ARGEMmy: an intermediate DSGE model calibrated/ estimated for Argentina: 2 policy rules are often better than 1

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This presentation

- Part I:
  - Model description
- Part II:
  - Preliminary Calibration and Bayesian estimation of ARGEMmy
- Part III:
  - Optimal policy rules under commitment and perfect information

Part I

Model description

ARGEMmy

- SOE DSGE model for the analysis of the Argentine macroeconomy and CB policy:
  - Larger than MEP used presently
  - Microfounded
  - Smaller than ARGEM
    - which includes investment and manufactured exports

ARGEMmy: main features

- Adaptable to different monetary/exchange rate policies, including
  - Managed Exchange Rate (MER):
    - 2 policy feedback rules:
      - Usual Taylor-like rule + FX intervention rule
  - Floating XR (FER)
  - Pegged XR (PER)
  - Non-feedback, `passive’ policy rules
- Includes a simple banking system:
  - Uncovered interest parity is derived from bank profit maximization

ARGEMmy: main features (2)

- Growth thru a permanent productivity shock,
- Risk premia for Banks and Gov
- Gov uses lump-sum taxes, foreign debt financing, CB quasi-fiscal surplus
- 3 Phillips equations:
  - Wages, domestic goods, imported goods

ARGEMmy: Market Structure

- Households, Domestic and Importing Firms:
  - Monopolistic Competitors with nominal rigidity
  - Set prices through:
    - Calvo-type nominal rigidity,
    - CEE-type full indexation for non-optimizers.
- Domestic sector firms:
  - price takers in factor/loan markets
- Export sector firms and banks:
  - perfectly competitive (no entry).

ARGEMmy: Exogenous shocks

- 14 shocks:
- 7 Domestic Shocks:
  - Productivity (Temporary and Permanent),
  - Consumption demand, Labor supply , Harvest,
  - Fraction of loan financed domestic firm cost bill,
  - Gov. Expenditures.
- 6 RW shocks
  - Productivity (Permanent),
  - Terms of Trade, Import goods inflation,
  - Risk free rate, Bank and Government risk premia.
- 1 Policy shock
  - Rate of nominal depreciation.

Households: Utility

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ z^C_{t+j} \log[C_{t+j}(h) - \xi C_{t+j-1}(h)] + \left[\overline{h} - \eta z^H_{t+j} \frac{h_{t+j}(h)^{1+\chi}}{1+\chi} \right] \right\},$$

- They decide on:
  - Consumption (C), Cash (M^0), Deposits (D), Wage (W)
- ξ: habit parameter
- z^C : Consumption shock
- z^H : Labor shock
- C is a CES bundle of Domestic and Imported bundles
- No Money in Utility

Households: Budget constraint

$$\frac{M_t^0(h)}{P_t}+\frac{D_t(h)}{P_t}=\frac{\Pi_t(h)}{P_t}+\frac{W_t(h)}{P_t}h_t(h)-\frac{T_t(h)}{P_t}+\frac{\Upsilon_t(h)}{P_t}$$
$$+\frac{M_{t-1}^0(h)}{P_t}+(1+i_{t-1})\frac{D_{t-1}(h)}{P_t}-\left[1+\tau_M\left(\frac{M_t^0(h)/P_t}{p_t^C C_t(h)}\right)\right]p_t^C C_t(h)$$

- M^0 , D: household Cash, Deposits
- T : lump-sum taxes
- Y : state-contingent income/cost
- τ_M(cash/absorption) is transactions cost function:

$$\tau_M(\varpi_t)\equiv a_M\varpi_t+\varpi_t^{-b_M}+c_M,\qquad a_M,b_M>0.$$

Domestic firms:

- Use bundles of labor (h), imported goods (N^D) to produce Q:

$$Q_t(i) = \epsilon_t (z_t h_t(i))^{b^D} N_t^D(i)^{1-b^D}$$

- Demand bank loans to finance stochastic fraction ς_t of their cost:

$$\min_{h_t(i), N_t^D(i)} \{(1+\varsigma_t i_{t-1}^L)[W_t h_t(i) + P_t^N N_t^D(i)]\}$$

- Productivity shocks:
  - ϵ_t : transitory (stationary)
  - z_t : permanent (unit root)

Growth

- permanent productivity shock in SOE: z_t
- permanent productivity shock in RW: z_t^{**}
- relative productivity: $z_t^\circ \equiv z_t^{**}/z_t$
- productivity growth rates:

$$\widehat{\mu}_t^z \equiv z_t/z_{t-1}, \qquad \widehat{\mu}_{t-1}^{z**} \equiv z_t^{**}/z_t^{**}$$

  - First: endogenous. Second: exogenous AR(1)
- SS productivity levels and growth rates in RW and SOE are equal:

$$z^\circ = 1, \text{ and } \mu^{z**} = \mu^z$$

Growth (2)

- Cointegration between RW and SOE permanent productivity shocks:

- RW: $$\widehat{\mu}_t^{z**} = \rho^{z**}\widehat{\mu}_{t-1}^{z**} + \varepsilon_t^{z**}$$

- SOE: $$\widehat{\mu}_t^{z} = \rho^{z}\widehat{\mu}_{t-1}^{z} + (1-\rho^{z})\widehat{\mu}_{t-1}^{z**} + \alpha_z \widehat{\overset{\circ}{z}}_{t-1} + \varepsilon_t^{z},$$

Banks

- Generate deposits D and loans L using domestic goods
- Cost function: $C_{t+1}^{B} = \frac{1}{2} b^{B} \left(\frac{L_t}{z_t P_t} \right)^2 \qquad (b^B > 0)$
- Obtain funds abroad: B_t^{*B}
- Invest in CB bonds: B_t^{CB}
- Balance sheet: $L_t + B_t^{CB} = D_t + S_t B_t^{*B}.$
- Bear XR risk since they repay foreign funds a period later
- Interest rate on funds borrowed abroad:

$$1 + i_t^B = (1 + i_t^{**}) \phi_t^{**B} \left[1 + p_B \left(\frac{S_t B_t^{*B}}{P_t Y_t} \right) \right],$$

Banks (2)

- Endogenous risk premium:

$$p_B(e_t b_t^{*B}/y_t) \equiv \frac{\alpha_1^B}{1-\alpha_2^B e_t b_t^{*B}/y_t}, \qquad \alpha_1^B > 0, \alpha_2^B > 0.$$

- FOCs for profit max gives
  - 1) loan supply as linear function of lending margin, and
  - 2) the model's UIP :

$$\frac{L_t^S}{z_t P_t} = \frac{1}{b^B}(i_t^L - i_t)$$

$$i_t = E_t \delta_{t+1} \left\{ (1+i_t^{**}) \phi_t^{**B} \left[1 + \varphi_B \left(\frac{S_t B_t^{*B}}{P_t Y_t} \right) \right] - 1 \right\},$$

$$\boxed{(\varphi_B(a) \equiv p_B(a) + a p_B'(a) = p_B(a)[1+\varepsilon_B(a)])}$$

Central Bank

- Passes quasi-fiscal surplus to Gov., so Balance sheet always holds:

$$M_t^0 + B_t^{CB} = S_t R_t^{*CB}$$

- Suite of possible monetary policies:
  - Managed XR (MER):
    - simultaneous intervention in CB bond ("money") and FX markets
  - Floating XR (FER)
  - Pegged XR (PER)

Central Bank (2)

- Balance sheet: $$M_t^0 + B_t^{CB} = S_t R_t^{*CB}$$
- With stationary variables:

$$b_t^{CB} = e_t r_t^{*CB} - m_t^0$$

Constant under PER regime

Constant under FER regime

- Both vary under MER regime, reflecting
  - Reserve Backing of CB liabilities (0 CB new worth)
  - Changing MRER (e)
  - Sterilization.
  - Money market equilibrium

$$m_t^0 = \left[\frac{b_M}{a_M + 1 - \frac{1}{1+i_t}} \right]^{\frac{1}{1+b_M}} p_t^C c_t$$

Monetary/XR policy (MER)

- I) CB uses 2 feedback rules
- Both can reflect inflation, output, and trade balance concerns
- 1) Interest rate:

$$1 + i_t = \left(\frac{\mu^{z**}\pi}{\beta}\right)^{1-h_0}(1 + i_{t-1})^{h_0}\left(\frac{\widetilde{\pi}_t^C}{\widetilde{\pi}_t^T}\right)^{h_1}\left(\frac{y_t}{y_t^T}\right)^{h_2}\left(\frac{e_t tb_t/y_t}{\gamma^{TBT}}\right)^{h_3}$$

- 2) Rate of nominal depreciation:

$$\delta_t = \left(\frac{\pi^T}{\pi^{**N}}\right)^{1-k_0}(\delta_{t-1})^{k_0}\left(\frac{\widetilde{\pi}_t^C}{\widetilde{\pi}_t^T}\right)^{k_1}\left(\frac{y_t}{y^T}\right)^{k_2}\left(\frac{e_t tb_t/y_t}{\gamma^{TBT}}\right)^{k_3}\left(\frac{e_t r_t^{*CB}/y_t}{\gamma^{CBT}}\right)^{k_5}\exp(\varepsilon_t^{\delta}).$$

Monetary/XR policy (2)

- II) Any or both can be substituted for non-feedback AR(1) rule:
- 1) Instead of interest rate rule:

$$b_t^{CB} = (b_{t-1}^{CB})^{\rho^\delta} (b^{CB})^{1-\rho^\delta} \exp(\varepsilon_t^{b^{CB}})$$

- or:

$$e_t r_t^{CB} = (e_{t-1} r_{t-1}^{CB})^{\rho^{r^{CB}}} (\gamma^{CBT} y)^{1-\rho^{r^{CB}}} \exp(\varepsilon_t^{r^{CB}})$$

- 2) For rate of nominal depreciation:

$$\delta_t = (\delta_{t-1})^{\rho^\delta} (\delta^T)^{1-\rho^\delta} \exp(\varepsilon_t^\delta)$$

- or, if not used above:

$$e_t r_t^{CB} = (e_{t-1} r_{t-1}^{CB})^{\rho^{r^{CB}}} (\gamma^{CBT} y)^{1-\rho^{r^{CB}}} \exp(\varepsilon_t^{r^{CB}})$$

Monetary/XR policy (FER)

- II) Floating XR:
- 1) Interest rate feedback rule:

$$1 + i_t = \left(\frac{\mu^{z**}\pi}{\beta}\right)^{1-h_0} (1 + i_{t-1})^{h_0} \left(\frac{\tilde{\pi}_t^C}{\tilde{\pi}_t^T}\right)^{h_1} \left(\frac{y_t}{y_t^T}\right)^{h_2} \left(\frac{e_t tb_t / y_t}{\gamma^{TBT}}\right)^{h_3}$$

- 2) Instead of nominal depreciation rule, CB keeps reserves at SS level: $r^{*CB} = r^0$
  - where r^0 is a parameter (1 equation and 1 variable less)

Monetary/XR policy (PER)

- III) Pegged XR:
  - 1) Instead of interest rate rule the CB keeps bonds at SS level: $b^{CB} = b^0$
    - where b^0 is a parameter (1 equation and 1 variable less)
  - 2) Nominal depreciation feedback rule:

$$\delta_t = \left(\frac{\pi^T}{\pi^{**N}}\right)^{1-k_0} (\delta_{t-1})^{k_0} \left(\frac{\widetilde{\pi}_t^C}{\widetilde{\pi}_t^T}\right)^{k_1} \left(\frac{y_t}{y^T}\right)^{k_2} \left(\frac{e_t tb_t / y_t}{\gamma^{TBT}}\right)^{k_3} \left(\frac{e_t r_t^{*CB} / y_t}{\gamma^{CBT}}\right)^{k_5} \exp(\varepsilon_t^\delta)$$

Part 2

Preliminary Calibration and Bayesian estimation of ARGEMmy

Imposed Ratios, parameters, endogenous Variables

- 8 Great ratios:

$$\gamma^{CBT} = \frac{er^{*CB}}{y} = 0.13, \ \gamma^{GT} = \frac{eb^{*G}}{y} = 0.2, \ \frac{eb^{*B}}{y} = 0.0658,$$

$$\frac{\ell}{y} = 0.23, \ \frac{g}{y} = 0.16, \ \frac{p^N N}{y} = 0.22, \ \frac{\tau_M c}{c} = 0.001, \ \frac{m^0}{y} = 0.08$$

- 8 Parameter values:

$$\beta = 0.999, \quad a_D = 0.8610526, \quad \chi = 0.7,$$

$$\varepsilon_G = 0.833397207, \quad \varepsilon_B = 1.15745156, \quad \varepsilon_M = 0.85$$

$$\rho^{z^C} = 0.85, \quad \rho^{r^{*CB}} = 0.1,$$

- 5 Endogenous variables:

$$y = 585.5, \quad 1 + i^L = 1.12^{0.25}, \quad 1 + i^G = 1.07^{0.25}, \quad (\pi^T)^4 = 1.065$$

Estimated parameters, Endogenous parameters/ratios

- 36 Estimated parameters (+ 11 or 5 for policy):

$$\theta^N, \quad \psi, \quad \theta^C, \quad b^D, \quad b^A, \quad \xi, \quad \alpha_D, \quad \alpha_N, \quad \alpha_W, \quad \alpha_z, \quad \rho^z,$$
$$\rho^{z^H}, \quad \rho^{\epsilon}, \quad \rho^{\varsigma}, \quad \rho^{z^A}, \quad \rho^{p^*}, \quad \rho^{\pi^{*N}}, \quad \rho^{i^*}, \quad \rho^{\phi^{*B}}, \quad \rho^{\phi^{*G}}, \quad \rho^{\mu^{z*}}, \quad \rho^{g},$$
$$\sigma^{z^H}, \quad \sigma^{\epsilon}, \quad \sigma^{\varsigma}, \quad \sigma^{z^A}, \quad \sigma^{p^*}, \quad \sigma^{\pi^{*N}}, \quad \sigma^{i^*}, \quad \sigma^{\phi^{*B}}, \quad \sigma^{\phi^{*G}}, \quad \sigma^{\mu^{z*}}, \quad \sigma^{g},$$
$$\sigma^{z^C}, \quad \sigma^{\varepsilon^z}, \quad \sigma^{\varepsilon^\delta},$$

- 28 Endogenous parameters and great ratios:

$$\theta, \quad \eta, \quad b^B, \quad \alpha_1^G, \quad \alpha_2^G, \quad \alpha_2^B \quad \alpha_1^B, \quad a_M, \quad b_M, \quad c_M,$$
$$\frac{q}{y}, \quad \frac{p^C c}{y}, \quad \frac{\overline{w}h}{y}, \quad \frac{etb}{y}, \quad \frac{(b^A ep^{**})^{\frac{1}{1-b^A}}/b^A}{y}, \quad \varpi \equiv \frac{m^0}{p^C c}, \quad \frac{q^{DX}}{y}, \quad \frac{n^D}{n}.$$

Individual parameter ranges for stability and determinacy
(keeping rest at baseline values)

- Interest rate rule (10 is max value tested):
  - h_0 lagged i — 0 to 10
  - h_1 consumption inflation — 0 to 6
  - h_2 output — 0 to 10
  - h_3 trade balance/GDP — -10 to –0.5
  - h_4 lagged trade balance/GDP — -0.5 to 0.5
- Depreciation rate rule:
  - k_0 lagged δ — 0 to 10
  - k_1 consumption inflation — -10 to 10
  - k_2 output — -10 to 10
  - k_3 trade balance/GDP — -10 to 10
  - k_4 lagged trade balance/GDP — -10 to 3
  - k_5 CB int. Res./GDP — outside of –0.1 to 0.1

Preliminary Bayesian estimation using Dynare

- Estimated 2 versions of model:
  - 2 policy feedback rules version
  - 1 feedback rule (for δ) and AR(1) (for er^{*CB})
- For the post-Convertibility period
  - 2002:3-2007.4 (22 observations)
- 10 Observable variables:
  - 6 rates of growth of Y, C+I, G, N, M_o, D,
  - MRER: e^C,
  - 3 inflation rates: π^D, π^N, π^W,

Comparison of 2 models:

- Log marginal data density was significantly greater for the 1 feedback rule model (for rate of nominal depreciation)
- Hence, Argentina's monetary policy in this period appears to be better characterized by a PER than a MER regime.
- Below I show results for the 1 feedback rule model:
  - Active feedback in the FX market
  - AR(1) for international reserves

Preliminary estimation results

Parameters

	prior mean	post. mean	confidence interval		prior	pstdev
k_0	0.20	-0.1657	-0.4853	0.0736	norm	2.000
k_1	-2.50	-1.6218	-2.9543	-0.6590	norm	4.500
k_2	-0.50	-0.3903	-0.6251	-0.0985	norm	2.000
k_3	-0.60	0.0026	-0.0002	0.0056	norm	1.000
k_4	-0.50	0.0018	-0.0013	0.0047	norm	1.000
k_5	0.15	0.0300	-0.1125	0.2004	norm	1.000
thetaN	1.30	1.1754	1.1477	1.2011	norm	3.000
thetaC	0.70	0.9895	0.8425	1.1286	norm	2.000
bD	0.89	0.8903	0.8774	0.9044	beta	0.050
bA	0.06	0.0729	0.0634	0.0792	beta	0.030
xi	0.85	0.8091	0.7979	0.8230	beta	0.040
psi	12.00	7.3116	4.6179	10.2944	norm	24.000
alphaD	0.70	0.6133	0.5889	0.6299	beta	0.100
alphaN	0.70	0.5889	0.5617	0.6270	beta	0.100
alphaW	0.70	0.5808	0.5559	0.6042	beta	0.100
alpha_z	0.01	0.0151	0.0119	0.0175	beta	0.015
rhoz	0.85	0.6733	0.6587	0.6863	beta	0.100

Preliminary estimation results (2)

Parameters	prior mean	post. mean	confidence interval		prior	pstdev
rhozH	0.60	0.5283	0.5150	0.5501	beta	0.100
rhoEpsilon	0.70	0.6563	0.6322	0.6820	beta	0.100
rhoVarsigma	0.55	0.5951	0.5680	0.6298	beta	0.100
rhozA	0.20	0.2270	0.1854	0.2579	beta	0.100
rhopStar	0.83	0.8169	0.8006	0.8318	beta	0.100
rhoPiStarN	0.65	0.5981	0.5833	0.6140	beta	0.100
rhoiStar	0.72	0.7275	0.7080	0.7489	beta	0.100
rhoPhiStarB	0.50	0.2790	0.2608	0.2904	beta	0.080
rhoPhiStarG	0.25	0.2733	0.2580	0.2881	beta	0.080
rhoMuzStar	0.50	0.5226	0.5025	0.5383	beta	0.090
rhog	0.80	0.8196	0.7992	0.8377	beta	0.060
Standard deviation of shocks						
eps_zC	0.250	0.1445	0.0975	0.1886	invg	Inf
eps_zH	0.600	0.6980	0.6448	0.7835	invg	0.2
eps_epsilon	0.200	0.3412	0.2179	0.4669	invg	Inf
eps_varsigma	0.040	0.0845	0.0109	0.1573	invg	Inf
eps_zA	0.080	0.0643	0.0185	0.1196	invg	Inf
eps_pStar	0.150	0.2416	0.1629	0.3189	invg	Inf
eps_piStarN	0.100	0.0925	0.0471	0.1391	invg	Inf
eps_iStar	0.400	0.3690	0.2974	0.4250	invg	0.2
eps_phiStarB	0.150	0.1763	0.1299	0.2211	invg	Inf
eps_phiStarG	0.050	0.0553	0.0112	0.1213	invg	Inf
eps_muzStar	0.100	0.0656	0.0378	0.0918	invg	Inf
eps_g	0.100	0.1267	0.0951	0.1575	invg	Inf
eps_varepsilonz	0.080	0.0818	0.0600	0.1032	invg	Inf
eps_varepsilondelta	0.100	0.0519	0.0244	0.0801	invg	Inf

Problems encountered

- Difficulties with posterior maximization...
- All kinds of Error messages.
  - The compute=6 option proved to be useful:
    - instead of using Newton type optimization to get the posterior mode, it uses MH simulation to update both an initial point in the parameter space with high posterior density value and an initial diagonal covariance matrix.
    - And then does the usual MH to obtain the posterior.
- However, I can’t say I’m confident on the results. Identification problems are abundant. Much more work ahead.

Part 3

Optimal policy rules under commitment and perfect information

Motivation

- What can lin-quad optimal control theory say on whether it may be optimal to use 2 policy feedback rules?
- I put ARGEMmy within a lin-quad optimal control under commitment and full information framework
  - Svensson and Woodford (2002), Levine, McAdam, Pearlman (2007)
- I computed optimal feedback rules and CB losses for
  - Different CB styles
  - Different policy regimes: MER, FER, PER

Optimal Control framework

- ARGEMmy can be written as:

$$B^{00} d_t = A^{00} E_t d_{t+1} + C^{00} d_{t-1} + F^2 E_t u_{t+1} - F^1 u_t + F^0 u_{t-1} + J^{00} \widetilde{Z}_t - D^{00} \chi^d_{t+1},$$

$$\widetilde{Z}_{t+1} = \widetilde{\widetilde{M}} \widetilde{Z}_t + \widetilde{\chi}^Z_{t+1},$$

UIP has $E_t \delta_{t+1}$

- d : endogenous;
- u : controls (i_t and δ_t)
- Z : exogenous disturbances
- χ : shocks

Optimal Control framework (2)

- Eliminating lagged variables:

$$\widetilde{k}_{t+1} = \widehat{M}\widetilde{k}_t + \widehat{S}\widetilde{d}_t + \widehat{I}u_t + x^*_{t+1}$$

$$A^{00}E_t\widetilde{d}_{t+1} = B^{00}\widetilde{d}_t - \widehat{C}\widetilde{k}_t + F^1u_t - F^2E_tu_{t+1} + D^{00}x^d_{t+1}$$

  - where k has the state variables and d the non-predetermined non-control variables:

$$\widetilde{k}_t = \begin{bmatrix} \widetilde{Z}_t \\ \overline{k}_t \end{bmatrix}, \quad \overline{k}_t = \begin{bmatrix} \overline{u}_t \\ \overline{V}_t \\ \overline{Y}_t \end{bmatrix}, \quad \widetilde{d}_t = \begin{bmatrix} V_t \\ Y_t \end{bmatrix}$$

Optimal Control framework (3)

- More compactly:

$$\widetilde{A} E_t s_{t+1} = \widetilde{B} s_t + \widehat{x}^*_{t+1},$$

  - where :

$$s_t = \begin{bmatrix} \widetilde{k}_t \\ \widetilde{d}_t \\ u_t \end{bmatrix}, \quad \widetilde{A} = \begin{bmatrix} I & 0 & 0 \\ 0 & A^{00} & F^2 \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} \widehat{M} & \widehat{S} & \widehat{I} \\ -\widehat{C} & B^{00} & F^1 \end{bmatrix}.$$

Optimal Control framework (4)

- Ad hoc CB loss function:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} L_t,$$

$$L_t = \omega_\pi \left(\widehat{\tilde{\pi}}_t^C\right)^2 + \omega_y (\hat{y}_t)^2 + \omega_{tb}\left(\widehat{tbr}_t\right)^2 + \omega_i\left(\hat{i}_t\right)^2 + \omega_\delta\left(\hat{\delta}_t\right)^2$$
$$+ \omega_{\Delta i}\left(\Delta\hat{i}_t\right)^2 + \omega_{\Delta i}\left(\Delta\hat{\delta}_t\right)^2 + \omega_{\Delta e}(\Delta\hat{e}_t)^2,$$

- CB prefers to remain close to SS:
  - No ‘potential’ or ‘natural’ GDP
- There are weights for:
  - π, y, trade balance ratio (tbr),
  - the 2 instruments (e.g. 0 lower bound on i)
  - Changes in the 2 instruments and the MRER

Optimal Control framework (5)

- FOC:

$$\tilde{A}s_{t+1} = \tilde{B}s_t$$

$$\tilde{B}'\lambda_{t+1} = \beta^{-1}\tilde{A}'\lambda_t - \Omega s_t.$$

- Reintroducing the stochastic shocks:

$$\begin{bmatrix} \tilde{A} & 0 \\ 0 & \tilde{B}' \end{bmatrix} E_t \begin{bmatrix} s_{t+1} \\ \lambda_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{B} & 0 \\ -\Omega & \beta^{-1}\tilde{A}' \end{bmatrix} \begin{bmatrix} s_t \\ \lambda_t \end{bmatrix} + \begin{bmatrix} \hat{x}^*_{t+1} \\ 0 \end{bmatrix},$$

More explicitly:

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & A^{00} & F^{2} & 0 & 0 \\ 0 & 0 & 0 & \widehat{M}' & -\widehat{C}' \\ 0 & 0 & 0 & \widehat{S}' & B^{00\prime} \\ 0 & 0 & 0 & \widehat{I}' & F^{1\prime} \end{bmatrix} \begin{bmatrix} \widetilde{k}_{t+1} \\ E_t \widetilde{d}_{t+1} \\ E_t u_{t+1} \\ E_t \lambda^k_{t+1} \\ \lambda^d_t \end{bmatrix}$$

$$= \begin{bmatrix} \widehat{M} & \widehat{S} & \widehat{I} & 0 & 0 \\ -\widehat{C} & B^{00} & F^{1} & 0 & 0 \\ -\Omega_{kk} & -\Omega_{kd} & -\Omega_{ku} & \beta^{-1} I & 0 \\ -\Omega_{dk} & -\Omega_{dd} & -\Omega_{du} & 0 & \beta^{-1} A^{00\prime} \\ -\Omega_{uk} & -\Omega_{ud} & -\Omega_{u} & 0 & \beta^{-1} F^{2\prime} \end{bmatrix} \begin{bmatrix} \widetilde{k}_t \\ \widetilde{d}_t \\ u_t \\ \lambda^k_t \\ \lambda^d_{t-1} \end{bmatrix} + \begin{bmatrix} x^*_{t+1} \\ D^{00} x^d_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Rearranging:

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & F^2 & 0 & A^{00} \\ 0 & -\widehat{C}' & 0 & \widehat{M}' & 0 \\ 0 & B^{00\prime} & 0 & \widehat{S}' & 0 \\ 0 & F^{1\prime} & 0 & \widehat{I}' & 0 \end{bmatrix} \begin{bmatrix} \widetilde{k}_{t+1} \\ \lambda_t^d \\ E_t u_{t+1} \\ E_t \lambda_{t+1}^k \\ E_t \widetilde{d}_{t+1} \end{bmatrix}$$

Predetermined

$$= \begin{bmatrix} \widehat{M} & 0 & \widehat{I} & 0 & \widehat{S} \\ -\widehat{C} & 0 & F^1 & 0 & B^{00} \\ -\Omega_{kk} & 0 & -\Omega_{ku} & \beta^{-1} I & -\Omega_{kd} \\ -\Omega_{dk} & \beta^{-1} A^{00\prime} & -\Omega_{du} & 0 & -\Omega_{dd} \\ -\Omega_{uk} & \beta^{-1} F^{2\prime} & -\Omega_{u} & 0 & -\Omega_{ud} \end{bmatrix} \begin{bmatrix} \widetilde{k}_t \\ \lambda_{t-1}^d \\ u_t \\ \lambda_t^k \\ \widetilde{d}_t \end{bmatrix} + \begin{bmatrix} x_{t+1}^* \\ D^{00} x_{t+1}^d \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Optimal Control framework (6)

- The solution, using the generalized Schur (QZ) decomposition is:

$$
\begin{aligned}
\tilde{k}_{t+1} &= G_{kk}^{opt}\tilde{k}_t + G_{kl}^{opt}\lambda_{t-1}^d + x_{t+1}^* \\
\lambda_t^d &= G_{lk}^{opt}\tilde{k}_t + G_{ll}^{opt}\lambda_{t-1}^d; \\
\boxed{u_t} &\boxed{= K_{uk}^{opt}\tilde{k}_t + K_{ul}^{opt}\lambda_{t-1}^d} \\
\lambda_t^k &= K_{kk}^{opt}\tilde{k}_t + K_{kl}^{opt}\lambda_{t-1}^d \\
\tilde{d}_t &= K_{dk}^{opt}\tilde{k}_t + K_{dl}^{opt}\lambda_{t-1}^d + D^0 x_{t+1}^d.
\end{aligned}
$$

Optimal controls

Minimum Loss

- CB loss at t=0:

$$\mathcal{L}_0 = \widehat{k}_0' V \widehat{k}_0 + \frac{\beta}{1-\beta} trace(V_{kk}\Sigma)$$

$$= \widetilde{k}_0' V_{kk} \widetilde{k}_0 + \frac{\beta}{1-\beta} trace(V_{kk}\Sigma)$$

$$= trace\left(V_{kk}\left(\widetilde{k}_0 \widetilde{k}_0' + \frac{\beta}{1-\beta}\Sigma\right)\right),$$

- where V solves Lyapunov:

$$V = \Omega^K + \beta G^{opt\prime} V G^{opt} \qquad \widehat{k}_{t+1} = G^{opt}\widehat{k}_t + \begin{bmatrix} x_{t+1}^* \\ 0 \end{bmatrix}$$

Minimum Loss (2)

- Since there's 2 parts to loss I calculate Loss:
- 1) assuming at t=0 system is at SS:

$$\mathcal{L}_0 = \frac{\beta}{1-\beta} trace\left(V_{zz}\widetilde{\Sigma}\right)$$

- 2) assuming k_0 corresponds to an (artificial) stagflationary scenario where variables are +/-1% from nsSS:

$$\mathcal{L}_1 = trace\left(V_{kk}\widetilde{k}_0\widetilde{k}_0'\right) + \mathcal{L}_0$$

Losses for alternative CB styles and policy regimes

Weights on loss function for different Central Bank styles

	BASELINE				INFLATION AWARE			
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
$\omega_{\Delta i}$	0	0.5	0	0.5	0	0.5	0	0.5
$\omega_{\Delta \delta}$	0	0	0.5	0.5	0	0	0.5	0.5
$\omega_{\Delta e}$	0	0	0	0	0	0	0	0
ω_{y}	1	1	1	1	1	1	1	1
ω_{e}	0	0	0	0	0	0	0	0
ω_{tb}	1	1	1	1	1	1	1	1
ω_{π}	1	1	1	1	2	2	2	2
ω_{i}	0	0	0	0	0	0	0	0
ω_{δ}	0	0	0	0	0	0	0	0

Losses for alternative CB styles and policy regimes

Weights on loss function for different Central Bank styles

	OUTPUT AWARE				TRADE BALANCE AWARE			
	I	*J*	*K*	*L*	*M*	*N*	*O*	*P*
$\omega_{\Delta i}$	0	0.5	0	0.5	0	0.5	0	0.5
$\omega_{\Delta \delta}$	0	0	0.5	0.5	0	0	0.5	0.5
$\omega_{\Delta e}$	0	0	0	0	0	0	0	0
ω_{y}	2	2	2	2	1	1	1	1
ω_{e}	0	0	0	0	0	0	0	0
ω_{tb}	1	1	1	1	2	2	2	2
ω_{π}	1	1	1	1	1	1	1	1
ω_{i}	0	0	0	0	0	0	0	0
ω_{δ}	0	0	0	0	0	0	0	0

Losses for alternative CB styles and policy regimes

Weights on loss function for different Central Bank styles

	BASELINE				INFLATION AWARE			
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Loss0 (thousands)								
MER (2 policy rules)								
	10.37	**80.71**	**48.35**	**74.45**	**12.61**	**85.16**	**50.54**	**76.53**
FER (Float)								
	5122.2	5122.3	5129.4	5129.5	5160.0	5160.1	5163.9	5164.0
PER (Peg)								
	5017.2	*5017.3*	*5020.7*	*5020.8*	*5030.9*	*5031.0*	*5034.3*	*5034.5*
Loss1 (thousands)								
MER (2 policy rules)								
	220.37	**2261.1**	**1596.3**	**2217.0**	**271.41**	**2377.4**	**1667.8**	**2281.7**
FER (Float)								
	51032	*51032*	*51116*	*51116*	*51455*	*51455*	*51606*	*51606*
PER (Peg)								
	100510	100510	100680	100680	100810	100810	101070	101070

Losses for alternative CB styles and policy regimes

Weights on loss function for different Central Bank styles

	OUTPUT AWARE				TRADE BALANCE AWARE			
	I	*J*	*K*	*L*	*M*	*N*	*O*	*P*
Loss0 (thousands)								
MER (2 policy rules)								
	27.35	**85.36**	**49.24**	**76.19**	**10.38**	**62.81**	**48.18**	**65.09**
FER (Float)								
	5123.7	5123.8	5130.7	5130.9	10104	10104	10130	10130
PER (Peg)								
	5017.8	*5017.9*	*5021.2*	*5021.4*	*9981.1*	*9981.3*	*9988.5*	*9988.6*
Loss1 (thousands)								
MER (2 policy rules)								
	579.42	**2364.0**	**1616.8**	**2258.2**	**220.42**	**1834.0**	**1591.9**	**1955.2**
FER (Float)								
	51048	*51048*	*51133*	*51133*	*100460*	*100460*	*100540*	*100540*
PER (Peg)								
	100520	100520	100690	100690	199800	199800	199900	199900

Other CB styles

- More extreme awareness:
  - Replace 2 by 5 for weights in each case
- Resistence to instrument departures from NSS
- Resistence to change in e

- Basically same results:
  - 2 policy rules gets lower Loss0 and Loss1

Sensitivity to parameters in CB style D (baseline)

- No nominal rigidity:
  - α_i =0 for i=W, D, N
- Lower nominal rigidity
  - α_D =0.2, or α_W =0.1, or α_N =0.1,
- Lower risk premium elasticity for banks:
  - ε^B =0.9 instead of 1.1575,
- Lower cash demand elasticity for households:
  - ε^M =0.7 instead of 0.85,
- Lower ES for imported goods:
  - θ^N =1.3 instead of 1.1754,
- Higher ES for labor types:
  - ψ =9 instead of 7.3116.

Sensitivity to parameters in CB style D (baseline)

- Again, basically same results:
  - 2 policy rules gets lower Loss0 and Loss1

Explicit optimal policy rules for baseline style

- MER:
  - No taste for inertia generates large policy responses.
  - Weights $\omega_{\Delta i}= 0.5$ or $\omega_{\Delta\delta}=0.5$, achieve inertial coefficients $=0.51$ or 0.34, respectively,
  - Weights $\omega_{\Delta i}= 0.5$ and $\omega_{\Delta\delta}=0.5$, achieve inertial coefficients $=0.41$ and $=-2.1$.
- In FER or PER very large weights on Δi ($\omega_{\Delta i}=1000$) or $\Delta\delta$ ($\omega_{\Delta\delta}=20$) are needed to obtain similar inertial coefficients.

Optimal Policy Rules

Coefficients on non-disturbance state variables $\overline{u}_t, \overline{V}_t, \overline{Y}_t$

	A		B		C		D	
$\hat{i}_{t-1}$	-0.0036	0.0014	0.5098	-0.176	-0.0001	0	0.4089	-0.0227
$\hat{\delta}^{M}_{t-1}$	-0.9441	-6.6518	-1.3004	-5.2117	-12.9423	0.3364	-3.0817	-2.0873
$\hat{r}^{*CB}_{t-1}$	2.4517	-0.9557	0.0298	-0.0452	0.0454	-0.0265	0.0156	-0.0156
$\hat{b}^{CB}_{t-1}$	-0.0001	0	0	0	0	0	0	0
$\hat{i}^{G}_{t-1}$	-3.7949	1.4793	-0.0461	0.0699	-0.0703	0.0409	-0.0242	0.0242
$\hat{b}^{*G}_{t-1}$	-3.7949	1.4793	-0.0461	0.0699	-0.0703	0.0409	-0.0242	0.0242
$\hat{i}^{B}_{t-1}$	-1.2474	0.4863	-0.0152	0.023	-0.0231	0.0135	-0.008	0.008
$\hat{p}^{C}_{t-1}$	12.4366	0.0607	0.2919	0.5539	1.4784	0.4337	0.3851	0.2901
$\hat{\mu}^{z}_{t-1}$	2.1446	8.1183	2.8784	3.4041	14.6635	0.4619	4.2279	1.0882
$\hat{\pi}^{W}_{t-1}$	-4.3468	2.0206	-0.0371	-0.0342	-0.4836	-0.0965	-0.0358	-0.0414
$\hat{\pi}^{D}_{t-1}$	-7.6711	8.8121	1.637	5.6468	13.8732	-0.1634	3.6589	2.3532
$\hat{\pi}^{N}_{t-1}$	-0.198	-2.5135	-0.9701	-0.8396	-2.9287	-0.1189	-1.3971	-0.1497
$\hat{z}^{\circ}_{t-1}$	1.8452	-0.6645	0.0709	-0.0245	0.1388	0.0003	0.0672	-0.0098
$\hat{e}_{t-1}$	1.14	5.6754	0.905	4.9854	12.7265	-0.2909	2.6363	2.1521
$\hat{\pi}^{C}_{t-1}$	-12.0442	-0.3051	-0.1793	-0.3616	-1.0477	-0.3103	-0.2409	-0.185
$\widehat{w}_{t-1}$	-3.6239	2.7997	0.8557	0.214	3.3303	-0.109	1.2473	-0.2006
$\hat{p}^{N}_{t-1}$	-1.2539	-4.6715	-2.5495	-0.443	-11.2417	0.0954	-3.6866	0.688

$\widehat{\pi}^{D}_{1,t-1}$	-6.2423	6.6717	2.9209	1.4474	3.4686	-0.1414	4.4591	-0.4571
$\widehat{\pi}^{D}_{2,t-1}$	4.2137	8.9129	6.3053	-1.0723	33.7782	-0.3367	8.8148	-2.7738
$\widehat{\pi}^{C}_{1,t-1}$	-13.4383	-0.5235	-0.0778	-0.188	-0.5604	-0.1686	-0.1121	-0.0927
$\widehat{\pi}^{C}_{2,t-1}$	-2.9559	-0.3741	-0.0135	-0.0602	-0.1383	-0.0506	-0.0261	-0.0279
$\widehat{\delta}^{M}_{1,t-1}$	6.2622	-6.5582	-2.9256	-1.4402	-3.4664	0.1476	-4.463	0.4609
$\widehat{\delta}^{M}_{2,t-1}$	-4.264	-8.8629	-6.3122	1.0769	-33.8074	0.3396	-8.8229	2.7777
$\widehat{\mu}^{z}_{1,t-1}$	-6.2423	6.6717	2.9209	1.4474	3.4686	-0.1414	4.4591	-0.4571
$\widehat{\mu}^{z}_{2,t-1}$	4.2137	8.9129	6.3053	-1.0723	33.7782	-0.3367	8.8148	-2.7738
$\widehat{e}_{1,t-1}$	-7.6235	-0.1394	1.7515	-4.0684	-10.3742	-0.1649	1.5123	-2.9453
$\widehat{e}_{2,t-1}$	11.2782	2.4174	3.6505	-2.7177	32.6929	-0.2107	4.6981	-2.4989
$\widehat{e}_{3,t-1}$	-4.545	-9.6137	-6.8011	1.1566	-36.4343	0.3632	-9.5079	2.9919
$\widehat{p}^{**X}_{0,t-1}$	1.8616	0.3696	-5.4797	10.7555	-0.4463	-0.1255	-3.5815	6.6866
$\widehat{p}^{**X}_{1,t-1}$	-7.6235	-0.1394	1.7515	-4.0684	-10.3742	-0.1649	1.5123	-2.9453
$\widehat{p}^{**X}_{2,t-1}$	11.2782	2.4174	3.6505	-2.7177	32.6929	-0.2107	4.6981	-2.4989
$\widehat{p}^{**X}_{3,t-1}$	-4.545	-9.6137	-6.8011	1.1566	-36.4343	0.3632	-9.5079	2.9919
$\widehat{c}_{t-1}$	1.2171	2.8893	2.8506	0.4119	11.9742	-0.1662	4.0169	-0.7222
$\widehat{b}^{*B}_{t-1}$	-1.2474	0.4863	-0.0152	0.023	-0.0231	0.0135	-0.008	0.008
$\widehat{i}^{L}_{t-1}$	-0.6854	0.1622	0.0274	0.0345	0.0635	-0.0141	0.0486	0.0064

Optimal Policy Rules

Coefficients on disturbance variables $\widetilde{Z}_t$

	A		B		C		D	
$\widehat{g}_t$	0.5257	-0.3924	0.1389	-0.0236	0.5322	-0.0359	0.1821	-0.0514
$\widehat{z}_t^C$	0.5071	-0.0832	0.2696	0.1965	0.668	0.0253	0.3563	0.0558
$\widehat{z}_t^H$	-0.4184	0.1522	-0.0448	-0.0348	-0.1664	-0.0101	-0.0624	-0.0095
$\widehat{\epsilon}_t$	4.2584	-3.6613	-1.2389	-0.1797	-5.2913	0.1519	-1.8115	0.3739
$\widehat{z}_t^A$	-2.1807	-8.4078	-5.4995	0.399	-27.3989	0.2829	-7.7851	2.1664
$\widehat{\varsigma}_t$	-0.0358	0.0088	-0.0004	0	-0.0044	-0.0011	-0.0003	-0.0001
$\widehat{\pi}_t^{**N}$	3.3759	1.5295	3.6661	-3.2152	5.4002	0.0126	3.388	-2.5576
$\widehat{i}_t^{**}$	-16.5854	6.465	-0.2015	0.3055	-0.3071	0.179	-0.1057	0.1059
$\widehat{\phi}_t^{**B}$	-1.1346	0.4423	-0.0138	0.0209	-0.021	0.0122	-0.0072	0.0072
$\widehat{\phi}_t^{**G}$	-5.1775	2.0182	-0.0629	0.0954	-0.0959	0.0559	-0.033	0.033
$\widehat{\mu}_t^{z**}$	3.8109	-1.4816	0.07	-0.0282	0.1298	-0.029	0.0609	0.0031
$\widehat{p}_t^{**X}$	0.5184	-0.2805	3.783	-6.7956	0.0725	-0.0115	2.1391	-4.0192
$\widehat{i}_{t-1}^{**}$	8.8831	-3.4626	0.1079	-0.1636	0.1645	-0.0959	0.0566	-0.0567
$\widehat{\mu}_{t-1}^{z**}$	0.5604	0.9251	0.81	-0.24	1.6612	0.306	0.8257	-0.2435

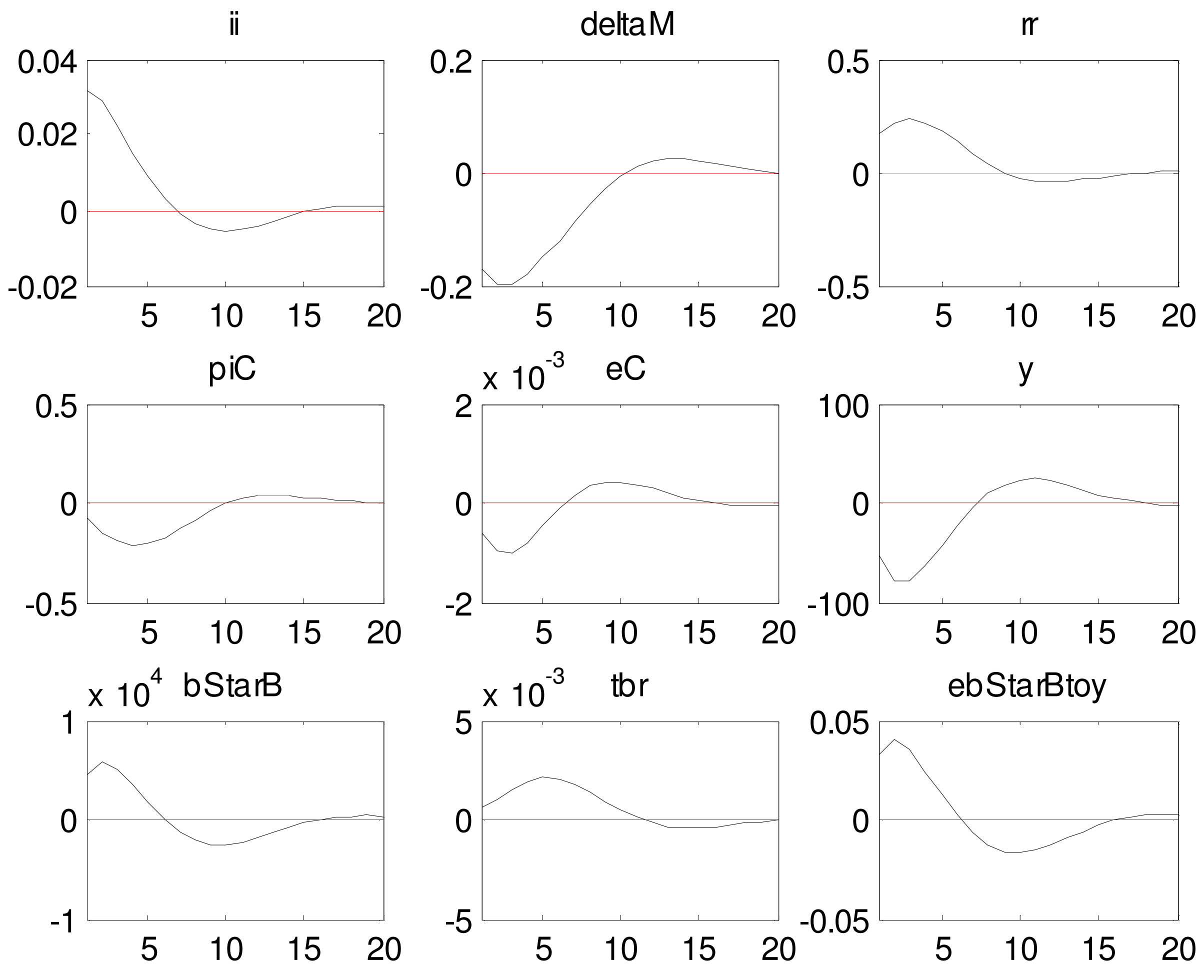
Coefficients on LMs corresponding to equations with expectational terms λ^{d}_{t-1}

λ^{c}_{t-1}	-0.7868	0.9884	-0.0211	0.0633	0.0155	0.0436	-0.0038	0.0248
$\lambda^{\lambda^0}_{t-1}$	98.9923	-40.4211	1.1717	-2.052	1.5667	-1.2282	0.5509	-0.7437
$\lambda^{b^{*B}}_{t-1}$	22.697	-20.588	0.4225	-1.2083	-0.352	-0.8137	0.0546	-0.4397
$\lambda^{i^L}_{t-1}$	-0.0193	0.0142	-0.0003	0.0008	-0.0001	0.0005	-0.0001	0.0003
λ^{t}_{t-1}	27.2273	-11.2455	0.3096	-0.5963	0.3804	-0.3595	0.1328	-0.2217
λ^{r}_{t-1}	-3.3148	-0.0323	-0.0207	-0.0392	-0.1331	-0.0364	-0.0273	-0.02
$\lambda^{\Gamma^W}_{t-1}$	5.9737	-5.8089	0.2098	-0.2147	0.293	-0.1397	0.1349	-0.0615
$\lambda^{\Psi^W}_{t-1}$	-1.7399	9.5285	-0.0638	0.8599	0.7985	0.6125	0.1446	0.3727
$\lambda^{\Gamma^D}_{t-1}$	10.9352	-11.6702	0.2884	-0.6743	0.013	-0.4579	0.0968	-0.2542
$\lambda^{\Psi^D}_{t-1}$	16.1871	-11.6189	0.3211	-0.6122	0.2239	-0.4003	0.14	-0.2226
$\lambda^{\Gamma^N}_{t-1}$	9.6918	-11.1071	0.2697	-0.6496	-0.0118	-0.4424	0.0861	-0.2463
λ^{Ψ^N}	10.6502	-7.4211	0.2232	-0.3698	0.2352	-0.2383	0.1136	-0.1332

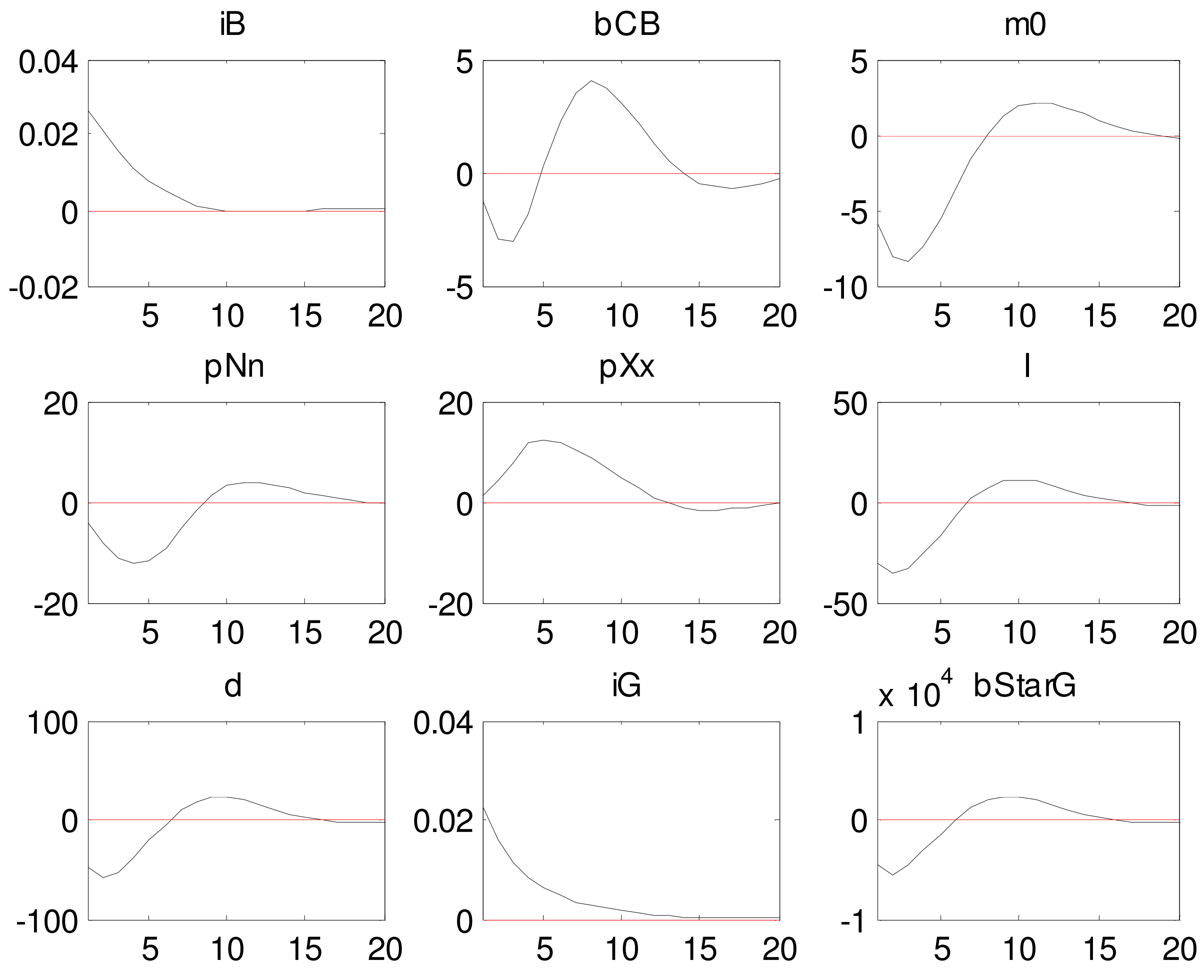
An illustration: a RW i* shock

- First, assume a Floating ER regime (FER)
- The Taylor Rule is initially:
  - $i=0.5*i(-1)+1.5\pi^{C}+1.5y+0.5*tbr+0.1*tbr(-1)$
  - Shock in i* is 0.0225
  - Contractionary effect on economy: y, π^{C}, l, d, , m^{0}, S fall.
  - i increases 0.03 (risk premium rises, since banks get more funds abroad, because deposits fall more than loans)
  - The lending rate and the real interest rate (rr) increase.

FER regime: response to i* shock



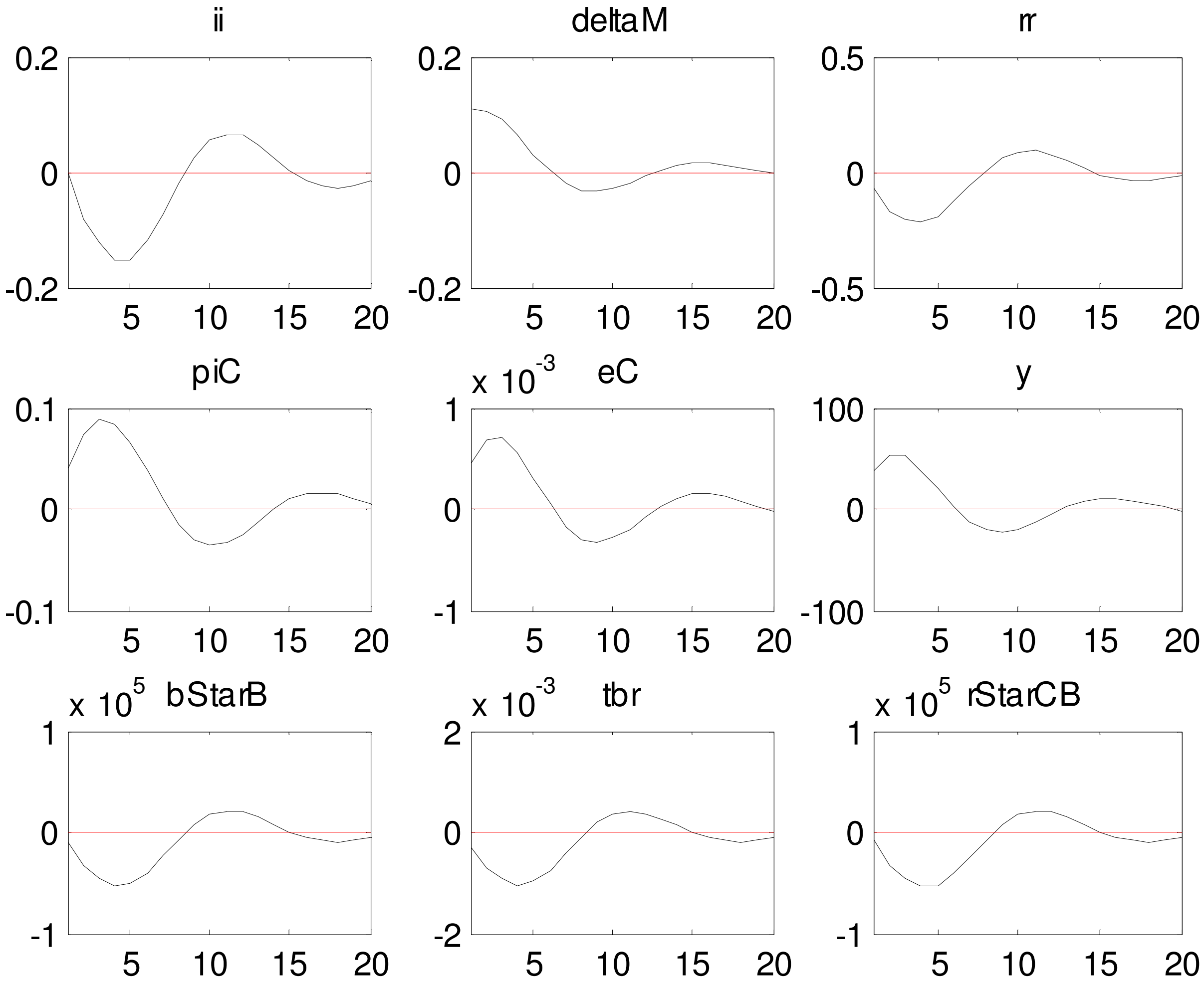
FER regime: response to i* shock (2)



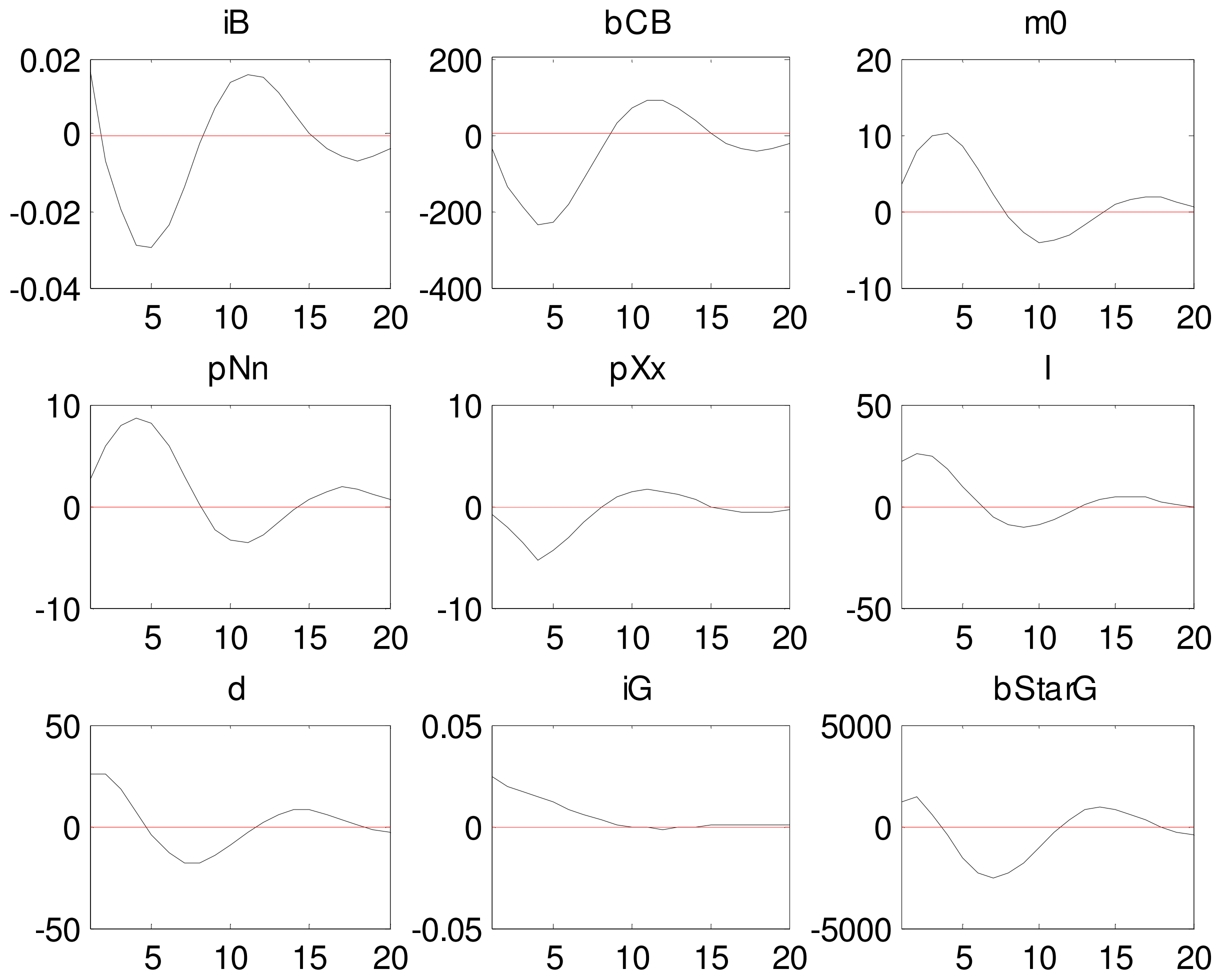
An illustration: a RW i* shock (2) Now, assume a MER regime

- Taylor Rule is the same.
- 2nd policy rule is:
  - $\delta=0.5*\delta(-1)-1.5\pi^{C}-1.2y-1.5*tbr-0.5*tbr(-1)+0.5*(e*r*^{CB}/y)$
- Now CB sells reserves and banks can cancel foreign debt instead of increasing it. This lowers their risk premium.
- The foreign currency interest rate for banks (iB), after an initial increase, actually falls below SS level, preventing the domestic currency i from rising on impact and actually falling.
- The potential recession has been turned into a boom!
- THIS HAS BEEN EXAGGERATED ON PURPOSE OF COURSE!

MER regime: response to i* shock



MER regime: response to i* shock (2)



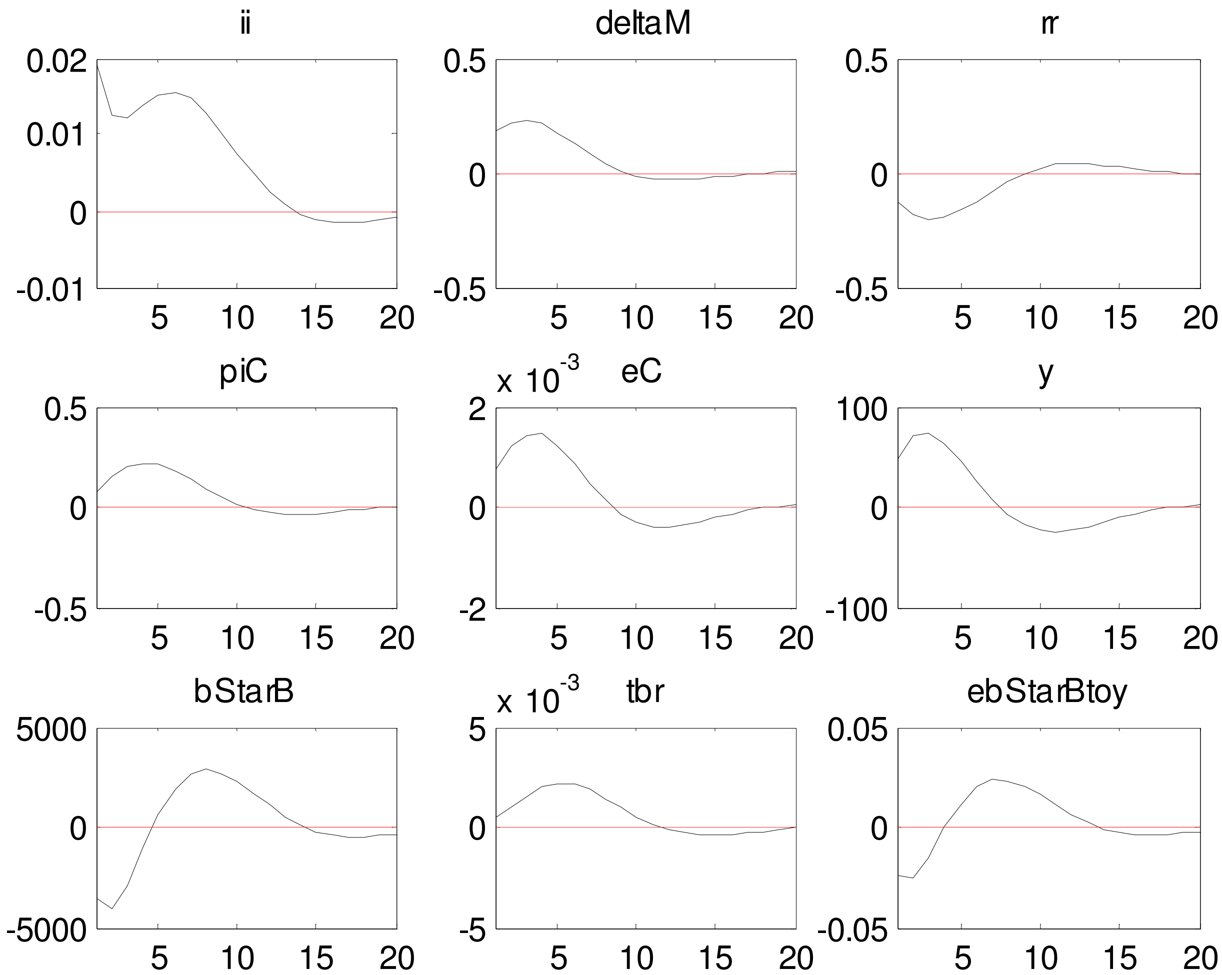
An illustration: a RW i* shock (3) Back to a FER regime

- The CB could have adopted a more benign TR.
  - $i=0.5*i(-1)+1.5\pi^{C} +1.5y-0.5*tbr-0.1*tbr(-1)$

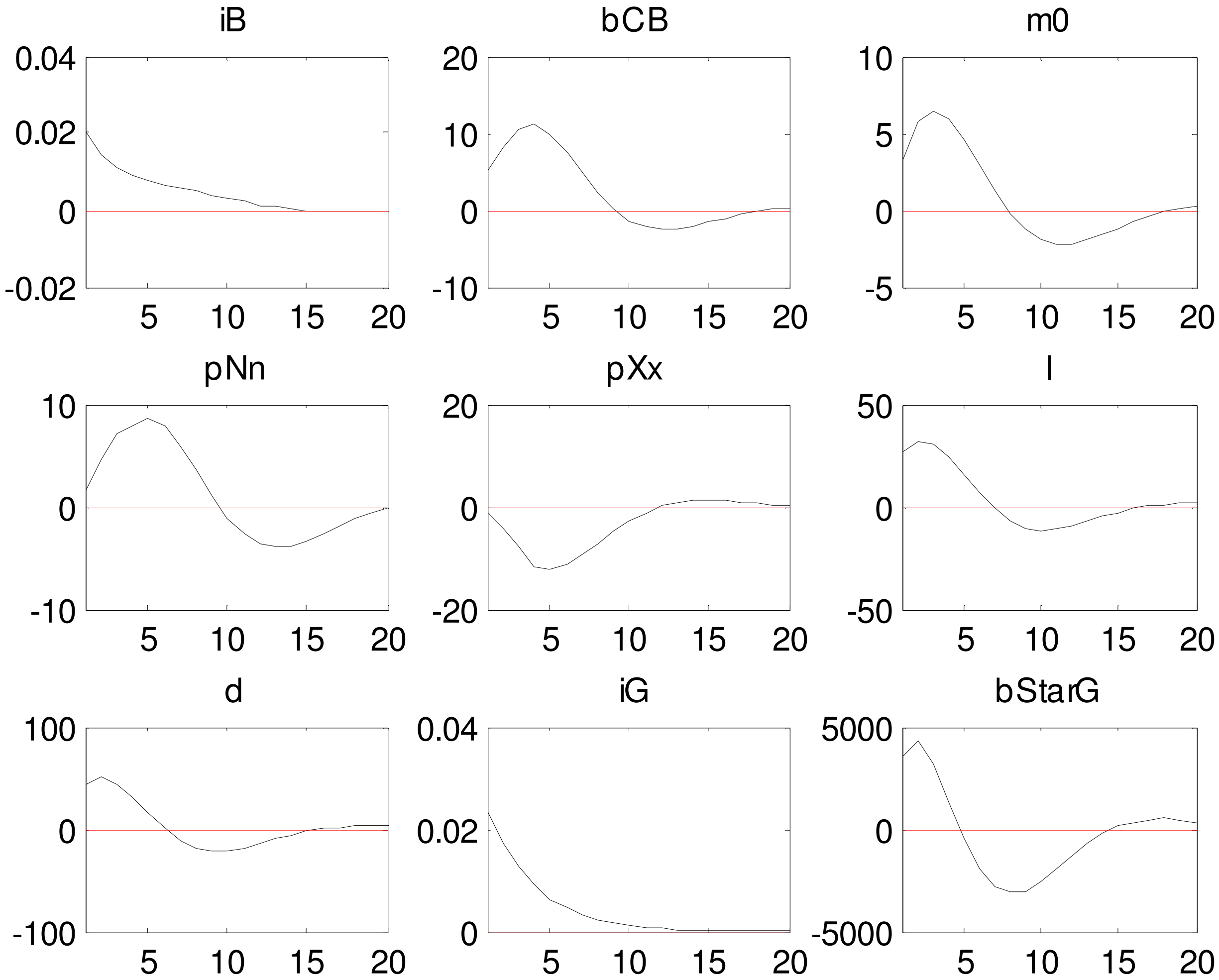
Changed signs

  - CB decreases i when tbr is above SS
  - This makes i rise less on impact and lifts the burden on Banks, who can cancel foreign debt and still lend more.
  - The potential recession has been turned into a boom again.
- The point is that the added flexibility of using a 2nd policy rule can help the CB fine tune so as to better achieve its overall objectives: in this case although y rises just a little more than with MER, inflation (piC) rises twice as much (versus MER).

FER regime: response to i* shock (3)



FER regime: response to i* shock (4)



Conclusion

- ARGEMmy is a DSGE model that appears prommisory in usefulness for policy analysis and projections.
- Banks are at the center of the stage. Financial flows are explicit.
- The CB intervenes simultaneously in the bond/money market and in the FX market.
- In a world in which CBs have to balance multiple goals, the use of 2 policy rules in a DSGE model appears to be potentially useful in integrating interest rate and exchange rate policies within a unified framework.

Thank you!