The Role of Interbank Markets in Monetary Policy: A Model with Rationing

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Abstract

This paper analyses the impact of asymmetric information in the interbank market and establishes its crucial role in the microfoundations of the monetary policy transmission mechanism. We show that interbank market imperfections induce an equilibrium with rationing in the credit market. This has three major implications: first, it reconciles the irresponsiveness of business investment to the user cost of capital with the large impact of monetary policy (magnitude puzzle), second, it shows that monetary policy affects long term credit (composition puzzle) and finally, that banks' liquidity positions condition their reaction to monetary policy (Kashyap and Stein liquidity puzzle).

Keywords: Banking, Rationing, Monetary Policy.

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1 Introduction

The aim of this paper is to understand how financial imperfections in the interbank market affect the monetary policy transmission mechanism and, more precisely, to explore whether the structure of the banking system has any effects beyond those of the classical *money channel*.

There are two basic empirical motivations for our work. On the one hand, Kashyap and Stein (2000) result, showing that the impact of monetary policy on a banks' amount of lending is stronger for banks with less liquid balance sheets, establishes the existence of imperfections in the interbank market. Such a *liquidity puzzle* is a challenge to the theoretical modelling of monetary policy channels based on highly efficient interbank markets, an assumption justified by the large volumes of transactions and the particularly low spreads observed on these markets.

On the other hand, the failure of existing theories of monetary transmission to explain a number of puzzles has also been a motivation for our work. Two of these puzzles, mentioned by Bernanke and Gertler (1995), are directly related to our work. First, empirical research has been unsuccessful in identifying a quantitatively important cost of capital effect on private spending, which has given rise to the so-called *magnitude puzzle,* whereby the aggregate impact of monetary policy is deemed excessively large, given the small elasticity of firms investment with respect to their cost of capital. Second, empirical evidence shows that, although monetary policy mainly affects short term interest rates, its main impact is on long term investment decisions, thus giving rise to the *composition puzzle*.

In this paper we show that, once we allow for interbank market imperfections, not only can we justify the Kashyap and Stein *liquidity puzzle*, but a new framework of analysis opens up, allowing for a better understanding of the *magnitude* and *composition puzzles*.

The interbank market allows banks to cope with liquidity shocks by borrowing and lending

from their peers, a function that, as it is assumed in this paper, the access to (inelastically supplied) deposits cannot fulfill. Our paper uncovers the important role of the interbank market as in an asymmetric information set-up by establishing the link between the imperfect functioning of the interbank market and the existence of rationing of banks and, in a cascading effect, of firms in the credit market. Our modelling of this effect allows us to establish that the relevance of imperfections in the interbank market for monetary policy depends on: (*i*) the dependence of firms on bank finance; (*ii*) the extent of *relationship lending*, in the sense of firms having access to funds through a unique bank; (*iii*) the heterogeneity of banks' liquidity positions, resulting from Treasury Securities (T-Bills) holdings resulting from past decisions and liquidity shocks originated in additional funding for existing projects.

The existence of credit rationing is of interest in our context, because, traditionally, the theory of credit rationing has been developed in a borrower-lender framework, better suited to the theory of banking than to the analysis of the transmission mechanism of monetary policy. In this paper we argue that credit rationing might also be an important part of the transmission mechanism. Introducing interbank market imperfections in the analysis of monetary policy seems a reasonable approach for two reasons. First, the interbank market is the first one to be exposed to the effects of monetary policy in the chain of effects that will generate the full impact of monetary policy. Second, it is worth considering an imperfect interbank market because Kashyap and Stein *liquidity puzzle* forces us to reconsider its supposedly perfect functioning and questions its purely passive role.

In order to analyze rigorously the effects of interbank markets imperfections on monetary transmission, we compare the transmission mechanism under two different scenarios: of symmetric and asymmetric information in the interbank market. The main lesson is that, under asymmetric information, the interbank market is unable to efficiently channel liquidity to sol-

vent illiquid banks and, as a consequence, there is *quantity rationing* in the bank loan market. The implications of credit rationing for monetary policy are straightforward. Under asymmetric information, monetary transmission may not be solely based on the *interest rate channel*, but may also depend on a *rationing channel*. When the Central Bank tightens its monetary policy, bank deposits decline and banks with less liquid balance sheets are forced to cut down on their lending. Thus, in an asymmetric information framework, an effect that cannot be accounted for in a symmetric information framework occurs: the interest rate effects combine with those of credit rationing and reinforce one another. This combination explains, on the one hand, that the effect of a monetary policy shock is larger than the one purely caused by interest rate movements, thus providing a justification for the *magnitude puzzle.* It also explains, on the other hand, that the effect of monetary policy is related not only to short term operations but also affects all types of loans, thus explaining the *composition puzzle.*

Our paper is related to several strands of the literature. As mentioned, our motivation stems from a number of empirical findings resulting from the concern with the traditional view of monetary policy, the *money view*, which explains the effects of monetary policy through the *interest rate channel*. Confronting this view, the *broad credit channel,* in its different variants (based either on the firms balance sheet and credit risk or on the banks inability to extend credit) assigns a more preeminent role to banks and asserts that the *supply of credit* plays a key role on the impact of monetary policy. One of its variants, the *lending view*, has focussed on the role of bank loans. Complementing the criticism to the *interest rate channel*, Mihov (2001) presents evidence that the banking system plays an important role in the propagation of monetary policy. Namely, he shows that the magnitude of monetary-policy responses of aggregate output is larger for countries with a higher ratio of corporate bank loans

to total liabilities. Moreover, he uses measures of banking industry health to conclude that the magnitude of monetary policy is larger for countries with less healthy banking systems, that is, that are subject to higher levels of financial imperfection. These are issues that a rigorous modeling of the banking sector should be able to clarify.

The *lending view* rests on the claim that there is a significant departure from the *Modigliani-Miller Theorem* for the banking firm, because financial markets are characterized by asymmetric information. When the Central Bank tightens its monetary policy, it forces banks to substitute away from reservable insured deposit financing and towards adverse selection prone forms of non deposit financing. This portfolio reallocation leads banks to adjust their asset holdings and this leads to a shift in the bank loan supply schedule, as argued by Stein (1998). Adopting such a perspective leads naturally to a theory of the *spread - augmented interest rate channel*. Some authors have indeed highlighted the influence of monetary innovations on the spread between the interest rate on bank loans and the risk free rate to justify the relevance of banks for monetary policy (see Kashyap and Stein (1994) and Stein (1998)). However, empirical research has faced great difficulties in showing that a contractionary monetary policy will increase the spread on bank loans. Berger and Udell (1992) document that, on the contrary, bank loan rate premia over treasury rates of equal duration decrease substantially when Treasury securities rates increase (and commitment loans do not explain this phenomenon), contrarily to the theoretical prediction.

A second strand of the empirical literature on monetary policy that is directly relevant for our analysis is concerned with the *magnitude* and *composition puzzles*. Regarding the *magnitude puzzle*, on the one hand, it has been extensively reported that the response of business investment to the user cost of capital tends to be unimportant relative to *quantity variables*, like financial structure and cash flow. These variables have been included frequently

as regressors in estimations, and generally have proven significant, suggesting that investment depends on variables other than the user cost of capital.[1] On the other hand, the empirical research on the macroeconomic effects of shifts in the interest rates controlled by the Central Bank shows that the real economy is powerfully affected by monetary policy innovations that induce relatively small movements in *policy rates*. See Angeloni *et al.* (2002) for additional empirical evidence on the *magnitude puzzle* in Europe.

Regarding the *composition puzzle* (Bernanke and Gertler (1995)), most monetary models predict that monetary policy should have its strongest influence on short-term interest rates and a relatively weaker impact on (real) long-term rates. Yet, the empirical evidence shows that the most rapid and strongest effect of monetary policy is on residential investment. This finding is surprising because residential investment is typically very long lived and therefore should not be sensitive to short term interest rates.

Finally, our paper is related to the borrower-lender relationship under asymmetric information and to the classical work of Stiglitz and Weiss (1981). Still, we do not consider second order stochastic dominance, so that our set-up is closer to Ackerloff (1970)'s market for lemons: once the interbank market is shown to be thin, in the sense that only fully collateralized loans are made in equilibrium, liquidity short banks are rationed and are forced to ration their clients.

The borrower-lender relationship under asymmetric information has been also explored in order to model interbank markets. Not surprisingly, many authors suggest that the interbank markets deliver an efficient allocation of bank reserves within the banking system (see for example Goodfriend and King (1988) and Schwartz (1992)). This will be the case if market

[1] See Hubbard (1998) and Schiantarelli (1995) for a review of this literature. Following the work by Kaplan and Zingales (1997) and Cleary (1999), some authors argue that investment cash-flow sensitivity may not be always interpreted as revealing the existence of financial constraints because investment demand is difficult to measure and may be positively correlated with cash flow.

participants are well informed to assess the solvency of any potential borrower. Still, under asymmetric information, the interbank market may lead to a second best allocation of liquidity as illustrated by Bhattacharya and Gale (1987), Rochet and Tirole (1996), Flannery (1996) and Freixas and Holthausen (2005). The empirical evidence tends to support the asymmetric information view of the market, as shown by Furfine (2001), who presents evidence that some monitoring is done by lenders in the interbank market, and Cocco *et al.* (2003), who document *relationship lending* in the interbank market.

The paper is organized as follows. We devote the next section to present the basic model and assumptions. We proceed by comparing the equilibrium under perfect information and asymmetric information. Section 5 evaluates the implications of our model for the monetary transmission mechanism and is followed by a short conclusion. The proofs of the main results are given in the Appendix.

2 The Model

This section presents a partial equilibrium model of the bank loan and interbank markets. Firms face liquidity shocks and rely on bank credit to raise external finance. In this way the firms' shocks become a demand for credit and a liquidity shock for the banks. As in Holmstrom and Tirole (1998) and Stein (1998), banks hold a large fraction of their assets as reserves and liquid securities to act as a buffer against liquidity shocks.[2] We assume that banks hold different amounts of securities and face different liquidity shocks. Owing to heterogeneity, there is a role for an interbank market to trade reserves as in Battacharya and Gale (1987). We will begin by developing a simple model in a perfect information set-up, and then proceed to introduce

[2] A model of how this buffer is build and how it is affected by monetary policy is studied in Stein (1998) and Van der Heuvel (2006).

asymmetric information on the banks' liquidity shocks.

Following the literature on monetary macroeconomics, we adopt a highly stylized view of how monetary authorities implement their policies. The Central Bank sets an interest rate at which it is willing to borrow or lend unlimited amounts of collateralized funds, with the collateral being liquid risk free assets as, for example, T-Bills. We call this interest rate the *policy rate* and denote it by r, with $r \geq 0$. We assume that households and firms hold money in the form of bank deposits which earn zero interest and provide payment services.[3] The alternative to holding money is holding T-Bills. We assume that arbitrage guarantees that the interest rate on Treasuries equals r. Hence the Central Bank, by controlling the interest rate, is able to affect the opportunity cost of holding deposits and we have a standard money demand which depends negatively on the risk free rate.

2.1 Firms

There is a continuum of firms with unit mass and there are three dates. Each firm has a fixed size project requiring an investment of one unit at date zero. At date one, each firm suffers a real shock and needs an amount ν of funds. When $\nu < 0$ the project generates a revenue for the firm and when $\nu > 0$ the firm experiences a cost overrun. For the sake of simplification, we also assume that firms can only be financed by bank loans and that, if the cost overrun is met, the project generates a certain return Y at date two; if it is not funded, the project is terminated and has no residual value.[4] We assume that the variable ν is random with a uniform distribution with support $[\underline{\nu}, \overline{\nu}]$, $\underline{\nu} \leq 0$, $\overline{\nu} > 0$ and $\overline{\nu} + \underline{\nu} > 0$. At date one, the firm

[3] We do not require that the interest rate on bank deposits is zero. We do require, though, that either the deposit interest rate is fixed or that the supply of bank deposits is inelastic with respect to the deposit rate quoted by an individual bank.

[4] This formulation is more extreme than it needs to be. All we need is that firms face some cost in liquidating projects early.

obtains an amount F of funds at an interest rate r_F.

We are mainly concerned with the effects occurring at date one, since it is at this point in time when monetary policy will impact the banks and firms decisions. In particular, we are interested in computing the firms' optimal liquidation, their borrowing and the interest rate sensitivity of their output.

Firms have a passive role as they are willing to borrow the amount of liquidity ν they require to fund their cost overrun and, therefore, $F = \nu$. At date zero, the firm asks for a unit loan and promises to repay $R_0 < Y$ at date two if its project is successful. In addition, the firm signs a credit line contract so that the interest rate on the date one bank loan that the firm demands is not renegotiable.

At date two, the profit of the firm, under this contract, is equal to $Y - R_0 - (1 + r_F)\nu$. A firm will default if and only if it cannot repay the bank at date two, that is, if the output Y cannot cover the sum of the repayments $R_0 + (1 + r_F)\nu$ for the two loans the firm has received. In other words, the firm will go bankrupt if its cost overrun ν is larger than $\frac{Y-R_0}{1+r_F}$. If the firm is unable to repay $R_0 + (1 + r_F)\nu$ at date two, the bank takes over the project at date one.[5] Nevertheless, if the bank taking repossession does not face financial frictions, it may not liquidate the project of the firm. Instead it will inject a cash flow provided that $\nu \leq \nu^*(r_F)$ with

$$\nu^*(r_F) \equiv \frac{Y}{1 + r_F}.$$

An interesting feature of our model is that the firm does not wait until date two to default.

Proposition 1 *The firm will be able to finance its project if and only if* $\nu \leq \nu^*(r_F) - \frac{R_0}{1+r_F}$;

[5] Alternatively, it is possible to assume that the firm renegotiates its debt and the bank appropriates the whole surplus.

otherwise the firm defaults and hands over its assets to the bank that will choose whether to continue (if $\nu \leq \nu^* (r_F)$) or liquidate (if $\nu > \nu^* (r_F)$) the firm's project.

2.2 Banks

We assume the existence of a continuum of identical banks. At date zero, banks collect an amount D_0 from depositors, invest in illiquid loans (of size 1) that finance the firms' investment projects and constitute a buffer of liquid securities (T-Bills) to face future liquidity shocks.

The value of the liquid securities of a bank at date one equals B_0 and we assume that liquidity holdings are distributed heterogeneously. Specifically, we consider that the variable B_0 is uniformly distributed across banks, with support $\left[\underline{B}, \overline{B}\right]$ and $0 < \underline{B} < \overline{B}$. The realizations of B_0 are hard information, as they appear in the banks' balance sheet, and because of this, we assume that they are observable. We denote the mean and variance of this distribution by $E[B_0] = \left(\overline{B} + \underline{B}\right)/2$ and $\sigma^2_{B_0}$. At date one, banks decide how much they hold in liquid securities until date two and, after trading, each bank ends up with an investment B_1 in T-Bills that, at date two, yields $(1 + r) B_1$, where r is the return of T-Bills.

At date one the deposit base of the bank institution changes. We denote bank deposits at date one by $D_1(r)$ and we assume that $dD_1(r)/dr < 0$. We represent the shift in deposits by

$$\mathcal{D} \equiv \mathcal{D}(r) \equiv D_1(r) - D_0.$$

Hence, the gross amount of liquidity available to a bank at date one is the sum of the market value of its T-Bills plus the net increase in deposits, that is $B_0 + \mathcal{D}$.

At date one, banks inherit a unit amount of bank loans and make additional loans equal to F (if projects do not default). For the sake of simplicity we assume that banks lend funds to a

set of firms with perfectly correlated projects. To all purposes this set of firms is treated as a unique firm, so that there is a one to one correspondence between the set of firms and the set of banks. Also, we assume a relationship banking framework so that, on the one hand, a bank has perfect knowledge of the firm liquidity shock ν, and on the other hand, a firm is captive from that bank and cannot switch to another one. We justify these assumptions by referring to both theoretical models and empirical evidence of *relationship banking* (See e.g. Boot (2000) for the former and Degryse and Ongena (2005) for the latter).

The difference between liquid assets and liquid liabilities at date one equals $F - (B_0 + \mathcal{D})$ and will be covered through access to the interbank market. A bank's net borrowing in the interbank market at date one will be denoted by L (positive or negative) and the corresponding interest rate by r_L. This interest rate may incorporate a risk premium because lenders in the interbank market are exposed to default risk. Lenders in this market diversify their interbank loan portfolio and obtain an effective rate of return ρ_L. Formally, let

$$\gamma = \begin{cases} \rho_L & if \;\; L < 0 \\ r_L & if \;\; L \geq 0 \end{cases} \qquad \text{with} \;\; r_L \geq \rho_L.$$

In order to avoid multiplicity of equilibria we assume that, when the cost of interbank funds is equal to the return on treasuries, banks do not borrow in the interbank market to invest in treasuries. Formally, we assume that $LB_1 \leq 0$ when $r_L = r$.

At date one the budget constraint of the bank yields

$$F + B_1 = B_0 + \mathcal{D} + L. \tag{1}$$

We focus on the interesting situation in which an interbank market develops. By this we

mean that some banks do not have enough liquidity to finance profitable projects.

Assumption A1: (Liquidity needs)

$$\underline{B} + \mathcal{D} < \min\left\{\overline{\nu}, \frac{Y}{1+r}\right\}.$$

We assume that the Central Bank does not set the interest rate r in such a way that it generates a liquidity crisis at date one. By this we mean that aggregate liquidity is enough to serve the aggregate demand for bank deposits.

Assumption A2: (No liquidity crisis)

$$E\left[B_0\right] + \mathcal{D} \geq 0.$$

2.3 The Firm - Bank Relationship

Recall that we are assuming that the bank has full information on the firm. We consider the case in which, at date zero, the firm and the bank sign a credit line contract which specifies the terms of the loan at date zero and the interest rate at date one. We assume that, at date zero, the market for bank loans is competitive and this implies that the bank loan rate at date one is set as equal to the interbank rate ($r_F = r_L$). We assume that the interest rate on the date one bank loan that the firm demands is not renegotiable. We consider explicitly the possibility that an individual bank is rationed in the interbank market and we allow for an upper bound equal to $\overline{L}$ on interbank borrowing. The profit function of the bank at date one depends on

whether the firm defaults or not. When the firm does not default, the problem of the bank is:

$$\begin{aligned} &\max_{\{L,B_1\}} R_0 + (1+r_L)\,F + (1+r)\,B_1 - (1+\gamma)\,L - D_1(r) \\ &s.t. \qquad (1) \qquad and \quad L \leq \overline{L}. \end{aligned} \tag{2}$$

When the firm defaults, then the bank appropriates the project, adds the assets of the firm to its own portfolio, and chooses whether to continue or liquidate the project. If the bank prefers to continue, its problem resembles the individual firm's problem in section 2.1:

$$\begin{aligned} &\max_{\{L,B_1\}} Y + (1+r)\,B_1 - (1+\gamma)\,L - D_1(r) \\ &s.t. \quad B_1 + \nu = B_0 + \mathcal{D} + L \\ &L \leq \overline{L}. \end{aligned} \tag{3}$$

If instead the bank liquidates the project at date one, then it obtains no profit from it and gets

$$(1+\rho_L)\,(B_0 + \mathcal{D}) - D_1(r) \tag{4}$$

as a return from its liquidity at date two.

Since there is perfect information inside the relationship, the financial contract between the firm and the bank does not affect the implementation of the project (this being a weaker version of the *Modigliani-Miller Theorem*, and assuming there is no cost associated to handing over the project as in Diamond and Rajan (2001)). Hence, our model is robust to alternative specifications of the credit contract signed by the firm and the bank. In the next result we show that, in order to obtain the decisions related to the project, it suffices to investigate the optimal decision taken by a single *Integrated Entity* which aggregates the firm and the bank.[6]

[6] We are not allowing for the possibilty of having partial liquidation of the projects. This extreme assumption

Lemma 1 *(Integrated Entity) The decisions regarding the defaulting threshold of the projects, the amount of securities B_1 and the amount of interbank loans L held by banks, obtained by solving separately the problems of the firm and bank are equal to the ones obtained by solving the problem of a single Integrated Entity which aggregates the firm and the bank. The profit function of an Integrated Entity which implements the project is given by*

$$\pi\left(\nu, \overline{L}, r_L, \rho_L, r\right) = Y - (1+\gamma)\left[\nu - B_0 - \mathcal{D}\right] - D_1(r) \tag{5}$$

while the profit derived from non-continuation is given by expression (4).

Proof. See the Appendix. ■

We can also define the defaulting threshold of the new entity.

Lemma 2 *The defaulting threshold for the Integrated Entity is equal to*

$$\begin{cases} \nu^*(\rho_L) & if \quad \nu \leq B_0 + \mathcal{D} \\ \nu^*(r_L) + \frac{r_L - \rho_L}{1+r_L}(B_0 + \mathcal{D}) & if \quad \nu > B_0 + \mathcal{D} \end{cases}$$

Proof. See the Appendix. ■

Notice that an illiquid entity may have a tougher liquidation policy than a liquid entity. As of now, we will (loosely) refer to the Integrated Entity as "bank".

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does not condition the qualitative results that we obtain. For example, if we consider the model in which the Integrated Entity can liquidate a fraction l of the projects and solves

$$\max \quad (1-l)Y + (1+r)B_1 - (1+\gamma)L - D_1(r)$$
$$s.t. \;\; B_1 + (1-l)\nu = B_0 + \mathcal{D} + L \quad and \quad L \leq \overline{L}$$

then this model has qualitative results identical to the one we solve.

3 Perfect Financial Markets

To close the model we study the interbank market. We analyze separately the cases of perfect and imperfect information in the interbank market. In this section we consider the case in which there is perfect information regarding the value of the cost overrun ν. Hence every bank knows the value of the cost overrun suffered by the projects financed by its peers and only banks that fully repay their interbank loans obtain funds in the interbank market. Hence, at date one, there is no risk premium and $\rho_L = r_L$. Provided that there is no liquidity shortage, then r_L equals the risk free rate r. Hence the defaulting threshold equals $\nu^*(r)$ and, as intuition suggests, it is independent of the liquidity position $(B_0 + \mathcal{D})$ of the banks.

The measure of projects that are liquidated is $[\overline{\nu} - \nu^*(r)] / (\overline{\nu} - \underline{\nu})$, while the measure of those that continue (whether property of the initial owners or property of the bank if they are unable to repay their debt) is $[\nu^*(r) - \underline{\nu}] / (\overline{\nu} - \underline{\nu})$. The aggregate output $\mathcal{Y}$ is therefore given by the proportion of firms with cost overruns below the threshold $\nu^*(r)$:

$$\mathcal{Y} = \frac{1}{\overline{\nu} - \underline{\nu}} \int_{\underline{\nu}}^{\nu^*(r)} Y d\nu. \tag{6}$$

The semi-elasticity of aggregate output depends on the effect of the interest rate on the liquidation threshold. The higher the interest rate the larger the number of firms that will be cut out of funds and forced to liquidate.

Our goal is to verify the existence of a *magnitude puzzle*. In order to measure the effect of interest rates over output, we compute:

- The semi-elasticity of aggregate output produced by the total number of firms with respect to the user cost of capital experienced by firms, which we denote by $\varepsilon_{uc}(r_F)$.[7]

[7] The general formula for a semi-elasticity of variable $\mathcal{Y}$ with respect to $\widetilde{r}$ is $\varepsilon(\widetilde{r}) = -\frac{d\mathcal{Y}}{d\widetilde{r}} \frac{1}{\mathcal{Y}}$.In what follows,

This semi-elasticity is obtained by aggregating the semi elasticity of the output produced by individual firms.

- The semi-elasticity of aggregate output with respect to the interest rate set by the Central Bank r, which we denote by $\varepsilon_r\ (r)$.

We define $M \equiv \varepsilon_r\ (r) - \varepsilon_{uc}\,(r_F)$ as a measure of the magnitude effect so that, the *magnitude puzzle* exists if and only if $M > 0$.

In the perfect markets case, the user cost of capital equals the riskless interest rate and $\varepsilon_{uc}\,(r_F)$ equals $\varepsilon_r\ (r)$.[8] Using expression (6), the common semi-elasticity is easily computed:

$$\varepsilon_r^*\,(r) = \varepsilon_{uc}^*\,(r) = \frac{\nu^*\,(r)}{1+r}\frac{1}{\nu^*\,(r) - \underline{\nu}}.$$

Thus, monetary policy affects aggregate output produced by firms through the interest rate channel: larger interest rates shift the defaulting threshold $\nu^*\,(r)$ and reduce the measure of projects with positive net present value. There is no *magnitude puzzle* because the magnitude effect is zero.

It is useful to compare the perfect markets case with the extreme case of no interbank market. In this case, banks which finance projects with $\nu > B_0 + \mathcal{D}$ would be unable to obtain funds to continue their projects and would have to default. Under this setup, a central issue for the transmission mechanism is whether monetary policy shocks affect the threshold $B_0 + \mathcal{D}$.

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we assume that the bank loan interest rate, r_F, is the sole determinant of the user cost of capital.

[8] Implicitly we are assuming that firms with a cost overrun $\nu \in \left(\frac{Y-R_0}{(1+r)}, \nu^*\,(r)\right]$ are reestructured by the bank and continue. Hence its defaulting threshold equals $\nu^*\,(r)$. Had we assumed that the firm is terminated and the bank takes over the project and we would have $\mathcal{Y} = \frac{1}{\overline{\nu}-\underline{\nu}} \int_{\underline{\nu}}^{\frac{Y-R_0}{(1+r)}} Y d\nu$. The results that we obtain do not depend on this assumption.

4 Asymmetric Information

In this section we assume the existence of asymmetric information on the firm's cost overrun ν. This cost overrun is known to the firm and its financing bank that, because of our assumption of *relationship banking*, has access to all relevant information. Obviously, banks that have not established any relation with the firm, cannot observe the value taken by ν, which is the source of asymmetric information.

The asymmetric information appears therefore in the interbank market in the contractual relationship between a bank and its peers. We will assume that this asymmetry allows insolvent banks to forbear and try to gamble for resurrection. Thus, as in Aghion *et al.* (1999) and Mitchell (2000) (where the accumulation of loan losses leads the bank to hide them by renewing its bad loans in order to stay afloat), accumulated loan losses produce a cascading effect here as well and triggers the bank gambling for resurrection. In order to model gambling for resurrection in a simplified way, we assume that bank managers have access to an alternative project at date one, which we refer to as *the private benefits project,* that yields, at date two, a pledgeable return, K, plus an amount of private benefits equal to ϑL.[9] In order to make the results on aggregate output directly comparable with the symmetric information case, we consider K as an asset and not as new production of the alternative project. We assume that depositors are senior with respect to interbank lenders and that $D_1(r) \leq K$, so that deposits are riskless. We interpret $K - D_1(r)$ as the bank's collateral in the interbank market.

The interpretation for the private benefits project is akin to Calomiris and Kahn (1991), in which managers have the opportunity to abscond with the funds and absconding is socially

[9] Alternatively, it is possible, although analytically more involved, to assume that the final outcome Y is a random variable. This, jointly with limited liability, provides the option-like structure of stockholders' profits, which would obviously lead to the same results.

wasteful. We assume that $0 < \vartheta < 2/3$, which implies that, the larger the amount of cash available to the bank manager, the larger its private benefit.[10]

Given an equilibrium in the interbank market characterized by a maximum amount of loan $\widetilde{L}$, a bank manager can always secure a loan of size $\widetilde{L}$, so that the value $\vartheta\widetilde{L}$ is its *reservation utility level*. Thus, after observing its project cost overrun ν, the bank will compare its profits level with the value of its private benefits, $\vartheta\widetilde{L}$, and choose whether to continue the project or to engage in the private benefits project. Obviously, since this project yields K, any bank obtaining a loan with a repayment $L(1+r_L)$ larger than $K - D_1(r)$, and choosing the private benefits project, will default. We refer to these banks as strategic defaulters. Remark, though, that the choice of strategical default is endogenous and depends upon each bank's liquidity position, given by ν, $\mathcal{D}$ and B_0, as well as on the interbank market interest rate r_L.

Let $\widehat{L}$ denote the present value of the banks' collateral that can be appropriated by the interbank lenders:

$$\widehat{L} \equiv \frac{K - D_1(r)}{1 + r_L}.$$

Later we prove that $r_L = \rho_L$, which means that we are discounting the value $K - D_1(r)$ using the risk free rate. Since a loan of size lower or equal to $\widehat{L}$ is fully collateralized, banks will always have access to such loans. Consequently, we denote the minimum amount of liquidity the bank is guaranteed to have access to as:

$$\widehat{F} \equiv B_0 + \mathcal{D} + \widehat{L}.$$

Notice that a bank's liquidity position at date one depends upon the value for B_0. This value is

[10] When the variables Y and K are random, the assumption that private benefits depend on the size of the interbank loans is justified by the fact that, when a bank has a large amount of interbank loans, it is more likely to be rescued either by the authorities or by its peers.

obtained from the previous period decisions (regarding the amount of T-Bills) as well as from the market (regarding T-Bills prices, if we had considered securities with a maturity equal to two periods). The Central Bank can affect the liquidity position of banks by conditioning their access to deposits (which affects the values for $\mathcal{D}$ and $\widehat{L}$) and influencing the interbank interest rate r_L (which influences $\widehat{L}$).

4.1 Contracts in the Interbank Market

We assume that banks compete in the interbank market. Because of asymmetric information, competition will be in terms of the contracts (L, r_L), since the profitability of a loan depends not only on its interest rate, r_L, but also on the amount granted, L.

Market behavior is captured in the following two stage game:

1. In stage one, lending banks simultaneously announce a menu of contracts $(L, r_L(L))$ to potential borrowers. Formally, this is defined by the set:

$$\Omega = \left\{ (L, r_L(L)) : L \in \mathcal{A} \subset R^+, r_L(L) : \mathcal{A} \to R^+ \right\}.$$

2. In stage two, the borrowing banks decide to accept or not one of the contracts offered by a specific bank. (For the sake of simplicity we suppose that, if they are indifferent among the contracts offered by banks, then they randomize among them).

A particular menu of contracts, the riskless competitive one, is particularly relevant to characterize the interbank equilibrium. It is defined as follows:

Definition 1 *The Riskless Competitive Contract Menu* $(RCCM)$ *is defined as:*

$$\Omega^P = \left\{ (L, r_L(L)) : L \in \left[0, \widehat{L}\right] \quad \textit{with } r_L = \rho_L \right\}.$$

We will show that, under some assumptions, the $RCCM$ characterizes the unique equilibrium set of contracts.

4.2 Equilibrium Set of Contracts

The reason why, under some conditions, the equilibrium is restricted to the $RCCM$ class of riskless, zero profit contracts is quite intuitive. Agents with a low liquidity need will always ask for a loan lower than $\widehat{L}$, which is riskless, and competition on any of these contracts will lead to $r_L = \rho_L$. Now, for contracts characterized by $L > \widehat{L}$, lenders know that the behavior of strategic defaulters will be to ask for the largest possible loan. Because of this, any loan contract with $\widetilde{L} > \widehat{L}$, will be dominated by a contract $\widetilde{L} - \varepsilon$ as this slight reduction of the loan size attracts only non-defaulting banks, so that it is always profitable. The difficulty is then to see whether deviations from the equilibrium associated with the $RCCM$ are possible, in which case no equilibrium would exist.[11] We show that, under some conditions, no contract outside the $RCCM$ class can make positive profits.

The following assumptions allow us to focus on the case where a pure strategy equilibrium with rationing in the interbank market exists. First, we disregard the case where collateral is enough to guarantee a riskless interbank market, because this is equivalent to reintroducing the perfect capital market we have already analyzed. This is why we make the following assumption:

Assumption A3: (Rationing)

$$K + \vartheta \frac{K - D_1(r)}{1 + r} < Y.$$

[11] This non-existence may occur because banks compete both on prices and quantities, and changes in quantities affects credit risk. This is related to Stahl (1988) and Yannelle (1987) papers, where double Bertrand competition may result in non-existence, and to Broecker (1990) where competition affects credit risk, leading to the non-existence of pure strategies equilibrium.

In a similar vein, we also want to discard the uninteresting case where banks have enough liquidity to cope with any type of cost overrun. Assumption A4 implies that, at least for large cost overruns, banks will have to borrow from the interbank market.

Assumption A4: (Liquidity needs')

$$\overline{B} + \frac{r}{1+r} D_1(r) - D_0 + \frac{K}{1+r} < \min\left\{\overline{\nu}, \frac{Y}{1+r}\right\}.$$

Assumption A4 implies that $\widehat{F} < \min\{\overline{\nu}, Y/(1+r)\}$, which means that it is more restrictive than assumption A1.

The last assumption states that the adverse selection problem is not negligible.

Assumption A5: (Existence)

$$\frac{3}{2}\left[\frac{Y - K - \vartheta\frac{K - D_1(r)}{1+r}}{1+r}\right] \leq \overline{\nu} - E\left[B_0 + \frac{K}{1+r} + \frac{r}{1+r} D_1(r) - D_0\right].$$

The intuition for this assumption is as follows. We can rewrite the term inside the brackets in left hand side of the above expression as

$$\frac{Y - D_1(r) - \vartheta\frac{K - D_1(r)}{1+r}}{1+r} + E[B_0 + \mathcal{D}] - E\left[B_0 + \mathcal{D} + \frac{K - D_1(r)}{1+r}\right]$$

which is the difference between what would be the "average" defaulting threshold without rationing, that is $\left[Y - D_1(r) - \vartheta\widehat{L}\right]/(1+r_L) + E[B_0 + \mathcal{D}]$, and the "average" defaulting threshold with rationing, $E\left[B_0 + \mathcal{D} + \widehat{L}\right]$, when the interbank rate equals r. Thus, the left hand side of the expression in assumption A5 is a measure of the inefficiency caused by rationing, while the right hand side is a proxy for the size of the mass of defaulters. Hence, assumption A5 guarantees that: (i) the mass of defaulters is sufficiently large, so that lenders have no incen-

tive to propose new contracts because the losses associated to such contracts are larger than the potential gains; (*ii*) the inefficiencies stemming from rationing and liquidation are not too large, as otherwise agents would have strong incentives to propose deviating contracts.

Proposition 2 *Under assumptions A2 to A5, the RCCM defines the unique equilibrium set of contracts that exists in the interbank market. The defaulting threshold for the Integrated Entity with liquidity* $B_0 + \mathcal{D}$ *equals* $\widehat{F}$.

Proof. See the Appendix. ■

In the next section we characterize the equilibrium in the interbank market and we restrict our attention to the case in which there is rationing. When this happens, on the one hand, banks with (relatively) more liquid balance sheets are able to finance the liquidity needs of their clients, and firms dependent on these banks obtain finance as long as they have projects with positive net present value. On the other hand, illiquid banks are unable to shield their loan portfolio. Firms captive of these banks are unable to obtain finance because they are being rationed and this entails the liquidation of profitable projects.

5 Interbank Market Equilibrium and Monetary Policy

In order to study monetary transmission under asymmetric information in the interbank market, we will start by clarifying the equilibrium concept in our set-up.

The agents' decision variables consist of their demand and supply of interbank loans (L) and liquidity holdings (B_1), and the equilibrium interest rate in the interbank market (r_L) is the one for which the aggregate excess demand for loans clears. Note that the level of interest rates (r) is exogenously given.

In order to compute the equilibrium, we aggregate the individual net demands for funds in

the interbank market. We must have $r_L \geq r$, otherwise there is excess demand in the market. When $r_L \geq r$, the individual net demand z for interbank funds by a bank with cost ν and liquidity B_0 is

$$z = \begin{cases} \nu + B_1(B_0, \nu) - B_0 - \mathcal{D} & if \quad \nu \leq B_0 + \mathcal{D} \\ \nu - B_0 - \mathcal{D} & if \quad B_0 + \mathcal{D} < \nu \leq \widehat{F} \\ \widehat{L} & if \quad \nu \geq \widehat{F} \end{cases}$$

where $B_1(B_0, \nu)$ denotes the holdings of treasuries, at the end of date one, by a bank which inherits B_0 and suffers a liquidity shock ν. When $r_L = r$ we have $B_1(B_0, \nu) \in [0, \infty)$ and when $r_L > r$ we have $B_1(B_0, \nu) = 0$ for all banks. The aggregate net demand for funds in the interbank market, Z, is the sum of (positive and negative) individual excess demands for loans.

Lemma 3 *Under assumptions A2 to A5 and $r_L \geq r$, the aggregate net demand for funds in the interbank market equals*

$$Z(r_L) = \frac{\Theta(r_L)}{\delta} + \frac{1}{\delta} \int_{\underline{B}}^{\overline{B}} \int_{\underline{\nu}}^{\overline{\nu}} B_1(B_0, \nu)\, d\nu dB_0$$

where

$$\Theta(r_L) = -\frac{1}{2} \int_{\underline{B}}^{\overline{B}} \left\{ \widehat{L}^2 - 2\left[\overline{\nu} - (B_0 + \mathcal{D})\right] \widehat{L} + \left[B_0 + \mathcal{D} - \underline{\nu}\right]^2 \right\} dB_0,$$

$\delta = \left(\overline{B} - \underline{B}\right)(\overline{\nu} - \underline{\nu})$ *and* $\int_{\underline{B}}^{\overline{B}} \int_{\underline{\nu}}^{\overline{\nu}} B_1(B_0, \nu)\, d\nu dB_0 \geq 0$, *and the function* $\Theta(r_L)$ *is decreasing in* r_L.

Proof. See the Appendix. ■

We will distinguish two different regimes on the basis of the existence or not of a spread in the interbank market above the *policy rate*. This is of interest as we will show that the effects of monetary policy differ in the two regimes.

- An excess liquidity regime occurs if there is no spread in the interbank market rate, that is $r_L = r$. This will be the case if any bank holds a positive amount of T-Bills in equilibrium, that is $B_1(B_0, \nu) > 0$.

- A liquidity shortage regime occurs if there is a spread $r_L - r > 0$ between the interbank market return r_L and the target rate r. In this case, it is not profitable for any bank to hold T-Bills, and $B_1(B_0, \nu) = 0$.

The two regimes can be distinguished according to the value taken by $\Theta(r)$, which is a proxy for the value of the aggregate net demand for funds when $B_1(B_0, \nu) = 0$ for all banks. If $\Theta(r) > 0$, then the economy is in the liquidity shortage regime; if instead $\Theta(r) \leq 0$, the economy is in the excess liquidity regime.

Lemma 3 allows to clarify the link between the amount of aggregate liquidity shocks (i.e. cost overruns) and the liquidity regime. For the economy to be in the excess liquidity regime, expression $\left\{\widehat{L}^2 - 2\left[\overline{\nu} - (B_0 + \mathcal{D})\right]\widehat{L} + \left[B_0 + \mathcal{D} - \underline{\nu}\right]^2\right\}$ must be positive on average. This will be the case when $(B_0 + \mathcal{D})$, the liquidity position of banks is large, so that the term $[\overline{\nu} - (B_0 + \mathcal{D})]$ is small, while the term $[B_0 + \mathcal{D} - \underline{\nu}]$ is large. In other terms, the occurrence of an excess liquidity regime, as well as the occurrence of a liquidity shortage one, will depend upon the position of $(B_0 + \mathcal{D})$ within the $[\overline{\nu}, \underline{\nu}]$ interval. This is quite intuitive, as we have assumed a uniform distribution for ν, $(B_0 + \mathcal{D})$ represents the banks aggregate liquidity supply and the aggregate (feasible) liquidity demand is driven by the extent of the liquidity shocks that are distributed in the $[\overline{\nu}, \underline{\nu}]$. Of course, the amount of collateral determining $\widehat{L}$ will also affect the aggregate economic regime, as it determines the maximum amount of a feasible interbank loan.

Proposition 3 *(Equilibrium in the Interbank Market) Under asymmetric information and assumptions A2 to A5, there exists a unique equilibrium in the interbank market, characterized*

by $r_L = \rho_L$. In the excess liquidity regime, we obtain $r_L = r$, while in the liquidity shortage regime the interbank market rate is given by:

$$r_L = \frac{K - D_1(r)}{(\overline{\nu} - E[B_0] - \mathcal{D}) - \sqrt{(\overline{\nu} - E[B_0] - \mathcal{D})^2 - \left[\sigma_{B_0}^2 + (E[B_0] + \mathcal{D} - \underline{\nu})^2\right]}} - 1.$$

Proof. See the Appendix. ■

Market clearing pins down the interbank rate, which represents the opportunity cost of funds for interbank lenders. As intuition suggests, the interbank rate equals r as long as the liquidity available in the interbank market is large enough. Otherwise liquidity is scarce and there is a spread between the interbank rate and the T-Bills rate that is purely liquidity driven, as interbank loans are fully collateralized. The arbitrage between the interbank market and the T-Bill market does not operate because T-Bills are in the hands of consumers and not in those of the banks. The lack of liquidity that creates the wedge between the T-Bills rate and the interbank rate has real implications on firms' access to credit, as the fringe of firms with a cost overrun ν in the interval $\left(\widehat{F}(r_L), \widehat{F}(r)\right)$ will be liquidated.

Notice that, as intuition suggests, the spread increases when the demand for liquidity increases, that is, for instance, when the value of collateral $K - D_1(r)$ increases. More interesting is the observation that a higher dispersion of T-Bills, σ_{B_0} implies a lower interbank market rate. This happens because a larger dispersion implies the existence of both more agents with excess liquidity and more rationed agents, generating an aggregate liquidity supply.

It is useful to note that, in the liquidity excess case, $\widehat{F}$ is independent of the interbank market rate which is equal to the T-Bills rate so that, for any r, we have a corresponding $\widehat{F}$. In the liquidity shortage case, the values for $\widehat{F}$ and r_L are jointly determined in equilibrium.

So far we have taken the decisions made by the bank at the initial date as given and we

have considered that the value of the liquidity holdings B_0 is exogenous. Would the results change significantly if we considered the possibility that banks could, at date zero, issue debt and choose optimally their liquidity holdings? The answer is no. Note that the information imperfections are present from the beginning. Therefore, at the initial date, lenders would anticipate the strategic behaviour of banks and would set a maximum size for the amount of debt that banks could issue (presumably equal to the present value of $K - D_0$). There is however an interesting difference with respect to our model. If banks anticipate that there will be a liquidity shortage in the interbank market at date one, then they would rather issue debt at date zero and invest the extra liquidity in T-Bills. In this way, banks would avoid paying high rates in the interbank market by bringing liquidity from outside the banking system, thereby minimizing the liquidity shortage problem. This may be the reason why we seldom observe significant spreads between the interbank and the policy rates.

6 Implications Regarding the Main Puzzles

The object of this section is to analyze to what extent our results fit in with the existing empirical findings.

6.1 Three Puzzles

In order to evaluate the effects of monetary policy we assess the effects of a shift in the policy rate r. Recall that $r_F = r_L = \rho_L$.

Proposition 4 *(Magnitude Puzzle) Under asymmetric information and assumptions A2 to A5, the aggregate effect of an interest rate shock is larger than the aggregate of individual effects of an increase in the user cost of capital.*

Proof. See the Appendix. ■

The magnitude effect is positive because $\varepsilon_r(r) > 0$ while $\varepsilon_{uc}(r_F) = 0$. In our framework, the magnitude effect hinges on the fact that the banking system (and not firms) determines endogenously the marginal projects that are undertaken in the economy. Firms take the threshold $\widehat{F}$ as exogenous and, because of asymmetric information, projects with cost overruns above $\widehat{F}$ are liquidated. Consequently, for these projects, the opportunity cost of funds is not $r_F = r_L$, but the shadow price of the credit constraint.

Recall that the defaulting threshold, for a rationed entity holding an amount B_0 in securities, equals $\widehat{F} = B_0 + [D_1(r) - D_0] + [K - D_1(r)] / (1 + r_L)$. This threshold reflects the availability of funds to the banking system and is influenced by monetary policy through two channels: (i) the *balance sheet channel* because monetary policy affects the present value of collateral $K - D_1(r)$; (ii) the deposit base of the banking system $D_1(r)$. Hence, when monetary policy is tightened, the value for $\widehat{F}$ declines, through the combination of these two effects, making the credit rationing constraint more severe with a higher number of projects being liquidated.

When there is a liquidity shortage, we must add, on top of the *rationing channel*, a *spread - augmented interest rate channel* similar to the one described by Stein (1998). Depending on the effect of policy rates on the interbank rate spread, that is depending on whether dr_L/dr is larger or inferior to one, the *spread - augmented interest rate channel* can amplify or mitigate the *rationing channel*.

Notice that we proceed to compare *within a given set-up* the semi-elasticity of the aggregate output with respect to the policy rate with the semi-elasticity with respect to the user cost of capital. The result is that the *magnitude puzzle* occurs as a consequence of asymmetric information and the resulting rationing in the credit market. As established in Section 3, no similar result holds in a perfect information set-up. A completely different exercise would be to

compare the elasticities across different set-ups. This would show how asymmetric information increases the semi-elasticities of output with respect to interest rates, but this would not be related in any way to the *magnitude puzzle*[12].

To conclude this section, it is worth exploring the connection between our theoretical model and the empirical results obtained by Kashyap and Stein (2000). They document that the impact of monetary policy on the lending behavior of banks is more pronounced for banks with less liquid balance sheets, where liquidity is measured by the absolute amount of liquid securities that the bank holds. Because of the construction of our model, it is not surprising that we obtain a result with the same flavour.

Proposition 5 *(Kashyap and Stein Liquidity Puzzle) Under asymmetric information and assumptions A2 to A5, the impact of shifts in the interest rate r on the supply of credit to firms is larger for banks with a smaller amount of T-Bills* (B_0).

Proof. See the Appendix. ■

Because our model is based on the existence of an imperfection in the interbank market that prevents perfect circulation of reserves from one bank to another, the result is not surprising. The previous proposition asserts the consistency of our framework to cope the issues captured by Kashyap and Stein (2000). Kashyap and Stein (2000) argue that their result is entirely driven by the smaller banks, which are those that are more affected by asymmetric information problems. We do not explore the different access to liquidity of large and small banks, because we assume that all banks have the same size. Note, however, that asymmetric information is the main factor responsible for the results in proposition 5.

[12] The effect would rather be connected with the financial accelerator obtained in related models (see Bernanke and Gertler (1990)).

Our approach based on the existence of an imperfection in the interbank market seems to be consistent with other empirical results. Indeed, if our framework is the correct one, banks having access to sources of reserves other than the interbank market, such as an internal capital market, should not react as much to shifts in interest rates. This is confirmed by the Ashcraft (2001)'s empirical analysis, who reports that loan growth of banks affiliated with multi-bank holding companies is much less sensitive to changes in the federal funds rate.

Our model allows for a discussion of the *composition puzzle*. As it is, our model does not distinguish between different maturities in the portfolio of loans, as there is a unique representative project facing a unique representative bank. Still, the extension to a well diversified portfolio of loans is straightforward. The effect of credit rationing will then be less dramatic as it will not lead to the liquidation of the firm's project. Instead, the bank will have to determine which loan applications are to be turned down. This will be done by comparing the overall profitability of granting *versus* denying the loan. As a consequence, it will depend upon relationship with the firm, cost to the bank of refusing a revolving loan compared to the cost of denying credit to a starting project, and other characteristics. Yet, a bank without sufficient liquidity to finance the cost overruns of its clients, will have to ration their clients regardless of the maturity of their investments, thus justifying the so called *composition puzzle*.[13]

[13] Angeloni *et al.* (2003) compare the European Area and the United States response patterns to a shift in monetary policy and find that they differ noticeably as to the composition of output changes. They conclude that in Europe investment is the predominant driving force of output changes, while in the United States the consumption contribution to output changes is larger. They label this difference as the *output composition puzzle*. Our model shows that the rationing channel amplifies the response of business output and investment to policy shocks (when compared with the perfect markets case). Arguably, business investment is more bank dependent in the European Area, and this explains why investment has such a preeminent role in Europe.

6.2 Financial Structure and Monetary Policy Transmission

Although our model is based on a number of restrictive assumptions, the main argument is quite intuitive and it is expected to carry out in more general frameworks. The *rationing channel* will affect a larger number of firms, the larger the degree of asymmetric information, the higher the interbank credit risk and the stronger the level of relationship banking. To verify the possible empirical predictions, we associate these characteristics of our model to variables that can be observed. We claim that asymmetric information is related to a lower level of development for financial markets and the importance of small banks. Also, the existence of credit risk in the interbank market can be measured by an index of bank health and the strong relationships that makes it too costly for firms in our model to switch from one bank to another implies, by assumption, a strong dependence on bank loans, which is related with the availability of alternative forms of finance.

Although testing our predictions is outside the scope of our contribution, it is interesting to relate our results with the ones obtained by Kashyap and Stein (1997), Cecchetti (1999) and Mihov (2001) that make similar points. Cecchetti (1999) builds indices on three key credit-channel factors, and uses these indices to build a summary statistic for the "predicted effectiveness of monetary policy". The definition of the summary statistic implies that larger values should be associated with more potent monetary policy, if the lending channel is important. Following Mihov (2001), we relate these results with the cumulative deviation of output from trend after a monetary policy shock. First, to illustrate the possible role that financial imperfection play in monetary policy, Figure 1 reports the relationship of the summary statistic proposed and discussed in Cecchetti (1999) and the magnitude of monetary policy responses. Although based on a limited number of observations, the diagram indicates a posi-

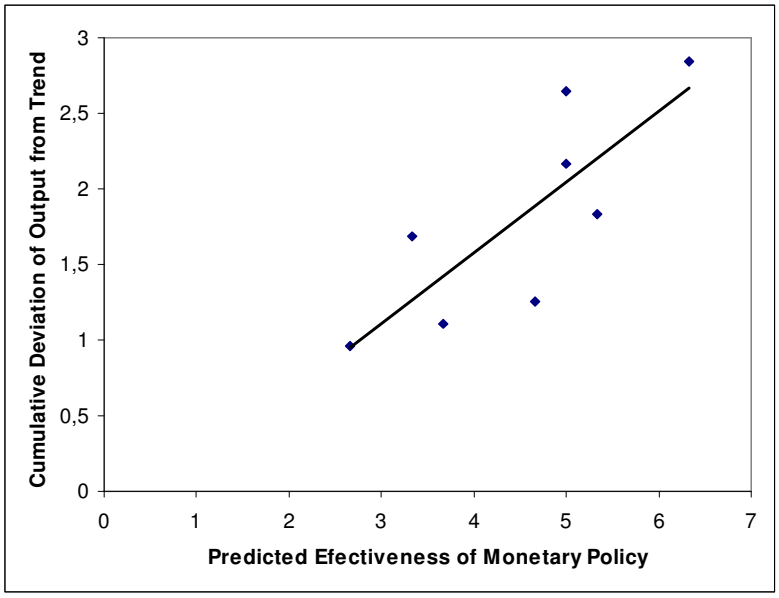

Figure 1: Cumulative Deviation from Trend and Predicted Efectiveness of Monetary Policy. Notes: "Predicted Effectiveness of Monetary policy" is a summary statistic proposed and discussed in Cecchetti (1999). See the details in the next Table. The "Cumulative Impulse Responses of Output to Interest Rate Shocks" is the cumulative response of impulse response of output to a 100-basis-point increase in the nominal interest rate from a vector autoregression containing output, price level and the short-term interest rate reported in Mihov (2001). Data for the United Kingdom, Netherlands, France, United States, Italy, Austria, Germany and Japan.

tive correlation. Second, Table 1 reports the Spearman's rank-order correlation coefficient for the relationship between the effects of monetary policy computed by Mihov (2001) and the indices presented by Cecchetti (1999), including the summary statistic "predicted effectiveness of monetary policy". So, combining their results, we obtain some preliminary empirical results that, we claim, provide support to the possibility of a *rationing channel*. Table 1 shows that, although the variables "Bank Health" and "Importance of Small Banks" are not relevant when taken in isolation, taken jointly with the variable "Availability of Alternative Finance", as we assume in our model and as reflected in the Cecchetti summary statistic, does provide evidence in line with our conclusions. This is promising for further research.

	Importance of Small Banks	Bank Health	Availability of Alternative Finance	Cecchetti Index of Predicted Effectiveness of Monetary Policy
Spearman's rank--order correlation Coefficient	0.17 (0.35)	0.4 (0.19)	0.69 (0.03)	0.83 (0.01)

Table 1: Measures of association between cumulative impulse responses of output to interest rate shocks and factors affecting the strength of the monetary transmission mechanism. Notes: The numbers in the brackets are exact or approximate p-values for the null hypothesis that there is no correlation between cumulative impulse responses and the variables in each column. "Importance of Small Banks" is based on "Banks per Million People", table 2 of Cecchetti (1999). "Bank Health" is calculated using the "Average Thomson Rating", table 3 of Cecchetti (1999). "Availability of Alternative Finance" is based on "Bank Loans as a Percentage of all Forms of Finance", table 4 of Cecchetti (1999). "Predicted Effectiveness of Monetary policy" is an average of the ranks of "Importance of Small Banks", "Bank Health" and "Availability of Alternative Finance". The "Cumulative Impulse Responses of Output to Interest Rate Shocks" is the cumulative response of impulse response of output to a 100-basis-point increase in the nominal interest rate from a vector autoregression containing output, price level and the short-term interest rate reported in Mihov (2001). Data for the United Kingdom, Netherlands, France, United States, Italy, Austria, Germany and Japan.

7 Conclusion

This paper primary aim is to draw attention to the key role the interbank market plays in the transmission of monetary policy. In our model, this role depends upon the existence of heterogeneous liquidity shocks for banks facing asymmetric information. In this way we explain how financial imperfections may account for some puzzles regarding the transmission mechanism of monetary policy. First, asymmetric information in the interbank market can generate rationing and helps justifying the *liquidity puzzle* presented by Kashyap and Stein (2000). Second, financial imperfections justify the existence of a *magnitude puzzle* because rationing creates a wedge between the shadow price of funds and the interest rate in the economy. Because banks themselves are rationed, there is no interest rate the borrowing firms can offer to entice banks into increasing their credit supply. Consequently, these firms that are bound to be liquidated do not appear as part of the demand for funds. Third, considerable advance has been made in explaining the *composition puzzle*.

Although we have focused on the *rationing channel*, our model can easily accommodate

other channels. For example, when there is liquidity shortage in the interbank market, equilibrium is characterized by a positive spread between the interbank market and the T-Bills rate, and monetary policy has the ability to influence the size of the spread. In this case, there is the possibility of obtaining a *spread-augmented interest rate channel* very similar to Stein (1998). Finally, our model has relevant empirical implications regarding future research on the determinants of business investment and bank loan behavior.

A Appendix

A.1 Proof of Lemma 1

Let

$$\Lambda \equiv \Lambda(r, B_0) \equiv B_0 + \mathcal{D}$$

The profit function of an entity that integrates the firm and the bank is obtained by aggregating the profits of the firm and bank and considering the restrictions of both entities. The problem of the Integrated Entity is

$$\begin{aligned} \max_{\{L,B_1\}} Y + (1+r) B_1 - (1+\gamma) L - D_1(r) \\ s.t.\ B_1 + \nu = \Lambda + L \\ L \leq \overline{L} \end{aligned} \tag{7}$$

We compare the solution to problem (7) with the solutions obtained with independent entities. There are two cases that we must consider.

In the first case, $\nu \leq \min\left\{\Lambda + \overline{L}, \nu^*(r_L) - \frac{R_0}{1+r_L}\right\}$ and the firm borrows ν and does not default. The profit of the firm equals $\Pi_F = Y - R_0 - (1+r_L)\nu$ and the bank solves problem

(2) with $F = \nu$.

- If $\gamma > r$ then $B_1 = 0$ and, if the bank continues its activity, its profit equals $\Pi_B = R_0 + (1+\gamma)\Lambda - D_1(r)$ and it satisfies the loan demand by the firm and sets $L = \nu - \Lambda \leq \overline{L}$. If the bank strategically defaults, it obtains $(1+\rho_L)\Lambda - D_1(r)$. Comparing both alternatives implies that the bank prefers to continue its activity because $R_0 > 0$.

- If $\gamma = r$ then $B_1 \geq 0$. Banks do not borrow interbank funds to invest in treasuries. If the bank continues its activity, its profit equals $\Pi_B = R_0 + (1+\gamma)\Lambda - D_1(r)$ and it satisfies the loan demand by the firm. The bank sets $L = \nu - \Lambda$ when $\nu \geq \Lambda$ or $L = 0$ and $B_1 = \Lambda - \nu$ if $\Lambda > \nu$. If the bank strategically defaults, it obtains $(1+\rho_L)\Lambda - D_1(r)$. Comparing both alternatives implies that the bank prefers to continue its activity because $R_0 > 0$.

We compare these results with the ones obtained by solving the problem from the Integrated Entity. If we solve problem (7), and take into account that the restriction $L \leq \overline{L}$ does not bind, the profit from the Integrated Entity equals $\Pi_F + \Pi_B$ which is larger than the payoff derived from strategic default. Hence the Integrated Entity continues the project. It uses the same decision rules as individual banks regarding the amounts of interbank loans L and treasuries B_1.

In the second case $\nu > \min\left\{\Lambda + \overline{L}, \nu^*(r_L) - \frac{R_0}{1+r_L}\right\}$. The firm defaults and the bank appropriates the project. In this case the profit of the bank is given by the solution to problem (3) which is the profit of an Integrated Entity. ■

A.2 Proof of Lemma 2

The profit from continuation is $\pi_{cont} = Y - (1-\gamma)\{\nu - \Lambda\} - D_1(r)$, while the profit from liquidation is $\pi_{liq} = (1+\rho_L)\Lambda - D_1(r)$. The bank's threshold will be the one for which $\Delta\pi = \pi_{cont} - \pi_{liq}$ equals zero.

When $\nu \leq \Lambda$, then $\gamma = \rho_L$ and $\Delta\pi = 0$ implies $Y = (1-\rho_L)\nu$, so that the defaulting threshold equals $\nu^*(\rho_L)$.

When $\nu > \Lambda$, then the Integrated Entity is a borrower and the defaulting threshold equals $\nu^*(r_L) + \frac{r_L - \rho_L}{1+r_L}\Lambda$. ■

A.3 Proof of Proposition 2

We use the concept of *Subgame Perfect Nash Equilibrium* ($SPNE$). We begin by the last stage of the game.

Remark 1 *Defaulting borrowers pool on the contract that grants the largest loan.*

Remark 2 *In every $SPNE$ lenders must earn exactly zero expected profits on every equilibrium contract.*

The former remark is obvious. Regarding the latter, note that the situation is akin to the standard Bertrand competition situation so that, if a contract makes profit, there is an incentive for competitors to undercut the interest rates and obtain a larger share of the market. Let

$$\kappa \equiv K - D_1(r).$$

Lemma 4 *In any $SPNE$ lenders must set $r_L = \rho_L$ and $L \leq \widehat{L}$ on every equilibrium contract*

Proof. Suppose that there is an $SPNE$ for which a measure of μ borrowers apply for a loan L at a rate $r_L > \rho_L$. Given remark 2 the expected profits made on this contract are null and two cases are possible:

- In the first case, $L > \frac{\kappa}{1+\rho_L}$ and there is a counterpart risk premium and some non defaulters signing the contract pay a premium because of strategic defaulters buying the contract. Then one of the lenders can offer a contract with a loan $L - \varepsilon$ and interest rate $r_L - \varepsilon'$, with $0 < \varepsilon < L - \frac{\kappa}{1+\rho_L}$ and $0 < \varepsilon' < r_L - \rho_L$, that attracts all (nondefaulting) borrowers with $\frac{\kappa}{1+\rho_L} < \nu < L - \varepsilon$. The profits made on this contract are positive since $\rho_L < r_L - \varepsilon'$. Hence $\rho_L \geq r_L$, but since $\rho_L > r_L$ is not possible we must have $r_L = \rho_L$.

- In the second case, $L \leq \frac{\kappa}{1+\rho_L}$ and there is no risk and remark 2 implies $r_L = \rho_L$.

We now prove that the maximum loan available in the interbank market equals $\widehat{L}$. To see this, suppose that the maximum loan was $\widetilde{L}$, with $\widetilde{L} > \widehat{L}$. Then it must be the case that all strategic defaulters choose $\widetilde{L}$. Given that $r_L = \rho_L$ then there is a lender making losses in equilibrium and this contradicts remark 2: we must therefore have a maximum loan $\widetilde{L}$ such that $\widetilde{L} \leq \widehat{L}$. But if $\widetilde{L} < \widehat{L}$, then one of the lending banks can earn positive profits by offering a contract with a loan $L \in \left(\widetilde{L}, \widehat{L}\right]$ with $r_L > \rho_L$ which contradicts remark 2. Hence the maximum loan available in the interbank market equals $\widehat{L}$. ■

Lemma 5 *Nondefaulting borrowers with $\nu - \Lambda \leq \widehat{L}$ receive a loan equal to $\nu - \Lambda$.*

Proof. Obviously if a nondefaulting borrower was rationed, a deviating bank could offer her a loan and make positive profits. The possibility of a borrower asking for a loan larger than $\nu - \Lambda$ is ruled out by the assumption that agents do not borrow from the interbank market to invest in T-Bills when $r \leq \rho_L$. ■

Lemma 6 *Defaulting borrowers sign a contract with a loan equal to* $\widehat{L}$ *at an interest rate* $r_L = \rho_L$.

Proof. This result is implied by Lemma 4 and the assumption that $\vartheta > 0$. ■

So far we have proved that the *RCCM* characterizes the only possible equilibrium. We now prove that equilibrium is characterized by rationing. For this we use assumption A3 to show that the defaulting threshold is determined by credit rationing.

Lemma 7 *Under assumptions A2 to A3 the "strategic defaulting threshold" for an Integrated Entity with available liquidity* $\Lambda = B_0 + \mathcal{D}$ *equals* $\frac{Y-\vartheta\widehat{L}-D_1(r)}{1+r_L} + \Lambda$, *which satisfies* $\widehat{F} < \frac{Y-\vartheta\widehat{L}-D_1(r)}{1+r_L} + \Lambda$.

Proof. The profit function of a borrowing Integrated Entity is equal to $\pi\left(\nu, \widehat{L}, r_L, r_L, r\right)$. In order to find the strategic defaulting threshold for the borrower we compute the value of ν such that $\pi\left(\nu, \widehat{L}, r_L, r_L, r\right) = \vartheta\widehat{L}$, which equals $\frac{Y-\vartheta\widehat{L}-D_1(r)}{1+r_L} + \Lambda$. This is enough to obtain a necessary and sufficient condition because the function $\pi\left(\nu, \widehat{L}, r_L, r_L, r\right) - \vartheta\widehat{L}$ is decreasing in ν.

Recalling that $\frac{\kappa}{1+r_L} = \widehat{L}$, because $\rho_L \geq r$ and $\vartheta > 0$, assumption A3 implies that $\kappa + \vartheta\widehat{L} + D_1(r) < Y$ which is equivalent to $\widehat{L} < \frac{Y-\vartheta\widehat{L}-D_1(r)}{1+r_L}$. This implies that $\widehat{F} < \frac{Y-\vartheta\widehat{L}-D_1(r)}{1+r_L} + \Lambda$ and, therefore, the defaulting threshold is equal to $\widehat{F}$. ■

Lemma 7 states that under assumptions A2 to A4 the interbank market rationing constraint becomes binding before the strategic default one does. Assumption A4 guarantees that the minimum value for $\widehat{F}$ is inferior to $\overline{\nu}$ and that there is rationing. The final step in the proof is to show that the *RCCM* is indeed an equilibrium. Let $\delta = \left(\overline{B} - \underline{B}\right)(\overline{\nu} - \underline{\nu})$.

Lemma 8 *Under assumptions A2 to A5 the RCCM defines the unique equilibrium set of contracts that exists in the interbank market*

Proof. If an equilibrium exists, the above lemmas show that it has to share every characteristic of the $RCCM$. So, in order to prove existence of the equilibrium we only have to establish that no deviation from $RCCM$ is profitable.

The proof is obvious when we consider deviations where the amount of the loan granted is lower than $\widehat{L}$. The proof for loans larger than $\widehat{L}$ is more complex and requires the use of assumption A5. We will show that a deviating bank cannot make positive profits by establishing that those profits have a negative upper bound that we denote by A.

A deviating bank offering a loan of size $\widetilde{L}$, with $\widetilde{L} > \widehat{L}$, will attract a fraction of rationed borrowers and all of the defaulting ones. Denote by $\widetilde{F} = \Lambda + \widetilde{L}$ the new bound on accessible bank liquidity when the new maximum interbank loan is $\widetilde{L} > \widehat{L}$.

The profit, denoted by Π, from offering a contract $\widetilde{L}$ consists of the profit made on the nondefaulting borrowers minus the losses made on the defaulting ones. This has an upper bound obtained by assuming that there are no defaulting borrowers with a cost overrun ν in the interval $\left(\widehat{F}, \widetilde{F}\right]$. Given A4 we have $\widehat{F} < \overline{\nu}$ for all banks and

$$\Pi \leq \frac{1}{\delta} \int_{\underline{B}}^{\overline{B}} \left\{ \int_{\widehat{F}}^{\widetilde{F}} \left[\widetilde{r}_L(\nu - \Lambda) - r_L\right][\nu - \Lambda]\, d\nu - \int_{\widetilde{F}}^{\overline{\nu}} \left[(1 + r_L)\widetilde{L} - \kappa\right] d\nu \right\} dB_0$$

where $\widetilde{r}_L(.)$ is a function of the size of the interbank loan.

Expression $[\widetilde{r}_L(\nu - \Lambda) - r_L][\nu - \Lambda]$ is bounded above by the maximum profit the lender can obtain, that is, the one that leaves the borrower at its reservation level, $\vartheta\widehat{L}$. This means that for each loan $\nu - \Lambda$, the profit is lower than $\pi\left(\nu, \widetilde{L}, r_L, r_L, r\right) - \vartheta\widehat{L}$. Let $S = \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\widehat{F}}^{\widetilde{F}} \pi\left(\nu, \widetilde{L}, r_L, r_L, r\right) d\nu dB_0$ denote the total surplus from nondefaulting borrowers. Then the maximum gain for a lender, Π, is (weakly) inferior to $A = S - \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\widetilde{F}}^{\overline{\nu}} \left[(1 + r_L)\widetilde{L} - \kappa\right] d\nu dB_0 - \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\widehat{F}}^{\widetilde{F}} \vartheta\widehat{L} d\nu dB_0$.

The final step of the proof is to show that $A \leq 0$. To see this note that:

$$\delta A = \int_{\underline{B}}^{\overline{B}} \int_{\widehat{F}}^{\widetilde{F}} Y - (1 + r_L)(\nu - \Lambda) - D_1(r)\, d\nu -$$
$$- \int_{\widetilde{F}}^{\overline{\nu}} \left[(1 + r_L)\widetilde{L} - \kappa\right] d\nu - \int_{\widehat{F}}^{\widetilde{F}} \vartheta \widehat{L} d\nu dB_0$$

Replacing $\kappa = (1 + r_L)\widehat{L}$, $\widetilde{F} = B_0 + \mathcal{D} + \widetilde{L}$, recalling that $\int_{\underline{B}}^{\overline{B}} B_0 dB_0 = E[B_0]\left(\overline{B} - \underline{B}\right)$ and rearranging yields:

$$A = \left\{ Y - (1 + r_L)\frac{\widetilde{L} + \widehat{L}}{2} - D_1(r) - (1 + r_L)\overline{\nu} + \right. \tag{8}$$
$$\left. (1 + r_L)\left(E[B_0] + \mathcal{D} + \widetilde{L}\right) - \vartheta \widehat{L} \right\} \frac{\left(\widetilde{L} - \widehat{L}\right)\left(\overline{B} - \underline{B}\right)}{\overline{\nu} - \underline{\nu}}$$

We will use assumption 5 to establish that $A < 0$. Since $\frac{\left(\widetilde{L} - \widehat{L}\right)\left(\overline{B} - \underline{B}\right)}{\overline{\nu} - \underline{\nu}} > 0$ it is only necessary to show that the expression inside the brackets is negative, that is:

$$Y - D_1(r) - \vartheta \widehat{L} + (1 + r_L)\frac{\widetilde{L} - \widehat{L}}{2} \leq (1 + r_L)\left(\overline{\nu} - E[B_0] - \mathcal{D}\right) \tag{9}$$

In order to do so, recall, first, assumption A5. Assumption A3 guarantees that the expression $\frac{3}{2}\left[\frac{Y - D_1(r) - \vartheta \frac{\kappa}{1+r_L} - \kappa}{1 + r_L}\right] + E\left[B_0 + \mathcal{D} + \frac{\kappa}{1+r_L}\right]$ is decreasing in r_L: It is easy to show that the derivative of this expression with respect to r_L equals $-\left[Y - D_1(r) - \vartheta \widehat{L} - \kappa\right] - \frac{\kappa}{(1+r_L)^2}\left(\frac{2}{3} - \vartheta\right) < 0$. Hence assumption A5 implies that

$$\frac{3}{2}\left[\frac{Y - D_1(r) - \vartheta \widehat{L}}{1 + r_L} - \widehat{L}\right] \leq \overline{\nu} - E\left[B_0 + \mathcal{D} + \frac{\kappa}{1 + r_L}\right].$$

Using this expression, the definition for $\widehat{L}$ and $\vartheta \widetilde{L} > \vartheta \widehat{L}$, we obtain

$$Y - D_1(r) - \vartheta\widehat{L} + \frac{Y - \vartheta\widetilde{L} - D_1(r) - \kappa}{2} \leq (1 + r_L)(\overline{\nu} - (E[B_0] + \mathcal{D})) \tag{10}$$

Second, note that Lemma 7 establishes that banks with $\nu > \frac{Y-\vartheta\widetilde{L}-D_1(r)}{1+r_L} + \Lambda$ default and, therefore, we know that lenders would only propose a value for $\widetilde{L}$ such that $\widetilde{L} \leq \frac{Y-\vartheta\widetilde{L}-D_1(r)}{1+r_L}$. This result, together with (10) implies (9). Hence we have proved that $A < 0$ and that the profit from a deviating strategy is negative. ■

■

A.4 Proof of Lemma 3

The sum of the individual excess demands for interbank loans equals

$$Z = \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\overline{\nu}} z d\nu dB_0 = \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\left\{\int_{\underline{\nu}}^{\Lambda}\nu + B_1(B_0,\nu) - \Lambda d\nu + \int_{\Lambda}^{\widehat{F}}\nu - \Lambda d\nu + \int_{\widehat{F}}^{\overline{\nu}}\widehat{L}d\nu\right\}dB_0$$
$$= \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\Lambda} B_1(B_0,\nu)\,d\nu dB_0 + \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\left\{-\Lambda\left[\widehat{F} - \underline{\nu}\right] + \int_{\underline{\nu}}^{\widehat{F}}\nu d\nu + \widehat{L}\left[\overline{\nu} - \widehat{F}\right]\right\}dB_0$$
$$= \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\Lambda} B_1(B_0,\nu)\,d\nu dB_0 + \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\left\{-\Lambda\left[\widehat{F} - \underline{\nu}\right] + \frac{(\widehat{F}+\underline{\nu})(\widehat{F}-\underline{\nu})}{2} + \widehat{L}\left[\overline{\nu} - \widehat{F}\right]\right\}dB_0$$
$$= \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\Lambda} B_1(B_0,\nu)\,d\nu dB_0 + \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\left\{\frac{1}{2}\left(\widehat{L} + \underline{\nu} - \Lambda\right)\left(\widehat{L} - \underline{\nu} + \Lambda\right) + \widehat{L}\left(\overline{\nu} - \Lambda - \widehat{L}\right)\right\}dB_0$$
$$= \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\Lambda} B_1(B_0,\nu)\,d\nu dB_0 + \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\left\{\frac{1}{2}\left[\widehat{L}^2 - (\Lambda - \underline{\nu})^2\right] - \widehat{L}^2 + \widehat{L}(\overline{\nu} - \Lambda)\right\}dB_0$$ which yields the result.

From the expression $\Theta(r_L) = -\frac{1}{2}\int_{\underline{B}}^{\overline{B}}\left\{\widehat{L}^2 - 2[\overline{\nu} - \Lambda]\widehat{L} + [\Lambda - \underline{\nu}]^2\right\}dB_0$,we derive:

$$\frac{d\Theta}{dr_L} = \left\{-\widehat{L}\int_{\underline{B}}^{\overline{B}} dB_0 + \int_{\underline{B}}^{\overline{B}}[\overline{\nu} - \Lambda]\,dB_0\right\}\frac{d\widehat{L}}{dr_L}$$

After substituting $\Lambda = B_0 + \mathcal{D}$, and integrating, it is possible to rewrite the above expression as follows:

$$\frac{d\Theta}{dr_L} = \left(\overline{B} - \underline{B}\right)\left\{\overline{\nu} - \left[E[B_0] + \mathcal{D} + \widehat{L}\right]\right\}\frac{d\widehat{L}}{dr_L}$$

since $\overline{\nu} > E[B_0] + \mathcal{D} + \widehat{L}$ (due to assumption A4) and $\widehat{L} = \kappa/(1 + r_L)$, we obtain $\frac{d\Theta}{dr_L} < 0$. ■

A.5 Proof of Proposition 3

Notice, first that no equilibrium is possible with $r_L < r$, as this would generate a zero supply and a positive demand in the interbank market.

Consider, next, the excess liquidity case, characterized by $\Theta(r) \leq 0$. In this case an equilibrium exists with $r_L = r$ and we have $B_1(B_0, \nu) \in [0, \infty)$. The equilibrium is unique, since $r_L > r$ would imply $B_1(B_0, \nu) = 0$ for all banks leading to an excess supply in the interbank market.

To prove existence and uniqueness in the liquidity shortage regime, characterized by $\Theta(r) > 0$, is more complex. Because $\widehat{L}$ tends to zero when r_L tends to infinity,

$$\Theta(r_L) = -\frac{1}{2}\int_{\underline{B}}^{\overline{B}} \left\{\widehat{L}^2 - 2[\overline{\nu} - \Lambda]\widehat{L} + [\Lambda - \underline{\nu}]^2\right\} dB_0$$

tends towards $-\frac{1}{2}\int_{\underline{B}}^{\overline{B}} [\Lambda - \underline{\nu}]^2 dB_0$. Thus, there exists a value of $\widetilde{r}_L$ for which $\Theta(\widetilde{r}_L) < 0$, and by continuity, this implies the existence of an interbank interest rate r_L, $r_L > r$, such that $\Theta(r_L) = 0$ The fact that $\Theta(.)$ is decreasing implies that the equilibrium is unique.

We now compute the equilibrium interest rate in the liquidity shortage regime. After substituting Λ by $B_0 + \mathcal{D}$, Θ can be rewritten as

$$\Theta(r_L) = \int_{\underline{B}}^{\overline{B}} -\frac{B_0^2}{2} - B_0\left\{\widehat{L} + \mathcal{D} - \underline{\nu}\right\} - \frac{\widehat{L}^2}{2} + \{\overline{\nu} - \mathcal{D}\}\widehat{L} - \frac{\{\mathcal{D} - \underline{\nu}\}^2}{2} dB_0$$

or, equivalently by introducing $H = \widehat{L} + \{\mathcal{D} - \underline{\nu}\}$ and $C = -\frac{\widehat{L}^2}{2} + (\overline{\nu} - \mathcal{D})\,\widehat{L} - \frac{(\mathcal{D}-\underline{\nu})^2}{2}$:

$$\begin{aligned}\Theta(r_L) &= \int_{\underline{B}}^{\overline{B}} -\frac{B_0^2}{2} - B_0 H + C dB_0 \\ &= \left\{-\frac{1}{6}\left(\overline{B}^2 + \overline{B}\underline{B} + \underline{B}^2\right) - H\frac{\overline{B}+\underline{B}}{2} + C\right\}\left(\overline{B} - \underline{B}\right)\end{aligned}$$

Now, using the expressions for the first and second moments of the uniform distribution, $\mu_{B_0} = \frac{\overline{B}+\underline{B}}{2}$ and $\sigma_{B_0}^2 = \frac{\overline{B}^2+\overline{B}\underline{B}+\underline{B}^2}{3} - \mu_{B_0}^2$, implies $\frac{1}{6}\left(\overline{B}^2 + \overline{B}\underline{B} + \underline{B}^2\right) = \frac{\sigma_{B_0}^2+\mu_{B_0}^2}{2}$. This allows a simpler expression for $\Theta(r_L)$

$$\Theta(r_L) = \left\{-\frac{\sigma_{B_0}^2 + E[B_0]^2}{2} - HE[B_0] + C\right\}\left(\overline{B} - \underline{B}\right)$$

Hence, $\Theta(r_L) = 0$ is equivalent to

$$\sigma_{B_0}^2 = -2HE[B_0] + 2C - E[B_0]^2.$$

Expression $2C = -\widehat{L}^2 + 2\{\overline{\nu} - \mathcal{D}\}\,\widehat{L} - \{\mathcal{D} - \underline{\nu}\}^2$ can be rewritten after adding and subtracting $\underline{\nu}$ as

$$\begin{aligned}&2C = -\widehat{L}^2 + 2\{\overline{\nu} + [\underline{\nu} - \underline{\nu}] - \mathcal{D}\}\,\widehat{L} - \{\mathcal{D} - \underline{\nu}\}^2 \\ &= -\widehat{L}^2 + 2\{\underline{\nu} - \mathcal{D}\}\,\widehat{L} - \{\mathcal{D} - \underline{\nu}\}^2 + 2(\overline{\nu} - \underline{\nu})\,\widehat{L} = \\ &= -\left\{\widehat{L} + \mathcal{D} - \underline{\nu}\right\}^2 + 2(\overline{\nu} - \underline{\nu})\,\widehat{L} = -H^2 + 2(\overline{\nu} - \underline{\nu})\,\widehat{L}\end{aligned}$$

Consequently, equilibrium is characterized, after replacing $2C$ and H, by the following

quadratic equation in $\widehat{L}$

$$\sigma_{B_0}^2 = 2\left(\overline{\nu} - \underline{\nu}\right)\widehat{L} - \left(E\left[B_0\right] + H\right)^2 = 2\left(\overline{\nu} - \underline{\nu}\right)\widehat{L} - \left\{\widehat{L} + E\left[\Lambda\right] - \underline{\nu}\right\}^2$$
$$\Longleftrightarrow -\widehat{L}^2 + 2\widehat{L}\left(\overline{\nu} - E\left[\Lambda\right]\right) - \left[\sigma_{B_0}^2 + \left(E\left[\Lambda\right] - \underline{\nu}\right)^2\right] = 0 \tag{11}$$

Now, notice that for a liquidity shortage to occur, we require $\Theta\left(r\right) > 0$, and, using the same calculations, this is equivalent to having $-\widehat{\widehat{L}}^2 + 2\widehat{\widehat{L}}\left(\overline{\nu} - E\left[\Lambda\right]\right) - \left[\sigma_{B_0}^2 + \left(E\left[\Lambda\right] - \underline{\nu}\right)^2\right] > 0$ where $\widehat{\widehat{L}}$ is defined as $\kappa/\left(1+r\right)$. In order for this quadratic expression to reach a positive value, this implies that (11) must have two distinct roots $\widehat{L} = \left(\overline{\nu} - E\left[\Lambda\right]\right) \pm \sqrt{\left(\overline{\nu} - E\left[\Lambda\right]\right)^2 - \left[\sigma_{B_0}^2 + \left(E\left[\Lambda\right] - \underline{\nu}\right)^2\right]}$, one larger and one smaller than $\widehat{\widehat{L}}$. We can rule out the root with the larger value, because $\widehat{\widehat{L}} < \widehat{L}$ would imply $r_L < r$. Replacing the solution in $\widehat{L} = \kappa/\left(1 + r_L\right)$ yields the value of r_L.

For the sake of completeness, note that

$\left(\overline{\nu} - E\left[\Lambda\right]\right)^2 - \left[\sigma_{B_0}^2 + \left(E\left[\Lambda\right] - \underline{\nu}\right)^2\right] > 0 \Leftrightarrow$

$\Leftrightarrow \left(\overline{\nu} - \underline{\nu}\right)\left[\overline{\nu} + \underline{\nu} - 2\left(E\left[B_0\right] + \mathcal{D}\right)\right] > \sigma_{B_0}^2 \Leftrightarrow \Theta(r) > 0$

and when $\left(\overline{\nu} - E\left[\Lambda\right]\right)^2 - \left[\sigma_{B_0}^2 + \left(E\left[\Lambda\right] - \underline{\nu}\right)^2\right] \leq 0$ we are in the no liquidity shortage case.

■

A.6 Proof of Proposition 4

Because, for each level of B_0, the measure of firms that are not liquidated is $\widehat{F} - \underline{\nu}$, aggregate output equals $\mathcal{Y} = \frac{1}{\delta}\int_{\underline{B}}^{\overline{B}}\int_{\underline{\nu}}^{\widehat{F}} Y d\nu dB_0$. When we compute $\varepsilon_r\left(r\right)$, we take into consideration that $\widehat{F}$ depends on r and we obtain

$$\varepsilon_r\left(r\right) = \frac{\frac{\widehat{L}}{1+r_L}\frac{dr_L}{dr} - \frac{r_L}{1+r_L}\frac{dD_1(r)}{dr}}{E\left[B_0\right] + \widehat{L} + \mathcal{D} - \underline{\nu}}$$

Two cases are possible. First, in the excess liquidity regime, $r_L = r$ implying $\frac{dr_L}{dr} = 1$.

Second case, in the liquidity shortage regime:

$$\frac{dr_L}{dr} = \frac{\frac{dD_1(r)}{dr}}{\widehat{L}} \left[(1 + r_L) \left(1 - \frac{\overline{\nu} - \underline{\nu}}{\sqrt{(\overline{\nu} - E[\Lambda])^2 - \left[\sigma_{B_0}^2 + (E[\Lambda] - \underline{\nu})^2\right]}} \right) - 1 \right] > 0.$$

This expression is positive because $\frac{dD_1(r)}{dr} < 0$ and, since $\overline{\nu} - \underline{\nu} > \overline{\nu} - E[\Lambda]$, then $1 - \frac{\overline{\nu} - \underline{\nu}}{\sqrt{(\overline{\nu} - E[\Lambda])^2 - \left[\sigma_{B_0}^2 + (E[\Lambda] - \underline{\nu})^2\right]}} < 0$. So, in both cases $\varepsilon_r(r) > 0$.

Firms take the defaulting threshold $\widehat{F}$ as exogenous (that is, as a constant that does not depend on r_F) and, therefore, $\varepsilon_{uc}(r_F) = 0$. The magnitude effect, measured by $M \equiv \varepsilon_r(r) - \varepsilon_{uc}(r_F)$, is therefore positive. ■

A.7 Proof of Proposition 5

The supply of credit to firms offered by banks with liquid assets equal to B_0 can be measured by $\pounds = \frac{1}{\overline{\nu} - \underline{\nu}} \int_{\underline{\nu}}^{\widehat{F}} \nu d\nu = \frac{\widehat{F}^2 - \underline{\nu}^2}{2(\overline{\nu} - \underline{\nu})}$. The semi-elasticity of the amount of bank lending with respect to the interest rate equals

$$\varepsilon_{\pounds} = -\frac{d\pounds}{dr}\frac{1}{\pounds} = \left[\frac{\widehat{L}}{1 + r_L}\frac{dr_L}{dr} - \frac{r_L}{1 + r_L}\frac{dD_1(r)}{dr} \right] \frac{2\widehat{F}}{\widehat{F}^2 - \underline{\nu}^2} > 0.$$

In order to evaluate the effect of B_0 over the impact of monetary policy, we compute

$$\frac{d\varepsilon_{\pounds}}{dB_0} = -2 \left[\frac{\widehat{L}}{1 + r_L}\frac{dr_L}{dr} - \frac{r_L}{1 + r_L}\frac{dD_1(r)}{dr} \right] \frac{\widehat{F}^2 + \underline{\nu}^2}{\left(\widehat{F}^2 - \underline{\nu}^2\right)^2} < 0$$

since $\frac{dr_L}{dr} > 0$, which is what we wanted to demonstrate. ■

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