to derive the residual income model for a dividend-paying firm, the clean-surplus-relationship relating dividends to earnings (E) is needed,

$$B_{t+k} = B_{t+k-1} + E_{t+k} - D_{t+k} \tag{19}$$

where B_{t+k} is book value of equity. See Ohlson [1995], and Feltham and Ohlson [1995] for further discussion of the clean surplus relationship. Solving for D_{t+k} in Equation 19 and substituting into Equation 3 yields

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{B_{t+k-1} + E_{t+k} - B_{t+k}}{(1+r)^k} \right\}$$

or

$$P_t = B_t + \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{E_{t+k} - r \cdot B_{t+k-1}}{(1+r)^k} \right\} - \mathcal{E}_t \left\{ \frac{B_{t+\infty}}{(1+r)^{\infty}} \right\}$$

$$= B_t + \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{E_{t+k} - r \cdot B_{t+k-1}}{(1+r)^k} \right\} \tag{20}$$

as $B_{t+\infty}/(1+r)^{\infty}$ is assumed to equal zero. The term $E_{t+k} - r \cdot B_{t+k-1}$ is typically referred to as abnormal earnings.

The derivation of the free cash flow valuation model is very similar, now with a financial assets relation in place of the clean surplus relation, to relate dividends to cash flows:

$$a_{t+k} = a_{t+k-1} + i_{t+k} + c_{t+k} - D_{t+k}. \tag{21}$$

Here a_{t+k} is financial assets net of financial obligations, i_{t+k} is interest revenues net of interest expenses, and c_{t+k} is cash flows realized from operating activities net of investments in operating activities, all of which can be positive or negative. Further, in the context of free cash flow models a net interest relation is often assumed,

$$i_{t+k} = r \cdot a_{t+k-1}. \tag{22}$$

See Feltham and Ohlson [1995] for further discussion. Solving for D_{t+k} in Equation 21, substituting into Equation 3, utilizing Equation 22 and assuming the discounted present value of financial assets a_{t+k} goes to zero as k increases, allows the derivation of the typical form of the free cash flow valuation model,

$$P_t = a_t + \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{c_{t+k}}{(1+r)^k} \right\} \tag{23}$$

analogously to the residual income valuation derivation.

V.B.1. The Residual Income Valuation and Free Cash Flow Valuation Models Under Dividend Augmentation

To solve for the residual income valuation equation when the investor augments cash payments with share liquidation, return to Equation 4, explicitly accounting for the number of shares held. The total cash payment to the shareholder in period $t + k$ equals

$N_{t+k-1}(f_{t+k}P_{t+k} + D_{t+k})$, the book value of the shareholder's total equity holdings in period $t+k$ equals $N_{t+k}B_{t+k}$, and the earnings on the shareholder's total holdings in period $t+k$ equals $N_{t+k-1}E_{t+k}$. Hence the clean surplus equation can be written as

$$N_{t+k}B_{t+k} = N_{t+k-1}\left(B_{t+k-1} + E_{t+k} - (f_{t+k}P_{t+k} + D_{t+k})\right)$$

or

$$N_{t+k-1}(1 - f_{t+k})B_{t+k} = N_{t+k-1}\left(B_{t+k-1} + E_{t+k} - (f_{t+k}P_{t+k} + D_{t+k})\right)$$

or

$$f_{t+k}P_{t+k} + D_{t+k} = B_{t+k-1} + E_{t+k} - (1 - f_{t+k})B_{t+k}. \tag{24}$$

Substituting for $(f_{t+k}P_{t+k} + D_{t+k})$ from Equation 24 into Equation 11 yields

$$P_t = \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right)\left(B_{t+k-1} + E_{t+k} - (1 - f_{t+k})B_{t+k}\right)}{(1+r)^k} \right\}$$

which can be re-written as

$$P_t = B_t + \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right)\left(E_{t+k} - r \cdot (1 - f_{t+k})B_{t+k-1}\right)}{(1+r)^k} \right\}. \tag{25}$$

Here $\left(\Pi_{i=1}^{\infty}\left[1 - f_{t+i}\right]\right) B_{t+\infty}/(1+r)^{\infty}$ is assumed to equal zero.[18] [19]

Once again, the derivation of the free cash flow valuation model is very similar to that of the residual income model, yielding

$$P_t = a_t + \sum_{k=1}^{\infty} \mathcal{E}_t \left\{ \frac{\left(\Pi_{i=0}^{k-1}\left[1 - f_{t+i}\right]\right) c_{t+k}}{(1+r)^k} \right\} \tag{26}$$

[18]Provided $0 < f_{t+i} < 1$ for all $i > 0$, this will be so under the usual assumptions.

[19]Notice that if $f_{t+i} = 0$ and $D_{t+i} = 0$ for all i, it follows that $B_{t+k} = B_{t+k-1} + E_{t+k}$ and the fundamental valuation equation, Equation 11, no longer holds. In fact, in this case "abnormal earnings" are expected to be zero as the expected earnings E_{t+k} equal rB_{t+k-1} and hence $P_t = B_t$.

assuming the discounted present value of the remaining financial assets $\left(\Pi_{i=1}^{k-1}\left[1-f_{t+i}\right]\right) a_{t+k}$ goes to zero as k increases.[20]

Notice that the residual income model for the limiting zero dividend case, $D_{t+i}=0$ for all i, Equation 25, is not equivalent to the residual income model under the positive dividend case, Equation 20. Similarly, the free cash flow model for the limiting zero dividend case, Equation 26, is not equivalent to the free cash flow model under the positive dividend case, Equation 23. That is, using conventional residual income and free cash flows formulas in the context of a zero dividend (zero cash payment) firm is incorrect, contrary to the common belief that an advantage over the dividend discount model of the free cash flow model is its application to zero dividend firms.[21] Careful comparison of these formulas reveals that Equation 20 and Equation 23 are upward biased in valuing zero-dividend stocks, as these equations omit additional discounting terms, such as the term $\left(\Pi_{i=0}^{k-1}\left[1-f_{t+i}\right]\right)$. These discounting terms are required to adjust for the share liquidation generating the cash payments under the zero dividend case. Also note that when the firm pays no cash the use of the residual income or free cash flow method does not alter the basic nature of fundamental valuation – the yield ratio variable must still be forecast into the future. Recall that for the zero-dividend case using a fixed value for f_{t+i}, say a 7% yield, produces a tautological price estimate.[22]

V.C. The Donaldson-Kamstra Valuation Method

The method of D&K [1996] for the dividend-payment case is an extension of the Gordon Growth model, and takes the discounted dividend growth model of Equation 13 and re-writes it as

$$P_t = D_t \sum_{k=0}^{\infty} \mathcal{E}_t \left\{ \Pi_{i=0}^{k} y_{t+i} \right\} \tag{27}$$

[20]Again, provided $0 < f_{t+i} < 1$ for all $i > 0$ this will be so under the usual assumptions.

[21]See, for instance, Hackel and Livnat [p.10, 1996].

[22]As Equation 25 and Equation 26 are algebraically identical to Equation 8 for the zero-dividend case, any fundamental price estimate derived from the residual income model or free cash flow model using a fixed yield ratio can also be re-written as a tautological price estimate, just as Equation 8 was rewritten in Section III A.

where $y_{t+i} = (1 + g^d_{t+i})/(1 + r_{t+i})$. The fundamental price of the D&K method is calculated by forecasting the range of possible evolutions of y_{t+i} up to some distant point in the future, period $t + I$, calculating $PV = D_t \sum_{k=0}^{I} \left(\Pi_{i=0}^{k} y_{t+i} \right)$ for each possible evolution of y_{t+i}, and averaging these values of PV across all the possible evolutions.[23]

V.C.1. The Donaldson-Kamstra Valuation Method Under Dividend Augmentation

To extend this method to the augmented dividend case, define $y^a_{t+i} = (1 - f_{t+i})(1 + g^a_{t+i})/(1+r_{t+i})$ where again $g^a_t \equiv (D_{t+1}+V_{t+1}-(D_t+V_t))/(D_t+V_t)$, and re-write Equation 17 as

$$P_t = A_t \sum_{k=0}^{\infty} \mathcal{E}_t \left\{ \Pi_{i=0}^{k} y^a_{t+i} \right\}. \tag{28}$$

Hence the D&K method requires forecasting $y^a_{t+i} = (1 - f_{t+i})(1 + g^a_{t+i})/(1 + r_{t+i})$. With historical values of the discount rates r_t, growth rates g^a_t, and yield ratios f_t, and a stable forecastible process for y^a_t, this method should produce reliable estimates of the fundamental value of the firm.

A key issue in applying the D&K method is determining the mean level of y^a_t. If this mean level is historically above 1 (as it typically will be for high growth firms) the problem faced is the same as that faced with the classic Gordon Growth model when average dividend growth (g^d) exceeds the discount rate (r) – the model does not apply, technically forecasting infinite prices. Conventional in the academic and practitioners' literatures is the assumption on high growth firms that their cash payment growth rates have several stages, permitting for instance, high growth firms to start with high dividend growth rates and then decelerate to a stable long-run rate. Extensions to the basic Gordon model along these lines include Hawkins [1977], Farrell [1985], Sorensen and Williamson [1985], Rappaport [1986], Hurley and Johnson [1994,1998], and Yao [1997]. These assumptions are often imposed in an ad hoc fashion, allowing a fixed period, say 5 years, to be high growth, after which a fixed low

[23] The value of I is chosen to produce a very small truncation error. Values of I=400 to 500 for annual data have been found by D&K [1996] to suffice.

growth will apply. The method of D&K does not impose any assumptions on how quickly the growth rate will drop, but does follow a similar convention by assuming that the long run average of the discounted dividend growth rate y reflects a stable low-growth state, well below 1. (The value for the discounted dividend annual growth rate for the S&P 500 index over 1952-1998 is roughly 0.94.) The speed of the drop in growth rates is determined by the parameters of the model estimated for y_t. A detailed description of the extension of the D&K [1996] method to the augmented dividend case is provided in the appendix.

VI. Conclusions

Fundamental valuation of a firm requires that shareholders must be able to extract cash payments from the firm. These payments may be provided by the firm issuing cash directly (dividend payments, share re-purchases) or by the shareholders constructing their own payment schedule by liquidating a portion of their holdings. I consider fundamental valuation of firms, including those that may have no history of cash payments to their shareholders. This limiting case of zero dividends complicates the task of valuing the firm, but does not change the nature of the task, and leads to an insight applicable to valuing any firm. Specifically, I establish here that *fundamental* valuation of a firm can be based on any variable that forms a stable long-run relationship with the fundamental price of the firm – a financial variable that is cointegrated with firm value. Such variables may include sales, revenues, or total assets of the firm, among others. Incorporating non-dividend information permits an estimate of the fundamental value of a dividend-paying firm which should be more reliable than an estimate based on dividends alone, as dividends are well-known to be smoothed and can provide a poor indicator of future cash payments to investors.

This paper provides re-statements of popular valuation methods including the Gordon Growth model, the residual income model, and the free cash flow model, to incorporate shareholder liquidation-augmented cash payments based on financial variables cointegrated with firm value. The extended versions of the valuation models provided here can be applied more broadly than the original versions. In particular, methods based on share liquidation-augmented cash payments can be applied to firms with zero dividends. The Gordon model and the D&K method can even be applied to firms with negative earnings, negative book value of equity, no free-cash flows, and even negative free-cash flows. The re-statements of these valuation methods extends their usefulness, exploiting non-dividend information in the valuation exercise.

This extension of the classic fundamental valuation formula also provides a new methodology for calculating the fundamental asset price of any firm, including "dot-com" firms and privately held firms, utilizing non-dividend information like sales explicitly. To implement classic valuation methods on unproven new firms requires a model calibration which implicitly assumes that in the long run the firm being priced will look like other firms, say the typical S&P 500 firm, so that high sales growth translates, ultimately, to large profits. Although this can only be described as speculative, valuation of any firm is inherently speculative in nature.

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Appendix
The Method of Donaldson and Kamstra Extended to the Augmented Dividend Case

A number of approaches can be taken to estimate the D&K [1996] valuation model, shown in Equation 27 (or in the case of an augmented dividend firm, Equation 28). With a very simple structure for the conditional expectation for discounted dividend growth (y_t in Equation 27) the expression can be solved analytically – for instance with discounted dividend growth a constant. But, as shown in D&K [1996], analytic solutions become complex for even simple ARMA models, and with sufficient non-linearity, the analytics can be intractable. For this reason a general solution algorithm based on the D&K [1996] method of Monte Carlo simulations is presented.

This method simulates y_t into the future to perform a numerical (Monte Carlo) integration to estimate the terms $\{\Pi_{k=0}^{i} y_{t+k}\}$ where $y_{t+k} = (1 + g_{t+k}^{d})/(1 + r_{t+k})$ in the classic case of a dividend-paying firm, and $y_{t+k} = y_{t+k}^{a} = (1 - V_{t+k}/P_{t+k})(1 + g_{t+k}^{a})/(1 + r_{t+k})$, $A_t = D_t + V_t$ and $g_t^a \equiv (D_{t+1} + V_{t+1} - (D_t + V_t))/(D_t + V_t)$, in the share liquidation-augmented dividend case. A general heuristic follows directly below.

Step I: Model $y_t,\ t = 1, ..., T$ as conditionally time-varying, for instance as an AR(k)-GARCH(p,q) process, and use the estimated model to make conditional mean forecasts $\hat{y}_t,\ t = 1, ..., T$ and variance forecasts, conditional on only data observed before period t. Ensure that this model is consistent with theory, for instance that the mean level of y is less than one. This mean value can be calibrated to available data, such as the S&P 500's mean annual y value of 0.94 over the 1952-1998 period. Recall, although analytic solutions are available for simple processes, the interest here is in a general solution algorithm applicable to virtually arbitrarily non-linear conditional processes for the discounted cash payment rate y.

Step IIa: Now simulate discounted cash payment growth rates. That is, produce ys that might be observed in period t given what is known at period $t - 1$. To do this for a given period t, simulate a population of J independent possible shocks (say draws from a normal distribution, mean 0 and appropriate variance) $\epsilon_{t,j},\ j = 1, ..., J$ and add these shocks separately to the conditional mean forecast $\hat{y}_t$ from Step I, producing $y_{t,j} = \hat{y}_t + \epsilon_{t,j},\ j = 1, ..., J$. This is a simulated cross-section of J possible realizations of y_t standing at time $t-1$, i.e. different paths the economy may take next period.

Step IIb: Use the estimated model from Step I to make the conditional mean forecast $\hat{y}_{t+1,j}$, conditional on only the j^{th} realization for period t, $y_{t,j}$ and $\epsilon_{t,j}$, and the data known at period $t-1$, to form $y_{t+1,j}$.

Step IIc: Repeat Step IIb to form $y_{t+2,j},\ y_{t+3,j},\ ...y_{t+I,j}$ for each of the J economies, where I is the number of periods into the future the simulation is truncated at. Form the perfect foresight present value $(P^*_{t,j})$ for each of the J possible economies,

$$P^*_{t,j} = A_t \left(y_{t,j} + y_{t,j}y_{t+1,j} + y_{t,j}y_{t+1,j}y_{t+2,j} + \cdots + \Pi^{I}_{i=0} y_{t+i,j} \right);\ j = 1, ..., J.$$

Provided I is chosen to be large enough, the truncated terms $\Pi^{K}_{i=0} y_{t+i,j},\ K = I+1, ..., \infty$ will be negligible. In practice I=500 is sufficient with annual data.

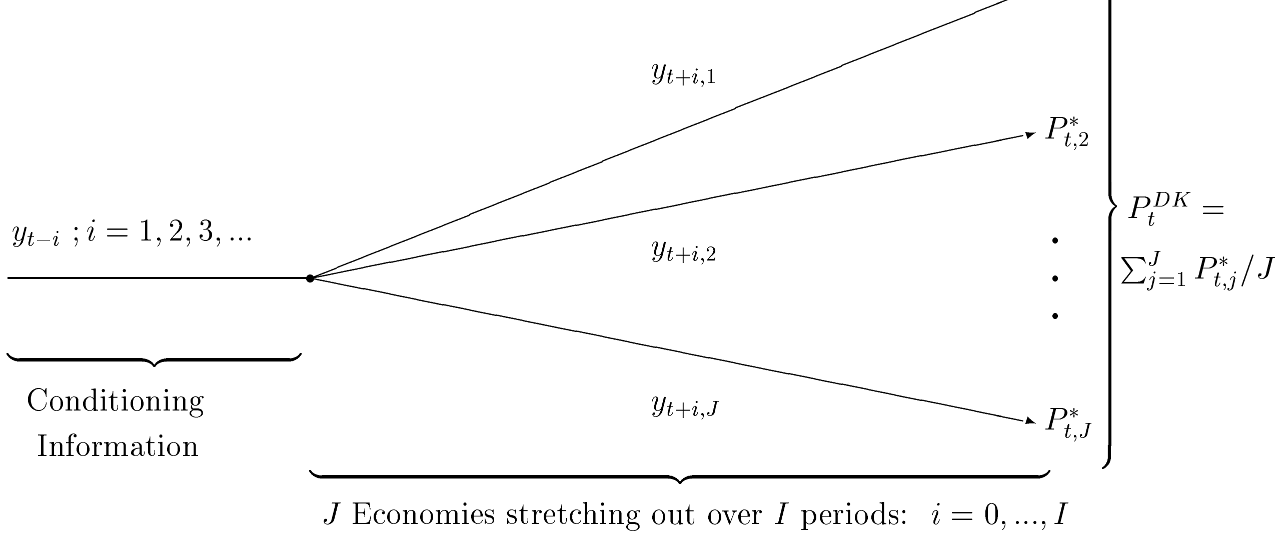
Step III: Calculate the D&K fundamental price for each $t = 1, ..., T$:

$$P^{DK}_t = \sum_{j=1}^{J} P^*_{t,j}/J. \tag{29}$$

These fundamental price estimates P^{DK}_t can be compared to the actual price (if market prices exist) at the beginning of period t to test for bubbles as in D&K [1996], or if the period t is the future, P^{DK}_t is the fundamental price forecast. This procedure is represented diagrammatically in Exhibit 1.

Exhibit 1

Diagram of D&K Monte Carlo Integration



The extension to the D&K [1996] procedure developed here revolves around an iterative procedure to recursively estimate the yield ratio f used in forming y for the share liquidation-augmented dividend case, $y_{t+k} = y^a_{t+k} = (1 - V_{t+k}/P_{t+k})(1 + g^a_{t+k})/(1 + r_{t+k})$. In the case of pricing a firm for which no market price is available (such as a firm yet to issue publicly traded shares), the appropriate yield ratio is not directly available. The yield ratio can, however, be derived by iteratively employing the the D&K [1996] method of evaluating fundamentals, as is described in Steps A-C directly below. Essentially, the price is estimated by approximating the yield ratio, applying the simulation algorithm Steps I-III, using the resulting price estimate to produce a new (approximate) yield ratio, and repeating until the yield ratio does not change from iteration to iteration.

Step A: Set the yield ratio f to 0 to produce $y^{(1)}_t = (1 + g^a_t)/(1 + r_t)$. Then apply Steps I through III above using $y^{(1)}_t$ producing initial price estimates $P^{DK,1}_t$. These price estimates are systematically biased to be too large, given that they were produced with an yield ratio estimate of zero leading to a discounted cash payment estimate $(1 + g^a_t)/(1 + r_t)$ that is too large. These price estimates will not be unbounded, however, as the simulation generates future ys with the right limiting mean level, below 1, as indicated in Step I above.

Step B: Calculate

$$y_t^{(2)} = \frac{(1 - V_t/P_t^{DK,1})(1 + g_t^a)}{1 + r_t}; \quad t = 1, ..., T$$

and apply Steps I through III again (now using $y_t^{(2)}$) to produce the second iteration price estimates $P_t^{DK,2}$. These price estimates are also biased to be too large, given that they were produced based on a price estimate $P_t^{DK,1}$ which is too large and hence a discounted cash payment estimate $(1 - V_t/P_t^{DK,1})(1 + g_t^a)/(1 + r_t)$ that is still too large. But $(1 - V_t/P_t^{DK,1})(1 + g_t^a) < (1 + g_t^a)$ so that Step B's price estimate $P_t^{DK,2}$ should be smaller than Step A's price estimate $P_t^{DK,1}$.

Step C: Repeat Step B to calculate $y_t^{(r)}$ and $P_t^{DK,r}$, for r equal to 3, 4, and so on. Continue until $P_t^{DK,r}$ converges i.e. ceases to change significantly from one iteration to another.

The final converged estimates are the fundamental price estimates. The iterative technique for forecasting fundamentals without use of market prices has to be adjusted for initial conditions after an IPO. With annual data for instance, the first year growth of A can be estimated with the annualized average quarterly growth rates.