Introducing Financial Frictions and Unemployment into a Small Open Economy Model*

Lawrence J. Christiano† Mathias Trabandt‡
Karl Walentin§

Sveriges Riksbank Working Paper Series No. 214

April 21, 2009

Abstract

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? What are the quantitative effects of increased financial risk on output and inflation? What drives the variation in the intensive and extensive margin of labor supply? What are the spillover effects of financial market disturbances to unemployment? In order to address these questions we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2008) in two important dimensions. First, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2007). Second, we include the search and matching framework of Mortensen and Pissarides (1994) and Gertler, Sala and Trigari (2009) into a small open economy model. We make a theoretical contribution by incorporating endogenous job separation in this rich framework. Finally, we estimate the full model using Bayesian techniques and illustrate the importance of the various frictions.

Keywords: small open economy, DSGE, financial frictions, labor market frictions, unemployment.
JEL codes: E0, E3, F0, F4, G0, G1, J6.

*Thanks to Malin Adolfson, Christopher Erceg, Jesper Hansson, Simon Gilchrist, Jesper Lindé, Skander Van den Heuvel, Mattias Villani, Peter Welz and seminar participants at Sveriges Riksbank, Oesterreichische Nationalbank, CCBS/WGEM workshop on 'Financial Sector in Macro-Forecasting', SED (Cambridge), ESEM (Milan), the Federal Reserve Board, CIED workshop on 'Financial Markets, International Capital Flows and Exchange Rates', University of St Andrews and University of Amsterdam. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

†Northwestern University, Department of Economics, 2001 Sheridan Road, Evanston, Illinois 60208, USA. Phone: +1-847-491-8231. E-mail: l-christiano@northwestern.edu.

‡Sveriges Riksbank, Research Division, 103 37 Stockholm, Sweden. Phone: +46-8-787 0438. E-mail: mathias.trabandt@riksbank.se

§Sveriges Riksbank, Research Division, 103 37 Stockholm, Sweden. Phone: +46-8-787 0491. E-mail: karl.walentin@riksbank.se

Contents

1. Introduction

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? In particular, what are the quantitative effects of increased financial risk on output and inflation? Furthermore, what drives the variation in the intensive and extensive margin of labor supply respectively? Moreover, what are the spillover effects of financial market disturbances to unemployment in a small open economy? In order to address these questions we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007, 2008) in two important dimensions.

First, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce 'entrepreneurs'. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, 'banks', can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are 'bankrupt'. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

Second, we include the labor market search and matching framework of Mortensen and Pissarides (1994), Hall (2005a,b,c), Shimer (2005a,b), Gertler and Trigari (2009, henceforth GT), Gertler, Sala and Trigari (2008) and Christiano, Ilut, Motto, and Rostagno (2007) into the small open economy model. We integrate this into our specific framework - which includes capital and monetary factors - following the labor market version of Gertler, Sala and Trigari (2008) (henceforth GST). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employed, it has unemployment and vacancies.

The labor market in our model is a modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model.

An important step forward is that we allow for endogenous separation of employees from their jobs. This has been done earlier, e.g. by den Haan, Ramey and Watson (2000), but not in a rich monetary DSGE model. The importance of time-varying separation rates is strongly motivated by empirical evidence, although for the U.S., by Fujita and Ramey (2007). For an analysis that focus entirely on the labor market and more fully document job separation in this type of model see Christiano, Trabandt and Walentin (2009).

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services. The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by 'employment agencies'.

Each employment agency retains a large number of workers. At the beginning of the period a fraction of workers is randomly selected to separate from the firm and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set. Then idiosyncratic shocks to workers' productivities are realized and endogenous separation decisions are made. A nice feature of this modelling is the high degree of symmetry with the modelling of entrepreneurial idiosyncratic risk and bankruptcy.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods. Each employment agency is permanently allocated to one of N different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same indexing rule used in our baseline model. The intensity of labor effort is determined by equating the worker's marginal cost to the agency's marginal benefit.

In addition to the main two new features described above, we integrate the following other new features into the model compared to Adolfson, Laséen, Lindé and Villani (2005, 2007, 2008): imported goods are directly used for export production, unit-root investment specific technological progress, working capital loans for all monopolists, possible price and wage dispersion in steady state, capital tax timing and allowances as well as the specification and estimation of a VAR that represents the foreign economy. These new features turn out to be useful when taking the model to the data.

We estimate the full model which contains the financial frictions as well as the and labor market frictions with Bayesian techniques.

The paper is organized as follows. In section 2 we describe the baseline small open economy model. Section 3 introduces financial frictions while section 4 incorporates labor market search and matching frictions into the model. Section 5 contains the estimation of the full model which include both financial and labor market frictions. Finally, section 6 concludes.

2. The Baseline Small Open Economy Model

This section describes an extension of the model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007, 2008) (henceforth ALLV), and presents a way to introduce financial frictions and search and matching in the labor market. Our baseline model makes some changes on the ALLV model:

- Exports are produced by using homogeneous imported goods in addition to homogeneous domestically produced goods.
- The price of investment goods is treated as a random variable with a unit root. Thus, growth in the model is driven by two independent unit root processes, one for neutral technology shocks and the other for technology shocks in the production of investment goods.
- Capital maintenance costs are deducted from capital income taxes, and physical depreciation is deducted at historic cost.
- The capital income tax rate is realized at the time the investment decision is made, not at the time when the payoff on investment is realized.

- Wages are indexed to the steady state growth rate of the economy, rather than to the current realization of technology shocks.

- All producers of specialized goods are assumed to require working capital loans.

- We allow for partial price and wage indexation which implies price and wage dispersion in steady state.

- The VAR that represents the foreign economy is estimated jointly with all other parameters in the Bayesian estimation.

2.1. Scaling of Variables

We adopt the following scaling of variables. The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{2.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods and C_t are final consumption goods) are scaled by z_t^+. Government consumption, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled z_t^+. Also, v_t is the shadow value in utility terms to the household of domestic currency and $v_t P_t$ is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by z_t^+ to induce stationarity. $\tilde{P}_t$ is the within-sector relative price of a good. w_t denotes the ratio between the (Nash) wage paid to workers $\tilde{W}_t$ and the "rental rate of homogenous labor" W_t in the labor market model. Finally, the expected discounted future surplus of a match to an employment agency, D_t^j is scaled like most other nominal variables. Thus,

$$
\begin{aligned}
k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ i_t^m = \frac{I_t^m}{z_t^+} \\
c_t^m &= \frac{C_t^m}{z_t^+}, c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+P_t},\ a_t \equiv \frac{S_tA_{t+1}^*}{P_tz_t^+}, \\
x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_tP_tz_t^+,\ (y_t =)\,\tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t},\ D_{z^+,t}^j \equiv \frac{D_t^j}{P_tz_t^+}.
\end{aligned}
$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_tP_{k',t},\ \bar{r}_t^k = \Psi_tr_t^k.$$

where $P_{k',t}$ is in units of the domestic homogeneous good. We define the following inflation rates:

$$
\begin{aligned}
\pi_t &= \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*}, \\
\pi_t^i &= \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},
\end{aligned}
$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'CPI'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$
\begin{aligned}
p_t^{m,x} &= \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t}, \\
p_t^x &= \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_tP_t^i}{P_t}.
\end{aligned}
\tag{2.2}
$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$, and in the production of final investment goods in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

We denote the real exchange rate by q_t :

$$q_t = \frac{S_t P_t^*}{P_t^c}. \tag{2.3}$$

2.2. Production of the Domestic Homogeneous Good

A homogeneous domestic good, Y_t, is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right]^{\lambda_{d,t}}, \ 1 \leq \lambda_{d,t} < \infty. \tag{2.4}$$

The domestic good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given.

The i^{th} intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi,$$

where $K_{i,t}$ denotes the labor services rented by the i^{th} intermediate good producer. Firms must borrow a fraction of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu_t^f R_t + 1 - \nu_t^f, \tag{2.5}$$

where W_t is the aggregate wage rate, R_t is the interest rate on working capital loans, and ν_t^f corresponds to the fraction that must be financed in advance.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t :

$$\begin{aligned} mc_t &= \frac{\tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k P_t\right)^{\alpha} \left(W_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}}{z_t^{1-\alpha} P_t} \\ &= \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}, \end{aligned} \tag{2.6}$$

where r_t^k is the nominal rental rate of capital scaled by P_t. Also, τ_t^d is a tax-like shock, which affects marginal cost, but does not appear in a production function. In the linearization of a version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

Productive efficiency dictates that another expression for marginal cost must also be satisfied:

$$
\begin{aligned}
mc_t &= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{MP_{l,t}} \\
&= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{\epsilon_t \left(1-\alpha\right) z_t^{1-\alpha} \left(k_{i,t} z_{t-1}^{+} \Psi_{t-1}/H_{i,t}\right)^{\alpha}} \\
&= \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t \left(1-\alpha\right) \left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^{\alpha}} \qquad (2.7)
\end{aligned}
$$

The i^{th} firm is a monopolist in the production of the i^{th} good and so it sets its price. Price setting is subject Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$
P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1},\ \tilde{\pi}_{d,t} \equiv \left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d},
$$

where κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in (0,1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t^c$ is the central bank's target inflation rate. Also, $\breve{\pi}$ is a scalar which allows us to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$). Note that we get price dispersion in steady state if $\varkappa_d > 0$ and $\breve{\pi}$ is different from the steady state value of π. See Yun (1996) for a discussion of steady state price dispersion.

With probability $1 - \xi_d$ the firm can change its price. The problem of the i^{th} domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$
E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{ P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j} \},
$$

subject to the requirement that production equal demand. In the above expression, υ_t is the multiplier on the household budget constraint. It measures the marginal value to the household of one unit of profits, in terms of currency. In the profit function, we replace the firm's output with the demand function:

$$
\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} Y_t = Y_{i,t},
$$

to obtain, after rearranging,

$$
E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d - 1}} - mc_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d - 1}} \},
$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ (X_{t,j} \tilde{p}_t)^{1-\frac{\lambda_d}{\lambda_d - 1}} - mc_{t+j} \left(X_{t,j} \tilde{p}_t \right)^{\frac{-\lambda_d}{\lambda_d - 1}} \},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t, \;\; X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{d,t+j} \cdots \tilde{\pi}_{d,t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, \;\; j > 0 \\ 1, \;\; j = 0. \end{cases} .$$

The i^{th} firm maximizes profits by choice of the within-sector relative price $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d - 1}+1}$, rearranging, and scaling we obtain:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_d \right)^j A_{t+j} \left[\tilde{p}_t X_{t,j} - \lambda_d mc_{t+j} \right] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j} \tilde{y}_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} \left(\beta \xi_d \right)^j A_{t+j} \lambda_d mc_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta \xi_d \right)^j A_{t+j} X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$\begin{aligned} K_t^d &\equiv E_t \sum_{j=0}^{\infty} \left(\beta \xi_d \right)^j A_{t+j} \lambda_d mc_{t+j} \\ F_t^d &= E_t \sum_{j=0}^{\infty} \left(\beta \xi_d \right)^j A_{t+j} X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$\begin{aligned} E_t \left[\psi_{z^+,t} \tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] &= 0 \\ E_t \left[\lambda_d \psi_{z^+,t} \tilde{y}_t mc_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] &= 0. \end{aligned}$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di \right]^{(1-\lambda_d)} \qquad (2.8) \\ &= \left[\left(1 - \xi_p \right) \tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p \left(\tilde{\pi}_{d,t} P_{t-1} \right)^{\frac{1}{1-\lambda_d}} \right]^{(1-\lambda_d)} \end{aligned}$$

After dividing by P_t and rearranging:

$$\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}=\left(\tilde{p}_t^d\right)^{\frac{1}{1-\lambda_d}}. \tag{2.9}$$

In sum, the equilibrium conditions associated with price setting are:[1]

$$E_t\left[\psi_{z^+,t}y_t+\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F_{t+1}^d-F_t^d\right] = 0 \tag{2.10}$$

$$E_t\left[\lambda_d\psi_{z^+,t}y_t mc_t+\beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}}K_{t+1}^d-K_t^d\right] = 0, \tag{2.11}$$

$$\mathring{p}_t=\left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d}+\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{2.12}$$

$$\left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{(1-\lambda_d)} = \frac{K_t^d}{F_t^d} \tag{2.13}$$

$$\tilde{\pi}_{d,t}\equiv\left(\pi_{t-1}\right)^{\kappa_d}\left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d}\left(\breve{\pi}\right)^{\varkappa_d} \tag{2.14}$$

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t=G_t+C_t^d+I_t^d+\int_0^1 X_{i,t}^d. \tag{2.15}$$

Here, C_t^d denotes intermediate goods used (together with foreign consumption goods) to produce final household consumption goods. Also, I_t^d is the number of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. Some of this good is used to add to the physical stock of capital, $\bar{K}_t$. The rest of the investment good is used in maintenance expenditures, which arise from the

[1] When we linearize about steady state and set $\varkappa_d=0$, we obtain,

$$\begin{aligned}\hat{\pi}_t-\hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1}-\hat{\bar{\pi}}_{t+1}^c\right)+\frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1}-\hat{\bar{\pi}}_t^c\right)\\ &-\frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t^c\\ &+\frac{1}{1+\kappa_d\beta}\frac{\left(1-\beta\xi_d\right)\left(1-\xi_d\right)}{\xi_d}\widehat{mc}_t,\end{aligned}$$

where a hat indicates log-deviation from steady state.

utilization of capital, $a(u_t)\bar{K}_t$. Here, u_t denotes the utilization rate of capital, with capital services being defined by:

$$K_t = u_t \bar{K}_t.$$

We adopt the following functional form for a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b(1-\sigma_a)u + \sigma_b((\sigma_a/2)-1), \tag{2.16}$$

where σ_a and σ_b are the parameters of this function. Finally, the integral in (2.15) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

2.3. Production of Final Consumption and Investment Goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}}\left(C_t^d\right)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}}\left(C_t^m\right)^{\frac{(\eta_c-1)}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}. \tag{2.17}$$

The representative firm takes the price of final consumption goods output, P_t^c, as given. Final consumption goods output is produced using two inputs. The first, C_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, C_t^m, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs in scaled form:

$$\begin{aligned} c_t^d &= (1-\omega_c)(p_t^c)^{\eta_c} c_t \\ c_t^m &= \omega_c\left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta c} c_t. \end{aligned} \tag{2.18}$$

In the usual way, the price of C_t is related to the price of the inputs by:

$$P_t^c = \left[(1-\omega_c)(P_t)^{1-\eta_c} + \omega_c(P_t^{m,c})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}.$$

After dividing by P_t, this becomes

$$p_t^c = \left[(1-\omega_c) + \omega_c(p_t^{m,c})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}. \tag{2.19}$$

The rate of inflation of the consumption good is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t\left[\frac{(1-\omega_c)+\omega_c(p_t^{m,c})^{1-\eta_c}}{(1-\omega_c)+\omega_c\left(p_{t-1}^{m,c}\right)^{1-\eta_c}}\right]^{\frac{1}{1-\eta_c}}. \tag{2.20}$$

We define investment to be the sum of investment goods, I_t, used in the accumulation of physical capital, plus investment goods used in capital maintenance:

$$\tilde{I}_t = I_t + a\left(u_t\right)\bar{K}_t.$$

Capital maintenance are expenses that arise from the utilization of capital. We discuss maintenance in subsection 2.5 below. Investment goods are produced by a representative competitive firm using the following technology:

$$\tilde{I}_t = \Psi_t\left[\left(1-\omega_i\right)^{\frac{1}{\eta_i}}\left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}}\left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}}.$$

The representative firm takes the price of the final investment good, P_t^i, as given. Investment goods are produced using two inputs. The first, I_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, I_t^m, is the homogeneous composite of specialized investment import goods discussed in the next subsection. The price of I_t^m is $P_t^{m,i}$. The representative firm takes the input prices, P_t and $P_t^{m,i}$ as given. To accommodate the observation that the price of investment goods relative to the price of consumption goods is declining over time, we assume that Ψ_t is a unit root process with positive drift. The details of the law of motion of this process is discussed below.

Profit maximization implies:

$$\begin{aligned}
P_t^i\left(\frac{\tilde{I}_t}{\Psi_t I_t^d}\right)^{\frac{1}{\eta_i}}\left(1-\omega_i\right)^{\frac{1}{\eta_i}} &= \frac{P_t}{\Psi_t} \\
P_t^i\left(\frac{\tilde{I}_t}{I_t^m}\right)^{\frac{1}{\eta_i}}\omega_i^{\frac{1}{\eta_i}}\left(\Psi_t\right)^{\frac{\eta_i-1}{\eta_i}} &= P_t^{m,i},
\end{aligned} \tag{2.21}$$

or,

$$\begin{aligned}
\left(p_t^i\right)^{\eta_i-1}\left(\frac{\tilde{I}_t}{\Psi_t}\right)^{\frac{\eta_i-1}{\eta_i}}\left(1-\omega_i\right)^{\frac{\eta_i-1}{\eta_i}} &= \left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}} \\
\left(\frac{p_t^i}{\Psi_t p_t^{m,i}}\right)^{(\eta_i-1)}\tilde{I}_t^{\frac{\eta_i-1}{\eta_i}}\omega_i^{\frac{\eta_i-1}{\eta_i}}\left(\Psi_t\right)^{\frac{\eta_i-1}{\eta_i}(\eta_i-1)} &= \left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}}
\end{aligned}.$$

Substituting these expressions into the production function for the final investment good, we obtain:

$$p_t^i = \left[\left(1-\omega_i\right) + \omega_i\left(p_t^{m,i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \tag{2.22}$$

Then, the inflation in the price of investment goods is:

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}}\left[\frac{\left(1-\omega_i\right)+\omega_i\left(p_t^{m,i}\right)^{1-\eta_i}}{\left(1-\omega_i\right)+\omega_i\left(p_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}}. \tag{2.23}$$

From here on we drop the notation $\tilde{I}_t$ and only refer to I_t, so that the production function for investment is:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}}.$$

Finally, the demand for imported investment, 2.21, in scaled form as follows:

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}} \right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) \tag{2.24}$$

The demand for domestic investment inputs in scaled form is as follows:

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i) \tag{2.25}$$

2.4. Exports and Imports

This section reviews the structure of imports and exports. Both activities involve Calvo price setting frictions, and so require the presence of market power. In each case, we follow the Dixit-Stiglitz strategy of introducing a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export and import sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which produces a specialized export good. Each monopolist produces the export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign, competitive retailers which create a homogeneous good that is sold to foreign citizens.

In the case of imports, specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. See Figure A for a graphical illustration.

We emphasize two features of this setup. First, before being passed on to final domestic users, imported goods must first be combined with domestic inputs. This is consistent with the view emphasized by Burstein, Eichenbaum and Rebelo (2005, 2007), that there are substantial distribution costs associated with imports. Second, there are pricing frictions in all sectors of the model. The pricing frictions in the homogeneous domestic good sector are standard, and perhaps do not require additional elaboration. We do need to elaborate on the pricing frictions in the part of the model related to imports and exports.

In all cases we assume that prices are set in the currency of the buyer ("pricing to market"). Pricing frictions in the case of imports help the model account for the evidence that exchange rate shocks take time to pass into domestic prices. Pricing frictions in the case of exports help the model to produce a hump-shape in the response of output to a monetary shock. To see this, it is useful to recall how a hump-shape is produced in a closed economy version of the model. In that version, the hump shape occurs because there are costs to quickly expanding consumption and investment demand. Consumption is not expanded rapidly because of the assumption of habit persistence in preferences and investment is not expanded because of the assumption that there are adjustment costs associated with changing the flow of investment. When the closed economy is opened up, another potential source of demand in the wake of a monetary policy shock is introduced, namely, exports. To explain this further it is convenient to adopt a highly simplified version of our model.

Suppose that P^x is the price of exports in foreign currency units, and the demand for exports, X, is given by

$$X = (P^x)^{-\varepsilon} A,$$

where A summarizes the impact on demand of foreign prices and output which we take as predetermined. With this type of demand curve, a monopolist who sells X will set P^x as a constant markup, μ, over marginal cost, MC. Given the denomination of P^x, we require that MC is denominated in foreign currency units. Thus,

$$P^x = \mu \times MC = \mu \times \frac{f\,(\text{domestic factor prices})}{S}.$$

The expression after the second equality emphasizes that the marginal costs of the exporter involve domestic factors of production. The costs of domestic factors of production do not rise much in the face of an expansionary monetary policy shock because of the slow response of demand and frictions in wage setting.[2] The term in the denominator is the nominal exchange rate, S. A positive monetary policy shock can be expected to result in an immediate depreciation in the currency, *i.e.*, jump in S. Thus, in the presence of sticky domestic factor costs, a depreciation in the exchange rate is expected to put downward pressure on P^x. Given the demand curve for exports, this is expected to create a surge in X. To the extent that this surge is strong enough, this can overwhelm the other factors in the model designed to create a hump-shaped response of output to an expansionary monetary policy shock. In sum, we introduce the frictions in the setting of P^x in order to help assure a hump-shape response of output to a monetary policy shock.

There are two additional observations worth making concerning the role of price frictions in the export sector. First, it is interesting to note that the price frictions in the import of

[2] The slow expansion of demand also plays a role here.

goods used as inputs into the production of exports work against us. These price frictions increase the need for price frictions in the export sector to damp the response of X to an expansionary domestic monetary shock. The reason is that in the absence of price frictions on imports, 'domestic factor prices' in the above expression would jump in the face of an expansionary monetary policy shock, as pass through from the exchange rate to the domestic currency price of imports of goods destined for export increases. From the perspective of achieving a hump-shaped response of output to an expansionary monetary policy shock, we suspect that it would be better to treat the import of goods destined for the export sector asymmetrically by supposing there are no price frictions in those goods.

The second observation on the role of price frictions in the export sector is related to the first. We make assumptions in the model that have the effect of also producing a hump-shape response of the exchange rate to an expansionary monetary policy shock. The model follows ALLV in capturing, in a reduced form way, the notion that holders of domestic assets require less compensation for risk in the wake of an expansionary monetary policy shock. As a result, the model does not display the classic Dornbusch 'overshooting' pattern in the exchange rate in response to a monetary policy shock. Instead, the nominal exchange rate rises slowly in response to an expansionary monetary policy shock. The slow response in the exchange rate reduces the burden on price frictions in P^x to slow the response of X in to an expansionary monetary policy shock.

2.4.1. Exports

We assume there is a total demand by foreigners for domestic exports, which takes on the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*.$$

In scaled form, this is

$$x_t = (p_t^x)^{-\eta_f}\, y_t^* \tag{2.26}$$

Here, Y_t^* is foreign GDP and P_t^* is the foreign currency price of foreign homogeneous goods. Also, P_t^x is an index of export prices, whose determination is discussed below. The goods, X_t, are produced by a representative, competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}}\, di\right]^{\lambda_x}. \tag{2.27}$$

Here, $X_{i,t}$, $i \in (0,1)$, are exports of specialized goods. The retailer that produces X_t takes its output price, P_t^x, and its input prices, $P_{i,t}^x$, as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^{x}}{P_{t}^{x}}\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} X_{t}. \tag{2.28}$$

Combining (2.27) and (2.28), we obtain:

$$P_{t}^{x} = \left[\int_{0}^{1} \left(P_{i,t}^{x}\right)^{\frac{1}{1-\lambda_{x,t}}} di\right]^{1-\lambda_{x,t}}.$$

The i^{th} specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_{x}^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{m}\right)^{\frac{\eta_{x}-1}{\eta_{x}}} + (1-\omega_{x})^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{d}\right)^{\frac{\eta_{x}-1}{\eta_{x}}}\right]^{\frac{\eta_{x}}{\eta_{x}-1}},$$

where $X_{i,t}^{m}$ and $X_{i,t}^{d}$ are the i^{th} exporter's use of the imported and domestically produced goods, respectively. We derive the marginal cost associated with the CES production function from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min \tau_{t}^{x} \left[P_{t}^{m,x} R_{t}^{x} X_{i,t}^{m} + P_{t} R_{t}^{x} X_{i,t}^{d}\right] + \lambda \left\{X_{i,t} - \left[\omega_{x}^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{m}\right)^{\frac{\eta_{x}-1}{\eta_{x}}} + (1-\omega_{x})^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{d}\right)^{\frac{\eta_{x}-1}{\eta_{x}}}\right]^{\frac{\eta_{x}}{\eta_{x}-1}}\right\},$$

where $P_{t}^{m,x}$ is the price of the homogeneous import good and P_{t} is the price of the homogeneous domestic output good. The first order conditions are:

$$\begin{aligned}
\tau_{t}^{x} R_{t}^{x} P_{t}^{m,x} &= \lambda X_{i,t}^{\frac{1}{\eta_{x}}} \omega_{x}^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{m}\right)^{\frac{-1}{\eta_{x}}} \\
\tau_{t}^{x} R_{t}^{x} P_{t} &= \lambda X_{i,t}^{\frac{1}{\eta_{x}}} (1-\omega_{x})^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{d}\right)^{\frac{-1}{\eta_{x}}} \\
X_{i,t} &= \left[\omega_{x}^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{m}\right)^{\frac{\eta_{x}-1}{\eta_{x}}} + (1-\omega_{x})^{\frac{1}{\eta_{x}}} \left(X_{i,t}^{d}\right)^{\frac{\eta_{x}-1}{\eta_{x}}}\right]^{\frac{\eta_{x}}{\eta_{x}-1}}
\end{aligned}$$

Use the first two conditions to solve for the inputs as a function of the exogenous variables and the multiplier:

$$\left(X_{i,t}^{m}\right)^{\frac{\eta_{x}-1}{\eta_{x}}} = \frac{\lambda^{\eta_{x}-1} X_{i,t}^{\frac{\eta_{x}-1}{\eta_{x}}} \omega_{x}^{\frac{\eta_{x}-1}{\eta_{x}}}}{\left(\tau_{t}^{x} R_{t}^{x} P_{t}^{m,x}\right)^{\eta_{x}-1}} \tag{2.29}$$

$$\left(X_{i,t}^{d}\right)^{\frac{\eta_{x}-1}{\eta_{x}}} = \frac{\lambda^{\eta_{x}-1} X_{i,t}^{\frac{\eta_{x}-1}{\eta_{x}}} (1-\omega_{x})^{\frac{\eta_{x}-1}{\eta_{x}}}}{\left(\tau_{t}^{x} R_{t}^{x} P_{t}\right)^{\eta_{x}-1}}. \tag{2.30}$$

Substitute these into the production function, to get:

$$X_{i,t} = \lambda^{\eta_{x}} X_{i,t} \left(\tau_{t}^{x}\right)^{-\eta_{x}} \left(\frac{\omega_{x}}{\left(R_{t}^{x} P_{t}^{m,x}\right)^{\eta_{x}-1}} + \frac{(1-\omega_{x})}{\left(R_{t}^{x} P_{t}\right)^{\eta_{x}-1}}\right)^{\frac{\eta_{x}}{\eta_{x}-1}}.$$

Nominal marginal cost is λ, so that real (in terms of the homogeneous final export good) marginal cost, mc_t^x, is

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{S_t P_t^x}\left[\omega_x \left(P_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\left(P_t\right)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}},$$

where

$$R_t^x = \nu_t^x R_t + 1 - \nu_t^x. \tag{2.31}$$

We rewrite the expression for marginal cost to get it in terms of stationary variables

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x}\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}, \tag{2.32}$$

where we have used

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c}\frac{P_t^c}{P_t}\frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x. \tag{2.33}$$

The i^{th}, $i \in (0,1)$, domestic exporting firm takes (2.28) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. In a given period, $1-\xi_x$ producers can reoptimize their price and ξ_x cannot. The firms that cannot optimize price, update their prices as follows:

$$P_{i,t}^x = \tilde{\pi}_t^x P_{i,t-1}^x,\ \tilde{\pi}_t^x = \left(\pi_{t-1}^x\right)^{\kappa_x}\left(\pi^x\right)^{1-\kappa_x-\varkappa_x}\left(\breve{\pi}\right)^{\varkappa_x}, \tag{2.34}$$

where $\kappa_x, \varkappa_x, \kappa_x + \varkappa_x \in (0,1)$.

The equilibrium conditions associated with price setting by exporters are analogous to the ones derived for domestic intermediate good producers:[3]

$$E_t\left[\psi_{z^+,t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}}\beta\xi_x F_{x,t+1} - F_{x,t}\right] = 0 \tag{2.35}$$

$$E_t\left[\lambda_x \psi_{z^+,t} q_t p_t^c p_t^x x_t mc_t^x + \beta\xi_x\left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x,t+1} - K_{x,t}\right] = 0, \tag{2.36}$$

$$\mathring{p}_t^x = \left[(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right)^{\lambda_x} + \xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_x}{1-\lambda_x}}\right]^{\frac{1-\lambda_x}{\lambda_x}} \tag{2.37}$$

[3] When we linearize around steady state and $\varkappa_{m,j} = 0$, equations (2.35)-(2.38) reduce to:

$$\begin{aligned}\hat{\pi}_t^x &= \frac{\beta}{1+\kappa_x\beta}E_t\hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\kappa_x\beta}\hat{\pi}_{t-1}^x \\ &\quad + \frac{1}{1+\kappa_x\beta}\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x}\widehat{mc}_t^x,\end{aligned}$$

where a hat over a variable indicates log deviation from steady state.

$$\left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{(1-\lambda_x)} = \frac{K_{x,t}}{F_{x,t}} \tag{2.38}$$

The quantity of the domestic homogeneous good used by specialized exporters is:

$$\int_0^1 X_{i,t}^d di,$$

and this needs to be expressed in terms of aggregates. Rewriting (2.30), one of the first order conditions of the foreign retailer who purchases the specialized export goods:

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}\left(1-\omega_x\right).$$

Integrating this expression:

$$\begin{aligned}\int_0^1 X_{i,t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right)\int_0^1 X_{i,t} di \qquad (2.39)\\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right) X_t \frac{\int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di}{\left(P_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}}.\end{aligned}$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$:

$$\mathring{P}_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di\right]^{\frac{\lambda_{x,t}-1}{-\lambda_{x,t}}}.$$

Then,

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} = \int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di,$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right) X_t \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}, \tag{2.40}$$

where

$$\mathring{p}_t^x \equiv \frac{\mathring{P}_t^x}{P_t^x},$$

and the law of motion of $\mathring{p}_t^x$ is given in (2.37).

We now simplify (2.40). Rewriting the second equality in (2.32), we obtain:

$$\frac{\lambda}{P_t\tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^c p_t^x}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t\tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t\frac{S_t P_t^*}{P_t^c}\frac{P_t^c}{P_t}\frac{P_t^x}{P_t^*}}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (2.40), we obtain:

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} Y_t^* \qquad (2.41)$$

We also require an expression for imported inputs for exports in terms of aggregates. Using (2.29) and (2.32),

$$X_{i,t}^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}}\right)^{\eta_x} X_{i,t}$$

The object on the left side of the equality is the quantity of the homogeneous import good used by the i^{th} specialized exporter. (This is to be distinguished from the output of the i^{th} specialized importer, which has the same notation.) The unweighted integral of $X_{i,t}^m$ is X_t^m because X_t^m is a homogeneous good. This is the same unweighted integral considered in (2.39). Using the result derived in (2.39), we obtain:

$$X_t^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}}\right)^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} X_t,$$

where the law of motion of $\mathring{p}_t^x$ is given in (2.37). Note how the impact of price dispersion operates in the previous expression. To produce a given total of the homogenous export good, X_t, one needs more of the homogeneous input good, X_t^m, to the extent that there is price dispersion. In that case $\mathring{p}_t^x < 1$ and ${}^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} > 1$, and more dispersion is reflected in a lower $\mathring{p}_t^x$.

After scaling the preceding expression and substituting out for X_t using the demand for exports (see (2.26)), we obtain

$$x_t^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}}\right)^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} y_t^* \qquad (2.42)$$

2.4.2. Imports

We now turn to a discussion of imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input (they 'brand name it') and supply that input monopolistically to domestic retailers. Importers are subject to Calvo price setting frictions. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment goods, and (iii) one produces an intermediate good used for the production of an input into the production of export goods.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is:

$$C_t^m = \left[\int_0^1 \left(C_{i,t}^m \right)^{\frac{1}{\lambda^{m,C}}} di \right]^{\lambda^{m,C}},$$

where $C_{i,t}^m$ is the output of the i^{th} specialized producer and C_t^m is an intermediate good used in the production of consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i^{th} intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. In the usual way, the demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}}.$$

We now turn to the producer of $C_{i,t}^m$, who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*}, \tag{2.43}$$

where

$$R_t^{\nu,*} = \nu_t^* R_t^* + 1 - \nu_t^*, \tag{2.44}$$

and R_t^* is the foreign nominal, intratemporal rate of interest. In addition, $\tau_t^{m,c}$ is a tax term. The notion here is that the intermediate good firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce $C_{i,t}^m$. There is no risk to this firm, because all shocks are realized at the beginning of the period, and so there is no uncertainty within the duration of the working capital loan about the realization of prices and exchanges rates.[4]

[4] We are somewhat uncomfortable with this feature of the model. The fact that interest is due and matters indicates that some time evolves over the duration of the loan. Our assumption that no uncertainty is realized over a period of significant duration of time seems implausible. We suspect that a more realistic representation would involve some risk. Our timing assumptions in effect abstract away from this risk, and we conjecture that this does not affect the first order properties of the model.

It is of interest to have a measure of the total imports of the intermediate good producers:

$$S_t P_t^* R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m, we substitute the demand curve into the previous expression:

$$\begin{aligned}
& S_t P_t^* R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}} di \\
= \;& S_t P_t^* R_t^{\nu,*} C_t^m \left(P_t^{m,c} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}} \int_0^1 \left(P_{i,t}^{m,c} \right)^{\frac{-\lambda^{m,C}}{\lambda^{m,C}-1}} \\
= \;& S_t P_t^* R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}},
\end{aligned}$$

where

$$\mathring{P}_t^{m,c} = \left[\int_0^1 \left(P_{i,t}^{m,c} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} \right]^{\frac{1-\lambda^{m,C}}{\lambda^{m,C}}}$$

We conclude that total imports account for by the consumption sector is:

$$S_t P_t^* R_t^{\nu,*} C_t^m \left(\mathring{p}_t^{m,c} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}}, \tag{2.45}$$

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}},$$

and $\mathring{p}_t^{m,c}$ is discussed below.

Now consider (ii). The production function for the domestic retailer of imported investment goods, I_t^m, is:

$$I_t^m = \left[\int_0^1 \left(I_{i,t}^m \right)^{\frac{1}{\lambda_t^{m,I}}} di \right]^{\lambda_t^{m,I}}.$$

The retailer of imported investment goods is competitive and takes output prices, $P_t^{m,i}$, and input prices, $P_{i,t}^{m,i}$, as given.

The producer of the i^{th} intermediate input into the above production function buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is also (2.43). Note that this implies the importing firm's cost is P_t^* (before borrowing costs and exchange rate conversion), which is the same cost for the specialized inputs used to produce C_t^m. This may seem inconsistent with the property of the domestic economy that domestically produced consumption and investment goods have different relative prices. We assume that (2.43) applies to both types of producer in order to simplify notation. Below, we suppose that the efficiency of imported investment goods grows over time, in a way that makes our assumptions about the relative costs of consumption and investment, whether imported or domestically produced.

The total value of imports associated with the production of investment goods is analogous to what we obtained for the consumption good sector:

$$S_t P_t^* R_t^{\nu,*} I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}}, \ \mathring{p}_t^{m,i} = \frac{P_{i,t}^{m,i}}{P_t^{m,i}}, \tag{2.46}$$

where $\mathring{p}_t^{m,i}$ is discussed below.

Now consider (iii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m, for the production of export goods is:

$$X_t^m = \left[\int_0^1 \left(X_{i,t}^m\right)^{\frac{1}{\lambda_t^{m,X}}} di\right]^{\lambda_t^{m,X}}.$$

The imported good retailer is competitive, and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost, (2.43). The total value of imports associated with the production of X_t^m is:

$$S_t P_t^* R_t^{\nu,*} X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}}, \ \mathring{p}_t^{m,x} = \frac{P_{i,t}^{m,x}}{P_t^{m,x}} \tag{2.47}$$

Each of the above three types of intermediate good firm is subject to Calvo price-setting frictions. With probability $1-\xi_{m,j}$, the j^{th} type of firm can reoptimize its price and with probability $\xi_{m,j}$ it sets price according to the following relation:

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j}, \ \tilde{\pi}_t^{m,j} \equiv \left(\pi_{t-1}^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}_t^c\right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \breve{\pi}^{\varkappa_{m,j}}. \tag{2.48}$$

for $j = c, i, x$.

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers:

$$E_t\left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t}\right] = 0 \tag{2.49}$$

$$E_t\left[\lambda_{m,j} \psi_{z^+,t} p_t^{m,j} mc_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t}\right] = 0, \tag{2.50}$$

$$\mathring{p}_t^{m,j} = \left[\left(1-\xi_{m,j}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_{m,j}} + \xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}}\right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \tag{2.51}$$

$$\left[\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right]^{(1-\lambda_{m,j})}=\frac{K_{m,j,t}}{F_{m,j,t}}, \tag{2.52}$$

for $j=c,i,x$.[5] Here,

$$\Xi_t^j=\begin{cases} c_t^m & j=c \\ x_t^m & j=x \\ i_t^m & j=i \end{cases}.$$

Real after tax marginal cost is

$$\begin{aligned} mc_t^{m,j} &= \tau_t^{m,j}\frac{S_t P_t^*}{P_t^{m,j}}R_t^{\nu,*}=\tau_t^{m,j}\frac{S_t P_t^* P_t^c P_t}{P_t^c P_t^{m,j} P_t}R_t^{\nu,*} \\ &= \tau_t^{m,j}\frac{q_t p_t^c}{p_t^{m,j}}R_t^{\nu,*} \end{aligned} \tag{2.53}$$

for $j=c,i,x$.

2.5. Households

Household preferences are given by:

$$E_0^j\sum_{t=0}^{\infty}\beta^t\left[\zeta_t^c\ln\left(C_t-bC_{t-1}\right)-\zeta_t^h A_L\frac{\left(h_{j,t}\right)^{1+\sigma_L}}{1+\sigma_L}\right]. \tag{2.54}$$

The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household owns the stock of net foreign assets and determines its rate of accumulation.

2.5.1. Technology for Capital Accumulation

The law of motion of the physical stock of capital is:

$$\bar{K}_{t+1}=(1-\delta)\bar{K}_t+\Upsilon_t F\left(I_t,I_{t-1}\right),$$

where

$$F\left(I_t,I_{t-1}\right)=\left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t,$$

[5] When we linearize around steady state and $\varkappa_{m,j}=0$,

$$\begin{aligned} \hat{\pi}_t^{m,j}-\hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_{m,j}\beta}E_t\left(\hat{\pi}_{t+1}^{m,j}-\hat{\bar{\pi}}_{t+1}^c\right)+\frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta}\left(\hat{\pi}_{t-1}^{m,j}-\hat{\bar{\pi}}_t^c\right) \\ &-\frac{\kappa_{m,j}\beta\left(1-\rho_\pi\right)}{1+\kappa_{m,j}\beta}\hat{\bar{\pi}}_t^c \\ &+\frac{1}{1+\kappa_{m,j}\beta}\frac{\left(1-\beta\xi_{m,j}\right)\left(1-\xi_{m,j}\right)}{\xi_{m,j}}\widehat{mc}_t^{m,j}, \end{aligned}$$

and

$$\begin{aligned}\tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]+\exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]-2\right\} \\ &= 0,\ x=\mu_{z^+}\mu_\Psi.\end{aligned} \tag{2.55}$$

Also,

$$\begin{aligned}\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]-\exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= 0,\ x=\mu_{z^+}\mu_\Psi.\end{aligned} \tag{2.56}$$

and

$$\begin{aligned}\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]+\exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= \tilde{S}'',\ x=\mu_{z^+}\mu_\Psi.\end{aligned}$$

Also,

$$\begin{aligned}F_1(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)-\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \\ &= 1,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi,\end{aligned}$$

and,

$$\begin{aligned}F_2(I_t,I_{t-1}) &= \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2 \\ &= 0,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi.\end{aligned}$$

Scaling,

$$\begin{aligned}F(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t \\ F_1(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}} \\ F_2(I_t,I_{t-1}) &= \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)^2\end{aligned}$$

In this notation, the law of motion of capital is written,

$$\bar{k}_{t+1}z_t^+\Psi_t=(1-\delta)\bar{K}_t z_{t-1}^+\Psi_{t-1}+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t,$$

or,

$$\bar{k}_{t+1}=\frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{2.57}$$

2.5.2. Household Consumption and Investment Decisions

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c \left(1 + \tau_t^c\right) = 0. \tag{2.58}$$

To define the intertemporal Euler equation associated with the household's capital accumulation decision, we need to define the rate of return on a period t investment in a unit of physical capital, R_{t+1}^k :

$$R_{t+1}^k = \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau_t^k\delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{2.59}$$

where it is convenient to recall

$$\frac{p_t^i}{\Psi_t}P_t = P_t^i,$$

the date t price of the homogeneous investment good. Here, $P_{k',t}$ denotes the price of a unit of newly installed physical capital, which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R_{t+1}^k represents the period $t+1$ payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets in (2.59) captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R_t^k in terms of scaled variables:

$$\begin{aligned}
R_{t+1}^k &= \frac{P_{t+1}\Psi_{t+1}}{P_t\Psi_{t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right] + (1-\delta)P_{k',t+1} + \tau_t^k\delta\frac{P_t}{P_{t+1}}P_{k',t}}{P_{k',t}} \\
&= \pi_{t+1}\frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)\Psi_{t+1}P_{k',t+1} + \tau_t^k\delta\frac{P_t}{P_{t+1}}\Psi_{t+1}P_{k',t}}{\Psi_{t+1}P_{k',t}}.
\end{aligned}$$

so that

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau_t^k\delta\frac{\mu_{\Psi,t+1}}{\pi_{t+1}}p_{k',t}}{p_{k',t}}. \tag{2.60}$$

Capital is a good hedge against inflation, except for the way depreciation is treated. A rise in inflation effectively raises the tax rate on capital because of the practice of valuing depreciation at historical cost. The first order condition for capital implies:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{2.61}$$

We differentiate the Lagrangian representation of the household's problem as displayed in ALLV, with respect to I_t :

$$-\upsilon_t P_t^i + \omega_t \Upsilon_t F_1 \left(I_t, I_{t-1}\right) + \beta \omega_{t+1} \Upsilon_{t+1} F_2 \left(I_{t+1}, I_t\right) = 0,$$

where υ_t denotes the multiplier on the household's nominal budget constraint and ω_t denotes the multiplier on the capital accumulation technology. In addition, the price of capital is the ratio of these multipliers:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}.$$

Expressing the investment first order condition in terms of scaled variables,

$$\begin{aligned} -\frac{\psi_{z^+,t}}{z_t^+} \frac{p_t^i}{\Psi_t} + \upsilon_t P_t P_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right] & \\ + \beta \upsilon_{t+1} P_{t+1} P_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right) \left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right)^2 &= 0. \end{aligned}$$

Now multiply by $z_t^+ \Psi_t$

$$\begin{aligned} -\psi_{z^+,t} p_t^i + \psi_{z^+,t} p_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right] & \\ + \beta \psi_{z^+,t+1} p_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{\Psi,t+1}\mu_{z^+,t+1} &= 0. \end{aligned} \quad (2.62)$$

Our first order condition for I_t appears to differ slightly from the first order condition in ALLV, equation (2.55), but the two actually coincide when we take into account the definition of f.

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = p_t^i a'\left(u_t\right),$$

or, in scaled terms,

$$\bar{r}_t^k = p_t^i a'\left(u_t\right). \quad (2.63)$$

The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

2.5.3. Financial Assets

The household does the economy's saving. Period t saving occurs by the acquisition of net foreign assets, A_{t+1}^*, and a domestic asset. The domestic asset is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to $t+1$, R_t. The first order condition associated with this asset is:

$$-\psi_{z^+,t} + \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}} \left[\frac{R_t - \tau_t^b \left(R_t - \pi_{t+1}\right)}{\pi_{t+1}}\right] = 0, \quad (2.64)$$

where τ_t^b is the tax rate on the real interest rate on bond income (for additional discussion of τ^b, see section 2.6.) A consequence of our treatment of the taxation on domestic bonds is that the steady state real after tax return on bonds is invariant to π.

In the model the tax treatment of domestic agents' earnings on foreign bonds is the same as the tax treatment of agents' earnings on foreign bonds. The scaled date t first order condition associated with A_{t+1}^*:

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1} [S_{t+1} R_t^* \Phi_t - \tau^b \left(S_{t+1} R_t^* \Phi_t - \frac{S_t}{P_t} P_{t+1} \right)]. \tag{2.65}$$

Recall that S_t is the domestic currency price of a unit of foreign currency. On the left side of this expression, we have the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the multiplier on the household's budget constraint, υ_t. The term in square brackets is the after tax payoff of the foreign asset, in domestic currency units. The first term is the period $t+1$ pre-tax interest payoff on A_{t+1}^*, which is $S_{t+1} R_t^* \Phi_t$. Here, R_t^* is the foreign nominal rate of interest, which is risk free in foreign currency units. The term, Φ_t represents a risk adjustment, so that a unit of the foreign asset acquired in t pays off $R_t^* \Phi_t$ units of foreign currency in $t+1$. The determination of Φ_t is discussed below. The remaining term in parentheses pertains to the impact of taxation on the return on foreign assets. If we ignore the term after the minus sign in parentheses, then we see that taxation is applied to the whole nominal payoff on the bond, including principle. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t+1$ coincides with the real value of the currency used to purchase the asset in period t. In particular, recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So, the period t real cost of the asset is S_t/P_t. The domestic currency value in period $t+1$ of this real quantity is $P_{t+1} S_t / P_t$.

We scale the first order condition, (2.65), by multiplying both sides by $P_t z_t^+ / S_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1} \mu_{z^+,t+1}} [s_{t+1} R_t^* \Phi_t - \tau_t^b \left(s_{t+1} R_t^* \Phi_t - \pi_{t+1} \right)], \tag{2.66}$$

where, recall,

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; s_t = \frac{S_t}{S_{t-1}}.$$

The risk adjustment term has the following form:

$$\Phi_t = \Phi \left(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t \right) = \exp \left(-\tilde{\phi}_a \left(a_t - \bar{a} \right) - \tilde{\phi}_s \left(E_t s_{t+1} s_t - s^2 \right) + \tilde{\phi}_t \right), \tag{2.67}$$

where, recall,

$$a_t = \frac{S_t A_{t+1}}{P_t z_t^+},$$

and $\tilde{\phi}_t$ is a mean zero shock whose law of motion is discussed below. In addition, $\tilde{\phi}_a$, $\tilde{\phi}_s$, $\bar{a}$ are positive parameters. In the steady state discussion in the appendix, we derive the equilibrium outcomes that a_t coincides with $\bar{a}$ and $\Phi_t = 1$ in non-stochastic steady state.

The dependence of Φ_t on a_t ensures, in the usual way, that there is a unique steady state value of a_t that is independent of the initial net foreign assets and capital of the economy. The dependence of Φ_t on the anticipated growth rate of the exchange rate is designed to allow the model to reproduce two types of observations. The first concerns observations related uncovered interest parity. The second concerns the hump-shaped response of output to a monetary policy shock.

We first consider interest rate parity. To understand this, consider the standard text book representation of uncovered interest parity:

$$R_t - R_t^* = E_t \log S_{t+1} - \log S_t + \phi_t,$$

where ϕ_t denotes the risk premium on domestic assets. A log linear approximation of our model implies the above expression in which ϕ_t corresponds to the log deviation of Φ_t about its steady state value of unity. Consider first the case in which $\phi_t \equiv 0$. In this case, a fall in R_t relative to R_t^* produces an anticipated appreciation of the currency. This drop in $E_t \log S_{t+1} - \log S_t$ is accomplished in part by an instantaneous depreciation in $\log S_t$. The idea behind this is that asset holders respond to the unfavorable domestic rate of return by attempting to sell domestic assets and acquire foreign exchange for the purpose of acquiring foreign assets. This selling pressure pushes $\log S_t$ up, until the anticipated appreciation precisely compensates traders in international financial assets holding domestic assets.

There are two types of evidence that the preceding scenario does not hold in the data. Vector autoregression evidence on the response of financial variables to an expansionary domestic monetary policy shock suggests that $E_t \log S_{t+1} - \log S_t$ actually rises for a period of time (see, e.g., Eichenbaum and Evans (1995)). Also, regressions of realized future exchange rate changes on current interest rate differentials fail to produce the expected value of unity. Indeed, the typical result is a statistically significant negative coefficient.

One interpretation of these results is that when the domestic interest rate is reduced, say by a monetary policy shock, then risk in the domestic economy falls and that alone makes traders happier to hold domestic financial assets in spite of their lower nominal return and the losses they expect to make in the foreign exchange market. Our functional form for ϕ_t is designed to capture this idea. According to this functional form, when a shock occurs which causes an anticipated appreciation in the level of the exchange rate, then the assessment of risk in the domestic economy, Φ_t, falls. Later, when we introduce financial frictions, we will have access to additional mechanism for achieving this outcome. A concern we have with the current model is its unfortunate implication that any shock that creates an expectation of a depreciation in the currency makes domestic financial assets seem less risky. As a general proposition, this seems implausible. When we consider financial frictions, we will have variables such as the bankruptcy rate, which falls in the wake of an expansionary monetary shock, and which may accomplish the same equilibrium outcome as the ALLV specification, though perhaps the mechanism is more plausible in this case.

When we turn to the regression interpretation of the uncovered interest parity result, it is useful to consider the regression coefficient:

$$\gamma = \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R_t^f\right)}{var\left(R_t - R_t^f\right)} = \overbrace{1}^{\text{in theory}} < \overbrace{0}^{\text{in data}}$$

$$\begin{aligned}
\gamma &= \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R_t^f\right)}{var\left(R_t - R_t^f\right)} \\
&= \frac{cov\left(R_t - R_t^f - \log \Phi_t, R_t - R_t^f\right)}{var\left(R_t - R_t^f\right)} \\
&= 1 - \frac{cov\left(R_t - R_t^f, \tilde{\phi}_s\left(R_t - R_t^f - \left(R - R^f\right)\right)\right)}{var\left(R_t - R_t^f\right)} \\
&= 1 - \tilde{\phi}_s
\end{aligned}$$

according to our linearized expression above. Then,

$$\gamma = 1 - \frac{cov\left(R_t - R_t^*, \phi_t\right)}{var\left(R_t - R_t^*\right)}.$$

$$\gamma = \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R_t^*\right)}{var\left(R_t - R_t^*\right)} = \frac{cov\left(R_t - R_t^* - \phi_t, R_t - R_t^*\right)}{var\left(R_t - R_t^*\right)},$$

according to our linearized expression above. Then,

$$\gamma = 1 - \frac{cov\left(R_t - R_t^*, \phi_t\right)}{var\left(R_t - R_t^*\right)}.$$

Thus, any specification of ϕ_t which causes it to have a positive covariance with the interest rate differential will help in accounting for the regression coefficient specification of the uncovered interest rate puzzle. That is, such a covariance could result in γ being negative. This motivates an alternative to the risk specification in (2.67):

$$\Phi_t = \Phi\left(a_t, R_t^* - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(a_t - \bar{a}\right) - \tilde{\phi}_s\left(R_t^* - R_t - \left(R^* - R\right)\right) + \tilde{\phi}_t\right), \quad (2.68)$$

where a variable without time subscript denotes the corresponding value in nonstochastic steady state. We use this specification in our benchmark model.

We now turn to the connection between Φ_t and the hump-shape response of output to an expansionary monetary policy shock. As explained in section 2.4, a key ingredient in obtaining this type of response lies in factors that slow the response of demand to an expansionary monetary policy shock. The response of foreign purchases of domestic goods in the wake of such a shock depends on how much the exchange depreciates. The mechanism we have described slows the depreciation, and this simultaneously reduces the expansion of foreign demand.

2.5.4. Wage Setting

Finally, we consider wage setting. We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t = \left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \ 1 \leq \lambda_w < \infty,$$

where h_j denotes the j^{th} household supply of labor services. Households are subject to Calvo wage setting frictions as in Erceg, Henderson and Levin (2000) (EHL). With probability $1 - \xi_w$ the j^{th} household is able to reoptimize its wage and with probability ξ_w it sets its wage according to:

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{2.69}$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}, \tag{2.70}$$

where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{(h_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \right],$$

where τ_t^y is a tax on labor income and τ_t^w is a payroll tax. Also, υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$. Substituting this into the objective function,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \, [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

It is convenient to recall the scaling of variables:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \; \tilde{y}_t = \frac{Y_t}{z_t^+}, \; w_t = \tilde{W}_t / W_t, \; z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Then,

$$\begin{aligned}
\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\
&= \frac{W_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i},
\end{aligned}$$

where

$$\begin{aligned}
X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}, \; i > 0 \\
&= 1, \; i = 0.
\end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi_t^c \cdots \pi_{t+i-1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \cdots \bar{\pi}_{t+i}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}$$

In steady state,

$$\begin{aligned}
X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w} \left(\bar{\pi}^i\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\bar{\pi}^i \mu_{z^+}^i} \\
&= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w - 1} \\
&\to 0,
\end{aligned}$$

in the no-indexing case, when $\breve{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$.

Simplifying using the scaling notation,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \, [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\upsilon_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}$$

$$+\psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

Differentiating with respect to w_t,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \lambda_w (1+\sigma_L) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)-1}$$

$$+\psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}$$

$$+\frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Solving for the wage rate:

$$\begin{aligned}
w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}} \\
&= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}
\end{aligned}$$

where

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L} \\
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}.
\end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\sigma_L} \\
&\quad + (\beta\xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \\
&\quad + \ldots
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \{\zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \zeta_{t+2}^h + \ldots\} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1},
\end{aligned}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \qquad (2.71)$$

Also,

$$\begin{aligned}
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + (\beta\xi_w)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \\
&\quad \times \left(\frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots
\end{aligned}$$

or,

$$\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots \} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
\end{aligned}$$

so that

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{2.72}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t}, \ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{2.73}$$

In addition to (2.73), we have following equilibrium conditions associated with sticky wages[6]:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{2.75}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t\right)^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{2.76}$$

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{2.77}$$

2.6. Fiscal and Monetary Authorities

For purposes of estimating our models, we must make assumptions about how policy was conducted in the historical sample. In the case of Sweden there was a break in policy in 1992. In the decade before 1992, the value of the Krona in relation to a basket of currencies was held fixed.[7] After 1992, there are three ways to represent monetary policy. One is to imagine that the Riksbank conducted policy with commitment with the object of maximizing the

[6] Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w = 0$, we obtain

$$E_t \begin{bmatrix} \eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \left(\hat{\pi}_t - \widehat{\bar{\pi}}_t^c\right) + \eta_4 \left(\hat{\pi}_{t+1} - \rho_{\widehat{\bar{\pi}}^c} \widehat{\bar{\pi}}_t^c\right) \\ +\eta_5 \left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + \eta_6 \left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c} \widehat{\bar{\pi}}_t^c\right) \\ +\eta_7 \hat{\psi}_{z^+,t} + \eta_8 \hat{H}_t + \eta_9 \hat{\tau}_t^y + \eta_{10} \hat{\tau}_t^w + \eta_{11} \hat{\zeta}_t^h \\ +\eta_{12} \hat{\mu}_{z^+,t} + \eta_{13} \hat{\mu}_{z^+,t+1} \end{bmatrix} = 0, \tag{2.74}$$

where

$$b_w = \frac{[\lambda_w \sigma_L - (1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w \xi_w \\ \left(\sigma_L \lambda_w - b_w \left(1 + \beta \xi_w^2\right)\right) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ (1-\lambda_w) \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w)\frac{\tau^y}{(1-\tau^y)} \\ -(1-\lambda_w)\frac{\tau^w}{(1+\tau^w)} \\ -(1-\lambda_w) \\ -b_w \xi_w \\ b_w \beta \xi_w \end{pmatrix}.$$

[7] There was some adjustment to the exchange because the basket of currencies was adjusted.

following criterion:

$$E_t \sum_{j=0}^{\infty} \beta^j \{ \left(100 \left[\pi_t^c \pi_{t-1}^c \pi_{t-2}^c \pi_{t-3}^c - (\bar{\pi}_t^c)^4\right]\right)^2 + \lambda_y \left(100 \log\left(\frac{y_t}{y}\right)\right)^2 + \lambda_{\Delta R} \left(400 \left[R_t - R_{t-1}\right]\right)^2$$
$$+\lambda_s \left(S_t - \bar{S}\right)^2 \}$$

This approach takes the parameters in the criterion, λ_y, $\lambda_{\Delta R}$ and λ_s as unknown parameters to be estimated. A second approach is to suppose that policy was Ramsey-optimal, that is that it was chosen with commitment to maximize the discounted social welfare criterion.. A virtue of this approach is that there are no policy parameters to be estimated. A third approach is to suppose that policy was conducted according to a Taylor rule of the following form (see ALLV):

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\,[\log\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right) \\ &\quad + r_y \log\left(\frac{gdp_{t-1}}{gdp}\right) + r_q \log\left(\frac{q_{t-1}}{q}\right)] + r_{\Delta\pi} \Delta \log\left(\frac{\pi_t^c}{\pi^c}\right) + r_{\Delta y} \Delta \log\left(\frac{gdp_t}{gdp}\right) + \varepsilon_{R,t}. \end{aligned} \tag{2.78}$$

Here too, the parameters would be taken as unknowns to be estimated. gdp denotes measured GDP in the data, which might be different from y in the model. In addition, $\bar{\pi}_t^c$ is an exogenous process that characterizes the central bank's consumer price index inflation target and its steady state value corresponds to the steady state of actual inflation.

For estimation to be done using a sample beginning with the 1980s, we have to take into account the break in policy in 1992. In order to avoid transition effects, we plan to drop data beginning with 1992Q4 and ending with a quarter after the new policy regime is in place. In formulating the likelihood function of the whole dataset, we plan to treat the pre- and post-break samples as independent datasets. Currently, we estimate the model on data from 1995q1-2008q1. The data discussed below refers to this period.

We model government consumption expenditures as

$$G_t = g_t z_t^+,$$

where g_t is an exogenous stochastic process, orthogonal to the other shocks in the model. We suppose that

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \varepsilon_t^g.$$

where $g = \eta_g Y$. We set $\eta_g = 0.3$, the sample average of government consumption as a fraction of GDP.

The tax rates in our model are:

$$\tau_t^k,\ \tau_t^b,\ \tau_t^y,\ \tau_t^c,\ \tau_t^w.$$

We briefly discuss the treatment of these tax rates. In the versions of our model without financial frictions, capital is accumulated and capital income accrues directly to the household. However, an observationally equivalent representation of the model has these activities occurring in the firm. This latter interpretation is the convenient one, when thinking about the data and, in particular, the measurement of τ^k. We set the tax rate on capital income, τ^k, to 0.25. We arrived at this number as follows. The statutory rate on household capital income is 30 percent and the statutory rate on corporate income is 28 percent. Combining these two numbers we conclude that the statutory rate on corporate and household income is 50 percent. Indirect evidence from Devereux, Griffith and Klemm (2002) suggests to us that the effective tax on capital income may be one half this amount, and this is why we set $\tau^k = 0.25$ in the model. We reach this conclusion because of Devereux, Griffith and Klemm observation that the effective corporate income tax is roughly 1/2 of the statutory rate and we adopt the rough approximation that the same applies to the household tax rate. Our assumption that τ^k is constant is also motivated by Devereux, Griffith and Klemm. Their measure of the corporate component of the effective capital income tax rate exhibits very little variation over our sample which is 1995-2005.

Now we turn to the tax rate on bonds, τ^b. We set $\tau^b = 0$ to be able to roughly match the pre-tax real rate on bonds of roughly 2.5% in the data. Setting $\tau^b = 0$ is required to get the interest rate on bonds to be this low, given the high GDP growth rate, log utility of consumption and β bounded below 1. We plan to investigate empirical measures of the effective tax rate on bonds in the future.

For evidence on τ^w we use the data collected by ALLV. Based on these data, we set the payroll tax rate, τ^w, to 0.35. Data on the value-added tax on consumption, τ^c, and the personal income tax rate that applies to labor, τ^y, are available from Statistics Sweden and indicate $\tau^c = 0.25$ and $\tau^y = 0.3$. These are the average values of the corresponding tax rates over the period 1995-2004. We hold these tax rates constant because they exhibit very little variability over this period.

2.7. Foreign variables

Below, we describe the stochastic process driving the foreign variables. Our representation takes into account our assumption that foreign output, Y_t^*, is affected by disturbances to z_t^+, just as domestic variables are. In particular, our model of Y_t^* is:

$$\begin{aligned} \log Y_t^* &= \log z_t^+ + \log y_t^* \\ &= \log y_t^* + \frac{\alpha}{1-\alpha} \log \psi_t + \log z_t, \end{aligned}$$

where $\log\left(y_t^*\right)$ is assumed to be a stationary process. We assume:

$$\begin{pmatrix} \log\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \log\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \end{pmatrix}$$
$$+ \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\psi,t} \end{pmatrix},$$

where the ε_t's are mean zero, unit variance, i.i.d. processes uncorrelated with each other. In matrix form,

$$X_t^* = AX_{t-1}^* + C\varepsilon_t,$$

in obvious notation. Note that the matrix C has 10 elements, so that the order condition for identification is satisfied, since CC' represents 15 independent equations.

We now briefly discuss the intuition underlying the zero restrictions in A and C. First, we assume that the shock, $\varepsilon_{y^*,t}$, affects the first three variables in X_t^*, while $\varepsilon_{\pi^*,t}$ only affects the second two and $\varepsilon_{R^*,t}$ only affects the third. The assumption about $\varepsilon_{R^*,t}$ corresponds to one strategy for identifying a monetary policy shock, in which it is assumed that inflation and output are predetermined relative to the monetary policy shock. Under this interpretation of $\varepsilon_{R^*,t}$, our treatment of the foreign monetary policy shock and the domestic one are inconsistent. In our model as currently formulated domestic prices are not predetermined in the period of a monetary policy shock. Second, note from the zeros in the last two columns of the first row in A and C, that the technology shocks do not affect y_t^*. This reflects our assumption that the impact of technology shocks on Y_t^* is completely taken into account by z_t^+, while all other shocks to Y_t^* are orthogonal to z_t^+ and they affect Y_t^* via y_t^*. Third, the A and C matrices capture the notion that innovations to technology affect foreign inflation and the interest rate via their impact on z_t^+. Fourth, our assumptions on A and C imply that $\log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right)$ and $\log\left(\frac{\mu_{z,t}}{\mu_z}\right)$ are univariate first order autoregressive processes driven by $\varepsilon_{\mu_\psi,t}$ and $\varepsilon_{\mu_z,t}$, respectively. This is a standard assumption made on technology shocks in DSGE models.

2.8. Resource Constraints

We begin by deriving a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production. Next, we consider the resource constraint.

2.8.1. Domestic Homogeneous Output: Production

Consider the unweighted average of the intermediate goods:

$$\begin{aligned}
Y_t^{sum} &= \int_0^1 Y_{i,t} di \\
&= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi \right] di \\
&= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{it}} \right)^{\alpha} H_{it} - z_t^+ \phi \right] di \\
&= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{it} di - z_t^+ \phi
\end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{2.79}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{2.80}$$

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^{\alpha} H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{2.81}$$

We replace aggregate homogeneous labor, H_t, with aggregate household labor, h_t, as follows:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right]. \tag{2.82}$$

2.8.2. Trade Balance

We begin by developing the link between net exports and the current account. Expenses on imports and net new purchases of foreign assets, A_{t+1}, must equal income from exports and from previously purchased net foreign assets:

$$S_t A_{t+1} + \textit{expenses} \text{ on imports}_t = \textit{receipts} \text{ from exports}_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*,$$

where Φ_t is as defined in (2.68) in our benchmark model. Expenses on imports correspond to the purchases of specialized importers for the consumption, investment and export sectors:

$$\textit{expenses} \text{ on imports}_t = S_t P_t^* R_t^{\nu,*} \left(C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right),$$

using (2.45), (2.46) and (2.47). Note the presence of the price distortion terms here. To understand these terms, recall that, for example, C_t^m is produced as a linear homogeneous function of specialized imported goods. Because the specialized importers only buy foreign goods, it is their total expenditures that interests us here. When the imports are distributed evenly across differentiated goods, then the total quantity of those imports is C_t^m, and the value of imports associated with domestic production of consumption goods is $S_t P_t^* R_t^{\nu,*} C_t^m$. When there are price distortions among imported intermediate goods, then the sum of the homogeneous import goods is higher for any given value of C_t^m. This is captured by the price distortion terms in the above expression.

We conclude that the current account can be written as follows:

$$
\begin{aligned}
& S_t A_{t+1}^* + S_t P_t^* R_t^{\nu,*} \left(C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
= \; & S_t P_t^x X_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*,
\end{aligned}
$$

where Φ_t is defined in section 2.5.3. Expressing the current account in scaled form,

$$
\begin{aligned}
& a_t P_t z_t^+ + S_t P_t^* z_t^+ R_t^{\nu,*} \left(c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
& \qquad = z_t^+ S_t P_t^x x_t + R_{t-1}^* \Phi_{t-1} S_t \frac{P_{t-1} z_{t-1}^+ a_{t-1}}{S_{t-1}},
\end{aligned}
$$

where, recall, $a_t = S_t A_{t+1}^* / (P_t z_t^+)$. Dividing by $P_t z_t^+$, we obtain

$$
\begin{aligned}
& a_t + q_t p_t^c R_t^{\nu,*} \left(c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
& \qquad = q_t p_t^c p_t^x x_t + R_{t-1}^* \Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}},
\end{aligned}
\tag{2.83}
$$

using (2.33).

We have already defined real, scaled GDP in terms of aggregate factors of production. It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (2.41),

$$
z_t^+ y_t = G_t + C_t^d + I_t^d + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} Y_t^*,
$$

or, after scaling by z_t^+ and using (2.18):

$$
\begin{aligned}
y_t \;\; = \;\; & g_t + (1-\omega_c) \left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} \right) (1-\omega_i) \\
& + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} y_t^*.
\end{aligned}
\tag{2.84}
$$

We now consider the restrictions across inflation rates implied by our relative price formulas. In terms of the expressions in (2.2) there are the restrictions implied by $p_t^{m,j}/p_{t-1}^{m,j}$, $j = x, c, i$, and p_t^x. The restrictions implied by the other two relative prices in (2.2), p_t^i and p_t^c, have already been exploited in (2.23) and (2.57), respectively. Finally, we also exploit the restriction across inflation rates implied by q_t/q_{t-1} and (2.3). Thus,

$$
\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \tag{2.85}
$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{2.86}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{2.87}$$

$$\frac{p_t^{x}}{p_{t-1}^{x}} = \frac{\pi_t^{x}}{\pi_t^{*}} \tag{2.88}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t\pi_t^{*}}{\pi_t^{c}}. \tag{2.89}$$

This completes the description of the baseline model.

2.9. Endogenous Variables of the Baseline Model

In the above sections we derived following 71 equations,

$$\begin{gathered}2.5, 2.6, 2.7, 2.10, 2.11, 2.12, 2.13, 2.14, 2.16, 2.18, 2.19, 2.20, 2.22, 2.23, 2.24,\\ 2.26, 2.31, 2.32, 2.34, 2.35, 2.36, 2.37, 2.38, 2.42, 2.44, 2.49, 2.50, 2.51, 2.52, 2.48,\\ 2.53, 2.55, 2.56, 2.57, 2.58, 2.60, 2.61, 2.62, 2.63, 2.64, 2.66, 2.75, 2.76, 2.77, 2.73,\\ 2.70, 2.71, 2.72, 2.78, 2.81, 2.82, 2.83, 2.84, 2.85, 2.86, 2.87, 2.88, 2.89, 2.68\end{gathered}$$

which can be used to solve for the following 71 unknowns:

$$\begin{aligned}\bar{r}_t^k, \bar{w}_t, R_t^{\nu,*}, R_t^f, R_t^x, R_t, mc_t, mc_t^x, mc_t^{m,c}, mc_t^{m,i}, mc_t^{m,x}, \pi_t, \pi_t^x, \pi_t^c, \pi_t^i, \pi_t^{m,c}, \pi_t^{m,i}, \pi_t^{m,x},\\ p_t^c, p_t^x, p_t^i, p_t^{m,x}, p_t^{m,c}, p_t^{m,i}, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, q_t, i_t, c_t, x_t, a_t, s_t, \psi_{z^+,t}, y_t\\ K_t^d, F_t^d, \tilde{\pi}_{d,t}, \mathring{p}_t, K_{x,t}, F_{x,t}, \tilde{\pi}_t^x, \mathring{p}_t^x, \{K_{m,j,t}, F_{m,j,t}, \tilde{\pi}_t^{m,j}, \mathring{p}_t^{m,j}; j = c, i, x\}, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k\\ \Phi_t, \tilde{S}_t, \tilde{S}_t', a(u_t), \mathring{w}_t, c_t^m, i_t^m, x_t^m, \pi_w.\end{aligned}$$

3. Introducing Financial Frictions into the Model

A number of the activities in the model of the previous section require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. In the model of the previous section financing requirements affect the allocations, but not very much. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household, who puts up the finances and later reaps the rewards. When real-world financial frictions are introduced into a model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

Although the literature shows how to introduce financial frictions much more extensively, here we proceed by assuming that only the accumulation and management of capital involves frictions. We will continue to assume that working capital loans are frictionless. At the end of this introduction, we briefly discuss the idea of introducing financial frictions into working capital loans. Our strategy of introducing frictions in the accumulation and management of capital follows the variant of the Bernanke, Gertler and Gilchrist (1999) (henceforth BGG) model implemented in Christiano, Motto and Rostagno (2003). The discussion here borrows heavily from the derivation in Christiano, Motto and Rostagno (2007) (henceforth CMR).

The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce 'entrepreneurs'. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, 'banks', can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. An alternative arrangement that is more efficient is one in which banks extend entrepreneurs a 'standard debt contract', which specifies a loan amount and a given interest payment. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are 'bankrupt'. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed. For a graphical illustration of the financing problem in the capital market, see Figure A.

The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneur's net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneur's assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

The ultimate source of funds for lending to entrepreneurs is the household. The standard debt contracts extended by banks to entrepreneurs are financed by issuing liabilities to households. Although individual entrepreneurs are risky, banks themselves are not. We suppose that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. Extensions of the model that introduce risk into banking have been developed, but it is not clear that the added complexity is justified.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

At the level of abstraction of the model, the capital stock includes both housing and business capital. As a result, the entrepreneurs can also be interpreted as households in their capacity of homeowners. An expanded version of the model would pull apart the household and business sectors to study each individually. Another straightforward expansion of the model would apply the model of financial frictions to working capital loans.

With this model, it is typically the practice to compare the net worth of entrepreneurs with a stock market quantity such as the Dow Jones Industrial Average. Whether this is really appropriate is uncertain. A case can be made that the 'bank loans' of entrepreneurs in the model correspond well with actual bank loans *plus* actual equity. It is well known that dividend payments on equity are very smooth. Firms work hard to accomplish this. For example, during the US Great Depression some firms were willing to sell their own physical capital in order to avoid cutting dividends. That this is so is perhaps not surprising. The asymmetric information problems with actual equity are surely as severe as they are for the banks in our model. Under these circumstances one might expect equity holders to demand a payment that is not contingent on the realization of uncertainty within the firm (payments could be contingent upon publicly observed variables). Under this vision, the net worth in the model would correspond not to a measure of the aggregate stock market, but to the ownership stake of the managers and others who exert most direct control over the firm. The 'bank loans' in this model would, under this view of things, correspond to the actual loans of firms (i.e., bank loans and other loans such as commercial paper) plus the outstanding equity. While this is perhaps too extreme, these observations highlight that there is substantial uncertainty over exactly what variable should be compared with net worth in the model. It is important to emphasize, however, that whatever the right interpretation is of net worth, the model potentially captures balance sheet problems very nicely.

Finally, we make some remarks on the introduction of financial frictions into working capital loans. It is possible to accomplish this with relatively little modification to the model, by following the strategy described in Fisher (1998). However, with this strategy the effects of financial frictions are quite modest, because the firms in the model which use working capital do not have assets. As a result, the balance sheet channel does not operate. We conjecture that for financial frictions in working capital to be interesting, the borrowing firms would need to have assets. One way this could be accomplished would be to assume that they use and own capital that is specific to their firm. In this way, fluctuations in the value of that capital induced by changes in asset prices would change their ability to borrow, and hence to produce. This strategy is algebra-intensive because of the fact that these firms also set their prices subject to Calvo frictions.

3.1. Modifying the Baseline Model

The financial frictions bring a net increase of two equations over the equations in the model of the previous section. In addition, they introduce two new endogenous variables, one related to the interest rate paid by entrepreneurs as well as their net worth. The financial frictions also allow us to introduce two new random variables. We now provide a formal discussion of the model.

As we shall see, entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumptions have been made in the model, which guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by a entrepreneur is proportional to his level of net worth. These are enough to guarantee the aggregation result.

3.1.1. The Individual Entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1}. The entrepreneur's net worth, N_{t+1}, constitutes his state at this time, and nothing else about his history is relevant. We imagine that there are many entrepreneurs for each level of net worth and that for each level of net worth, there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with a particular level of net worth, N_{t+1}. The entrepreneur combines this net worth with a bank loan, B_{t+1}, to purchase new, installed physical capital, $\bar{K}_{t+1}$, from capital producers. The loan the entrepreneur requires for this is:

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \tag{3.1}$$

The entrepreneur is required to pay a gross interest rate, Z_{t+1}, on the bank loan at the end of period $t+1$, if it is feasible to do so. After purchasing capital the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital, $\bar{K}_{t+1}$, into $\bar{K}_{t+1}\omega$. Here, ω is a unit mean, lognormally and independently distributed random variable across entrepreneurs. The variance of $\log \omega$ is σ_t^2. The t subscript indicates that σ_t is itself the realization of a random variable. This allows us to consider the effects of an increase in the riskiness of individual entrepreneurs. We denote the cumulative distribution function of ω by $F(\omega;\sigma)$. and its partial derivatives as e.g. $F_\omega(\omega;\sigma)$, $F_\sigma(\omega;\sigma)$

After observing the period $t+1$ shocks, the entrepreneur sets the utilization rate, u_{t+1}, of capital and rents capital out in competitive markets at nominal rental rate, $P_{t+1} r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in (2.16). The entrepreneur then sells the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic shock ω earns a return (after taxes), of $R_{t+1}^k \omega$, where R_{t+1}^k is defined in (2.59). Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k.

After entrepreneurs sell their capital, they settle their bank loans. At this point, the resources available to an entrepreneur who has purchased $\bar{K}_{t+1}$ units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1}$. There is a cutoff value of ω, $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{3.2}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

to the bank, which is less than $Z_{t+1} B_{t+1}$. In this case, the bank monitors the entrepreneur, at cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

We note briefly that the definition of R_{t+1}^k lacks some realism because it does not take into account the deductibility of interest payments. With the more realistic treatment of interest, the after tax rate of return on capital would be changed from (2.59) to include deductibility of interest payments for firms:

$$
\begin{aligned}
R_{t+1}^{k} &= \frac{\left(1-\tau_{t}^{k}\right)\left[u_{t+1}r_{t+1}^{k}-\frac{p_{t+1}^{i}}{\Psi_{t+1}}a(u_{t+1})-(Z_{t+1}-1)\frac{B_{t+1}}{P_{t}P_{k',t}\bar{K}_{t+1}}\right]P_{t+1}}{P_{t}P_{k',t}} \\
&\quad+\frac{(1-\delta)P_{t+1}P_{k',t+1}+\tau_{t}^{k}\delta P_{t}P_{k',t}}{P_{t}P_{k',t}} \\
&= \frac{\left(1-\tau_{t}^{k}\right)\left[u_{t+1}r_{t+1}^{k}-\frac{1}{\Psi_{t+1}}a(u_{t+1})-\bar{\omega}_{t+1}R_{t+1}^{k}+\frac{B_{t+1}}{P_{t}P_{k',t}\bar{K}_{t+1}}\right]P_{t+1}}{P_{t}P_{k',t}} \\
&\quad+\frac{(1-\delta)P_{t+1}P_{k',t+1}+\tau_{t}^{k}\delta P_{t}P_{k',t}}{P_{t}P_{k',t}},
\end{aligned}
$$

by (3.2). With this representation, R_{t}^{k} is a function of features of the loan contract. This will change the choice of optimal contract, discussed below. We plan to explore the implications of this in future work.

Banks obtain the funds loaned in period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest, R_{t}. The subscript on R_{t} indicates that the payoff to households in $t+1$ is not contingent on the period $t+1$ uncertainty. This feature of the relationship between households and banks is simply assumed. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as (2.64).

We suppose that there is competition and free entry among banks, and that banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs.[8] It follows that the bank's cash flow in each state of period $t+1$ is zero, for each loan amount.[9] For loans in the amount, B_{t+1}, the bank receives gross interest, $Z_{t+1}B_{t+1}$, from the $1-F\left(\bar{\omega}_{t+1};\sigma_{t}\right)$ entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is:

$$
\left[1-F\left(\bar{\omega}_{t+1};\sigma_{t}\right)\right]Z_{t+1}B_{t+1}+(1-\mu)\int_{0}^{\bar{\omega}_{t+1}}\omega dF\left(\omega;\sigma_{t}\right)R_{t+1}^{k}P_{t}P_{k',t}\bar{K}_{t+1}=R_{t}B_{t+1},
$$

or, after making use of (3.2) and rearranging,

$$
\left[\Gamma(\bar{\omega}_{t+1};\sigma_{t})-\mu G(\bar{\omega}_{t+1};\sigma_{t})\right]\frac{R_{t+1}^{k}}{R_{t}}\varrho_{t}=\varrho_{t}-1 \tag{3.3}
$$

[8] If banks also had access to state contingent securities, then free entry and competition would imply that banks earn zero profits in an ex ante expected sense from the point of view of period t.

[9] Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state by state zero profit condition quoted in the text.

where

$$G(\bar{\omega}_{t+1};\sigma_t) = \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega;\sigma_t).$$
$$\Gamma(\bar{\omega}_{t+1};\sigma_t) = \bar{\omega}_{t+1}\left[1 - F(\bar{\omega}_{t+1};\sigma_t)\right] + G(\bar{\omega}_{t+1};\sigma_t)$$
$$\varrho_t = \frac{P_t P_{k',t}\bar{K}_{t+1}}{N_{t+1}}.$$

The expression, $\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)$ is the share of revenues earned by entrepreneurs that borrow B_{t+1}, which goes to banks. Note that $\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) = 1 - F(\bar{\omega}_{t+1};\sigma_t) > 0$ and $G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) = \bar{\omega}_{t+1}F_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) > 0$. It is thus not surprising that the share of entrepreneurial revenues accruing to banks is non-monotone with respect to $\bar{\omega}_{t+1}$. BGG argue that the expression on the left of (3.3) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1} = \omega^*$, say. The expression is increasing for $\bar{\omega}_{t+1} < \omega^*$ and decreasing for $\bar{\omega}_{t+1} > \omega^*$. Thus, for any given value of ϱ_t and R^k_{t+1}/R_t, generically there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy (3.3). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of the inverted 'U' shape. This is because, according to (3.2), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. As discussed below, the equilibrium contract is one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period $t+1$ shock that drives R^k_{t+1} up. The fraction of entrepreneurs that experience bankruptcy is $F\left(\bar{\omega}_{t+1};\sigma_t\right)$, so it follows that a shock which drives up R^k_{t+1} has a negative contemporaneous impact on the bankruptcy rate. According to (2.59), shocks that drive R^k_{t+1} up include anything which raises the value of physical capital and/or the rental rate of capital.

As just noted, we suppose that the equilibrium debt contract maximizes entrepreneurial welfare, subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt, B_{t+1} and a state $t+1-$contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty}\left[R^k_{t+1}\omega P_t P_{k',t}\bar{K}_{t+1} - Z_{t+1}B_{t+1}\right]dF(\omega;\sigma_t)}{R_t N_{t+1}}$$
$$= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty}\left[\omega - \bar{\omega}_{t+1}\right]dF(\omega;\sigma_t)R^k_{t+1}P_t P_{k',t}\bar{K}_{t+1}}{R_t N_{t+1}}$$
$$= E_t\left\{\left[1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)\right]\frac{R^k_{t+1}}{R_t}\right\}\varrho_t,$$

after making use of (3.1), (3.2) and

$$1=\int_0^\infty \omega dF(\omega;\sigma_t)=\int_{\bar{\omega}_{t+1}}^\infty \omega dF(\omega;\sigma_t)+G(\bar{\omega}_{t+1};\sigma_t).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to R_tN_{t+1}), subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t,\{\bar{\omega}_{t+1}\}} E_t\left\{[1-\Gamma(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t+\lambda_{t+1}\left([\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1\right)\right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The first order conditions for this problem are:

$$\begin{aligned}
E_t\left\{[1-\Gamma(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}\left([\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}-1\right)\right\} &= 0\\
-\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}[\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)-\mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t} &= 0\\
[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1 &= 0,
\end{aligned}$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that we assume $\lambda_{t+1}>0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the first order conditions reduce to:

$$E_t\left\{\begin{array}{c}[1-\Gamma(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}+\frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)-\mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}\\ \left([\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}-1\right)\end{array}\right\} = 0, \tag{3.4}$$

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t)-\mu G(\bar{\omega}_{t+1};\sigma_t)]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1 = 0, \tag{3.5}$$

for $t=0,1,2,...\infty$ and for $t=-1,0,1,2,...$ respectively.

Since N_{t+1} does not appear in the last two equations, we conclude that ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs, regardless of their net worth. The results for ϱ_t implies that

$$\frac{B_{t+1}}{N_{t+1}}=\varrho_t-1,$$

i.e. that an entrepreneur's loan amount is proportional to his net worth. Rewriting (3.1) and (3.2) we see that the rate of interest paid by the entrepreneur is

$$Z_{t+1}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{N_{t+1}}{P_tP_{k',t}\bar{K}_{t+1}}}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{1}{\varrho_t}}, \tag{3.6}$$

which is the same for all entrepreneurs, regardless of their net worth.

3.1.2. Aggregation Across Entrepreneurs and the Risk Premium

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth, N_{t+1}. Then, aggregate average net worth, $\bar{N}_{t+1}$, is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1}) \, dN_{t+1}.$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N, where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \tag{3.7}$$

where $\bar{K}_t^N$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t) \, dN_t.$$

Multiplying (3.7) by $f(N_t)$ and integrating over all entrepreneurs,

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.$$

Writing this out more fully:

$$\begin{aligned} V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &- \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t. \end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$. Substituting:

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t).$$

After V_t is determined, each entrepreneur faces an identical and independent probability $1 - \gamma_t$ of being selected to exit the economy. With the complementary probability, γ_t, each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is simply $\gamma_t \bar{V}_t$. A fraction, $1-\gamma_t$, of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer, W_t^e. This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or,

$$\begin{aligned} \bar{N}_{t+1} &= \gamma_t \{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \} \\ & \quad + W_t^e. \end{aligned} \tag{3.8}$$

3.2. Solving the Financial Frictions Model

In this subsection we indicate how the equilibrium conditions of the baseline model must be modified to accommodate financial frictions. We then consider the problem of solving for the model's steady state.

3.2.1. Equilibrium Conditions

Consider the households. Households no longer accumulate physical capital, and the first order condition, (2.61), must be dropped. No other changes need to be made to the household first order conditions. Equation (2.64) can be interpreted as applying to the household's decision to make bank deposits. The household equations, (2.57) and (2.62), pertaining to investment can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the first order condition of many identical, competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables, noting that W_t^e is set so that its scaled counterpart is constant:

$$n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+}, \; w^e = \frac{W_t^e}{P_t z_t^+}.$$

Dividing both sides of (3.8) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} \left[R_t^k p_{k',t-1} \bar{k}_t - R_{t-1} \left(p_{k',t-1} \bar{k}_t - n_t \right) - \mu G \left(\bar{\omega}_t; \sigma_{t-1} \right) R_t^k p_{k',t-1} \bar{k}_t \right] + w^e, \tag{3.9}$$

for $t = 0, 1, 2, ...$. Equation (3.9) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payments to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, eq. (3.5), can be expressed in terms of the scaled variables as:

$$\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t) = \frac{R_t}{R_{t+1}^k}\left(1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}\right), \tag{3.10}$$

for $t = -1, 0, 1, 2, \ldots$. The optimality condition for bank loans is (3.4).

The household's first order condition associated with the accumulation of capital, (2.61), must be dropped. The output equation, (2.82), does not have to be modified. Instead, the resource constraint for domestic homogenous goods (2.84) needs to be adjusted for the monitoring costs:

$$\begin{aligned} y_t - d_t &= g_t + (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i}\left(i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \\ &+ \left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t-1}}}\left(p_t^x\right)^{-\eta_f} y_t^*, \end{aligned} \tag{3.11}$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t;\sigma_{t-1}) R_t^k p_{k',t-1}\bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

When we later take the model to data measured GDP is taken to be the left-hand side of eq. (3.11), $gdp_t = y_t - d_t$.

Account has to be taken of the consumption by exiting entrepreneurs. The net worth of these entrepreneurs is $(1-\gamma_t)V_t$ and we assume a fraction, $1-\Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ, is consumed by the exiting entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta\frac{1-\gamma_t}{\gamma_t}(n_{t+1} - w^e) z_t^+ P_t$$

from the right side of (2.17). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

We now turn to the risk premium on entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate, R_t, which he loses by applying it to capital rather than just depositing it in the bank. The average payment by all entrepreneurs to the bank is the entire object in square brackets in equation (3.8). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the risk premium in the model. Another is the excess of the interest rate paid by entrepreneurs who are not bankrupt, over R_t :

$$Z_{t+1} - R_t = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t,$$

according to (3.6).

The financial frictions brings a net increase of 2 equations (we add (3.4), (3.9) and (3.10), and delete (2.61)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72 variables. The financial frictions also introduce additional shocks, σ_t and γ_t.

4. Introducing Unemployment into the Model

This section replaces the model of the labor market in our baseline model with the search and matching framework of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005a,b). We integrate the framework into our environment - which includes capital and monetary factors - following the Gertler, Sala and Trigari (2008) (henceforth GST) strategy implemented in Christiano, Ilut, Motto, and Rostagno (2007). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees. In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employed, it has unemployment and vacancies.

The labor market in our alternative labor market model is a modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model. A key difference is that we allow for endogenous separation of employees from their jobs, as in e.g. den Haan, Ramey and Watson (2000). In what follows, we first provide an overview and after that we present the detailed decision problems of agents in the labor market.

4.1. Sketch of the Model

As in the discussion of section 2.2, we adopt the Dixit-Stiglitz specification of homogeneous goods production. A representative, competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists, who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions in the baseline model.

In the baseline model, the homogeneous labor services are supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services (see section 2.2).

The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by 'employment agencies'. See Figure A for a graphical illustration. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected.[10]

Each employment agency retains a large number of workers. At the beginning of the period a fraction, $1-\rho$, of workers is randomly selected to separate from the agency and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods. The employees of an agency are represented by a union at negotiations. This assumption has no consequences except that it makes clear which wage (i.e. the collectively negotiated wage) will apply to workers arriving at the agency during the duration of the wage contract. As an alternative, we also consider the case when each worker bargains with the employer on a unilateral basis (see section 4.4.2). Each employment agency is permanently allocated to one of N different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same rule used in our baseline model.

Next, each worker realizes an idiosyncratic productivity shock and workers with a shock below an endogenously determined cutoff separate into unemployment. The cutoff level of productivity is chosen relative to a particular surplus criterion,in our main case maximizing the surplus of the employment agency. The intensity of each worker's labor effort is then determined by an efficiency criterion. To explain how labor intensity is chosen, we discuss the implications of increased intensity for the worker and for the employment agency. The utility function of the household in the present labor market model is a modified version of (2.54):

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{ \zeta_{t+l}^c \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \left[\sum_{i=0}^{N-1} \frac{\left(\varsigma_{i,t+l}\right)^{1+\sigma_L}}{1+\sigma_L} \left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i\right)\right] l_{t+l}^i \right] \}, \quad (4.1)$$

where $\left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i\right)\right] l_{t+l}^i$ is the quantity of people working in cohort i and $\varsigma_{i,t}$ is the intensity with with each worker in cohort i works. As in GST, we follow the family household construct of Merz (1995) in supposing that each household has a large number of workers. Although

[10] An alternative (perhaps more natural) formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem.

the individual worker's labor market experience - whether employed or unemployed - is determined in part by idiosyncratic shocks, the household has sufficiently many workers that the total fraction of workers employed, L_t, as well as the fractions allocated among the different cohorts, $[1-\mathcal{F}(\bar{a}_t^i)]\, l_t^i$, $i=0,...,N-1$, is the same for each household. We suppose that all the household's workers are supplied inelastically to the labor market (i.e., labor force participation is constant). Each worker passes randomly from employment with a particular agency to unemployment and back to employment according to the endogenous probabilities described below.

The household's currency receipts arising from the labor market are:

$$\left(1-\tau_t^y\right)\left(1-L_t\right) P_t b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i \overbrace{\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i}^{\text{quantity of people working in cohort } i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} \tag{4.2}$$

where W_t^i is the nominal wage rate earned by workers in cohort $i=0,...,N-1$. The index, i, indicates the number of periods in the past when bargaining occurred most recently. As in our baseline model, there is a labor income tax τ_t^y and a payroll tax τ_t^w that affect the after-tax wage. Note that we implicitly assume that labor intensity, $\varsigma_{i,t}$, is cohort-specific. This is explained below. The presence of the term involving b^u indicates the assumption that unemployed workers receive a payment of $b^u z_t^+$ final consumption goods. The unemployment benefits are financed by lump sum taxes.

Let the price of labor services, W_t, denote the marginal gain to the employment agency that occurs when an individual worker raises labor intensity by one unit. Because the employment agency is competitive in the supply of labor services, W_t is taken as given and is the same for all agencies, regardless of which cohort it is in. Labor intensity equates the worker's marginal cost to the agency's marginal benefit:

$$W_t \frac{\mathcal{E}_t^i}{1-\mathcal{F}_t^i} = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\upsilon_t} \tag{4.3}$$

for $i=0,...,N-1$. Here,

$$\mathcal{E}_t^i \equiv \mathcal{E}\left(\bar{a}_t^i;\sigma_{a,t}\right) \equiv \int_{\bar{a}_t^i}^{\infty} a d\mathcal{F}\left(a;\sigma_{a,t}\right) \tag{4.4}$$

$$\mathcal{F}_t^i = \mathcal{F}\left(\bar{a}_t^i;\sigma_{a,t}\right) = \int_0^{\bar{a}_t^i} d\mathcal{F}\left(a;\sigma_{a,t}\right). \tag{4.5}$$

and $\mathcal{E}_t^i/\left(1-\mathcal{F}_t^i\right)$ is the average productivity of a worker in working in cohort i (i.e., who has survived the endogenous productivity cut). Division by $1-\mathcal{F}_t^i$ is required in (4.3) so that the expectation is relative to the distribution of a conditional on $a \geq \bar{a}_t^i$. To understand the expression on the right of (4.3), note that the marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is $\zeta_t^h A_L \varsigma_{i,t}^{\sigma_L}$. This is converted to currency units by dividing by the multiplier, υ_t, on the household's nominal budget constraint. Scaling (4.3) by $P_t z_t^+$yields:

$$\bar{w}_t \frac{\mathcal{E}_t^i}{1 - \mathcal{F}_t^i} = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\psi_{z^+,t}} \tag{4.6}$$

Labor intensity will be potentially different across cohorts because $\mathcal{E}_t^i / \left(1 - \mathcal{F}_t^i\right)$ in (4.6) is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (4.6) and that some workers will endogenously separate. Note, that the ratio

$$\frac{\mathcal{E}_t^i}{\left(1 - \mathcal{F}_t^i\right) \varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since all other variables in (4.6) are not indexed by cohort.

Finally, the employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies, v_t^i. The vacancy posting costs associated with v_t^i are:

$$\frac{\kappa z_t^+}{2} \left(\frac{Q_t^\iota v_t^i}{\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i} \right)^2 \left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i,$$

units of the domestic homogeneous good. Here, $\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i$ denotes the number of employees in the i^{th} cohort after endogenous separations have occurred and $\kappa z_t^+/2$ is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate. Also, Q_t is the probability that a posted vacancy is filled. The functional form of our cost function nests GT and GST when $\iota = 1$. With this parameterization the cost function is in terms of the number of people hired, not the number of vacancies per se. We interpret this as reflecting that the GT and GST specifications emphasize internal costs (such as training and other) of adjusting the work force, and not search costs. In models used in the search literature (see, e.g., Shimer (2005a)), vacancy posting costs are independent of Q_t, i.e., $\iota = 0$. We also plan to investigate this latter case. We suspect that the model implies less amplification in response to expansionary shock in the case, $\iota = 0$. In a boom, Q_t can be expected to fall, so that with $\iota = 1$, costs of posting vacancies decrease in the GT specification.

4.2. Employment-Agency

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $\tilde{W}_{i,t}$, as in (2.69):

$$\tilde{W}_{i,t} = \tilde{\pi}_{w,t} \tilde{W}_{i-1,t-1}, \;\; \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}\right)^{\kappa_w} \left(\bar{\pi}_t\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}, \tag{4.7}$$

for $i = 1, ..., N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. After wages are set, employment agencies in cohort i decide on endogenous separation, post vacancies to attract new workers in the next period and supply labor services, $l_t^i \varsigma_{i,t}$, into competitive labor markets.

4.2.1. Employment-Agency Problem

To understand how agencies bargain and how they make their employment decisions, it is useful to consider $F(l_t^0, \omega_t)$, the value function of the representative employment agency in the cohort that negotiates its wage in the current period. The arguments of F are the agency's workforce after beginning-of-period exogenous separations and new arrivals, l_t^0, and an arbitrary value for the nominal wage rate, ω_t. We are thus interested in the employment agency's problem after the wage rate has been set, when endogenous separations take place, followed by the setting of vacancies. To simplify notation, we leave out arguments of F that correspond to economy-wide variables. We find it convenient to adopt a change of variables. We suppose that the employment agency chooses a particular monotone transform of vacancy postings, which we denote by $\tilde{v}_t^j$:

$$\tilde{v}_t^j \equiv \frac{Q_t^\iota v_t^j}{\left(1 - \mathcal{F}_t^j\right) l_t^j},$$

where $1 - \mathcal{F}_t^j$ denotes the fraction of the beginning-of-period t workforce in cohort j which remains after endogenous separations. The agency's hiring rate is related to $\tilde{v}_t^j$ by:

$$\chi_t^j = Q_t^{1-\iota} \tilde{v}_t^j. \tag{4.8}$$

The timing in the endogenous separation model is that at the beginning of period t, exogenous separations occur, and new arrivals occur. Then, if this is a bargaining period, bargaining occurs. Then, idiosyncratic productivities are realized and a cutoff productivity, $\bar{a}_t^j$, is determined. Thus, the fraction of the current workforce in cohort j that is let go is $\mathcal{F}_t^j$ and the fraction that survives is $1 - \mathcal{F}_t^j$. So, if l_t^j is the work force just after exogenous separations and new arrivals, then

$$\left(1 - \mathcal{F}_t^j\right) l_t^j$$

is the size of the workforce after endogenous separations. The law of motion of the work force in each cohort is:

$$l_{t+1}^{j+1} = \left(\chi_t^j + \rho\right) \left(1 - \mathcal{F}_t^j\right) l_t^j, \tag{4.9}$$

for $j = 0, 1, ..., N-1$, with the understanding here and throughout that $j = N$ is to be interpreted as $j = 0$ and where l_{t+1}^{j+1} is the workforce after new arrivals and exogenous separations in period $t+j$. Expression (4.9) is deterministic, reflecting the assumption that the agency employs a large number of workers. After endogenous separations, agencies post vacancies.

The value function of the employment agency is:

$$
\begin{aligned}
F\left(l_t^0,\omega_t\right) &= \sum_{j=0}^{N-1}\beta^j E_t\frac{\upsilon_{t+j}}{\upsilon_t}\max_{\tilde{v}_{t+j}^j}\Big[\int_{\bar{a}_{t+j}^j}^{\infty}\left(W_{t+j}a-\Gamma_{t,j}\omega_t\right)\varsigma_{j,t+j}\overbrace{d\mathcal{F}\left(a\right)}^{\text{`fraction' of } l_{t+j}^j \text{ with productivity } a} \\
&- \overbrace{P_{t+j}\frac{\kappa z_{t+j}^+}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}\left(1-\mathcal{F}_{t+j}^j\right)}^{\text{costs are proportional to workforce after current period separations}}\Big]l_{t+j}^j \\
&+\beta^N E_t\frac{\upsilon_{t+N}}{\upsilon_t}F\left(l_{t+N}^0,\tilde{W}_{t+N}\right),
\end{aligned}
$$

where $\varsigma_{j,t}$ is assumed to satisfy (4.6). Simplifying,

$$
\begin{aligned}
F\left(l_t^0,\omega_t\right) &= \sum_{j=0}^{N-1}\beta^j E_t\frac{\upsilon_{t+j}}{\upsilon_t}\max_{\tilde{v}_{t+j}^j}[\left(W_{t+j}\mathcal{E}_{t+j}^j-\Gamma_{t,j}\omega_t\left[1-\mathcal{F}_{t+j}^j\right]\right)\varsigma_{j,t+j} \\
&-P_{t+j}\frac{\kappa z_{t+j}^+}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}\left(1-\mathcal{F}_{t+j}^j\right)]l_{t+j}^j \\
&+\beta^N E_t\frac{\upsilon_{t+N}}{\upsilon_t}F\left(l_{t+N}^0,\tilde{W}_{t+N}\right),
\end{aligned}
\tag{4.10}
$$

Here,

$$
\Gamma_{t,j}=\begin{cases}\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}, & j>0 \\ 1 & j=0\end{cases}. \tag{4.11}
$$

Also, $\tilde{W}_{t+N}$ denotes the Nash bargaining wage rate that will be negotiated when the agency next has an opportunity to do so. At time t, the agency takes $\tilde{W}_{t+N}$ as given.

Writing out (4.10):

$$
\begin{aligned}
F\left(l_t^0,\omega_t\right) &= \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}}\{\left[\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_t-P_t\frac{\kappa z_t^+}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left(1-\mathcal{F}_t^0\right)\right]l_t^0 \\
&+\beta E_t\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{t+1}-P_{t+1}\frac{\kappa z_{t+1}^+}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right] \\
&\times\left(\chi_t^0+\rho\right)\left[1-\mathcal{F}_t^0\right]l_t^0 \\
&+\beta^2 E_t\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{t+2}-P_{t+2}\frac{\kappa z_{t+2}^+}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right] \\
&\times\left(\chi_{t+1}^1+\rho\right)\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right)l_t^0 \\
&+\ldots+ \\
&+\beta^N E_t\frac{\upsilon_{t+N}}{\upsilon_t}F\left(l_{t+N}^0,\tilde{W}_{t+N}\right).
\end{aligned}
$$

The agency chooses vacancies to solve the problem in (4.10). We impose the following property:

$$F\left(l_t^0,\omega_t\right)=J\left(\omega_t\right)l_t^0, \tag{4.12}$$

where $J(\omega_t)$ is not a function of l_t^0. The function, $J(\omega_t)$, is the surplus that a employment agency bargaining in the current period enjoys from a match with an individual worker, when the current wage is ω_t. For convenience, we omit the expectation operator E_t below. Let

$$\begin{aligned}
J\left(\omega_t\right) \quad = \quad & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}}\{\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_tz_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \qquad (4.13)\\
& +\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right]\times\\
& \left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(1-\mathcal{F}_t^0\right)\\
& +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right]\times\\
& \left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_{t+1}^{1-\iota}+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left[1-\mathcal{F}_t^0\right]\\
& +...+\\
& +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times\\
& \left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right)\}.
\end{aligned}$$

4.2.2. Vacancy Decision

We require expressions for the vacancy posting decisions of the employment agencies. We derive optimal vacancy posting decisions of employment agencies by differentiating (4.13) with respect to $\tilde{v}_t^0$ and multiply the result by $\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}$, to obtain:

$$
\begin{aligned}
0 &= -P_t z_t^{+} \kappa \left(\tilde{v}_t^0\right)^{\varphi-1} \left[1-\mathcal{F}_t^0\right] \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&+\beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t \left[1-\mathcal{F}_{t+1}^1\right]\right) \varsigma_{1,t+1} - P_{t+1} z_{t+1}^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_{t+1}^1\right)^{\varphi} \left(1-\mathcal{F}_{t+1}^1\right)\right] \times \\
&\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left[1-\mathcal{F}_t^0\right] \\
&+\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t \left[1-\mathcal{F}_{t+2}^2\right]\right) \varsigma_{2,t+2} - P_{t+2} z_{t+2}^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_{t+2}^2\right)^{\varphi} \left(1-\mathcal{F}_{t+2}^2\right)\right] \times \\
&\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \left[1-\mathcal{F}_{t+1}^1\right] \left[1-\mathcal{F}_t^0\right] \\
&+ \ldots + \\
&+\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right) \times \\
&\left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \cdots \left[1-\mathcal{F}_t^0\right] \} \\
&= J\left(\omega_t\right) - \left(W_t \mathcal{E}_t^0 - \omega_t \left(1-\mathcal{F}_t^0\right)\right) \varsigma_{0,t} + P_t z_t^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_t^0\right)^{\varphi} \left[1-\mathcal{F}_t^0\right] \\
&-P_t z_t^{+} \kappa \left(\tilde{v}_t^0\right)^{\varphi-1} \left[1-\mathcal{F}_t^0\right] \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}
\end{aligned}
$$

Since the latter expression must be zero, we conclude:

$$
\begin{aligned}
J\left(\omega_t\right) &= \left(W_t \mathcal{E}_t^0 - \omega_t \left(1-\mathcal{F}_t^0\right)\right) \varsigma_{0,t} - P_t z_t^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_t^0\right)^{\varphi} \left[1-\mathcal{F}_t^0\right] \\
&+P_t z_t^{+} \kappa \left(\tilde{v}_t^0\right)^{\varphi-1} \left[1-\mathcal{F}_t^0\right] \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&= \left(W_t \mathcal{E}_t^0 - \omega_t \left(1-\mathcal{F}_t^0\right)\right) \varsigma_{0,t} + P_t z_t^{+} \kappa \left[\left(1-\frac{1}{\varphi}\right) \left(\tilde{v}_t^0\right)^{\varphi} + \left(\tilde{v}_t^0\right)^{\varphi-1} \frac{\rho}{Q_t^{1-\iota}}\right] \left[1-\mathcal{F}_t^0\right].
\end{aligned}
$$

Next, we obtain simple expressions for the vacancy decisions from their first order necessary conditions for optimality. Multiplying the first order condition for $\tilde{v}_{t+1}^1$ by

$$
\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \frac{1}{Q_{t+1}^{1-\iota}},
$$

we obtain:

$$
\begin{aligned}
0 &= -\beta \frac{\upsilon_{t+1}}{\upsilon_t} P_{t+1} z_{t+1}^{+} \kappa \left(\tilde{v}_{t+1}^1\right)^{\varphi-1} \left[1-\mathcal{F}_{t+1}^1\right] \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \frac{1}{Q_{t+1}^{1-\iota}} \left[1-\mathcal{F}_t^0\right] \\
&+\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t \left(1-\mathcal{F}_{t+2}^2\right)\right) \varsigma_{2,t+2} - P_{t+2} z_{t+2}^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_{t+2}^2\right)^{\varphi} \left[1-\mathcal{F}_{t+2}^2\right]\right] \times \\
&\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \left[1-\mathcal{F}_{t+1}^1\right] \left[1-\mathcal{F}_t^0\right] \\
&+ \ldots + \\
&+\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right) \times \\
&\left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \cdots \left[1-\mathcal{F}_t^0\right].
\end{aligned}
$$

Substitute out the period $t+2$ and higher terms in this expression using the first order condition for $\tilde{v}_t^0$. After rearranging, we obtain,

$$\frac{P_t z_t^+ \kappa \left(\tilde{v}_t^0\right)^{\varphi-1}}{Q_t^{1-\iota}} = \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\begin{array}{c} \left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t \left[1-\mathcal{F}_{t+1}^1\right]\right) \varsigma_{1,t+1} \\ +P_{t+1} z_{t+1}^+ \kappa \left(1-\mathcal{F}_{t+1}^1\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^1\right)^{\varphi} + \left(\tilde{v}_{t+1}^1\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}}\right] \end{array} \right].$$

Following the pattern set with $\tilde{v}_{t+1}^1$, multiply the first order condition for $\tilde{v}_{t+2}^2$ by

$$\left(\tilde{v}_{t+2}^2 Q_{t+2}^{1-\iota} + \rho\right) \frac{1}{Q_{t+2}^{1-\iota}}.$$

Substitute the period $t+3$ and higher terms in the first order condition for $\tilde{v}_{t+2}^2$ using the first order condition for $\tilde{v}_{t+1}^1$ to obtain, after rearranging,

$$\frac{P_{t+1} z_{t+1}^+ \kappa \left(\tilde{v}_{t+1}^1\right)^{\varphi-1}}{Q_{t+1}^{1-\iota}} = \beta \frac{\upsilon_{t+2}}{\upsilon_{t+1}} \left[\begin{array}{c} \left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t \left[1-\mathcal{F}_{t+2}^2\right]\right) \varsigma_{2,t+2} \\ +P_{t+2} z_{t+2}^+ \kappa \left(1-\mathcal{F}_{t+2}^2\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+2}^2\right)^{\varphi} + \left(\tilde{v}_{t+1}^2\right)^{\varphi-1} \frac{\rho}{Q_{t+2}^{1-\iota}}\right] \end{array} \right].$$

Continuing in this way, we obtain,

$$\frac{P_{t+j} z_{t+j}^+ \kappa \left(\tilde{v}_{t+j}^j\right)^{\varphi-1}}{Q_{t+j}^{1-\iota}} = \beta \frac{\upsilon_{t+j+1}}{\upsilon_{t+j}} \left[\begin{array}{c} \left(W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1} - \Gamma_{t,j+1}\omega_t \left[1-\mathcal{F}_{t+j+1}^{j+1}\right]\right) \varsigma_{j+1,t+j+1} \\ +P_{t+j+1} z_{t+j+1}^+ \kappa \left(1-\mathcal{F}_{t+j+1}^{j+1}\right) \left[\begin{array}{c} \left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi} \\ +\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi-1} \frac{\rho}{Q_{t+j+1}^{1-\iota}} \end{array} \right] \end{array} \right],$$

for $j = 0, 1, ..., N-2$. Now consider the first order necessary condition for the optimality of $\tilde{v}_{t+N-1}^{N-1}$. After multiplying this first order condition by

$$\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}},$$

we obtain,

$$\begin{aligned} 0 \;=\; & -\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t} P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \cdots \\ & \cdots \left(\tilde{v}_{t+N-2}^{N-2} Q_{t+N-2}^{1-\iota} + \rho\right) \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}} \left[1-\mathcal{F}_{t+N-2}^{N-2}\right] \cdots \left[1-\mathcal{F}_t^0\right] \\ & +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \times \\ & \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \cdots \left[1-\mathcal{F}_t^0\right]\} \end{aligned}$$

or,

$$P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \frac{1}{Q_{t+N-1}^{1-\iota}} = \beta \frac{\upsilon_{t+N}}{\upsilon_{t+N-1}} J\left(\tilde{W}_{t+N}\right).$$

Making use of our expression for J, we obtain:

$$P_{t+N-1}z_{t+N-1}^{+}\kappa\left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1}\frac{1}{Q_{t+N-1}^{1-\iota}}=\beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}\left[\begin{array}{c}\left(W_{t+N}\mathcal{E}_{t+N}^{0}-\tilde{W}_{t+N}\left(1-\mathcal{F}_{t+N}^{0}\right)\right)\varsigma_{0,t+N}\\+P_{t+N}z_{t+N}^{+}\kappa\left[\begin{array}{c}\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+N}^{0}\right)^{\varphi}\\+\left(\tilde{v}_{t+N}^{0}\right)^{\varphi-1}\frac{\rho}{Q_{t+N}^{1-\iota}}\end{array}\right]\left[1-\mathcal{F}_{t+N}^{0}\right]\end{array}\right].$$

The above first order conditions apply over time to a group of agencies that bargain at date t. We now express the first order conditions for a fixed date and different cohorts:

$$\begin{aligned}P_{t}z_{t}^{+}\kappa\left(\tilde{v}_{t}^{j}\right)^{\varphi-1}\frac{1}{Q_{t}^{1-\iota}} &= \beta\frac{\upsilon_{t+1}}{\upsilon_{t}}[\left(W_{t+1}\mathcal{E}_{t+1}^{j+1}-\Gamma_{t-j,j+1}\tilde{W}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1}\\ &\quad +P_{t+1}z_{t+1}^{+}\kappa\left(1-\mathcal{F}_{t+1}^{j+1}\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}+\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)],\\ &\quad \text{for } j=0,...,N-2.\end{aligned}$$

Scaling the above equation one obtains the following scaled first order optimality conditions:

$$\begin{aligned}\kappa\left(\tilde{v}_{t}^{j}\right)^{\varphi-1}\frac{1}{Q_{t}^{1-\iota}} &= \beta\frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}}[\left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{j+1}-G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1}\\ &\quad +\kappa\left(1-\mathcal{F}_{t+1}^{j+1}\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}+\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)],\\ &\quad \text{for } j=0,...,N-2,\end{aligned}\tag{4.14}$$

where

$$\begin{aligned}G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}}\left(\frac{1}{\mu_{z^{+},t-i+1}}\right)\cdots\left(\frac{1}{\mu_{z^{+},t+1}}\right),\ i\geq 0,\\ w_{t} &= \frac{\tilde{W}_{t}}{W_{t}},\ \bar{w}_{t}=\frac{W_{t}}{z_{t}^{+}P_{t}}.\end{aligned}\tag{4.15}$$

Also,

$$G_{t,j}=\begin{cases}\frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}\left(\frac{1}{\mu_{z^{+},t+1}}\right)\cdots\left(\frac{1}{\mu_{z^{+},t+j}}\right) & j>0\\ 1 & j=0\end{cases}.\tag{4.16}$$

The scaled vacancy first order condition of agencies that are in the last period of their contract is:

$$\begin{aligned}\kappa\left(\tilde{v}_{t}^{N-1}\right)^{\varphi-1}\frac{1}{Q_{t}^{1-\iota}} &= \beta\frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}}[\left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{0}-w_{t+1}\bar{w}_{t+1}\left(1-\mathcal{F}_{t+1}^{0}\right)\right)\varsigma_{0,t+1}\\ &\quad +\kappa\left(1-\mathcal{F}_{t+1}^{0}\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^{0}\right)^{\varphi}+\left(\tilde{v}_{t+1}^{0}\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)].\end{aligned}\tag{4.17}$$

4.2.3. (Marginal) Surplus

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables:

$$\begin{aligned} J_{z^+,t} &= \sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+j} \frac{\mathcal{E}_{t+j}^j}{1-\mathcal{F}_{t+j}^j} - G_{t,j} w_t \bar{w}_t \right) \varsigma_{j,t+j} - \frac{\kappa}{\varphi} \left(\tilde{v}_{t+j}^j \right)^{\varphi} \right] \Omega_{t+j}^j \\ &\quad + \beta^N \frac{\psi_{z^+,t+N}}{\psi_{z^+,t}} J_{z^+,t+N} \frac{\Omega_{t+N}^N}{1-\mathcal{F}_{t+N}^0}. \end{aligned} \quad (4.18)$$

We also require the derivative of J with respect to ω_t, i.e. the marginal surplus of the employment agency with respect to the negotiated wage. By the envelope condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and only be concerned with the direct impact of ω_t on J. Taking the derivative of (4.13):

$$\begin{aligned} J_{w,t} &= -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\ &\quad -\beta \frac{\upsilon_{t+1}}{\upsilon_t} \Gamma_{t,1} \varsigma_{1,t+1} \left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\ &\quad -\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \Gamma_{t,2} \varsigma_{2,t+2} \left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+2}^2\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\ &\quad -\ldots - \beta^{N-1} \frac{\upsilon_{t+N-1}}{\upsilon_t} \Gamma_{t,N-1} \varsigma_{N-1,t+N-1} \left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+1}^{N-2}+\rho\right) \times \\ &\quad \left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left[1-\mathcal{F}_t^0\right]. \end{aligned}$$

Let,

$$\Omega_{t+j}^j = \begin{cases} \left(1-\mathcal{F}_{t+j}^j\right) \prod_{l=0}^{j-1} \left(\chi_{t+l}^l+\rho\right)\left(1-\mathcal{F}_{t+l}^l\right) & j>0 \\ 1-\mathcal{F}_t^0 & j=0 \end{cases}. \quad (4.19)$$

It is convenient to express this in recursive form:

$$\begin{aligned} \Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_{t+1}^1 = \left(1-\mathcal{F}_{t+1}^1\right)\left(\chi_t^0+\rho\right)\overbrace{\left(1-\mathcal{F}_t^0\right)}^{\Omega_t^0}, \\ \Omega_{t+2}^2 &= \left(1-\mathcal{F}_{t+2}^2\right)\left(\chi_{t+1}^1+\rho\right)\overbrace{\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_t^0\right)\left(1-\mathcal{F}_{t+1}^1\right)}^{\Omega_{t+1}^1}, \ldots \end{aligned}$$

so that

$$\Omega_{t+j}^j = \left(1-\mathcal{F}_{t+j}^j\right)\left(\chi_{t+j-1}^{j-1}+\rho\right)\Omega_{t+j-1}^{j-1},$$

for $j = 1, 2, \ldots$. It is convenient to define these objects at date t as a function of variables dated t and earlier for the purposes of implementing these equations in Dynare:

$$\begin{aligned} \Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_t^1 = \left(1-\mathcal{F}_t^1\right)\left(\chi_{t-1}^0+\rho\right)\overbrace{\left(1-\mathcal{F}_{t-1}^0\right)}^{\Omega_{t-1}^0}, \\ \Omega_t^2 &= \left(1-\mathcal{F}_t^2\right)\left(\chi_{t-1}^1+\rho\right)\overbrace{\left(\chi_{t-2}^0+\rho\right)\left(1-\mathcal{F}_{t-2}^0\right)\left(1-\mathcal{F}_{t-1}^1\right)}^{\Omega_{t-1}^1} \end{aligned}$$

so that

$$\Omega_t^j = \left(1 - \mathcal{F}_t^j\right)\left(\chi_{t-1}^{j-1} + \rho\right)\Omega_{t-1}^{j-1}.$$

Then, in terms of scaled variables we obtain:

$$J_{w,t} = -\sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} G_{t,j} \Omega_{t+j}^j \varsigma_{j,t+j}. \tag{4.20}$$

4.3. Worker Problem

We now turn to the worker. For the bargaining problem, we require the worker's value function before they know if they will survive the endogenous separation. It is convenient to begin by defining the worker's value function after they have survived the endogenous separation. We do so first. We then derive the value function of an unemployed worker, and finally we consider the value function of the employed worker before endogenous separations occur.

The period t value of being a worker in an agency in cohort i is V_t^i :

$$\begin{aligned} V_t^i \quad = \quad & \Gamma_{t-i,i} \tilde{W}_{t-i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\upsilon_t} \\ & + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}\left(\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right) V_{t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right) U_{t+1}\right), \end{aligned} \tag{4.21}$$

for $i = 0, 1, ..., N-1$. Here, ρ is the exogenous probability of remaining with the agency in the next period and $\left(1 - \mathcal{F}_{t+1}^{i+1}\right)$ is the endogenous probability of remaining with the agency. Also, U_t is the value of being unemployed in period t. The values, V_t^i and U_t, pertain to the beginning of period t, after job separation and job finding has occurred. Scaling V_t^i by $P_t z_t^+$, we obtain:

$$\begin{aligned} V_{z^+,t}^i \quad = \quad & G_{t-i,i} w_{t-i} \bar{w}_{t-i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}} \\ & + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right) U_{z^+,t+1}\right], \end{aligned} \tag{4.22}$$

for $i = 0, 1, ..., N-1$, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i,\ U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.$$

In our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have:

$$\mathcal{M}_{t+j} = \left(1 - \mathcal{F}_t^0\right) \cdots \left(1 - \mathcal{F}_{t+j}^j\right), \tag{4.23}$$

for $j = 0, ..., N-1$. Then, the derivative of V, which we denote by $V_w^0(\omega_t)$, is:

$$\begin{aligned} V_w^0(\omega_t) &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}. \end{aligned} \tag{4.24}$$

Note that ω_t has no impact on the intensity of labor effort. This is determined by (4.6), independent of the wage rate paid to workers.

The value of being an unemployed worker is U_t :

$$U_t = P_t z_t^+ b^u \left(1-\tau_t^y\right) + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} [f_t V_{t+1}^x + (1-f_t) U_{t+1}], \tag{4.25}$$

where f_t is the probability that an unemployed worker will land a job in period $t+1$. Also, V_t^x is the period $t+1$ value function of a worker who finds a job, before it is known which agency the job is found with:

$$V_{z^+,t}^x = \sum_{i=0}^{N-1} \frac{\chi_{t-1}^i \left(1-\mathcal{F}_{t-1}^i\right) l_{t-1}^i}{m_{t-1}} V_{z^+,t}^{i+1}, \tag{4.26}$$

after scaling. Here, the total number of new matches is given by:

$$m_t = \sum_{j=0}^{N-1} \chi_t^j \left(1-\mathcal{F}_t^j\right) l_t^j. \tag{4.27}$$

In (4.26),

$$\frac{\chi_{t-1}^i \left(1-\mathcal{F}_{t-1}^i\right) l_{t-1}^i}{m_{t-1}}$$

is the probability of finding a job in an agency which was of type i in the previous period, conditional on being a worker who finds a job in t.

Scaling (4.25),

$$U_{z^+,t} = b^u \left(1-\tau_t^y\right) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [f_t V_{z^+,t+1}^x + (1-f_t) U_{z^+,t+1}]. \tag{4.28}$$

This value function applies to any unemployed worker, whether they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of an exogenous separation, or because they arrived because of an endogenous separation.

Finally, we consider the value function of a worker before they know whether they will survive the endogenous separation cut. We denote this value function by $\tilde{V}_t^j$:

$$\tilde{V}_t^j = \mathcal{F}_t^j U_t + \left(1-\mathcal{F}_t^j\right) V_t^j.$$

4.4. Bargaining Problem

The employment-agency negotiates with the workers about the nominal wage. In what follows we consider two alternative bargaining setups: union bargaining and individual bargaining.

4.4.1. Union Bargaining

The $i = 0$ cohort of agencies in period t solve the following Nash bargaining problem:

$$\max_{\omega_t} \left(\tilde{V}_t^0 - U_t\right)^{\eta_t} J\left(\omega_t\right)^{(1-\eta_t)} = \max_{\omega_t} \left[\left(1 - \mathcal{F}_t^0\right)\left(V_t^0 - U_t\right)\right]^{\eta_t} J\left(\omega_t\right)^{(1-\eta_t)} \tag{4.29}$$

where $V^0\left(\omega_t\right) - U_t$ is the match surplus enjoyed by a worker and η_t is the bargaining power of workers which we allow to follow an exogenous time-varying process. We denote the wage that solves this problem by $\tilde{W}_t$. Note that $\tilde{W}_t$ takes into account that intensity will be chosen according to (4.6) as well as (4.7). The first order condition associated with this problem is:

$$\eta_t V_{w,t} J_{z^+,t} + (1 - \eta_t)\left[V_{z^+,t}^0 - U_{z^+,t}\right] J_{w,t} = 0, \tag{4.30}$$

after division by $z_t^+ P_t$.Our interpretation of the Nash bargaining problem is that the bargain is between the employment agency and a union which represents the 'average worker'. The worker's interests are summarized by $\tilde{V}_t$ and take into account that with some probability the worker will separate at some time during the contract. The worker's outside option is unemployment, and so its surplus is $\tilde{V}_t - U_t$. The agency's surplus corresponds to J_t and this takes into account that workers who arrive in the future, while the contract remains in force, will be paid the wage rate that solves the bargaining problem, (4.29). In addition, if bargaining with the employment agency breaks down and the union takes all the workers, l_t^0, into unemployment, then the value of the agency drops to zero. This is because not only are current revenues from l_t^0 set to zero, but the agency's ability to ever hire in the future is eliminated when l_t^0 is set to zero.

4.4.2. Individual Bargaining

We now consider an alternative formulation of the bargaining problem, in which there is no union. In the alternative formulation, we imagine that bargaining occurs among a continuum of worker-agency representative pairs. Each bargaining session takes the outcomes of all other bargaining sessions as given. Because each bargaining session is atomistic, each session ignores its impact on the wage earned by workers arriving in the future during the contract. We assume that those future workers are simply paid the average of the outcome of all bargaining sessions. Since each bargaining problem is identical, the wage that solves each problem is the same and so the average wage coincides with the wage that solves the bargaining problem. There is an important distinction between the atomistic and the union approach. When the Nash bargaining problem is optimized with respect to the wage, the impact on the wage earned by future arriving workers is ignored. The outside option of the worker in the alternative scenario is the same as before, it is the unemployment state, U_t. The outside option of the agency is also the same as before, namely zero. To see this, note that the agency's present discounted value of profits, $F\left(l_t^0, \omega_t\right)$, still has the following form, which is linear in l_t^0 :

$$F\left(l_t^0, \omega_t\right) = J\left(\omega_t\right) l_t^0.$$

Suppose that each worker in l_t^0 is identified with a point, i, on the interval, $i \in [0, l_t^0]$. Then, profits can be written in terms of each individual worker as follows:

$$F\left(l_t^0, \omega_t\right) = \int_0^{l_t^0} J\left(\omega_t^i, i\right) di,$$

where ω_t^i denotes the wage negotiated by worker i. We adopt the Riemann interpretation of this integral:

$$F\left(l_t^0, \omega_t\right) = \lim_{M \to \infty} \sum_{j=1}^{M} J\left(\omega_t^{i_j}, i_j\right)\left(i_j - i_{j-1}\right),$$

where $i_0 = 0$, $i_0 < i_1 < ... < i_M = l_t^0$. Thus, in the finite-but-large-value of M case, we interpret $i_1, ..., i_M$ as the M workers. We suppose that if the bargaining session involving the i_j^{th} worker fails to reach an agreement, then $J\left(\omega_t^{i_j}, i_j\right) = 0$. For this reason, in the atomistic version of the Nash bargaining problem, we set the outside option of the agency to zero.

We now turn to the computation of J_w for our alternative formulation. We consider the surplus associated with a single worker and denote the wage received by that worker by ω_t. We denote the average across the wages received by all workers by $\tilde{\omega}_t$. Then,

$$\begin{aligned}
J\left(\omega_t\right) \quad = \quad & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left(W_t \mathcal{E}_t^0 - \omega_t\left(1 - \mathcal{F}_t^0\right)\right) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left(1 - \mathcal{F}_t^0\right) \\
& + \beta \frac{\upsilon_{t+1}}{\upsilon_t}[\left(W_{t+1}\mathcal{E}_{t+1}^1 \varsigma_{1,t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1 - \mathcal{F}_{t+1}^1\right)\right)\left(\chi_t^0 + \rho\right)\left(1 - \mathcal{F}_t^0\right) \\
& - \Gamma_{t,1}\omega_t\left(1 - \mathcal{F}_{t+1}^1\right)\varsigma_{1,t+1}\rho\left(1 - \mathcal{F}_t^0\right) - \Gamma_{t,1}\tilde{\omega}_t\left(1 - \mathcal{F}_{t+1}^1\right)\varsigma_{1,t+1}\chi_t^0\left(1 - \mathcal{F}_t^0\right)] \\
& ...
\end{aligned}$$

To simplify the notation and given that we only want this expression for the purpose of computing J_w, we drop all terms that do not involve ω_t:

$$\begin{aligned}
J(\omega_t) &= -\omega_t\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\
&+\beta\frac{\upsilon_{t+1}}{\upsilon_t}[-\Gamma_{t,1}\omega_t\varsigma_{1,t+1}\rho\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right)] \\
&+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[-\Gamma_{t,2}\omega_t\varsigma_{2,t+2}\right]\rho^2\left(1-\mathcal{F}_{t+2}^2\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\
&+...+ \\
&+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\left[-\Gamma_{t,N-1}\omega_t\varsigma_{N-1,t+N-1}\right]\rho^{N-1}\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right) \\
&+\text{terms not involving } \omega_t.
\end{aligned}$$

So that,

$$\begin{aligned}
J_{w,t} &= -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\
&+\beta\frac{\upsilon_{t+1}}{\upsilon_t}[-\Gamma_{t,1}\varsigma_{1,t+1}\rho\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right)] \\
&+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[-\Gamma_{t,2}\varsigma_{2,t+2}\right]\rho^2\left(1-\mathcal{F}_{t+2}^2\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\
&+...+ \\
&+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\left[-\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\right]\rho^{N-1}\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right),
\end{aligned}$$

which (after scaling) is identical to (4.20) with the understanding that in the definition of Ω_{t+j}^j, $\chi_{t+l}^l = 0$. To implement this alternative version of the model, we simply use this definition of $J_{w,t}$ together with the previous definition of J_t in the equation that characterizes the solution to the Nash bargaining problem. This is the only change required to implement this alternative version of the model. An alternative representation of J_w is convenient, and highlights how employment agencies now discount future wages in the same way as the household does in V_w :

$$\begin{aligned}
J_{w,t} &= -\mathcal{M}_{t,0}\varsigma_{0,t} \\
&-(\beta\rho)\mathcal{M}_{t,1}\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1} \\
&-(\beta\rho)^2\mathcal{M}_{t,2}\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2} \\
&-... \\
&-(\beta\rho)^{N-1}\mathcal{M}_{t,N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}.
\end{aligned} \tag{4.31}$$

It is interesting to compare $J_{w,t}$ and $V_{w,t}$:

$$V_{w,t} = \sum_{j=0}^{N-1}(\beta\rho)^j\mathcal{M}_{t,j}\varsigma_{j,t+j}\frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w}\Gamma_{t,j}\frac{\upsilon_{t+j}}{\upsilon_t}.$$

Note that one is just the minus of the other, if we ignore the tax wedge. That is, absent the tax wedge a change in the wage simply reallocates resources between the agency and the worker. In our baseline case, this is not true because the agency and the worker discount the future differently. This implies that if there were no restrictions on the intertemporal pattern of wage payments in the baseline model, then it would be desirable to shift wages into the present. When we take into account the tax wedge, increases in the wage take resources away from the agency and only incompletely transfer them to households. As a result, we conjecture that the presence of the tax wedge causes the equilibrium pre-tax wage to be smaller.

4.5. Separation Decision

In this section we consider either an employer, worker or total surplus criterion for determining the a cutoff in period t. We begin by discussing the cutoff for cohort $j = 0$, denoted $\bar{a}^0$. We identify each worker with a productivity level, a, in the current period and this allows us to define a total surplus for the employment agency and worker for each a. Because of the linearity in our environment, it must be that when we integrate over the surplus of all the individual a's, we arrive at the aggregate surplus implicit in our construction of the Nash bargaining problem:

$$s_w \left(1 - \mathcal{F}_t^0\right) \left(V_t - U_t\right) + s_e J_t = \int_{\bar{a}}^{\infty} s_t(a) dF\left(a\right), \tag{4.32}$$

where $s_t\left(a_t\right)$ denotes the surplus of the match associated with a worker with productivity, a. Here, recall that $(1 - \mathcal{F}_t^0)$ denotes the fraction of matches that remain after endogenous separation. Accordingly $(1 - \mathcal{F}_t^0)\left(V_t - U_t\right)$ is the average surplus of workers in l_t^0, over all values of a. We arrive at this expression from the fact that each worker with $a \geq \bar{a}_t$ experiences the same surplus, $V_t - U_t$, and workers with $a < \bar{a}_t$ enjoy zero surplus. Similarly, J_t denotes the average surplus of a match to an employment agency across all $a \geq \bar{a}$. The parameters s_w, $s_e \in \{0; 1\}$ allow for a variety of interesting separation decisions. If $s_w = 0$ and $s_e = 1$ the a cutoff is determined by maximizing the employer surplus. Further, if $s_w = 1$ and $s_e = 1$ the cutoff is determined by maximizing the total surplus. Let $s_t\left(a\right)$ be defined as follows:

$$s_t\left(a\right) = s_w \left(1 - \mathcal{F}_t^0\right) \left(V_t^0 - U_t\right) + s_e \left[\left(W_t a - \omega_t\right) \varsigma_{0,t} \left(1 - \mathcal{F}_t^0\right) - P_t z_t^+ \frac{\kappa}{\varphi} \left(\tilde{v}_t^0\right)^{\varphi} \left(1 - \mathcal{F}_t^0\right) + D_t^0\right],$$

where D_t^0 denotes:

$$\begin{aligned}
D_t^0 &= \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right]\\
&\quad\times\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_t^0\right)\\
&\quad+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right]\\
&\quad\times\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right)\\
&\quad+...+\\
&\quad+\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+N-1}^{N-1}+\rho\right)\\
&\quad\times\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right)\}\\
&= J_t-\left[\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_tz_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left(1-\mathcal{F}_t^0\right)\right], \qquad (4.33)
\end{aligned}$$

so that after imposing $s_t\left(\bar{a}_t^0\right)=0$ we obtain:

$$s_e\left[J_t-W_t\left(\mathcal{E}_t^0-\bar{a}_t^0\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}\right]+s_w\left(1-\mathcal{F}_t^0\right)\left(V_t^0-U_t\right)=0$$

Dividing by $P_tz_t^+$, we obtain:

$$s_e\left[J_{z^+,t}-\bar{w}_t\left[\mathcal{E}_t^0-\bar{a}_t^0\left(1-\mathcal{F}_t^0\right)\right]\varsigma_{0,t}\right]+s_w\left(1-\mathcal{F}_t^0\right)\left(V_{z^+,t}^0-U_{z^+,t}\right)=0. \qquad (4.34)$$

We proceed similarly for the cutoff levels $\bar{a}_t^j$ for $j=1,..,N-1$.We and arrive at the following cutoff conditions (see appendix for details) We conclude that the efficiency conditions for cohorts, $j=1,..,N-1$ are:

$$s_w(V_{z^+,t}^j-U_{z^+,t})+s_e\left[\left(\bar{w}_t\bar{a}_t^j-G_{t-j,j}\bar{w}_{t-j}w_{t-j}\right)\varsigma_{j,t}-\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}+\frac{D_{z^+,t}^j}{1-\mathcal{F}_t^j}\right]=0. \qquad (4.35)$$

Thus, the equilibrium conditions determining $\bar{a}_t^j$, for $j=0,...,N-1$ are (4.34) and (4.35).

We now turn to the expression for $D_{z^+,t}^j$, $=1,..,N-1$. With $j=1$:

$$\begin{aligned}
&\frac{D_t^1}{P_tz_t^+}=\beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\frac{1}{P_{t+1}z_{t+1}^+}\left[\begin{array}{c}\left(W_{t+1}\mathcal{E}_{t+1}^2-\Gamma_{t-1,2}\omega_{t-1}\left(1-\mathcal{F}_{t+1}^2\right)\right)\varsigma_{2,t+1}\\-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^2\right)\end{array}\right]\\
&\times\left(\chi_t^1+\rho\right)\left(1-\mathcal{F}_t^1\right)\\
&+\beta^2\frac{\psi_{z^+,t+2}}{\psi_{z^+,t}}\frac{1}{P_{t+2}z_{t+2}^+}\left[\begin{array}{c}\left(W_{t+2}\mathcal{E}_{t+2}^3-\Gamma_{t-1,3}\omega_{t-1}\left(1-\mathcal{F}_{t+2}^3\right)\right)\varsigma_{3,t+1}\\-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^3\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^3\right)\end{array}\right]\\
&\times\left(\chi_t^1+\rho\right)\left(1-\mathcal{F}_t^1\right)\left(\chi_{t+1}^2+\rho\right)\left(1-\mathcal{F}_{t+1}^2\right)\\
&+...+\\
&+\beta^{N-1}\frac{\psi_{z^+,t+N-1}}{\psi_{z^+,t}}\frac{J\left(\tilde{W}_{t+N-1}\right)}{P_{t+N-1}z_{t+N-1}^+}\left(\chi_t^1+\rho\right)\cdots\left(\chi_{t+N-2}^{N-1}+\rho\right)\\
&\times\left(1-\mathcal{F}_{t+N-2}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^1\right)\},
\end{aligned}$$

or, generalizing to arbitrary $j \in (1, N-1)$:

$$D_{z^+,t}^j \equiv \frac{D_t^j}{P_t z_t^+} = \overbrace{\begin{array}{c}\beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\begin{array}{c} \left(\bar{w}_{t+1} \mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1} \bar{w}_{t-j} w_{t-j} \left(1 - \mathcal{F}_{t+1}^{j+1}\right)\right) \varsigma_{j+1,t+1} \\ -\frac{\kappa}{\varphi} \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi} \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \end{array} \right] \\ \times \left(\chi_t^j + \rho\right) \left(1 - \mathcal{F}_t^j\right)\end{array}}^{\equiv 0, \text{ for } j = N-1} \tag{4.36}$$

$$+\beta^2 \frac{\psi_{z^+,t+2}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+2} \mathcal{E}_{t+2}^{j+2} - G_{t-j,j+2} \bar{w}_{t-j} w_{t-j} \left(1 - \mathcal{F}_{t+2}^{j+2}\right)\right) \varsigma_{j+2,t+2} - \frac{\kappa}{\varphi} \left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi} \left(1 - \mathcal{F}_{t+2}^{j+2}\right) \right]$$

$$\times \left(\chi_{t+1}^{j+1} + \rho\right) \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \times \left(\chi_t^j + \rho\right) \left(1 - \mathcal{F}_t^j\right)$$

$$+ ... +$$

$$+\beta^{N-j} \frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t}} J_{z^+,t+N-j} \left(\chi_t^j + \rho\right) \cdots \left(\chi_{t+N-(j+1)}^{N-1} + \rho\right) \left(1 - \mathcal{F}_{t+N-(j+1)}^{N-1}\right) \cdots \left(1 - \mathcal{F}_t^j\right) \}.$$

4.6. Resource Constraint in the Unemployment Model

We assume that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation, (2.82), unchanged, and we alter the resource constraint:

$$\underbrace{y_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 \left[1 - \mathcal{F}_t^j\right] l_t^j}_{gdp_t} = g_t + c_t^d + i_t^d \tag{4.37}$$

$$+ \left(R_t^x\right)^{\eta_x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1 - \omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) \left(p_t^x\right)^{-\eta_f} y_t^*.$$

We consider the left-hand side of this equation to be measured GDP when we take the model to the data.

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m \left(1 - L_t\right)^{\sigma} v_t^{1-\sigma}, \tag{4.38}$$

where

$$L_t = \sum_{j=0}^{N-1} \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{4.39}$$

and σ_m is the productivity of the matching technology.

In our environment, there is a distinction between effective hours and measured hours. Effective hours is the hours of each person, adjusted by their productivity, a. Recall that the average productivity of a worker in working in cohort j (i.e., who has survived the endogenous productivity cut) is $\mathcal{E}_t^j / \left(1 - \mathcal{F}_t^j\right)$. The number of workers who survive the productivity cut in cohort j is $\left(1-\mathcal{F}_t^j\right) l_t^j$, so that our measure of total effective hours is:

$$
\begin{aligned}
H_t &= \sum_{j=0}^{N-1} \varsigma_{j,t} \mathcal{E}_t^j l_t^j, && (4.40) \\
\mathcal{E}\left(\bar{a}_t^j; \sigma_{a,t}\right) &= \int_{\bar{a}_t^j}^{\infty} a d\mathcal{F}\left(a; \sigma_{a,t}\right) = 1 - prob\left[v < \frac{\log\left(\bar{a}_t^j\right) + \frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}} - \sigma_{a,t}\right], && (4.41)
\end{aligned}
$$

where $prob$ refers to the standard normal distribution and eq. (4.41) simply is eq. (4.4) spelled out under the assumption of log-normal distribution of idiosyncratic productivities. We also need to spell out eq. (4.5):

$$
\begin{aligned}
\mathcal{F}\left(\bar{a}^j; \sigma_a\right) &= \int_0^{\bar{a}^j} d\mathcal{F}(a; \sigma_a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma_a^2}{\sigma}} \exp^{\frac{-v^2}{2}} dv && (4.42) \\
&= prob\left[v < \frac{\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma_a^2}{\sigma_a}\right].
\end{aligned}
$$

Total measured hours is:

$$
H_t^{meas} = \sum_{j=0}^{N-1} \varsigma_{j,t} \left(1-\mathcal{F}_t^j\right) l_t^j.
$$

The job finding rate is:

$$
f_t = \frac{m_t}{1-L_t}. \tag{4.43}
$$

The probability of filling a vacancy is:

$$
Q_t = \frac{m_t}{v_t}. \tag{4.44}
$$

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$
v_t = \frac{1}{Q_t^{\iota}} \sum_{j=0}^{N-1} \tilde{v}_t^j \left(1-\mathcal{F}_t^j\right) l_t^j.
$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (4.44), (4.27) and (4.8). This completes the derivations of the alternative representation of the labor market. The unemployment part also brings the two additional shocks η_t and $\sigma_{a,t}$ into the model.

Depending on parameters we define the following models:

Model	Parameter
employer surplus model	$s_w = 0,\ s_e = 1$
total surplus model	$s_w = 1,\ s_e = 1$
exogenous separations model	$F = 0$; equations (4.41), (4.42), (4.34 and 4.35) become $\mathcal{E}_t^j = \mathcal{E}^j$, $\mathcal{F}_t^j = \mathcal{F}^j$ and $\bar{a}_t^j = \bar{a}^j\ \forall t \geq 0$.

4.7. Labor Market Frictions in the Baseline Model

This subsection summarizes the equations of the labor market that define the equilibrium and how they are integrated with the baseline model. The equations include the N efficiency conditions that determines hours worked, (4.6); the law of motion of the workforce in each cohort, (4.9); the first order conditions associated with the vacancy decision, (4.14), (4.17), $j = 0, ..., N-1$; the derivative of the employment agency surplus with respect to the wage rate, (4.20); scaled agency surplus, (4.18; the value function of a worker, $V^i_{z^+,t}$, (4.22); the derivative of the worker value function with respect to the wage rate, (4.24); the growth adjustment term, $G_{t,j}$ (4.16); the scaled value function for unemployed workers, (4.28); first order condition associated with the Nash bargaining problem, (4.30); the (suitably modified) resource constraint, (2.84); the equations that characterize the productivity cutoff for job separations, (4.34) (4.35); the equations that characterize $D^j_{z^+,t}$ (4.36); the value of finding a job, (4.26); the job finding rate, (4.43); the probability of filling a vacancy, (4.44); the matching function, (4.27); the wage updating equation for cohorts that do not optimize, (4.7); the equation determining total employment, (4.39); the equation determining Ω^j_{t+j}, (4.19); the equation determining the hiring rate, χ^i_t (4.8); the equation determining the number of matches (the matching function), (4.38); the definition of total effective hours (4.40); the equations defining $\mathcal{M}^j_t$, (4.23); the equations defining $\mathcal{F}^j_t$, (4.42); the equations defining $\mathcal{E}^i_t$, (4.41).

The following additional endogenous variables are added to the list of endogenous variables in the baseline model:

$$l^j_t, \mathcal{E}^j_t, \mathcal{F}^j_t, \varsigma_{j,t}, \mathcal{M}^j_t, \bar{a}^j_t, \tilde{v}^j_t, G_{t,j}, Q_t, \Omega^j_{t+j}, J_{w,t}, w_t, J_{z^+,t}, V^j_{z^+,t}, U_{z^+,t}, V^0_{w,t},$$
$$V^x_{z^+,t}, f_t, D^j_{z^+,t}, m_t, v_t, \chi^j_t, \tilde{\pi}_{w,t}, L_t,$$

We drop the equations from the baseline model that determines wages, eq. (2.75), (2.76), (2.77),(2.73) and (2.70).

4.8. The Full Model: Labor Market Frictions in the Financial Frictions Model

Finally, in this subsection, we integrate financial frictions and unemployment together into what we call the full model.

The equations which describe the dynamic behavior of the model are those of the baseline model discussed in section 2 plus those discussed in the financial frictions model specified in section 3.2.1 plus those discussed in the unemployment model presented in subsection 4.7. Finally, the resource constraint needs to be adjusted to include monitoring as well as vacancy posting costs.

5. Estimation

We estimate the full model which includes both financial and labor market frictions using Bayesian techniques. We choose the version of the labor market where bargaining is atomistic, as described in subsection 4.4.2 and where endogenous breakups are determined using employer surplus.[11]

5.1. Calibration

We calibrate and later estimate our model using Swedish data. The time unit is a quarter. Parameters that are related to "great ratios" and other observable quantities in the data are calibrated. These include the discount factor β and the tax rate on bonds τ_b which are calibrated to yield a real interest of rate of 2.8 percent annually[12]. We calibrate the capital share α to 0.35, a standard value in the literature that yields a capital-output ratio of roughly 2 on an annual basis.

Three observable ratios are chosen to be exactly matched throughout the estimation, and accordingly we recalibrate three corresponding parameters for each parameter draw: We set the depreciation rate δ to match the ratio of investment over output, $p_i i/y$, entrepreneurial survival rate γ to match the net worth to assets, $n/(p_{k'}k)$, ratio[13] and finally we set the disutility of labor scaling parameter A_L to fix the fraction of their time that individuals spend working.

	Parameter description	Posterior median	Moment	Moment value
δ	Depreciation rate of capital	0.0159	$p_i i/y$	0.169
γ	Entrepreneurial survival rate	0.972	$n/(p_{k'}k)$	0.5
A_L	Scaling of disutility of work	32.4	$L\varsigma$	0.25

Table 0. Matched moments and corresponding parameters.

Sample averages are used when available, e.g. for the various import shares ω_i, ω_c, ω_x (obtained from input-output tables), the remaining tax rates, the government consumption share of GDP, η_g, growth rates of technology (using investment prices to disentangle neutral from investment-specific technology) and several other parameters. We set the steady state real exchange rate $\varphi = 0.648$ to match the export share $p_x x/y$ of 0.438 in the data. We

[11] For a comparison of the dynamics of the model across the various separation criteria, see Christiano, Trabandt and Walentin (2009).

[12] The sample average of the ex-post real rate is lower than 2.8 percent, but ruling out negative values of τ_b and β not too close to 1 this is as low as we can manage to go.

[13] We used micro data to calculate the average equity/total assets during the sample period both for all Swedish firms and for only the stock market listed firms. In the first case book values where used, and in the second case market value of equity was used. Both ratios where close to 0.5.

simply use the inflation target stated by Sveriges Riksbank to calibrate the steady value of the inflation target.

We let the markup of export good producers λ_x be low so as to avoid double marking up of these goods. All other price markups are set to 1.2, following a wide literature. We require full working capital financing in all appropriate sectors. We set ϑ_w so that there is full indexation of wages to the steady state real growth. The indexation parameters $\varkappa^j$, $j = d, x, mc, mi, mx, w$ are set so that there is no indexation to the inflation target, but instead to $\breve{\pi}$ which is set equal to the steady state inflation. This implies that we actually do not allow for partial indexation in this estimation, which would result in steady state price and wage dispersion.

For the financial block of the model we set $F(\bar{\omega})$ equal to the sample average bankruptcy rate. W_e/y has no other noticeable effect than jointly with γ determining the $n/(p_{k'}k)$ and is set arbitrarily.

For the labor block, $1 - L$ is set to the sample average unemployment rate, the length of a wage contract N to annual negotiation frequency, ρ and $\mathcal{F}$ is set so that it takes an unemployed person, on average, 3 quarters to find a job (i.e. $f = 1/3$), in line with the evidence presented in Forslund and Johansson (2007) for completed unemployment spells. Holmlund (2006) present evidence of unemployment duration for all unemployment spells being slightly higher, around 4 quarters. The matching function parameter σ is set so that number of unemployed and vacancies have equal factor shares in the production of matches. σ_m is calibrated to match the probability $Q = 0.9$ of filling a vacancy within a quarter. We assume hiring costs, and not search costs by setting $\iota = 1$ and thereby follow GST. We are reinforced in this calibration by the limited importance of search costs that has been documented using Swedish microdata by Carlsson, Eriksson and Gottfries (2006). The calibrated values are displayed in Table 1.

Parameter	Value	Description
α	0.35	Capital share in production
β	0.999	Discount factor
ω_i	0.43	Import share in investment goods
ω_c	0.25	Import share in consumption goods
ω_x	0.35	Import share in export goods
η_g	0.3	Government consumption share on GDP
φ	0.648	Steady state real exchange rate
τ_k	0.25	Capital tax rate
τ_w	0.35	Payroll tax rate
τ_c	0.25	Consumption tax rate
τ_y	0.30	Labor income tax rate
τ_b	0.0	Bond tax rate
μ_z	1.0059	Steady state growth rate of neutral technology
μ_ψ	1.0003	Steady state growth rate of investment technology
$\bar{\pi}$	1.005	Steady state gross inflation target
λ_x	1.05	Export price markup
λ_j	1.2	Price markups, $j = d, mc, mi, mx$
$\nu_t^*, \nu_t^x, \nu_t^f$	1	Working capital shares
ϑ_w, κ_w	0	Wage indexation to real growth trend and lagged inflation
$\varkappa^j$	$1 - \kappa^j$	Indexation to inflation target for $j = d, x, mc, mi, mx, w$
$\breve{\pi}$	1.005	Third indexing base
$F(\bar{\omega})$	0.0063	Steady state bankruptcy rate
W_e/y	0.001	Transfers to entrepreneurs
L	1-0.075	Steady state fraction of employment
N	4	Number of agency cohorts/length of wage contracts
ρ	0.974	Exogenous survival rate of a match
$\mathcal{F}$	0.001	Endogenous breakup rate of a match
σ	0.5	Unemployment share in matching technology
σ_m	0.5475	Level parameter in matching function
ι	1	Employment adj. costs dependence on tightness

Table 1. Calibrated parameters.

5.2. Choice of priors

The priors are displayed in tables A1 and A2. The general approach has been to choose diffuse priors, with the exceptions to this rule detailed below.

For the exogenous technology processes where we use tight priors (a standard deviation of 0.075) on the persistence parameters and a mode at 0.85. For the Calvo price stickiness parameters we use a mode of 0.75 (corresponding to annual price setting) and tight priors. For habit formation we follow a wide literature by setting the prior mode at 0.65. For the Taylor rule we use the same priors as ALLV (where appropriate). Regarding the parameters for indexation to past inflation we are agnostic and use a diffuse beta prior centered at 0.5.

We follow Smets and Wouters (2003) in setting a prior for σ_a around 0.2.

The persistence of the entrepreneurial parameters γ_t and σ_t have the same priors as the technology processes. The prior mode for μ is set to yield a 1.5% annual external finance premium, as this is the sample average. We choose a diffuse prior so as to let data determine the elasticity of the finance premium in terms of basis points, as this is what affects the dynamics of the economy.[14]

For the labor block we use a diffuse prior for σ_L centered around 2, implying a Frisch elasticity of 1/2. For the fraction of GDP spent on vacancy costs we use a prior with a mode of 0.1% corresponding to a value of κ around 2.[15] We set the mode for the replacement rate for unemployed workers, $bshare$, slightly above the average statutory replacement ratio (0.71), after tax, for this time period. The reason to put the prior above the statutory rate is that the latter ignores the utility value of leisure and any private unemployment insurance, which is reasonably common.

5.3. Data

We estimate the model using Swedish data. Our sample period is 1995Q1-2008Q1. All real quantities are in per capita terms. We use the same 15 macro variables as in ALLV. Further, we use 4 additional data series. First, we add the time series for government consumption. Second, we add a time series for stock prices (the 'OMX Stockholm PI' index, formerly 'SAX All Shares') scaled by the domestic price level as a measure of real net worth. Third, we match a proxy for the spread between the risk-free rate and the loan rate entrepreneurs face. In particular, we compute the spread between the interest rate on all outstanding loans to non-financial corporations and the interest rate on government bonds with a maturity of 6 months (the latter measured as an average over the past 6 month period).[16] The choice of bond duration and averaging is made to match the duration and contract date of the corporate debt. Fourth, we include the official time series for the unemployment rate.

We match the levels of the following 6 (nominal) time series:

$$R_t^{data}, \pi_t^{data}, \pi_t^{c,data}, \pi_t^{i,data}, \pi_t^{*,data}, R_t^{*,data}.$$

[14] In this way we are not constrained by the assumption for the functional form of the idiosyncratic risk.

[15] Formally the steady state recruitment share is defined as

$$recruitshare = \frac{\frac{\kappa}{2} N \tilde{v}^2 l}{y}$$

[16] Ideally one would like to match interest data on newly issued loans for the same maturity as in the model. Unfortunately, such data is not available for Sweden.

For the remaining 13 time series we take logs and first differences.

$$\Delta \ln(W_t/P_t)^{data}, \Delta \ln C_t^{data}, \Delta \ln I_t^{data}, \Delta \ln q_t^{data}, \Delta \ln H_t^{data}, \Delta \ln Y_t^{data}, \Delta \ln X_t^{data}$$
$$\Delta \ln M_t^{data}, \Delta \ln Y_t^{*,data}, \Delta \ln G_t^{data}, \Delta \ln N_t^{data}, \Delta Spread_t^{data}, \Delta \ln Unemprate_t^{data}.$$

In addition we demean each first-differenced time series because in our sample variables such as output, consumption, real wages, investment, exports, imports, stock prices grow on average at substantially different rates. The model, however, allows for two different real long-run growth rates only. In order to match these different trends in the data the estimation would be likely to result in a series of negative or positive shocks for some stationary exogenous process. We want to avoid this and therefore demean the data. After the estimation we compare the growth rates of the data with those implied by the model.

See Figure C in the Appendix for plots of the above data used in the estimation.

5.4. Shocks

In total, there are 22 exogenous stochastic variables in the model. 11 of these evolve according to AR(1) processes:

$$\epsilon, \Upsilon, \bar{\pi}^c, \zeta^c, \zeta^h, \tilde{\phi}, \sigma, \gamma, g, \eta, \sigma_a$$

Further, we have 6 shock processes that are i.i.d.:

$$\tau^d, \tau^x, \tau^{mi}, \tau^{mc}, \tau^{mx}, \varepsilon_R.$$

Finally, the last 5 shock processes are assumed to follow a VAR(1):

$$y^*, \pi^*, R^*, \mu_z, \mu_\Psi.$$

In the estimation we only allow for 19 shocks - same as the number of matched time series. Accordingly we do not allow three shocks present in the theoretical model: the inflation target shock $\bar{\pi}^c$, the shock to bargaining power η and the shock to the standard deviation of idiosyncratic productivity of workers σ_a. Indeed for our sample, 1995-2008, the de jure inflation target has been in place the entire period and has been constant. η also seems superfluous as we already have the standard labor supply shock - the labor preference shock ζ^h.

5.5. Measurement errors

Similarly to Adolfson, Laséen, Lindé and Villani (2007,2008) we allow for measurement errors, except for the domestic and foreign nominal interest rate, since Swedish macro data is measured with substantial noise. We calibrate the standard deviations of the measurement errors so that they correspond to 10% of the variance of each data series. As can be seen in Figure C in the Appendix the size of the measurement errors are small: data and the smoothed series of the model without measurement errors are almost indistinguishable, with a few exceptions.[17]

5.6. Measurement equations

Below we report the measurement equations we use to link the model to the data. First differences are written in percentages so model variables are multiplied by 100 accordingly. Furthermore our data series for inflation and interest rates are annualized, so we make the same transformation for the model variables i.e. multiplying by 400:[18]

$$
\begin{aligned}
R_t^{data} &= 400(R_t - 1) - \vartheta_1 400(R - 1) \\
R_t^{*,data} &= 400(R_t^* - 1) - \vartheta_1 400(R^* - 1)
\end{aligned}
$$

$$
\begin{aligned}
\pi_t^{data} &= 400 \log \pi_t - \vartheta_1 400 \log \pi + \varepsilon_{\pi,t}^{me} \\
\pi_t^{c,data} &= 400 \log \pi_t^c - \vartheta_1 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{i,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \\
\pi_t^{*,data} &= 400 \log \pi_t^* - \vartheta_1 400 \log \pi^* + \varepsilon_{\pi^*,t}^{me},
\end{aligned}
$$

where $\varepsilon_{i,t}^{me}$ denote the measurement errors for the respective variables. In addition, we introduce the parameter $\vartheta_1 \in \{0, 1\}$ and $\vartheta_2 \in \{0, 1\}$ which allows us to handle demeaned and non-demeaned data. In particular, our data for inflation and interest rates is not demeaned, and we therefore set $\vartheta_1 = 0$. An alternative specification would be that we use demeaned inflation and interest rates which would require to set $\vartheta_1 = 1$ in order to correctly match the data with the model.

We use demeaned first-differenced data for the remaining variables. This implies setting the second indicator variable $\vartheta_2 = 1$.

[17] We have experimented by estimating the size of the measurement error for the the most problematic time series - the interest rate spread. The estimated value is lower than the prior mean (which is set to the calibrated value of 10% of the variance), and the parameter estimates are virtually indentical.

[18] Note that in the data we measure $\pi_t^{data} = 400(\log P_t^{data} - \log P_{t-1}^{data})$. In the model, we have defined $\pi_t = \frac{P_t}{P_{t-1}}$. Matching data with the model results in the above measurement equations for inflation.

$$
\begin{aligned}
\Delta \ln Y_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln \left(y_t - p_t^i a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - d_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 \left[1 - \mathcal{F}_t^j\right] l_t^j \right)) - \\
&\quad \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{y,t}^{me} \\
\Delta \ln Y_t^{*,data} &= 100(\ln \mu_{z^+,t} + \Delta \ln y_t^*) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{y^*,t}^{me} \\
\Delta \ln C_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln c_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{c,t}^{me} \\
\Delta \ln X_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln x_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{x,t}^{me} \\
\Delta \ln q_t^{data} &= 100\Delta \ln q_t + \varepsilon_{q,t}^{me} \\
\Delta \ln H_t^{data} &= 100\Delta \ln H_t^{meas} + \varepsilon_{H,t}^{me} \\
\Delta \ln M_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln \text{ Imports}_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
&= 100 \left[\ln \mu_{z^+,t} + \Delta \ln \left(\begin{array}{c} c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} \\ +i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \\ +x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \end{array} \right) \right] - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
\Delta \ln I_t^{data} &= 100 \left[\ln \mu_{z^+,t} + \ln \mu_{\psi,t} + \Delta \ln i_t\right] - \vartheta_2 100(\ln \mu_{z^+} + \ln \mu_\psi) + \varepsilon_{I,t}^{me} \\
\Delta \ln G_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln g_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{g,t}^{me}
\end{aligned}
$$

Note that neither measured GDP nor measured investment include investment goods used for capital maintenance. The reason is that the documentation for calculation of the Swedish National Accounts (SOU (2002)) indicate that these are not included in the investment definition (and the national accounts are primarily based on the expenditure side). To calculate measured GDP we also exclude monitoring costs and recruitment costs. Note that it is measured GDP that enters the Taylor rule.

The demeaned real wage is measured by the demeaned employment-weighted average Nash bargaining wage in the model:

$$
w_t^{avg} = \frac{1}{L} \sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j} \bar{w}_{t-j}
$$

Given this definition the measurement equation for wages is:

$$
\Delta \ln(W_t/P_t)^{data} = 100\Delta \ln \frac{\tilde{W}_t}{z_t^+ P_t} = 100(\ln \mu_{z^+,t} + \Delta \ln w_t^{avg}) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{W/P,t}^{me}
$$

Finally, we measure demeaned net worth and unemployment as follows:

$$
\begin{aligned}
\Delta \ln N_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln n_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{N,t}^{me} \\
\Delta \ln Spread_t^{data} &= 100\Delta \ln(Z_{t+1} - R_t) = 100\Delta \ln \left(\frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t \right) + \varepsilon_{Spread,t}^{me} \\
\Delta \ln Unemp_t^{data} &= 100\Delta \ln(1 - L_t) + \varepsilon_{Unemp,t}^{me}.
\end{aligned}
$$

5.7. Estimation results

We obtain the estimation results using a random walk Metropolis-Hasting chain with 250 000 draws after a burn-in of 150 000 draws and with an acceptance rate of 0.21. Substantial analysis has been spent on ensuring that the Hessian used for the Metropolis-Hasting algorithm approximates the curvature of the likelihood well. The quality of the Hessian is documented in the Computational Appendix. We estimate 28 structural parameters, 10 AR1 coefficients, 16 parameters for the VAR describing the foreign economy and 19 standard deviations of shocks.

Figure D presents the smoothed values for the shock processes. None of them contain any obvious trend.

5.7.1. Posterior parameter values

We start by commenting briefly on the parameter estimates. See the prior-posterior tables, Table A1 and A2, in the Appendix. We focus our discussion on the posterior median which is used for all computations below. Note that the posterior median and the posterior mean are virtually indistinguishable, with the exception of the shock variances. Their posterior distribution inherit some skewness from the inverse-gamma prior and the means are therefore higher than the median.

The Calvo price rigidity parameters indicate substantial variation with domestic prices being the most rigid ($\xi_d = 0.91$), and prices for imported investment goods and imported inputs for export production being quite flexible as they are optimally re-set more often than annually ($\xi_{mi} = 0.72$ and $\xi_{mx} = 0.71$). With the exception of domestic prices these estimates are substantially below earlier work on Swedish data by Adolfson (2008). Both the later sample and the additional internal propagation in our model might contribute to this difference. We find only a moderate degree of indexation to lagged inflation, from 1/4 to 1/2, with the exception of $\kappa_x = 0.59$.

The estimated Taylor rule parameters are in line with the literature. The posterior median of the curvature of capacity utilization $\sigma_a = 0.21$ is substantially higher than Smets and Wouters (2003) find for Euro data, but still allows for substantial variation in utilization.

The posterior median of μ of 0.4 is below the prior mean of 0.55, indicating that the elasticity of the interest rate spread, in terms of basis points, is slightly lower than implied sample average and by the functional form assumption we have made.

Moving on to the labor block we find a replacement ratio of 0.88, i.e. substantially higher than the statutory replacement rate of the public Swedish unemployment insurance. The recruitment costs as a fraction of GDP is estimated to be a slightly below 1/6 of a percent, corresponding to $\kappa = 2.4$. We are in a unique position to estimate the curvature of the increasing disutility of labor supply as our model and fact that we match data series for both hours and employment, allow for good identification. Our posterior median of σ_L is 1.7 (i.e. a Frisch elasticity of 0.59), roughly in line with what the macro literature without extensive margin has used, but below most micro estimates.

As we will see in the IRFs below the posterior median of $\tilde{\phi}_s = 1.3$, barely generates a hump-shaped response of the exchange rate to monetary policy shocks. This is in contrast to the response to the exchange rate at the prior mean.

A number of parameters of interest are functions of the explicitly estimated parameters. First of all, three parameters follow from the calibration of various ratios as reported in Table 0. The last column of that table reports the parameter values at the posterior mean, and they are all reasonable. Two labor parameters are worth mentioning: The bargaining power of workers, η, is solved for to yield a steady state unemployment rate matching the sample average. The value of η at the posterior mean is roughly 1/3.

We note from the prior-posterior table (or the corresponding plots, provided in a separate Computational Appendix) that data is informative for all parameters, with a couple of exceptions: data seem to contain no information on κ_{mx}, η_i or ρ_{μ_Ψ} and very little regarding r_π, η_f, η_x and ρ_g.

5.7.2. Model Moments and Variance Decomposition

In Table A3 we present a comparison of data and model means and standard deviations for the observed time series. We note a substantial variation of real growth rates in the data, which is the reason why we demeaned the growth rates in the first place, before matching the data. There is small but noticeable tendency for the standard deviations implied by the model to be higher than in the data. We note one large failure in this respect: the model implied volatility of the growth of the interest rate spread, which also spills over of the investment growth volatility. We believe the main reason for this overprediction is the imperfect measure of the spread in the data which drives up the estimated volatility of the idiosyncratic risk shock.

We compute the asymptotic variance decomposition and present it in Table A4. The first thing to note is the importance of the stationary (neutral) technology shock: It is the most important shock for GDP and quite important for all inflation rates. It also explains roughly 50% of the variation in hours worked and more than 20% of unemployment variation. The labor preference shock is by far the most prominent in explaining wages and unemployment. The entrepreneurial wealth (survival) shock drives 3/4 of investment and is thereby also the most important shock for the nominal interest rate and explains 14% of output fluctuations. Finally, it is interesting to note the substantial spillovers from the two financial shocks to unemployment.

5.7.3. Impulse responses

Finally, we plot impulse responses at the posterior median for all shocks. For comparison purposes we plot the IRFs, for the same fixed parameter vector, for smaller versions of the model as well. Only one parameter is recalibrated between models: α has to be re-set to keep the capital-output ratio unchanged in the baseline and unemployment model specifications.[19]

We start with a general comment before analyzing a few key impulse responses. Somewhat surprisingly the IRFs of the model with unemployment frictions is remarkably similar to the more traditional EHL modelling of the labor market used in our baseline specification. For the same reason the full model is similar to the financial friction model, as unemployment frictions is the only difference between them. This observation applies to most shocks and all variables plotted but one. The clear exception is real wages, where the unemployment model imply more volatile wages (for the same given nominal wage rigidity). Obviously the unemployment model is still richer than the EHL model in that it has explicit implications for unemployment and hours per worker.

The IRFs for the monetary policy shock is reasonably standard: A 30 basis point temporary increase in the nominal interest rate is amplified by the financial frictions. Entrepreneurial net worth is reduced both because of the falling price of capital and because of the surprise disinflation that increases the real value of the nominal debt. Accordingly the interest rate risk spread increase by 9 basis points. Comparing across model we see how the increased spread cause the expected amplification in the response of investment. We note that our assumption regarding the country risk premium implies that the real exchange rate moves substantially less than one-for-one with the nominal interest rate, and in an almost hump-shaped manner. The policy shock implies an increase in unemployment from the steady state value of 7.5% to a maximum of 7.6% after 3 quarters.

The response to a stationary technology shock in our estimated model is novel: Output actually falls initially in response to a positive shock, and this is true for all four model

[19] The absence of financial frictions in these two versions of the model imply that the required rate of return on capital is substantially lower. We therefore set $\alpha = 0.1952$ to keep the capital-output ratio constant.

specifications. The reason for this response is the very strong decrease capital utilization (not plotted) and in hours worked.[20] The latter is contributed to both by an increase in unemployment and a decrease in hours worked per employee. In terms of demand components it is net exports (not plotted) that decrease strongly because of a substantial appreciation of the real exchange rate. Comparing across models we note that financial frictions dampens the response of investment as net worth of entrepreneurs initially falls. This is a standard result for supply shocks in terms of the nominal debt contract / Fisher debt deflation mechanism.

Finally, for the estimated parameter values the entrepreneurial wealth (or survival) shock has some of the characteristics of a classic demand shock: It drives up CPI inflation and output (the former result is parameter dependent). The interesting part is that it moves consumption and investment in opposite directions, which is a similarity with the stationary investment-specific shock (see Justiniano, Primiceri and Tambalotti (2008)). The key difference versus the investment-specific shock is that the wealth shock implies an increase in net worth, as well as a decrease in real wages. In particular the former characteristic makes the entrepreneurial wealth shock a more plausible candidate for explaining the main part of the increase in investment both in the late 1990's and in the last boom when financial data is included in the analysis (also see the smoothed shock values in Figure D).

6. Conclusion

This paper incorporates two important extensions of the emerging standard monetary DSGE model in a small open economy setting. We add financial frictions in the accumulation of capital in a well established way, based on Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2007). We then add labor market frictions building on a large literature where we are closest to Gertler, Sala and Trigari (2008). We made an important contribution to the literature by endogenizing the job separation decision in this rich setting.

We estimate the full model, which contains the financial frictions as well as the search and matching frictions, with Bayesian techniques. Surprisingly, the dynamics of the model with unemployment frictions is remarkably similar to the established labor market modelling by Erceg, Hendersson and Levin (2000), which has no extensive margin of employment. We also note that the entrepreneurial wealth shock plays a very large role in the variance decomposition - it is the main determinant of investment and also very important for GDP and the nominal interest rate. An interesting question to be analyzed further is the relative

[20] The size (and existence) of the initial decrease in GDP depends inversely on the estimated parameter for variable capacity utilization σ_a, while the duration of the decrease mainly depends on the domestic price rigidity parameter ξ_d.

importance of investment-specific shock vs. the entrepreneurial wealth shock, contrasting the results in the present paper with Justiniano, Primiceri and Tambalotti (2008).

References

[1] Adolfson, Malin, Stefan Laséen, Jesper Linde and Mattias Villani (2005), "The Role of Sticky Prices in An Estimated Open Economy DSGE Model: A Bayesian Investigation", Journal of the European Economic Association Papers and Proceedings, Vol 3(2-3), 444-457.

[2] Adolfson, Malin, Stefan Laseén, Jesper Lindé and Mattias Villani (2007), "Bayesian Estimation of Open Economy DSGE Model with Incomplete Pass-Through", Journal of International Economics, Vol 72, 481-511.

[3] Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2008), "Evaluating An Estimated New Keynesian Small Open Economy Model", Journal of Economic Dynamics and Control, Vol 32(8), 2690-2721.

[4] Altig, David, Christiano, Lawrence J, Martin Eichenbaum and Jesper Linde (2004), "Firm-Specific Capital, Nominal Rigidities and the Business Cycle", manuscript, Northwestern University.

[5] Barro, Robert (1977), "Long-Term Contracting, Sticky Prices and Monetary Policy," Journal of Monetary Economics, Vol 3(3), 305-316.

[6] Bernanke, Ben, Mark Gertler and Simon Gilchrist. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," Handbook of Macroeconomics, edited by John B. Taylor and Michael Woodford, 1341-1393. Amsterdam, New York and Oxford: Elsevier Science, North-Holland.

[7] Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2005), "Large Devaluations and the Real Exchange Rate", Journal of Political Economy, Vol 113, 742–784 .

[8] Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2007), "Modeling Exchange Rate Passthrough After Large Devaluations", Journal of Monetary Economics, Vol 54(2), 346-368.

[9] Carlsson, Mikael, Stefan Eriksson and Nils Gottfries (2006), "Testing Theories of Job Creation: Does Supply Create its Own Demand", Sveriges Riksbank Working Paper 194.

[10] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2003). "The Great Depression and the Friedman-Schwartz Hypothesis," Journal of Money, Credit, and Banking, Vol 35, 1119-1198.

[11] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno (2007), "Shocks, Structures or Monetary Policies? The Euro Area and US After 2001," manuscript, European Central Bank.

[12] Christiano, Lawrence J., Cosmin Ilut, Roberto Motto and Massimo Rostagno (2007), "Monetary Policy and Stock Market Boom-Bust Cycles", manuscript, Northwestern University.

[13] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin (2009), "A Monetary Business Cycle Model With Labor Market Frictions", mimeo.

[14] den Haan, Wouter, Garey Ramey and Joel Watson (2000), "Job Destruction and Propagation of Shocks", American Economic Review, Vol 90(3), 482-498.

[15] Devereux, Michael P., Rachel Griffith and Alexander Klemm (2002), "Corporate income tax reforms and international tax competition," Economic Policy, CEPR, CES, MSH, Vol. 17, 449-495, October.

[16] Eichenbaum, Martin and Charles L. Evans (1995), "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," The Quarterly Journal of Economics, Vol. 110(4), 975-1009.

[17] Erceg, Christopher J., Henderson, Dale, W., and Andrew T. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts," Journal of Monetary Economics, Vol 46, 281-313.

[18] Fisher, Irving (1933), "The Debt-Deflation Theory of Great Depressions," Econometrica, Vol 1, 337-357.

[19] Fisher, Jonas (1998), "Credit market imperfections and the heterogeneous response of firms to monetary shocks", Federal Reserve Bank of Chicago, Working Paper Series, 96-23.

[20] Forslund, A. and K. Johansson (2007), "Random and stock-flow models of labour market matching - Swedish evidence", Working paper 2007:11, IFAU

[21] Fujita, Shigeru and Garey Ramey (2007), "The Cyclicality of Separation and Job Finding Rates", Working Paper 07-19, Federal Reserve Bank of Philadelphia.

[22] Gertler, Mark, Luca Sala and Antonella Trigari (2008), "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," Journal of Money, Credit and Banking, Vol 40(8), 1713-1764.

[23] Gertler, Mark and Antonella Trigari (2009), "Unemployment Fluctuations with Staggered Nash Bargaining," Journal of Political Economy, Vol 117(1), 38-86.

[24] Hall, Robert (2005a), "Employment Fluctuations with Equilibrium Wage Stickiness," American Economic Review, Vol 95(1), 50-65.

[25] Hall, Robert (2005b), "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market," Review of Economics and Statistics, Vol 87(3), 397-407.

[26] Hall, Robert (2005c), "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years," NBER Macroeconomics Annual, M.Gertler and K. Rogoff editors, MIT Press, 101-137.

[27] Holmlund, Bertil (2006), "The Rise and Fall of Swedish Unemployment," In M. Werding (ed.), Structural Unemployment in Western Europe: Reasons and Remedies, MIT Press.

[28] Justiniano, Alejandro, Giorgio Primiceri and Andrea Tambalotti (2008), "Investment Shocks and Business Cycles", Staff Report no 322, Federal Reserve Bank of New York.

[29] Levin, Andrew., David Lopez-Salido (2004), "Optimal Monetary Policy with Endogenous Capital Accumulation", manuscript, Federal Reserve Board.

[30] Levin, Andrew, Alexei.Onatski, John Williams and Noah Williams (2005), "Monetary Policy under Uncertainty in Microfounded Macroeconometric Models," NBER Macroeconomics Annual 2005, Gertler, M., Rogoff, K., eds. Cambridge, MA: MIT Press.

[31] Merz, Monika (1995), "Search in the Labor Market and the Real Business Cycle", Journal Monetary Economics, Vol 36, 269-300.

[32] Mortensen, Dale and Christopher Pissarides (1994), "Job Creation and Job Destruction in the Theory of Unemployment," Review of Economic Studies, Vol 61, 397-415.

[33] Shimer, Robert (2005a), "The Cyclical Behavior of Equilibrium Unemployment Vacancies, and Wages: Evidence and Theory," American Economic Review, Vol 95(1), 25-49.

[34] Shimer, Robert (2005b), "Reassessing the Ins and Outs of Unemployment," manuscript, University of Chicago.

[35] SOU, (2002), "Beräkningsrutiner för nationalräkenskaperna", SOU 2002:118, appendix 3, in "Utveckling och förbättring av den ekonomiska statistiken", Fritzes.

[36] Yun, Tack (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," Journal of Monetary Economics, Vol 37(2), 345-370.

A. Tables and Figures

	Prior distr.	Prior mean	Prior s.d.	Post. mean	Post. median	Post. s.d.	5 %	95 %
ξ_d	β	0.750	0.0750	0.906	0.905	0.0217	0.8772	0.9481
ξ_x	β	0.750	0.0750	0.794	0.797	0.0412	0.7285	0.8621
ξ_{mc}	β	0.750	0.0750	0.842	0.843	0.0404	0.7834	0.9129
ξ_{mi}	β	0.750	0.0750	0.722	0.723	0.0399	0.6552	0.7863
ξ_{mx}	β	0.750	0.0750	0.706	0.709	0.0599	0.6122	0.8069
κ_d	β	0.500	0.1500	0.264	0.256	0.0959	0.1083	0.4139
κ_x	β	0.500	0.1500	0.587	0.589	0.1200	0.4031	0.7967
κ_{mc}	β	0.500	0.1500	0.347	0.339	0.1222	0.1432	0.5375
κ_{mi}	β	0.500	0.1500	0.412	0.404	0.1276	0.1980	0.6173
κ_{mx}	β	0.500	0.1500	0.484	0.483	0.1449	0.2474	0.7256
κ_w	β	0.500	0.1500	0.288	0.282	0.0944	0.1274	0.4357
σ_L	Γ	2.000	0.5000	1.703	1.672	0.2238	1.3531	2.0620
b	β	0.650	0.1000	0.774	0.779	0.0565	0.6885	0.8688
S''	N	8.000	2.0000	9.000	8.960	1.7790	6.0450	11.8860
σ_a	Γ	0.200	0.0750	0.219	0.209	0.0548	0.1347	0.3032
ρ_R	β	0.850	0.1000	0.903	0.904	0.0156	0.8777	0.9284
r_π	N	1.700	0.1000	1.740	1.740	0.0984	1.5776	1.9005
$r_{\Delta\pi}$	N	0.300	0.1000	0.086	0.086	0.0297	0.0363	0.1346
r_y	N	0.125	0.0500	0.128	0.126	0.0333	0.0729	0.1817
$r_{\Delta y}$	Γ	0.050	0.0250	0.064	0.063	0.0155	0.0379	0.0881
η_i	Γ	1.500	0.2500	1.469	1.453	0.2483	1.0460	1.8522
η_f	Γ	1.500	0.2500	1.404	1.390	0.2184	1.0416	1.7466
η_c	Γ	1.500	0.2500	1.219	1.205	0.2070	0.8765	1.5533
η_x	Γ	1.500	0.2500	1.570	1.555	0.2588	1.1535	1.9909
$\tilde{\phi}_s$	Γ	1.500	0.1500	1.282	1.278	0.1195	1.0820	1.4710
$recruitshare, \%$	Γ	0.100	0.0750	0.138	0.135	0.0261	0.0959	0.1782
$bshare$	β	0.750	0.0750	0.882	0.884	0.0260	0.8402	0.9233
μ	β	0.550	0.2000	0.409	0.390	0.1479	0.1686	0.6437
ρ_{μ_z}	β	0.500	0.1500	0.725	0.736	0.1022	0.5695	0.8927
ρ_{μ_Ψ}	β	0.500	0.1500	0.489	0.487	0.1460	0.2477	0.7273
ρ_ε	β	0.850	0.0750	0.870	0.872	0.0278	0.8263	0.9175
ρ_Υ	β	0.850	0.0750	0.947	0.949	0.0175	0.9208	0.9771
ρ_{ζ^c}	β	0.850	0.0750	0.778	0.785	0.0693	0.6683	0.8881
ρ_{ζ^h}	β	0.850	0.0750	0.843	0.852	0.0532	0.7603	0.9264
$\rho_{\tilde{\phi}}$	β	0.850	0.0750	0.862	0.869	0.0543	0.7800	0.9504
ρ_g	β	0.850	0.0750	0.835	0.840	0.0716	0.7249	0.9570
ρ_σ	β	0.850	0.0750	0.759	0.763	0.0527	0.6766	0.8461
ρ_γ	β	0.850	0.0750	0.870	0.874	0.0484	0.7919	0.9476
$a11$	N	0.500	0.5000	1.087	1.096	0.1382	0.8575	1.3034
$a22$	N	0.000	0.5000	-0.048	-0.051	0.1597	-0.3104	0.2162
$a33$	N	0.500	0.5000	0.542	0.539	0.1473	0.3094	0.7991
$a12$	N	0.000	0.5000	-0.039	-0.042	0.1827	-0.3327	0.2700
$a13$	N	0.000	0.5000	-0.378	-0.344	0.2797	-0.8343	0.0411
$a21$	N	0.000	0.5000	0.219	0.206	0.1379	-0.0018	0.4386
$a23$	N	0.000	0.5000	-0.241	-0.224	0.2420	-0.6339	0.1522
$a24$	N	0.000	0.5000	-0.057	-0.063	0.2572	-0.4676	0.3788
$a31$	N	0.000	0.5000	0.240	0.235	0.0660	0.1330	0.3519
$a32$	N	0.000	0.5000	0.034	0.034	0.0419	-0.0355	0.1013
$a34$	N	0.000	0.5000	-0.014	-0.037	0.1535	-0.2433	0.2526
$c21$	N	0.000	0.5000	-0.072	-0.075	0.1776	-0.3595	0.2278
$c31$	N	0.000	0.5000	0.108	0.108	0.0513	0.0214	0.1919
$c32$	N	0.000	0.5000	-0.012	-0.011	0.0413	-0.0801	0.0537
$c24$	N	0.000	0.5000	-0.165	-0.165	0.3574	-0.7333	0.4423
$c34$	N	0.000	0.5000	0.030	0.027	0.1289	-0.1860	0.2338

Table A1. Estimation results.Parameters

	Prior distr.	Prior mean	Prior s.d.	Post. mean	Post. median	Post. s.d.	5 %	95 %
μ_z	Inv-Γ	0.150	Inf	0.109	0.106	0.0290	0.0614	0.1538
μ_Ψ	Inv-Γ	0.150	Inf	0.084	0.078	0.0311	0.0374	0.1278
ε	Inv-Γ	0.500	Inf	1.143	1.129	0.1461	0.9077	1.3742
Υ	Inv-Γ	0.500	Inf	0.469	0.465	0.0613	0.3706	0.5665
ζ^c	Inv-Γ	0.150	Inf	0.270	0.262	0.0609	0.1737	0.3639
ζ^h	Inv-Γ	0.150	Inf	0.385	0.382	0.0604	0.2832	0.4786
$\tilde{\phi}$	Inv-Γ	0.150	Inf	0.280	0.267	0.0995	0.1186	0.4316
ε_R	Inv-Γ	0.150	Inf	0.085	0.084	0.0104	0.0685	0.1008
g	Inv-Γ	0.500	Inf	0.703	0.695	0.0794	0.5739	0.8304
τ^d	Inv-Γ	0.150	Inf	0.598	0.479	0.3458	0.2062	1.2612
τ^x	Inv-Γ	0.150	Inf	0.203	0.183	0.0856	0.0756	0.3341
τ^{mc}	Inv-Γ	0.150	Inf	0.631	0.517	0.3482	0.1885	1.1455
τ^{mi}	Inv-Γ	0.150	Inf	0.108	0.102	0.0335	0.0581	0.1554
τ^{mx}	Inv-Γ	0.150	Inf	0.497	0.434	0.2470	0.1597	0.8771
γ	Inv-Γ	0.500	Inf	0.603	0.584	0.1426	0.3718	0.8249
σ	Inv-Γ	0.500	Inf	5.338	5.296	0.6842	4.1837	6.3780
y^*	Inv-Γ	0.500	Inf	0.203	0.202	0.0308	0.1541	0.2542
π^*	Inv-Γ	0.500	Inf	0.189	0.188	0.0233	0.1516	0.2268
R^*	Inv-Γ	0.500	Inf	0.234	0.231	0.0549	0.1433	0.3213

Table A2. Estimation results. Standard deviation of shocks

	Means		Standard Deviations	
	Data	Model	Data	Model
Domestic. Inflation	1.56	2.00	1.97	2.30
CPI inflation	1.48	2.00	1.38	2.42
Invest. price inflation	1.42	1.88	2.03	2.78
Nom. interest rate	3.82	4.82	1.73	1.91
GDP growth	0.61	0.61	0.44	0.76
Real wage growth	0.69	0.61	0.79	0.92
Consumption growth	0.52	0.61	0.53	0.76
Investment growth	1.09	0.64	1.42	3.11
Real exch. rate growth	0.04	0.00	2.30	2.46
Total hours growth	0.05	0.00	0.46	0.40
Gov. cons. growth	0.06	0.61	0.81	0.79
Exports growth	1.52	0.61	1.36	1.41
Import growth	1.37	0.61	1.48	1.97
Stock market growth	1.96	0.61	9.94	12.28
Interest spread growth	-0.88	0.00	10.26	81.10
Unemployment growth	-1.16	0.00	3.61	5.87
Foreign GDP growth	0.45	0.61	0.30	0.28
Foreign inflation	1.84	2.00	0.81	0.84
Foreign nom. int. rate	3.89	4.82	1.04	1.16

Table A3. Data and model moments.

Shocks/Variables	Pid	Pic	Pii	R	dy	dw	dc	di	dq	dH	dG	dexp	dimp	dn	dspread	dunemp
Stat. neutr. tech.	14.9	13.8	15.3	18.9	21.0	6.5	5.7	0.5	11.3	52.1	0.0	1.3	25.9	0.6	0.4	22.8
Stat. invest. tech.	3.6	3.9	4.7	20.6	7.9	0.9	5.6	13.1	3.6	7.3	0.0	0.8	3.3	53.3	27.7	3.0
Consumption pref.	0.1	0.2	0.5	6.2	9.4	0.1	63.4	1.7	1.1	3.7	0.0	0.2	2.2	0.3	0.2	1.2
Labor pref.	7.2	6.7	7.4	9.1	14.4	48.9	2.8	0.2	5.4	9.7	0.0	0.6	18.1	0.3	0.2	54.4
Monetary policy	0.5	0.7	0.8	6.7	1.1	1.5	0.9	2.4	0.4	0.5	0.0	0.3	1.0	2.2	1.1	2.3
Gov. consumption	0.0	0.0	0.0	0.5	4.3	0.1	0.1	0.1	0.0	1.5	85.1	0.0	0.5	0.1	0.0	0.2
Domestic markup	64.8	35.3	20.1	4.9	1.1	27.4	1.5	0.0	0.4	0.5	0.0	0.1	0.5	0.1	0.2	2.1
Export markup	0.0	0.0	0.0	0.6	2.9	0.2	0.1	0.1	0.0	1.0	0.0	62.9	4.6	0.1	0.0	0.5
Cons. import mkup	0.1	32.4	0.0	6.3	1.7	0.8	1.4	0.4	2.3	1.0	0.0	0.1	6.6	0.6	0.3	0.1
Invest. import mkup	0.0	0.0	28.0	0.0	0.8	0.1	0.0	0.1	0.0	0.2	0.0	0.0	1.3	0.1	0.1	0.2
Export import mkup	0.1	0.1	0.3	0.8	9.9	0.6	0.1	0.0	0.5	3.2	0.0	10.0	20.0	0.1	0.1	1.6
Entrepreneur risk	0.0	0.0	0.0	0.0	3.0	0.9	0.0	3.7	0.0	3.6	0.0	0.0	1.3	10.1	54.1	3.4
Entrepreneur survival	0.9	1.5	3.5	21.9	14.3	2.6	10.7	74.3	5.8	1.0	0.0	1.7	4.6	25.5	15.5	3.0
Risk premium	0.3	2.0	11.8	3.3	2.4	0.1	1.1	1.2	56.1	0.8	0.0	7.3	3.1	0.0	0.0	0.7
Unit-root neutr. tech.	0.1	0.1	0.1	0.2	2.2	2.0	1.7	0.1	0.1	0.1	4.0	1.4	0.7	0.0	0.0	0.7
Unit-root invest. tech.	0.0	0.0	1.5	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.4	0.1	0.0	0.2	0.1	0.0
Foreign output	0.0	0.1	0.7	0.1	0.1	0.0	0.0	0.0	4.2	0.0	0.0	0.8	0.5	0.0	0.0	0.0
Foreign inflation	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.1	0.0	3.0	0.3	0.0	0.0	0.0
Foreign nom.int. rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A4. Variance decomposition (asymptotic).

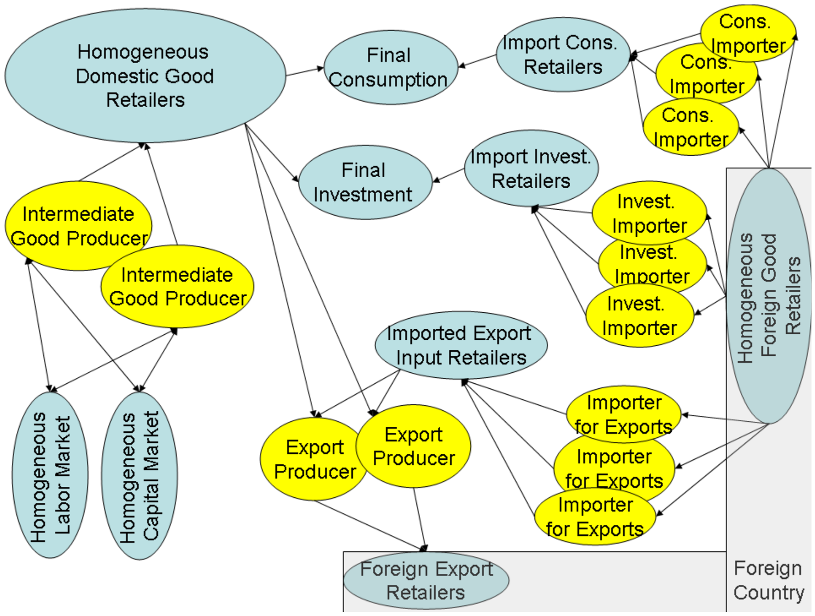


Figure A. Graphical illustration of the goods production part of the model.

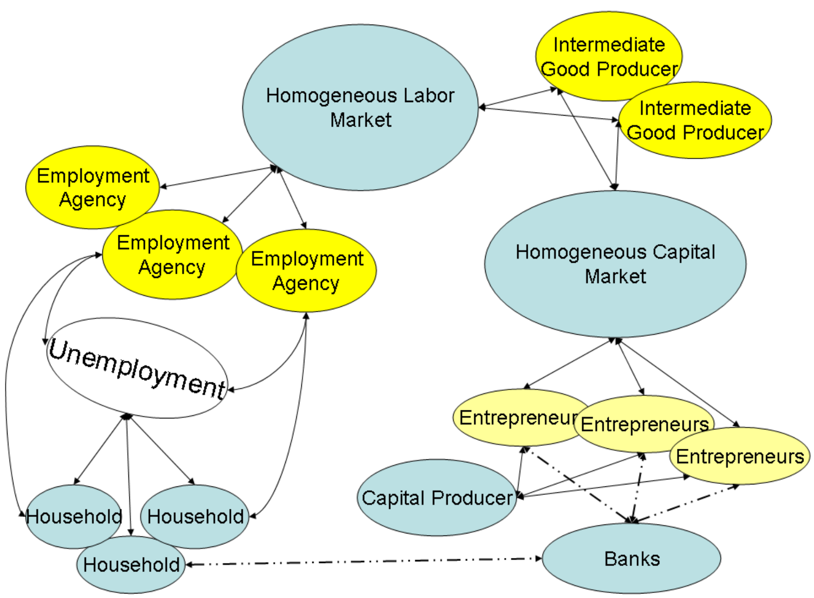


Figure B. Graphical illustration of the labor and capital markets of the model.

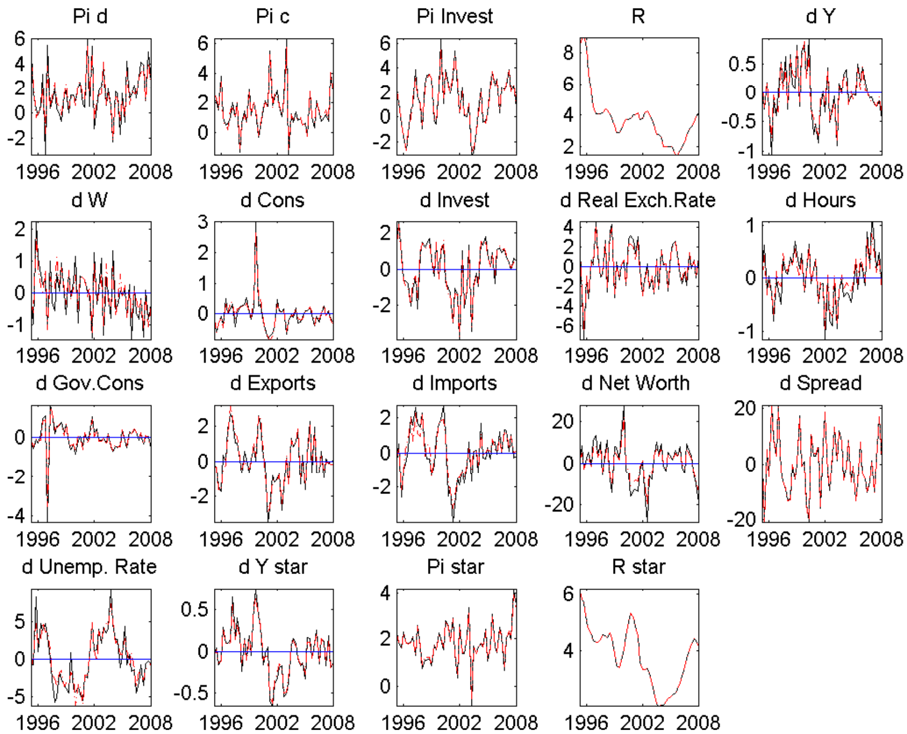


Figure C. Data series used in estimation (solid black) and smoothed variables without measurement error (in dashed red).

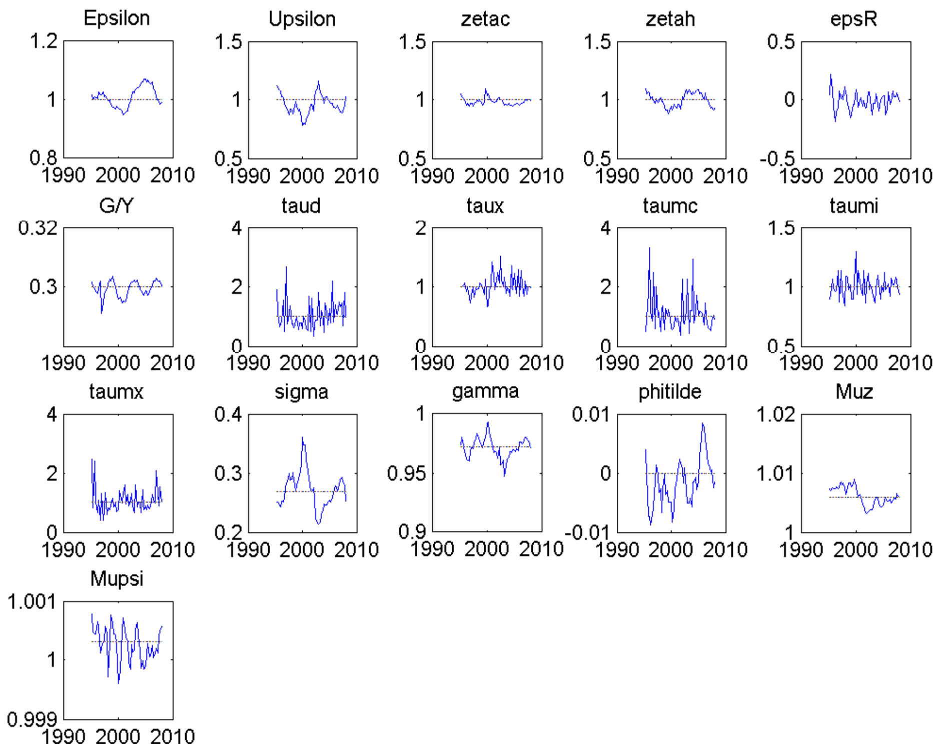
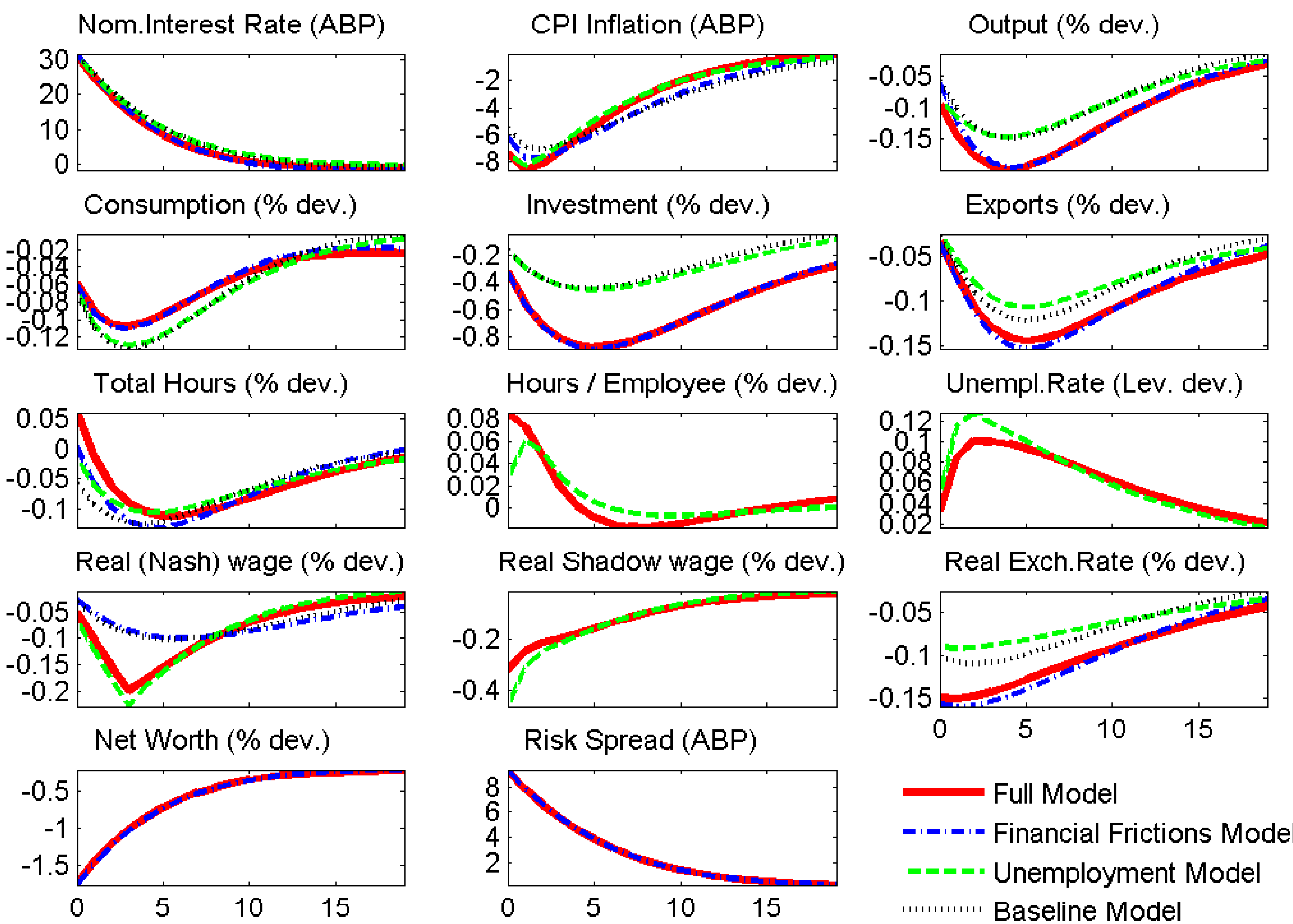
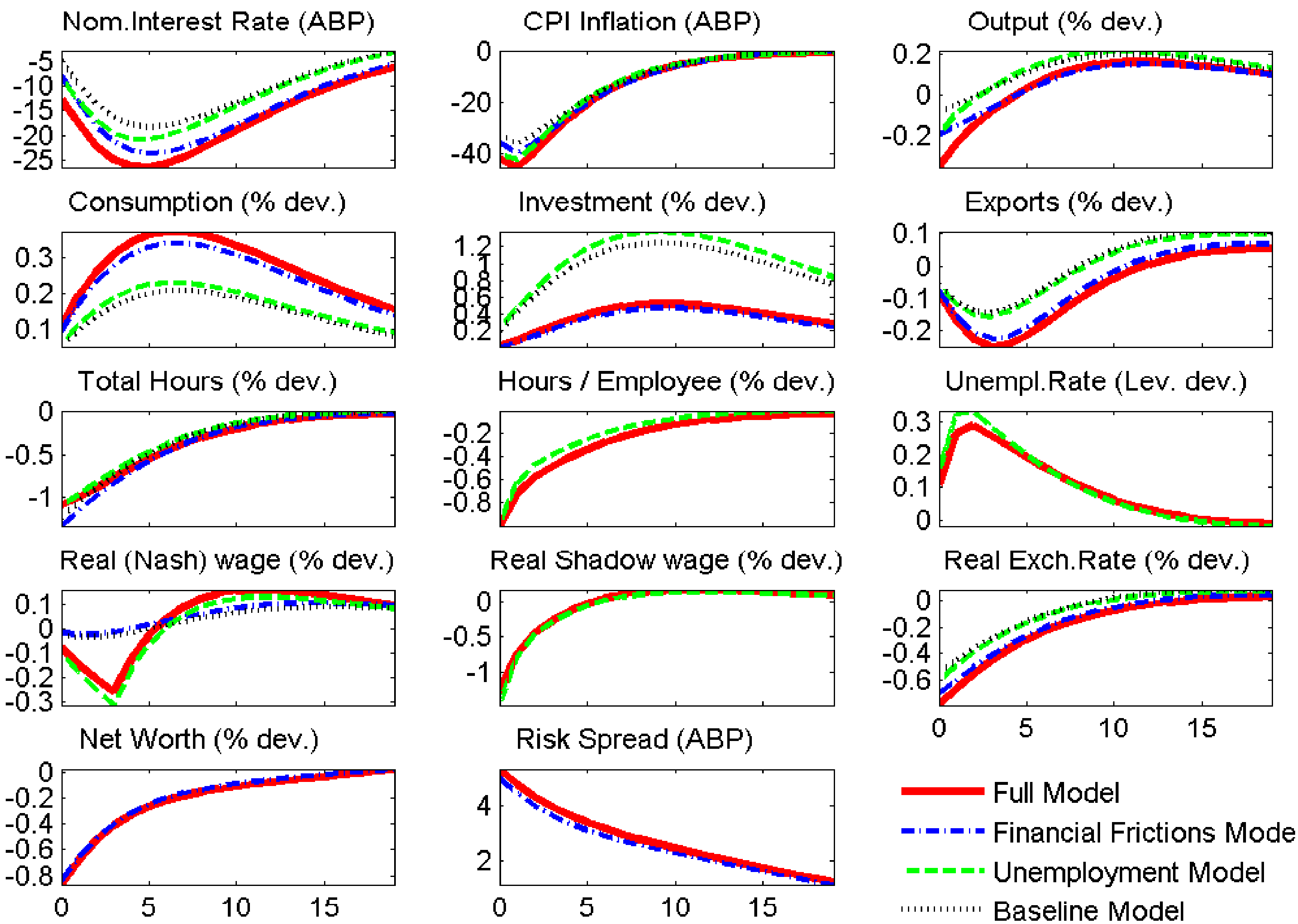


Figure D. Smoothed shock processes (with the exception of epsR which is the innovation to the monetary policy rule).

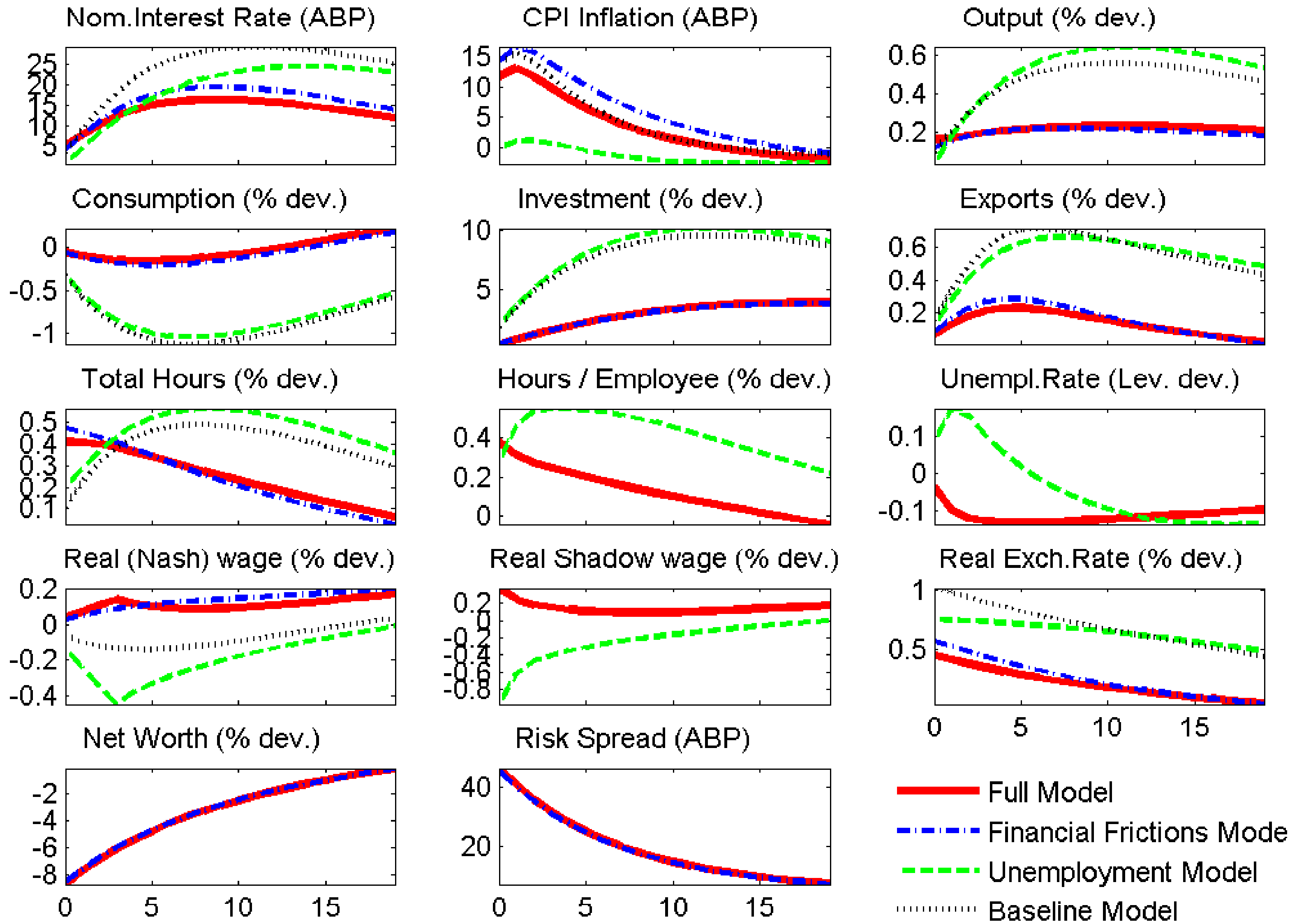
Monetary Policy Shock



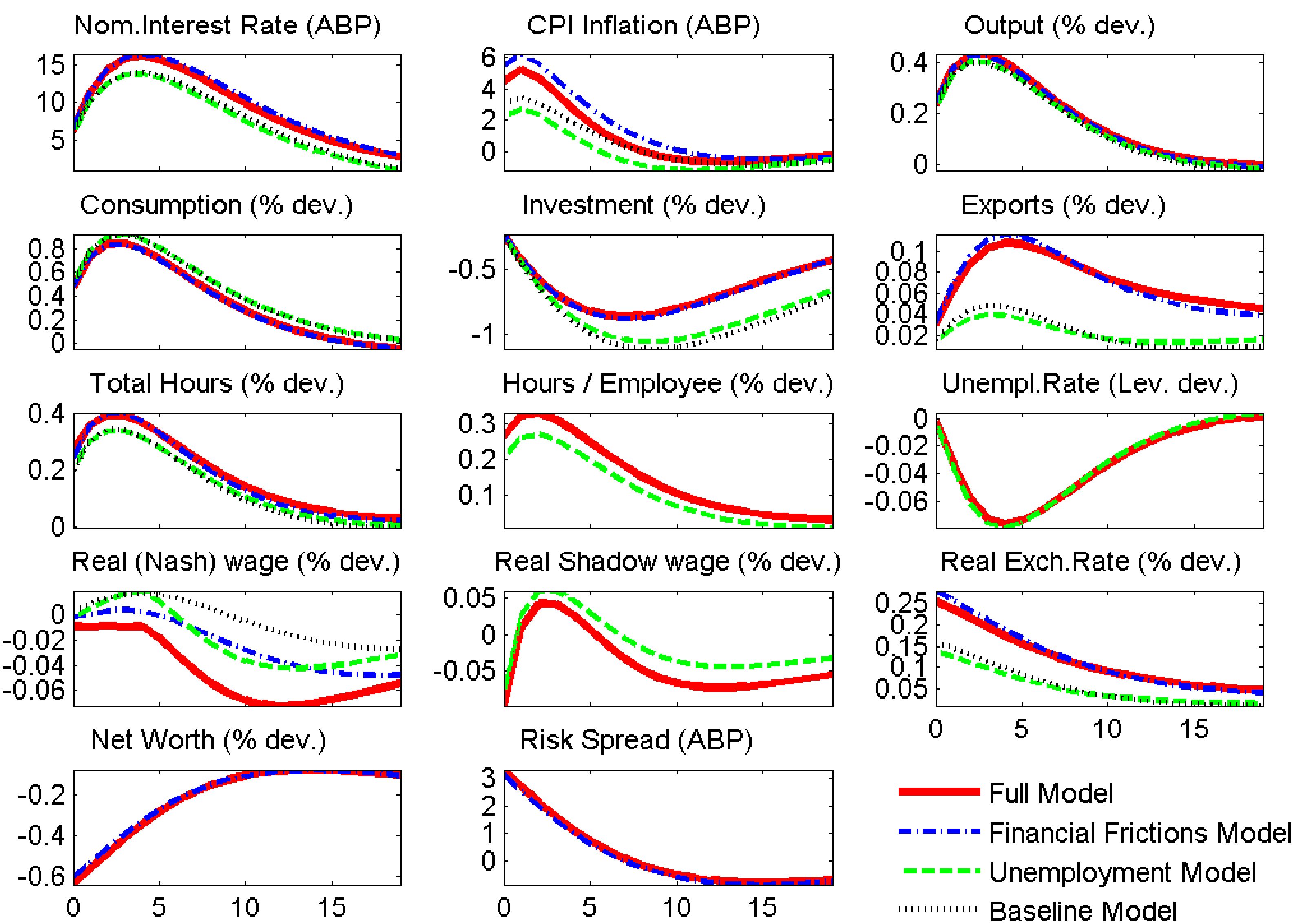
Stationary Neutral Technology Shock

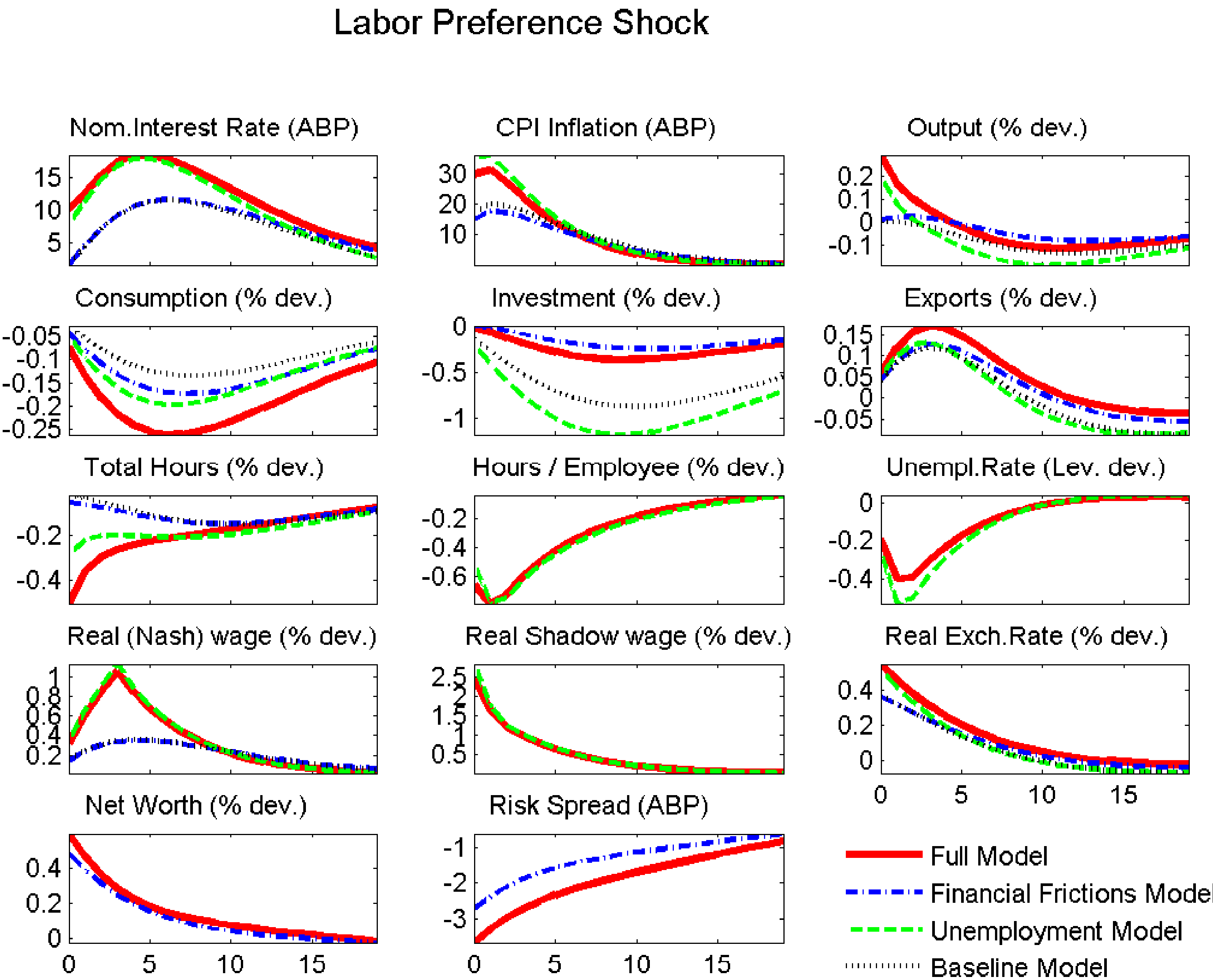


Stationary Investment Technology Shock

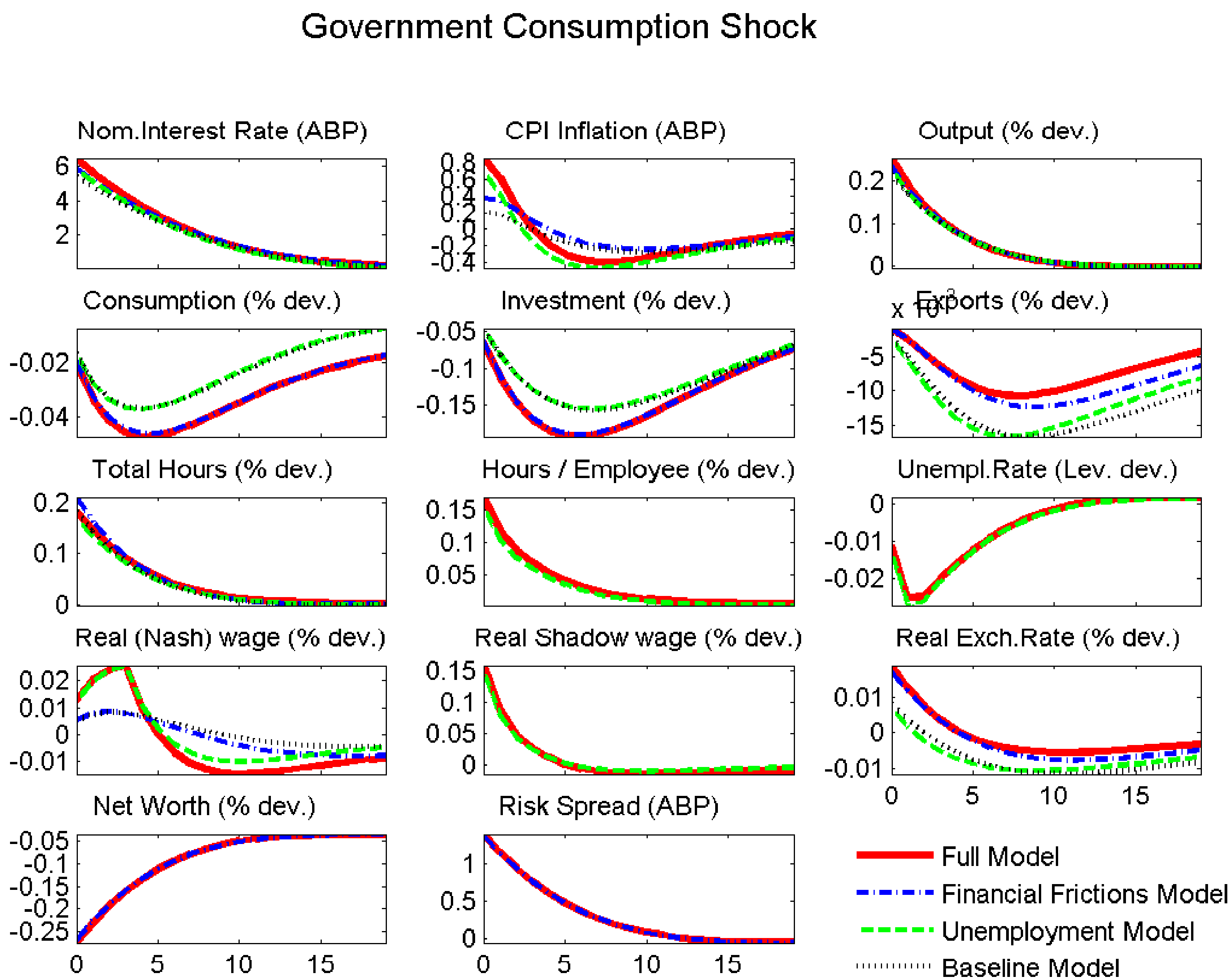


Consumption Preference Shock



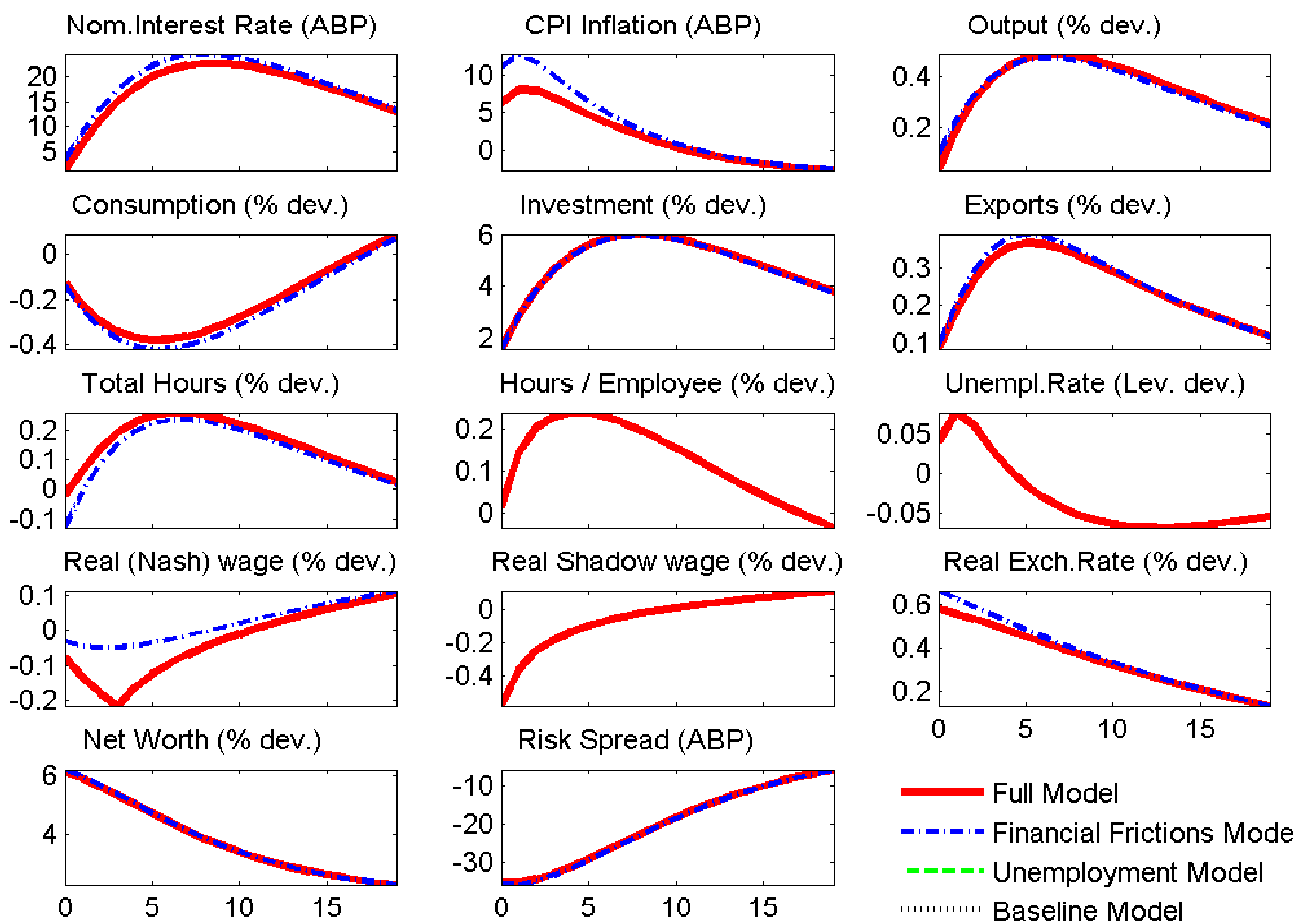


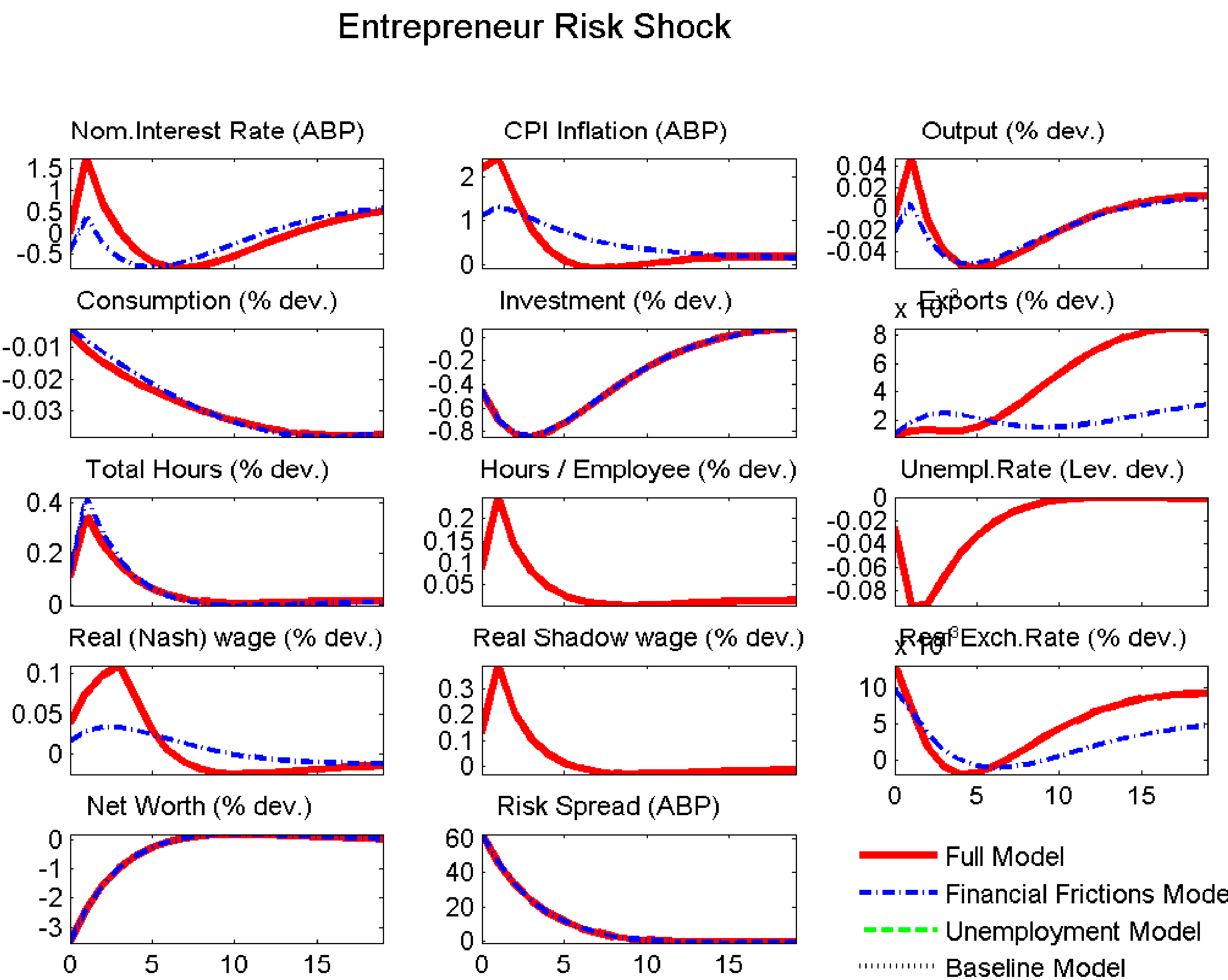
Labor Preference Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model



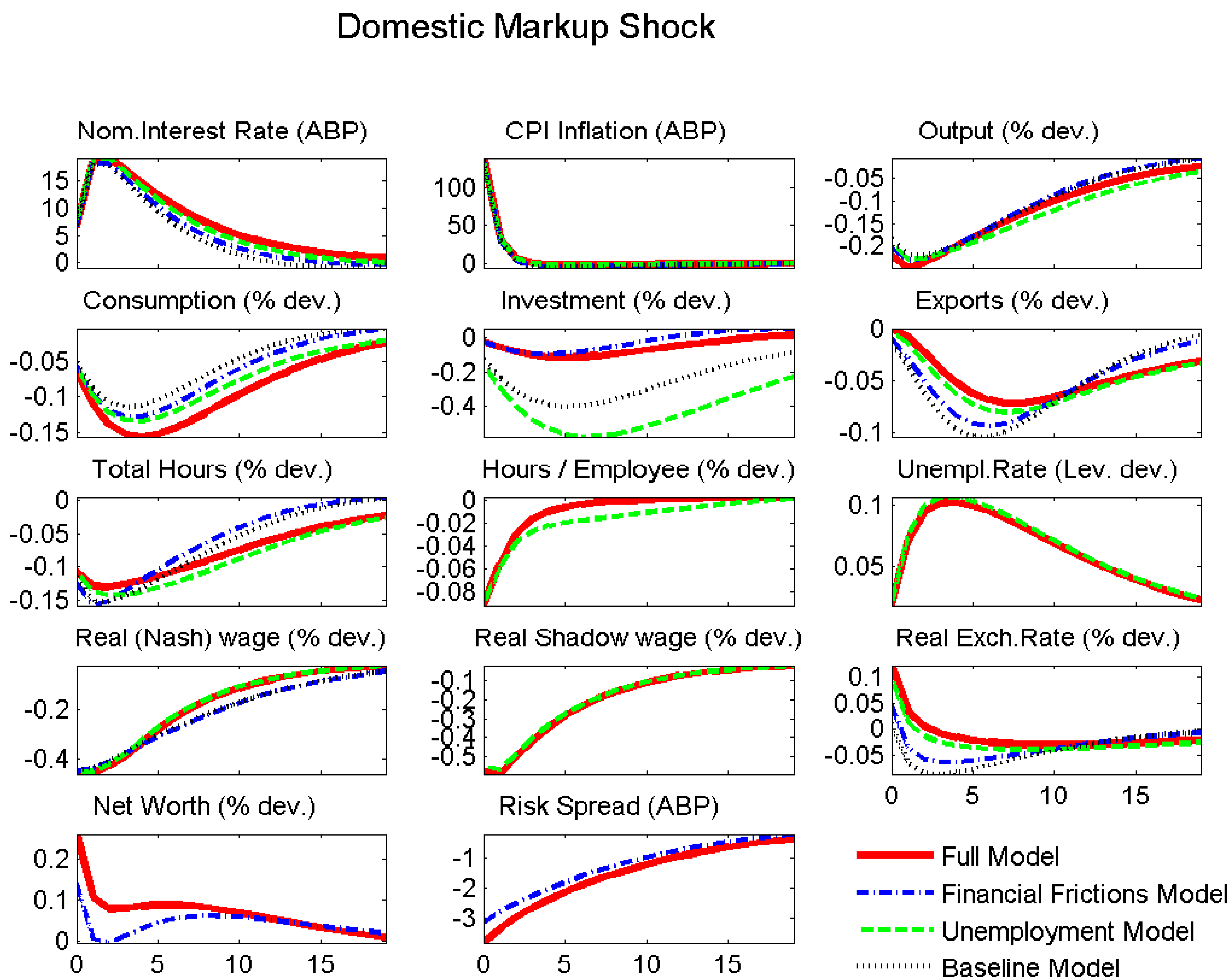
Government Consumption Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

Entrepreneur Survival Prob. Shock

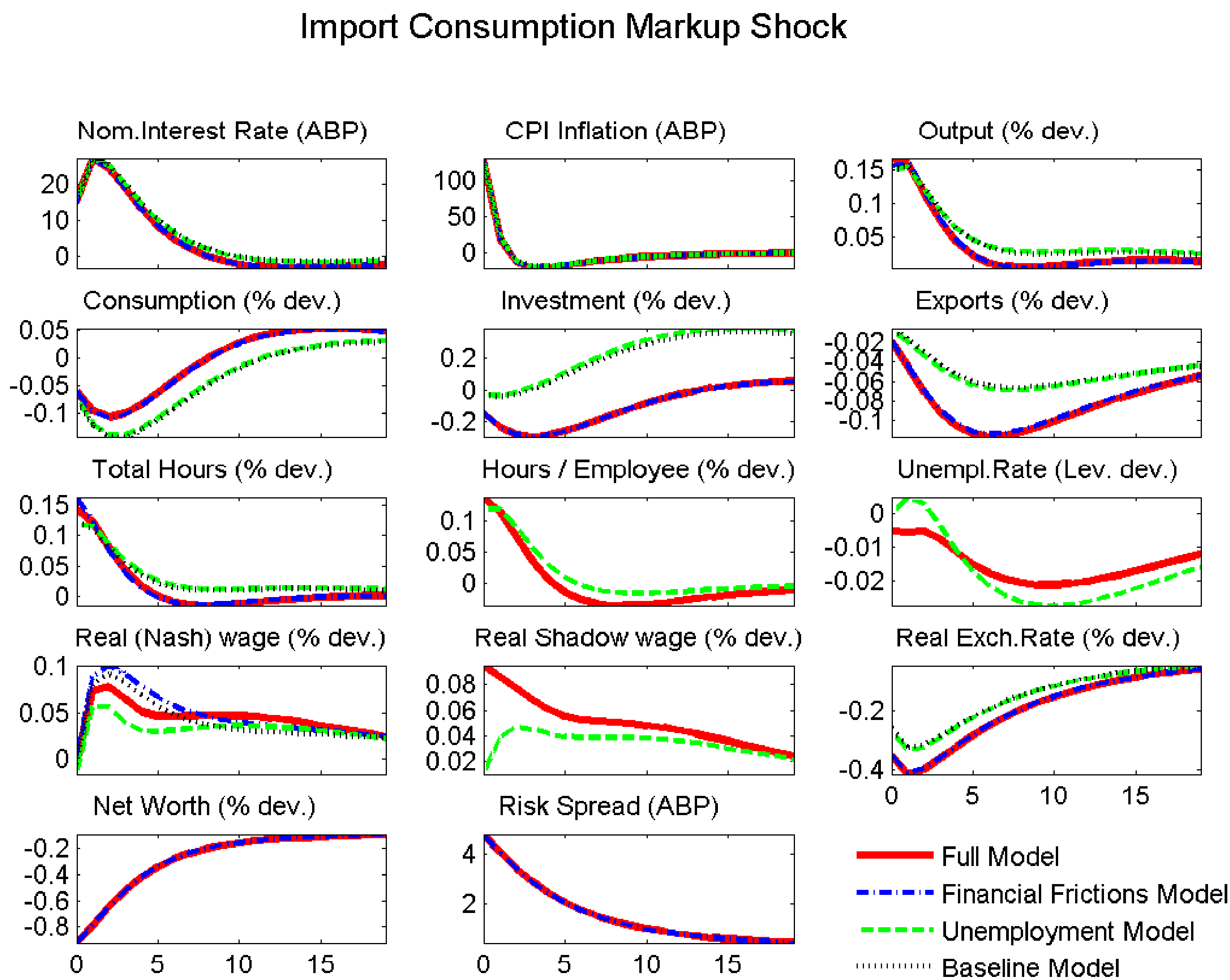




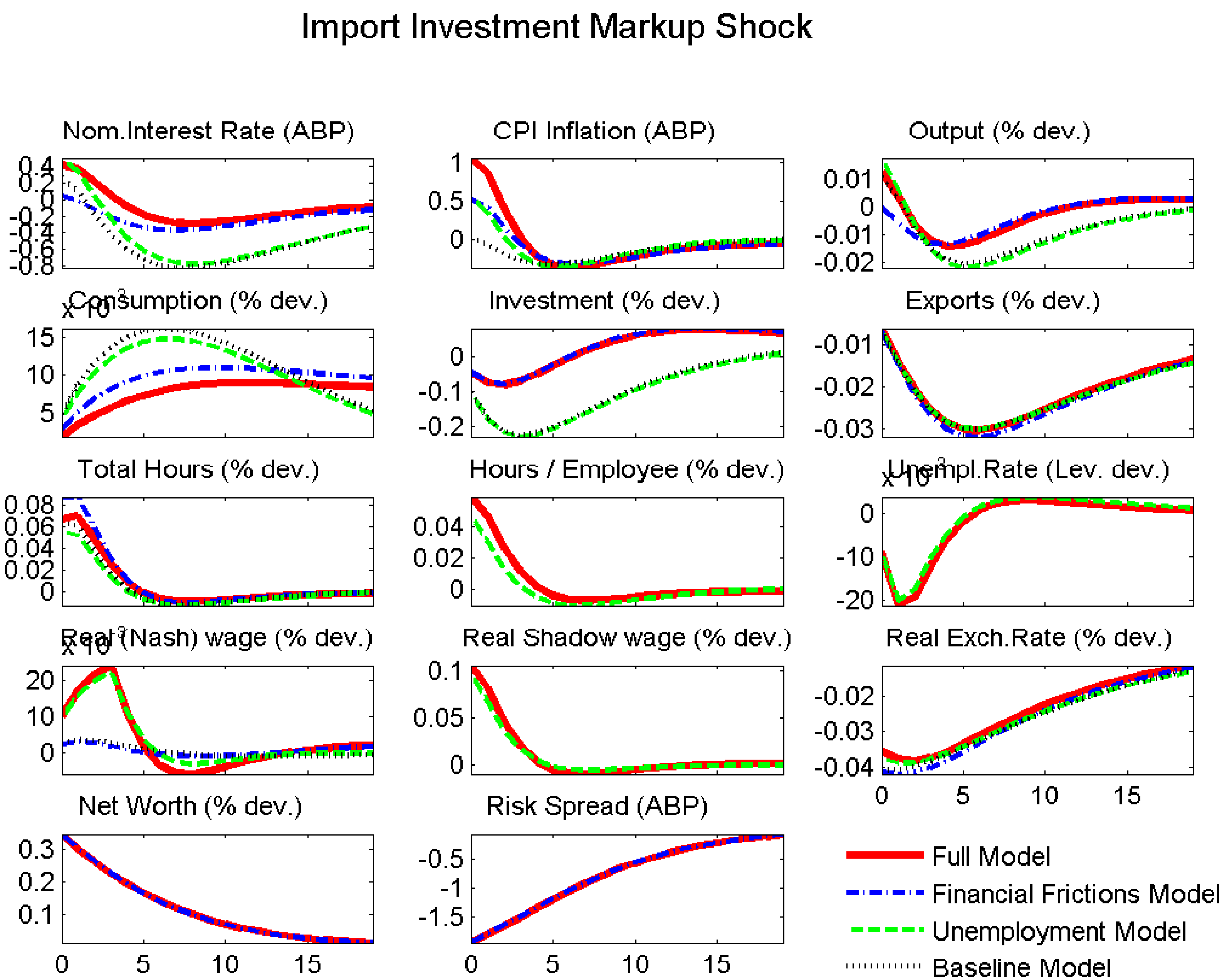
Entrepreneur Risk Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Mode
Unemployment Model
Baseline Model



Domestic Markup Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

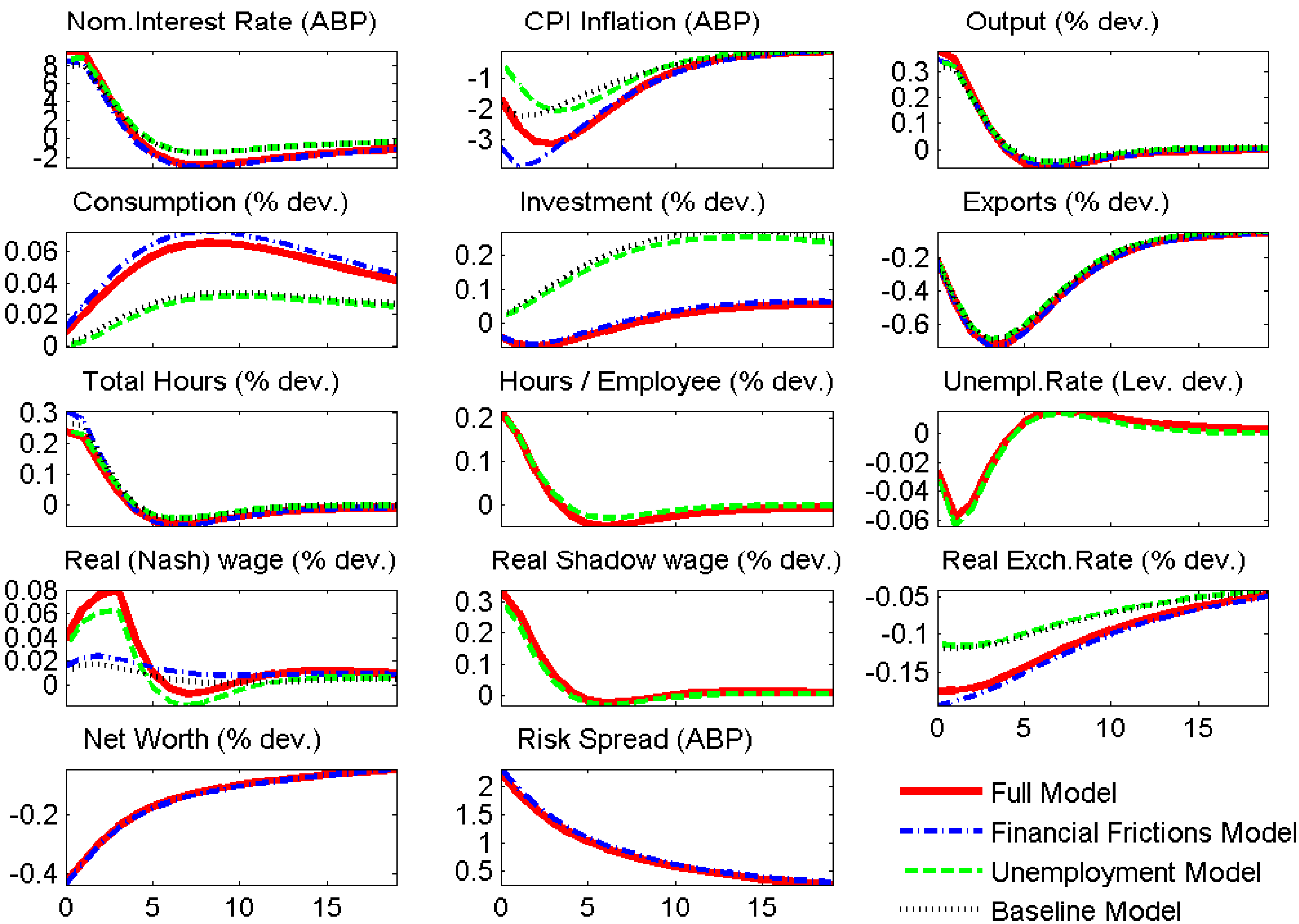


Import Consumption Markup Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

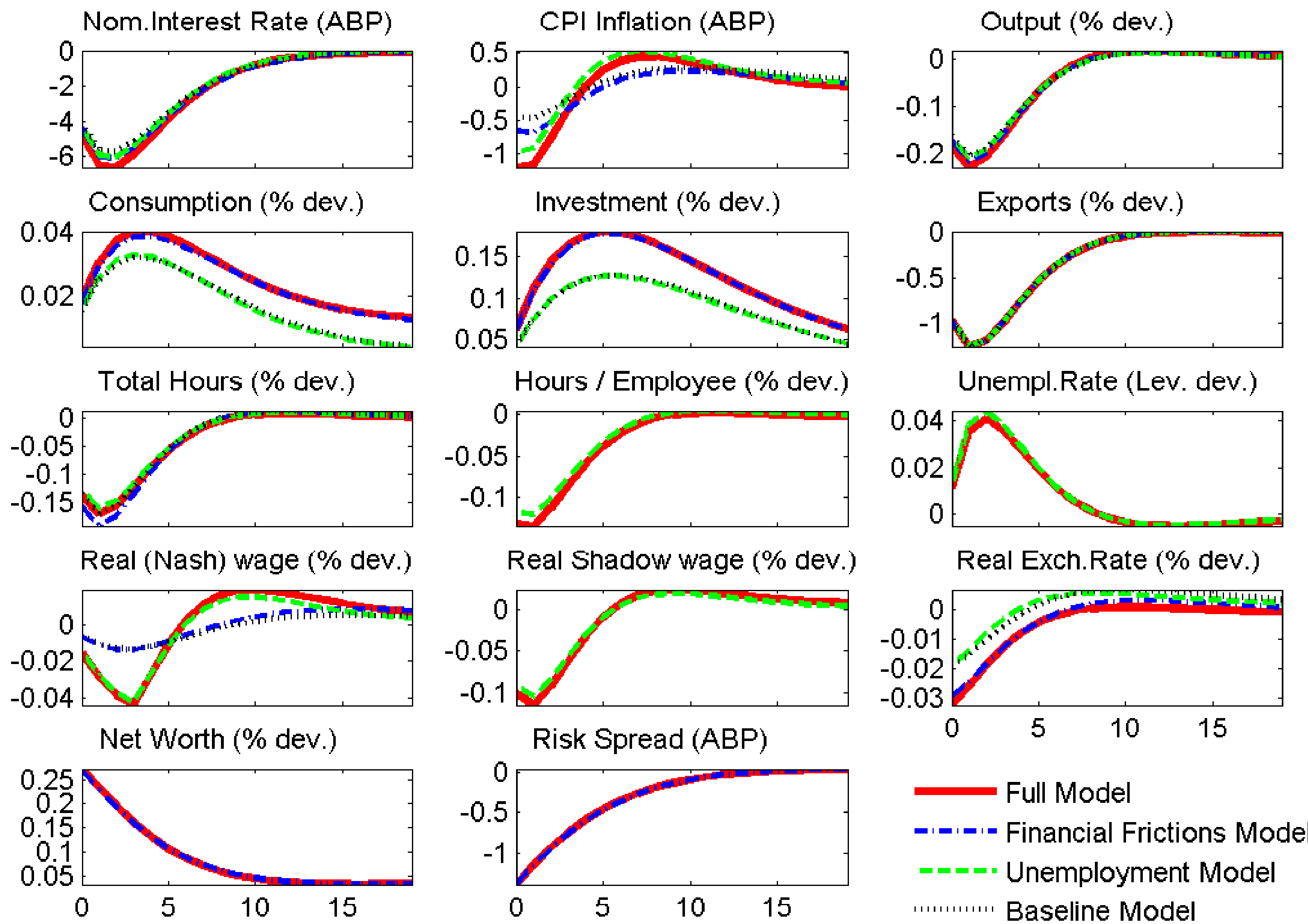


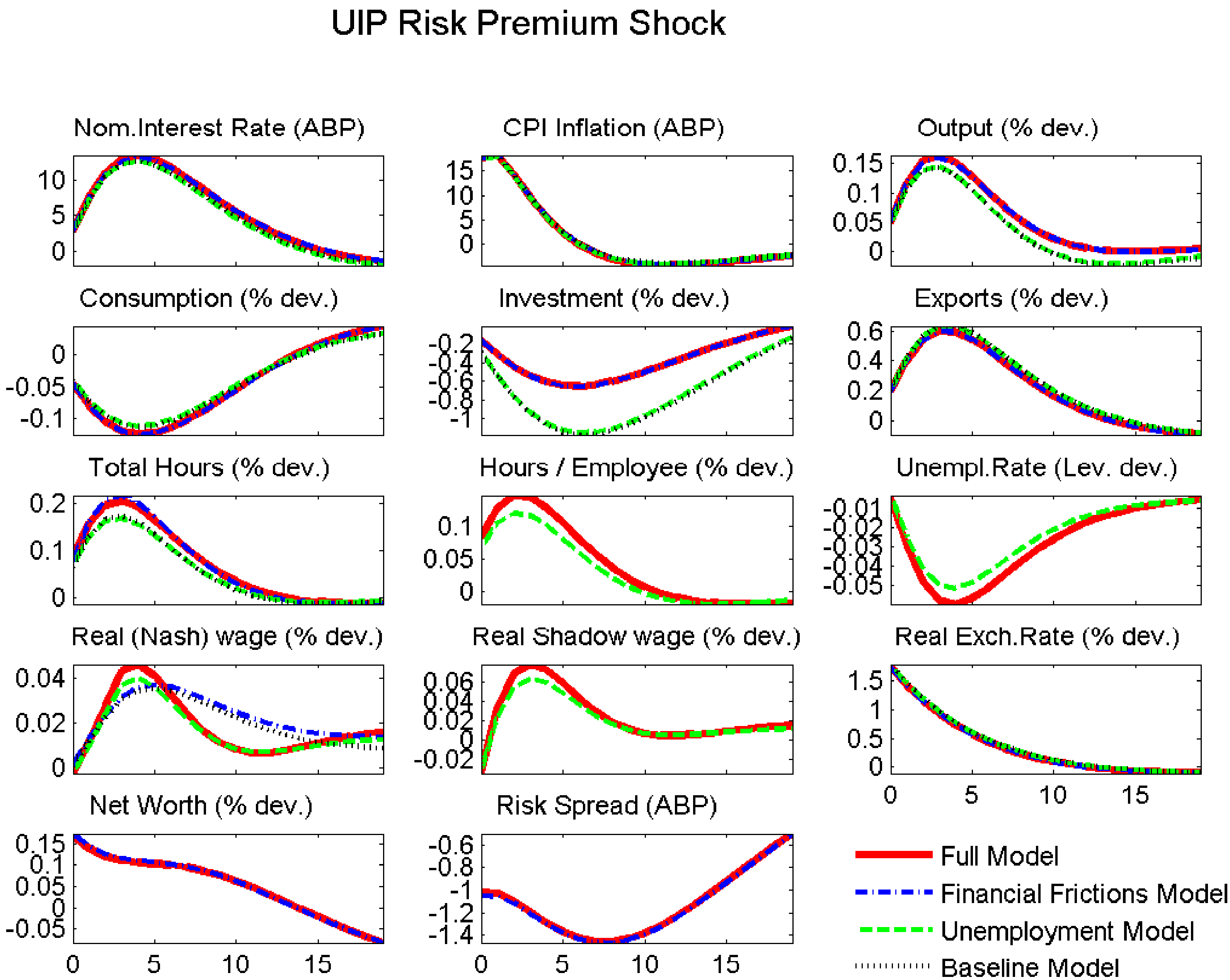
Import Investment Markup Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

Import Export Markup Shock

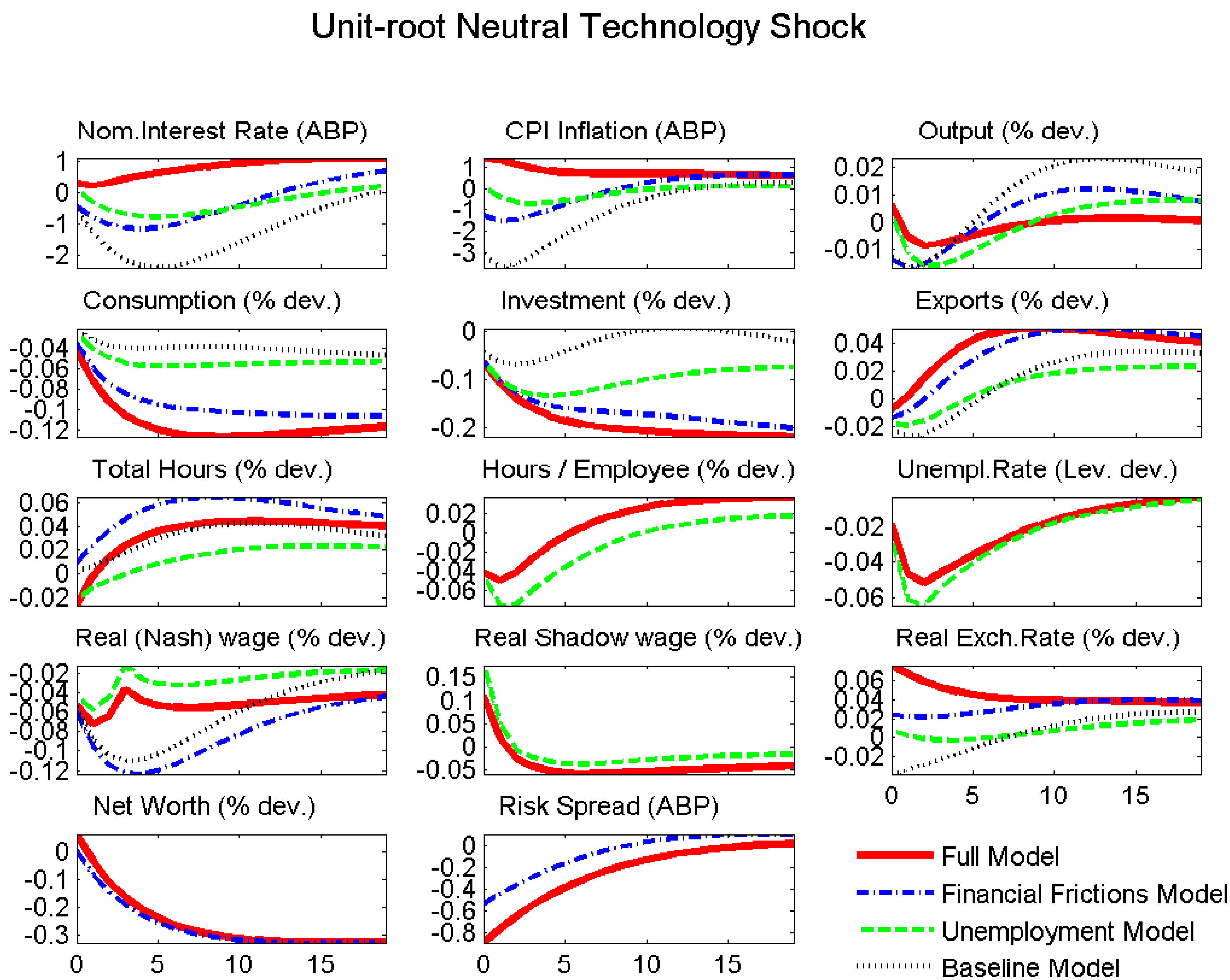


Export Markup Shock

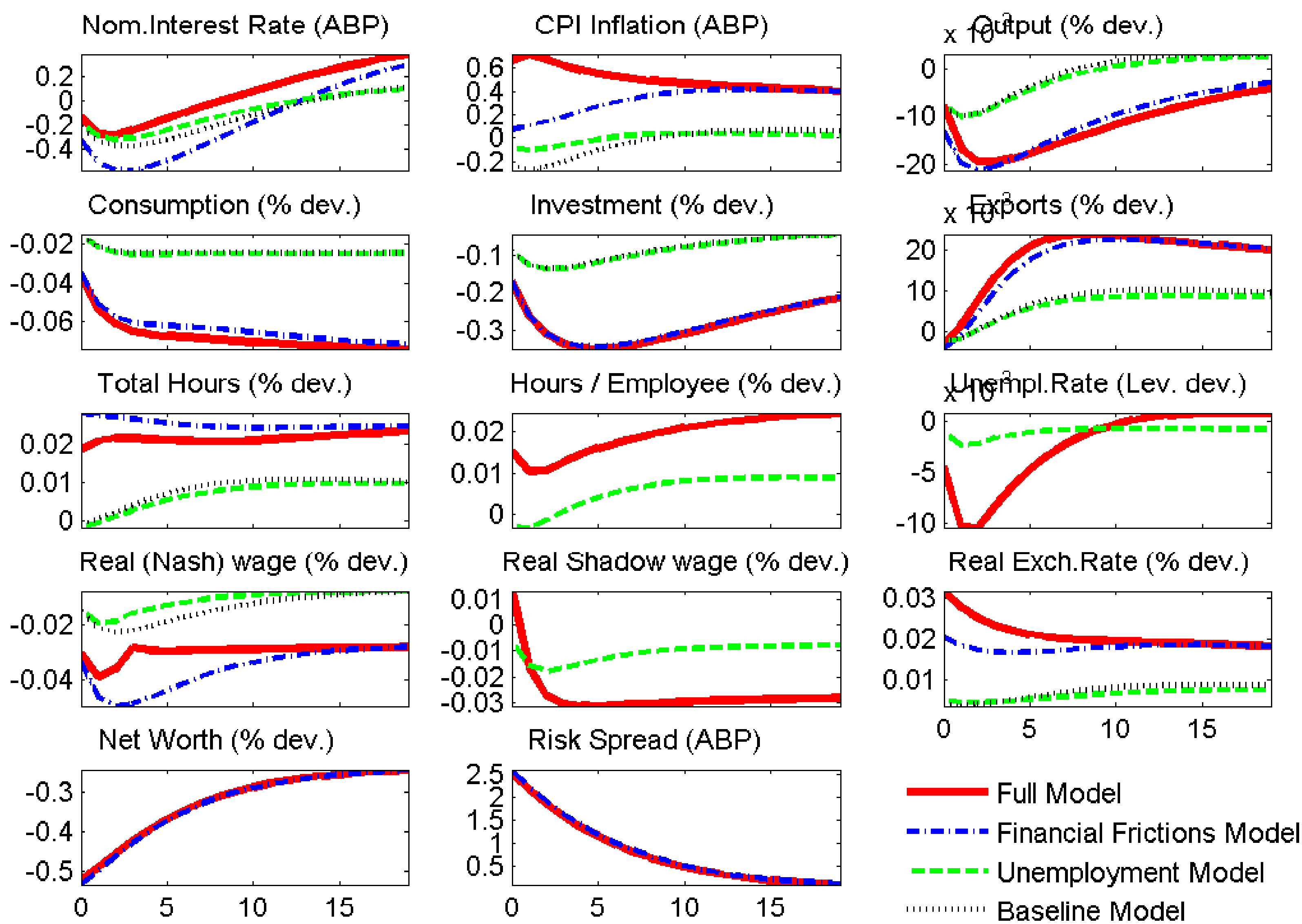


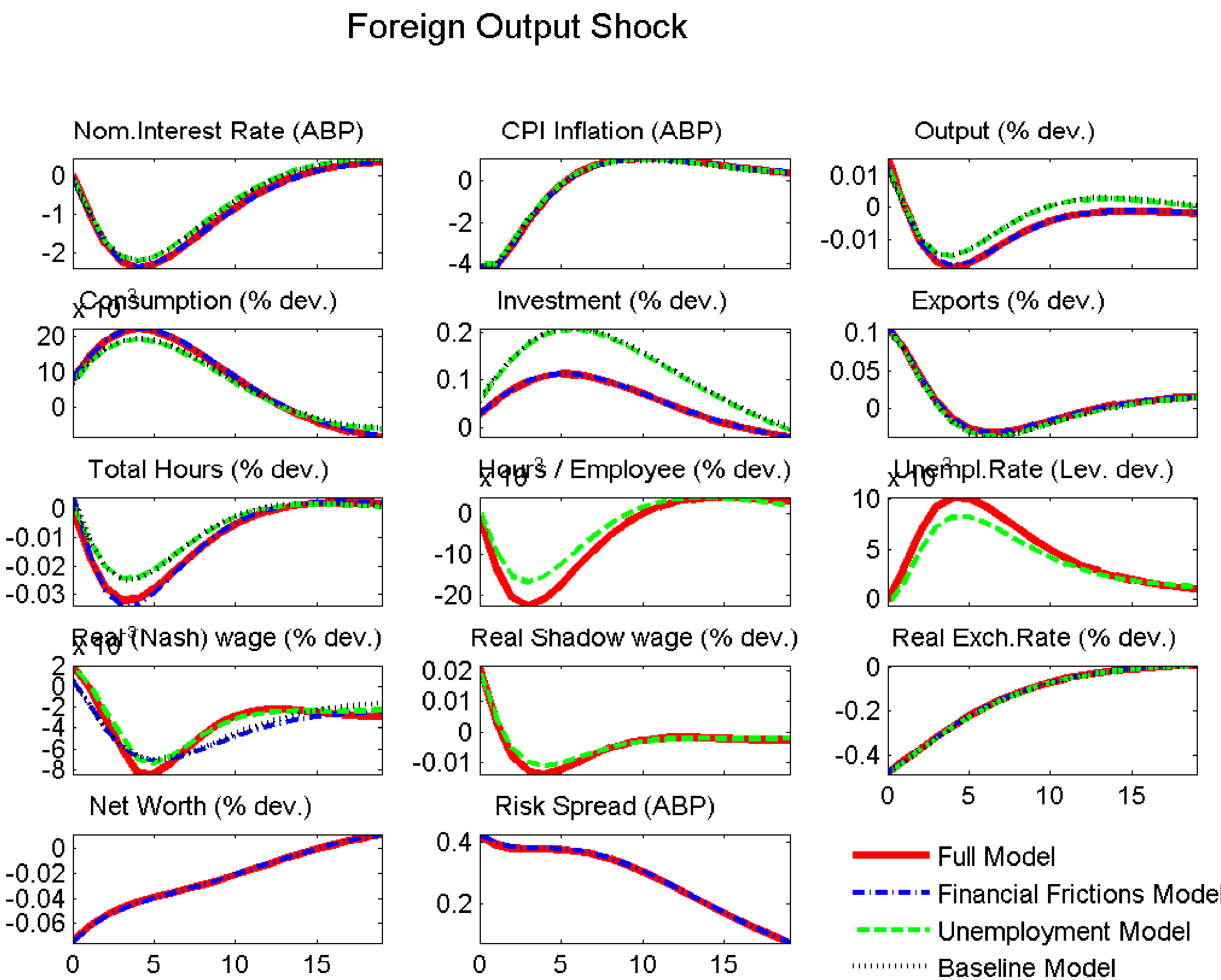


UIP Risk Premium Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model


Unit-root Neutral Technology Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

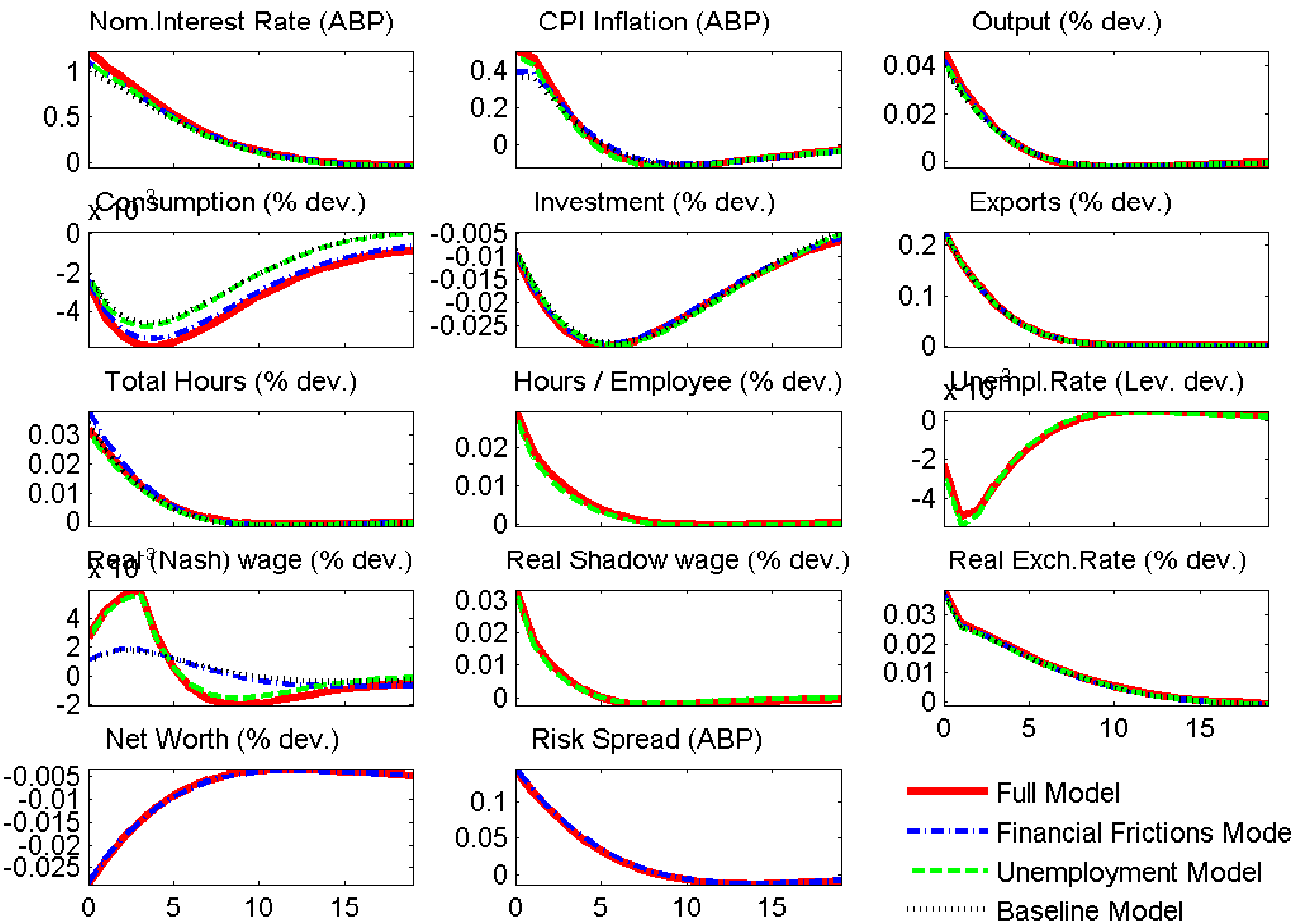
Unit-root Investment Technology Shock

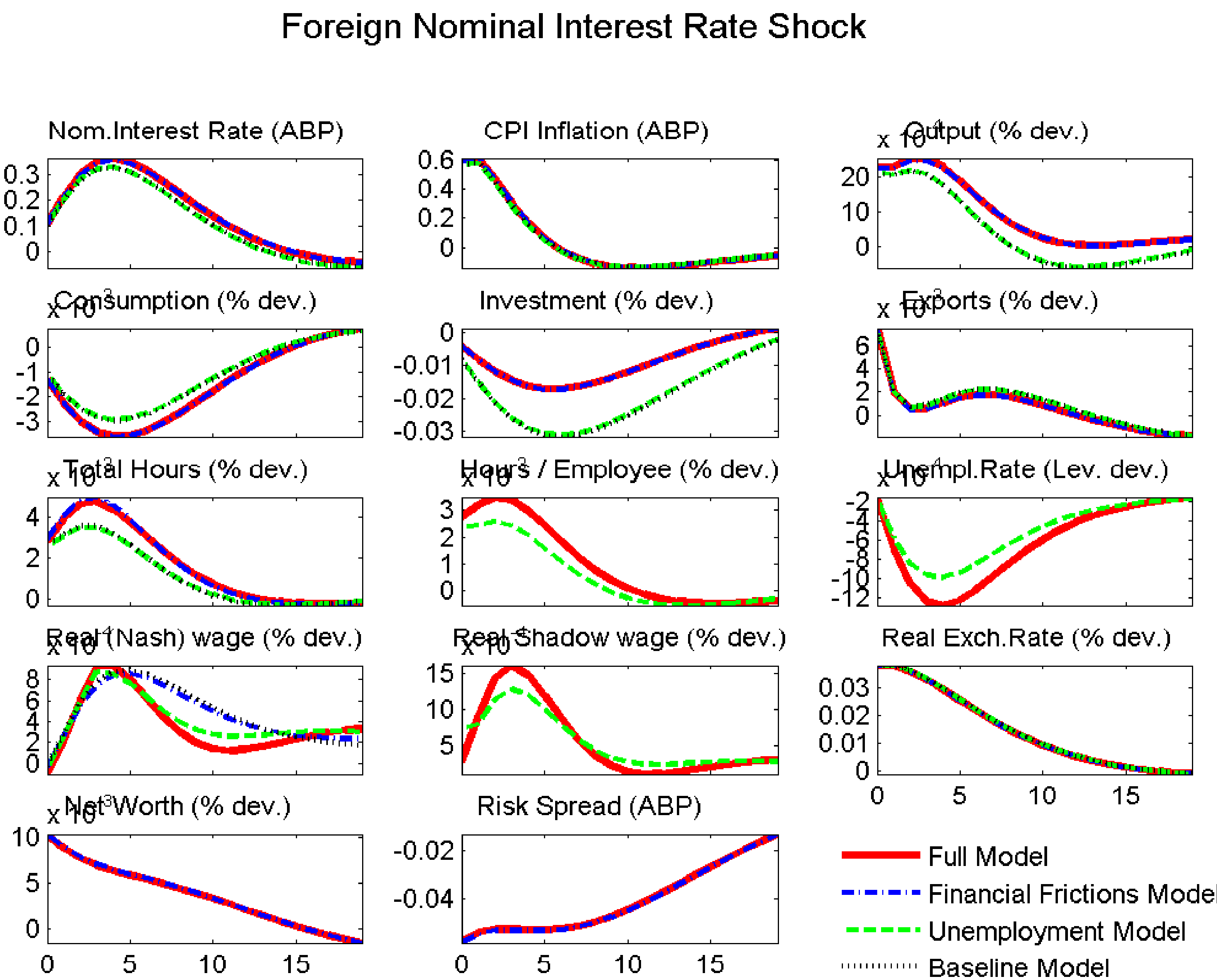




Foreign Output Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model

Foreign Inflation Shock




Foreign Nominal Interest Rate Shock
Nom.Interest Rate (ABP)
CPI Inflation (ABP)
Output (% dev.)
Consumption (% dev.)
Investment (% dev.)
Exports (% dev.)
Total Hours (% dev.)
Hours / Employee (% dev.)
Unempl.Rate (Lev. dev.)
Real (Nash) wage (% dev.)
Real Shadow wage (% dev.)
Real Exch.Rate (% dev.)
Net Worth (% dev.)
Risk Spread (ABP)
Full Model
Financial Frictions Model
Unemployment Model
Baseline Model