# Nonaddictive Habit Formation and the Equity Premium Puzzle

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**Abstract:** I analyze a model in a simple representative-agent economy with one risky and one riskless asset, populated by habit-forming consumer-investors. These consumer-investors exhibit nonaddictive habit formation in the sense that the current consumption rate of the consumer-investors can fall below the habit-forming past consumption rate. I endogenize the real riskless rate of return in this representative-agent economy and find that the equity premium puzzle is resolved for values of the coefficient of relative risk aversion, the discount rate, and the intensity of nonaddictive habit formation, which are validated by previous empirical or survey-based studies. Nonaddictive habit formation studied here complements and extends current research on habit-forming preferences. Given a constant investment opportunity set, I find that the real riskless rate in the economy increases with relative risk aversion of the consumer and decreases as the habit formation intensity increases. The historically observed volatility of the real riskless rate is matched for one set of parameter values. Extensions with time-varying investment opportunity sets could explain the low risk-free rate and the relatively large variability of the market return over the variability of the risk-free rate through time.

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**Nonaddictive Habit Formation and the Equity Premium Puzzle**

## 1. Introduction

I analyze a model in a simple representative agent economy with one risky and one riskless asset wherein consumer-investors exhibit habit formation. When the current consumption rate is lower than the habit-forming consumption rate, the consumer-investor suffers a sense of deprivation. Any relative increase in the consumption rate as compared to the habit-forming consumption rate would result in a sense of elation. However, as Duesenberry (1967) argues in his seminal work on consumer behavior, an inherent drive or proclivity of the habit-forming consumer for higher consumption would mean that the sense of deprivation on a decline in the consumption rate is much stronger than the sense of elation from a corresponding increase in the consumption rate. I formalize this notion of habit formation.

In a discrete-time set-up, Abel (1990) has recently introduced habit formation wherein the period utility of the consumer is a function of current consumption as well as a habit-forming consumption from the previous period. This can be called "short-memory" habit formation. In the continuous-time framework, the representative agent's utility is a function of the current consumption rate if the current consumption rate is no different from the habit-forming past consumption rate. However, if the current consumption rate is lower than the habit-forming past consumption rate, the consumer suffers a utility penalty. This is a stronger notion of habit-formation than Abel's (1990) "short-memory" habit formation since the habit-forming consumption rate can be shown to persist for a considerable length of time.

Other recent habit-formation models in the literature (e.g., Constantinides 1990, Sundaresan 1989) do not permit a decline in consumption below a habit-forming, subsistence level of consumption. My set-up permits such declines in the consumption rate below the habit-forming consumption rate. The habit-forming, subsistence level of consumption in Constantinides (1990), for example, is an exponentially weighted sum of past consumption rates, first introduced by Ryder and Heal (1973). In contrast, the precursors to my work are Duesenberry (1967) and Abel (1990). Thus, in my model all the weight is placed on the most recent habit-forming consumption rate. I use preferences that are a sub-set of a broader class of habit-forming, von Neumann-Morgenstern preferences defined by Detemple and Zapatero (1991). Under this classification, Constantinides (1990) and Sundaresan (1989) model addictive habit-formation while I study nonaddictive habit-formation.

This definition of habit formation also addresses the concern over the lack of intertemporal complementarity expressed first by Hicks (1965). Hicks summarized intertemporal complementarity (which I here call "habit-formation") succinctly as: "The sacrifice [in current consumption] which one would be willing to make to fill a gap [in the consumption stream at some later time] must normally be much greater than what it would be worthwhile to incur for a mere extra." The utility penalty I subject the representative agent to whenever there is a decline in the consumption rate is thus, consistent with Hicks (1965), much greater than the utility reward the agent will enjoy when there is an increase in the consumption rate. Ingersoll (1992) studies a class of non-time-additive felicity functions such that consumers' choices are characterized by both intertemporal substitution and complementarity where typically one would think of intertemporal substitution as the shorter-run phenomenon. Short-run intertemporal substitution, or the idea that abnormally high consumption in the recent past would satiate the consumer and increase current felicity, is studied by Hindy and Huang (1993) as "local substitution" while I study complementarity wherein sustained past consumption, higher than current consumption, would lower current felicity due to habit-formation. Working with nonseparable preferences of several different forms, Heaton (1995) finds evidence for the local substitution of consumption with habit-formation (or complementarity) occurring over longer periods of time.

More recently, Campbell and Cochrane (1995) have used an i.i.d. consumption growth driving process and added a slow-moving external habit to the standard power utility function. Their external habit is such that an individual's habit level depends on the history of aggregate consumption rather than the individual's own past consumption. Thus, their external habit is not to be confused with habit formation as understood in the current literature but as "catching up with the Jones's" so labeled by Abel (1990).[1] In my model, an exogenous wealth process that follows geometric Brownian motion is the driving force, and an optimal consumption policy solved for by the representative agent, to maximize expected utility, generates the consumption growth process and the riskless interest rate process endogenously.

The nonaddictive habit-forming preferences are put to the test for solving a well-known puzzle in the finance and economics literature, namely, the equity premium puzzle. Simply stated, the equity premium puzzle is that the consumption rate appears to be too smooth to justify the mean equity premium observed in the U.S. economy. Over the last hundred years, the return on equity has exceeded the return on short-term T-bills by about 6 percent on an average. For the period 1889-1978, Mehra and Prescott (1985) show that this equity premium is too large to be

[1] An interesting application of habit-formation as defined in Abel (1990) to the nature of correlation between saving and growth across countries and within countries over time is available in Carroll, Overland, and Weil (1995), while Lettau and Uhlig (1995) answer the question, "Can habit formation be reconciled with business cycle facts?" using the Campbell and Cochrane (1995) construction of external habit.

explained by a conventional asset pricing model like the Lucas (1978) model, or that inordinately high values of the coefficient of relative risk aversion are required to explain the equity premium. This paper shows that the definition of habit formation used here helps greatly in improving upon previous attempts made to resolve this puzzle. The motivation for the approach outlined here is twofold: (a) to give expression to Hicks's (1965) criticism of the independence assumption implicit in separable utility functions by using a utility-penalty based approach and (b) to improve upon the results of the Ryder and Heal (1973) model of complementarity in the utility function[2] used by Constantinides (1990) to solve the equity premium puzzle.

I use constant relative risk averse (CRRA) preferences, which are rendered non-time-additive when the notion of habit formation is introduced. To ensure comparability with the Mehra and Prescott (1985) economy, the utility function is based on constant relative risk aversion (CRRA) preferences but with a utility penalty for a decline in the consumption rate as compared to the habit-forming, past consumption rate. The rest of the paper reads as follows: Section 2 describes the production economy for which the model is developed. Section 3 outlines an alternative representation of the representative consumer's problem, which is then solved in Section 4 to yield the optimal consumption policy. In Section 5, I demonstrate a possible resolution of the equity premium puzzle. In Section 6, I provide a comparative dynamic analysis to study variations in the riskless interest rate and the equity premium. Section 7 concludes the paper by indicating some ideas for future research.

[2] Duffie and Epstein (1992) generalize the habit-forming utilities first used by Ryder and Heal (1973). Duffie and Skiadas (1994) mention that extending such a generalization to the case in which the utility function is state- and time-dependent is possible.

## 2. The Economy and Assumptions

The economy has only one consumption-investment good. The representative agent has wealth W(t) at time t denominated in consumption-good units. The representative agent's consumption-rate of the good in the economy is c(t). Thus, c(t)dt units of the good are consumed in time dt. The risky, constant-returns-to-scale technology has a rate of return over time dt of $\mu dt + \sigma dz(t)$ where $\mu \in \Re$ and $\sigma \in \Re_+$ are constants and dz(t) is the increment of a standard Wiener process. Thus, the change in wealth at time dt is

$$dW(t) = [\mu W(t) - c(t)]dt + \sigma W(t)dz(t). \tag{1}$$

Given a consumption policy c(t), for $t \geq 0$, the expected lifetime utility of the infinitely lived representative agent is given by

$$E\left[\int_0^\infty e^{-\rho t}\left(\frac{1}{\gamma}(c(t))^\gamma dt + [I(dc(t) > 0)g + I(dc(t) < 0)b]dc(t)\right)\right], \tag{2}$$

where I(•) is an indicator function: its value is one if the argument is true and zero otherwise, and ρ is the discount rate in the economy. The change in the consumption rate over time dt is denoted dc(t) and is given by

$$dc(t) = [c(t) - c(t - dt)]. \tag{3}$$

The representative agent in this economy has constant-relative-risk-averse (CRRA) preferences, but her utility exhibits habit persistence in the following sense: if there is a change in the consumption rate, the consumer suffers a sense of deprivation when there is a decline ($dc(t) < 0$). This is captured by a utility penalty $b = \theta \frac{du(c(t))}{dc(t)}$, $\theta > 1$, where $\frac{du(c(t))}{dc(t)}$ is the marginal utility of consumption, and if $dc(t) > 0$, the consumer receives a utility reward $g = \frac{du(c(t))}{dc(t)}$. In this sense, the current consumption is habit-forming.[3]

The motivation for modeling habit-forming preferences in this manner is provided by the earlier insightful work of Duesenberry (1967). In his macroeconomic model, he showed that a drive for higher consumption leads to a "ratchet effect" on consumption due to learning and habit formation. Consumers are reluctant to reduce consumption but have no hesitancy in increasing consumption. My interpretation of Duesenberry's (1967) habit-forming preferences is twofold: (a) the reluctance to reducing consumption is modeled as a utility penalty[4] for any decline in the consumption rate as compared to the habit-forming consumption rate; and (b) the utility penalty on a decline in consumption rate is larger than the corresponding reward for an increase in the consumption rate, thus capturing the reluctance to reducing consumption as well as the inherent drive for increasing consumption. The higher the reluctance to reducing consumption rates, the larger is the utility penalty.

Fishburn and Kochenberger (1979) report evidence from empirically assessed power utility functions that utility gains from increases in present wealth are much lower than corresponding utility losses from decreases in present wealth. This follows from their two-piece von Neumann-Morgenstern utility functions. In my one consumption-investment good set-up, it is analogous to lower utility gains from increments in the consumption rate than utility losses from corresponding reductions in the habit-forming consumption rate.

[3] Such asymmetry in utility gains and losses of income has been studied by Friedman and Savage (1948). Marshall (1993) more recently modeled extremely small costs of consumption adjustment. In my case, the sense of deprivation leads to a utility cost or penalty, and the sense of elation to a corresponding, but smaller utility gain.

[4] In an extreme case of such a utility penalty, Dybvig (1993) defines utility to be minus infinity, if consumption ever falls.

I express the utility penalty and the utility reward in terms of marginal utility of consumption to indicate that while the reward is just the marginal utility of consumption (since the increase in consumption rates is to be expected, given the proclivity for higher consumption), the penalty is much higher than just the marginal utility of consumption (since the decline in consumption rates is painful once the consumer is used to a certain habit-forming consumption rate). θ, the ratio of the utility penalty to the utility reward, indicates the intensity of habit formation and is consistent with the notion of habit formation in Duesenberry (1967), described as a "ratchet effect" on consumption. Though there is no hindrance to increasing consumption rates, the greater utility penalty for subsequent declines in the consumption rate will deter the representative agent from adopting a consumption policy that allows for consumption rate increases to be unbounded.

The central planner in this representative-agent economy solves the following expected utility maximization problem [A]:

$$\max_{\{c(t)\}} E\left[\int_0^{\infty} e^{-\rho t}\left(\left[\frac{1}{\gamma}(c(t))^{\gamma}\right]dt + \left[I(dc(t)>0)g + I(dc(t)<0)b\right]dc(t)\right)\right] \quad \textbf{(4a)}$$

subject to: $dW(t) = \left[\mu W(t) - c(t)\right]dt + \sigma W dz(t)$, and **(4b)**

$dc(t) = G(t)$, if $dc(t) > 0$ and $dc(t) = B(t)$, if $dc(t) < 0$. **(4c)**

G(t) and B(t) are finite variation processes that keep track of the increases and decreases of the consumption rate, respectively.

The habit-forming preferences of the representative agent I model here fall within the broader class of such preferences studied by Detemple and Zapatero (1991). In their set-up, the preferences of the representative agent can be represented by the von Neumann-Morgenstern index:

$$E\int_0^T e^{-\int_0^t \rho_s ds} u(c(t), z(t))dt, \tag{5}$$

where $z(t) = z_0 e^{-\psi t} + \delta \int_0^t e^{-\psi(t-s)} c_s ds,\ z_0 \geq 0,$ where $\rho(t)$ is the discount rate, $u(.,.)$ is the instantaneous utility function, and $z(t)$ represents the standard of living of the decision maker. The parameters $\psi$ and $\delta$ are non-negative constants, and $z(0)$ represents the perceived standard of living at the initial date, an exogenous non-negative quantity. The standard of living satisfies

$$dz(t) = (\delta c(t) - \psi z(t))dt,\ z_o \geq 0; \tag{6}$$

$\psi$ and $\delta$ measure the persistence of the past and the intensity of consumption habits, respectively. In this case, $z(t) = 0$ since $\psi = \delta = z(0) = 0$, but the standard of living is separately modeled in the form of a habit-forming consumption rate, which results in the utility penalty $b = \theta \frac{du(c(t))}{dc(t)}$ whenever there is a decline in the consumption rate. $\theta > 1$ measures the intensity of consumption habits, which I call the intensity of habit formation.

Detemple and Zapatero (1992) provide a formal justification,[5] in a more general case, for a functional link between the standard of living process and the marginal utility process. They illustrate the separable nonlinear case for $u(c,z)$ using Cobb-Douglas utility. In Detemple and Zapatero (1991), however, we learn that Cobb-Douglas utility is also a good example of modeling

[5] We thank an anonymous referee for suggesting very pertinent references for this formal justification.

nonaddictive habit-forming preferences. The formal justification applies here since what we have in this paper is an example of the separable nonlinear case where the standard of living is separated from the utility function and the utility penalty represented as a very simple function of the marginal utility of consumption.

My habit-forming preferences complement and extend recent research on habit formation in several ways:

a) Abel (1990) models habit formation using a period utility function $u(c(t), v(t)) = [c(t)/v(t)]^{(1-\gamma)}/(1-\gamma)$, $\gamma > 0$, where $v(t)=[c(t-1)]^{\eta}$, $\eta=1$. In his discrete time model, the period utility is a function of the current consumption level and the consumption level in the previous period. My utility function is a continuous time analog where utility at any time t is a function of the current consumption rate, c(t), and the past consumption rate, c(t-dt). My solution indicates that this past consumption rate can stay unchanged over an extended period of time and so is called the habit-forming consumption rate.

b) The Constantinides (1990) general equilibrium set-up does not endogenize the real riskless interest rate in the economy. I endogenize the real riskless interest rate. Since the utility function u(c(t), z(t)) is a function of present and past consumption in the Constantinides (1990) model, the relative risk aversion (RRA) coefficient has to be defined in terms of an atemporal gamble that changes wealth and not in terms of a gamble that changes consumption. Thus, the RRA coefficient is a function of wealth and z(t) and only approximately equal to $(1 - \gamma)$. In my model, since the utility function itself is independent of past consumption, the curvature of the constant-relative-risk-averse utility function is captured by the parameter $\gamma$, and the RRA coefficient is exactly equal to $(1 - \gamma)$.

c) Constantinides (1990) and Sundaresan (1989) model addictive habit formation wherein an infinite utility penalty is stipulated for any consumption below the subsistence level of consumption, z(t). In my model, the consumption rate is permitted to fall below the habit-forming consumption rate, carrying a finite utility penalty. The representative agent never suffers an infinite expected utility penalty and exhibits nonaddictive habit formation as defined by Detemple and Zapatero (1991).

d) Defining u(c, z) = v(c, z) in a linear model, Constantinides (1990) and Sundaresan (1989) have habit formation driving a wedge between the relative risk aversion of the representative agent and the intertemporal elasticity of substitution in consumption. However, such separation of risk and time preferences is not a necessary condition for habit formation. My CRRA preferences with a utility penalty and a utility reward for declines and increases in the consumption rate, respectively, satisfy von Neumann-Morgenstern axioms and mix risk and time preferences as in Selden (1978) such that the coefficient of relative risk aversion is inversely related to the elasticity of intertemporal substitution in consumption. Consistent with Constantinides (1990), my habit-forming preferences yield a time-nonseparable but state-separable representation of tastes.

The aggregate wealth constraint in equation (4b) can be understood as follows: each investor in the economy invests a fraction α(t) of his wealth at time t in the risky production process. The investor is also borrowing or lending at a non-constant (stochastic) riskless interest rate, r(t). The wealth constraint on any individual investor in this economy is[6]

$$d\underline{W} = \left[\left\{(\mu - r(t))\alpha(t) + r(t)\right\}\underline{W}(t) - \underline{c}(t)\right]dt + \sigma\alpha(t)\underline{W}(t)dz(t), \tag{7}$$

[6] This is similar to the treatment in Merton (1971).

where $\underline{W}$ is the current level of the investor's wealth and $\underline{c}$ is the current rate of the individual's consumption. Aggregation across such identical investors in the economy gives the wealth-constraint in equation (4b). At the aggregate level $\alpha(t) = 1$. Thus, a central planner in such an economy will have only one investment opportunity, namely, the risky production process.

My model predicts that the consumption rate will not be continuously changed. The optimal consumption policy will be to maintain the consumption rate constant until such time as the deviation of the wealth-to-consumption ratio from its frictionless (i.e., the "no utility penalty or reward" case where the representative agent is not habit-forming in consumption) value is large enough to accommodate the utility penalty, given favorable or unfavorable changes in wealth and a utility reward when there is an increase in the consumption rate. The purpose of the optimization procedure discussed below is to determine an indicator function p(t) and threshold values of the indicator function at which the consumption rate is changed given the current consumption rate and the level of current wealth. The consumption policy parameters to be determined will be:

H: threshold value that determines when the consumption rate should be increased,

L: threshold value that determines when the consumption rate should be decreased.

## 3. Alternative Representation of the Problem

The representative agent problem in [A] is a nonlinear stochastic control problem that is easier to solve using control theory (as in Harrison 1985) when represented as follows:[7]

$$J\big(W(t),c(t)\big) = \max_{\{c(t)\}} E\int_{t}^{\infty} e^{-\rho t}\frac{1}{\gamma}\big(c(t)\big)^{\gamma}\,dt\,; \tag{8}$$

$$dW(t) = \big[\mu W(t) - c(t)\big]dt + \sigma W dz(t) - \theta B(t)dt, \text{ where} \tag{9}$$

$$dc(t) = G(t), \text{ if } dc(t) > 0; \text{ and} \tag{10a}$$

$$dc(t) = B(t), \text{ if } dc(t) < 0. \tag{10b}$$

G(t) and B(t) are finite variation processes that keep track of the increases and decreases in the consumption rate, respectively. From a control theory point of view the consumer will use changes in the consumption rate (G(t) and B(t)) to control the variable p(t) [to be determined later]. The controls work as follows:

a) increase c(t) by G(t) when p(t) = H(c(t)), and

b) decrease c(t) by B(t) when p(t) = L(c(t)).

The consumer maximizes the discounted utility function in equation (8) subject to the wealth constraint in equation (9). It is important to note that there is a shadow rate of interest r(t) in the economy at the aggregate level. It can be represented as a function of any one of the variables representing the evolution of the economy. It is actually computed as a result of the

[7] The first-order conditions of optimality of this problem coincide with the first-order conditions of the problem in [A]. This suggests that the two problems are equivalent. It is only suggestive because no verification theorem to this effect is being provided. For the equivalence of the first-order necessary conditions for problem [A] and the alternate representation in equations (8) through (10) above, see Appendix I.

utility maximization problem being solved. In order to calculate the value of the shadow riskless rate of interest, the expression for $\alpha(t)$ that maximizes[8]

$$J_W\left[((\mu-r)\alpha+\gamma)W-c\right]+\frac{1}{2}J_{WW}\alpha^2\sigma^2W^2 \tag{11}$$

is derived. Equation (11) suppresses the time subscript t. This is like the treatment in Merton (1969) in that we are maximizing the objective function where the expression for $\alpha(t)$ is

$$\alpha(t)=\frac{-(\mu-r(t))}{\frac{J_{WW}(t)}{J_W}W(t)\sigma^2}. \tag{12}$$

At the aggregate level, the riskless asset is in zero net supply and $\alpha(t) = 1$. From equation (12), the expression for r(t), the shadow interest rate in the economy, is then

$$r(t)=\mu+\frac{J_{WW}(t)}{J_W(t)}W(t)\sigma^2. \tag{13}$$

In an economy where the wealth process follows a geometric Brownian motion with drift $\mu$, the risk premium at time t is given by $(\mu - r(t))$. The aggregate wealth in the economy fluctuates in keeping with the stochastic differential equation (9). Hicks's (1965) concept of "a sacrifice needed to fill a gap in consumption" is operationalized as a proportional cost for any reduction in consumption compared to the past level of consumption.[9] Here, the idea from Dumas (1988) that such proportional costs in the state-space (or the (W,c) plane) imply an open area (and we assume it to be a single connected one) where it is optimal not to change

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[8] Equation (11) is the part of the Hamilton-Bellman-Jacobi (HBJ) equation containing terms in $\alpha(t)$. The HBJ equation can be written as $\frac{1}{2}J_{WW}\alpha^2\sigma^2W^2+J_W\left[((\mu-r)\alpha+\gamma)W-c\right]-\rho J+\frac{1}{\gamma}c^\gamma=0$.

[9] For a recent attempt of a similar nature using small non-convex (e.g., fixed) costs of consumption adjustment, see Marshall (1993).

consumption rates is used. So, holding c(t) constant, the aggregate wealth evolves in keeping with equation (9). The time that elapses[10] between any two consecutive hits at either boundary results in the constant, habit-forming, consumption rate. The boundaries of this region are the two edges of a two-dimensional cone with its vertex at the origin. (See Figure 1.) Within the region, when the wealth process reaches the upper edge, the consumer decides to increase the consumption rate. When the lower edge is reached, the consumer reduces the consumption rate. My habit-forming preferences lead to such a consumption policy. In contrast, Hindy and Huang (1993) use preferences embodying the idea of "local substitution" that lead to periodic consumption[11] only when the marginal value of average past consumption is higher than the marginal value of wealth. In this case, the choice of action at the two limiting boundaries of the region is permitted by a *consumption policy*[12] that can be understood as explained below.

In the (W,c) plane there exists a cone with its vertex at the origin. The upper edge of this cone (of slope $h$) is the "abundant limit": when the existing pair (W,c) is such that the economy is at this limit, an instantaneous decision is made to increase consumption by G(t). The consumption increases by an infinitesimal amount, enough to just move back into the region within the cone (along a horizontal line). The lower edge of the cone (of slope $\ell$) is the "austere limit": when the existing pair (W,c) is such that the economy is at this lower limit, the central planner

[10] The expected time until the wealth process hits either boundary when starting it at a point on the "Merton-line" where the wealth-to-consumption ratio is equal to one can be calculated. (See pages 191-192 in Karlin and Taylor 1981.) Preliminary computations indicate that for parameter values that resolve the equity premium puzzle, expectations range from one to several weeks.

[11] In a related context, periodic consumption *changes* is a feature in models of durable good purchases with transaction costs; see, for example, Grossman and Laroque (1990). The optimal consumption policy in these models is periodic consumption in gulps (stock) while my consumption policy involves continuous consumption with periodic changes in the consumption rate (flow).

[12] A key finding in Sundaresan (1989) is that the optimal consumption behavior involves considerable "smoothing"; consumers tend to keep their optimally determined current consumption close to their past consumption standards. The infinitesimal consumption changes in my model are justified on this basis.

instantaneously decides to decrease consumption by B(t). The decrease is infinitesimal, enough to push the economy within the cone (along a line of slope $\theta$).

Within the open region bounded by the cone, the consumption rate is constant. This is a direct consequence of the friction introduced in the model. If there were no penalties or rewards for changing consumption, each change in wealth would be accompanied by a commensurate change in consumption (defining the "Merton-line"). The effect of having to incur a utility penalty for reducing consumption[13] is twofold: (a) There will be an upper bound on the wealth-to-consumption ratio depending on the penalty for reducing consumption; (b) the wealth-to-consumption ratio cannot go below a lower bound because the consumer reduces consumption when adverse wealth shocks compel the consumer to do so. When aggregate wealth follows a Markov diffusion process and is the single source of shocks in this economy, such lower and upper bounds on the wealth-to-consumption ratio[14] can be solved for using the idea of regulated Brownian motion (Harrison 1985). Since the penalty is a proportional cost, a good source for the treatment is Constantinides (1986). The representative agent's problem [A] in equations (4a) - (4c) of welfare optimization, keeping this background in mind, can be solved as a problem of nonlinear stochastic control when represented as equations (8) - (10).

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[13] As mentioned in Davis and Norman (1990), "any attempt to apply Merton's strategy in the face of transaction costs would result in immediate penury, since incessant trading is necessary to hold the portfolio on the Merton line. There must in such a case be some 'no-transaction' region inside which the portfolio is sufficiently 'out of line' to make trading worthwhile."

[14] In a model of optimal consumption and portfolio selection in which consumption services are generated by holding a durable good that is illiquid (in the sense that a transaction cost must be paid when the good is sold), Grossman and Laroque (1990) show that it is optimal for the consumer to wait until a large change in wealth occurs before adjusting his consumption. They also come up with boundary values of the consumption-to-wealth ratio at which the consumer sells the durable good for a smaller or larger durable good depending on unfavorable or favorable movements in the wealth process, respectively.

The barrier policy necessary here is [L(c(t)),H(c(t))], which is obtained by applying a two-sided regulator. The problem can now be formulated by using a substitution that leads us to an analytical, closed-form solution. A new variable $p(t) = W(t)/c(t)$ and a new function

$$F(p) = \frac{J(W,c)}{c^{\gamma}} \tag{14}$$

are chosen, and two numbers, $\ell$ and $h$, are chosen, such that $L(c(t)) = \ell < H(c(t)) = h$ and (dropping the subscript (t)) the rate of consumption increases only when $p = W/c = h$ and the rate of consumption decreases only when $p = W/c = \ell$. The variable p(t) is the ratio of wealth to consumption, which is the correct variable to control,[15] since deviations from unity (the Merton 1969 frictionless benchmark for this ratio), are caused due to my habit-forming preferences. The optimal consumption policy is determined by the values of this ratio at the edges of the two-dimensional cone-shaped region on either side of unity. (Refer to Figure 1.)

## 4. Solution and Optimal Consumption Policy

The solution is to find the optimal consumption policy by optimizing the two values $\ell$ and $h$. It is possible to find an analytical solution for the functions J and F. The function J exists.[16] The Hamilton-Bellman-Jacobi equations below give the conditions to be satisfied by functions J and F within the cone:

$$0 = \frac{1}{\gamma}c^{\gamma} - \rho J + J_W(\mu W - c) + \frac{1}{2}J_{WW}\sigma^2 W^2; \tag{15}$$

[15] The formal proof for the optimal consumption policy indicated here (with proportional costs or utility penalties like ours) appears in Davis and Norman (1990).
[16] The proof for this assertion is found in Dumas (1991).

$$0=\frac{1}{\gamma}-\rho F(p)+F'(p)(\mu p-1)+\frac{1}{2}F''(p)\sigma^2 p^2 . \tag{16}$$

If the trajectory of the values of F is to remain continuous with probability one, since applying the regulator results in an instantaneous movement to the target position, the value of expected discounted utility must match at the arrival and departure points (with probability one). This happens when implementing the "barrier policy" at both edges of the cone where the triggers for the two-sided regulators H and L are applied. Thus, when W/c = $\ell$,

$$J(W,c)=J(c-B,W-\theta B), \tag{17}$$

or

$$0=-J_c(W,c)-\theta J_W(W,c), \tag{18}$$

and when W/c = $h$,

$$J(W,c)=J(c+G,W), \tag{19}$$

or

$$0=J_c(W,c). \tag{20}$$

In terms of conditions on the function F, equation (17) transforms into:

$$\frac{F'(\ell)}{F(\ell)}=\frac{\gamma}{\ell+\theta}, \tag{21}$$

and equation (19) transforms into:

$$\frac{F'(h)}{F(h)}=\frac{\gamma}{h}. \tag{22}$$

Now, for given (or optimized) values of $h$ and $\ell$, we can calculate the functions J and F. For example, the ordinary differential equation (15) has a general solution:

$$F(p;C_1,C_2)=\frac{1}{\rho\gamma}+C_1N_1\left(\frac{2}{\sigma^2 p}\right)+C_2N_2\left(\frac{2}{\sigma^2 p}\right), \tag{23}$$

where $C_1$ and $C_2$ are integration constants, and

$$N_i(y)=e^{-y}y^{\pi_i}M(a_i,b_i,y) \text{ for } i=1,2, \tag{24}$$

where $M(a_i, b_i, y)$ is the confluent hypergeometric function,[17] $\pi_1$, $\pi_2$ are solutions of:

$$\frac{1}{2}\pi(\pi-1)\sigma^2-\pi(\mu-\sigma^2)-\rho=0, \tag{25}$$

and

$$a_i=\frac{\mu-\sigma^2-\frac{1}{2}\pi_i\sigma^2}{\frac{1}{2}\sigma^2}, \tag{26}$$

$$b_i=\frac{\mu-\sigma^2-\frac{1}{2}\pi_i\sigma^2}{\frac{1}{2}\sigma^2}. \tag{27}$$

Since $C_1$ and $C_2$ are functions of $\ell$ and $h$ each, on the basis of conditions like (17) through (20), we can find expressions for them. However, to get an optimal consumption policy, we need to get optimal values for $\ell$ and $h$. The nature of the problem is such (with proportional costs) that the function F and integration constants $C_1$ and $C_2$ will reach an optimum simultaneously for

[17] $M(a_i, b_i, y)$ is a power series solution. It is part of a power series solution to Kummer's equation, a well-studied equation in the theory of ordinary differential equations. It is given by:
$M(a_i,b_i,y)=1+\frac{a_i}{b_i}y+\frac{a_i(a_i+1)y^2}{b_i(b_i+1)2!}+\frac{a_i(a_i+1)(a_i+2)y^3}{b_i(b_i+1)(b_i+2)3!}+\dots$ For a good treatment of confluent hypergeometric functions, see Abramowitz and Stegun (1965).

the same set of optimized values $\ell$ and $h$. At this optimum, the derivatives of the integration constants will be zero. Using this fact, the first order conditions with respect to $\ell$ and $h$ for equations (15) and (16) are examined. These are conditions for the optimum. We then have

$$\frac{F''(\ell;C_1,C_2)}{F'(\ell;C_1,C_2)}=\frac{\gamma-1}{\ell+\theta}, \tag{28}$$

or

$$\frac{J_c}{J_W}=\frac{J_{cc}+J_{Wc}}{J_{Wc}+J_{WW}}, \tag{29}$$

and

$$\frac{F''(h;C_1,C_2)}{F'(h;C_1,C_2)}=\frac{\gamma-1}{h}, \tag{30}$$

or

$$J_{cc}=-J_{Wc}. \tag{31}$$

Equations (29) and (31) are second-order conditions on J taken with respect to W and c, using the first-order conditions on J given by equations (18) and (20), respectively.

Thus, the conditions necessary and sufficient for optimizing the slopes $\ell$ and $h$ are (21), (22), (28), and (30). The constants $C_1$ and $C_2$ appear in the system linearly. The four equations taken simultaneously help us not only to eliminate $C_1$ and $C_2$ but also give us a system of two non-linear equations in the unknowns $\ell$ and $h$. These can be solved using the Newton-Raphson technique.

## 5. Resolution of the Equity Premium Puzzle

The model used is versatile. The analytical, closed-form general equilibrium solution to the central planner's problem leaves *numerical* computation for the constants of integration alone. The model has predictive ability. It predicts movements in the real riskless rate for the economy with a given investment opportunity set. Besides, it predicts an average growth rate of consumption and the variance of the growth rate of consumption. This is made possible by employing a discrete approximation of the wealth process. The wealth process can be expressed as

$$dW(t) = \mu^*\left(W(t)dt + \sigma^*\left(W(t)\right)dz(t)\right). \tag{32}$$

Let W(t), a Markov diffusion process, be a solution to the Ito stochastic differential equation (32). Let $\mu^*(W(t))$ and $\sigma^*(W(t))$ be the drift and diffusion of the process, respectively. An approximation of the process W(t) can be computed by approaching equation (32) by a time discretized equation of the type

$$W_{t(i+1)} = f\left(W_{t(i)}, z\right), \tag{33}$$

where {i = 0, 1, ...} is, for example, a fixed mesh, at time t. The discretization scheme chosen converges in quadratic mean to the process (32) and is called the Euler scheme.[18] The proof for convergence is described in Duffie and Singleton (1993). Discrete-time simulations of stochastic differential equations of the form (32) are conducted. Simulated solutions to (32) are calculated by breaking each unit of time into n equal subdivisions and approximating the solution W(t) by $W^n(t) = W^n(nt)$, where $W^n(nt)$ is generated by a difference equation of the form

[18] See Milshtein (1978) and Talay (1984) for other numerical schemes that are available in the literature.

$$\Delta W^n(k+1) = W^n(k) + \Delta W^n(k). \tag{34}$$

$\Delta W^n(k)$ is a stochastic step at time interval k, and $W^n(0)$ is the initial observation. Among the various steps suggested in the approximation literature are the Euler scheme and the Milshtein scheme, which are first- and second-order schemes, respectively. I choose the Euler scheme:

$$\Delta W^n(k) = \frac{1}{n}\mu^*\left(W^n(k)\right) + \frac{1}{\sqrt{n}}\sigma^2\left(W^n(k)\right) \in (k), \tag{35}$$

where $\in(k)$ is an i.i.d. sequence of N(0, 1) random numbers. This is the approximation step that is typically used in discrete-time approximations to continuous-time processes for asset prices. It is employed here to simulate the wealth process in a production economy.

The wealth process can be simulated using Monte Carlo methods. In particular, the evolution of the natural log of the wealth process is studied. Thus, the drift term is a function of wealth, but the diffusion term is a constant since equation (9) can also be written as

$$d\ln W(t) = \left[\mu - \frac{1}{2}\sigma^2 - \frac{c(t)}{W(t)}\right]dt + \sigma dz(t) - \frac{\theta B(t)}{W(t)}dt. \tag{36}$$

The exercise undertaken thereafter is similar to the calibration exercise attempted by Mehra and Prescott (1985) in order to calculate the risk premium in the economy. The mean and variance of the wealth process are chosen to match corresponding historical values of the S & P 500 series returns during the 1889-1978 period. Given the rate of return on the risky asset as its mean value, the riskless rate in the economy is predicted by the model. The mean value of the real riskless rate in the economy is used to calculate the risk premium. The inclusion of a penalty on reducing the rate of consumption (due to habit formation) in the model leads us to a choice of values for $(1 - \gamma)$, the coefficient of relative risk aversion, and $\rho$, the discount rate in the economy. The Monte Carlo simulations conducted enable a calibration exercise (see Appendix II for details). Different sets of $(\rho, \gamma, \theta)$ as parametric values can generate different values for (a) the mean riskless rate in the economy, (b) the mean growth rate of consumption, and (c) the variance of the growth rate of consumption. Monte Carlo simulations conducted indicate that the values of $(1 - \gamma)$ for given values of $\rho$ and $\theta$ vary from 1.5 to 3.0 in order to predict the risk premium in the economy so as to match the historic value observed over the period 1889-1978 by Mehra and Prescott (1985) (see Table 1). The results also provide comparative dynamics for variation in the endogenous riskless rate with changes in the values chosen for $\gamma$ and $\theta$ (see Table 2).

## 6. Nonaddictive Habit Formation and the Riskless Interest Rate

The main results of this paper are summarized in Table 1. In Table 2, I present the results in detail for some of the parametric values in Table 1.

The central result of this paper is the resolution of the equity premium puzzle enabled by my definition of nonaddictive habit formation. Unlike the Abel (1990) and Constantinides (1990) papers, the utility function specified here enables this resolution with traditionally accepted definitions of the discount factor and the coefficient of relative risk aversion.[19] The result emerges from the analysis presented in Table 1. For a given value of risk aversion, the intensity of habit formation required to resolve the equity premium puzzle increases as the discount rate increases. For example, from Table 1, for $(1 - \gamma) = 2.20$, $\theta = 4.5$ resolves the puzzle for $\rho = 0.025$,

[19] See Table 1 for values of the parameters that resolve the puzzle. In Abel (1990), since utility is measured for consumption ratios (e.g., $c(t)/c(t-1)$), the discount factor normally used needs suitable modification. In Constantinides (1990), $(1-\gamma)$ is only approximately equal to the coefficient of relative risk aversion, since habit formation drives a wedge between the intertemporal elasticity of substitution in consumption and the coefficient of relative risk aversion for the representative consumer.

but a much higher value of $\theta = 45$ is necessary to resolve the puzzle when the discount rate is $\rho =$ 0.037. The puzzle is resolved for $\theta = 4.2$ only because the CRRA is much higher (3.0) when compared to the original value of CRRA (2.2). In other words, if $\rho = 0.007$, and CRRA = 2.20, the puzzle would be resolved at a much lower intensity level of habit formation than $\theta = 4.2$.

How does one interpret $\theta$? Can we place a metric on this habit-formation parameter? That is, based on some independent study, can we arrive at economically meaningful values for this parameter? Empirical work undertaken by Fishburn and Kochenberger (1979) on two-piece von Neumann-Morgenstern utility functions of the type used in this paper indicates that constant relative risk-averse utility functions gave the best fits in the case of utility functions that were convex below the reference point and concave above. More significantly, the utility function below the reference point was generally steeper than the utility function above, with a median below-to-above slope ratio of about 4.8. The intensity of habit formation, denoted by $\theta$, is such a below-to-above slope ratio where the reference point is the past, habit-forming, consumption rate. The Fishburn and Kochenberger (1979) study provides a useful benchmark for economically meaningful $\theta$ values.

The next result provides comparative dynamics between the risk-premium and the intensity of habit formation in the economy. At given levels of $\rho$ and $(1 - \gamma)$ (i.e., a given discount rate and coefficient of relative risk aversion), a higher intensity of habit formation is reflected in a higher utility penalty for a reduction in the consumption rate. From part (i) of Table 2 we observe that the riskless rate in the economy is a decreasing function of the utility penalty for a given reduction in the consumption rate. Since the utility penalty increases as $\theta$ increases, the consumer is more averse to a proportional loss of wealth. Thus, $-[WJ_{WW}/J_W]$, the relative risk aversion measure, is higher, implying that $[WJ_{WW}/J_W]$ is lower. Examining equation (13), we find that at the aggregate level, r(t) would be lower. In my set-up, this implies that with a stronger manifestation of habit formation comes a higher risk premium. Consumers now "wait and see" longer before they change consumption. A stronger manifestation of the habit formation characteristic implies smoother consumption in comparison to the wealth series in the economy.

In growing economies, stronger habit formation implies a higher savings rate. If the consumer chooses between consumption and investment, stronger habit formation implies a higher investment rate in such economies. Habit formation provides a strong justification for a consumption series that is smoother than the wealth series. In a recent paper, Carroll, Overland, and Weil (1995) introduce habit formation as in Abel (1990) to show that it can change the effect of favorable productivity shocks on the saving rate from negative to positive.[20] My model confirms the result that if habit formation were absent as an observed characteristic of consumer behavior, there would be no utility penalty for reducing consumption and all changes in wealth would cause immediate changes in consumption such that the ratio of consumption to wealth in the economy would remain constant.

Next, I analyze variations in the risk premia with changes in the levels of risk aversion. For a given discount rate, the riskless rate in the economy increases as the risk aversion increases (see part (ii) of Table 2), and the preference of the consumer now shifts in favor of borrowing and lending at the riskless rate. This result is surprising. If the risky return in the economy were allowed to fluctuate *ex ante* in this economy,[21] we would have had the more intuitive result that

[20] They argue that such a mechanism may be responsible for the high and rising saving rates in the fast-growing East Asian countries over the past thirty years.
[21] In future research, allowing for a time-varying opportunity set would be the next step to examine this result more thoroughly.

the riskless rate should *decrease* as the consumers become *more* risk averse. This counter-intuitive result seems to be a consequence of the fact that we hold the risky return constant *ex ante*. The result is robust for various levels of $\theta$, the utility penalty. In other words, whenever such a penalty is suffered, the result is independent of $\theta$.

Yet another result needs emphasis: from Table 1 it is clear that a higher value of the discount rate ($\rho = 0.048$) permits resolution of the puzzle at a lower value of CRRA of 1.56 when compared to the value of $(1 - \gamma) = 2.10$ necessary when the discount rate is lower ($\rho = 0.037$). The comparative dynamics are starkly seen for variations in $\rho$, $(1 - \gamma)$, and $\theta$ in the summary results wherein five sets of parameter values are presented that resolve the puzzle fully. However, this still leaves unresolved the issue of higher-than-expected volatility of the riskless interest rate. Abel (1990), for example, finds that the standard deviation of the real riskless interest rate in the economy is too high at 17.87% compared to the Mehra and Prescott (1985) value of 5.67% for the 90-year period 1889-1978.

In my results, the standard deviation of the real riskless interest rate of 5.67% reported by the Mehra and Prescott (1985) study was matched only in the case[22] of a low discount rate of $\rho = 0.007$, high CRRA of $(1 - \gamma) = 3.00$, and $\theta = 4.2$. In all other cases, the standard deviation was too high, in the range of 45% to 65%. Thus, this observation of higher-than-expected volatility of the real riskless interest rate remains unresolved in these cases. Since the risky return is held constant in this model, any computation of the standard deviation of the risk premium would essentially be contributed by variation in the real riskless interest rate. Therefore, the standard deviation of the risk premium was not computed.

[22] The value of $\theta$ of 4.2 is comparable in magnitude to the value of 4.8 arrived at in the Fishburn and Kochenberger (1979) study.

## 7. Conclusion

Constantinides (1990) has used addictive habit formation in an expected utility framework, upsetting a conventional paradigm. Is habit formation or habit persistence adding value to existing theory? The first step in answering this question has been taken in the present paper. The introduction of a habit-formation parameter to denote nonaddictive habit-forming preferences in expected utility theory has quantified habit formation just as the coefficient of relative risk aversion in earlier times quantified the concept of risk aversion.[23] The assumption of non-time separable utility made here is in the same spirit as Kydland and Prescott (1982). They study firm behavior as we do consumer behavior. Their idea of "time to build" is translated here as "time to consume." The efficacy of this notion of nonaddictive habit formation in resolving the equity premium puzzle has also been demonstrated. My analysis complements the work of Constantinides (1990) on addictive habit formation and extends Abel (1990) insomuch as it indicates a possible resolution of the equity premium puzzle assuming nonaddictive habit formation rather than Abel's (1990) "catching up with the Jones's" kind of preferences. The paper also parallels the modeling approach in Dumas (1988). He deals with capital formation, while here the emphasis is on the theory of consumption. Since consumption growth is non-i.i.d., the first-order correlation between respective changes in consumption growth is another unknown[24] such a calibration exercise could predict. This is possible, with the number of parameters on hand, if the generalized method of moments approach is used.[25]

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[23] See the survey-based findings on the coefficient of relative risk aversion in Blume and Friend (1973).
[24] This would enable the type of analysis undertaken by Mankiw and Zeldes (1989) analyzing the consumption of stockholders and non-stockholders.
[25] See Hansen and Singleton (1982).

I have formalized a notion of habit formation first defined by Duesenberry (1967) that an inherent drive or proclivity of the consumer for higher consumption would mean that the sense of deprivation on a decline in the consumption rate is much stronger than the sense of elation on a corresponding increase in the consumption rate. I show that this notion fits the definition of nonaddictive habit formation under the broad class of habit formation models defined in Detemple and Zapatero (1991). Further, the habit-formation parameter values that resolve the equity premium puzzle are the same order of magnitude as those of a comparable parameter calculated in empirical work on power utility functions by Fishburn and Kochenberger (1979).

Using the recent Campbell and Cochrane (1995) construct of external habit (which is to be contrasted with habit-forming preferences as discussed in the literature so far), Lettau and Uhlig (1995) show that utility functions with such a habit give rise to a consumption volatility puzzle in place of asset pricing puzzles when agents can choose consumption and labor optimally in response to more fundamental shocks. Replacing external habit with the idea of nonaddictive habit formation in a model with persistent technology shocks and stochastic labor may help validate their results using habit-forming preferences.

Finally, the implications of the model for growing and declining economies needs to be studied in greater detail. Such underlying trends can be built into the wealth process for the production economy. The savings rate in an economy can then be related to the habit-formation parameters of consumer-investors. However, a general equilibrium model like this one needs to be developed for a *time-varying* investment opportunity set. In other words, instead of holding the return on the risky asset constant as I have done here, we need future research to allow *both* the real riskless rate and the return on the risky asset to vary with time.

## Appendix I: Derivation of necessary conditions

The original problem set-up in equations (4a) to (4c) leads to the following boundary conditions:

when $\frac{W}{c} = \ell,$

$$J(W,c) = J(c - \theta B, W - B) \text{ or} \tag{38}$$

$$0 = -\theta J_c - J_W \tag{39}$$

when $\frac{W}{c} = h,$

$$J(W,c) = J(c + G, W) \text{ or} \tag{40}$$

$$0 = J_c(W,c)\,. \tag{41}$$

Now, taking second-order conditions with respect to J, we get from equation (39)

$$\frac{J_c\left[J_{Wc} + J_{WW}\right] - J_W\left[J_{cc} + J_{Wc}\right]}{\left(J_c\right)^2} = 0, \tag{42}$$

and from equation (40)

$$J_{cc} + J_{Wc} = 0. \tag{43}$$

We can write equation (42) as

$$\frac{J_c}{J_W} = \frac{J_{cc} + J_{Wc}}{J_{Wc} + J_{WW}} \tag{44}$$

and equation (43) as

$$J_{\alpha c} = -J_{Wc}. \tag{45}$$

Equations (43) and (44) are the same as the corresponding boundary conditions for the reformulated problem given by equations (29) and (31). In the comparison, it is now clear that the two problems—the original and reformulated—are equivalent in that the necessary conditions of the reformulated problem in Equations (8) - (10) are the same as the necessary conditions of the original problem [A] in Equations (4a) - (4c).

**Appendix II: Simulation methodology**

The experiment design used for the computation is reported in compliance with the standards for reporting of computation-based results laid out in Hoaglin and Andrews (1975). There are two broad segments in the experimental design:

1. The calculation of the optimal values of the "abundant limit" ($h$), the "austere limit"($\ell$), and the constants of integration $C_1$ and $C_2$ was done using the Newton-Raphson technique. These values correspond to a chosen vector of values $(\gamma_o, \rho_o, \theta_o)$ of the coefficient of relative risk aversion for a given set of values for the discount factor and the utility penalty for reductions in the habit-forming consumption rate.

2. The calculation of the unknown endogenous variables in my model, which are

(a) the real riskless rate, r, in the production economy,

(b) the mean rate of consumption growth, $\underline{c}$, for a sampling every quarter of the discrete approximation to the continuous-time process, and

(c) the variance of the rate of consumption growth, $\sigma_c$, for the same sampling method as in (b), given a set of parameters $(\gamma_o, \rho_o, \theta_o)$. To do so, I use the Monte Carlo method to simulate the wealth process over time. In other words, for each set of parameters we simulate several sample paths of the wealth process.

Experiments were conducted as follows:[26]

1. Preliminary Simulations: Coarse Grid

Here, the objective is to search for a triplet $(\gamma, \rho, \theta)$ that through simulation gives sample values of the mean growth rate of consumption, the standard deviation of the growth rate of consumption, and the real riskless that which match historically observed values in the Mehra and Prescott (1985) study. The first set of simulations was conducted using four values for $\rho$ and ten values each for $\theta$ and $\gamma$. Thus, we had 400 (10x10x4) sets of $(\gamma, \rho, \theta)$ values for which we could generate 400 values of $(r, \underline{c}, \sigma_c)$. This provided a first approximation for calibrating the model or adjusting the parameter values so that $(r, \underline{c}, \sigma_c)$ attained close-to-desirable values of the unknown endogenous variables: $r = 0.008$/year, $\underline{c} = 0.018$/year, and $\sigma_c = 0.0357$/year.

2. Final Simulations: Fine Grid

The set of values of $(r, \underline{c}, \sigma_c)$, or first approximations from the preliminary simulations, provided values of $(\gamma, \rho, \theta)$ around which a fine grid could be constructed for the final simulations. Each simulation run used 10,800 random numbers generated using a random number generator. For 90 years of data (1889-1978), this meant the consumer was permitted to change consumption once every three days. As an additional step, I also tried matching the standard deviation of the real riskless rate with historically observed values in Mehra and Prescott (1985), apart from the three moments mentioned above.

[26] I thank an anonymous referee for helping us with a clear and concise description of my simulation methodology.

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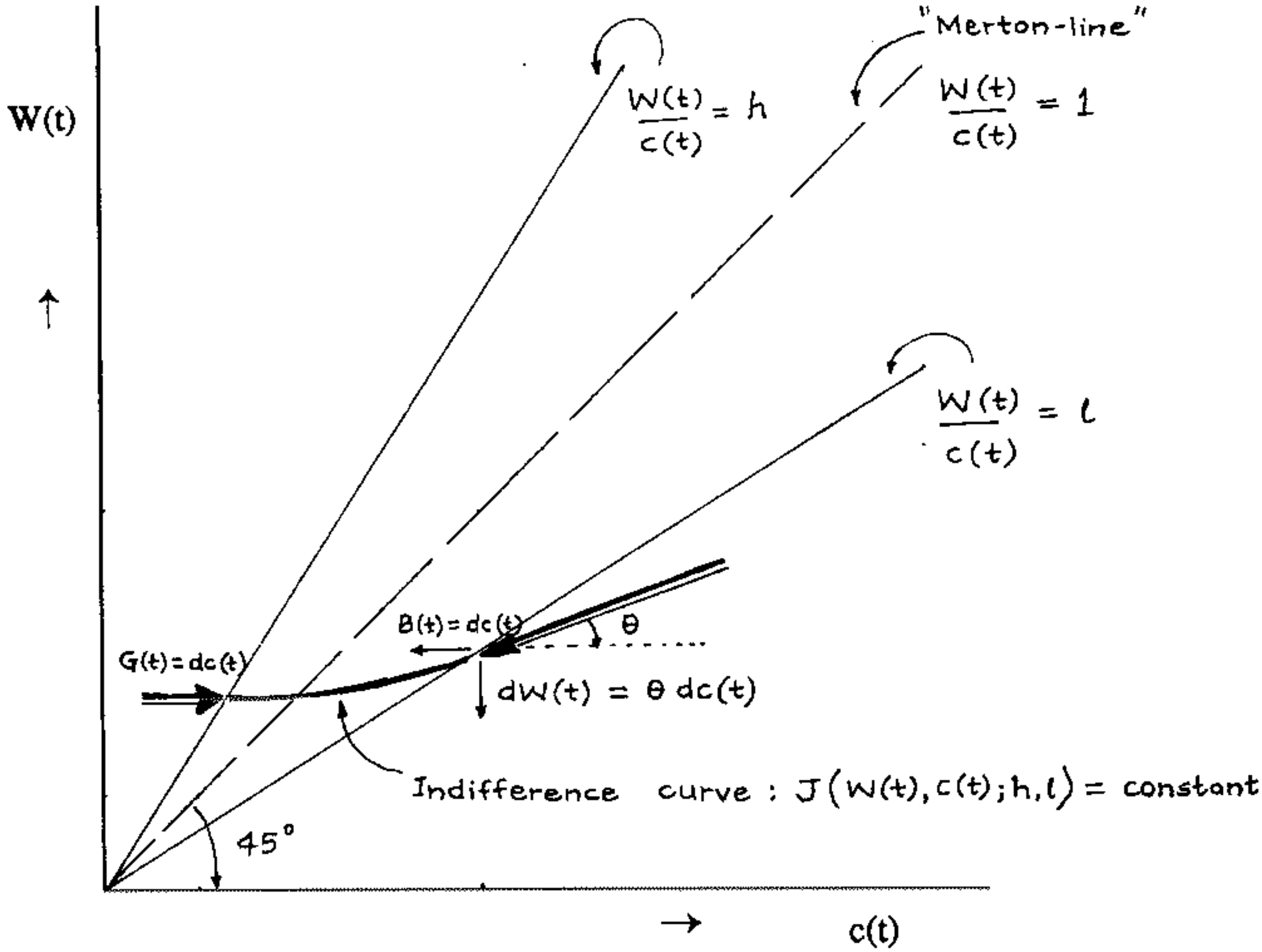


**Solution: Representative agent's expected utility maximization problem.**

FIGURE 1

## Table 1

### Monte Carlo Simulation Results

Values for ($\rho$, $(1 - \gamma)$, $\theta$) that resolve the equity premium puzzle

| | Discount Rate ($\rho$) | CRRA $(1 - \gamma)$ | Habit-Formation Parameter ($\theta$) |
|---|---|---|---|
| 1. | 0.048 | 1.56 | 30 |
| 2. | 0.037 | 2.10 | 30 |
| 3. | 0.037 | 2.20 | 45 |
| 4. | 0.025 | 2.20 | 4.50 |
| 5. | 0.007 | 3.00 | 4.20 |

These values lead to a calibrated model wherein

(i) real riskless rate $r = 0.008$ per annum,
(ii) mean growth rate of consumption = 0.018/year,
(iii) standard deviation of the growth rate of consumption = 0.0357/year.

## Table 2

### COMPARATIVE DYNAMICS
(i) Risk Premium vs. Habit Formation
(ii) Risk Premium vs. CRRA

**(i) RISK PREMIUM VS. HABIT FORMATION**
For: $\mu = 0.07$; $\sigma = 0.165$; $\rho = 0.007$; $\theta = 3.0$.

| $\theta$ | $h$ | $\ell$ | $C_1$ | $C_2$ | Riskless Rate | Risk Premium |
|---|---|---|---|---|---|---|
| 10 | 76.93 | 36.79 | -6.4416 | 65.0819 | -0.0223 | 0.0923 |
| 5 | 68.8 | 37.75 | -5.9244 | 66.0052 | 0.0010 | 0.0690 |
| 4 | 66.78 | 38.07 | -5.7871 | 66.2442 | 0.0097 | 0.0603 |
| 3 | 64.35 | 38.48 | -5.6287 | 66.5162 | 0.0185 | 0.0515 |

**(ii) RISK PREMIUM VS. CRRA**
For: $\mu = 0.07$; $\sigma = 0.165$; $\rho = 0.048$; $\theta = 30$.

| CRRA | $h$ | $\ell$ | $C_1$ | $C_2$ | Riskless Rate | Risk Premium |
|---|---|---|---|---|---|---|
| 1.7 | 47.94 | 14.90 | -0.1449 | 13.8022 | 0.0218 | 0.0482 |
| 1.5 | 47.84 | 14.12 | -0.1224 | 15.2193 | 0.0051 | 0.0649 |
| 1.4 | 48.00 | 13.76 | -0.1129 | 16.0023 | -0.0084 | 0.0784 |
| 1.3 | 48.34 | 13.41 | -0.1042 | 16.8588 | -0.0193 | 0.0893 |

Note: The utility penalty for reductions in the habit-forming consumption rate, $\theta$, is also the habit-formation parameter. The risk premium is calculated as the difference between the return on the risky asset, $\mu$, and the real riskless interest rate, $\gamma = \mu + \left(\frac{J_{WW}}{J_W}\right) W\sigma^2$, at any instant of time, t.