Inflation and Intermediation Frictions[1]

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Abstract

We assess the long-run impact of inflation on economic activity in a model with frictions in trade and in financial intermediation. Intermediaries transform liabilities into commercial loans for working capital. In equilibrium inflation impairs this asset transformation process, so moderate or even low inflation levels can generate inefficiencies in the allocation of labor in addition to the usual savings distortions. Three predictions emerge from the analysis: Inflation is negatively associated to financial and real activity; The marginal impact of inflation is stronger for low inflation rates; Two inflation thresholds exist, which are lower in economies with greater intermediation frictions.
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1 Introduction

Stability in the value of money, taken to mean low and steady inflation, is a priority of central banks sometimes manifested in explicit inflation ceilings.[2] What motivates this concern about inflation? A typical view is that excessive inflation, even if perfectly anticipated, hurts real activity especially by impairing the performance of financial markets. This view is supported by a number of empirical studies (see the surveys in [9, 20]) among which is the recent work in [10], which highlights three regularities: There is cross-section evidence of a negative association between long-run inflation and financial activity; The marginal impact of inflation diminishes rapidly; There is a discrete drop in activity at high inflation levels (thresholds).

Our study presents a theory of the inflation-finance relationship whose predictions are consistent with the observations above. We find that anticipated inflation, even at modest levels, can degrade the asset transformation process performed by financial intermediaries. To the extent that this restricts the supply of commercial credit, then it disrupts real activity. In addition, the theory identifies two inflation thresholds. These findings rest on the assumptions of frictions in trading as well as in intermediation, which motivate three main features of the model: Money is needed to support trade on input and output markets; Producers need short-term credit for routine operations (working capital); Intermediation is a costly activity (noninterest costs).

We adopt a model where money is essential for trade and there is no role for personal debt. Inflation is tied to a fixed and predictable money growth process. There are two production technologies. Workers can sell "home-produced" output directly to consumers, or can supply labor to a representative firm defined by a more efficient technology. The firm markets its production after workers leave and cannot commit to repay IOUs. Such delay creates an explicit need for working capital and commercial loans, because anonymous workers demand monetary compensation. This motivates the introduction of financial intermediation technologies (intermediaries) that sell bonds and provide short-term credit to the firm in exchange for claims to its net worth. We model intermediation frictions by assuming intermediaries sustain a random cost to lend. We identify the friction's severity according to first order stochastic dominance of cost distributions.

We focus on stationary competitive equilibria, and obtain three strong predictions. First, higher rates of (anticipated) inflation are associated to lower levels of working capital

[2] For example, the European Central Bank defines price stability as an annual increase in the (harmonized) CPI for the euro area of below 2%.

and financial depth, i.e., the ratio of commercial loan supply to private consumption. This occurs for two reasons. Real savings fall with inflation, even if savers move away from money and into higher-return illiquid bonds. In addition, inflation raises the interest costs of liabilities, which lowers the profitability of lending. As a consequence, a smaller number of intermediaries lend, hence a smaller portion of their liabilities is transformed into loans. So, inflation degrades financial activity by amplifying the impact of noninterest expenses and reducing the share of liabilities transformed into working capital.

Second, the loan supply monotonically falls with inflation but its marginal impact on financial activity declines as inflation rises. If inflation is sufficiently low, then working capital constraints do not bind, and inflation simply distorts savings decisions. Above a moderate inflation level, instead, the firm cannot borrow enough working capital. In this constrained situation real wages fall below the marginal value product, which distorts labor supply decisions. This intensive margin inefficiency grows as inflation rises, because intermediaries further limit their lending and (real) wages keep falling. We then reach a second inflation level beyond which the firm offers workers their reserve wage, which is simply the value of home production. At this point some labor is reallocated away from market production, which brings about an additional, extensive margin, inefficiency. The two inflation thresholds depend on frictions in intermediation as well as on trading frictions. The thresholds are smaller when average noninterest costs are higher. An interpretation is that inflation amplifies the severity of frictions in the allocation of financial resources. In this sense, the model suggests price instability is more harmful to economies where financial markets operate less smoothly.

Uncovering the links between inflation and macroeconomic activity is a classic theme in monetary economics, so our work fits into a vast literature. Our study ties especially into three recent strands of theoretical works focusing on the link between financial intermediation and macroeconomic activity. A first strand deals with the theoretical possibility of inflation thresholds; see the survey in [9]. The idea in this literature is that inflation restricts financial flows by amplifying pre-existing intermediation frictions. Examples are the banking models in [5, 11], where excessive inflation disrupts credit markets' functioning by exacerbating the impact of informational asymmetries. Inflation lowers assets' real returns and, at high levels, creates adverse selection problems leading to equilibrium credit rationing. An inflation threshold also arises in [3], due to a shift of resources into a non-productive liquid asset. Our analysis identifies inflation thresholds, also, though they are tied to intermediation costs and not to borrowers' heterogeneity and credit rationing.

A second strand of research has introduced intermediation in the model in [19]. Cen-

tral to this literature is the notion that depository institutions improve social welfare by supplying liquidity for consumption purposes. For instance, in [16] banks have a safekeeping role, mediating trade among agents by issuing liabilities that are safer than money. In [7] banks make consumer loans and some inflation is socially optimal, as it removes incentives to default on loan repayment. So, credit is rationed only if inflation is sufficiently low. In [6], banks improve welfare, even at the Friedman rule, by insuring agents against idiosyncratic productivity shocks and allowing them to economize on money balances. We extend this literature by introducing financial institutions that supply credit for production purposes, a theme that is also found in [13].

Finally, our work ties into a theoretical literature on financial development and economic activity (e.g. see [18] or [23]). In the model, intermediaries sustain technological innovation by financing the operations of the most efficient production technology. If we identify greater intermediation costs with less developed financial systems, then the model predicts a positive association between financial and economic development.

We proceed as follows. Section 2 lays out the model. Section 3 discusses the efficient allocation. Sections 4 and 5 study the problems of various economic units, equilibrium is discussed in Section 6 and Section 7 concludes.

2 The environment

We adopt a model where money is used in trade and there is no role for personal debt, based primarily on [1, 8, 19]. Preferences are quasilinear and agents interact in two sequential markets, as in [19], and markets are anonymous and competitive, as in [1, 8].

Time is discrete and the horizon is infinite, starting with date 1. There is a unit-mass continuum of identical infinitely-lived utility-maximizing agents who consume a perishable good, and discount even to odd dates with factor $\beta \in (0,1)$. Preferences differ across dates, so we define trading cycles by $t = 1, 2, ...$ each including an odd and an even date. On even dates agents can work and consume, and their preferences are $u_e(c_e) - h_e$, which is the utility from consuming $c_e \geq 0$ of someone else's production and disutility from $h_e \geq 0$ labor. On odd dates a random shock, i.i.d. across agents and time, partitions the population into a fraction $\frac{\alpha}{2}$ of consumers, $\frac{\alpha}{2}$ workers and $1 - \alpha$ idle agents. Consumers cannot work but derive $u(c)$ utility from $c \geq 0$ consumption. Workers do not wish to consume but can supply $h \geq 0$ labor at utility loss $-g(h)$. Idle agents neither wish to consume nor can work. The functions u_e and u satisfy the Inada conditions, and g is strictly convex with $g'(0) = 0$.

We follow [19] in assuming labor is the only factor of production and workers own a

"home production" technology; this allows workers to transform, on every date, h labor into h goods and market the output to consumers. We introduce an additional technology on odd dates. Workers can supply labor to a profit-maximizing firm defined by the "market production" technology $Y : [0, \infty) \to [0, \infty)$ that is strictly increasing and concave, satisfies the Inada conditions, and displays non-increasing production elasticity.[3]

Sellers and buyers are price takers on output and input markets. On each date there are infinitely many spatially separated trading groups, each defining a market and including countably many anonymous agents, as in [8]. It is assumed the firm must pay workers before output is sold but cannot issue IOUs.[4] Since workers do not wish to consume, goods are perishable and anonymity rules out credit, workers will demand monetary compensation. Hence, the firm must borrow working capital. This creates an explicit role for intermediaries.

There is a large number N of profit-maximizing intermediaries owned by agents in equal non-marketable shares. Intermediaries are technologies that facilitate loans to the firm, as follows. On even dates they sell nominal bonds using the receipts to finance the firm's operation, on the next odd date. Lenders acquire non-marketable claims to the firm's net worth (private equity). This asset transformation process is called commercial lending. We assume the following commitments: on even dates the firm repays the loan, intermediaries repay bonds, and profits are distributed as dividends.

Commercial lending is costly. On odd dates an intermediary can transform a dollar into $1 - \gamma$ dollars of loan. The value $\gamma \in [0, 1]$ is random, i.i.d. across intermediaries with c.d.f. G. It identifies the portion of liabilities that must be dissipated to overcome frictions (informational, enforcement, etc.) in lending.[5] So, we say that economy x has higher intermediation frictions than economy y if $G_x(\gamma) < G_y(\gamma)$ for each $\gamma \in (0, 1)$ (G_x strictly first-order stochastically dominates G_y)

Let $\bar{M}_t$ denote the fiat money stock at the start of trading cycle t. It is supplied

[3] We consider this a representative firm even if Y is not homogeneous of degree one. Such a feature can be recovered by adding an "entrepreneurial" factor in fixed supply to redistribute profits as factor payments (see [22]). Non-increasing elasticity $\frac{d \ln Y}{d \ln H}$ is standard (e.g., [21]).

[4] For a similar assumption see [12, 15]. One can imagine production and retail operations are sequential so sales receipts accrue after workers leave. For the inability to issue IOUs assume agents can costlessly counterfeit IOUs, so no one would accept them, as in [4].

[5] Non-interest expenses are a realistic feature of intermediation (e.g. see [14] for banks). We assume these expenses flow as lump-sum compensation to idle agents. Alternatively we can keep the money idle or give lump-sum transfers to everyone at the end of odd dates. All we need is intermediation costs should neither alter the money supply (hence prices), nor agents' consumption/labor decisions and wages.

by an authority whose policy is a time-invariant gross rate of money growth π, publicly announced. Money is injected via deterministic lump-sum transfers $(\pi - 1)\bar{M}_t$ on even dates, so $\bar{M}_{t+1} = \pi \bar{M}_t$ money is available at the end of cycle t.

3 Efficient allocation

To set the stage for the analysis, consider the allocation selected by a planner who treats agents identically. We call it the *efficient allocation*. The planner maximizes the expected lifetime utility of a representative agent subject to the physical and technological constraints. This means the optimal plan solves a sequence of identical static problems. The planner chooses non-negative values $\{c, c_e, h, h_e\}$ and $\theta \in [0, 1]$ to solve

$$\begin{aligned} &\text{Maximize:} \quad \tfrac{\alpha}{2}[u(c) - g(h)] + u_e(c_e) - h_e \\ &\text{Subject to:} \quad \tfrac{\alpha}{2}c = Y(\tfrac{\alpha}{2}h\theta) + \tfrac{\alpha}{2}h(1-\theta) \text{ and } c_e = h_e. \end{aligned}$$

The two constraints reflect aggregate and technological feasibility. The second is obvious. To understand the first notice that in market one there are $\frac{\alpha}{2}$ workers and $\frac{\alpha}{2}$ consumers. Each consumer gets c goods and each worker supplies h labor. Feasibility depends on the technology adopted. The planner can assign workers to either technology, θ of them to market production and the remainder to home production. Hence, $H = \frac{\alpha}{2}h\theta$ labor goes to market production, and the rest to home production. Since $Y'(0) > 1$ some labor is always allocated to market production. All labor is efficiently allocated to market production when the following assumption is imposed.

Assumption 1 *$Y'(H^*) > 1$ for H^* defined in (1).*

Lemma 1 *The efficient allocation is stationary across trading cycles and is defined by the vector $(c, h, c_e, h_e) = (c^*, h^*, c_e^*, h_e^*)$ that uniquely satisfies*

$$\begin{aligned} &u'(c^*)Y'(H^*) = g'(h^*), \text{ with } c^* = \tfrac{2}{\alpha}Y(H^*) \text{ and } H^* = \tfrac{\alpha}{2}h^*, \\ &u_e'(h_e^*) = 1, \text{ with } c_e^* = h_e^*. \end{aligned} \tag{1}$$

On each date the planner equates marginal utility of income to marginal labor disutility. On odd dates, the planner equates the marginal products of the two technologies, if possible. Assumption 1 ensures it is socially beneficial to allocate all labor to market production; this generates a clear-cut role for financial intermediation. The result in Lemma 1 also motivates our focus on monetary allocations that are stationary.

Definition 2 *An allocation for a monetary economy is stationary if the aggregate real money stock and per-capita consumption are positive and invariant across trading cycles.*

We omit subscripts for trading cycles and let a prime identify the consecutive cycle. So, if p_o and p_e are nominal prices in odd and even dates of cycle t, then p_o' and p_e' refer to $t+1$. Also, we work with real variables, dividing nominal variables by p_e. Let $p = \frac{p_o}{p_e}$ denote the odd-date real price of goods. At the start of an arbitrary cycle, $\bar{m} = \frac{\bar{M}}{p_e}$ is the real money stock, and (m, b) is the real financial portfolio of an agent, money balances $m \geq 0$ and bonds $b \geq 0$. Stationarity and money market clearing require

$$m = m', \; b = b' \text{ and } m + b = \bar{m}. \tag{2}$$

Hence, $\bar{m} = \bar{m}'$ and monetary policy pins down inflation,

$$\frac{p_e'}{p_e} = \frac{\bar{M}'}{\bar{M}} = \pi. \tag{3}$$

Finally, let $\tau = (\pi - 1)\bar{m}$ denote the real balance transfer on even dates.

The timing of a trading cycle is as follows. An agent starts the odd date with portfolio (m, b). Random shocks are realized: the agent ends up in some state $s = c, w, n$ (consumer, worker, idle) and intermediaries draw their shocks γ. Then, intermediaries lend, the firm hires labor, produces, pays workers, and then sells its output. Next, the even date starts. The agent has portfolio (m_s, b_e) reflecting his odd-date activity. Production and trade occurs, the firm repays lenders, intermediaries pay bondholders and dividends, agents get τ and buy bonds. Then, a new trading cycle starts.

4 Financial intermediation

Intermediaries facilitate loans to the firm, a process that generates *interest costs* from maintaining liabilities, and *noninterest costs* from transforming liabilities into assets. On each even date N intermediaries sell nominal bonds at par to agents. Suppose each agent buys $\frac{b}{N} > 0$ bonds from every intermediary (later we prove this is optimal). Each intermediary has thus $\frac{b}{N} = \int_0^1 \frac{b}{N} dx$ liabilities. The bond contract is a non-negotiable and non-transferable contingent claim to currency. If the intermediary lends, then the bond pays gross nominal interest $i \geq 1$ on the next even date. Otherwise, it pays its face value on the next odd date. This is a simple way to introduce a nominal financial asset in addition to money: the bond has an expected return greater than money, but is illiquid and is risky (see [6] for a model with interest-bearing riskless transaction accounts). Indeed, the bond either generates a low return on the odd date, or a high return on the even date.

Consider an odd date, when the intermediary draws the random cost shock γ and chooses how much to lend. Lenders acquire claims to a share of the firm's net worth. Payments accrue after output is sold, on the even date. It turns out that an intermediary will either lend all it has, or not at all because their profit maximization problem is linear.

Let the (endogenous) expected gross rate of return on a dollar loan be R. Given γ and i, the profit per dollar loan is $R(1-\gamma)-i$. Define

$$\gamma^* = 1 - \tfrac{i}{R}. \tag{4}$$

If $\gamma \leq \gamma^*$, then the intermediary lends all it has. Otherwise, no loan is made. The assumption of random cost is a simple device to endogenize investment activity. If not all intermediaries lend in a cycle, then only a fraction of bonds is transformed into loans and it pays interest. However, the remaining fraction is not kept idle, since savers are paid the face value on odd dates and can spend it to buy consumption. We say there is intermediation only if the loan supply is positive. If $b = 0$, then intermediaries cannot lend. If $b > 0$ but intermediaries do not lend, then bonds are like money.

Given noninterest costs, financial intermediation requires a positive spread between loan return and interest cost of liabilities, $R > i$. Suppose this inequality holds and everyone buys $\frac{b}{N}$ bonds from each intermediary. We use Kolmogorov's law of large numbers to derive the following result.

Lemma 3 *Suppose $1 \leq i < R$ and agents buy $\frac{b}{N}$ bonds from each intermediary. The share of intermediaries that lend and the loan supply converge to the expectations $G(\gamma^*)$ and $b\int_0^{\gamma^*}(1-\gamma)dG(\gamma)$ as $N \to \infty$. Real per-capita dividends, nominal rate of return on bonds, and intermediaries' payments converge to the values ξ, r and ζ defined by*

$$\begin{aligned} \xi &= b\int_0^{\gamma^*}[R(1-\gamma)-i]dG(\gamma), \\ r &= 1 + G(\gamma^*)(i-1), \\ \zeta &= \tfrac{b}{1-\alpha}\int_0^{\gamma^*}\gamma dG(\gamma). \end{aligned} \tag{5}$$

Given stationarity, the key endogenous financial variables can be treated as being deterministic and fixed. The portion of active intermediaries converges to $G(\gamma^*)$, so a fraction $1-G(\gamma^*)$ of bonds pays its face value on odd dates. Noninterest costs amount to $b\int_0^{\gamma^*}\gamma dG(\gamma)$ so intermediaries pay ζ to idle agents, and pay a dividend ξ.

5 Agents

Agents maximize expected lifetime utility. Let V be the value function of an agent at the start of a trading cycle (odd date, before shocks are realized), and V_e at the start of an

even date. We conduct the analysis supposing that each agent buys $\frac{b}{N} \geq 0$ bonds from each intermediary.

5.1 Even dates

The agent's problem is recursive, so we formalize it with a functional equation. Given portfolio (m_s, b_e) at the start of even dates, let $h_s \geq 0$ denote the labor choice. We have

$$\begin{aligned} V_e(m_s, b_e) = & \max_{(c_e, h_s, m', b') \geq 0} \{u_e(c_e) - h_s + \beta V(m', b')\} \\ \text{s.t.} \quad & h_s = c_e + \pi(m' + b') - [m_s + b_e i + \xi + \tau]. \end{aligned} \tag{6}$$

The resource constraint holds with equality due to non-satiation.[6]

Financial resources depend on current payments and assets. Payments include lump-sum real balance transfers τ and dividends ξ. The agent has a return i on $b_e = bG(\gamma^*)$ bonds and m_s money, where

$$m_s = m + b[1 - G(\gamma^*)] \begin{cases} +\zeta & \text{if} \quad s = n \\ -pc & \text{if} \quad s = c \\ +\delta(h) & \text{if} \quad s = w. \end{cases} \tag{7}$$

On odd dates only $G(\gamma^*)$ intermediaries lend, so agents can spend $b[1 - G(\gamma^*)]$ out of their initial bond holdings. Agents start with m money, then they either stay idle or become consumers or workers. Idle agents get ζ from intermediaries (noninterest cost). Consumers spend pc. Working h hours generates income

$$\delta(h) = h \times \max\{p, \omega\},$$

given the choice between home production (price p), or market production (wage ω). So, the endogenous variable p not only defines the inverse of the value of money, but it also defines and endogenous reserve wage.

Funds available on even dates can be used for consumption $c_e \geq 0$ or savings, $b' \geq 0$ bonds or $m' \geq 0$ balances.[7] Savings are carried into the next cycle, so are adjusted by gross inflation π. The agent compensates shortages in funds by working h_s hours, which generates h_s income. Thus, under the premise that $h_s \geq 0$, we have

$$V_e(m_s, b_e) = m_s + b_e i + \xi + \tau + \max_{c_e} [u_e(c_e) - c_e] + \max_{(b', m')} [\beta V(m', b') - \pi(m' + b')]. \tag{8}$$

[6] Since $h_j \geq 0$ we work under the conjecture that the right hand side of the constraint in (6) is non-negative. In equilibrium this simply requires that $(u_e')^{-1}(1)$ is sufficiently large (see later).

[7] Short-selling is not allowed. Agents cannot lend to each other because the structure of the environment severs all future links among current trade partners (see [8]). Anonymity rules out credit with the firm.

Given that V_e and V exist and are differentiable (see [2]), the Envelope theorem implies

$$\frac{\partial V_e(m_s,b_e)}{\partial m_s} = 1 \quad \text{and} \quad \frac{\partial V_e(m_s,b_e)}{\partial b_e} = i, \tag{9}$$

i.e., the marginal value of assets is their return and is independent of the agent's wealth $m_s + b_e$. This is due to linear labor disutility and unconstrained labor endowments, which imply agents can produce any amount at constant marginal cost. So, they make up for any shortage in wealth by working more. Consequently, even-date consumption is deterministic and independent of wealth. From (8) we have

$$c_e = c_e^*. \tag{10}$$

Market clearing requires

$$\frac{\alpha}{2}(h_c + h_w) + (1-\alpha)h_n = c_e^* \tag{11}$$

where h_s satisfies the constraint in (6). We can thus write

$$\begin{aligned} V_e(m_s, b_e) &= \max_{(m',b')} \left[\beta V(m',b') - \pi(m'+b')\right] + u_e(c_e^*) - c_e^* + \xi + \tau + m_s + b_e i \\ &= V_e(0,0) + m_s + G(\gamma^*)bi, \end{aligned} \tag{12}$$

i.e., expected utility V_e is the current value of wealth $m_s + G(\gamma^*)bi$, plus the continuation payoff $V_e(0,0)$. As in [19], the choice of savings is disentangled from the agent's wealth. It follows that if V is strictly concave, then agents start each cycle with identical portfolios $(m',b') = (m,b)$. The amount and composition of savings will depend on the relative return on money/bonds and on their relative liquidity.

The first order condition (omitting the multiplier) is

$$1 \geq \frac{\beta}{\pi} \times \frac{\partial V(m',b')}{\partial x} \quad \text{for } x = b', m'. \tag{13}$$

The marginal cost of assets is one unit of consumption. The right hand side defines expected marginal benefits, discounted and deflated. Marginal benefits cannot surpass costs and hinge on the asset's yield but *also* on its use in trade. Indeed, consumers need money on odd dates, but some bonds cannot be spent. So, the marginal benefit of money and bonds depends on liquidity constraints, as we show next.

5.2 Odd dates

The problem for an agent starting a trading cycle with portfolio (m,b) is

$$\begin{aligned} V(m,b) = \quad & \tfrac{\alpha}{2} \max_{c\geq 0} \left[u(c) + V_e(m_c,b_e)\right] + \tfrac{\alpha}{2} \max_{h\geq 0} \left[-g(h) + V_e(m_w,b_e)\right] \\ & +(1-\alpha)V_e(m_n,b_e) \\ \text{s.t.} \quad & pc \leq m + b[1-G(\gamma^*)]. \end{aligned} \tag{14}$$

The budget constraint includes the portion $1 - G(\gamma^*)$ of bonds that pays on odd dates.

Workers do not wish to consume and, since goods are perishable, demand monetary compensation. A worker chooses $h \geq 0$ to maximize the payoff $V_e(m_w, b_e) - g(h)$. The necessary and sufficient first-order condition for an interior solution is

$$g'(h) = \frac{\partial V_e(m_w, b_e)}{\partial m_w} \frac{\partial m_w}{\partial h}.$$

Using (7) and (9), the odd-date individual labor supply is determined by

$$g'(h) = \max\{\omega, p\}. \tag{15}$$

An hour of work yields p real balances with home production, and ω with market production. Earnings can be spent on the next even date, when the price of money and marginal labor disutility are both one. Since workers can substitute odd- for even-date labor, they supply labor until the marginal disutility $g'(h)$ reaches $\max\{\omega, p\}$. If

$$\omega \geq p, \tag{16}$$

then workers are willing to supply labor to the firm (if the firm pays the reservation wage, then the worker is indifferent). If $\omega < p$, then the worker selects home production. So, we use $\theta \in [0, 1]$ to denote the endogenous share of workers who supply labor to the firm. Corner solutions $\theta = 0, 1$ occur when ω is unequal to p.

To find ω consider the firm. It demands $H \geq 0$ labor given the (real) price p, wage ω, and cash on hand $b \int_0^{\gamma^*} (1 - \gamma) dG(\gamma)$ from loans to solve

$$\begin{array}{ll} \text{Maximize:} & pY(H) - \omega H \\ \text{Subject to:} & \omega H \leq b \int_0^{\gamma^*} (1 - \gamma) dG(\gamma). \end{array}$$

Given that the constraint may be binding, labor demand satisfies

$$\omega \leq pY'(H). \tag{17}$$

Labor is paid the marginal value product only if the working capital constraint does not bind. This depends on the loan supply, which hinges on savings b and lending rule γ^*. The firm can hire labor only if $\omega \geq p$. Since $\omega = g'(h)$ then (17) becomes

$$g'(h) \leq pY'(H). \tag{18}$$

If $\omega = pY'(H)$, then $\omega > p$ as long as $Y'(H) > 1$. As we will see, this holds due to Assumption 1. If $\omega < pY'(H)$, then H must be sufficiently small for $\omega \geq p$ to hold. This suggests the loan supply affects the firm's operation, hence its net worth.

Labor market clearing requires

$$H = \theta \tfrac{\alpha}{2} h. \tag{19}$$

Given the definition of θ and $\frac{\alpha}{2}$ consumers, goods' market clearing is

$$c = \begin{cases} \frac{2}{\alpha} Y(H) + (1-\theta)h & \text{if } p \leq \omega, \\ h & \text{otherwise.} \end{cases} \tag{20}$$

The consumer chooses $c \geq 0$ to solve

$$\begin{array}{ll} \text{Maximize:} & u(c) + V_e(m_c, b_e) \\ \text{Subject to:} & pc \leq m + b[1 - G(\gamma^*)]. \end{array}$$

The first-order condition (omitting the multiplier on the constraint) is

$$u'(c) \geq -\tfrac{\partial V_e(m_c, b_e)}{\partial m_c} \tfrac{\partial m_c}{\partial c} = p. \tag{21}$$

Consumption depends on p and available liquidity, money and bond receipts on odd dates. Let $\hat{c}(p)$ denote unconstrained consumption—(21) is an equality—and define $\hat{m} = p\hat{c}(p)$. If $m + b[1 - G(\gamma^*)] < \hat{m}$, then the consumer is liquidity constrained, so $c < \hat{c}(p)$.

Lemma 4 *In any stationary monetary economy we have*

$$c = \min\{\tfrac{m + b(1 - G(\gamma^*))}{p}, \hat{c}(p)\} \tag{22}$$

where $c = \hat{c}(p)$ uniquely satisfies $u'(c) = p$, and $u'(c) > p$ for all $c < \hat{c}(p)$.

Corollary 5 *If $p = \frac{g'(h)}{Y'(H)} = u'(c)$, then $c = \hat{c} = c^*$ and the allocation is efficient.*

Agents consume at most the efficient quantity c^*, and whether they do so depends on the price p. We know from the Planner's problem that at $c = c^*$ marginal labor disutility equals marginal utility of income, $g'(h) = u'(c)Y'(H)$. If the firm hires some workers, then $\omega \leq pY'(H)$ where $\omega = g'(h)$. If $g'(h) = pY'(H)$ and $p = u'(c)$, then workers are paid the marginal value product, and this equals the marginal utility of income. Here, all workers are employed at the firm and the allocation is efficient.

This discussion suggests three types of inefficiencies can arise in the monetary economy. First, we have a *consumption inefficiency* when $p < u'(c)$, i.e., when buyers' liquidity constraints bind. This is a standard inefficiency in monetary models (e.g., see [19]) and implies $g'(h) = pY'(H) < u'(c)Y'(H)$. Two additional inefficiencies can arise in our model, depending on the allocation of labor. They result in $g'(h) < pY'(H) < u'(c)Y'(H)$.

A *labor effort inefficiency* arises when $\omega < pY'(H)$, i.e., when working capital constraints bind. It affects negatively the firm's production because workers optimally choose to supply less than the profit-maximizing labor. An additional *production inefficiency* arises when $\omega = p$. It hurts production along the extensive margin to the extent that some labor gets reallocated to the less efficient home production. In Section 6 we demonstrate that these inefficiencies arise progressively as inflation rates increase.

5.3 Portfolio choices and market clearing

To determine optimal savings we need $\frac{\partial V(m,b)}{\partial x}$ for $x = m, b$. Using (7), (12), and (14)

$$V(m,b) = \tfrac{\alpha}{2}[u(c) - pc] + \tfrac{\alpha}{2}[\delta(h) - g(h)] + (1-\alpha)\zeta + m + br + V_e(0,0), \tag{23}$$

where h satisfies (15) and c satisfies (22). The lifetime utility V is a function of savings $m + br$, continuation payoff $V_e(0,0)$, and expected surplus from odd-date trades. With probability $\frac{\alpha}{2}$ the agent either enjoys utility $u(c)$ by spending pc, or earns $\delta(h)$ by suffering disutility $g(h)$.

Clearly, $\frac{\partial c}{\partial m} = \frac{1}{p}$ if agents are liquidity constrained (else, zero). So, from (23):

$$\frac{\partial V(m,b)}{\partial m} = \begin{cases} 1 + \frac{\alpha}{2}(\frac{u'(c)}{p} - 1) & \text{if } m + b[1 - G(\gamma^*)] < \hat{m} \\ 1 & \text{otherwise.} \end{cases} \tag{24}$$

We also have:

$$\frac{\partial V(m,b)}{\partial b} = \begin{cases} r + \frac{\alpha}{2}(\frac{u'(c)}{p} - 1)[1 - G(\gamma^*)] & \text{if } m + b[1 - G(\gamma^*)] < \hat{m} \\ r & \text{otherwise.} \end{cases} \tag{25}$$

It can be verified that V is concave.[8]

Using (24)-(25) in (13), and using r from (5), the choice of savings must satisfy

$$\begin{aligned} 1 \geq \quad & \tfrac{\beta}{\pi}[1 + \tfrac{\alpha}{2}(\tfrac{u'(c)}{p} - 1)] \\ 1 \geq \quad & \tfrac{\beta}{\pi}\left\{[1 + \tfrac{\alpha}{2}(\tfrac{u'(c)}{p} - 1)][1 - G(\gamma^*)] + iG(\gamma^*)\right\}. \end{aligned} \tag{26}$$

The first line is the choice of $m' \geq 0$ money and the second of $b' \geq 0$ bonds. If the amounts are positive, then we have equalities. In making these choices, agents compare the unit price of each asset on left hand side to the (expected marginal) benefit discounted by $\frac{\beta}{\pi}$.

[8] One can prove that the Hessian of V is negative semi-definite. Thus, we cannot ensure strict concavity of V. In that case, to ensure degeneracy of the distribution of portfolios on odd dates, we can focus on portfolio selections that are symmetric.

The benefit of money is $1+\frac{\alpha}{2}(\frac{u'(c)}{p}-1)$. It is stochastic due to random consumption shocks. It exceeds money's yield (one) only if $u'(c)>p$, when the liquidity constraint binds. The extra component is a liquidity premium, which grows with the severity of liquidity constraints (smaller c) and with the probability of consumption shocks (higher α). The benefit of bonds is a convex combination of the benefit of money and the interest payment i. This combination depends on intermediaries' activity summarized by $G(\gamma^*)$, which pins down the portion of bonds that pay i on even dates, while the remaining portion pays one on odd dates.

It should be clear that buying bonds from a subset of intermediaries cannot improve a saver's payoff. Cost shocks are i.i.d., so intermediaries offer identical assets, and dealing with a subset of intermediaries introduces uncertainty in the bonds' portfolio return. Since agents are risk averse, full diversification must be weakly optimal. We can now link rates of return on assets to conditions necessary to sustain intermediation.

Lemma 6 *In any stationary monetary economy we must have $\pi \geq \beta$.*

(a) If $i=\frac{\pi}{\beta}$, then

$$p=\frac{\alpha u'(c)}{\alpha+2(i-1)}, \tag{27}$$

$$c=\frac{m}{p}+\frac{b}{p}[1-G(\gamma^*)]; \tag{28}$$

(b) If $i<\frac{\pi}{\beta}$, then $b=0$; If $i>\frac{\pi}{\beta}$, then $m=0$.

The ratio $\frac{\pi}{\beta}$ is the nominal yield of a what we call an *illiquid government bond*, which is a non-marketable claim to currency traded only on even dates (see [8]). So, $\frac{1}{\beta}$ is the real (or shadow) interest rate. If the real rate of return on money $\frac{1}{\pi}$ exceeds the shadow interest rate, then agents would accumulate money, which is inconsistent with stationarity. Hence, we need $\pi \geq \beta$.

To explain (a)-(b) note that the expected return on bonds is

$$r=1+(i-1)G(\gamma^*)$$

since $G(\gamma^*)$ is the 'invested' portion and the rest is equivalent to money. If $i<\frac{\pi}{\beta}$, then the return on bond is less than on illiquid government bonds: so, it is best to hold money (and perhaps illiquid government bonds), but not bonds. If $i>\frac{\pi}{\beta}$, then bonds pay more than illiquid government bonds. But the marginal benefit from holding illiquid government bonds must match at least that from holding money (first line in (26)), so no one holds money. The value $i=\frac{\pi}{\beta}$ is necessary for indifference between money and bonds. Here,

bonds dominate money in rate of return since $r > 1$, but are not as liquid. In this case (26) gives (27).

5.4 Returns on loans and on bonds

In this subsection we determine the key financial variable γ^* by first finding the optimal interest rate i offered by intermediaries, and then the rate of return on loans R.

Lemma 7 *The intermediary's optimal bond contract specifies*

$$i = \frac{\pi}{\beta}. \tag{29}$$

This is an arbitrage condition. The intermediary minimizes interest costs by making agents indifferent to holding its bonds. In offering the bond contract the intermediary takes as given the returns on all other assets. The intermediary's expected profit falls in the interest cost of liabilities, so it offers $i = \frac{\pi}{\beta}$, which leaves agents indifferent to buying bonds, money, or illiquid government bonds (Lemma 6).

We now find the lending rule γ^* as function of inflation, prices, and labor. Loan market clearing is

$$\omega H = b \int_0^{\gamma^*} (1-\gamma) dG(\gamma), \tag{30}$$

so the firm's net worth is $pY(H)$, which is income from sales plus working capital (assets) minus wages (liabilities). We have $pY(H) = Rb \int_0^{\gamma^*} (1-\gamma) dG(\gamma)$ as intermediaries own all shares to the firm's net worth. Hence,

$$R = \frac{pY(H)}{\omega H}, \tag{31}$$

i.e., the gross rate of return on loans is the average product of labor. Finally, (4), (29) and (31) give

$$\gamma^* = 1 - \frac{\pi}{\beta} \times \frac{\omega H}{pY(H)}. \tag{32}$$

We can already see why inflation may affect lending. Suppose working capital constraints are not binding, $\omega = pY'(H)$. Here, R is the inverse of production elasticity (assumed non-increasing in the scale of operations). If output falls with inflation, then the interest cost of liabilities rises relative to the return on loans and γ^* falls. Of course, the working capital may bind so we define equilibrium differentiating the two cases.

Definition 8 *Given a policy (π, τ), a stationary monetary equilibrium is a list of values $(c, h, c_e, h_s, b, m, p, \omega, i, \theta, \gamma^*)$ consistent with optimality and market clearing. In particular, (2)-(4), (6), (11), (14), and (18)-(30) must hold. We say that the equilibrium is* ***unconstrained*** *if (18) is an equality, and* ***constrained*** *otherwise.*

Thus, in any equilibrium the following holds: the reserve wage p satisfies (27), b and m satisfy (28), and loan market clearing (30) holds. In unconstrained equilibrium $\omega = pY'(H)$ with $\omega > p$. Constrained equilibrium arises when (30) is violated for $\omega = pY'(H)$ even if agents save only with bonds. In this case $m = 0$ and $\omega < pY'(H)$ with $\omega \geq p$.

6 Existence and characterization of equilibrium

We discuss equilibrium in three steps, starting with economies where financial intermediation is unnecessary. Here inflation simply distorts savers' decisions. Then, we introduce intermediation but assume it is costless (frictionless). Here, inflation affects also the credit supply, but not the asset transformation process of intermediaries. Finally, we show that with intermediation frictions inflation degrades the asset transformation process, also.

6.1 Finance is inessential

Suppose the firm can either commit to pay workers after output is sold, or there is some way to enforce such a promise. In this case the firm needs no working capital and in equilibrium $pY'(H) = \omega = g'(h)$.[9] Obviously, intermediaries have no role to play, do not lend, hence $b = 0$ because so bonds are equivalent to money.

In equilibrium all labor is supplied to the firm, $\theta = 1$. Hence $c = \frac{2}{\alpha}Y(H)$ and $H = \frac{\alpha}{2}h$. Since savings are $m = pc$ and the first line in (26) holds with equality, then there is a unique equilibrium value h that satisfies

$$\frac{\pi}{\beta} - 1 = \frac{\alpha}{2}\left(\frac{u'(c)Y'(H)}{g'(h)} - 1\right). \tag{33}$$

We obtain the standard result that inflation acts as a tax that distorts savings decisions. Hence, we have the standard consumption inefficiency for $\pi > \beta$. This is obvious from (33), since the bracketed term grows with π. Using the implicit function theorem we have $\frac{\partial h}{\partial \pi} < 0$, so odd-date consumption falls with inflation. The price p, the real wage ω and savings m all fall with inflation, also. Consequently, the Friedman rule is optimal because the allocation is efficient as $\pi \to \beta^+$. This is immediately established by examining (33) and Corollary 5.

Figure 1 reports odd date consumption across inflation, c(unc.), for a baseline parameterization given by $\alpha = 0.1$, $\beta = 0.95$, $u(c) = 2c^{0.5}$, $g(h) = h^2$, and $Y(H) = H^{0.9}$. It follows that in unconstrained equilibrium ($\omega = pY'(H)$) production elasticity is independent of inflation, being constant at 0.9, and efficient consumption is $c^* = .925$. The figure

[9] Since the firm makes profits in equilbrium, it also pays dividend ζ to agents.

illustrates the optimality of the Friedman rule, and, for comparison purposes, reports consumption when the market production technology is unavailable, denoted c(no inter.).

6.2 Frictionless finance

Suppose the firm can neither commit to pay workers after output is sold, nor can such a promise be enforced. In this case the firm must pay workers before output is sold, so intermediation matters. However, we rule out intermediation frictions by assuming that making loans is a costless process. So we set $G(0) = 1$, i.e., each dollar of liabilities can be transformed into a dollar loan.

The consequence is that if intermediaries lend, then their liabilities are identical to illiquid government bonds. Savers receive payment i with certainty on the next even date. There are three implications. In equilibrium agents always hold some money because bonds never pay a low return on odd dates, so $m > 0$. Second, if an intermediary lends, then all intermediaries do so. So, the loan supply equals the bond demand b, i.e.,

$$b = \omega H$$

Third, inflation does not affect the intermediaries' asset transformation process. It does affect the supply of commercial loans, however, because it distorts savings decisions. This discussion is summarized in the following statement.

Proposition 9 *Let $G(0) = 1$. There exist a unique equilibrium for all $\pi > \beta$. It can be described by partitioning the set of inflation rates as follows:*

(a) if $\pi \in (\beta, \hat{\pi}]$, then we have unconstrained equilibrium;

(b) if $\pi \in (\hat{\pi}, \bar{\pi}]$, then we have constrained equilibrium with $\theta = 1$;

(c) if $\pi > \bar{\pi}$, then we have constrained equilibrium with $\theta < 1$.

Corollary 10 *Higher inflation is associated to lower equilibrium savings and output, and the Friedman rule sustains the efficient allocation.*

The model pins down two long-run inflation thresholds $\hat{\pi}$ and $\bar{\pi}$, moderate and high. The working capital constraint binds when π reaches $\hat{\pi}$. At this point production distortions start to emerge, in addition to savings distortions. Intuitively, as π rises above β, interest costs rise relative to the return on loans, which is nonincreasing, and intermediaries' profits fall.

As π reaches $\hat{\pi}$, lending becomes unprofitable unless the rate of return on loans increases. Recall that in equilibrium R corresponds to the average product of labor. Therefore an increase in return can be obtained by paying wages below the marginal value product, i.e., $\omega < pY'(H)$. Of course, this results in workers reducing their supply of labor below the profit maximizing level. However, everyone supplies labor to the firm since for $\pi \in (\beta, \bar{\pi})$ we have $\omega > p$. As inflation keeps rising above the first threshold $\hat{\pi}$, wages keep falling below the marginal value product, reaching the reserve value p for $\pi = \bar{\pi}$.

For inflation rates beyond the second threshold $\bar{\pi}$, lending is unprofitable even if workers are paid the reserve wage. Consequently, for $\pi > \bar{\pi}$ the firm further reduces its scale of operations by hiring only a portion $\theta < 1$ of the available labor supply. By decreasing the scale of operations further, the firm can increase the average product of labor, hence the return to the intermediary. In this segment of the parameter space intermediation declines further. This decline is accompanied by an additional decline in real activity due to a gradual shift to the less efficient home production. From here on, the level of real economic activity is lower than before. This theoretical finding mirrors those obtained in [11, 17].

Why does the 'frictionless finance' model predict a negative association between inflation and financial activity? The reason is savings fall. As inflation rates increase agents reduce their exposure to the inflation tax by holding a smaller fraction of savings in money and in bonds, which are also nominal. This decline in savings reduces spending and the firm's scale of operation, and—for low to moderate inflation rates (when the working capital constraint does not bind)—also the return from lending. Since $\gamma = 0$ for all intermediaries in this version of the model, all intermediaries lend as long as $R \geq \frac{\pi}{\beta}$. As π rises above β, output falls, i grows, R can only decrease, and for $\pi = \hat{\pi}$ we have $R = i$, i.e., borrowing constraints bind. From that point on, inflation is positively associated with the real return on loans, because real wages keep falling and are set below the marginal value product. This explains the nonlinear decline in intermediation activity.

What we see is that if the equilibrium rate of return on commercial loans is non-increasing in inflation, as it happens here, then even moderate inflation can significantly lower real activity. Though inflation does not degrade the asset transformation process—each dollar collected through bond sales is transformed into a dollar loan to the firm—it leads to shortages in commercial credit. Excessive inflation, $\pi > \bar{\pi}$, is associated to an additional drop in activity because financial flows decrease and a reallocation of labor to a less efficient technology.

We conclude with a remark. As $\pi \to \beta$ we are at the Friedman rule $i = 1$ and, since

the equilibrium is unconstrained the allocation is efficient (Lemma 1 and (27)). Of course, this rests on the availability of intermediation, without which the firm could not borrow working capital. This is interesting because it contrasts with the finding in [7], where the Friedman rule destroys intermediation, but sustains the efficient allocation.

Illustration. Consider Figure 2. The solid curve labeled c reports odd-date consumption for an economy without intermediation frictions. The working capital constraint starts to bind when inflation reaches 5.6%. From there on we have constrained equilibrium; as a comparison, unconstrained equilibrium consumption is the dashed 'continuation' curve labeled c(unc.). As inflation grows above 5.6%, real wages keep falling. When inflation reaches 64.2%, ω equals the reservation wage p, and intermediation declines further.

Real savings $b + m$ monotonically decline with inflation. Though not shown, m and b both decline, and their ratio is constant in unconstrained equilibrium. In constrained equilibrium the ratio of bonds to money holding falls. The impact of inflation on intermediation activity is nonlinear. It is strong at low inflation rates, and weak at high rates. In unconstrained equilibrium loans fall by almost 5%, on average, with each additional inflation point. In constrained equilibrium the impact is reduced to 1.7% with each additional inflation point, on average. Financial depth, measured by the ratio b/c is most strongly affected by inflation for low inflation levels.

6.3 Frictional finance

Now we go back to the case when intermediation is costly. In this case the loan supply is less than b because not all intermediaries lend. We will prove that greater inflation will be associated to a reduction in aggregate savings and it can disrupt the asset transformation process by decreasing the portion of liabilities that are transformed into working capital. We take this to mean that inflation amplifies the severity of intermediation frictions.

For this economy, we can prove a proposition similar to Proposition 9. However, note that as long as $\gamma^* < 1$, then some bonds pay a low return on odd dates. So in equilibrium agents may choose not to hold money as they can still access liquidity on odd dates.

Proposition 11 *Let $G(0) \neq 1$. There exist a unique equilibrium for all $\pi > \beta$. It can be described by partitioning the set of inflation rates as follows:*

(a) if $\pi \in (\beta, \tilde{\pi}]$, then we have unconstrained equilibrium;

(b) if $\pi \in (\tilde{\pi}, \bar{\pi}]$, then we have constrained equilibrium with $\theta = 1$;

(c) if $\pi > \bar{\pi}$, then we have constrained equilibrium with $\theta < 1$.

The interval $(\beta, \tilde{\pi}]$ is non-trivial only if $\alpha < \tilde{\alpha}$, with $\tilde{\alpha} > 0$. In addition, $\frac{\partial \tilde{\pi}}{\partial \alpha} < 0$.

Corollary 12 *Inflation reduces equilibrium savings and output, and the Friedman rule sustains the efficient allocation if $\alpha < \tilde{\alpha}$. The association between inflation and financial activity is nonlinear.*

Again, economic activity slows down faster and the process of financial intermediation breaks down further if inflation is excessive. A difference with the result in the earlier proposition is that agents may now hold no money at all. To see why, recall that in equilibrium agents are indifferent to holding money or bonds. Bonds offer greater return but are not as liquid as money, so agents may want to hold some money. The amount held depends on odd-date trade opportunities (governed by α) and by the purchasing power of currency (governed by π). Trade opportunities increase with α. All else equal, with higher α values agents want to insure more against consumption risk, holding a greater share of their portfolios in money. The same occurs with low inflation, which is when money better retains its purchasing power.

The intuition above also explains why α must be sufficiently small for unconstrained equilibria to be possible. With greater α agents carry more money and less bonds, a portfolio choice that affects the availability of commercial credit, also. There is a level of α, denoted $\tilde{\alpha}$, beyond which agents hold a substantial fraction of savings in money even if $i = 1$. Such hoarding of money reduces the loan supply enough to constrain the firm's ability to raise working capital. Hence, the allocation is inefficient even if the economy is at the Friedman rule.

The lesson is that with a poorly functioning financial system, inefficiency may arise for $i = 0$ because shortages of commercial loans may arise even if nominal interest rates are set to zero.[10] This brings us to consider one last question. What is the link between inflation thresholds and intermediation frictions?

Proposition 13 *Economies with more severe financial intermediation frictions display lower inflation thresholds.*

As inflation rises intermediaries face higher interest costs of liabilities as well as lower rate of return on loans. In response to this, intermediaries reduce their exposure to

[10] Monetary transfers to intermediaries may change this result.

noninterest costs by lowering γ^*. This behavior reduces the portion of liabilities transformed into working capital, which is why higher intermediation frictions are associated to lower inflation rate thresholds. As a consequence, greater intermediation friction are associated to lower financial activity, and lower real activity. This last finding suggests that the greater is the extent of frictions in finance, the lower is financial and economic development, and the more beneficial is price stability.

Illustration. Figures 1, 3 and 4 illustrate equilibria with intermediation frictions.[11] Consumption, behaves similarly in economies with and without intermediation frictions (compare Figures 1 and 2). Working capital constraints bind for $\pi = 1.01$. Differences emerge in agents' savings behavior and intermediaries' lending behavior, reported in Figure 3. As inflation grows, money's purchasing power falls, so agents reduce the money component of their portfolios in favor of bonds. Of course, agents also save less, $m+b$ falls. As inflation rises intermediaries attract savers by raising the interest rate i, so liabilities become more expensive. Since intermediaries must also sustain noninterest expenses, both inflation thresholds fall relative to economies without intermediation frictions (compare Figures 1 and 2). The working capital constraint binds at 1% inflation (versus 5.6%), and the second inflation threshold above which some labor is reallocated to a less efficient use is 60% (versus 64.2%).

Figure 4 shows that γ^* falls and it does so nonlinearly. Intermediaries progressively reduce noninterest costs to offset higher interest rates. Scrapping the more expensive loans implies equilibrium average noninterest costs fall. This means that as inflation rises the fraction of liabilities transformed into short-term commercial loans falls as well. The curve labeled "loans/c" illustrates the negative association between inflation and financial depth. When inflation exceeds 1%, the firm's working capital constraint binds, and the equilibrium wage falls below the marginal value product. Observe also that inflation and financial activity have a nonlinear relationship. The marginal impact of inflation on the asset transformation process is strong at low inflation levels, but it progressively declines.

7 Final remarks

We have assessed the long-run impact of inflation on economic activity in a model with frictions in trade and in financial intermediation. The latter type of frictions is modeled as random noninterest expenses. Inflation magnifies the impact of intermediation frictions and it degrades the equilibrium asset transformation process of intermediaries. If such

[11] The variable γ is uniformly distributed on $[0,1]$, so $G(\gamma^*) = \gamma^*$ and $\int_0^{\gamma^*}(1-\gamma)dG(\gamma) = \gamma^*(1-\frac{\gamma^*}{2})$.

a drop in financial activity results in credit shortages, then real activity declines due to inefficiencies in the allocation of labor. The theory captures three empirical regularities. It predicts a negative association between inflation and equilibrium financial activities. It suggests a declining marginal impact of inflation on financial and real activities—high for low inflation and low for high inflation. Finally, the theory predicts two inflation thresholds exist, each of which is associated to a specific inefficiency in the allocation of labor (within or across productive sectors). These inflation thresholds are negatively related to the severity of intermediation frictions.

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Appendix

Proof of Lemma 1

It is obvious that $c_e = h_e = h_e^*$ with $u_e'(h_e^*) = 1$ are optimal choices for the planner. To find the optimal values c^*, h^*, and θ^* rewrite the remainder of the planner's problem substituting for $c = \frac{2}{\alpha}\left[Y(\frac{\alpha}{2}h\theta) + \frac{\alpha}{2}h(1-\theta)\right]$. We have:

$$\max_{\theta\in[0,1],h\geq 0} u\left(\frac{2}{\alpha}\left(Y(\tfrac{\alpha}{2}h\theta) + \tfrac{\alpha}{2}h(1-\theta)\right)\right) - g(h)$$

Suppose for a moment that we have an interior solution $\theta^* \in (0,1)$. Then, the first order sufficient and necessary conditions for optimality are:

$$(h): \quad u'(c)\,\frac{2}{\alpha}\left[Y'(\tfrac{\alpha}{2}\theta h)\tfrac{\alpha}{2}\theta + \tfrac{\alpha}{2}(1-\theta)\right] = g'(h)$$

$$(\theta): \quad u'(c)\,\frac{2}{\alpha}\left[Y'(\tfrac{\alpha}{2}\theta h)\tfrac{\alpha}{2}h - \tfrac{\alpha}{2}h\right] = 0$$

The second equation implies that θ^* and h^* must satisfy $Y'(\frac{\alpha}{2}\theta^* h^*) = 1$. The first equation implies $u'(c^*) = g'(h^*)$ with c^* given by aggregate feasibility.

Instead, if the corner $\theta^* = 1$ is the optimal planner's choice, then from the second FOC h^* must satisfy $Y'(\frac{\alpha}{2}\theta h^*) > 1$ for all $\theta \leq 1$ (Y is strictly concave). From the first FOC we see that h^* must satisfy $u'(c^*)\,Y'(\frac{\alpha}{2}h^*) = g'(h^*)$, with $c^* = \frac{2}{\alpha}Y(\frac{\alpha}{2}h^*)$. Hence, Assumption 1 implies it is socially beneficial to allocate all labor to market production, so we define $H^* = \frac{\alpha}{2}h^*$.■

Proof of Lemma 3. Suppose $1 \leq i < R$ and fix an arbitrary intermediary $j \in \{1, ..., N\}$. Let $\bar{b} = \frac{b}{N}$ for each j. Clearly $\gamma_j \in [0, \gamma^*]$ with probability $G(\gamma^*)$ and $\gamma_j \notin [0, \gamma^*]$ with probability $1 - G(\gamma^*)$. Let the random indicator function

$$\chi_{[0,\gamma^*]}(\gamma_j) = \begin{cases} 1 & \text{if } \gamma_j \in [0,\gamma^*] \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

where $\chi(\gamma_j) = 1$ denotes the event "intermediary j lends" (omit $[0, \gamma^*]$ when understood). Since γ^* is independent of j and $\{\gamma_j\}_{i=1}^N$ are i.i.d., then$\{\chi(\gamma_j)\}_{j=1}^N$ are i.i.d. Clearly, the expectation $E[\chi(\gamma_j)] = \int_0^{\gamma^*} dG(\gamma) = G(\gamma^*)$, which is simply the probability of lending.

By Kolmogorov's strong law of large numbers we have

$$\tfrac{1}{N}\textstyle\sum_{j=1}^N \chi(\gamma_j) \xrightarrow{b.s.} G(\gamma^*) \quad \text{for } N \to \infty, \tag{35}$$

i.e., $G(\gamma^*)$ is the fraction of intermediaries that lend on each date.

If $\gamma_j \in [0, \gamma^*]$, then intermediary j lends all it has. It transforms all its bonds $\frac{b}{N}$ into a loan, supplying $\frac{b(1-\gamma_j)}{N}$ funds to the firm and sustaining noninterest cost $\frac{b\gamma_j}{N}$. Clearly, $\gamma_j\chi(\gamma_j)$ is a random variable and $E[\gamma_j\chi(\gamma_j)] = \int_0^{\gamma^*} \gamma dG(\gamma)$. Aggregate noninterest costs are $\frac{b}{N}\sum_{j=1}^N \gamma_j\chi(\gamma_j)$, and we have

$$\frac{b}{N}\sum_{j=1}^N \gamma_j\chi(\gamma_j) \xrightarrow{a.s.} b\int_0^{\gamma^*} \gamma dG(\gamma) \quad \text{for } N \to \infty.$$

The aggregate loan supply is $\frac{1}{N}\sum_{j=1}^N b(1-\gamma_j)\chi(\gamma_j)$, and

$$\frac{b}{N}\sum_{j=1}^N (1-\gamma_j)\chi(\gamma_j) \xrightarrow{a.s.} b\int_0^{\gamma^*} (1-\gamma) dG(\gamma) \quad \text{for } N \to \infty.$$

If intermediary j lends, then its profit is $\frac{b}{N}[R(1-\gamma_j)-i]$ (zero otherwise). Define the random variable $[R(1-\gamma_j)-i]\chi(\gamma_j)$, so $E\left[[R(1-\gamma_j)-i]\chi(\gamma_j)\right] = \int_0^{\gamma^*}[R(1-\gamma)-i]dG(\gamma)$. Aggregate profits are $\frac{b}{N}\sum_{j=1}^N [R(1-\gamma_j)-i]\chi(\gamma_j)$. So,

$$\frac{b}{N}\sum_{j=1}^N [R(1-\gamma_j)-i]\chi(\gamma_j) \xrightarrow{a.s} b\int_0^{\gamma^*}[R(1-\gamma)-i]dG(\gamma) \quad \text{for } N \to \infty.$$

Since every agent gets an equal share of profits of each intermediary, the first line of (5) follows.

To find the expected rate of return on bonds r (for an agent who divides equally bonds among all N intermediaries) given in the second line in (5), note that

$$1 + \frac{i-1}{N}\sum_{j=1}^N \chi(\gamma_j) \xrightarrow{a.s} 1 + (i-1)G(\gamma^*) \quad \text{for } N \to \infty.$$

Finally, $1-\alpha$ is the idle population fraction on odd dates. So, intermediaries' compensation to idle agents is the third line of (5).■

Proof of Lemma 4. In a stationary monetary economy $m + b > 0$. Use (9) and notice $\frac{\partial m_c}{\partial c} = -p$ by (7). Letting λ denote the multiplier on the budget constraint, the first line in (21) is $u'(c) + \frac{\partial V_e(m_c,b_e)}{\partial m_c}\frac{\partial m_c}{\partial c} = \lambda p$, i.e., $u'(c) = (1+\lambda)\,p$. If $\lambda = 0$ then $c = \hat{c}$, with $u'(\hat{c}) = p$. If $\lambda > 0$, then $c = \frac{m+b[1-G(\gamma^*)]}{p} < \hat{c}$, because $u'' < 0$. Since $\lim_{c\to 0} u'(c) = \infty$ we have $c > 0$ for $m + b > 0$.

Recall from Lemma 1 that there is a unique c^* (with associated h^* and H^*) that solves the planner's problem; it satisfies $u'(c^*)Y'(H^*) = g'(h^*)$. If the firm is active, then from (17) and (15) we have $pY'(H) \geq \omega$ and $\omega = g'(h)$. So, if $pY'(H) = g'(h)$ and $p = u'(c)$, then $u'(c)Y'(H) = g'(h)$, i.e., the monetary allocation is efficient.■

Proof of Lemma 6. Consider a stationary monetary economy, i.e., $(m, b) = (m', b')$ and $b + m > 0$. The first line of (26) requires

$$\tfrac{\pi}{\beta} \geq 1 + \tfrac{\alpha}{2}[\tfrac{u'(c)}{p} - 1]. \tag{36}$$

Clearly, (22) and $u'' < 0$ imply $\frac{u'(c)}{p} - 1 \geq 0$. So, we must have $\pi \geq \beta$.

Consider i. The second expression in (26) can be rearranged as

$$\tfrac{\pi}{\beta} - G(\gamma^*)i \geq \{1 + \tfrac{\alpha}{2}(\tfrac{u'(c)}{p} - 1)\}[1 - G(\gamma^*)] \tag{37}$$

Suppose $G(\gamma^*) > 0$, i.e., some intermediaries lend. We have three cases:
(a) $i < \frac{\pi}{\beta}$: The LHS of (37) is

$$\tfrac{\pi}{\beta} - G(\gamma^*)i > \tfrac{\pi}{\beta}[1 - G(\gamma^*)] \quad \geq \{1 + \tfrac{\alpha}{2}(\tfrac{u'(c)}{p} - 1)\}[1 - G(\gamma^*)].$$

In the second step we have used (36). Hence, (37) holds strictly and $b = 0$.
(b) $i > \frac{\pi}{\beta}$: The LHS of (37) is $\frac{\pi}{\beta} - G(\gamma^*)i < \frac{\pi}{\beta}[1 - G(\gamma^*)]$. Given (36), then we must have $\frac{\pi}{\beta} > 1 + \frac{\alpha}{2}(\frac{u'(c)}{p} - 1)$ or (37) would not hold. So, $m = 0$.
(c) $i = \frac{\pi}{\beta}$: The expressions in (26) are equivalent, so must hold with equality since we need $m + b > 0$. Hence we can have $b \geq 0$ and $m \geq 0$. Since (36) is an equality, either line in (26) can be rearranged as (27). We have $u'(c) \geq p$. Using (27) we see $\lim_{\pi \searrow \beta} c = \hat{c}$. Since $\frac{\partial c}{\partial \pi} < 0$, the budget constraint must bind, i.e., (28) must hold.■

Proof of Lemma 7

Consider a bond contract. The intermediary chooses i to maximize the expected profit $\int_0^{\gamma^*}[R(1-\gamma) - i]dG(\gamma)$, given $\gamma^* = 1 - \frac{i}{R}$ from (4), the expected return R, and the agents' participation constraints. From Lemma 6, the participation constraint is $i \geq \frac{\pi}{\beta}$. Indeed, $b = 0$ if $i < \frac{\pi}{\beta}$. Thus, the intermediary solves the constrained problem

$$\max_i \left[R \int_0^{\gamma^*} (1 - \gamma) dG(\gamma) - iG(\gamma^*) \right] + \lambda(i - \tfrac{\pi}{\beta}),$$

where $\lambda \geq 0$ is a Kuhn-Tucker multiplier. We have the first order condition

$$R(1 - \gamma^*)G'(\gamma^*)\tfrac{\partial \gamma^*}{\partial i} - G(\gamma^*) - iG'(\gamma^*)\tfrac{\partial \gamma^*}{\partial i} + \lambda = 0.$$

Since $\gamma^* = 1 - \frac{i}{R}$ we get $-G(\gamma^*) + \lambda = 0$. Clearly, $G(\gamma^*) \geq 0$, so the $\lambda > 0$ and $i = \frac{\pi}{\beta}$.■

Next, we list a useful result.

Lemma 14 *Define* $R(H) = \frac{Y(H)}{Y'(H)H}$. *We have:*

(a) $R(H) > 1$ *for all* $H > 0$,

(b) $\lim_{H\to 0} R(H) = 1$, *and*

(c) $R'(H) \geq 0$ *for all* $H \leq H^*$.

Proof of Lemma 14

(a) Since $Y(H)$ is strictly concave $Y(H_0) < Y(H) + Y'(H)(H_0 - H)$ for all $0 \leq H_0 < H$. Since $Y(0) = 0$ let $H_0 = 0$, so $Y(H) > Y'(H)H$. By assumption $Y'(H) > 1$ for all $H \leq H^*$. Also, $H \leq H^*$ for every $\pi > \beta$, by Lemma 4. Hence, $\frac{Y(H)}{H} > Y'(H) > 1$ for all $H \in [0, H^*]$.

(b) Applying l'Hospital rule we have $\lim_{H\to 0} R(H) = \frac{\lim_{H\to 0} Y'(H)}{\lim_{H\to 0} Y'(H)+Y''(H)H} = 1$,

(c) Production elasticity is $\frac{Y'(H)/Y(H)}{1/H} = \frac{1}{R(H)}$; it is non-increasing, by assumption.■

Proof of Proposition 9

Let $\pi > \beta$. If all labor is supplied to the firm, then we have $H = H(h) = \frac{\alpha}{2}h$, $c = c(h) = \frac{2}{\alpha}Y(H)$, and p must satisfy (27). In what follows we omit the argument h from $H(h)$ when understood. Let $p = p(h)$ with

$$p(h) = \frac{\alpha\beta u'(\frac{2}{\alpha}Y(H))}{\alpha\beta + 2(\pi - \beta)}.$$

We have $p'(h) < 0$ because $u'' < 0$. We also have $m = p(h)c > 0$ for all $\pi > \beta$, since $G(\gamma^*) = 1$ for all $\gamma^* \geq 0$.

Suppose $b > 0$ and $\gamma^* \geq 0$. Using $\omega = g'(h)$ from (15), the inequality in (17) is

$$g'(h) \leq p(h)Y'(H). \tag{38}$$

From (31) we have $R = \frac{pY(H)}{\omega H}$. From (4) and (29) we have $\gamma^* = 1 - \frac{\pi}{\beta}\frac{1}{R}$. Thus we define the following:

1. if $g'(h) = \omega = p(h)Y'(H)$, then $R = R(h) = \frac{Y(H)}{Y'(H)H}$ and $\gamma^* = \gamma(\pi, h) = 1 - \frac{\pi}{\beta}\frac{1}{R(h)}$; (here $p < \omega$)

2. if $g'(h) = \omega < p(h)Y'(H)$, then $R = \hat{R}(h) = \frac{p(h)Y(H)}{g'(h)H}$ and $\gamma^* = \hat{\gamma}(\pi, h) = 1 - \frac{\pi}{\beta}\frac{1}{\hat{R}(h)}$; (here $p < \omega$)

3. if $\omega = p$ then $R = \breve{R}(h) = \frac{Y(H)}{H}$ and $\gamma^* = \breve{\gamma}(\pi, h) = 1 - \frac{\pi}{\beta}\frac{H}{Y(H)}$.

Clearly, $\hat{R}(h) > R(h)$ because $\omega < p(h)Y'(H)$. This implies $\hat{\gamma}(\pi, h) > \gamma(\pi, h)$. Lemma 14 indicates that $R'(h) \geq 0$. Thus, $\frac{\partial \gamma(\pi,h)}{\partial h} \geq 0$. Also (omitting the arguments) we have

$$\hat{R}'(h) = p' \times \frac{Y}{g'H} + \frac{\partial(Y/H)}{\partial h} \times \frac{p}{g'} - \frac{g''}{g'} \times \frac{pY}{g'H} < 0$$

since $p'(h) < 0$, $g''(h) > 0$, and $\frac{Y(H)}{H}$ falls in H (by concavity). Hence, $\frac{\partial \hat{\gamma}(\pi,h)}{\partial h} < 0$.

We proceed by discussing equilibrium with intermediation, unconstrained, constrained for $p < \omega$ and for $p = \omega$.

Unconstrained equilibrium. Denote by $h(\pi)$ the unique value $h > 0$ that satisfies

$$g'(h) - p(h)Y'(H) = 0. \tag{39}$$

To prove that $h(\pi)$ exists and is unique for all $\pi > \beta$ notice that for $\pi = \beta$ (39) is satisfied by $h = h^*$, because $p(h) = u'(c(h))$ and $g'(h^*) = u'(c(h^*))Y'(H^*)$ by earlier assumption. Thus, $h(\beta) = h^*$. Then, use the Intermediate Value Theorem. Finally, one can apply the Implicit Function Theorem to obtain $h'(\pi) < 0$. So $h(\pi) \leq h^*$ for all $\pi \geq \beta$.

For $H > 0$ we need $p \leq \omega$. That is, we need

$$g'(h) \geq p(h). \tag{40}$$

Recall $h(\pi) \leq h^*$, $h'(\pi) < 0$ and $Y'(H) > 1$ for all $h \leq h^*$. Hence, if $h = h(\pi)$, then (40) holds with strict inequality. So, if $\omega = pY'(H)$, then $p < \omega$ for all $\pi > \beta$.

Given $h = h(\pi)$, loan market clearing (30) pins down

$$b = \omega H = g'(h)H.$$

So $\frac{\partial b}{\partial \pi} < 0$. Clearly, in this economy without noninterest costs, loans are made if $\gamma^* \geq 0$. So, we must verify that if $h = h(\pi)$, then $\gamma^* \geq 0$ for some $\pi > \beta$. Recall that if $\omega = pY'(H)$, then $R = R(H)$ and $\gamma^* = \gamma(\pi, h)$.

Given $h = h(\pi)$, then we have

$$\frac{\partial \gamma^*}{\partial \pi} = \frac{\partial \gamma(\pi, h)}{\partial \pi} + \frac{\partial \gamma(\pi, h)}{\partial h}\frac{\partial h}{\partial \pi} < 0$$

because $h'(\pi) < 0$, $\frac{\partial \gamma(\pi,h)}{\partial \pi} < 0$ and we have established $\frac{\partial \gamma(\pi,h)}{\partial h} \geq 0$. Since $R(H) \geq 1$, $h(\pi) \in (0, h^*)$, and $h'(\pi) < 0$, it follows that there is a unique value $\hat{\pi} > \beta$, that satisfies $\frac{\hat{\pi}}{\beta}\frac{1}{R(H)} = 1$. Consequently, $\gamma(\pi, h) = 0$ if $\pi = \hat{\pi}$ and $\gamma(\pi, h) > 0$ for $\pi < \hat{\pi}$. We conclude that if $\pi \in (\beta, \hat{\pi}]$, then there exists a unique $h = h(\pi) \in (0, h^*)$ such that $\omega = pY'(H) > p$ and $\gamma^* \geq 0$. Finally, notice that $\frac{b}{m} = \frac{g'(h)H}{pc} = \frac{\alpha}{2R(H)}$, so $\frac{\partial(b/m)}{\partial \pi} < 0$.

To complete the proof of existence we must verify that all equilibrium conditions are satisfied. We have proved already that there is a unique pair $m, b > 0$ that satisfies (27), (32), (30), and (28) given that (18) holds with equality. The only equations left to satisfy are money market clearing (2) and (3), which pin down the nominal prices $\{p_{e,t}, p_{o,t}\}_{t=0}^{\infty}$. Finally, (5) and the constraint in (6) determine a unique $h_s \geq 0$ for all s, given assumptions on the function u_e ensuring that $c_e = (u_e')^{-1}(1)$ is large enough. In these circumstances it is a matter of algebra to show that all markets clear.

Constrained equilibrium with $p < \omega$. Consider $\pi > \hat{\pi}$. Here, $\gamma(\pi, h) < 0$ when $h = h(\pi)$, and therefore $\omega = pY'(H)$ cannot be an equilibrium. So, consider a constrained equilibrium where $\gamma^* = 0$, i.e., intermediaries are indifferent to making loans. In this equilibrium, $p = p(h)$ and $\omega = g'(h)$, as before but (38) must be an inequality since $\omega < pY'(H)$ in constrained equilibrium. Thus, we have $R = \hat{R}(H)$ and $\gamma^* = \hat{\gamma}(\pi, h)$. If an equilibrium h exists for $\pi > \hat{\pi}$, then it must solve $\hat{\gamma}(\pi, h) = 0$.

Clearly, as $\pi \to \hat{\pi}$, then $h = h(\pi)$ satisfies $\hat{\gamma}(\pi, h) = \gamma(\pi, h) = 0$. Again, one can use the Intermediate Value Theorem to demonstrate that there is a unique positive $h = \hat{h}(\pi)$ that satisfies $\hat{\gamma}(\pi, h) = 0$ for all $\pi > \hat{\pi}$. Since $\frac{\partial \hat{\gamma}(\pi,h)}{\partial h} < 0$ and $\frac{\partial \hat{\gamma}(\pi,h)}{\partial \pi} < 0$, then $\hat{h}'(\pi) < 0$. It should also be clear that since $\hat{\gamma}(\pi, h) > \gamma(\pi, h)$ for all h, then $\gamma(\pi, h) < 0 = \hat{\gamma}(\pi, h)$ for $h = \hat{h}(\pi)$. Finally, $\hat{h}(\pi) < h(\pi)$ because $h(\pi)$ solves $g'(h) = p(h)Y'(H)$, while $\hat{h}(\pi)$ must satisfy $g'(h) < p(h)Y'(H)$ (and $g'' > 0$). We conclude that for all $\pi > \hat{\pi}$, we have a unique $h = \hat{h}(\pi) < h(\pi)$ that satisfies $g'(h) < p(h)Y'(H)$ and $\hat{\gamma}(\pi, h) = 0$.

Note that we need $p(h) \leq g'(h)$ for $H > 0$. If $h = \hat{h}(\pi)$, then (40) is violated for π sufficiently large. This is because $u'(0) = \infty > g'(0)$. So, there exists a unique value $\bar{\pi} > \hat{\pi}$ that satisfies (40) with equality, when $h = \hat{h}(\pi)$. We conclude that if $\pi \in (\hat{\pi}, \bar{\pi}]$, then there is a unique value $h = \hat{h}(\pi) < h(\pi)$ such that $p(h) \leq g'(h) < p(h)Y'(H)$, $b > 0$ and $\gamma^* = \hat{\gamma}(\pi, h) = 0$. We have $\frac{b}{m} = \frac{\alpha}{2\hat{R}(H)}$, so $\frac{\partial(b/m)}{\partial \pi} > 0$ because $\frac{\partial \hat{R}(H)}{\partial h}\frac{\partial h}{\partial \pi} > 0$.

Constrained equilibrium with $p = \omega$. Now let $\pi > \bar{\pi}$. The above discussion implies that if $h = \hat{h}(\pi)$, i.e., $\gamma^* = \hat{\gamma}(H) = 0$ and $\omega < pY'(H)$, then $p(h) > \omega(h) = g'(h)$. Clearly, if $h > \hat{h}(\pi)$, then $\hat{\gamma}(H) < 0$. And if $h < \hat{h}(\pi)$, then $\hat{\gamma}(H) > 0$, $\omega < pY'(H)$, but $p(h) > \omega(h)$ since $p'(h) < 0 < g'(h)$. We conclude that for all values of $\pi > \bar{\pi}$ then we cannot have $H = H(h) = \frac{\alpha}{2}h$. Consequently, H must fall. We need to look for an equilibrium in which the firm hires only a portion of available labor to satisfy $\omega = p$, while all other workers engage in home production. For convenience we will say that $H = H(\theta, h) = \theta\frac{\alpha}{2}h$ with $\theta \in (0, 1]$.

It follows that odd-date consumption must satisfy $c = c(\theta, h)$ with

$$c(\theta, h) = \frac{2}{\alpha} Y(\theta \frac{\alpha}{2} h) + (1-\theta)h.$$

Therefore, now we have

$$p(\theta, h) = \frac{\alpha\beta u'(c(\theta, h))}{\alpha\beta + 2(\pi - \beta)}.$$

Now we have that since $p(\theta, h) = \omega$, then $R = \breve{R}(h) = \frac{Y(H)}{H}$ and $\gamma^* = \breve{\gamma}(\pi, h) = 1 - \frac{\pi}{\beta}\frac{H}{Y(H)}$. So, in equilibrium $h = \bar{h}(\theta, \pi)$ where $\bar{h}(\theta, \pi)$ is the unique solution to $\breve{\gamma}(\pi, h) = 0$. Given $H = H(\theta, h) = \theta\frac{\alpha}{2}h$ we have that for any $\theta \in (0, 1]$ $\frac{\partial \bar{h}(\theta,\pi)}{\partial \pi} < 0$. Hence, if $\pi > \bar{\pi}$, then there exists a unique $h = \bar{h}(\theta, \pi) < \hat{h}(\pi) < h(\pi)$.

The value θ is found by solving $p = \omega$, i.e.,

$$p(\theta, \bar{h}(\theta, \pi)) = g'(\bar{h}(\theta, \pi)),$$

We have that $\theta < 1$.■

Proof of Proposition 11

The proof follows that of Proposition 9. Consider $\pi > \beta$ and suppose $b, m \geq 0$ and $m + b > 0$. As before, with financial intermediation and odd-date market production we have $c = \frac{2}{\alpha}Y(H)$, $H = \frac{\alpha}{2}h$, $p = p(h)$, $\omega = g'(h)$, and (38) must hold.

Unconstrained equilibrium. Suppose $b > 0$, $m \geq 0$ and $g'(h) = pY'(H)$. In this case $R = R(H)$ and $\gamma^* = \gamma(H)$. As shown earlier, $h = h(\pi)$ satisfies (38) with equality, in which case $p(h) < \omega(h)$ for all $\pi > \beta$. Earlier result imply $\frac{\partial \gamma^*}{\partial \pi} < 0$, and $\gamma(H) = 0$ for $\pi = \hat{\pi}$. Loan market clearing (30) is

$$g'(h)H = b\int_0^{\gamma^*}(1-\gamma)dG(\gamma). \tag{41}$$

Given $h = h(\pi)$, the budget constraint (28) is

$$b = \frac{pc-m}{1-G(\gamma^*)} \quad = \left[\frac{2g'(h)Y(H)}{\alpha Y'(H)} - m\right]\frac{1}{1-G(\gamma^*)}$$

rearranged, using (41), as $m = hg'(h)\phi(h, \pi)$ with

$$\phi(h, \pi) = R(H) - \frac{\alpha}{2}\frac{1-G(\gamma^*)}{\int_0^{\gamma^*}(1-\gamma)dG(\gamma)}. \tag{42}$$

Clearly, if $\phi(h, \pi) \geq 0$ then $m \geq 0$. So, consider the function ϕ.

We know that $R(H) > 1$ and $\frac{\partial R(H)}{\partial \pi} \leq 0$. The partial derivative relative to π of $-\frac{\alpha}{2}\frac{1-G(\gamma^*)}{\int_0^{\gamma^*}(1-\gamma)dG(\gamma)}$ is proportional to

$$-\left\{-G'(\gamma^*)\int_0^{\gamma^*}(1-\gamma)dG(\gamma) - [1-G(\gamma^*)](1-\gamma^*)G'(\gamma)\right\}\frac{\partial \gamma^*}{\partial \pi} < 0.$$

Thus $\frac{\partial \phi(h,\pi)}{\partial \pi} < 0$. Notice that $h = h^* = h(\beta)$ for all $\pi \leq \beta$ (from (22)) since no buyer wishes to consume more than c^*. So, $\lim_{\pi\to 0}\gamma(H) = 1$, hence $\lim_{\pi\to 0}\phi(h,\pi) > 0$. Also, $\lim_{\pi\to\hat{\pi}}\gamma(H) = 0$ and so $\lim_{\pi\to\hat{\pi}}\phi(h,\pi) < 0$. From the Intermediate Value Theorem, it follows that there is a value $\tilde{\pi} \in (0,\hat{\pi})$ such that if $\pi \leq \tilde{\pi}$, then $\phi(h,\pi) \geq 0$. One can show there is a unique value $\tilde{\alpha} > 0$ that solves $\phi(h^*,\beta) = 0$ (use the Intermediate Value Theorem). Since $\frac{\partial \phi(h^*,\beta)}{\partial \alpha} < 0$, it follows that $\tilde{\pi} > \beta$ for all $\alpha < \tilde{\alpha}$. So, if $\alpha \in (0,\tilde{\alpha})$, then for all $\pi \in (\beta,\tilde{\pi}]$ there is a unique value $h = h(\pi) > 0$ that satisfies $g'(h) = p(h)Y'(H) > p(h)$, $b > 0$, $m \geq 0$ and $\gamma^* = \gamma(H) > 0$.

Constrained equilibrium. Now consider $\pi > \tilde{\pi}$. If $h = h(\pi)$ then $m < 0$. Therefore, $b > 0$ and $\omega = pY'(H)$ cannot be an equilibrium. So, consider a constrained equilibrium with $b > m = 0$ and $g'(h) < p(h)Y'(H)$. As before, $p = p(h)$ and $\omega = g'(h)$, but now $R = \hat{R}(H)$ and $\gamma^* = \hat{\gamma}(H)$.

We will prove that, for some $\pi > \tilde{\pi}$, there exists a unique h that solves $m = 0$, i.e.,

$$\hat{\phi}(h,\pi) \equiv \quad \hat{R}(H) - \frac{\alpha}{2}\frac{1-G(\gamma^*)}{\int_0^{\gamma^*}(1-\gamma)dG(\gamma)} = 0.$$

Using earlier results, we can establish that there is a unique $h = \tilde{h}(\pi) \in (0,h^*)$ such that $\hat{\phi}(h,\pi) = 0$.

Notice that $\hat{\phi}(h,\tilde{\pi}) = \phi(h,\tilde{\pi}) = 0$, which is when $\omega = g'(h) = pY'(H)$ and $m = 0$, so that $h = \tilde{h}(\tilde{\pi}) = h(\tilde{\pi})$ and $\hat{R}(H) = R(H)$. We also know that $\phi(h,\pi) < 0$ for all $\pi > \tilde{\pi}$. Therefore we must have $\tilde{h}(\pi) < h(\pi)$ since $\hat{R}'(H) < 0$. This immediately implies that if $h = \tilde{h}(\pi)$ then we have $\omega = g'(h) < pY'(H)$ and $\hat{R}(H) > R(H)$. Also, if $h = \tilde{h}(\pi)$ then $\gamma^* > 0$ for all $\pi \in (\tilde{\pi},\hat{\pi}]$; indeed, if $h = h(\pi) > \tilde{h}(\pi)$ then we have $\gamma^* \geq 0$ for all $\pi \in (\tilde{\pi},\hat{\pi}]$.

Thus, suppose that $h = \tilde{h}(\pi)$. Since $\hat{R}'(H) < 0$ we have that $\frac{\partial \gamma^*}{\partial h} \leq 0$. The derivative of the second (negative) term of $\hat{\phi}$ is proportional to

$$-\left\{-G'(\gamma^*)\int_0^{\gamma^*}(1-\gamma)dG(\gamma) - [1-G(\gamma^*)](1-\gamma^*)G'(\gamma)\right\}\frac{\partial \gamma^*}{\partial h} \leq 0$$

So, $\frac{\partial \hat{\phi}}{\partial h} \leq 0$. It is immediate that $\frac{\partial \hat{\phi}}{\partial \pi} < 0$ since $\frac{\partial \gamma^*}{\partial \pi} < 0$. It follows that $\tilde{h}'(\pi) < 0$ (implicit function theorem). This last result implies there exists a value $\pi_1 > \tilde{\pi}$ such that $p(h) \geq \omega = g'(h)$ for all $\pi \geq \pi_1$. Therefore $H = 0$ for all $\pi > \pi_1$.

Finally, when $h = \tilde{h}(\pi)$ then $\gamma^* > 0$ for $\pi = \hat{\pi}$ and there is a unique $\pi_2 > \hat{\pi}$ such that $\gamma^* \leq 0$ for all $\pi \geq \pi_2$. So $H = 0$ for all $\pi > \pi_2$. Notice that $\pi_1 \leq \pi_2$. To see why this is so, consider the contradictory statement $\pi_1 > \pi_2$. In this case, for all $\pi \in (\pi_2, \pi_1)$ we would have $\gamma^* \leq 0$ and $p(h) < \omega = g'(h)$. However, when $\gamma^* \leq 0$ no loans are made and so we must have $\omega = 0$, a contradiction. Thus let $\bar{\pi} = \min(\pi_1, \pi_2) = \pi_1$.

If $\pi \in (\tilde{\pi}, \bar{\pi}]$, then there is a unique $h = \tilde{h}(\pi) < h(\pi)$ such that $p \leq \omega = g'(h) < pY'(H)$, $b > m = 0$ and $\gamma^* \geq 0$. We have $\tilde{h}'(\pi) < 0$. Notice that $\frac{\partial \gamma^*}{\partial \pi} = \frac{\partial \gamma^*}{\partial H}\frac{\partial H}{\partial \pi} < 0$.

Constrained equilibrium without $p = \omega$. This proof is as in Proposition 9.

Finally, we verify that the association between inflation and financial activity is non-linear. Denote $R' = \frac{\partial R}{\partial \pi}$, notice that $\frac{\partial \gamma^*}{\partial \pi} = -\frac{R - \pi R'}{\beta R^2}$. Above, we have proved that $\frac{\partial \gamma^*}{\partial \pi} < 0$. From the proof of Proposition 9**,** we know that in unconstrained equilibrium $R = R(h)$ so $R' \leq 0$. In constrained equilibrium $R = \hat{R}(h)$, so $R' > 0$. Therefore $\frac{\partial \gamma^*}{\partial \pi}$ has a greater absolute value in unconstrained equilibrium, than in constrained equilibrium.■

Proof of Proposition 13

Let G_2 first-order-stochastically dominate G_1, i.e., $G_2(\gamma) < G_1(\gamma)$ for each $\gamma \in (0, 1)$. We say that financial intermediation frictions intensify when we move from G_1 to G_2. If $h = h(\pi)$, then $\tilde{\pi}$ satisfies $\phi(h, \pi) = 0$ (see (42)). The absolute value of the second (negative) term in $\phi(h, \pi)$ is larger when $G = G_2$ than when $G = G_1$. This is because $1 - G_2(\gamma^*) > 1 - G_1(\gamma^*)$ and

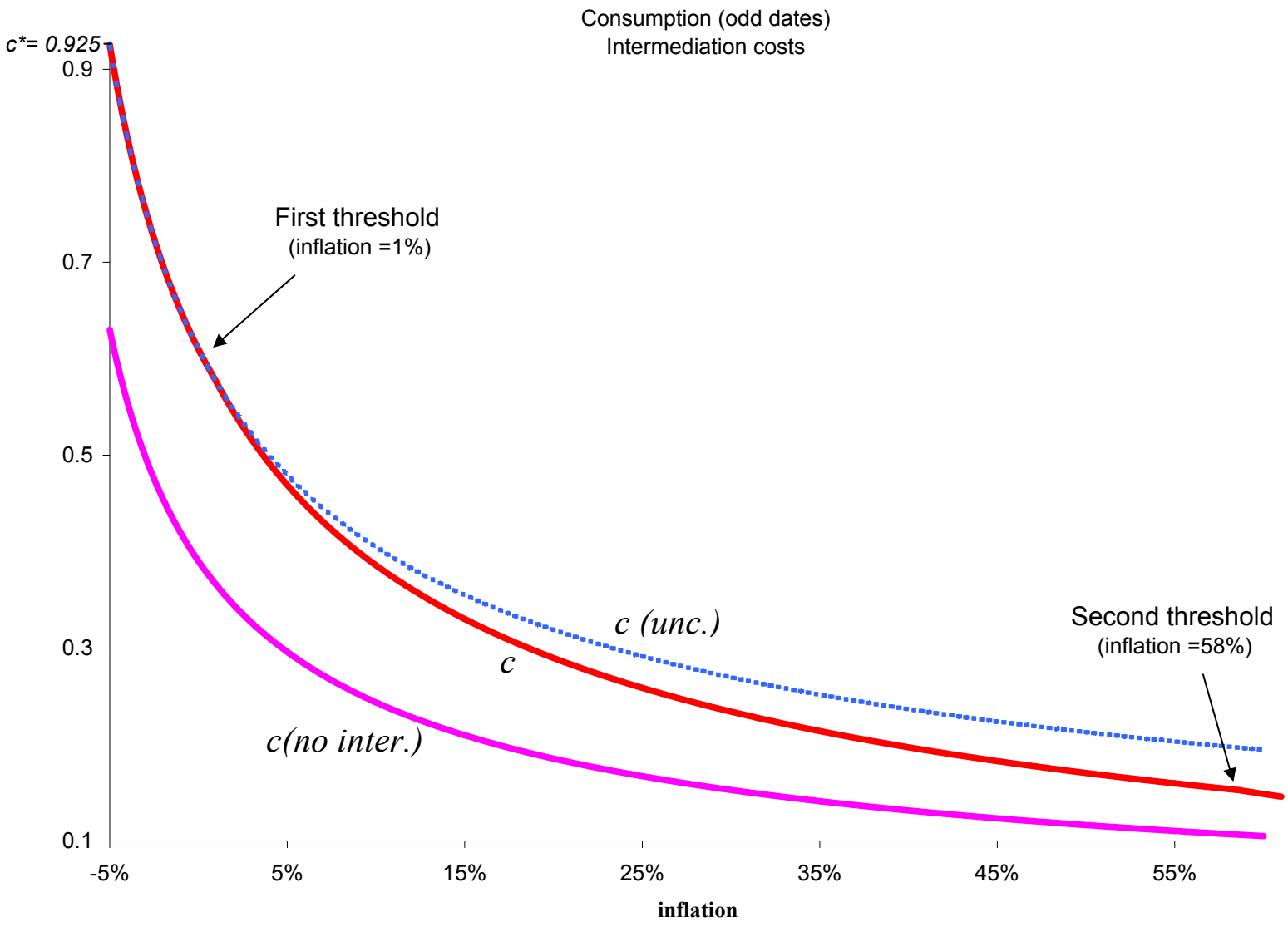
$$\int_0^{\gamma^*} (1 - \gamma) dG_2(\gamma) < \int_0^{\gamma^*} (1 - \gamma) dG_1(\gamma).$$

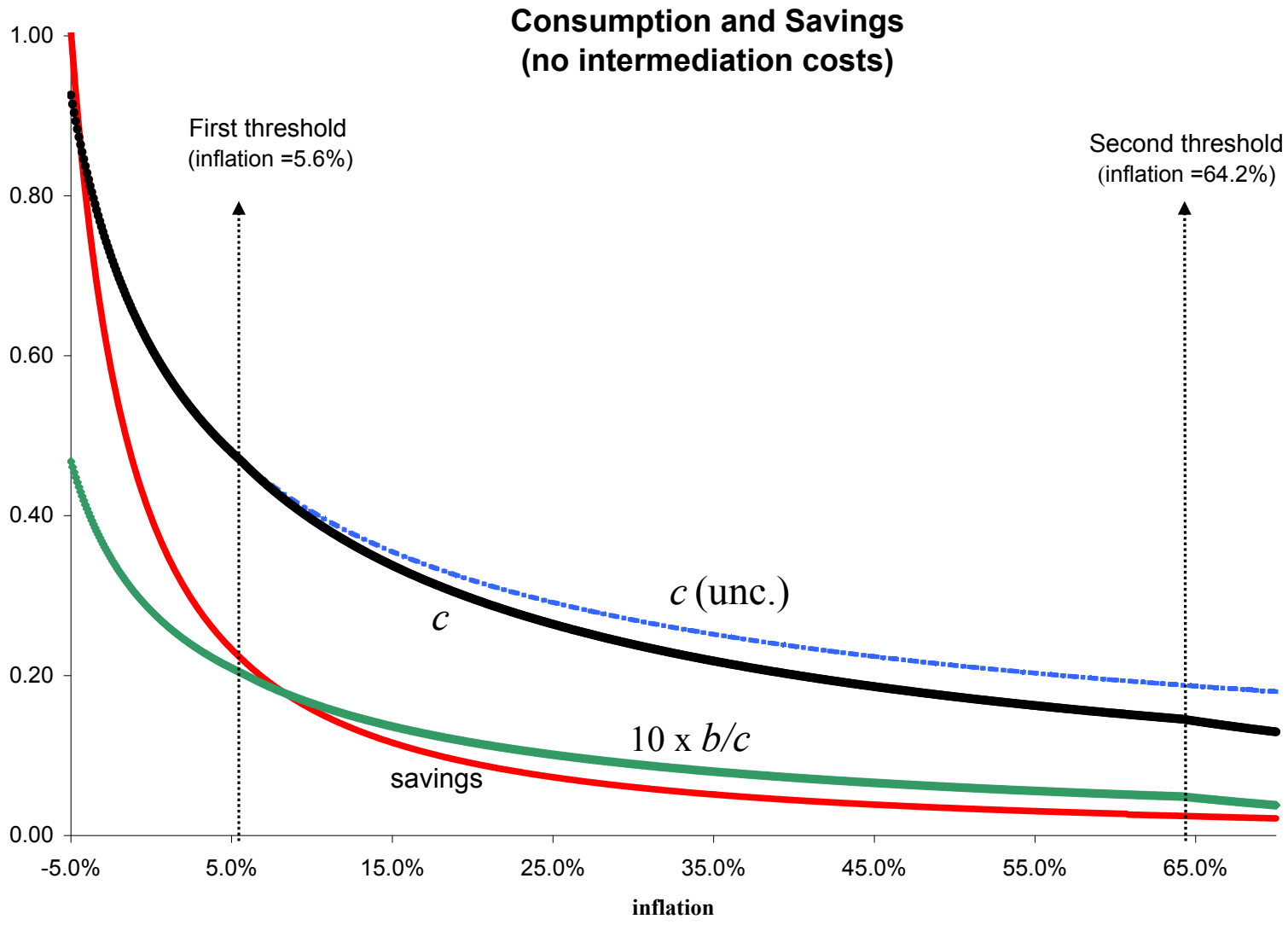
Denote by $\tilde{\pi}_i$ the unique value of π that satisfies $\phi(h, \tilde{\pi}_i) = 0$ when $G = G_i$, for $i = 1, 2$. Recall that $\frac{\partial \phi(h,\pi)}{\partial \pi} < 0$ for all $\pi > \beta$. It follows that $\tilde{\pi}_2 < \tilde{\pi}_1$.

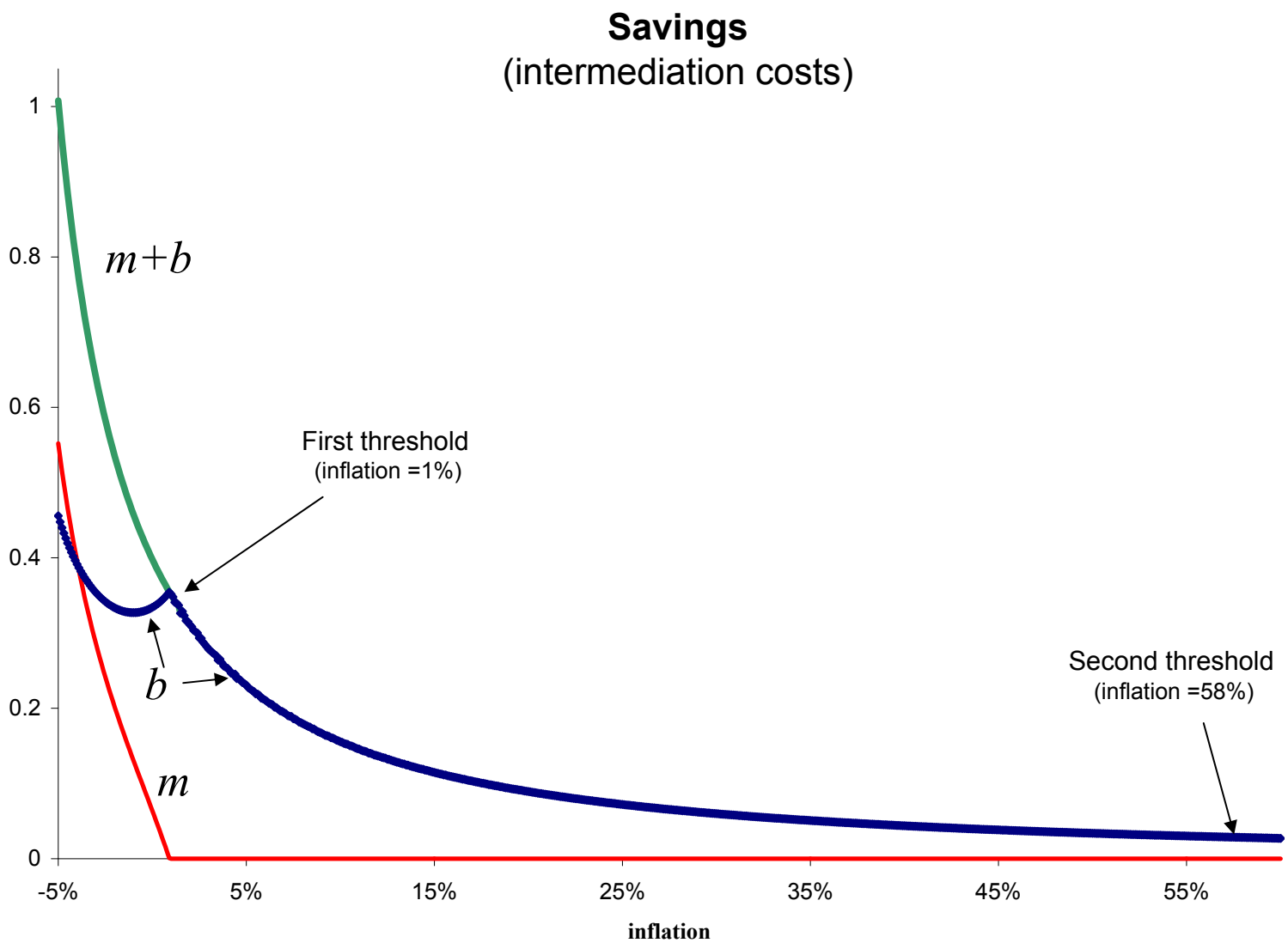
Next, examine the relationship between degree of financial market frictions and $\bar{\pi}$. Recall that $\bar{\pi} = \pi_1$ as defined in the proof of proposition 11. From that proof we know that for any $\pi > \tilde{\pi}$, the equilibrium individual labor effort $h = \tilde{h}(\pi)$ solves

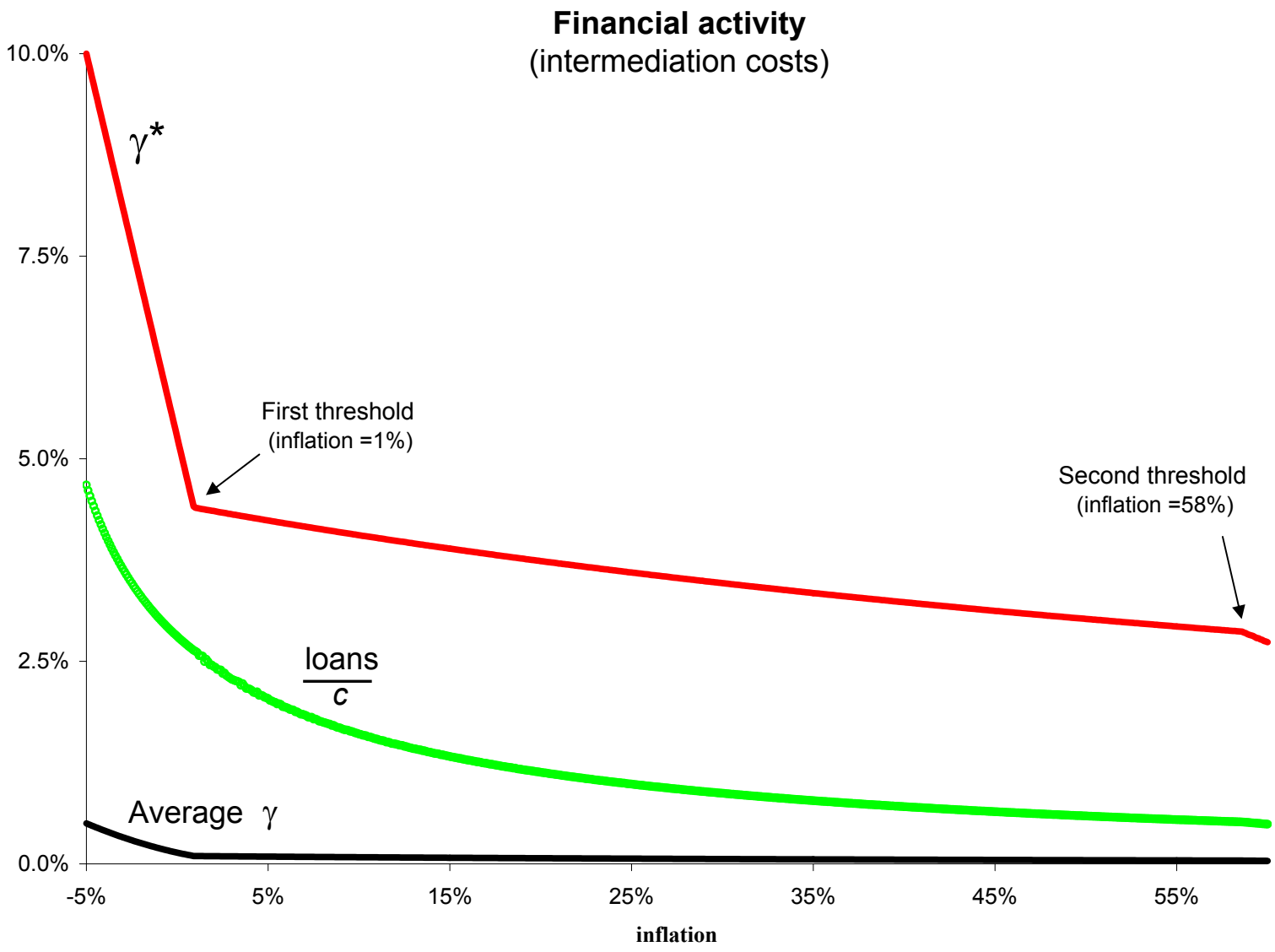
$$\hat{\phi}(h, \pi) = \hat{R}(H) - \frac{\alpha}{2} \frac{1 - G(\gamma^*)}{\int_0^{\gamma^*} (1 - \gamma) dG(\gamma)} \quad = 0.$$

Since $\hat{R}'(H) < 0$, we have that $\tilde{h}(\pi; G_2) < \tilde{h}(\pi; G_1)$. In addition we have that $g''(h) > 0$ and $p'(h) < 0$ and $\frac{\partial p(\tilde{h}(\pi))}{\partial \pi} > 0$. Hence, $p(\tilde{h}(\pi; G_2)) > p(\tilde{h}(\pi; G_1))$ and therefore $\bar{\pi}(G_2) < \bar{\pi}(G_1)$. ■


Consumption (odd dates)
Intermediation costs
c*= 0.925
0.9
0.7
0.5
0.3
0.1
First threshold
(inflation =1%)
Second threshold
(inflation =58%)
c (unc.)
c
c(no inter.)
-5%
5%
15%
25%
35%
45%
55%
inflation


Consumption and Savings
(no intermediation costs)
First threshold
(inflation =5.6%)
Second threshold
(inflation =64.2%)
1.00
0.80
0.60
0.40
0.20
0.00
-5.0%
5.0%
15.0%
25.0%
35.0%
45.0%
55.0%
65.0%
inflation
c (unc.)
c
10 x b/c
savings


Savings
(intermediation costs)
1
0.8
0.6
0.4
0.2
0
m+b
First threshold
(inflation =1%)
b
m
Second threshold
(inflation =58%)
-5%
5%
15%
25%
35%
45%
55%
inflation


Financial activity
(intermediation costs)
10.0%
7.5%
5.0%
2.5%
0.0%
γ*
First threshold
(inflation =1%)
Second threshold
(inflation =58%)
loans
c
Average γ
-5%
5%
15%
25%
35%
45%
55%
inflation