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Abstract: Using data on workers' flows into and out of employment, unemployment, and not-in-the-labor-force, I construct transition probabilities between "employment" and "unemployment" that can be used in the calibration of economies such as Krusell and Smith's (1998). I show that calibration in Krusell and Smith has some counterfactual features. Yet the gains from adopting alternative calibrations in terms of matching the data are not very large, unless one assumes that the duration of unemployment spells is well above what is usually assumed in the literature.

JEL classification: J21

Key words: calibration, labor market flows

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1 Introduction

In the influential paper "Income and Wealth Heterogeneity in the Macroeconomy" Krusell and Smith (1998) evaluate the aggregate implications of heterogeneity in income and wealth. In their setup, Krusell and Smith (henceforth KS) assume that the aggregate shock evolves exogenously according to a two-state Markov process, where one state represents good times (expansions) and the other bad times (recessions). The aggregate shock affects both the economy's productivity and the individual agents' probability of being employed. These probabilities, in turn, determine the aggregate level of unemployment in the economy, according to the following equation:

$$u_s \frac{\pi_{ss'00}}{\pi_{ss'}} + (1-u_s)\frac{\pi_{ss'10}}{\pi_{ss'}} = u_{s'}, \tag{1}$$

where (using KS's notation) u_s is unemployment in state s, $\pi_{ss'\epsilon\epsilon'}$ denotes the joint probability of transition from state (z_s, ϵ) to state $(z_{s'}, \epsilon')$, $\pi_{ss'}$ denotes the marginal probability of transition from state z_s to state $z_{s'}$, and the ratio of the two, $\frac{\pi_{ss'\epsilon\epsilon'}}{\pi_{ss'}}$, denotes the conditional probability. The aggregate state z_s can be either good ($z_s = z_g$) or bad ($z_s = z_b$). The idiosyncratic state ϵ can be either equal to 0 when the agent is unemployed, or equal to 1 if the agent is employed.

KS calibrate the conditional transition probabilities $\frac{\pi_{ss'\epsilon\epsilon'}}{\pi_{ss'}}$ in Eq. (1) (which I will sometimes refer to as "the πs") so that aggregate unemployment is constant within good and bad times. That is, u always equals u_g in good times and u_b in bad times. The rationale for this choice lies in its computational convenience: If aggregate unemployment is fully determined by the aggregate state z_s, it needs not be an additional state variable entering the agents' problem. KS state:

> "By virtue of the law of large numbers, the only exogenous source of aggregate uncertainty in the economy is the aggregate productivity shock. More specifically, the number of agents who are unemployed always equals u_g in good times and u_b in bad times." (page 872)

In general, not all transition probabilities between employment and unemployment will imply that aggregate unemployment is constant in good and bad times. To see why, note that the law of motion (1) has to hold for all pairs (s, s'). Since there are four such pairs, u_g and u_b would have to satisfy four equations. Given *any* set of πs, this is clearly not possible. Hence, KS's calibration of the πs must be such that two of the four equations are linear combinations of the other two.

This note will address three questions: First, is this calibration consistent with the data? Using data on workers' flows into and out of employment, unemployment, and not-in-the-labor-force (henceforth E, U, and NLF, respectively) from Bleakley, Ferris, and Fuhrer (1999), I construct transition probabilities between "employment" ($\epsilon = 1$) and "unemployment" ($\epsilon = 0$) from QIV-1967 to QIII-1998. I investigate whether these transition probabilities observed in the data, conditional on the pattern of the aggregate state (s, s') (i.e., going from an expansion to a recession, etc.), are consistent with KS's calibration of the πs entering Eq. (1). I also compare the actual path of unemployment with that implied by Eq (1) using the πs from KS's calibration. Second, how does the calibration in KS compare to existing alternative in the literature in terms of matching the data? Finally, can one improve on this calibration, and are the gains in terms of matching the data large enough to outweight its computational convenience?

2 The Calibration of the Transition Probabilities in KS

KS do not give explicit values for all the πs in the paper, but provide the necessary information to back them out. KS state that (pages 876-7): i) "$u_g = .04$ and $u_b = .10$"; ii) "the average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times" (this implies that $\frac{\pi_{gg01}}{\pi_{gg}} = \frac{2}{3}$ and $\frac{\pi_{bb01}}{\pi_{bb}} = \frac{2}{5}$); iii) "$\frac{\pi_{gb00}}{\pi_{gb}} = 1.25\frac{\pi_{bb00}}{\pi_{bb}}$ and $\frac{\pi_{bg00}}{\pi_{bg}} = .75\frac{\pi_{gg00}}{\pi_{gg}}$".

Using the fact that $\frac{\pi_{ss'01}}{\pi_{ss'}} + \frac{\pi_{ss'00}}{\pi_{ss'}} = 1$, one obtains from ii) that $\frac{\pi_{gg00}}{\pi_{gg}} = \frac{1}{3}$ and $\frac{\pi_{bb00}}{\pi_{bb}} = \frac{3}{5}$. In turn, iii) implies $\frac{\pi_{gb00}}{\pi_{gb}} = \frac{3}{4}$ and $\frac{\pi_{bg00}}{\pi_{bg}} = \frac{1}{4}$. Using equation (1) for the pairs (g, g) and (b, b) one can also obtain that $\frac{\pi_{gg10}}{\pi_{gg}} = \frac{1}{36}$ and $\frac{\pi_{bb'10}}{\pi_{bb}} = \frac{2}{45}$. The parameters $\frac{\pi_{bg10}}{\pi_{bg}}$ and $\frac{\pi_{gb10}}{\pi_{gb}}$ can be recovered using precisely the restriction that the aggregate unemployment rate jumps from u_g to u_b (u_b to u_g) whenever the aggregate state moves from g to b (b to g). Using this condition, one obtains that $\frac{\pi_{bg10}}{\pi_{bg}} = \frac{1}{60}$

and $\frac{\pi_{gb10}}{\pi_{gb}} = \frac{7}{96}$. All remaining πs can be found using $\frac{\pi_{ss'01}}{\pi_{ss'}} + \frac{\pi_{ss'00}}{\pi_{ss'}} = \frac{\pi_{ss'11}}{\pi_{ss'}} + \frac{\pi_{ss'10}}{\pi_{ss'}} = 1$. Column (A) of Table 1 summarizes the values for the conditional transition probabilities $\frac{\pi_{ss'\epsilon\epsilon'}}{\pi_{ss'}}$ in KS.

The values for the πs imply that

$$\frac{\pi_{bg10}}{\pi_{bg}} < \frac{\pi_{gg10}}{\pi_{gg}} < \frac{\pi_{bb10}}{\pi_{bb}} < \frac{\pi_{gb10}}{\pi_{gb}} \qquad (2)$$

and

$$\frac{\pi_{bg00}}{\pi_{bg}} < \frac{\pi_{gg00}}{\pi_{gg}} < \frac{\pi_{bb00}}{\pi_{bb}} < \frac{\pi_{gb00}}{\pi_{gb}}. \qquad (3)$$

The probability of being unemployed next period ($\epsilon' = 0$) is the highest when the aggregate state moves from b to g, and the lowest when the aggregate state moves from g to b. This is the case regardless of whether one is currently employed ($\epsilon = 1$) or not ($\epsilon = 0$). In other words, agents have greater chances of finding jobs when the economy moves from a recession to an expansion than when it stays in an expansion. While this feature of the calibration may seem counterintuitive, it is needed to guarantee that the aggregate unemployment rate jumps immediately to the new steady state.

3 How Well Does KS's Calibration Match the Data?

Figure 1 shows that the pattern of the transition probabilities (πs) in the data are somewhat at odds with inequalities (2) and (3). In the data the direction of the inequalities is by and large reversed. The plots in Figure (1) show $\frac{\pi_{ss'01}}{\pi_{ss'}}$ and $\frac{\pi_{ss'00}}{\pi_{ss'}}$ (at the quarterly frequency) when $\epsilon = 0$ denotes either being unemployed or not-in-the-labor-force ($U + NLF$, top), or only unemployed (U, bottom), from 1967 to 1998. For each plot, vertical solid and dashed lines denote the beginning and the end of NBER-defined recessions. The transition probabilities are computed using data on flows from Bleakley, Ferris, and Fuhrer (1999), described in the data appendix.[1]

[1]Bleakley, Ferris, and Fuhrer (1999) also provide a detailed analysis of the transition probabilities, which they call "escape rates", between employment (E) and unemployment (U). Their transition probabilities are disaggregated for reason of unemployment (that is, quit, termination, layoff, etc.).

Regardless of how the state $\epsilon = 0$ is defined, for each of the five recessions from 1967 to 1998 $\frac{\pi_{ss'01}}{\pi_{ss'}}$ is lower at the beginning of the recession than at the end and, conversely, is higher at the beginning of an expansion than at the end: $\frac{\pi_{bg10}}{\pi_{bg}} > \frac{\pi_{gb10}}{\pi_{gb}}$. Whenever $\epsilon = 0$ includes both U as well as NLF, the cyclical pattern of $\frac{\pi_{ss'00}}{\pi_{ss'}}$ is not at all clear. Whenever $\epsilon = 0$ includes only U, the cyclical pattern of $\frac{\pi_{ss'00}}{\pi_{ss'}}$ is quite the same as that of $\frac{\pi_{ss'10}}{\pi_{ss'}}$.

How well does KS's calibration do in reproducing the historical path of aggregate unemployment rate? That is, if we input in equation (1) the πs computed in the previous section, do we obtain a path for the unemployment rate that is similar to that observed in the data? This question is not straightforward to answer, because one has to take a stand on what unemployment ($\epsilon = 0$) represents. The state $\epsilon = 0$ includes more than only those classified as unemployed by the Bureau of Labor Statistics (see the discussion in İmrohoroğlu 1989), but most likely less than the sum of workers that are unemployed and not-in-the-labor-force: Those individuals that are not-in-the-labor-force because they are not active participants in the labor market are not affected by the cyclical behavior of the economy that is the subject of KS's study. Fortunately, if one focuses on the behavior of aggregate unemployment at the business cycle frequencies this measurement issue does not make nearly as much a difference as when one focuses on the actual series. The top chart in Figure 2 plots the times path of two different definitions of the unemployment rate together with their respective Hodrick-Prescott (1997, henceforth HP) trend. The HP trend captures low frequency movemens in the series, possibly due to demographic factors (see Bleakley, Ferris, and Fuhrer, 1999). The solid line corresponds to the unemployment rate computed including those agents that are not-in-the-labor-force ($\frac{U + NLF}{U + NLF + E}$) while the dotted lines excludes these agents ($\frac{U}{U + E}$). The chart shows that the two definitions of the unemployment rate are very different. First, the rate of unemployment including individuals is on average 33% higher than when these individuals are excluded. Second, the trends in the two series exhibit a different patterns. The bottom chart of Figure 2 shows that the behavior of the HP-filtered series is very similar, however.[2] At business cycle

[2] I added the average rate of unemployment in KS, which is 7%, to the HP-filtered series.

frequencies the pattern of the aggregate unemployment rate is about the same regardless of how one measures $\epsilon = 0$. The correlation between the two detrended (that is, HP-filtered) series is .98. The remainder of this note focuses on detrended unemployment rates, and in particular on the broader measure (the one including agents that are not in the labor force). Note that the HP-filtered unemployment rate is very persistent, as discussed also in Shimer (2003), with an autocorrelation coefficient of .94.

Chart (A) in Figure 3 compares the HP-filtered unemployment rate (dotted line) with that implied by equation (1) using KS's transition probabilities (solid line). I will refer to the latter series as "KS's unemployment rate". The average unemployment rate in the two series is by construction the same, 7%.[3] There are two major discrepancies between the HP-filtered and KS's unemployment rates. First, the swings in the HP-filtered unemployment rate between recessions and expansions are much milder than those in KS's. The difference between the maximum and the minimum for the two series are 3.2% and 6%, respectively. The difference between the maximum and the minimum for the unfiltered series (including agents that are not-in-the-labor-force) is 8.6% (see Figure 1). However, this difference reflects more the downward trend in the series than cyclical movements: The maximum is reached at the end of the 1975 recession while the minimum is reached at the end of the sample period. The second discrepancy between the HP-filtered and KS's unemployment rates is that in the former the unemployment rate declines (rises) only gradually in expansions (recessions), while the latter by construction jumps immediately to the new steady state. Specifically, the discrepancy is much more noticeable during expansions than during recessions.

[3] KS's calibration of π_{gg} and π_{bb} is such that recessions and expansions last on average the same time, eight quarters. As is well known, in the data recessions are shorter than expansions, at least using the NBER definitions. In this exercise I condition on the aggregate state. Therefore KS's assumption of symmetry does not affect the path of the unemployment rate computed using equation (1).

4 Evaluating Alternative Calibrations

This section compares KS's calibration with potential alternatives, and asks the questions: Can alternative calibrations do better than KS's in terms of matching the data? Is the improvement substantial enough that one should give up the computational convenience resulting from KS's calibration?

İmrohoroğlu (1989) provides an alternative calibration of the transition probabilities. İmrohoroğlu assumes that steady state unemployment in the good and the bad state is 4% and 12% respectively. She also assumes that the duration of unemployment in the good and the bad state is 10 and 14 weeks, respectively, while KS assume a longer duration in both states (1.5 quarters ≈ 19.5 weeks in the good state and 2.5 quarters ≈ 32.5 weeks in the bad state). Moreover, she assumes that the conditional probabilities are only a function of next period's aggregate state s': Hence, $\frac{\pi_{gb\epsilon\epsilon'}}{\pi_{gb}} = \frac{\pi_{bb\epsilon\epsilon'}}{\pi_{bb}}$ and $\frac{\pi_{bg\epsilon\epsilon'}}{\pi_{bg}} = \frac{\pi_{gg\epsilon\epsilon'}}{\pi_{gg}}$. Column (B) of Table 1 provides the conditional transition probabilities used by İmrohoroğlu in parenthesis. These probabilities are not directly comparable with KS's because İmrohoroğlu's model period is six weeks, as opposed to one quarter. The figures outside the parenthesis in the second column of Table 1 are approximately (see footnote 4) İmrohoroğlu's figures converted to a quarterly period.[4] Chart (B) in Figure 3 compares the HP-filtered and KS's unemployment rates (solid and dotted lines, respectively) with that obtained from equation (1) using these quarterly transition probabilities (dash-and-dotted line). [5] I will refer to the latter series as "I-q's unemployment rate". By

[4]This conversion is not straightforward, because in good times the duration of unemployment is less than one quarter. I proceeded as follows. Call $\Pi_{6w}^{s'}$ the matrix of conditional transition probabilities in İmrohoroğlu when next period's aggregate state is s'. İmrohoroğlu's model period is roughly a half of KS's. Therefore,I obtained the quarterly probabilities as

$$\Pi^g_{\text{quarterly}} = \pi_{gg} * \Pi^g_{6w} \times \Pi^g_{6w} + (1 - \pi_{gg}) * \Pi^b_{6w} \times \Pi^g_{6w},$$
$$\Pi^b_{\text{quarterly}} = \pi_{bb} * \Pi^b_{6w} \times \Pi^b_{6w} + (1 - \pi_{bb}) * \Pi^g_{6w} \times \Pi^b_{6w}.$$

The quarterly figures imply roughly the same steady state unemployment in the good and the bad state (4% and 12% respectively) as in İmrohoroğlu, but a longer duration of the unemployment spell (15.8 weeks in the good state and 19.6 weeks in the bad state).

[5]To be precise, the notation in equation (1) is consistent only with the case where the unemployment rate swtiches immediately to the new steady state as teh aggregate state changes. A more

construction, I-q's unemployment rate has larger variation between good and bad times (8%) than both KS's and the HP-filtered series.

I also construct an alternative set of probabilities using the same assumptions as KS about duration of the unemployment spell and average unemployment in good and bad states, but where I assume as in İmrohoroğlu that the conditional probabilities are only a function of next period's aggregate state: $\frac{\pi_{gb\epsilon\epsilon'}}{\pi_{gb}} = \frac{\pi_{bb\epsilon\epsilon'}}{\pi_{bb}}$ and $\frac{\pi_{bg\epsilon\epsilon'}}{\pi_{bg}} = \frac{\pi_{gg\epsilon\epsilon'}}{\pi_{gg}}$. This calibration, referred to as "KS-2", is reported in the column (C) of Table 1. The corresponding implied unemployment rate, referred to as "KS-2's unemployment", is plotted in chart (C) of Figure 3 (dash-and-dotted line). While KS-2's unemployment does not jump immediately to the new steady state like KS's, the difference is not very large. During expansions (recessions) KS-2's unemployment declines (rises) very rapidly to the new steady state. Conversely, the decline in the HP-filtered unemployment, particularly during expansions, is only gradual.

The reason for the fast convergence to the new steady state is the following. Under the assumption that next period's aggregate state is the same as today's ($s' = s$) equation (1) can be rewritten as an AR(1) process:

$$(u'_s - \hat{u}_s) = (\frac{\pi_{ss00}}{\pi_{ss}} - \frac{\pi_{ss10}}{\pi_{ss}})(u_s - \hat{u}_s) \tag{4}$$

where u'_s is next period's unemployment rate, and $\hat{u}_s$ is the steady state unemployment in state s, which is given by the formula:

$$\hat{u}_s = \frac{\frac{\pi_{ss10}}{\pi_{ss}}}{1 - \frac{\pi_{ss00}}{\pi_{ss}} + \frac{\pi_{ss10}}{\pi_{ss}}}. \tag{5}$$

Unless one is willing to assume durations of the unemployment spell that are much larger than those assumed by either KS or İmrohoroğlu, the autoregressive coefficient is going to be relatively small. Under KS's calibration the autoregressive coefficient is .31 in good times and .56 in bad times. Therefore the unemployment rate will quickly reach the new steady, especially during expansions.

general formulation would be:

$$u\frac{\pi_{ss'00}}{\pi_{ss'}} + (1-u)\frac{\pi_{ss'10}}{\pi_{ss'}} = u'.$$

In order to achieve a slower convergence of the unemployment rate one needs to assume values for $\frac{\pi_{ss00}}{\pi_{ss}}$ that are closer to one than those assumed by KS. Such high figures for $\frac{\pi_{ss00}}{\pi_{ss}}$ would be consistent for instance with the inclusion of agents that are not-in-the-labor-force in the definition of the state $\epsilon = 0$, as shown in the top chart of Figure 1. An example of such calibration, which I refer to as "HDU" (high duration of unemployment), is given in column (D) of Table 1. The figures chosen for $\frac{\pi_{ss00}}{\pi_{ss}}$ are .86 and .87 for good and bad states, respectively. These numbers are roughly consistent with those plotted in the top chart of Figure 1, and imply a high autoregressive coefficient (about .85) but also an extremely high duration of unemployment: namely 7.1 quarters in expansions and 7.7 quarters in recessions. Since I keep the level of unemployment in the ballpark of the figures in KS (I assume 10% unemployment in bad states, and 6% in bad states), the implied values for $\frac{\pi_{ss10}}{\pi_{ss}}$ are very small: .009 in the good state and .014 in the bad state. These numbers are much smaller than those plotted in the top chart of Figure 1. This is not surprising of course, since the steady state unemployment implied by the numbers in the top chart of Figure 1 is between 30% and 40%, rather than between 4% and 10%. The unemployment rate implied by the HDU calibration, referred to as "HDU's unemployment", is plotted in chart (D) of Figure 3 (dash-and-dotted line). As expected, HDU's unemployment falls during expansions and rises during recessions only gradually, consistently with the HP-filtered data.

The conclusion that one needs to allow for much higher duration of unemployment spells to account for the business cycle features of unemployment echos that in Cole and Rogerson (1999). Cole and Rogerson's analysis is much broader in scope than the one conducted here, as they consider an array of business cycle facts that includes the time series features not only of unemployment, but also of job destruction and creation. Also, Cole and Rogerson use data on job flows as in Davis, Haltiwanger and Schuh (1996) rather than the worker flows used here. Finally, these authors derive a mapping between the structural parameters of a matching model á la Mortensen and Pissarides (1994) and the reduced form parameters of equation (1), that is, the πs. Yet, part of the analysis in Cole and Rogerson is similar to the one in this paper, as it focuses on these reduced form parameters (in their notation $\frac{\pi_{ss10}}{\pi_{ss}}$ is λ_s and $1 - \frac{\pi_{ss00}}{\pi_{ss}}$ is p_s). And so are some of the conculsions they reach, namely that one needs values of $\frac{\pi_{ss00}}{\pi_{ss}}$ above .67 (values of p_s below .33) in order to match the business cycle facts. These values imply a duration of unemployment spells that are on average above three quarters. Cole and Rogerson also report that the average job destruction rate in the data implies values of $\frac{\pi_{ss10}}{\pi_{ss}}$ (λ_s) around .055, that is, values that are in between those reported in the top chart of Figure (1) ($\epsilon = 0$ corresponding to $U + NLF$) and in the bottom chart ($\epsilon = 0$ corresponding to U only). These figures for $\frac{\pi_{ss10}}{\pi_{ss}}$ and $\frac{\pi_{ss00}}{\pi_{ss}}$ imply that the unemployment rate, averaging across good and bad states, is about 15%, more than twice as high as that assumed in KS. In summary, the findings in this paper, as well as in Cole and Rogerson's, suggest that a successful calibration of the πs should incorporate in the definition of the state $\epsilon = 0$ at least part of the agents that are not-in-the-labor-force, which in turn implies both a higher duration of unemployment and a higher steady state unemployment rate than are usually assumed.

5 Conclusions

Using data on flows into and out of employment, unemployment, and not-in-the-labor-force, I have constructed transition probabilities between "employment" ($\epsilon = 1$) and "unemployment" ($\epsilon = 0$) that can be used in the calibration of economies such as Krusell and Smith's. I have shown that KS's calibration has a few counterfactual features. Specifically, the feature that unemployment jumps immediately to the new steady state when the economy moves from a recession to an expansion, or from an expansion to a recession, is at odds with the data. And so are the patterns of the transition probabilities that deliver such jumps in aggregate unemployment: Namely the fact that the probability of being unemployed next period is the highest when the aggregate state moves from the bad to the good state, and the lowest when the aggregate state moves from the good to the bad state. However, I have also shown that the implications for aggregate unemployment of alternative calibrations is not very different, as long as the assumed duration of unemployment spells does not depart substantially from what has been previously assumed in the literature. Therefore, applied researchers may as well follow KS's calibration, which conveniently implies that the aggregate unemployment rate is no longer an additional exogenous state variable in the agent's problem. It is possible to calibrate the transition probabilities in such a way that the implied aggregate unemployment falls during expansions and rises during recessions only gradually, as in the data. These transition probabilities would be consistent with the inclusion of (at least part of) the agents that are not-in-the-labor-force in the definition of the state "unemployed". However, they imply values for the duration of the unemployment spell that are larger than three quarters, well above what is usually assumed in the literature.

A Data Appendix

The monthly data on workers' flows into and out of employment, unemployment, and not-in-the-labor-force were constructed by Bleakley, Ferris, and Fuhrer (henceforth BFF, 1999). BFF constructed the flows, which measure changes in the employment status of individuals, using the monthly Current Population Surveys (CPS) from January 1976 to March 1999. BFF obtained the data prior to 1976 from Blanchard and Diamond (1990). Section II and the appendix in BFF provide a detailed explaination of how the flows are constructed, which I will briefly summarize here. For each month CPS workers are matched with the same individuals in the previous month's survey. Since not all workers can be matched, matched workers are reweighted to represent the U.S. population. The gross flows are computed by adding up all (matched) workers' transitions in employment status. The gross flows are then adjusted for seasonal factors, misclassification (using the methodology in Abowd and Zellner, 1985), and methodological breaks in the survey's design (in 1994).

I compute the monhtly transition probabilities from the gross flows as follows. I first take a nine-month centered moving average of the flows, following BFF. Next, I define two states: "employment" ($\epsilon = 1$), which always coincedes with the employment status E, and "unemployment" ($\epsilon = 0$), which coincedes either with the employment status U or with $U + NLF$. The transition probability $\pi_{\epsilon\epsilon'}$ is then constructed as the mass of agents that transition from state ϵ to state ϵ during the period, divided by the mass of agents in state ϵ at the beginning of the period. Naturally, $\pi_{\epsilon 0} + \pi_{\epsilon 1} = 1$ whenever "unemployment" ($\epsilon = 0$) is defined as $U+NLF$. Whenever "unemployment" ($\epsilon = 0$) coincides with the status U, $\pi_{\epsilon 0}+\pi_{\epsilon 1}$ adds to one minus the proportion of agents that transitions from ϵ into NLF. The resulting transition probabilities are monthly. I computed the quarterly transition probabilities by multiplying the monthly transition matrices for each month in the quarter. For instance, if Π_1, Π_2, and Π_3, are the transition matrices for October, November, and December 1967, the quarterly transition matrix for QIV 1967 is computed as $\Pi_1 \times \Pi_2 \times \Pi_3$.[6] The monthly data for the transition probabilities exhibit the same pattern as the quarterly data.

The "unemployment rate" is computed as the mass of agents in state $\epsilon = 0$ divided by the mass of agents in states $\epsilon = 1$ and $\epsilon = 0$ (at the end of the period). This quantity is defined as u' in equation (1). The HP filter uses a smoothing coefficient equal to 1600, which is standard in the literature for quarterly data.

[6]Whenever $\epsilon = 0$ coincides with U only, for this procedure to be correct the Π matrix must be a 3×3 matrix that includes NLF as one of the states.

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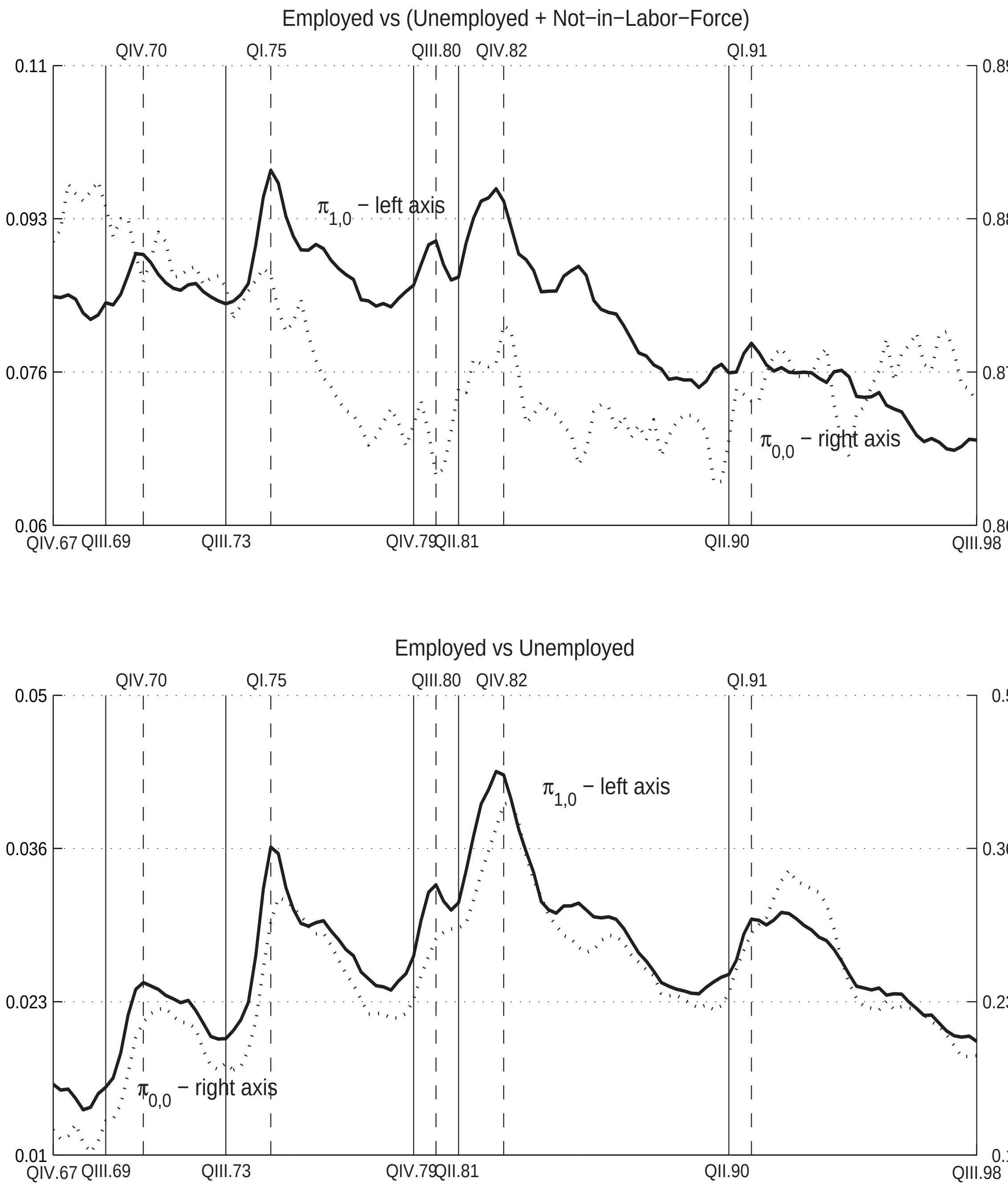
Table 1: Transition Probabilities (πs): Different Calibrations

Transition Probabilities	(A) KS	(B) I-q (İmrohoroğlu)	(C) KS-2	(D) HDU
$\frac{\pi_{gg'00}}{\pi_{gg'}}$	$\frac{1}{3} \approx .333$	.177 (.400)	.333	.860
$\frac{\pi_{gg'10}}{\pi_{gg'}}$	$\frac{1}{36} \approx .028$	.034 (.025)	.028	.009
$\frac{\pi_{bb'00}}{\pi_{bb'}}$	$\frac{3}{5} = .600$	.338 (.570)	.600	.870
$\frac{\pi_{bb'10}}{\pi_{bb'}}$	$\frac{2}{45} \approx .044$	.090 (.060)	.044	014
$\frac{\pi_{bg'00}}{\pi_{bg'}}$	$\frac{1}{4} = .250$	.177 (.400)	.333	.860
$\frac{\pi_{bg'10}}{\pi_{bg'}}$	$\frac{1}{60} \approx .017$	.034 (.025)	.028	.009
$\frac{\pi_{gb'00}}{\pi_{gb'}}$	$\frac{3}{4} = .750$	.338 (.570)	.600	.870
$\frac{\pi_{gb'10}}{\pi_{gb'}}$	$\frac{7}{96} \approx .073$	.090 (.060)	.044	014

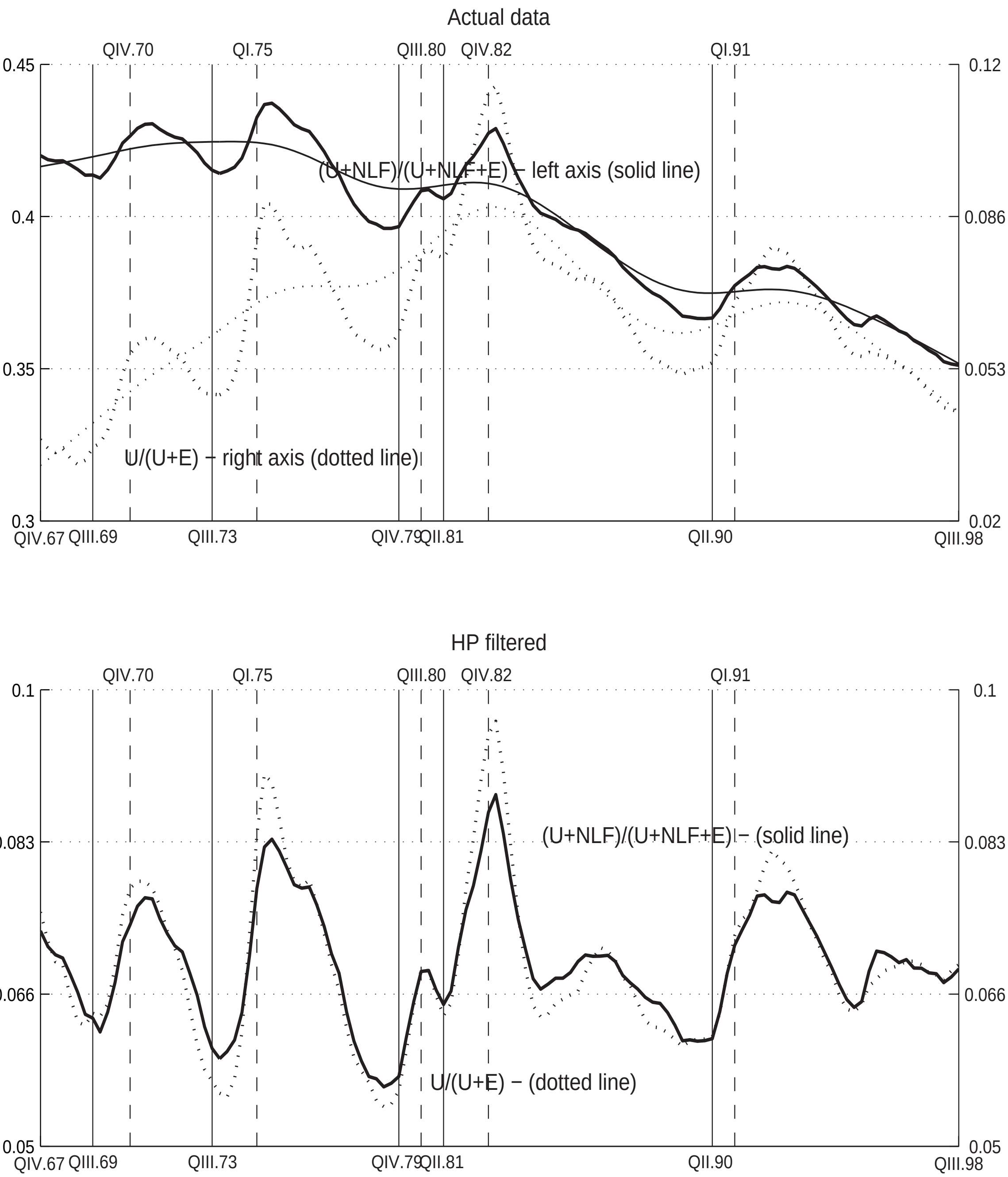
Notes: The Table shows conditional transition probabilities obtained under different calibrations. Column (A) shows the figures used in KS's calibration. Column (B) shows in parenthesis the figures used in İmrohoroğlu (1989)'s calibration. Outside the parenthesis are İmrohoroğlu's figures translated at the quarterly frequency (see footnote 4). Columns (C) and (D) present alternative calibrations.

Figure 1: Transition Probabilities (πs): The Data



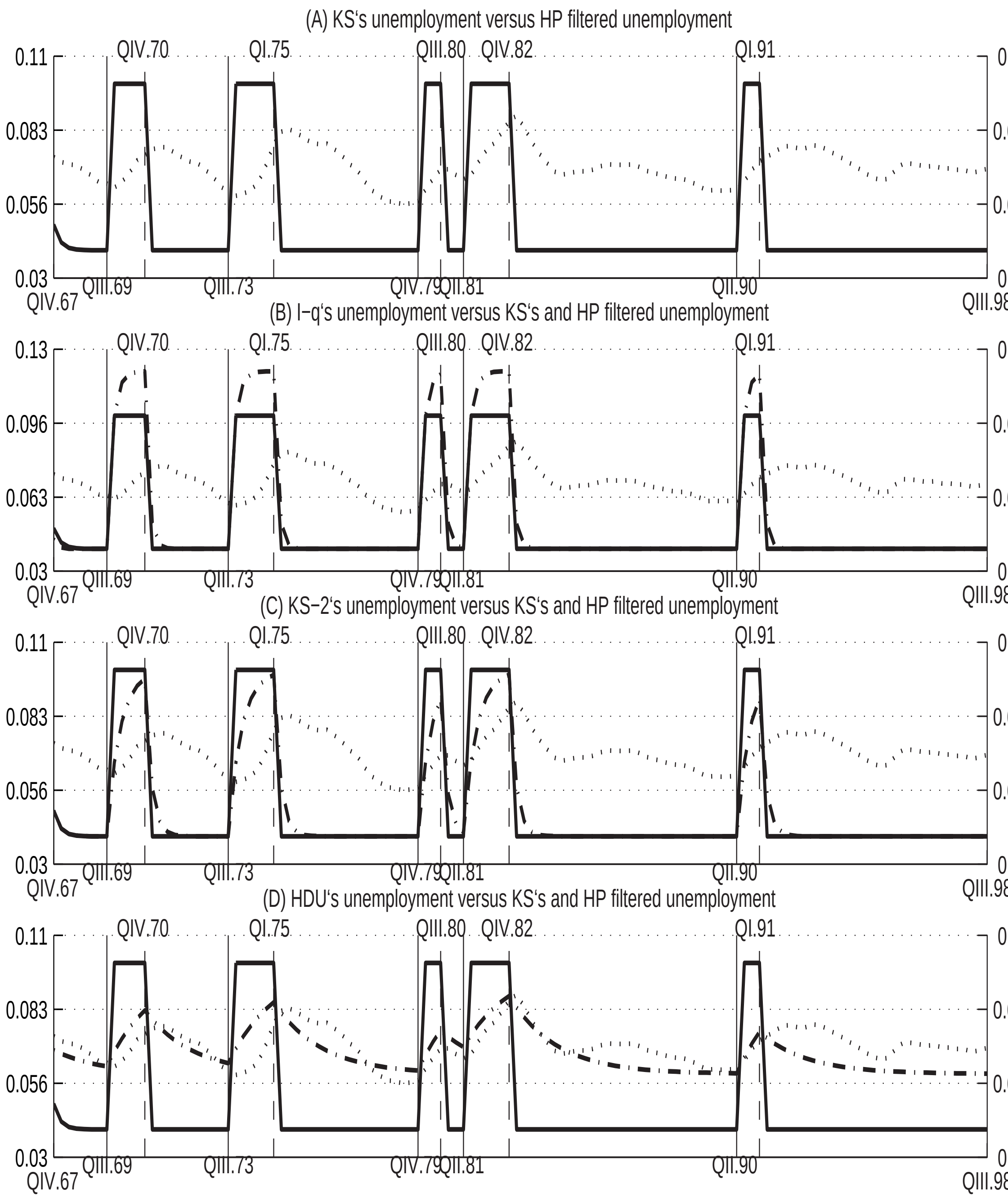
Notes: The plots show the transition probabilities $\frac{\pi_{ss'01}}{\pi_{ss'}}$ and $\frac{\pi_{ss'00}}{\pi_{ss'}}$ (at the quarterly frequency) when $\epsilon = 0$ denotes either being unemployed or out-of-the-labor-force (top), or only unemployed (bottom). The transition probabilities are constructed using data on workers' flows in and out of employment (E), unemployment (U), and not-in-the-labor-force (NLF), described in the data appendix A. Vertical solid and dashed lines denote the beginning and the end of NBER-defined recessions.

Figure 2: Unemployment Rate: Actual and HP-filtered



Notes: The top chart plots the path of two different definitions of the unemployment rate together with their respective Hodrick-Prescott trend. The solid line corresponds to the unemployment rate computed including those agents that are not-in-the-labor-force ($\frac{U+NLF}{U+NLF+E}$) while the dotted lines excludes these agents ($\frac{U}{U+E}$). U, E and NLF correspond to the mass of agents that are unemployed, employed, and not-in-the-labor-force, respectively. The bottom chart of Figure 2 shows the HP-filtered series. Vertical solid and dashed lines denote the beginning and the end of NBER-defined recessions.

Figure 3: Implications of Different Calibrations of the Transition Probabilities for the Agrgegate Unemployment Rate.



Notes: Chart (A) compares the HP-filtered unemployment rate (dotted line) with KS's unemployment rate (solid line), namely the unemployment rate obtained from equation (1) using the quarterly transition probabilities shown in column (A) of Table 1. Charts (B), (C), and (D) compare the HP-filtered and KS's unemployment rates (solid and dotted lines, respectively) with the unemployment rate (dash-and-dotted line) obtained from equation (1) using the transition probabilities shown in columns (B), (C), and (D) of Table 1, respectively. Vertical solid and dashed lines denote the beginning and the end of NBER-defined recessions.