The Long-Run Real Effects of Monetary Policy: Keynesian Predictions from a Neoclassical Model

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Abstract: In this paper we integrate Diamond's (1965) model of neoclassical production and capital with Wallace's (1984) model of monetary policy in order to study the real effects of two types of monetary policy actions: open market operations and changes in reserve requirements. We show that a permanent easing of open market or reserve policy can produce permanent increases in both the inflation rate and the level (but not the growth rate) of output. We also describe conditions under which the effects of these policies on real interest rates and output can be large relative to their effects on the rate of inflation. When we compare the effects of the two types of policies we find that open market operations are the policy tool that minimizes the change in the inflation rate necessary to achieve a given change in the level of output.

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1 Introduction

A generation ago, the conventional wisdom among macroeconomists held that changes in monetary policy that produced permanent changes in the rate of inflation could also produce essentially permanent changes in the levels of real interest rates and real output. Within the framework of the Keynesian models that were popular at the time this prediction could be justified only by assuming that economic agents were victims of persistent money illusion. This point was made forcefully by Friedman (1968) and Phelps (1972), who argued that analyses of the effects of policy changes that were based on this assumption were unsound and unconvincing. These arguments were extended by neoclassical economists such as Lucas (1972) and Sargent and Wallace (1976), who developed rational expectations models in which persistent money illusion was explicitly ruled out. In these models, permanent changes in the inflation rate induced by monetary policy had limited real effects in the short run and no real effects in the long run. As the rational expectations locomotive gained steam the notion that money was long-run superneutral became the consensus view among macroeconomists.

In recent years the linear structural models that were once the most common vehicles for the study of the monetary policy implications of rational expectations have been superseded by general equilibrium optimizing models. The standard framework for monetary policy analysis is now the infinite-horizon representative agent model augmented by placing money in the utility function or imposing a cash-in-advance constraint. From a purely analytical perspective money is not always long-run superneutral in this class of model. As the models are usually specified, however, any departures from long-run superneutrality are empirically inconsequential.[1] Thus, the popularity of these models has done nothing to

[1] See Sidrauski (1967), in which money is analytically superneutral, and Danthine, Donaldson and Smith (1987). For additional discussion see Section 6.2 below.

shake macroeconomists' belief in the long-run ineffectiveness of monetary policy.

While the empirical evidence in favor of long-run superneutrality has never been as overwhelming as the balance of opinion on the subject might suggest, particularly strong reasons for entertaining doubts have appeared during the last few years. Recent advances in methods for econometric analysis of time series data have made it possible to conduct more definitive tests of hypotheses involving long-run relationships.[2] When these methods have been applied to the study of the real effects of monetary policy they have frequently turned up evidence that [1] money is not long-run superneutral and [2] permanent increases in the rates of money growth and/or inflation lead to permanent decreases in real interest rates and permanent increases in the level of output. King and Watson (1992), for example, report that for the U.S. during the postwar period "the data do not appear to be consistent with the hypothesis that, over the long run, money is superneutral or that nominal interest rates move one-for-one with inflation." When they employ the common strategy of identifying their models by assuming recursivity they find that permanent increases in the inflation rate produce substantial increases in the level of output.[3] Similar results are reported by Weber (1994), who uses the same methodology but looks at postwar data for the G-7 countries. Bullard and Keating (1995), who look at postwar inflation-output relationships across a sample of almost 60 OECD countries, find that for countries with relatively low inflation rates the long-run effect of inflation on the level of output is typically positive. Ahmed and Rogers (1997) who look at U.S. data over a much longer sample period, conclude that "in the *long run*, the effect of inflation on investment and output is positive ... and the investment rate, and hence the real interest rate, are not independent of inflation." [Their emphasis.][4]

[2] See King and Watson (1992) and Fisher and Seater (1993).

[3] A revised version of this paper has recently been published in the *Economic Quarterly* of the Federal Reserve Bank of Richmond. [King and Watson (1997).] King and Watson (1994) use similar methodology to study the long-run relationship between inflation and unemployment in U.S. postwar data. They conclude that a "traditional Keynesian identification" fits the data as well as the other identification schemes they study and yields "large estimated long-run trade-offs between inflation and unemployment".

[4] The results reported in most of these papers are not unambiguous: there is evidence of significant departures from superneutrality for some identification schemes but not others and/or for some countries but not others, etc. However, we think these results indicate, at minimum, that [1] the evidence against long-run superneutrality is far stronger than is widely believed and [2] the presumptive direction of departures from superneutrality is consistent with our theoretical results, at least for countries with relatively low inflation

In this paper, we demonstrate that an alternative class of general equilibrium models can allow permanent changes in the money growth and inflation rates to have effects on the steady-state levels of real interest rates and output whose magnitudes are potentially large. The directions of these effects are consistent with key aspects of the Keynesian conventional wisdom: if the money growth and inflation rates increase, then the real interest rate falls and the level of output rises. Thus, these models can account for the aforementioned empirical evidence concerning the long-run real effects of changes in monetary policy. The models to which we refer are extensions of the model studied by Diamond (1965) — a model with overlapping generations of agents, neoclassical production and capital, and government debt. The overlapping-generations structure of this model allows its parameters to be chosen in a way that guarantees the existence of equilibria in which government debt can be unbacked, in the sense of being unaccompanied by an equal-present-value stream of future government surpluses. We take a simple specification of this model and augment it with a source of demand for fiat money, a government budget constraint, and exogenous technological progress. The combination of fiat money, unbacked government bonds and a government budget constraint provides a mechanism that allows changes in the growth rate of the stock of fiat money to have significant effects on the level of the real interest rate. In addition, neoclassical production/capital and exogenous technological progress allow changes in the level of real interest rate to be associated with changes in the growth rate of real output that are persistent but not permanent. The steady-state real output growth path shifts to a new level and the actual growth rate of real output must change temporarily, but persistently, in order to produce the required change in the level. The long-run real output growth rate is unaffected, however, since it is determined by the exogenous rates of population growth and

rates (see our concluding remarks). In addition, the results just described are quite consistent with the large empirical-finance literature which finds that nominal interest rates tend to rise less than one-for-one with inflation: see, for example, Fama and Gibbons (1982), Mishkin (1981, 1984, 1992) and Kandel, Ofer and Sarig (1996). While most of these papers look at interest rates on bonds, there is similar evidence concerning rates of return on other financial assets, Boyd, Levine and Smith (1997), for example, find that "for countries with average annual inflation rates of less than 15 percent, there is no significant relationship between the rate of inflation and the nominal return on equity." They cite a number of earlier studies that reach similar conclusions.

technological progress.

The ability of overlapping-generations models to allow changes in monetary policy to have permanent effects on the real interest rate has been known for many years. Wallace (1984) constructed a model in which the existence of a reserve requirement allows fiat money to coexist with return-dominant government debt. He used the model to show that a permanent open market purchase (a permanent decrease in the ratio of bonds to money) would cause a permanent reduction in the real rate of interest.[5] In some respects, however, Wallace's results were quite inconsistent with the conventional wisdom: a permanent open market purchase also causes a permanent decrease in the rate of inflation. Espinosa and Russell (1998) use Wallace's model to show that this perverse relationship between changes in the inflation rate and changes in the real rate of interest is an artifact of the same assumption that produced the closely related phenomenon of "unpleasant arithmetic" [Sargent and Wallace (1981)]: the assumption that the initial steady-state real interest rate is higher than the steady-state growth rate of real output.[6]

[5] The notion that open market operations can have permanent real effects may disconcert some readers. The most likely source of this discomfort is that one result of an open market operation is an immediate change in the nominal stock of money, and in most general equilibrium models — including this one — a strictly one-time change in the nominal money stock has no long-run effect on real variables. That is, money is neutral in the long run. However, under the definition of an "open market operation" that used in the literature on which this paper draws, such an operation represents a *permanent* change in the ratio of the stock of government bonds to the stock of fiat money — a change which necessitates, through the government's deficit-finance constraint, both permanent changes in the real stocks of bonds and/or money and a permanent change in growth rate of the fiat money stock. In fact, the models in this literature can be reparameterized so that the exogenous policy change is a change in the growth rate of the nominal stock of fiat money, in which case the changes in both the bonds/money ratio and the real stocks of money and bonds become endogenous adjustments that are necessary to allow the government to continue to finance its deficit. If money is not long-run *super*neutral then a policy change of this type should be expected to have real effects. Parameterizing monetary policy in terms of the bonds/money ratio has the virtue of resolving an indeterminacy that arises in these models. Typically, there are two equilibria associated with a given inflation rate, but these equilibria involve very different bonds/money ratios.

[6] For discussions of the role of this assumption see Darby (1984) and Miller and Sargent (1984). Miller and Wallace (1985) and Miller and Todd (1995) study multi-country versions of the Wallace (1984) model under the high-real-interest-rate assumption. These models also deliver forms of "unpleasant arithmetic."

Bhattacharya, Guzman, Huybens and Smith (1997) study a model that is similar to ours except for its assumptions about money demand and financial intermediation. These assumptions allow a given specification of open market policy to support multiple steady states involving different rates of inflation. In steady states where the initial inflation rate is high a tightening of monetary policy can produce a lower rate of inflation even when real interest rate exceeds the output growth rate. This is possible because in these steady states lower rates of money growth and inflation increase government revenues from money creation.

In this paper, we extend the analysis conducted by Espinosa and Russell (1998) in a number of different directions. We replace the pure-exchange model they borrowed from Wallace (1984) with a Diamond-style model of neoclassical production and capital augmented by exogenous technological progress. The new model has two important advantages. First and foremost, it can be used to study the effects of changes in monetary policy on the level and growth rate of real output, and also on the levels of the capital stock and investment. In addition, the new model allows us to produce these real effects without introducing any heterogeneity within generations. This simplifies certain aspects of the analysis, facilitates welfare comparisons, and allows the model to be extended to less-stylized environments in which the agents live and work for many periods.[7]

Both Wallace (1984) and Espinosa and Russell (1998) confine themselves to studying the effects of changes in monetary policy conducted via open market operations. In this paper, however, we also investigate the impact of changes in reserve requirements. In particular, we show that while the effects of reserve ratio changes are generally similar to those of open market operations, there are at least two major differences. First, the use of reserve ratio changes as the instrument of policy broadens the range of policy parameter values over which the conventional monetary-policy wisdom holds true — that is, the range over which policy changes that increase the inflation rate also reduce the real interest rate. Second, the change in the reserve ratio that is necessary to produce a given change in the real interest rate always produces a larger change in the inflation rate than the open-market operation that is needed to achieve the same real interest rate target. This finding may help explain the Federal Reserve System's historical preference for open market operations.

Our third innovation is that we are able to organize the bulk of our analysis of the model

Although the version of the model these authors employ can produce equilibria with low real interest rates, their analysis of monetary policy focuses on high-real-rate equilibria and situations involving unpleasant arithmetic. In addition, they cannot produce situations in which the interest rate on government bonds is lower than the ouput growth rate but the real interest rate facing private borrowers exceeds the output growth rate. Empirical evidence suggests that these are the situations that are empirically relevant (see below).

[7] An extension of this type is pursued by Bullard and Russell (1998b) using a model that must be analyzed via computational methods.

around the behavior of the government's seigniorage revenue function, which we refer to as the "seigniorage Laffer curve." This allows us to provide both more complete descriptions of the regions of the parameter space where the effects in which we are interested arise and more satisfying intuitive explanations of the sources of these effects.

A fourth and final innovation involves our assumptions about the relationship between the real interest rate on government debt and the marginal product of capital. Monetary policy can have real effects of the type we describe only if the real interest rate on government debt is lower than the output growth rate, and only if the government can issue and maintain a stock of unbacked debt. (The former condition is necessary and sufficient for the latter situation to be possible.) Both these assumptions seem eminently plausible. During the postwar period, average real interest rates on U.S. Treasury bonds have been substantially lower than the average U.S. output growth rate, and the federal government has maintained a large debt while running an average primary surplus close to zero.[8] However, in the models of Wallace (1984) and Espinosa and Russell (1998) the real rate of return on private liabilities is equal to the real interest rate on government bonds. This presents a problem, because in U.S. data the average real return rates on many types of private liabilities — notably, diversified portfolios of common stocks — exceed the average real output growth rate by a substantial margin.

The analysis conducted in this paper follows that of our predecessors by assuming that all credit arrangements are intermediated. However, we introduce a real cost of intermediating private liabilities that drives a wedge between the real rate of return on these liabilities — the marginal product of capital — and the real interest rate on government debt. Our results indicate that under relatively weak assumptions, any specification of our model that has equilibria in which the real interest rate on government debt is lower than the real output growth rate also has equilibria from which a change in monetary policy has "conventional"

[8] See Bullard and Russell (1998a) and Espinosa and Russell (1998). The average output growth rate has been close to 3 percent, while the average real interest rate on short-term government debt has been less than one percent before taxes and less than zero percent after taxes. The average primary (net of interest) surplus has been approximately 0.1 percent of GDP.

effects — even if the equilibrium marginal product of capital begins and remains higher than the output growth rate. (See the example presented in Section 7 below.) Thus, while our intermediation-cost assumption does not represent any contribution to the theory of financial intermediation, it does serve to make the point that the empirical observation that the marginal product of capital is higher than the output growth rate is not inconsistent with the existence of monetary-policy effects of the type we describe.

The next section of the paper lays out the model that provides the framework for our analysis. In Sections 3 and 4 we use this model to study the effects of open market operations and changes in reserve requirements, respectively; in Section 5, we compare the effects of these two policy instruments. In Section 6 we discuss the implications of our results for the effects of monetary policy on nominal interest rates and for the ability of policy to produce relatively large changes in real interest rates and output. We also comment on the relationship between our model and models that produce Tobin effects. In Section 7 we present a parametric example that illustrates our basic findings. Section 8 concludes the paper.

2 The model

2.1 The environment

At each discrete date $t \geq 1$ a positive number of identical two-period-lived agents are born. This number N_t grows at a gross rate of $n \geq 1$ per period, so that $N_t = n^t N_0$, where $N_0 > 0$ (see below). These agents are endowed with a single unit of labor in the first period of their lives and have no endowment of any kind in the second period. Young agents supply their labor inelastically at any positive real wage, so the total supply of labor at any date is equal to the total population of young agents at that date.

The preferences of the agents are assumed to have the property that the amount agents save is invariant to the rate(s) of return on the assets available to them. In particular, each young agent saves an amount $s\,w_t$, where s represents the fraction of his income he devotes to saving and w_t represents the real wage rate at date t. We assume $s \in (0,1)$.[9]

At each date there are an indeterminate number of competitive, zero-profits firms that produce the single consumption good and obtain loans from competitive financial intermediaries (see below) in order to finance the acquisition of physical capital. Physical capital consists of consumption goods that were stored by agents during the previous period. The firms use the Cobb-Douglas production technology

$$Y_t = F_t(K_t, L_t) = \lambda^{(1-\alpha)(t-1)} K_t^{\alpha} L_t^{1-\alpha}.$$

Here Y_t represents date t output of the single good, which can be consumed at date t or stored and used as capital at date t+1. In addition, K_t represents the total stock of capital used in production at date t, L_t represents the total quantity of labor used at that date, $\alpha \in (0,1)$, and $\lambda \geq 1$. If $\lambda > 1$ then there is exogenous technical progress at a gross rate of λ per period.

The marginal product of capital r_t is given by

$$r_t = \lambda^{(1-\alpha)(t-1)} \alpha k_t^{\alpha-1},$$

where $k_t \equiv K_t/L_t$. In a competitive equilibrium, arbitrage requires the gross private real lending rate R_t to satisfy $R_t = 1 + r_{t+1} - \delta$, where δ is the net rate of depreciation. We assume that goods stored at date t depreciate at net rate δ from date t to t+1 whether or not they are used in production at the beginning of the latter date. For simplicity, however, we shall set $\delta = 1$, so that $R_t = r_{t+1}$.[10] We then have

$$k_t = \lambda^{t-1} \left(\frac{R_{t-1}}{\alpha} \right)^{\frac{1}{\alpha-1}},$$

[9] This assumption follows Wallace (1984) and Espinosa and Russell (1998). It is adopted primarily for purposes of analytical tractability. There is, however, considerable empirical evidence that aggregate gross saving is relatively insensitive to changes in the real rate of return. Given our endowment assumptions, a utility function that will generate this behavior is $u(c_1, c_2) = \log c_1 + \frac{1}{1+\rho} \log c_2$, where $\rho > -1$. This function produces $s = \frac{1}{2+\rho}$.

[10] This assumption is not unreasonable in a two-period model: if we think of a period as thirty years, an annual depreciation rate of 0.1 corresponds to a per-period rate of almost 0.96.

so aggregate demand for capital/loans at date t is given by

$$K_{t+1}(R_t) = (\lambda n)^t N_1 \left(\frac{R_t}{\alpha}\right)^{\frac{1}{\alpha-1}}.$$

The real wage rate is given by

$$w_t = \lambda^{(1-\alpha)(t-1)}(1-\alpha)k_t^{\alpha}.$$

Using the expression for k_t, we have

$$w_t = \lambda^{t-1}(1-\alpha)\left(\frac{R_{t-1}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.$$

It follows that the aggregate savings function is

$$S_t(R_{t-1}) = (\lambda n)^{t-1} N_1\, s \left(\frac{R_{t-1}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.$$

Note that $S_t(\cdot)$ is a function of R_{t-1}, which determines agents' income at date t, but not of R_t^d, which is the gross real rate of return they will receive on assets acquired at date t (see below). In addition, since $Y_t = L_t \lambda^{(1-\alpha)(t-1)} k_t^{\alpha}$, real output at each date t can be written

$$Y_t(R_{t-1}) = (\lambda n)^{t-1} N_1 \left(\frac{R_{t-1}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}.$$

At date 1, there are N_0 "initial old" agents who are endowed with H_0 nominal units of fiat currency and $\mathcal{B}_0$ units of one-period nominal government bonds (payable in fiat currency) which are due at that date. At each date $t \geq 1$ the government must finance a fixed positive real net-of-interest deficit $G_t > 0$ by a combination of currency and bond seigniorage. The value of G_t is assumed to grow at gross rate $\Psi \equiv \lambda n$ per period. The government's budget constraint is

$$G_t = p_t[(H_t - H_{t-1}) + (P_t^b \mathcal{B}_t - \mathcal{B}_{t-1})].$$

Here H_t represents the nominal stock of fiat currency at date t (so that $M_t \equiv p_t H_t$ is the real stock of currency) and $\mathcal{B}_t$ represents the nominal face value of the stock of bonds (so that $B_t = p_t \mathcal{B}_t$ is the real face value of the bond stock). The nominal price of a unit-face-value nominal bond is P_t^b, and $R_t^{nom} = 1/P_t^b$ is the nominal interest rate on government bonds.

Of course, the government budget constraint can be met only if $p_t > 0$ for all t. If bonds, money and capital are to coexist, moreover, we must have

$$P_t^b = \frac{R_t^m}{R_t^b} \leq 1\,,$$

where R_t^b is the gross real interest rate on government bonds (see below).

The assets available to young agents are fiat currency and deposits issued by a competitive banking system. At each date, an indeterminate number of banks make real loans to the firms and/or purchase one-period nominal bonds from the government. The intermediary faces a nonnegative proportional cost c on its loans to the firms; this cost is incurred at the date the loans are repaid.[11] Government bonds, in contrast, are intermediated costlessly. As a result, we have

$$R_t^d = R_t - c = R_t^b.$$

where R_t is the gross real interest rate on loans to the firms, R_t^b is the gross real interest rate on government bonds and R_t^d is the gross real interest rate on bank deposits.

The government also requires the banking system to hold reserves of fiat currency against its deposits. If the reserve requirement is binding, then the gross real deposit interest rate R_t^d is given by

$$R_t^d = (1-\theta)R_t^b + \theta R_t^m,$$

where R_t^m is the gross real rate of return on currency — $R_t^m = p_{t+1}/p_t = 1/\Pi_t$, where p_t is the goods price of a unit of fiat currency at date t and Π_t is the gross inflation rate from dates t to $t{+}1$ — and θ is the required reserve ratio. A binding reserve requirement also requires $M_t = \theta\, S_t(R_{t-1})$ and $P_t^b < 1$; thus, when the reserve requirement is binding fiat currency is held only by banks and only to satisfy reserve requirements.

Finally, credit-market clearing requires

$$S_t(R_{t-1}) - K_{t+1}(R_t) = M_t + P_t^b B_t\,.$$

[11]We think of c as a proxy for intermediation costs of all types, including the information and diversification costs associated with the existence of default risk on private liabilities. It can also be viewed, in part, as a proxy for the equity premium.

In steady-state equilibria, which are the only type we will study, the rate-of-return variables R_{t-1}, R_t^b, R_t^m, and P_t^b will be date-invariant constants R, R^b, R^m, and P^b. In addition, all nominal variables will grow at a gross rate of $\Psi\Pi$ per period, where $\Pi = 1/R^m$. All real variables will grow at a gross rate of Ψ per period, except for the real wage rate w_t and the capital-labor ratio k_t, which will grow at a gross rate of λ. The equilibrium conditions for a steady state can be expressed in terms of the situation at date 1.

We begin by defining the steady-state capital-demand (or private asset-supply) function

$$K_2(R) \equiv \Psi N_1 \left(\frac{R}{\alpha}\right)^{\frac{1}{\alpha-1}}, \tag{1}$$

with

$$K_2'(R) = \frac{\Psi}{\alpha(\alpha-1)} N_1 \left(\frac{R}{\alpha}\right)^{\frac{2-\alpha}{\alpha-1}} < 0 \tag{2}$$

and

$$K_2''(R) = \frac{2-\alpha}{[\alpha(\alpha-1)]^2} \Psi N_1 \left(\frac{R}{\alpha}\right)^{\frac{3-2\alpha}{\alpha-1}} > 0\,. \tag{3}$$

Note that these functions have a 2-subscript because the capital goods that are employed at date 2 must be acquired by households at date 1. In addition, we define the steady-state saving (or asset demand) function

$$S_1(R) \equiv N_1\, s\,(1-\alpha) \left(\frac{R}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \tag{4}$$

with

$$S_1'(R) = -N_1\, s\, \left(\frac{R}{\alpha}\right)^{\frac{1}{\alpha-1}} < 0 \tag{5}$$

and

$$S_1''(R) = \frac{N_1\, s}{\alpha(1-\alpha)} \left(\frac{R}{\alpha}\right)^{\frac{2-\alpha}{\alpha-1}} > 0\,. \tag{6}$$

Lemma 1 *For $R \in (0, \Psi + c]$, [1] $S_1'(R) > K_2'(R)$ and [2] $S_1''(R) < K_2''(R)$.*

2.2 Equilibria

We will confine our attention to steady state equilibria with a binding reserve requirement, which we will call *binding steady states*. The equilibrium conditions for a binding steady state are

$$0 < \theta \leq 1\,, \tag{7}$$

$$R^b = R - c \tag{8}$$

$$P^b = \frac{R^m}{R^b} < 1\,, \tag{9}$$

$$R^d = (1-\theta)R^b + \theta R^m, \tag{10}$$

$$M_1 = \theta\, S_1(R)\,, \tag{11}$$

$$S_1(R) - K_2(R) = M_1 + P^b B_1\,, \tag{12}$$

$$\Psi G_1 = (\Psi - R^m)\, M_1 + (\Psi - R^b)\, P^b B_1\,, \tag{13}$$

$$G_1 = (M_1 + P^b B_1) - p_1(H_0 + \mathcal{B}_0)\,, \tag{14}$$

Given a positive value of G_1 and nonnegative values of H_0, $\mathcal{B}_0$ and c, a binding steady state consists of positive values of θ, R, R^b, R^d, R^m, P^b, M_1, and p_1 and a nonnegative value of B_1 that satisfy these conditions.

Note that we can define

$$Y_1(R) = N_1 \left(\frac{R}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

with

$$Y_1'(R) = \frac{N_1}{\alpha - 1}\left(\frac{R}{\alpha}\right)^{\frac{1}{\alpha-1}} < 0\,.$$

Thus, the steady-state output level is strictly decreasing in the steady-state real interest rate. However, output grows at gross rate of Ψ in a steady state, along with most other real variables (see above).

2.3 Supplementary assumptions

Henceforth we will drop the date subscripts on functions and variables. We will also make three important supplementary assumptions:

I. *There is a* laissez faire *steady state with a low real interest rate.* The values of α, s and c are consistent with the existence of an $\underline{R} \in (0, \Psi + c)$ such that $S(\underline{R}) - K(\underline{R}) = 0$.

II. *There is no nonbinding steady state.* The equation $[\Psi - (R - c)]\ [S(R) - K(R)] = \Psi G$ has no real solutions on $R \in (\underline{R}, \Psi + c)$.

III. *Intermediation costs are not too high.* The values of α, s and c satisfy

$$1 + \frac{c}{\Psi} < \frac{1}{s\,(1-\alpha)}\,.$$

The Lemma establishes that $S'(R) > K'(R)$ for $R \in (0, \Psi + c]$ — that is, the aggregate outside-asset demand function $S(R) - K(R)$ is strictly increasing on $(0, \Psi + c]$. Consequently, Assumption I holds if and only if there exists a *laissez faire* ($\theta = G = 0$) steady state in which unbacked government liabilities are valued, which is to say iff $S(\Psi + c) - K(\Psi + c) > 0$. Equations (2) and (4) can be used to show that a necessary and sufficient condition for this to be the case is

$$s\left(1 + \frac{c}{\Psi}\right) > \frac{\alpha}{1-\alpha}\,; \tag{15}$$

they also imply

$$\underline{R} = \frac{\alpha}{1-\alpha}\frac{\Psi}{s}\,. \tag{16}$$

Assumption I also implies that for at least some positive values of θ the equation $(1 - \theta)S(R) - K(R) = 0$, which characterizes a binding steady state in which there are no government bonds, has a unique solution $\underline{R}_\theta \in (\underline{R}, \Psi + c)$. Equations (2) and (4) imply

$$\underline{R}_\theta = \frac{\underline{R}}{(1-\theta)}\,. \tag{17}$$

The least upper bound of the set of values of θ consistent with Assumption I is

$$\theta_{\max} \equiv 1 - \frac{\alpha}{s\,(1-\alpha)}\frac{1}{1 + \frac{c}{\Psi}}\,. \tag{18}$$

Assumption II implies that the government cannot finance its deficit without imposing a binding reserve requirement.

Assumption III is a technical assumption that is used in the proof of Lemma 1 and holds for virtually any plausible (which is to say, relatively low) values of c, given that Assumption I holds. Assumptions I and III collectively imply

$$\frac{\alpha}{1-\alpha} < s\left(1+\frac{c}{\Psi}\right) < \frac{1}{1-\alpha}\,.$$

3 Open market operations

Define $\beta \equiv P^b \mathcal{B}_t / H_t$. We will view β, the ratio of the market value of the stock of government bonds to the stock of fiat currency, as a monetary policy indicator that can be manipulated through open market operations. Our assumptions regarding B_1 imply $\beta \geq 0$. We will refer to an increase/decrease in β as a tightening/easing of monetary policy; we will justify this interpretation by demonstrating that when θ is fixed, changes in β always change the real interest rate in the same direction (see below).[12]

Equation (11) and the definitions of β and P^b are readily seen to imply

$$P^b B = \beta\theta\, S(R)\,; \tag{19}$$

this equation, along with equation (11) itself, allows us to rewrite equations (12) and (13) as

$$\left[1-\theta\left(1+\beta\right)\right] S(R) - K(R) = 0 \tag{20}$$

and

$$\Psi G = \theta\, S(R)\,\left\{\left(\Psi - R^m\right) + \beta\left[\Psi - (R-c)\right]\right\}, \tag{21}$$

respectively.

Equations (11) and (12) imply

$$P_b B = (1-\theta)\, S(R) - K(R)\,, \tag{22}$$

[12]Wallace (1984) and Espinosa and Russell (1998) use the ratio of the *face value* of the bond stock to the stock of currency as their open-market-operations parameter; that is, they define $\beta \equiv \mathcal{B}_t/H_t$. A brief explanation of the pros and cons of these alternative parametrizations is presented in Appendix A.

and equation (19) then implies that $\beta\theta\, S(R) = (1-\theta)\, S(R) - K(R)$. This equation can be combined with equation (21) to produce the government revenue function

$$G_\beta(R, R^m; \theta) \equiv (\Psi - R^m)\, \theta\, S(R) + [\Psi - (R-c)]\, [(1-\theta)\, S(R) - K(R)]\,. \tag{23}$$

Our analysis of the effects of open market operations is based on the properties of this function. When R_m and θ are fixed, we will refer to the function as the *seigniorage Laffer curve.* The seigniorage Laffer curve describes the dependence of the level of government seigniorage revenues on the value of the gross real interest rate R.[13] The revenues from currency seigniorage are $(\Psi - R^m)\, \theta\, S(R)$, while $[\Psi - (R-c)]\, [(1-\theta)S(R) - K(R)]$ represents the revenues from bond seigniorage.

Given θ, a binding steady state can be characterized as values of R and R^m such that $\underline{R}_\theta < R < \Psi + c$, $0 \leq R^m < R$, and $G_\beta(R, R^m; \theta) = \Psi G$. In what follows we will think of the seigniorage Laffer curve primarily as a function of R, and we will think of its domain as $[\underline{R}_\theta, \Psi + c]$. However, in a binding steady state we must have $R_m < R^d = R - c$, and if $R^m > \underline{R}_\theta$ then there are points on the seigniorage Laffer curve to the left of R^m. These points are not potential equilibrium points.

As is readily seen, the slope of the seigniorage Laffer curve is

$$\frac{\partial G_\beta}{\partial R} = (\Psi - R^m)\, \theta\, S'(R) + [\Psi - (R-c)]\, [(1-\theta)S'(R) - K'(R)] - [(1-\theta)S(R) - K(R)]\,. \tag{24}$$

Proposition 1 establishes three important facts about the seigniorage Laffer curve:

Proposition 1 *The function $G_\beta(R, R^m; \theta)$ is [1] strictly concave in R on $[\underline{R}_\theta, \Psi + c]$, [2] positive and downward-sloping in R at $R = \Psi + c$, [3] positive and upward-sloping in R at $R = \underline{R}_\theta$ if*

$$\theta < \gamma\, \theta_{\max}, \tag{25}$$

where

$$\gamma \equiv \frac{1 + \frac{c}{\Psi}}{\left(1 + \frac{c}{\Psi}\right) + \frac{\alpha}{1-\alpha}}\,. \tag{26}$$

[13] Note that our "government revenue function" actually returns the product of government revenue and the gross real growth rate Ψ.

Henceforth we will add

IV. $\theta < \gamma\theta_{\max}$.

to our list of supplementary assumptions. Note that $0 < \gamma < 1$ and that γ is increasing in c with a least upper bound of unity. Also, if $c = 0$ then $\gamma = 1 - \alpha$. Assumption IV is sufficient, but not necessary, for part [3] of Proposition 1 to hold.[14]

Assumption IV ensures that for given values of θ and R^m the seigniorage Laffer curve is upward-sloping at its left endpoint (see Figure 1). Under this assumption, the curve associated with each admissible vector (θ, R^m) is a positive-valued, downward-opening paraboloid on $(\underline{R}_\theta, \Psi]$, and thus has a unique peak on the interior of this domain.

Equations (11) and (22) and the definitions of β, B and M allow us to write

$$\beta_\theta(R) = \frac{(1-\theta)\,S(R) - K(R)}{\theta\,S(R)}. \tag{27}$$

It follows that, for any given value of θ, the relationship between β and R is monotone increasing (see Theorem 1). Consequently, the seigniorage Laffer curve provides an indirect description of the relationship between the value of β and the level of revenue from seigniorage.

We can use the seigniorage Laffer curve to analyze the effects of an open market operation (a permanent change in β, with θ held fixed) on the equilibrium value of R^m. Given θ, the value of β determines the steady-state value of R.[15] There is then a unique value of R^m such that $G_\beta(R, R^m; \theta) = \Psi G$. This value of R^m determines the identity of the initial seigniorage Laffer curve. A change in β produces a change in R that causes government revenue to move up or down along the curve. An adjustment in R_m is then necessary to shift the curve in the opposite direction far enough to restore revenues to their equilibrium level.[16]

[14]The sufficiency of Assumption IV for part [3] of Proposition 1 is established under the very conservative assumption that $R^m = 0$. As R^m increases the required condition becomes weaker, and it can be shown that if $R_m \geq \underline{R}_\theta$ then part [3] holds for all values of θ consistent with Assumption I. (See the proof of Proposition 1.)

[15]This value can be obtained by inverting $\beta_\theta(R)$.

[16]As we noted in the introduction, characterizing monetary policy in terms of β avoids a problem of nonuniqueness that arises in models of this type. For given values of θ and R^m (or equivalently θ and

Theorem 1 establishes that Assumptions I-IV are sufficient to ensure that if we start with a binding steady state in which the initial value of β is relatively low, then a small increase in β will always produce a new steady state with a higher real interest rate and a lower inflation rate:

Theorem 1 *Given $\theta \in (0,1)$, suppose there exists at least one $\beta \geq 0$ that will support a binding steady state. Call this value β', and call R' the associated value of R . Then there are unique values $\beta_\theta^{\min} \in [0, \beta']$ and $R_\beta^{\min} \in [\underline{R}_\theta, R']$ which are the minimum values of β and R that will support a binding steady state, and unique values $\beta_\theta^{\max} > \beta'$ and $R_\theta^{\max} \in (R', \Psi + c]$ which are the maximum values of β and R that will support a binding steady state. Moreover, any $\beta \in [\beta_\theta^{\max}, \beta_\theta^{\max}]$ will support a unique binding steady state with $R \in [\underline{R}_\theta, \Psi + c]$, where the relationship between the value of β and the steady-state value of R is monotone increasing. Finally, there exists $\widetilde{\beta}_\theta \in (\beta_\theta^{\max}, \beta_\theta^{\max})$ such that if $\beta \in [\beta_\theta^{\min}, \widetilde{\beta}_\theta)$, then any increase in β that leaves $\beta \leq \widetilde{\beta}_\theta$ will support a unique binding steady with a higher value of R^m, while if $\beta \in (\widetilde{\beta}_\theta, \beta_\theta^{\max})$, then any increase in β that leaves $\beta \leq \beta_\theta^{\max}$ will support a binding steady state with a lower value of R^m.*

The corollary to Theorem 1 establishes that the theorem is nonvacuous — that is, that under Assumption I there are always reserve ratios and values of G that satisfy Assumptions II-IV as well as the hypothesis of Theorem 1. These values of G can be financed with a binding reserve requirement, but not without one.

Corollary 1 *There exists $G > 0$ and $\theta \in (0, \theta_{\max})$ such that $\Psi G < [\Psi - (R - c)]\ [S(R) - K(R)]$ for all $R \in [\underline{R}, \Psi + c]$, but $\Psi G = (\Psi - R^m)\, \theta\, S(R) +\ [\Psi - (R - c)]\ [(1 - \theta)\, S(R) - K(R)]$ for some $R \in [\underline{R}, \Psi + c]$, and $R^m \in (0, \underline{R})$.*

Figure 1 displays a situation consistent with Theorem 1. The initial steady state supports a real interest rate of R^* and is on the upward-sloping portion of the relevant Laffer curve. The associated gross real return rate on money (the inverse of the gross inflation rate) is R^{m*}. The monetary authority tightens open market policy (increases β) in order to increase the steady-state real interest rate to $\widehat{R}$. At this interest rate the government has excess seigniorage revenue, so the inflation rate adjusts downward (R^m adjusts upward to $\widehat{R}^m$) to

Z, where Z represents the gross growth rate of the stock of fiat money) there are always two values of R that succeed in financing G, but these two values involve very different values of β. Note that the monotonicity of the relationship between β and R follows from the fact that $S(R) - K(R)$ is strictly increasing on $[\underline{R}_\theta, \Psi]$; see the proof of Theorem 1.

reduce flow of revenue. If the authority continues to tighten, however, the resulting increases in R^m eventually shifts the seigniorage Laffer curve down to a level at which it is tangent to the horizontal line that represents the government's revenue needs. The real interest rate associated with this tangency point is $\widetilde{R}_\theta$ and the corresponding real money return rate is $\widetilde{R}^m_\theta$. Further decreases in the inflation rate will not produce enough revenues to finance the real deficit.[17] If the monetary authority wishes to tighten further it must allow the inflation rate to rise, which will shift the seigniorage Laffer curve upward. The resulting steady states will support real interest rates like $\overline{\widetilde{R}}$ to the right of $\widetilde{R}_\theta$ and will occur on the downward-sloping sides of the relevant Laffer curves.

4 Changes in the required reserve ratio

Equations (19) and (23) imply

$$\theta_\beta(R) = \frac{S(R) - K(R)}{(1+\beta)\, S(R)}\,, \tag{28}$$

which expresses the equilibrium relationship between θ and R when the monetary authority allows θ to vary but holds β fixed. It is readily seen that $\theta'_\beta(R) > 0$ on $[R, \Psi + c)$, so that an increase in the reserve ratio always represents a tightening of policy in the sense of increasing the real interest rate. Equations (13), (19) and (28) can be used to define

$$G_\theta(R, R_m; \beta) \equiv \frac{S(R) - K(R)}{1+\beta}\, \left[(\Psi - R_m) + \beta\left\{\Psi - (R - c)\right\}\right], \tag{29}$$

which is the fixed-β seigniorage Laffer curve. This curve is defined on $[\underline{R}, \Psi + c]$ and is positive everywhere on this domain except at $\underline{R}$, where it is zero. It slope is

$$\frac{\partial G_\theta}{\partial R} = \frac{1}{1+\beta}\left(\left[(\Psi - R_m) + \beta\left\{\Psi - (R - c)\right\}\right]\left[S'(R) - K'(R)\right] - \beta\left[S(R) - K(R)\right]\right). \tag{30}$$

Proposition 2 establishes four important facts about this alternative seigniorage Laffer curve:

[17] Thus $\widetilde{\Pi}_\theta = 1/\widetilde{R}^m_\theta$, the steady-state gross inflation rate associated with $\widetilde{R}_\theta$, is the lowest inflation rate that will allow the government to finance its deficit without increasing the required reserve ratio.

Proposition 2 *The function $G_\theta(R, R^m; \beta)$ is [1] strictly concave in R on $[\underline{R}, \Psi + c]$ and [2] upward-sloping in R at $R = \underline{R}$. In addition, [3] for any $R^m \in [0, \Psi + c)$, there exists $\overline{\beta} > 0$ such that the function is upward-sloping in R at $R = \Psi$ if $\beta < \overline{\beta}$ and downward-sloping in R at $R = \Psi$ if $\beta > \overline{\beta}$, and [4] if $\beta = 0$, then $G_\theta(R, R^m; \beta)$ is upward-sloping on $[\underline{R}, \Psi + c)$ for any $R_m \in [0, \Psi + c)$, but if $\beta > 0$ there exists an $R^m \in [0, \Psi + c)$ such that the function is downward-sloping in R at $R = \Psi$ if $R \geq R_m$.*

Property [3] is associated with the fact that when $\beta = 0$ all seigniorage revenues come from currency seigniorage — a source of revenue that always becomes more productive as R rises — while as $\beta \to \infty$ the fixed-β seigniorage Laffer curve converges to the no-reserve-requirements Laffer curve $[\Psi - (R - c)]\ [S(R) - K(R)]$, which is downward-sloping at $R = \Psi + c$. An immediate implication of property [3] is that $G_\theta(R, R^m; \beta)$ has an interior peak in R — that is, a peak on $R \in (\underline{R}, \Psi + c)$ — iff $\beta > \overline{\beta}$. Similarly, an implication of property [4] is that if $\beta > 0$, then it is always possible to choose R^m high enough so that $G_\theta(R, R^m; \beta)$ has an interior peak in R.[18] Note that the fixed-β seigniorage Laffer curves are generally similar to the fixed-θ curves, except that they are zero-valued rather than positive-valued at the endpoints of their domains and may not always have downward-sloping regions.

Theorem 2 describes some of the implications of Proposition 2 for the effects of monetary policy conducted by changing the required reserve ratio:

Theorem 2 *Suppose that a given value of β supports a binding steady state for at least one value of θ. Then there exist a minimum value $\underline{\underline{R}}_\beta$ and a maximum value $\overline{\overline{R}}_\beta$, with $\underline{R} < \underline{\underline{R}}_\beta < \overline{\overline{R}}_\beta \leq \Psi$, and an associated minimum value $\underline{\theta}_\beta$ and maximum value $\overline{\theta}_\beta$, with $0 < \underline{\theta}_\beta < \overline{\theta}_\beta < 1$, that are consistent with the existence of such a steady state; moreover, any $\theta \in [\underline{\theta}_\beta, \overline{\theta}_\beta]$ will support a steady state involving $R \in [\underline{R}_\beta, \overline{R}_\beta]$, where the relationship between θ and R is one-to-one and strictly increasing. In addition, unless conditions [1] and [2] below hold for this value of β, there exists an $\widetilde{R}_\beta \in (\underline{R}_\beta, \overline{R}_\beta)$ and $\widetilde{\theta}_\beta \in (\underline{\theta}_\beta, \overline{\theta}_\beta)$ such that any increase in θ starting from $\theta \in (\underline{\theta}_\beta, \widetilde{\theta}_\beta)$ that leaves $\theta < \widetilde{\theta}_\beta$ will produce an increase in R^m, while any increase in θ starting from $\theta \in [\widetilde{\theta}_\beta, \overline{\theta}_\beta)$ will have the opposite effect. Finally, if [1] $G_\theta(\Psi, 0, \beta) \geq \Psi G$ and [2] $(\partial/\partial R) G_\theta(\Psi + c, \widehat{R}^m_\beta, \beta) \geq 0$, where $\widehat{R}^m_\beta$ solves $G_\theta(\Psi + c, R^m, \beta) = \Psi G$, then $\overline{R}_\beta = \Psi$ and any increase in θ starting from $\theta \in [\underline{\theta}_\beta, \overline{\theta}_\beta)$ will produce an increase in R^m.*

[18]Note that if $R^m > \underline{R}$ then the seigniorage Laffer curve crosses the no-reserve-requirement curve from below, and the former curve may conceivably peak at a point where in lies below the latter curve. Such a peak cannot be an equilibrium point, since we cannot have $R^m > R$ in equilibrium.

When conditions [1] and [2] do not hold, the implications of Theorem 2 are similar to those of Theorem 1. In this case the interval of feasible values of the policy parameter (in this case θ) can be divided into two subintervals: an interval of relatively low values (and associated real interest rates) from which tightening policy will reduce the inflation rate, and an interval of higher values from which it will have the opposite effect. When conditions [1] and [2] hold, however, tightening policy always reduces the inflation rate.

The corollary to Theorem 2 demonstrates that it is always possible for the monetary authority to set $\beta = 0$, and that conditions [1] and [2] always hold at $\beta = 0$ and for a range of relatively low values of β:

Corollary 2 *Suppose there is at least one positive value of β that supports a binding steady state. Then any lower (non-negative) value of β will also support a binding steady state. In addition, there exists $\widehat{\beta} > 0$ such that conditions [1] and [2] hold for $\beta \in [0, \widehat{\beta})$.*

Thus, if the monetary authority wishes it can always set β at a level low enough to insure that an increase in the required reserve ratio will always be disinflationary.

Figure 2 displays a situation in which conditions [1] and [2] hold. (Situations of the opposite type produce diagrams closely analogous to Figure 1.) The initial equilibrium occurs at a real interest rate of R^* and a real fiat money return rate of R^{m*}. The associated Laffer curve is upward-sloping throughout its length. If the monetary authority tightens policy by increasing the required reserve ratio, then the level of seigniorage revenues will rise. A reduction in the inflation rate (an increase in R^m to $\widehat{R}^m$) will be needed to reduce the flow of revenues; this will produce a downward shift in the seigniorage Laffer curve and an increase in the real interest rate (to $\widehat{R}$). In this case, however, moves to tighten policy always increase the government's seigniorage revenues. As a result, increases in the real interest rate are invariably associated with declines in the inflation rate (increases in R^m), all the way out to $R = \Psi + c$.

5 Comparing the effects of the two policy instruments

While open market operations have been the principal tool of monetary policy in recent years, the required reserve ratio has also been used, on occasion, as an instrument of policy. The conventional wisdom holds that [1] the effects of an increase in the reserve ratio are generally similar to those of an open market sale, but that [2] reserve ratio changes have disadvantages that explain the Federal Reserve's revealed preference for open market operations. As Kareken (1960) points out, however, the disadvantages of using the reserve ratio as a policy tool that are typically cited by the Fed (and repeated in most money-and-banking textbooks) are not very convincing.[19]

In Theorem 3 we provide an explanation for both of these two elements of the conventional wisdom regarding the effects of changes in reserve requirements. We show that whenever the initial values of β and R fall in the conventional-wisdom range for the current value of θ, then a marginal increase in θ will definitely cause the inflation rate to fall. We also show that when the monetary authority uses a change in the reserve ratio to change the equilibrium real interest rate, the resulting change in the inflation rate is invariably larger than it would have been if the authority had engineered the same change in the real rate via open market operations. Thus, a monetary authority that desired to manipulate the real interest rate (countercyclically, or for some other purpose) while minimizing the variability of the inflation rate would have reason to prefer open market operations to reserve ratio changes.

The analysis that leads to Theorem 3 begins by using the equilibrium conditions of the model to express R_m as a function of R and the policy parameter to be held fixed, so that the policy parameter of interest is endogenized. In the case where θ is fixed and policy is

[19]Kareken observes that "official reasons for making day-to-day adjustments in members banks' reserve positions by means of open market sales and purchases will not stand up under close scrutiny, and the same can be said of the post-Accord record on reserve ratios." He presents a direct assault on the most frequently-cited reason for the Fed's reluctance to conduct policy by changing the reserve ratio — the argument that substantial changes in the ratio have large, discontinuous effects, while frequent small changes would unduly complicate banks' reserve-management problems.

conducted via open market operations we have

$$\Psi G = (\Psi - R^m)\,\theta\, S(R) + [\Psi - (R-c)]\ [(1-\theta)S(R) - K(R)]\ , \tag{31}$$

which produces

$$\left(\frac{\partial R^m}{\partial R}\right)_\beta = \frac{(\Psi - R^m)\,\theta\, S'(R) + [\Psi - (R-c)]\ [(1-\theta)S'(R) - K'(R)] - [(1-\theta)\,S(R) - K(R)\,]}{\theta\, S(R)}\,. \tag{32}$$

Notice that the numerator of this expression is equal to the slope of the fixed-β seigniorage Laffer curve. In the case where β is fixed and policy is conducted via reserve ratio changes we have

$$\Psi G = \frac{S(R) - K(R)}{1+\beta}\ [(\Psi - R^m) + \beta\ \{\Psi - (R-c)\}]\ , \tag{33}$$

which produces

$$\left(\frac{\partial R_m}{\partial R}\right)_\theta = \frac{[(\Psi - R_m) + \beta\ \{\Psi - (R-c)\}]\ \frac{S'(R)-K'(R)}{1+\beta} - [(1-\theta)\,S(R) - K(R)]}{\theta\, S(R)}\,. \tag{34}$$

In constructing the last derivative we have used equations (23) and (28) to produce an expression that that is readily compared with $(\partial R^m/\partial R)_\beta$.

Theorem 3 *Suppose θ and β support a binding steady state with $R < \Psi + c$. Then $(\partial R^m/\partial R)_\theta > (\partial R^m/\partial R)_\beta$. In addition, if $\beta \le \widetilde{\beta}_\theta$ (or equivalently, $R < \widetilde{R}_\theta$) in the initial steady state, so that $\partial R^m/\partial\beta > 0$, then $\partial R^m/\partial\theta > 0$.*

6 The power of monetary policy

6.1 The effect of monetary policy on nominal interest rates

The gross nominal interest rate is $R^{nom} \equiv R/R^m$, so that we have

$$\frac{\partial R^{nom}}{\partial\beta} = \frac{R^m\frac{\partial R}{\partial\beta} - R\frac{\partial R^m}{\partial\beta}}{(R^m)^2} \quad\text{and}\quad \frac{\partial R^{nom}}{\partial\theta} = \frac{R^m\frac{\partial R}{\partial\theta} - R\frac{\partial R^m}{\partial\theta}}{(R^m)^2}.$$

It follows immediately that if $\partial R^m/\partial\beta = 0$ then $\partial R^{nom}/\partial\beta > 0$, and similarly for $\partial/\partial\theta$ — that is, if a tightening of monetary policy does not change inflation rate, then it causes the nominal interest rate to rise. Moreover, Proposition 2 implies that for any value of θ consistent with the existence of a steady state, there will always be a range of values of β at which an open market purchase increases the nominal interest rate, and Proposition 4 implies that for any binding steady state involving a sufficiently large value of β, there will always be a range of values of θ at which an increase in the reserve ratio has the same effect.[20]

In this model, there is a simple quantity-theoretic relationship between the steady-state inflation rate and the steady-state money supply (stock of fiat currency) growth rate: if we let Z represent the gross growth rate of the money supply, then $Z = \Pi\Psi$, where $\Pi = 1/R^m$ is the gross inflation rate. Thus our results about the inflationary and real effects of monetary policy conducted via open market operations or reserve ratio changes can be reformulated as results about the real effects of policy conducted by means of changes in the money supply growth rate. Suppose, for example, that we imagine that the government holds the reserve ratio fixed and uses changes in the money supply growth rate to manipulate the real interest rate. In this case, if the initial real interest rate lies in the relatively-low range $[\underline{R}_\theta, \widetilde{R}_\theta)$ then a decrease in the money supply growth rate will cause the inflation rate to fall and the real interest rate to rise. If R is sufficiently close to $\widetilde{R}_\theta$, moreover, then the decline in the inflation rate will be more than offset by the increase in the real interest rate and the nominal interest rate will fall. Indeed, as R approaches $\widetilde{R}_\theta$ from below the elasticity of the real interest rate with respect to a change in the money-supply growth rate (or inflation rate) becomes arbitrarily large. Thus the range of values of R (and β or θ) just to the left of the peak of the relevant seigniorage Laffer curve is a range where monetary policy is very powerful, in the sense that a small decrease in the money supply growth and inflation rates rate can produce a large increase in the real interest rate.

The logic of this model gives us good reasons to expect that the monetary authority might often find itself operating in the range of policy parameter values from which marginal changes in policy have effects that are both conventional and powerful. First, steady states on

[20] As we have noted, however, some specifications may not support binding steady states involving values this large.

the left (relatively low real interest rate) side of the seigniorage Laffer curve are dynamically stable, while steady states on the right side are not. [See Sargent (1987), pp. 261-262; his analysis generalizes straightforwardly to this somewhat-more-complicated model.] Second, agents' welfare is strictly increasing in the real interest rate. As a result, given a choice of stable steady states the monetary authority is likely to prefer steady states near the peak of the seigniorage Laffer curve, since these are the ones that support the highest real interest rates.[21]

6.2 The Tobin effect

One well-known alternative mechanism that might allow changes in monetary policy to have real effects that are generally consistent with the conventional wisdom is the Tobin (1965) effect. Under this mechanism, an increase in the money supply growth rate produces an increase in the inflation rate, which causes agents to substitute out of fiat money and into physical capital. The resulting increase in the stock of physical capital drives down its marginal rate of return, which is equal to the real interest rate.[22]

The trouble with Tobin effects is that they are inherently small. As we have just noted, the Tobin effect mechanism relies on inflation-induced substitution out of fiat money and into physical capital. However, the stock of fiat money is very small relative to the stock of capital, and the empirical evidence suggests that it is not particularly sensitive to changes in the rate of inflation. As a result, it is hard to imagine that the Tobin effect mechanism could allow moderate changes in the money growth and inflation rates to produce real effects of any significance.[23] Attempts to produce Tobin effects in calibrated models seem to confirm

[21]It may be dangerous to make to much of this welfare result, since if there is heterogeneity among the members of each generation there may be agents who do not benefit from higher interest rates. See Wallace (1984).

[22]Orphanides and Solow (1990) provide a survey of the literature on Tobin effects. See also Ahmed and Rogers (1996) and Azariadis and Smith (1996).

[23]The U.S. monetary base makes up roughly two percent of total U.S. net assets, most of which consist of physical capital. Hoffman and Raasche (1991) estimate the nominal-interest elasticity of the real monetary base as -0.3 to -0.4. This estimate implies that a one percent increase in the nominal interest rate would cause a decrease in desired real money balances, and thus an increase in the demand for other assets, equal to roughly 0.2 percent of net asset demand. The decrease in the real interest rate required to accommodate this asset demand increase would presumably be extremely small.

this view.[24]

In our model, changes in the inflation rate do not affect real activity by producing substitution out of fiat money. Indeed, under our simplifying assumptions changes in the inflation rate have no direct effect on the demand for fiat money, and decreases in the inflation rate that have "conventional" real effects actually cause real money demand to fall rather than rise.[25] Decreases in the inflation rate matter in our model because they produce declines in currency seigniorage revenues that must be offset by increased revenues from bond seigniorage. While currency seigniorage revenues are also small in absolute terms, the ratio of currency seigniorage revenues to revenues from bond seigniorage is an order of magnitude larger than the ratio of fiat money balances to the capital stock.[26] Moreover, as the the real interest rate increases private credit demand falls and the stock of government bonds rises; this "tax base effect" of the increase in the real rate tends to push bond seigniorage revenues up rather down. As a result, it takes a relatively large increase in the real interest rate to allow the "tax rate effect" [the decline in bond seigniorage revenues as R^b rises and $(\Psi - R^b)$ falls] to overwhelm the tax rate effect and produce a decline in revenues.

7 An example

In this section we provide an example that illustrates our basic results. The example specification is $s = 0.44$, $\alpha = 0.3$, $\lambda = 1.02$ and $n = 1.01 \Rightarrow \Psi \doteq 1.03$, and $c = 0.05$. The initial monetary policy setting is $\theta = 0.025$ and $\beta \doteq 0.7877$, and the initial deficit to be financed is $G_1 \doteq 0.0004426 \Rightarrow \Psi G_1 = 0.000456$. The equilibrium (gross) rates of return are $R^* = 1.0504$ and $\Pi^* = 1.05 \Rightarrow R^{m*} \doteq 0.9524$, and the associated gross nominal interest

[24] See Danthine, Donaldson, and Smith (1987) and Romer (1986).

[25] This happens because a policy-induced decline in the real interest rate increases real labor income, and money demand is proportional to real labor income.

[26] During the postwar period, currency seigniorage revenues have averaged approximately 0.4 percent of GDP while bond seigniorage revenues have been roughly four times larger. These calculations are based on the average values of M/Y and B/Y during the postwar period, as well as the average postwar values of $\Psi - R^m$ and $\Psi - R^b$. [See Bullard and Russell (1998b).] They are also based on the assumption that the entire postwar debt has been unbacked by expected future surpluses. If a substantial portion of the debt has been backed then the postwar ratio of currency to bond seigniorage revenues may have been much larger.

rate is $R^{nom*} \doteq 1.1029$. This equilibrium is on the left side, near the peak, of the initial seigniorage Laffer curve $G_\beta(R, R^{m*}, \theta)$. Note that the gross real interest rate paid by private borrowers (the gross marginal product of capital) exceeds the gross output growth rate by more than 2 percentage points, while the gross real interest rate on government debt, which is $R^{b*} = R^* - c \doteq 1.0004$, is almost 3 percentage points lower than the output growth rate. If we let $N_1 = 1$ then $Y_1^* \doteq 0.5845$.

Now suppose the monetary authority wishes to reduce the real interest rate using open market operations, and is willing to accept a 50-basis-point increase in the steady-state rate of inflation. It conducts open market purchases until $\beta \doteq 0.4892$, which produces equilibrium gross real rates of $\widehat{\Pi} = 1.055 \Rightarrow \widehat{R}^m \doteq 0.9479$ and $\widehat{R} = 1.0423$. The associated gross nominal interest rate is $\widehat{R}^{nom} \doteq 1.0996$. This equilibrium is on the left side of the new seigniorage Laffer curve $G_\beta(R, \widehat{R}^m, \theta)$. The inflation rate has increased by 50 basis points, but since the real interest rate has fallen by approximately 81 basis points the nominal interest rate has actually decreased (by approximately 33 basis points). The level of output rises to $\widehat{Y}_1 \doteq 0.5864$, which is approximately 0.33 percent above its initial level. The effects of this experiment on the seigniorage Laffer curve and the equilibrium real interest rate are displayed in Figure 3.

Next, suppose the monetary authority starts from the same initial equilibrium, but uses a decrease in the reserve requirement (holding β fixed) in order to reduce the equilibrium real interest rate and increase the equilibrium level of output by exactly the same amounts as in the experiment described above. These targets require it to reduce the required reserve ratio to $\widetilde{\theta} \doteq 0.02083$. The equilibrium gross real rates are $\widetilde{R} = \widehat{R} = 1.0423$ and $\widetilde{\Pi} = 1.0651 \Rightarrow \widetilde{R}^m \doteq 0.9389$, and the associated nominal interest rate is $\widetilde{R}^{nom} \doteq 1.1101$. In this case the desired real-interest-rate reduction (81 basis points) requires an inflation-rate increase of almost twice its size (151 basis points), so the nominal interest rate increases (by 72 basis points). The effects of this experiment on the seigniorage Laffer curve and the equilibrium real interest rate are displayed in Figure 4: note that the seigniorage Laffer curves $G_\theta(R, R^{m*}, \beta)$ and $G_\theta(R, \widetilde{R}^m, \beta)$ are strictly increasing on $[\underline{R}, \Psi + c]$.

8 Concluding remarks

In this paper, we have integrated the model of monetary policy devised by Wallace (1984) with Diamond's (1965) neoclassical model of production and capital. The result is a general equilibrium model in which monetary policy can have long-run real effects consistent with the conventional wisdom: an "easing" of policy, engineered by open market sales or by decreases in the required reserve ratio, increases the rate of inflation while decreasing the real rate of interest. The decline in the real interest rate produces a permanent increase in the level of output, and thus persistent but ultimately temporary increases in the growth rate of output. We show that, under certain conditions, the decrease in the real interest rate that is produced by a move to ease policy can be large enough to allow the nominal interest rate to fall. Under these conditions the ratio of the increase in the real interest rate to the associated decrease in the inflation rate can become quite large, which means that monetary policy can have substantial real effects. We also show that changes in the required reserve ratio are both similar to and different from open market operations as tools of monetary policy. On one hand, whenever an open market sale affects inflation and real interest rates in a manner qualitatively consistent with the conventional wisdom, an increase in the required reserve ratio will also do so. On the other hand, if the reserve ratio is used as a policy instrument, then the change in the inflation rate necessary to achieve a given change in the real interest rate will be larger than the inflation-rate change needed to achieve the same real-interest-rate change by means of an open market sale.

The empirical evidence on superneutrality that provides much of the motivation for our analysis was collected principally from countries with relatively low inflation rates. Boyd, Levine and Smith (1997) and a number of other researchers find that for countries with relatively high inflation rates, increases in the inflation rate tend to reduce the level and/or growth rate of output but have little effect on real interest rates. This evidence suggests that the actual effect of inflation on real interest rates and output is a combination of an effect of the type we describe, which tends to weaken as the inflation rate rises (see Section 6.1) and an adverse affect on output that becomes significant only at relatively high inflation rates and may be caused by the tendency of high inflation to disrupt financial markets.[27] Azariadis and Smith (1996) describe a model in which increases in inflation have a Tobin effect on interest rates and output at low inflation rates that is overwhelmed at higher inflation rates by a financial-intermediation-disruption effect. Integrating the financial intermediation assumptions of their model with our alternative mechanism for delivering Tobin-type effects would be a potentially interesting extension of our analysis.

[27] Boyd, Levine and Smith (1997) comment that "much of the relevant literature indicates that the negative correlation between inflation and growth performance derives largely from the experiences of relatively high inflation economies ... ." They go on to identify a number of papers of this type, emphasizing Bullard and Keating (1995) and Bruno and Easterly (1998). As we have noted, their own results are consistent with these findings. They also find that "at moderate rates of inflation, marginal increases in predictable inflation are not matched by increases in nominal equity returns. ... However, in economies where inflation rates are sufficiently high, nominal stock returns move essentially one-for-one with further increases in the rate of inflation." They also report evidence that for countries with high inflation rates, increases in inflation rates tend to reduce the volume of activity in financial markets.

Appendix A

A Parameterizing open market policy

As we point out in footnote 12, Wallace (1984) and Espinosa and Russell (1998) use the ratio of the *face value* of the bond stock to the stock of currency as their open-market-operations parameter. If we call the latter ratio β_0, then we have $\beta = \beta_0 \left(R/R^m\right)$. There is a clear sense in which the Wallace *et al.* procedure is more realistic. An open market sale unambiguously increases β_0 but does not unambiguously increase β: it is conceivable that an increase in β_0 might cause a decrease in the gross nominal interest rate R/R^m large enough to cause β to decline.

Using β as the open market operations parameter has two closely related advantages. First, it produces a dramatic simplification of the analysis. Second, while it is easy to show that a marginal increase in β always increases the real interest rate (see below), a marginal increase in β_0 may not always do so: when β and β_0 move in opposite directions, a marginal increase in β_0 will cause a marginal decline in R, and vice-versa. Thus, whichever parametrization of policy we choose, it will always be possible for the monetary authority to find that a "tightening" of policy (an increase in R) requires an open market purchase. However, if the authority simply conducts whatever open market purchases or sales turn out to be necessary to move β in the desired direction, it can be confident that R will change in the same direction. It seems to us that this story is generally consistent with the Federal Reserve's operating procedures in recent years: the FOMC directs the trading desk at the New York Fed to conduct the open market operations necessary to achieve a target interest rate, without specifying the nature (purchases or sales) or amount of these operations.

One consequence of our parametrization is that if the monetary authority attempts to change the required reserve ratio while holding β fixed — which is the way we model monetary policy when the reserve ratio is used as the policy instrument — then it will typically also have to conduct small open market purchases or sales, because the associated change in the nominal interest rate will change the value of β_0.

B Proofs:

Proof of Lemma 1: [1] Equations (1) and (4) imply

$$\frac{K'(R)}{S'(R)} = \frac{1}{s\,(1-\alpha)}\,\frac{\Psi}{R}\,;$$

as $s \in (0,1)$, $\alpha \in (0,1)$, and $0 \leq c < \Psi\left([s\,(1-\alpha)]^{-1} - 1\right)$ by assumption, we have $K'(R)/S'(R) > 1$ on $(0, \Psi + c]$. As both $K'(R)$ and $S'(R)$ are negative, we have $K'(R) < S'(R)$ on $(0, \Psi + c]$.

[2] Equations (3) and (6) imply

$$\frac{K''(R)}{S''(R)} = \frac{2-\alpha}{s\,(1-\alpha)}\,\frac{\Psi}{R}\,.$$

Since $\alpha \in (0,1)$ implies $2-\alpha > 1$ we have $K''(R)/S''(R) > 1$ on $(0, \Psi + c]$ (see above). This time $K''(R)$ and $S''(R)$ are positive, so we have $K''(R) > S''(R)$ on $(0, \Psi + c]$.

Proof of Proposition 1: [1] We need to show that

$$\frac{\partial}{\partial R}\left(\frac{\partial}{\partial R} G_\beta(R, R^m; \theta)\right) < 0\,.$$

Differentiation of equation (24) establishes that the expression in question is

$$(\Psi - R^m)\,\theta\,S''(R) + [\Psi - (R - c)]\;[(1-\theta)S''(R) - K''(R)] - 2\,[(1-\theta)S'(R) - K'(R)]\;.$$

Sufficient would be

$$\Psi\,\theta\,S''(R) - 2\,[(1-\theta)S'(R) - K'(R)]\; < 0,$$

since $\Psi\,\theta\,S''(R) \geq (\Psi - R^m)\,\theta\,S''(R)$ for $R^m \in [0, \Psi)$, and $(\Psi - R - c)\,[(1-\theta)S''(R) - K''(R)] < 0$ on $R \in (\underline{R}_\theta, \Psi + c)$. The latter inequality follows from the fact that $K''(R) > S''(R)$ on $(\underline{R}_\theta, \Psi + c)$, which was established in Lemma 1.

Now

$$\Psi\,\theta\,S''(R) - 2\,[(1-\theta)S'(R) - K'(R)]\; < 0$$

$\Leftrightarrow$

$$\Psi\,\theta < 2\left[(1-\theta)\frac{S'(R)}{S''(R)} - \frac{K'(R)}{S''(R)}\right].$$

Equations (1), (5), and (6) imply

$$\frac{S'(R)}{S''(R)} = -(1-\alpha)R \quad \text{and} \quad \frac{K'(R)}{S''(R)} = -\frac{\Psi}{s},$$

which gives us

$$\Psi\,\theta < 2\left[\frac{\Psi}{s} - (1-\theta)\,(1-\alpha)R\right]$$

$$\Leftrightarrow \frac{R}{\Psi} < \frac{\frac{1}{s}-\frac{\theta}{2}}{(1-\theta)\,(1-\alpha)}.$$

We have $R < \Psi + c \Leftrightarrow R/\Psi < 1 + c/\Psi$, so sufficient would be

$$1 + \frac{c}{\Psi} < \frac{\frac{1}{s}-\frac{\theta}{2}}{(1-\theta)\,(1-\alpha)}.$$

Assumption I gives us

$$1 + \frac{c}{\Psi} < \frac{1}{s\,(1-\alpha)},$$

so sufficient would be

$$\frac{1}{s\,(1-\alpha)} < \frac{\frac{1}{s}-\frac{\theta}{2}}{(1-\theta)\,(1-\alpha)} \Leftrightarrow \frac{1-\theta}{s} < \frac{1}{s} - \frac{\theta}{2} \Leftrightarrow s < 2\,.$$

[2] Equation (24) implies that at $R = \Psi + c$ the slope of the seigniorage Laffer curve is

$$(\Psi - R^m)\,\theta\,S'(\Psi + c) - \,[(1-\theta)S(\Psi + c) - K(\Psi + c)]\,.$$

We know that $(1-\theta)S'(R) - K'(R) > 0$ on $(\underline{R}_\theta, \Psi + c)$ and that $(1-\theta)S(\underline{R}_\theta) - K(\underline{R}_\theta) = 0$, where $\underline{R}_\theta < \Psi$. It follows that $(1-\theta)S(\Psi + c) - K(\Psi + c) > 0$.

[3] Equation (24) implies that at $R = \underline{R}_\theta$ the slope of the seigniorage Laffer curve is

$$(\Psi - R^m)\,\theta\,S'(\underline{R}_\theta) + [\Psi - (\underline{R}_\theta - c)]\,\,[(1-\theta)S'(\underline{R}_\theta) - K'(\underline{R}_\theta)]\,.$$

We need to show that this is positive. Since $S'(R) < 0$ and $R^m \in [0, \Psi)$, sufficient would be

$$\Psi\,\theta\,S'(\underline{R}_\theta) + [\Psi - (\underline{R}_\theta - c)]\,\,[(1-\theta)S'(\underline{R}_\theta) - K'(\underline{R}_\theta)] > 0,$$

which is equivalent to

$$\Psi\,\theta < [\Psi - (\underline{R}_\theta - c)]\;[\frac{K'(\underline{R}_\theta)}{S'(\underline{R}_\theta)} - (1-\theta)]\,. \tag{B.1}$$

Using equations (2), (5), (16) and (17) we have

$$\frac{K'(R)}{S'(R)} = \frac{1}{s\,(1-\alpha)}\,\frac{\Psi}{R} \quad \text{and} \quad \underline{R}_\theta = \frac{\alpha}{1-\alpha}\,\frac{\Psi}{s\,(1-\theta)}\,,$$

so

$$\frac{K'(\underline{R}_\theta)}{S'(\underline{R}_\theta)} = \frac{1-\theta}{\alpha} \quad \text{and} \quad \frac{K'(\underline{R}_\theta)}{S'(\underline{R}_\theta)} - (1-\theta) = (1-\theta)\,\frac{1-\alpha}{\alpha}\,.$$

This leaves us with

$$\Psi\,\theta < [\Psi - (\underline{R}_\theta - c)]\;(1-\theta)\,\frac{1-\alpha}{\alpha}\,. \tag{B.2}$$

Now

$$\Psi - (\underline{R}_\theta - c) = \Psi\,\left[1 - \frac{\alpha}{1-\alpha}\,\frac{1}{s\,(1-\theta)}\right] + c,$$

producing the condition

$$\theta < \left\{\left[1 - \frac{\alpha}{1-\alpha}\,\frac{1}{s\,(1-\theta)}\right] + \frac{c}{\Psi}\right\}(1-\theta)\,\frac{1-\alpha}{\alpha} =$$

$$\left[(1-\theta)\frac{1-\alpha}{\alpha} - \frac{1}{s}\right] + \frac{c}{\Psi}\left[(1-\theta)\,\frac{1-\alpha}{\alpha}\right]$$

which resolves to

$$\theta < \frac{(1-\alpha) - \frac{\alpha}{s} + \frac{c(1-\alpha)}{\Psi}}{1 + \frac{c(1-\alpha)}{\Psi}} = \frac{\left(1+\frac{c}{\Psi}\right) - \frac{\alpha}{s(1-\alpha)}}{\frac{1}{1-\alpha} + \frac{c}{\Psi}} = \frac{\left(1+\frac{c}{\Psi}\right) - \frac{\alpha}{s(1-\alpha)}}{(1+\frac{c}{\Psi}) + \frac{\alpha}{1-\alpha}}\,.$$

Now define

$$\widetilde{\theta} \equiv \frac{\left(1+\frac{c}{\Psi}\right) - \frac{\alpha}{s(1-\alpha)}}{(1+\frac{c}{\Psi}) + \frac{\alpha}{1-\alpha}}.$$

We have

$$\widetilde{\theta} = \frac{\left(1+\frac{c}{\Psi}\right)}{(1+\frac{c}{\Psi}) + \frac{\alpha}{1-\alpha}}\,\theta_{\max}. \;\square$$

Supplementary note: If $R^m > 0$, then the threshold value of θ will be higher than $\widetilde{\theta}$. In particular, if $R^m > 0$ then the appropriately revised version of inequality (B.2) is

$$(\Psi - R^m)\,\theta < (\Psi - \underline{R}_\theta - c)\,(1-\theta)\,\frac{\alpha}{1-\alpha}]$$

or equivalently

$$\frac{\theta}{1-\theta} < \frac{(\Psi - \underline{R}_\theta - c)}{(\Psi - R^m)} \frac{1-\alpha}{\alpha} \,.$$

If $R^m \geq \underline{R}_\theta$ then sufficient would be

$$\frac{\theta}{1-\theta} < \frac{1-\alpha}{\alpha} \Leftrightarrow \theta < 1-\alpha$$

and it is readily seen that this condition holds for any $\theta \in (0, \theta_{\max})$. Thus, in this case there does not need to be any additional restriction on θ.

Proof of Theorem 1:

Equation (27) implies

$$\beta_\theta'(R) = \frac{[(1-\theta)\, S'(R) - K'(R)]\, \theta\, S(R) - \theta\, S'(R)\, [(1-\theta)\, S(R) - K(R)]}{[\theta\, S(R)]^2} \,.$$

Since $S'(R) > 0$, and $(1-\theta)\, S(R) - K(R) \geq 0$ on $[\underline{R}_\theta, \Psi + c]$, Lemma 1 implies that $\beta_\theta'(R) > 0$ on $[\underline{R}_\theta, \Psi + c]:$ for fixed θ, the relationship between β and R is monotone increasing.

By hypothesis, there exist some $R_0 \in (\underline{R}_\theta, \Psi + c)$ and $R_0^m \in (0, R)$, with $\beta_0 \equiv \beta_\theta(R_0) \geq 0$, such that $G_\beta(R_0, R_0^m; \theta) = \Psi G$. Since $G_\beta(R, R^m; \theta)$ is strictly increasing in R^m, we have $G_\beta(R_0, 0; \theta) > \Psi G$.

Case A: Suppose $\theta\, S(\underline{R}_\theta) < G$; we then have $G_\beta(\underline{R}_\theta, 0; \theta) < \Psi G$. By Proposition 1, $G_\beta(R, R^m; \theta)$ is upward-sloping at $\underline{R}_\theta$ and strictly concave in R. It follows that there exists a unique value $\underline{\underline{R}}_\theta \in (\underline{R}_\theta, R_0)$ at which $G_\beta(R, 0; \theta) = \Psi G$. We must have $G(R, 0; \theta) < \Psi G$ at each $R < \underline{\underline{R}}_\theta$; it follows that no such value of R can support a steady state, since this would require $R^m < 0$. Since $\underline{\underline{R}}_\theta > \underline{R}_\theta$ and $\beta = 0$ at $\underline{R}_\theta$, the monotone-increasing relationship between β and R implies $\underline{\underline{\beta}}_\theta > 0$. Since $S'(R) < 0$, we have $\theta\, S(\Psi + c) < \theta\, S(\underline{R}_\theta)$ and thus $\theta\, S(\Psi + c) < G$; an argument analogous to the one just presented implies that $R^m = 0$ is associated with an $\overline{\overline{R}}_\theta \in (R_0, \Psi + c)$ and a $\overline{\overline{\beta}}_\theta > \beta_0$ which represent the maximum values of R and β that support a steady state.

Note that equation (23) can be rewritten

$$G_\beta(R, R^m; \theta) = [(R-c) - R^m]\, \theta\, S(R) + [\Psi - (R-c)]\, [S(R) - K(R)] \,. \qquad \text{(B.3)}$$

By Assumption II we know that there exists $R^m_0 \in (0, \Psi)$ such that $G(R, R^m_0; \theta) < \Psi G$ for all $R \in [\underline{R}_\theta, \Psi + c]$. Let $D(R^m; \theta)$ represent the value of $G(R, R^m; \theta) - \Psi G$ at the (unique) value of R at which $(\partial/\partial R)\, G(R, R^m_0; \theta) = 0$. We know $D(0; \theta) > 0$ and $D(R^m_0; \theta) < 0$. In addition, $D(R^m; \theta)$ is continuous in R^m, and it follows that if we decrease R^m by an arbitrarily small amount then we shift $G(R, R^m; \theta)$ upward by a range of positive but arbitrarily-small values. This means that the maximum value of $G(R, R^m; \theta)$ will increase by an arbitrarily small amount. It follows that there is a unique value $\widetilde{R}^m_\theta \in (0, R^m_0)$ at which $D(R^m; \theta) = 0$. Since $G(R, R^m; \theta)$ is strictly concave in R, the value $\widetilde{R}_\theta$ that solves $G(\widetilde{R}_\theta, \widetilde{R}^m_\theta; \theta) = \Psi G$ must be a tangency value. In addition, since $G(\widetilde{R}_\theta, R^m; \theta) > \Psi G$ for $R^m < \widetilde{R}^m_\theta$, and since $(\Psi - R_m)\, \theta S(\underline{R}_\theta) < \Psi \theta\, S(\underline{R}_\theta) < \Psi G$ and $(\Psi - R_m) \theta S(\Psi + c) < (\Psi - R_m) \theta S(\underline{R}_\theta) < \Psi \theta S(\underline{R}_\theta) < \Psi G$ for any $R^m > 0$, it follows that for any $R^m \in [0, \widetilde{R}^m)$ there is a steady state involving $R \in [\underline{R}_\theta, \widetilde{R}^m)$ and another steady state involving $R \in (\widetilde{R}^m, \Psi + c)$. Moreover, since $G(R, R^m; \theta)$ is continuous in both R and R^m, as R^m increases the value of R in the lower steady state declines continuously and the value of R in the high steady state rises continuously. Finally, the monotone-increasing relationship between R and β implies that there is a $\widetilde{\beta}_\theta > 0$ such that the value of R^m increases as β increases from $\underline{\underline{\beta}}_\theta$ to $\widetilde{\beta}_\theta$ and decreases as β increases from $\widetilde{\beta}_\theta$ to $\overline{\overline{\beta}}_\theta$.

Case B: Suppose $\theta\, S(\underline{R}_\theta) > G$. Assumption II implies $[\Psi - (\underline{R}_\theta - c)]\, [S(\underline{R}_\theta) - K(\underline{R}_\theta)] < \Psi G$, and equation (B.1) then implies that for R^m sufficiently close to $\underline{R}_\theta$ we also have $G(\underline{R}_\theta, R^m_\theta; \theta) < \Psi G$. It follows that there is a unique value $\underline{R}^m_\theta \in (0, \underline{R}_\theta)$ such that $G(\underline{R}_\theta, \underline{R}^m_\theta; \theta) = \Psi G$. Equation (23) demonstrates that the associated steady state involves $\beta = 0$. Since $G(\underline{R}_\theta, \underline{R}^m_\theta; \theta) > G(\underline{R}_\theta, R^m; \theta)$ for $R^m > \underline{R}^m_\theta$, we have existence of $\widetilde{\beta}_\theta > 0$ and $\widetilde{R}_\theta > \underline{R}_\theta$, and comparative statics on $[\underline{R}_\theta, \widetilde{R}_\theta)$ and $\beta \in [0, \widetilde{\beta}_\theta)$ as above.

Case B1: If $\theta\, S(\Psi + c) > G$ then there exists $\underline{\underline{R}}^m \in (0, \Psi)$ such that $(\Psi - \underline{\underline{R}}^m)\, \theta\, S(\Psi + c) = \Psi G$, and the maximum values of R and β that support binding steady states are $\overline{\overline{R}}_\theta = \Psi + c$ and $\overline{\overline{\beta}}_\theta = [(1 - \theta) S(\Psi + c) - K(\Psi + c)]/[\theta S(\Psi)]$, respectively.

Case B2: If $\theta\, S(\Psi + c) < G$ then there are values $\overline{\overline{R}}_\theta < \Psi + c$ and $\overline{\overline{\beta}}_\theta = [(1 - \theta) S(\overline{\overline{R}}_\theta) - K(\overline{\overline{R}}_\theta)]/[\theta S(\overline{\overline{R}}_\theta)]$ that support a steady state with $R^m = 0$; by analogy with Case A above (but on the other side of the Laffer curve), these are the maximum values of R and β that support binding steady states.

In both these cases, comparative statics on $(\widetilde{R}_\theta, \overline{\overline{R}}_\theta) \Leftrightarrow (\widetilde{\beta}_\theta, \overline{\overline{\beta}}_\theta)$ operate and are established as in Case A. $\square$

Proof of Corollary 1: Define

$$G_{LF}(R) = \frac{[\Psi - (R-c)]\,[S(R) - K(R)]}{\Psi}.$$

Under Assumption 1, this function is positive on the interior of $[\underline{R}, \Psi + c]$. It is also readily seen to be strictly concave on this interval, equal to zero but strictly increasing at $R = \underline{R}$, and equal to zero but strictly decreasing at $R = \Psi$. Let $\widehat{R} \in (\underline{R}, \Psi + c)$ solve $(\partial/\partial R)\, G_{LF}(R) = 0$; the preceding analysis implies that $\widehat{R}$ exists and is unique. Let $\widehat{G} = G_{LF}(\widehat{R})$; the preceding analysis also implies that $\widehat{G}$ is the maximum value of G_{LF} on $[R, \Psi]$, so that if $G > \widehat{G}$ then $[\Psi - (R-c)]\,[S(R) - K(R)] < \Psi G$ for all $R \in (\underline{R}, \Psi)$. Assumption I implies $\theta_{\max} > 0$; it also implies the existence of at least one $\widehat{\theta} \in (0, \theta_{\max})$ such that $\underline{R_{\widehat{\theta}}} \in (\underline{R}, \widehat{R}]$. Choose $\widehat{\theta}$ and $\widehat{R}^m \in (0, \widehat{R})$. Let

$$\widehat{\widehat{G}} = (\Psi - \widehat{R}^m)\widehat{\theta} S(\widehat{R}) + \left[\Psi - \left(\widehat{R} - c\right)\right] \left[(1-\widehat{\theta})S(\widehat{R}) - K(\widehat{R})\right] .$$

We have $\widehat{\widehat{G}} = \widehat{G} + (\widehat{R} - \widehat{R}^m)\widehat{\theta} S(\widehat{R}) > \widehat{G}$. In addition, $\underline{R_{\widehat{\theta}}} < \widehat{R}$ implies $(1-\widehat{\theta})S(\widehat{R}) - K(\widehat{R}) \geq 0$; since $\widehat{\beta} = [(1-\widehat{\theta})S(\widehat{R}) - K(\widehat{R})]/[\theta\, S(\widehat{R})]$, we also have $\widehat{\beta} \geq 0$.

Proof of Proposition 2: [1] We need to show that

$$\frac{\partial}{\partial R}\left(\frac{\partial}{\partial R} G_\theta(R, R^m; \beta)\right) < 0$$

for $R \in [\underline{R}, \Psi + c]$. Differentiation of equation (30) reveals that the expression in question is

$$\frac{1}{1+\beta}\left([(\Psi - R_m) + \beta(\Psi - R)]\,[S''(R) - K''(R)] - 2\beta[S'(R) - K'(R)]\right).$$

The result then follows from Lemma 1.

[2] At $R = \underline{R}$ we have $S(\underline{R}) - K(\underline{R}) = 0$. Equation (30) then reduces to

$$\frac{\partial G_\theta}{\partial R} = \frac{1}{1+\beta}\left([(\Psi - R_m) + \beta(\Psi - \underline{R})]\,[S'(\underline{R}) - K'(\underline{R})]\right),$$

which is positive as above.

[3] At $R = \Psi + c$ equation (30) becomes

$$\frac{\partial G_\theta}{\partial R} = \frac{[(\Psi - R_m)[S'(\Psi + c) - K'(\Psi + c)] - \beta\,[S(\Psi + c) - K(\Psi + c)]]}{1 + \beta}\ .$$

Assumption I and Lemma 1 imply that this derivative is positive at $\beta = 0$. As $\beta \to \infty$, however, $\partial G_\theta / \partial R \to -\,[S(\Psi + c) - K(\Psi + c)] < 0$. Moreover, the numerator of $\partial G_\theta / \partial R$ is strictly decreasing in β, so it follows that there is a unique positive value of β at which $\partial G_\theta / \partial R = 0$, with $\partial G_\theta / \partial R < 0$ for smaller values of β and $\partial G_\theta / \partial R > 0$ for larger values.

[4] When $\beta = 0$ equation (30) becomes

$$\frac{\partial G_\theta}{\partial R} = (\Psi - R_m)\,[S'(R) - K'(R)],$$

which is positive on $R \in [\underline{R}, \Psi + c]$ for any $R^m < \Psi$. However, when $\beta > 0$ it is always possible to find R^m below but sufficiently close to Ψ such that

$$(\Psi - R_m)[S'(\Psi) - K'(\Psi)] < \beta\,[S(\Psi) - K(\Psi)],$$

which implies $(\partial / \partial R)\, G_\theta(\Psi, R^m; \theta) < 0$.

Proof of Theorem 2: Inspection of equation (29) reveals that $G_\theta(R, R^m; \beta)$ is strictly decreasing in R^m and is equal to zero at $R = \underline{R}$. Proposition 2 then implies that if there are any binding steady states then there must be a steady state with $R^m = 0$. As in Theorem 1, the minimum values of R and θ must be the values associated with $R^m = 0$; in this case, however, we must have $\underline{\underline{R}}_\beta > \underline{R}$ and $\underline{\underline{\theta}}_\beta > 0$.

Case A: Suppose $\beta > 0$ and $G_\theta(R, 0; \beta)$ is downward-sloping at $R = \Psi + c$. Then by property 3 from Proposition 2, the function $G_\theta(R, R^m; \beta)$ must be a downward-opening paraboloid for any value of R^m.

Case A1: Suppose $G_\theta(\Psi + c, 0; \beta) > \Psi G$. In this case, since $G_\theta(\Psi + c, \Psi; \beta) = 0$, there must be an $R^m \in (0, \Psi)$ such that $G_\theta(\Psi + c, R^m; \beta) = \Psi G$, and it follows that $\Psi + c$ and $[S(\Psi + c) - K(\Psi + c)]/[(1 + \beta)S(\Psi + c)]$ are the maximum values of R and θ, respectively. We may proceed as in Theorem 2 to demonstrate the existence of an $\widetilde{R}_\beta \in (\underline{\underline{R}}_\beta, \Psi + c)$ and $\widetilde{\theta}_\beta \in (\underline{\underline{\theta}}_\beta, \overline{\overline{\theta}}_\beta)$ that separate the region where $\partial R^m / \partial \theta > 0$ from the region where $\partial R^m / \partial \theta > 0$.

Case A2: Suppose $G_\theta(\Psi + c, 0; \beta) < \Psi G$. In this case the maximum values of R and θ are the values associated with the steady state on the right side of the seigniorage Laffer curve when $R^m = 0$, but the analysis is otherwise identical.

Case B: Suppose $\beta > 0$ and $G_\theta(R, 0; \beta)$ is upward-sloping at $R = \Psi + c$. *Case B1:* Suppose $G_\theta(\Psi+c, 0; \beta) > \Psi G$. Let $\widehat{R}^m$ represent the value of R^m at which $G_\theta(\Psi+c, R^m; \beta) > \Psi G$. *Case B1a:* Suppose $G_\theta(R, R^m; \beta)$ is upward-sloping or flat at $R = \Psi + c$ and $R^m = \widehat{R}^m$. (In this case conditions [1] and [2] hold.) Then we know by Property 3 that $G_\theta(R, R^m; \beta)$ is upward-sloping in R at $R = \Psi + c$ for any lower values of R^m. It follows that there are no steady states for higher values of R^m, and that the maximum values of R and R^m are $\Psi + c$ and $\overline{\overline{\theta}}_\beta = [S(\Psi+c) - K(\Psi+c)]/[(1+\beta)S(\Psi+c)]$. It also follows that every seigniorage Laffer curve that supports a steady state is upward-sloping throughout $R \in [R, \Psi]$, and thus that no seigniorage Laffer curve supports more than one steady state. Consequently, any increase in θ that leaves $\theta \leq \overline{\overline{\theta}}_\beta$ must be accompanied by an increase in R^m. *Case B1b:* Suppose $G_\theta(R, R^m; \beta)$ is downward-sloping at $R = \Psi + c$ and $R^m = \widehat{R}^m$; this case is essentially identical to Case A1 above. *Case B2:* Suppose $G_\theta(\Psi + c, 0; \beta) < \Psi G$. In this case we know that $G_\theta(R, 0; \beta)$ must be downward-sloping in R at $R = \Psi + c$, and the analysis proceeds as in Case A2 above.

Case C: Suppose $\beta = 0$. Then we must have $G_\theta(\Psi + c, 0; 0) \geq \Psi G$, and we know that $G_\theta(R, R^m; 0)$ is upward-sloping throughout $R \in [R, \Psi + c]$ for every $R^m \in (0, \Psi + c)$. (It follows that conditions [1] and [2] hold.) This case is essentially identical to case B1a.

Proof of Corollary 2: By hypothesis, we have a $\beta_0 > 0$ such that $G_\theta(R_0, R_0^m; \beta_0) = \Psi G$ for some $R_0 \in (\underline{R}, \Psi + c)$ and some $R_0^m \in (0, R_0)$. Differentiation of equation (29) reveals that

$$\frac{\partial}{\partial \beta} G_\theta(R, R^m; \beta) = \frac{R^m - (R - c)}{1 + \beta} \left[S(R) - K(R) \right],$$

so G_θ is decreasing in β whenever $R \in (\underline{R}, \Psi + c]$ and $R^m \in (0, R - c)$. Consequently, we have $G_\theta(R_0, R_0^m; \beta) > \Psi G$ for any $\beta \in [0, \beta_0)$. And since $G_\theta(\underline{R}, R^m; \beta) = 0$ for any relevant values of R^m and β, we know that there is a steady state involving $R \in (\underline{R}, R_0)$ for any such value of β. [Note that if $G_\theta(R, R_0^m; \beta) = \Psi G$ involves $R_0^m \geq R - c$, we can always replace R_0^m with some $R^m \in [0, \underline{R} - c)$.]

It follows from Proposition 2 that if we choose β above but sufficiently close to zero, we can ensure that there are steady states for some values of R^m, and also that the seigniorage Laffer curve is upward-sloping at $R = \Psi + c$ except for R^m sufficiently close to Ψ. These two properties imply that for β sufficiently close to zero, the curve is upward-sloping at the value of R^m such that $G_\theta(\Psi + c, R^m; \beta) = \Psi G$.

We know that for β sufficiently close to zero, the value of R^m that supports a steady state when $R = \Psi + c$ is discretely smaller than Ψ. And since $\partial G_\theta / \partial \beta < 0$ at $R = \Psi + c$, we also know that as β increases this value of R^m falls continuously. We know that for β sufficiently close to zero, the value of R_m at which $\partial G_\theta / \partial R = 0$ at $R = \Psi + c$ can be driven arbitrarily close to Ψ. We can also show that as β increases this value falls continuously: if $\partial G_\theta / \partial R = 0$ at $R = \Psi + c$, we must have

$$(\Psi - R^m)[S'(\Psi + c) - K'(\Psi + c)] = \beta\,[S(\Psi + c) - K(\Psi + c)];$$

total differentiation with respect to R^m and β then produces

$$\frac{dR^m}{d\beta} = -\frac{S'(\Psi + c) - K'(\Psi + c)}{S(\Psi + c) - K(\Psi + c)} < 0.$$

The analysis presented above establishes that there are two possibilities as β increases from zero: *Case 1:* At the value of β at which the steady state with $R = \Psi + c$ involves $R^m = 0$, the value of R^m at which $\partial G_\theta / \partial R = 0$ is greater than zero. In this case, conditions [1] and [2] hold for any values of β consistent with a steady state. *Case 2:* There is some positive value of β at which the value of R^m that supports a steady state involving $R = \Psi + c$ also involves $\partial G_\theta / \partial R = 0$. For lower values of β, conditions [1] and [2] hold; for higher values, they do not. □

Proof of Theorem 3: We have

$$\frac{\partial G}{\partial R} = \frac{1}{1+\beta}\left([(\Psi - R_m) + \beta\,[\Psi - (R - c)]]\,[S'(R) - K'(R)] - \beta\,[S(R) - K(R)] \right).$$

The expression in parentheses can be rewritten

$$[(\Psi - R_m) + \beta\,[\Psi - (R - c)]]\;[\theta S'(R) + (1-\theta) S'(R) - K'(R)] - \beta\,[\theta S(R) + (1-\theta) S(R) - K(R)]$$

which can be further rewritten

$$\beta\,\left((\Psi - R_m)\theta S'(R) + [\Psi - (R - c)]\;[(1-\theta) S'(R) - K'(R)] - [(1-\theta) S(R) - K(R)]\right) +$$

$$(1-\beta)(\Psi - R_m)\theta S'(R) + \beta\,[\Psi - (R - c)]\;\theta S'(R) + (\Psi - R_m)[(1-\theta) S'(R) - K'(R)] - \;\beta\theta S(R)\,.$$

We know that when $\beta \leq \widetilde{\beta}_\theta$ the term multiplying β in the first expression is non-negative — it is $\partial G_\beta(R, R^m; \theta)$ — so this term can be rewritten

$$(1-\beta)(\Psi-R_m)\theta S'(R)+\beta\,[\Psi-(R-c)]\;\theta S'(R)+(\Psi-R_m)[(1-\theta)S'(R)-K'(R)]-\beta\theta S(R)+\beta\Omega,$$

where $\Omega \geq 0$. This expression can be rewritten

$$(\Psi-[(1-\beta)R_m+\beta(R-c)])\,\theta S'(R)+(\Psi-R_m)[(1-\theta)S'(R)-K'(R)]-[(1-\theta)S(R)-K(R)]+\beta\Omega,$$

using the fact that $P_b B = (1-\theta)S(R) - K(R) = \beta\theta S(R)$, or equivalently

$$(\Psi-R_m)\theta S'(R)+[\Psi-(R-c)]\,[(1-\theta)S'(R)-K'(R)]-\;[(1-\theta)S(R)-K(R)]+$$
$$\beta\,[R_m-(R-c)]\;\theta S'(R)+[(R-c)-R_m]\,[(1-\theta)S'(R)-K'(R)]+\beta\Omega,$$

which is

$$[(R-c)-R_m]\,[(1-\theta)\,S'(R)-K'(R)]-\beta\,[(R-c)-R_m]\;\theta\,S'(R)+(1+\beta)\,\Omega>0.$$

Thus we have $\partial G_\theta/\partial R > 0$ at any values of θ and β that support a steady state at which $\beta \leq \widetilde{\beta}_\theta$.

We want to show that $(\partial R^m/\partial R)_\theta > (\partial R^m/\partial R)_\beta$, which is equivalent to

$$\frac{(\Psi-R_m)+\beta\,[\Psi-(R-c)]}{1+\beta}\,[S'(R)-K'(R)]>(\Psi-R_m)\,\theta\,S'(R)+[\Psi-(R-c)]\;[(1-\theta)S'(R)-K'(R)]\,.$$

Multiplying both sides by $1+\beta$ and cancelling a number of common terms produces

$$\beta\,[R_m-(R-c)]\;\theta\,S'(R)>[R_m-(R-c)]\,[(1-\theta)\,S'(R)-K'(R)]$$

$\Leftrightarrow$

$$(1-\theta)\,S'(R)-K'(R)\;-\beta\theta\,S'(R)>0$$

which is true for any $R \in (\underline{R}, \Psi + c)$. $\square$

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