

*A Supply and Demand Model of the College Admissions Problem**

Hector Chade[†] Gregory Lewis[‡] Lones Smith[§]

First version: July 2003

This version: November 11, 2008

Abstract

We consider a decentralized college admissions problem with uncertainty. We assume that a continuum of heterogeneous students apply to two colleges. College application choices are nontrivial because they are costly and their evaluations are noisy. Colleges set admissions standards for signals of student caliber.

We develop the supply and demand framework where admissions standards act like prices that allocate scarce slots to students. We explore comparative statics in the game between colleges, and find that the better college is affected by the lesser college's admissions standard in this decentralized world. Our analysis is complicated by the students' application portfolio shifts. Noisy applications not only distort outcomes, but even strategies: We find that the best students apply to the best college, and the best college sets higher standards only when the colleges differ sufficiently in quality, *and* the lesser school is not too small.

Applying the model, we find that racial affirmative action at the better college is optimally met by a discriminatory admissions policy at the weaker school; and that binding early admissions programs are more effective than non-binding programs when a college competes with a rival that does not use early admissions.

*An earlier version was called "The College Admissions Problem with Uncertainty." Lones is grateful for the financial support of the National Science Foundation. We have benefited from seminars at BU, UCLA, Georgetown, HBS, the 2006 Two-Sided Matching Conference (Bonn), 2006 SED (Vancouver), 2006 Latin American Econometric Society Meetings (Mexico City), and 2007 American Econometric Society Meetings (New Orleans).

[†]Arizona State University, Department of Economics, Tempe, AZ 85287-3806.

[‡]Harvard University, Department of Economics, Cambridge, MA 02138.

[§]University of Michigan, Economics Department, Ann Arbor, MI 48109-1220.

1 Introduction

The college admissions process has lately been the object of much scrutiny, both from academics and in the popular press. This interest owes in part to the strategic nature of college admissions, as schools use the tools at their disposal to attract the best students. Those students, in turn, respond most strategically in making their application decisions. This paper examines the joint behavior of students and colleges in a matching framework.

We develop and flesh out an equilibrium model of the college admissions process, with decentralized matching of students and two colleges — one better and one worse, respectively, called 1 and 2. The model captures two previously unexplored aspects of the ‘real-world’ problem. First, each college application is costly, and second, colleges only observe a noisy signals of each student’s caliber. We assume that colleges seek to fill their capacity with the best students possible. Students meanwhile must solve a nontrivial portfolio choice problem. This tandem of noisy caliber and costly applications feeds the intriguing conflict at the heart of the student choice problem: Gamble on Harvard, settle for Michigan, or apply to Harvard while insuring with Michigan. By the same token, college standards are endogenous, reflecting student preferences and their capacities.

By analyzing how college and students interact in equilibrium, we make four main contributions. First we determine when students sort by caliber into colleges. Second, we provide a graphical framework for the analysis of the system. Third, we show how cost and noise induce an indirect interdependency between the colleges, producing externalities between their decisions. Finally, we apply the model to the topical issues race-based and early admissions. We show that race-based affirmative action by one college produces an acceptance curse effect at the other. We also compare binding and non-binding early admissions programs and explain why better colleges may desire neither.

A central question addressed in this paper is: Do the best students apply to the best colleges, and does college 1 impose a higher admission standard than 2. Student sorting requires that two forces cooperate. First, student applications must increase in their caliber. Specifically, we argue that this means that: *(i)* the best students apply just to college 1; *(ii)* the middling/strong students insure by applying to both colleges 1 and 2; *(iii)* the middling/weak students apply just to college 2; and finally, *(iv)* the weakest students apply nowhere. We show that this need not occur in equilibrium.

Next, it may be easier to gain admission to college 1 if its capacity is too large for

its caliber niche — as with a large high quality state college. For in that case a curious inversion may arise, as college 2 may screen applicants more tightly than college 1. College standards reflect not only the quality of the college but also their capacity.

Our first contribution is an analysis of college-student sorting. If colleges are close in quality, then sorting may be *impossible* in well-behaved signal distributions. Also, college size and quality are substitutes, since a smaller worse college may set higher admissions standards in equilibrium. We prove that only sorting equilibria exist if the colleges differ sufficiently in quality, and the higher ranked school is not too small.

Our second contribution is methodological: We provide an intuitive graphical analysis of the student choice problem that fully captures the application equilibrium. We hope that this framework will prove a tractable workhorse for future work on this subject. It embeds both the tradeoffs found in the search-theoretic problems analyzed by Chade and Smith (2006), and the colleges' choice of capacity-filling admission standards.

In our third contribution, we uncover some new externalities between the colleges. Absent noise, the better college does not care about decisions made by the lesser. But we show that admissions standards at *both* colleges fall if college 2 raises its capacity. Harvard is thus affected by admissions policies at the University of Chicago. Moreover, the better ranked college profits from higher application costs charged by *either* school.

Our fourth contribution are two applications of our framework. We show that when the better college introduces affirmative action policies to increase diversity, the weaker college should optimally discriminate against minority students unless it too has a preference for diversity. This result stems from an “acceptance curse” effect: A student’s enrollment at the weaker college is bad news since it indicates that she was not admitted at the better college, but is especially bad news if that student was advantaged by affirmative action at the better college. Moreover, greater student diversity at one college necessarily comes at the expense of reduced diversity at its rival.

Next, our framework affords an analysis of early admissions programs. We show that top colleges benefit little from them. Further, binding early admissions programs are more effective for a college than non-binding programs, as any outcome a college may achieve with a non-binding program can also be achieved with a binding one.

The paper is related to several strands of literature. Gale and Shapley initiated the college admissions problem in their classic 1962 work in the economics of matching. As the prime example of many-to-one matching, it has long been in the province of

cooperative game theory (e.g. Roth and Sotomayor (1989) and (1990)). Our model differs by the assumption that matching is decentralized and subject to two frictions — the application cost and the noisy evaluation process. To analyze these features in a simple fashion, we posit homogeneous preferences and consider a model with only two colleges. With these restrictions, we view our model as a tractable benchmark analysis of what is otherwise a difficult equilibrium problem.

We focus on the sorting question. This has been an organizing question of the two-sided matching literature since Becker (1973). Shimer and Smith (2000), Smith (2006), Anderson and Smith (2007) and Chade (2006) have analyzed sorting in models with search frictions under alternative informational assumptions. In these papers, both sides play symmetric roles. But in this many-to-one college admissions setting, the sides play different roles, as colleges control standards while students choose application sets. Other recent papers have examined matching under in a decentralized or noisy world, but have instead asked under which conditions stable matchings will emerge (Niederle and Yariv (2007), Chakraborty, Citanna, and Ostrovsky (2007)).

The student portfolio problem embedded in the model is a special case of the simultaneous search problem solved in Chade and Smith (2006). Here, we use their solution to characterize the optimal student application strategy. However, the acceptance chances here are endogenous, since any one student's acceptance probability depends on which of her peers also applies to that school. Thus, this paper is also contributes to the literature on equilibrium models with nonsequential or directed search (e.g. Burdett and Judd (1983), Burdett, Shi, and Wright (2001), and Albrecht, Gautier, and Vroman (2003)), as well as Kircher and Galenianos (2006). Another related paper is that of Nagypál (2004), who analyzes a model in which colleges know student types, but students themselves can only learn their type through normal signals. Arguably, neither students nor colleges know the true talent; however, we feel that students have the informational edge. And finally, Chade (2006) introduced the acceptance curse notion with type uncertainty. Lee (2007) recently shows that early admissions programs can mitigate such effects.

The paper is organized as follows. The model is found in Section 2. Section 3 presents the equilibrium analysis, focusing on the question of whether sorting occurs. Section 4 presents comparative statics results. Section 5 applies our framework to race-based admissions policies, while Section 6 takes up early admission. Section 7 concludes.

2 An Overview of the Environment

We impose very little structure and concentrate on the essential features of the problem. We ignore the important consideration of heterogeneity in preferences of students over colleges or vice versa. Instead, we focus on two key frictions. First, student applications to colleges are costly. In practice, such costs can be quite high, as attested by the recent popularity of the “common application”, whose sole purpose is to lower the cost of multiple applications.¹ Without application costs, there is no role for student choice.

Second, signals of student calibers are noisy. This informational friction creates uncertainty on the student side, and a filtering problem for colleges. It captures the difficulty faced by market participants, with students choosing “insurance schools” and “long shots”, and colleges trying to infer the best students from noisy signals. Without noise, sorting would be trivial: Better students would apply and be admitted to better colleges, for their caliber would be correctly inferred and they would be accepted. As we will see, sorting is less easily achieved with both costs and noise. Indeed, there is a richer role for student choice with both application costs and noisy outcomes.

We also make two other key modeling choices. First, we assume just two colleges. This is done for the sake of tractability. The n -college problem is very important, but will remain a challenging open problem in this literature for many years to come. We also fix their capacity. This is most defensible in the short run, and so it is best to interpret our model as focusing on the “short run” analysis of college admissions. We also assume that students apply to all their colleges first, and then colleges decide simultaneously whom to admit. However, we later briefly explore the possibility of “early admissions”.

The final important assumption is that signals of student calibers are conditionally independent. This is justified if — *before* applying to college — students are apprised of all variables common to their applications, such as any standardized achievement score — ACT/SAT/GMAT, or their GPA. This allows them to determine their caliber. Then based on this, they decide on specific schools to apply to, adding costly and idiosyncratic elements to their applications such as college-specific essays and interviews. Students are uncertain as to how these idiosyncratic elements will be evaluated. We treat the resulting signal as conditionally independent across colleges.

¹The “common application” is a general application form that is used by over 150 colleges in an effort to simplify college applications.

3 The Model

There are two colleges 1 and 2 with *capacities* κ_1 and κ_2 , and a unit mass of students with *calibers* t whose distribution has a density $f(x)$ over $[0, \infty)$. We avoid trivialities, and assume that college capacity is insufficient for all students, as $\kappa_1 + \kappa_2 < 1$. Each college application costs a student $c > 0$. Preferences coincide, with all students preferring college 1. Everyone receives payoff 1 for attending college 1, $u \in (0, 1)$ for college 2, and no payoff for not attending college. To avoid trivialities, we later bound application costs above. Students maximize expected college payoff less application costs. College payoff equals the average enrolled student caliber *times* the measure of students enrolled.

Students know their caliber, and colleges do not. Appendix A.1 shows how our results can be at once re-interpreted if x is a student’s signal of his own caliber. Colleges 1 and 2 each just observes a noisy conditionally independent signal of each applicant’s caliber. In particular, they do not know where else students have applied. Signals σ are drawn from a conditional density function $g(\sigma|x)$ on a subinterval of \mathbb{R} , with cdf $G(\sigma|x)$. We assume that $g(\sigma|x)$ is continuous and obeys the *strict* monotone likelihood ratio property (MLRP). So $g(\tau|x)/g(\sigma|x)$ is increasing in the student’s type x for all signals $\tau > \sigma$.

Students apply simultaneously to either, both, or neither college. This strategy selects for each caliber x , a college application menu $S(x)$ in $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. Colleges receive student applications, and having already decided upon the sets of student signals to accept.² They intuitively should use admission standards to achieve their objective functions — college i admitting students above a threshold signal $\underline{\sigma}_i$. This follows if better signals come from better expected students. Appendix A.2 proves this property, even though college 2 faces an *acceptance curse*: it sometimes accepts a reject of college 1.

For a fixed admission standard, we want to ensure that very high quality students are almost never rejected, and very poor students are almost always rejected. For this, we assume that for a fixed signal σ , we have $G(\sigma|x) \rightarrow 0$ as $x \rightarrow \infty$ and $G(\sigma|x) \rightarrow 1$ as $x \rightarrow 0$. For instance, exponential signals have this property $G(\sigma|x) = 1 - e^{-\sigma/x}$. A large signal family is the location family, in which the conditional cdf of signals σ is given by $G((\sigma - x)/\rho)$, for any smooth cdf G and $\rho > 0$ — eg. normal, logistic, Cauchy, or uniformly distributed signals. The strict MLRP then holds if $\log G'$ is strictly concave.

²Alternatively, colleges could first commit to an admission standard. This yields the same equilibria until we study affirmative action (proof omitted), and lends itself to some sharper analysis. In the interests of a unified treatment throughout the paper, we proceed in the simultaneous move world.

4 Equilibrium

An equilibrium is a triple $(S^e(\cdot), \underline{\sigma}_1^e, \underline{\sigma}_2^e)$ such that

- (a) Given $(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$, $S^e(x)$ is an optimal college application portfolio for each x ,
- (b) Given $(S^e(\cdot), \underline{\sigma}_j^e)$, college i 's payoff is maximized by admissions standard $\underline{\sigma}_i^e$.

In a *sorting equilibrium*, colleges' and students' strategies are *monotone*. This means that the better college is more selective ($\underline{\sigma}_1^e > \underline{\sigma}_2^e$) and higher caliber students are increasingly aggressive in their portfolio choice — namely, $S^e(x)$ is increasing in x under the “strong set order” ranking $\emptyset \prec \{2\} \prec \{1, 2\} \prec \{1\}$. This order captures an intuitive increasing aggressiveness in student applications: The weakest apply nowhere; better students apply to the “easier” college 2; even better ones “gamble” by applying also to college 1; the top students dispense with a college 2 “insurance” application. Alternatively, monotone strategies ensure the intuitive result that the distribution of student calibers at college 1 first-order stochastically dominates that of college 2 (see Lemma 3 in Appendix A.8), so that all top student quantiles are larger at college 1. This is the most compelling notion of student sorting in our environment with noise.

Our concern with a sorting equilibrium may be motivated on efficiency grounds. If there are complementarities between student caliber and college quality, so that welfare is maximized by assigning the best students to the best colleges, any decentralized matching system must necessarily satisfy sorting to be (constrained) efficient. Since formalizing this idea would add notation and offer little additional insight, we have abstracted from these normative issues and focus on the positive analysis of the model.

5 The Student Optimization Problem

We begin by examining the student application decision, taking the college thresholds as known and fixed. The problem of selecting an optimal college application set for a given set of acceptance probabilities is in general hard, but an algorithm has recently been provided by Chade and Smith (2006). In our two college case, the solution is somewhat straightforward, and may be depicted graphically. From the graph, we can easily deduce sufficient conditions for monotone student behavior.

Consider the portfolio choice problem for a student facing the admission chances $0 \leq \alpha_1, \alpha_2 \leq 1$. The expected payoff of applying to both colleges is $\alpha_1 + (1 - \alpha_1)\alpha_2u$. The marginal benefit MB_{ij} of adding college i to a portfolio of college j is then:

$$MB_{21} \equiv [\alpha_1 + (1 - \alpha_1)\alpha_2u] - \alpha_1 = (1 - \alpha_1)\alpha_2u \quad (1)$$

$$MB_{12} \equiv [\alpha_1 + (1 - \alpha_1)\alpha_2u] - \alpha_2u = \alpha_1(1 - \alpha_2u) \quad (2)$$

The optimal application strategy is then given by the following rule:

- (a) Apply nowhere if costs are prohibitive: $c > \alpha_1$ and $c > \alpha_2u$.
- (b) Apply just to college 1, if it beats applying just to college 2 ($\alpha_1 \geq \alpha_2u$), and nowhere ($\alpha_1 \geq c$), and to both colleges ($MB_{21} < c$, i.e. adding college 2 is worse).
- (c) Apply just to college 2, if it beats applying just to college 1 ($\alpha_2u \geq \alpha_1$), and nowhere ($\alpha_2u \geq c$), and to both colleges ($MB_{12} < c$, i.e. adding college 1 is worse).
- (d) Apply to both colleges if this beats applying just to college 1 ($MB_{21} \geq c$), and just to college 2 ($MB_{12} \geq c$), for then, these solo application options respectively beat applying to nowhere, as $\alpha_1 > MB_{12} \geq c$ and $\alpha_2u > MB_{21} \geq c$ by (1)–(2).

This optimization problem admits an illuminating and rigorous graphical analysis. The left panel of Figure 1 depicts three critical curves: $MB_{21} = c$, $MB_{12} = c$, $\alpha_1 = u\alpha_2$. From (1) and (2), we see that all three curves share a crossing point, since $MB_{21} = MB_{12}$, when $\alpha_1 = u\alpha_2$. Since $MB_{12} = c(1 - c) < c$, this crossing point lies above and right of the point $\alpha_1 = u\alpha_2 = c$, below which applying anywhere is dominated.

Throughout the paper, we assume that $c < u(1 - u)$. For if not, then the curves $MB_{21} = c$ and $MB_{12} = c$ cross a second time inside the unit square.³ The analysis then trivializes because multiple college applications need not occur.

Cases (a)–(d) partition the unit square into regions of (α_1, α_2) that correspond to each portfolio choice, suggestively denoted Φ, C_2, B, C_1 . These regions are shaded in the right panel of Figure 1. This picture summarizes the optimal portfolio choice of a student with arbitrary admissions chances (α_1, α_2) .

³For if $\alpha_2 = 1$, then $MB_{21} = c$ and $MB_{12} = c$ respectively force $\alpha_1 = 1 - (c/u)$ and $\alpha_1 = c/(1 - u)$. Now, $1 - (c/u) > c/(1 - u)$ exactly when $c < u(1 - u)$.

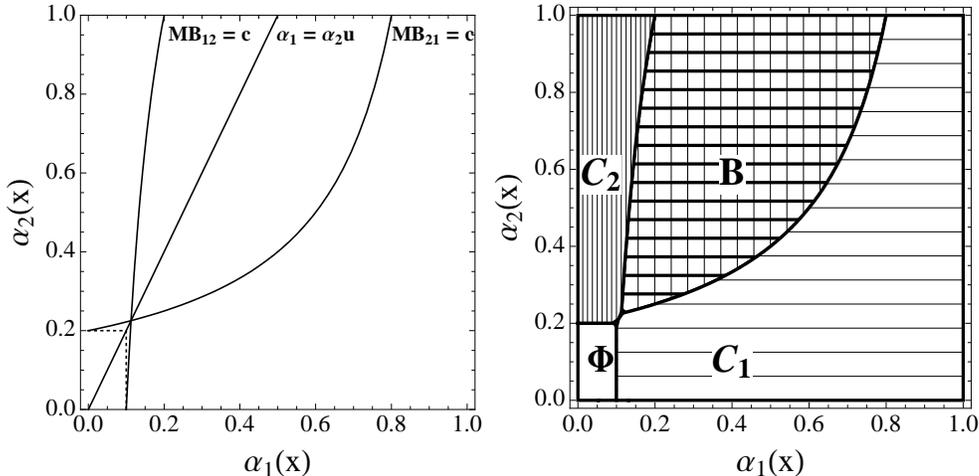


Figure 1: **Optimal Decision Regions.** The left panel depicts (i) a dashed box, inside which applying anywhere is dominated; (ii) the indifference line for solo applications to colleges 1 and 2; and (iii) the marginal benefit curves $MB_{12} = c$ and $MB_{21} = c$ for adding colleges 1 or 2. The right panel shows the optimal application regions. A student in the blank region Φ does not apply to college. He applies to college 2 only in the vertical shaded region C_2 ; to both colleges in the hashed region B , and to college 1 only in the horizontal shaded region C_1 .

For an alternative insight into the student optimization, we could apply the marginal improvement algorithm of Chade and Smith (2006). There, a student first decides whether she should apply anywhere. If so, she asks which college is the best singleton. In Figure 1 at the left, college 1 is best right of the line $\alpha_1 = u\alpha_2$, and college 2 is best left of it. Next, she asks whether she should apply anywhere else. Intuitively, there are two distinct reasons for applying to both colleges that we can now parse: Either college 1 is a “stretch” (as a gamble) school — namely, added second as a lower-chance higher payoff option — or college 2 is a “safety school”, added second for insurance. In Figure 1, these are the parts of region B above and below the line $\alpha_1 = u\alpha_2$, respectively.

6 Admission Chances and Student Calibers

Let us now fix the thresholds $\underline{\sigma}_1$ and $\underline{\sigma}_2$ set by college 1 and college 2. Student x 's acceptance chance at college i is now given by $\alpha_i(x) \equiv 1 - G(\underline{\sigma}_i|x)$. Since a higher caliber student generates stochastically higher signals, $\alpha_i(x)$ is increasing in x . In fact, it is a *smoothly monotone onto* function — namely, it is strictly increasing and differentiable, with $0 < \alpha_1(x) < 1$, and the limit behavior $\lim_{x \rightarrow 0} \alpha_1(x) = 0$ and $\lim_{x \rightarrow \infty} \alpha_1(x) = 1$.

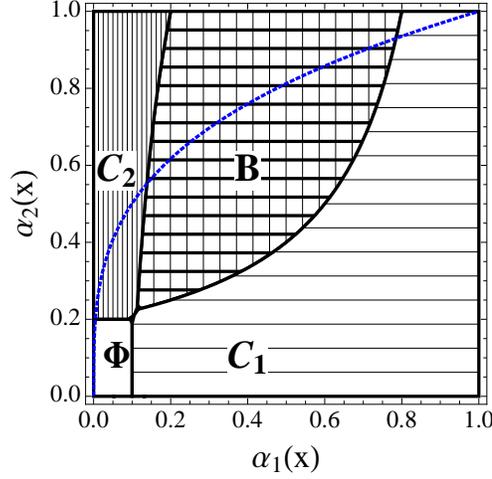


Figure 2: **The Acceptance Function with Exponential Signals.** The figure depicts the acceptance function $\psi(\alpha_1)$ for the case of exponential signals. Students apply to nowhere (Φ), college 2 only (C_2), both colleges (B) and college 1 only (C_1) as caliber x increases. Student behavior is therefore monotone for the acceptance function depicted.

Taking the acceptance chances as given, each student of caliber x faces the portfolio optimization problem of §5. She must choose a set $S^e(x)$ of colleges to apply to, and accept the offer of the best school that admits her. We now translate the sets Φ, C_2, B, C_1 of acceptance chance vectors into corresponding sets of *calibers*. Let \mathcal{C}_1 be the set of calibers that apply just to college 1. Likewise define \mathcal{C}_2 and \mathcal{B} .

Key to our graphical analysis is a quantile-quantile function relating student admission chances at the colleges: Since $\alpha_i(x)$ strictly rises in the student's type x , a student's admission chance α_2 to college 2 is strictly increasing in his admission chance α_1 to college 1. Inverting the admission chance in the type x , the inverse function $\xi(\alpha, \sigma)$ is the student type accepted with chance α given the admission standard σ , namely $\alpha \equiv 1 - G(\sigma|\xi(\alpha, \sigma))$. This yields an implied differentiable *acceptance function*

$$\alpha_2 = \psi(\alpha_1|\underline{\sigma}_1, \underline{\sigma}_2) = 1 - G(\sigma_2|\xi(\alpha_1, \sigma_1)) \quad (3)$$

Lemma 1 *The acceptance function rises in college 1's standard $\underline{\sigma}_1$ and falls in college 2's standard $\underline{\sigma}_2$, and tends to 0 and 1 as the thresholds approach their extremes.*

The proof of this and all results are in the appendix. To best characterize acceptance functions, we now define a function $h : [0, 1] \rightarrow [0, 1]$ as (*weakly*) *regular* if $h(\alpha)$ is a (*weakly*) increasing function on $[0, 1]$ with $h(0) = 0$, $h(1) = 1$, and the secant slopes

$h(\alpha)/\alpha$ and $(1 - h(\alpha))/(1 - \alpha)$ (weakly) fall in α . We find that this description fully captures how our Bayesian acceptance chances relate to one another.

Theorem 1 (a) *If $\underline{\sigma}_1 > \underline{\sigma}_2$, then the acceptance function $\alpha_2 = \psi(\alpha_1)$ is regular.*
 (b) *For any smoothly monotone onto function $\alpha_1(x)$, and any regular function h , there exists a continuous signal density $g(\sigma|x)$ with the strict MLRP, and thresholds $\underline{\sigma}_1 > \underline{\sigma}_2$, for which admission chances of student x to colleges 1 and 2 are $\alpha_1(x)$ and $h(\alpha_1(x))$.*

This result gives a complete characterization of how student admission chances at two ranked universities should compare; the properties of this curve are testable implications. It says that if a student is so good that he is guaranteed to get into college 2, then he is likewise a sure bet at college 1; likewise, if he is so bad that college 1 surely rejects him, then college 2 perforce rejects him too. More subtly, as a student's caliber rises, the ratio of his acceptance chances at college 1 to college 2 rises, as does the ratio of his rejection chances at college 2 to college 1. And conversely, these are the only restrictions placed on acceptance chances by an informative signalling structure.

For an application, suppose that caliber signals have the exponential density $g(\sigma|x) = (1/x)e^{-\sigma/x}$. The acceptance function is then given by the geometric function $\psi(\alpha_1) = \alpha_1^{\underline{\sigma}_2/\underline{\sigma}_1}$ (see Figure 2). Lemma 1 follows. This is increasing and concave — and so regular — when college 2 has a lower admission standard. In general, the acceptance relation for the location family is easily seen to be $\psi(\alpha_1) = 1 - G((\underline{\sigma}_2 - \underline{\sigma}_1)/\rho + G^{-1}(1 - \alpha_1))$.

Omitted from Lemma 1 is another intuitive but loose property of the acceptance function. It is closer to the diagonal when signals are noisier, and farther from it with more accurate signals. For an extreme case, as we approach the noiseless case, a student is either acceptable to neither college, both colleges, or just college 2 (assuming that it has a lower admission standard). In other words, the ψ function tends to a function passing through $(0, 0)$, $(0, 1)$, and $(1, 1)$.⁴ For the location family, this notion is precise: The acceptance function $\psi(\alpha_1)$ rises in the signal accuracy $1/\rho$ (see Persico (2000)). Easily, the acceptance function tends to the top of the box $\psi(\alpha_1) = 1 - G(-\infty) = 1$ as $\rho \rightarrow 0$, and to the diagonal $\psi(\alpha_1) = 1 - G(0 + G^{-1}(1 - \alpha_1)) = \alpha_1$ as $\rho \rightarrow \infty$. Near this extreme case, the student behavior is surely monotone, since as the student caliber rises, we proceed in sequence through the regions Φ , C_2 , B , and finally C_2 . We next explore the intermediate cases with noise, and find when students use monotone strategies.

⁴The limit function is not well-defined: If a student's type is known, just these three points remain.

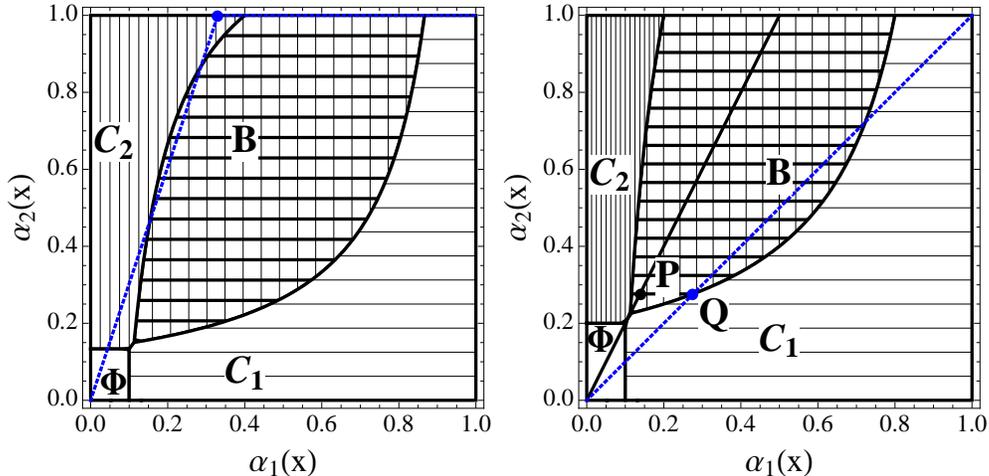


Figure 3: **Non-Monotone Behavior.** In the left panel, the signal structure induces a piecewise linear acceptance function. Student behavior is non-monotone, since there are both low and high caliber students who apply to college 2 only (C_2), while intermediate ones insure by applying to both. In the right panel, equal thresholds at both colleges induce an acceptance function along the diagonal, $\alpha_1 = \alpha_2$. Student behavior is non-monotone, as both low and high caliber students apply to college 1 only (C_1), while middling caliber students apply to both. Such an acceptance function also arises when caliber signals are very noisy.

7 Portfolio Changes Across Student Calibers

As a student's caliber rises, his admission chance at college 1 rises proportionately faster than at college 2. Indeed, the ratio $\alpha_1(x)/\alpha_2(x)$ strictly rises in x by Lemma 1. This intuitively skews the optimal portfolio in §5 towards college 1 as the student caliber rises. This monotone behavior is depicted in Figure 2. We now explore this character.

In general, there are two reasons for non-sorting. First, the acceptance function may multiply cross the $MB_{12} = c$ curve, as in Figure 3. This defies sorting: some caliber x gambles up by applying to both colleges, but a higher caliber $y > x$ plays it safe by applying to college 2 only. Figure 3 depicts a non-monotone sequence of application sets $\Phi, \{2\}, \{1, 2\}, \{2\}, \{1, 2\}, \{1\}$ as caliber rises. This can happen since the marginal benefit MB_{12} in (2) rises in the expected payoff α_1 of college 1 and falls in the expected payoff $\alpha_2 u$ of college 2. So if $\alpha_2 u$ rises faster than α_1 , then a better student may drop college 1 from her portfolio. We show that if $u \leq 0.5$, then this cannot happen. If a student includes college 1 in his portfolio, then any higher caliber student also does.

The next problematic case for sorting applies when college 1 is insufficiently more selective than college 2. For an intuition, assume that both colleges impose the same

standards, thereby inducing an acceptance function along the diagonal $\alpha_2 = \alpha_1$ (as in Figure 3). In this case, the worst students who apply anywhere will choose college 1, since $\alpha_1 > \alpha_2 u$. It is clearly impossible to preclude this behavior on the basis of primitives of the student optimization alone. What is needed is that the (endogenous) admission standard at college 1 be sufficiently higher than at college 2.

Lemma 2 (Monotone Applications) *Student behavior is monotone in caliber if*
 (a) *College 2 has payoff $u \leq 0.5$, so that if a student applies to college 1, then any better student will also apply to college 1, and*
 (b) *College 2 imposes a low enough admissions standard relative to college 1 so that if a student applies to college 2, then any worse student applies to college 2 or nowhere.*

The proof in the appendix of (a) argues that the marginal benefit locus $MB_{12} = c$ has a rising secant slope if $u \leq 0.5$, and thus it can only cross the acceptance function ψ once — since ψ has the falling secant property. The proof of (b) shows that we need merely insist that ψ cross high enough above a known acceptance point $(\bar{\alpha}_1, \bar{\alpha}_2)$ in region B . A sufficient condition for this awaits the analysis of the college behavior in §8.

8 Equilibrium via Supply and Demand Analysis

Each college i must choose an admission standard $\underline{\sigma}_i$ as a best response to its rival's threshold $\underline{\sigma}_j$, accounting for optimal student behavior. With a continuum of students, the resulting *enrollment* \mathcal{E}_i at colleges $i = 1, 2$ is a non-stochastic number:

$$\mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2) = \int_{\mathcal{B} \cup \mathcal{C}_1} \alpha_1(x) f(x) dx \quad (4)$$

$$\mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2) = \int_{\mathcal{C}_2} \alpha_2(x) f(x) dx + \int_{\mathcal{B}} \alpha_2(x) (1 - \alpha_1(x)) f(x) dx, \quad (5)$$

suppressing the dependence of the sets \mathcal{B} , \mathcal{C}_1 and \mathcal{C}_2 on the student application strategy. To understand (4) and (5), observe that caliber x student is admitted to college 1 with chance $\alpha_1(x)$, to college 2 with chance $\alpha_2(x)$, and finally to college 2 but not college 1 with chance $\alpha_2(x)(1 - \alpha_1(x))$. Also, anyone that college 1 admits will enroll automatically, while college 2 only enrolls those who either did not apply or got rejected from college 1.

We now wish to explore enrollment functions (4)–(5) as demand curves, the admissions standards as prices, and the supplies κ_1 and κ_2 as vertical supply curves. The admission rate of any student obviously falls in its anticipated admission standard — the *standards effect*. Theorem 2 below exploits a compounding *portfolio effect* — that demand also falls due to an application portfolio shift, visible in Figures 2 and 3. Each college’s applicant pool shrinks in its own admissions threshold, and expands in its rival’s. Together, we deduce the natural property of demand curves:

Theorem 2 (The Falling Demand Curve) *As a college raises its admission standard, its enrollment falls, and thus enrollment of its rival’s enrollment rises.*

Assume monotone student behavior. Careful inspection of Figures 2 and 3 reveals that college 1’s applicant pool not only shrinks but also improves when either admission standard rises: Indeed, it just loses its worst applicants. By contrast, college 2 loses both better and worse applicants — the top students who also applied to college 1 will deem their college 2 insurance no longer worth it, while the worst students who simply shot a solo application to college 2 will drop out altogether.

Since capacities act as vertical supply curves, we have now justified a supply and demand analysis, in which the colleges are selling differentiated products:

$$\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2) \quad \text{and} \quad \kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$$

For now, let us ignore the possibility that some college might not fill its capacity. Then equilibrium without excess capacity requires that both markets clear $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$ and $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$. Since each enrollment (demand) function is falling in its own threshold, we may invert these equations. This yields for each school i the threshold that responds to its rival’s admissions threshold $\underline{\sigma}_j$ so as to fill their capacity κ_i :

$$\underline{\sigma}_1 = \Sigma_1(\underline{\sigma}_2, \kappa_1) \qquad \underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1, \kappa_2) \qquad (6)$$

Given the discussion of the enrollment functions, we can treat Σ_i as if it is a *best response function* of college i . It rises in its rival’s admission standard and falls in its own capacity. In other words, the admissions standards at the two colleges are strategic complements. Figure 4 depicts an equilibrium as a crossing of these increasing best response functions. By way of contrast, observe that without noise or without application costs, the better

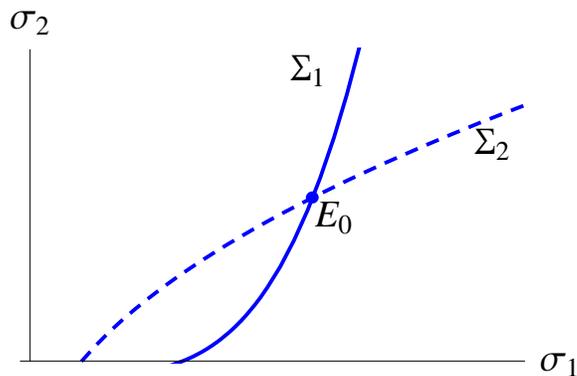


Figure 4: **Best Responses and a Stable Equilibrium.** The functions Σ_1 (solid) and Σ_2 (dashed) give pairs of thresholds so that colleges 1 and 2 fill their capacities in equilibrium.

college is completely insulated from the actions of its lesser rival — Σ_1 is vertical. The equilibrium analysis is straightforward, and there is necessarily a unique equilibrium. In either case, the applicant pool to college 1 is independent of what college 2 does. When the application signal is perfectly noiseless, just the top students apply to college 1. When applications are free, all students apply to college 1, and will enroll if accepted.

With application costs and noise, Σ_1 is upward-sloping, and application pools depends on both college thresholds. When college 2 adjusts its admission standards, the student incentives to gamble on college 1 are affected. This feedback is critical in our paper. It leads to richer interaction among the colleges, and perhaps multiple equilibria.

In Figure 4, the best response function Σ_1 is steeper than Σ_2 at the crossing point. Let us call any such college equilibrium *stable*. It is robust in the following sense: Suppose that whenever enrollment falls below capacity, the college relaxes its admission standards, and vice versa. Then this dynamic pushes us back to the equilibrium. So at this theoretical level, admission thresholds act as prices in a Walrasian tatonnement. Unstable equilibria should be rare: They require that a college's best response be more affected by the other school's admission standard than its own. But the reinforcing standards effect is absent when one's rival adjusts its admission standard.

Theorem 3 (Existence) *A stable equilibrium exists. College 1 fills its capacity. Also, there exists $\bar{\kappa}_1(\kappa_2, c) < 1 - \kappa_2$ satisfying $\lim_{c \rightarrow 0} \bar{\kappa}_1(\kappa_2, c) = 1 - \kappa_2$ such that if $\kappa_1 \leq \bar{\kappa}_1(\kappa_2, c)$, then college 2 also fills its capacity in any equilibrium. If $\kappa_1 > \bar{\kappa}_1(\kappa_2, c)$, then college 2 has excess capacity in some equilibrium.*

For some insight, we choose the capacity $\bar{\kappa}_1$ given κ_2 so that when college 2 has no standards, both colleges exactly fill their capacity. This borderline capacity is less than $1 - \kappa_2$ since a positive mass of students — perversely, those with the highest calibers — applies just to college 1, and some are rejected. (This happens whenever the admission chance into college 1 is at least $1 - c/u$, by (1).) It may be surprising that some college spaces can go unfilled in equilibrium despite insufficient capacity for the applicant pool. Essentially, if college 1 is “too big” relative to college 2, then college 2 is left with excess capacity. There is excess demand for college slots, and yet due to the informational frictions, there is also excess supply of slots at college 2, even at “zero price”.

When college 2 has excess capacity, it optimally accepts all applicants. Since college 1 maintains an admissions standard, college behavior is monotone. But this forces $\alpha_2 = 1$ for all students, and so the acceptance function traverses the top side of the unit square in Figure 2. In other words, as student caliber rises, they apply in order to colleges $\{2\}$, then $\{1, 2\}$, and finally $\{1\}$. Altogether, this is a sorting equilibrium. We next attack the harder problem of finding conditions for sorting equilibria without excess capacity.

9 Do Colleges and Students Sort in Equilibrium?

We now wish to explore more thoroughly when sorting arises in equilibrium. In other words, when is our portfolio effect monotone in the student caliber? In our new language, when do “richer” students shift towards more expensive goods? Hearken back to the sufficient conditions in Lemma 2, where we argued that a sufficiently dominant college 1 ($u \leq 0.5$) and a low enough admission standard at college 2 were *sufficient* for monotone student behavior. We now show that these conditions are also *necessary* for sorting to occur *in equilibrium* when the admission standards clear the market (6).

Theorem 4 (Non-Sorting in Equilibrium)

- (a) [COLLEGE 2 IS TOO GOOD] *For any payoff $u > 0.5$ and any capacities $\kappa_1, \kappa_2 > 0$ with $\kappa_1 + \kappa_2 < 1$, a continuous signal density $g(\sigma|x)$ with the MLRP exists for which an equilibrium exists having $\underline{\sigma}_1 > \underline{\sigma}_2$ but non-monotone student behavior.*
- (b) [COLLEGE 2 IS TOO SMALL] *There exists $\bar{\kappa}_2(\kappa_1) > 0$ so that college 2 sets a higher admissions standard than college 1 in a stable equilibrium, for any capacity $\kappa_2 \leq \bar{\kappa}_2(\kappa_1)$.*

We tackle part (a) constructively in the appendix, starting with the acceptance function depicted in the left panel of Figure 3 and then appealing to Theorem 1 to find some signal distribution that generates it. The message of part (b) is that even a bad college 2 may maintain higher standards if it is sufficiently small. Because college 2 has higher admissions standards and lower payoff, it is no applicant’s first choice. But some students will still insure themselves with an application to college 2. If college 2 is sufficiently small, it may fill its capacity with these insurance applicants.

Thus far, we have found conditions under which sorting fails in some equilibrium. We next finish the picture and give sufficient conditions for the reverse conclusion that college matching entails sorting. Towards this conventional wisdom, we complete Lemma 2, providing a sufficient condition for its part (b). This is a partial converse of Theorem 4.

Theorem 5 (Sorting Equilibrium) *There exists $\underline{\kappa}_1(\kappa_2) > 0$ such that if $\kappa_1 \leq \underline{\kappa}_1(\kappa_2)$ and $u < 0.5$ — namely, college 1 is not too big and college 2 is not too good — then there are only sorting equilibria and neither college has excess capacity.*

10 Changing College Sizes and Application Costs

We now continue to explore the supply and demand metaphor, and derive some basic comparative statics. The potential multiplicity of equilibria renders a comparative statics exercise difficult.⁵ The analysis of this section applies to all stable equilibria, and in particular to any unique sorting equilibrium. In Figure 5, we present the equilibrium effects of increases in the capacity of college 1 (left panel) and college 2 (right panel).

Theorem 6 (College Capacity) *An increase in either college’s capacity lowers both college admissions thresholds.*

Indeed, let us consider what happens if college 2 raises its capacity κ_2 . For a given admission standard $\underline{\sigma}_1$, this pushes down $\underline{\sigma}_2$. But the marginal student that was indifferent between applying to college 2 only (C_2) and both colleges (B) now strictly prefers college 2 only, and the set of applicants to college 1 shrinks. This portfolio reallocation is optimally met by a drop in the admission standards of college 1. The lesser-ranked college thus imposed this “externality” upon the better college in equilibrium.

⁵It is not easy to ensure uniqueness of equilibrium. One case in which this holds is when c is sufficiently small. This follows by continuity from the uniqueness of equilibrium in the costless case.

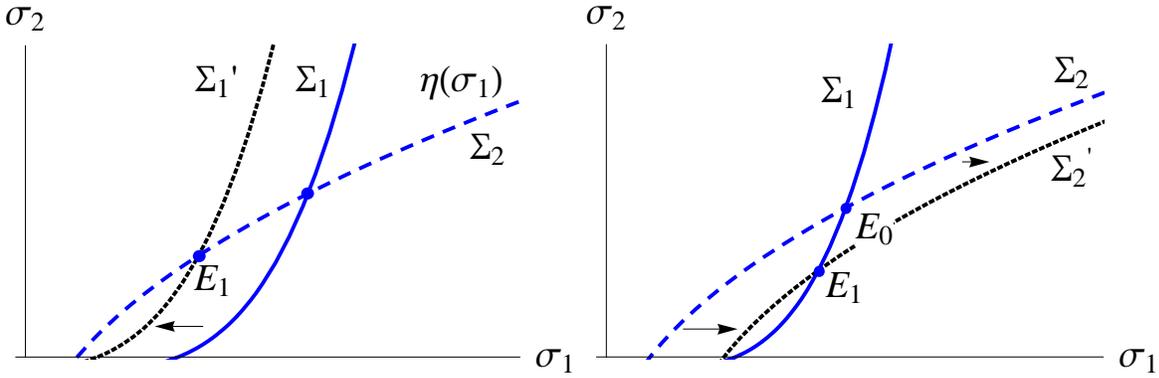


Figure 5: **Equilibrium Comparative Statics.** The figure illustrates how the equilibrium is affected by changing capacities κ_1, κ_2 . The best response functions Σ_1 (solid) and Σ_2 (dashed) are drawn. The left panel considers a rise in κ_1 , shifting Σ_1 left, thereby lowering both college thresholds. The right panel depicts the analogous rise in κ_2 , and shift Σ_2 .

Next, we turn to changes in application costs. Since we have assumed these are the same across colleges, we simply consider *small* unilateral increases.

Theorem 7 (Application Costs) *If the application costs at any college i slightly rise, then both admissions standards fall. If equilibrium is also sorting, then the caliber distribution of students enrolled at college 1 stochastically improves.*

The logic for the first part is straightforward. When applications costs at a college rise, its applicant set shrinks, and it must in turn reduce its standards to compensate. Once again, this affects rival colleges. For example, if applications costs at college 2 rise, then it must lower its own standards. But now a student previously indifferent between applying to college 2 only and to both colleges will choose college 2 only, and so college 1's set of applicants also shrinks. College 1 is forced to lower its standards.

The corresponding result that increased costs at college 1 lower standards at college 2 is not true. On the one hand, given higher costs at college 1, fewer students “gamble upwards”, and college 2 gains expected enrollment from these students. On the other hand, since college 1's admissions standards fall, college 2 loses top applicants who no longer need to insure by applying there. The overall effect is ambiguous.

For some insight into the last part, we show that higher costs help the top college in the special case of a sorting equilibrium. When the application cost at college 1 rises, its weakest applicants — for whom at college 1 was a stretch school — will now pass

on this gamble, and apply to college 2 only. The quality of the applicant pool and thus the enrolled students at the better college rises. Put simply, higher applications costs attenuates the effect of noise on the better college, since it benefits from a tougher self-screening process undertaken by students.

More surprisingly, the better college also benefits from higher costs at the worse one. Since college 2 attracts fewer applicants, it must drop its admissions threshold to fill its capacity. It is now easier to gain admission into college 2, and the marginal benefit (2) of a stretch application to college 1 falls. Its weakest applicants drop out, and thus the caliber distribution of its applicant pool and enrolled student body rises. We cannot make similar inferences about college 2. Higher applications costs there not only prunes its worst students, but also those at the very top for whom it was insurance.

11 Affirmative Action and the Acceptance Curse

We now modify our model to address the topical issue of affirmative action in college admissions. We flesh out the equilibrium implications of this new objective, and hopefully illustrate the power and flexibility of our framework. We show how affirmative action at either college affects both. Amending the model, posit that a fraction ρ of students is minority, and $1 - \rho$ is majority, and that they share a common caliber distribution. Assume that students honestly report their race on their applications. Colleges wish to promote a more *diverse* student body, and so we adjust their payoffs to reflect this. College i earns a bonus $\delta_i \geq 0$ for each enrolled minority student.

Since race is observable, the colleges may set different thresholds for the two groups. Let these standards be $(\underline{\sigma}_1, \underline{\sigma}_2)$ and $(\underline{\sigma}_1 - \Delta_1, \underline{\sigma}_2 - \Delta_2)$ respectively for majority and minority groups.⁶ At each college, the expected payoff of the marginal admits from the two groups should be equal. This gives us two new equilibrium conditions:

$$E[X + \delta_1 | \sigma = \underline{\sigma}_1 - \Delta_1, \textit{minority}] = E[X | \underline{\sigma}_1, \textit{majority}] \quad (7)$$

$$E[X + \delta_2 | \sigma = \underline{\sigma}_2 - \Delta_2, \textit{minority, accepts}] = E[X | \underline{\sigma}_2, \textit{majority, accepts}] \quad (8)$$

where X is the random student caliber. So as with third degree price discrimination,

⁶Here, Δ_i can be interpreted as bonus points given to minority applicants, as in the old undergraduate admissions policy of the University of Michigan, struck down by the Supreme Court in *Gratz v Bollinger*.

the colleges equate the shadow cost of capacity across groups.

Simply by examining equation (7) we see one common thread. College 1 always sets $\Delta_1 > 0$, advantaging minority applicants, since the expected caliber of its marginal minority admit must be δ_1 smaller than its marginal majority admit — so $\Delta_1 > 0$.

Next, we suppose that college 2 is indifferent about affirmative action — so $\delta_2 = 0$. Then it should penalize minority applicants, due to an *acceptance curse* (Chade 2006). Since some students only enroll in college 2 upon rejection by college 1, this event is informative about their caliber. Moreover, this is an even stronger negative signal of a minority candidate, since he was rejected at college 1 despite its affirmative action policy. Then by (8), it should penalize minority students, setting $\Delta_2 < 0$.⁷

Finally, assume that college 2 and not college 1 has a preference for diversity — so $\delta_1 = 0$. College 2 gives a bonus $\Delta_2 > 0$ to minority students, and college 1 responds with a bonus $\Delta_1 > 0$, even absent a preference for diversity or an acceptance curse. Rather, a higher admission chance at college 2 lowers the marginal student’s benefit (1) of applying to college 1. So college 1 loses its weakest minority applicants, and sets $\Delta_1 > 0$.

Theorem 8 (Feedback Effects of Affirmative Action)

- (a) *If some college has a preference for diversity, then college 1 favors minority students.*
- (b) *If just college 1 has such a preference, then college 2 penalizes minority students.*
- (c) *If just college 2 has such a preference, then college 2 favors minority students.*

12 Early Admissions

In a final application of our model, we show how the possibility of early admissions affects the behavior of students and colleges. We amend the game as follows. For simplicity, we assume that just one college i can employ early admissions, setting an early standard $\underline{\sigma}_i^E$ and a regular standard $\underline{\sigma}_i^R$. The other college j only employs regular admissions with standard $\underline{\sigma}_j^R$. We assume a non-binding or *early action* admissions policy, so that a student admitted at a college early may still apply to other colleges during the regular

⁷In this case, we can also unambiguously determine how student behavior adjusts. Since minority students face a laxer admission standard at college 1 and a stiffer one at college 2, their acceptance relation falls, by Lemma 1-(b). The opposite holds for majority students. So in a sorting equilibrium, as seen in Figure 2, minority students apply more aggressively and majority students less so.

period.⁸ We also assume that students may not re-apply to a college that has previously rejected them.⁹ This automatically follows if early applicants are weakly favored.

The student strategy S^e specifies both an early application decision, and conditional on the outcome of that decision, a regular application decision.¹⁰

Student behavior is straightforward to analyze: there are seven undominated strategies, and for given admissions standards, each caliber picks the one that gives the highest expected payoff. We provide more details as needed below. Given this, colleges must maximize their payoffs. College j best responds as before, choosing its regular admission standard to fill its capacity. College i must also choose its pair of early and regular standards to meet the capacity condition, but in addition must equate the expected caliber of early and regular admits:

$$E[X|\sigma = \underline{\sigma}_i^E, \text{applies early, accepts}] = E[X|\sigma = \underline{\sigma}_i^R, \text{applies regular, accepts}] \quad (9)$$

This condition need not hold if there are no applicants in one of the groups (i.e. no early or no regular) — this is a corner solution.

We now illustrate how early admissions may be used as a tool for attracting and sorting students. First, we show that if the better college uses early admissions, in equilibrium it will always penalize early applicants. To see this, fix college 2’s standards $\underline{\sigma}_2^R$ and consider two possible responses by college 1. On the one hand, college 1 could set arbitrarily high standards for early applicants $\underline{\sigma}_1^{E_0} = \infty$, so that no one applies early. Then its regular threshold $\underline{\sigma}_1^{R_0}$ is determined by the capacity condition as usual, and its marginal applicant is \underline{x}^0 .

On the other hand, it could favor early applicants with $\underline{\sigma}_1^{E_1} \leq \underline{\sigma}_1^{R_1}$. Then everyone who applies to college 1 will do so early, so $\underline{\sigma}_1^{R_1}$ is payoff irrelevant and $\underline{\sigma}_1^{E_1}$ is set by the capacity condition. Now, the sequential application {1 early, 2 if rejected} has higher payoff than the simultaneous application {1, 2}, and so if $\underline{\sigma}_1^{E_1} = \underline{\sigma}_1^{R_0}$, more calibers would gamble up and the marginal applicant to college 1 would be lower than before

⁸We have also analyzed the binding or *early decision* case. The results are similar.

⁹In practice, early applicants may be also be “deferred” to the regular admissions season. This is primarily done for enrollment management purposes; and since this issue plays no part in our model, we abstract away from it.

¹⁰In the model where we interpret x as the student’s noisy signal of his caliber, that student’s posterior expected caliber will also drop upon rejection.

$(\underline{x}^1 < \underline{x}^0)$.¹¹ In fact, this holds even though college 1 will set higher early standards $\underline{\sigma}_1^{E_1} > \underline{\sigma}_1^{R_0}$ to meet the capacity condition. But comparing these options, college 1 attracts a more selective application pool by shutting out early applicants than it does by favoring them, and so it will never be a best response for college 1 to favor early applicants.¹² It follows that it will always penalize early applicants in equilibrium.

This is an example of a college using early admissions to sort students. Next, we consider how college 2 can use early admissions to attract or **poach** students from college 1. Suppose that college 2 introduces a non-binding early action program, with lower early admissions standards ($\underline{\sigma}_2^E \leq \underline{\sigma}_2^R$). It is still strictly better to apply early and so no-one will ever apply to college 2 in the regular period. Next, look at the contingent behavior of early applicants. If accepted early at college 2, a student will apply to college 1 regular only if its marginal gain $\alpha_1(x)(1-u)$ exceeds its marginal cost c , or $\alpha_1(x) > c/(1-u)$. If rejected at college 2, he will apply to college 1 if $\alpha_1(x) > c$.

In light of this, consider the early application decision. There are three sets of calibers to consider. First are the calibers with $\alpha_1(x) < c$. Since they will never apply to college 1, they simply compare the costs and benefits of applying to college 2, applying if $\alpha_2^E(x)u > c$, and otherwise applying nowhere. Second are those with $c \leq \alpha_1(x) \leq c/(1-u)$. By the analysis in the paragraph above, students of these calibers choose between applying to college 1 only, and the contingent plan of applying to college 2 early, and to college 1 only if rejected early at college 2. Marginal analysis shows they apply early if $\alpha_2^E(x) \geq c/(u-\alpha_1(x)+c)$. Last are the high calibers with $\alpha_1(x) > c/(1-u)$. They will apply to college 1 regardless, and so as usual they apply also to college 2 early if the marginal benefit MB_{21} exceeds the cost c .

This partitions the space into four regions, depicted in the right panel of Figure 6. For comparison, the left panel shows the corresponding regions in the regular admissions game where a caliber x has admissions chances $(\alpha_1(x), \alpha_2(x) = \alpha_2^E(x))$ in both cases. Notice that the early action program discourages some successful early applicants from applying to the better school. In particular, if we compare the left and right panels, we see that students on the solid part of the acceptance function in the right panel of the figure used to apply to both colleges, but now don't bother sending an application to college 1 if accepted early at college 2. These students have in some sense been "stolen"

¹¹See also Theorem 3 of Chade and Smith (2006).

¹²The best response will satisfy equation (9), or have $\underline{\sigma}_1^E$ high enough that no-one applies early.

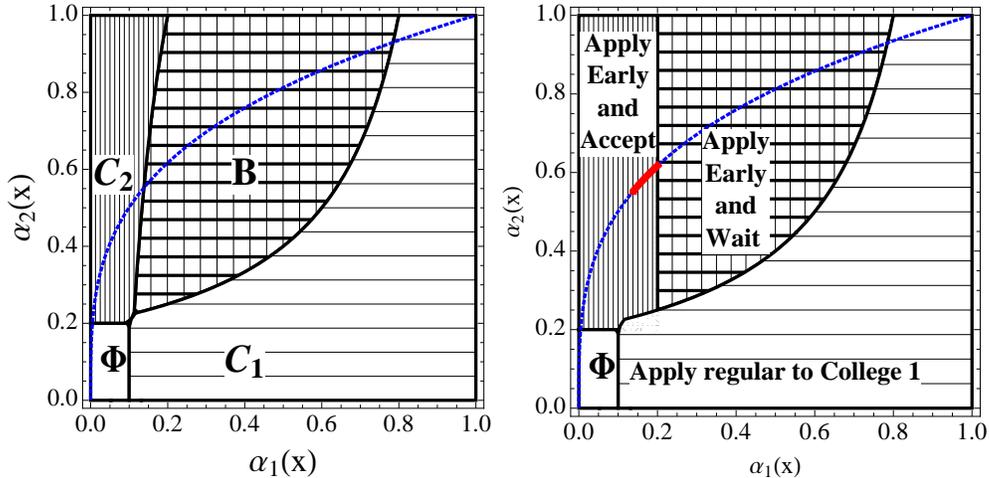


Figure 6: **Capturing Students with Early Action.** The left and right panels depict the optimal student strategy regions under regular and early action programs respectively, with exponential signals and equal admissions standards. Early action enlarges the set of students who will attend college 2 if admitted from the set C_2 in the left panel to the superset of students who will apply early and accept their offer in the right panel. Relative to the regular admissions process, college 2 is able to “capture” some of the students that college 1 would usually have had first option on. These students are shown here as the solid part of the acceptance function.

from college 1’s application pool by college 2, through the use of early admissions.

College 2 can maximize its chances of stealing students from college 1 by favoring early applicants. This will clearly only be worthwhile if they are high-caliber students. To formalize this intuition, consider raising the early standard to ε above the regular standard. Then only middle types with $c \leq \alpha_1(x) \leq c/(1 - u)$ will continue to apply early — these are the students who can be stolen. If at this point the LHS of (9) is higher than the RHS, so that the early applicants are better conditional on enrollment than the regular applicants, the corner solution where early applicants are favored may be optimal (this is a necessary condition). Notice that early applicants are not subject to an acceptance curse, which is one reason why this condition may hold.

This analysis of student behavior squares well with many aspects of real-world college admissions. On the student side, many applicants do in fact “settle” if admitted early to a fairly good school, rather than incurring the costs of sending off another batch of applications to try and secure a marginal improvement. For the colleges, the student stealing rationale has historically been cited as one motivation for early admissions programs, as schools that employ them feel they enjoy a competitive advantage (Karabel

2006). Early admissions have the added benefit of increasing the colleges' matriculation rate, as the "stolen" students will enroll with certainty. Finally, there is strong evidence that early applicants are favored in practice, as our model suggests would be optimal for the colleges (Avery, Fairbanks, and Zeckhauser 2003).¹³

A Appendix: Proofs

A.1 Students' Uncertainty about their Own Calibers

We have assumed that students know their caliber. We now prove that all the results obtain if they only see a noisy signal of their caliber. We assume a density $p(t)$ of types on $[\underline{t}, \bar{t}]$. A student does not observe t but only a signal X with density $f(x|t)$. Similarly, a college observes a signal σ (conditionally independent from the student's signal) from an applicant with density $\gamma(\sigma|t)$. Both $f(x|t)$ and $\gamma(\sigma|t)$ satisfy MLRP, so that each is log-supermodular. Thus, the associated cdf $\Gamma(\sigma|t)$ is also log-supermodular.

Let $p(t|x)$ be the posterior belief of a student who observed x , and suppose that each college $i = 1, 2$ sets a threshold $\underline{\sigma}_i$. Then

$$\alpha_i(x) = \int_{\underline{t}}^{\bar{t}} [1 - \Gamma(\underline{\sigma}_i|t)]p(t|x)dt = 1 - \int_{\underline{t}}^{\bar{t}} \Gamma(\underline{\sigma}_i|t)p(t|x)dt.$$

Define $G(\sigma|x) \equiv 1 - \int_{\underline{t}}^{\bar{t}} \Gamma(\sigma|t)p(t|x)dt$, so that $\alpha_i(x) = 1 - G(\underline{\sigma}_i|x)$. Then (i) G is a cdf as a function of σ ; (ii) G is decreasing in x ; and (iii) $1 - G(\sigma|x)$ is log-supermodular in (σ, x) (as the integral of a product of log-supermodular functions). So results continue by reinterpreting all statements made about calibers as referring to signals of their caliber.

Lemma 3 now states that the cdf of accepted students' signals at college 1 dominates that of college 2 in the sense of first-order stochastic dominance. Since (i) the set of applicants (based on their signals) at college 1 is higher than that at college 2 in the strong set order, and (ii) the cdf $P(t|x) = \int_{\underline{t}}^t p(s|x)ds$ is decreasing in x , the cdf of accepted *calibers* at college 1 also dominates that of college 2.

¹³In addition to the empirical analysis in the cited paper, some schools, such as Duke, explicitly state that they look favorably on early applicants on their websites.

A.2 Colleges Optimally Employ Admissions Thresholds

Let $\chi_i(\sigma)$ be the expected value of the student's caliber given that he applies to college i , his signal is σ , and he accepts. College i optimally employs a threshold rule if, and only if, $\chi_i(\sigma)$ increases in σ . For college 1 this is immediate, since $g(\sigma|x)$ enjoys the MLRP property. We prove this for college 2, since it faces an acceptance curse. We assume that students of calibers in set \mathcal{C}_i apply to college i only, and in \mathcal{B} apply to both colleges.¹⁴

$$\chi_2(\sigma) = \frac{\int_{\mathcal{C}_2} xg(\sigma|x)f(x)dx + \int_{\mathcal{B}} xG(\underline{\sigma}_1|x)g(\sigma|x)f(x)dx}{\int_{\mathcal{C}_2} g(\sigma|x)f(x)dx + \int_{\mathcal{B}} G(\underline{\sigma}_1|x)g(\sigma|x)f(x)dx} \quad (10)$$

It is easy to show that $\chi_2(\sigma)$ is less than the expectation without the cdf's G — because being accepted by a student reduces college 2's estimate of his caliber, as there is a positive probability that the student was rejected by college 1; i.e., college 2 suffers an *acceptance curse* effect. Write (10) as $\chi_2(\sigma) = \int_{\mathcal{B} \cup \mathcal{C}_2} xh_2(x|\sigma)dx$ using indicator function notation:

$$h_2(x|\sigma) = \frac{(I_{\mathcal{C}_2}(x) + I_{\mathcal{B}}G(\underline{\sigma}_1|x))g(\sigma|x)f(x)}{\int_{\mathcal{B} \cup \mathcal{C}_2} (I_{\mathcal{C}_2}(t) + I_{\mathcal{B}}G(\underline{\sigma}_1|t))g(\sigma|t)f(t)dt}, \quad (11)$$

Then the 'density' $h_2(x|\sigma)$ has the MLRP. Therefore, $\chi_2(\sigma)$ increases in σ .

A.3 Acceptance Function and Signals: Proof of Lemma 1

Since $G(\underline{\sigma}_1|x)$ is continuously differentiable in x , the acceptance function is continuously differentiable on $(0, 1]$. Given $\alpha \equiv 1 - G(\sigma|\xi(\alpha, \sigma))$, partial derivatives have positive slopes $\xi_\alpha, \xi_\sigma > 0$. Differentiating (3),

$$\begin{aligned} \frac{\partial \psi}{\partial \alpha_1} &= -G_x(\underline{\sigma}_2|\xi(\alpha_1, \underline{\sigma}_1))\xi_\alpha(\alpha_1, \underline{\sigma}_1) > 0 \\ \frac{\partial \psi}{\partial \underline{\sigma}_1} &= -G_x(\underline{\sigma}_2|\xi(\alpha_1, \underline{\sigma}_1))\xi_\sigma(\alpha_1, \underline{\sigma}_1) > 0 \\ \frac{\partial \psi}{\partial \underline{\sigma}_2} &= -g(\underline{\sigma}_2|\xi(\alpha_1, \underline{\sigma}_1)) < 0 \end{aligned}$$

Properties of the cdf G imply $\psi(0, \underline{\sigma}_1, \underline{\sigma}_2) \geq 0$ and $\psi(1, \underline{\sigma}_1, \underline{\sigma}_2) = 1$. The limits of ψ as thresholds approach the supremum and infimum owe to limit properties of G .

¹⁴We assume that students employ pure strategies, which follows from our analysis of the student optimization in §5. Measurability of sets \mathcal{B} and \mathcal{C}_2 owe to the continuity of our functions $\alpha_i(x)$ in §6.

A.4 Acceptance Function Shape: Proof of Theorem 1

(\Rightarrow) THE ACCEPTANCE FUNCTION IS REGULAR. First, $G(\sigma|x)$ and $1 - G(\sigma|x)$ are strictly log-supermodular in (σ, x) since the density $g(\sigma|x)$ obeys the strict MLRP.¹⁵ Since $x = \xi(\alpha_1, \underline{\sigma}_1)$ is strictly increasing, $G(s|\xi(\alpha_1, \underline{\sigma}_1))$ and $1 - G(s|\xi(\alpha_1, \underline{\sigma}_1))$ are then strictly log-supermodular in (s, α_1) . Thus, the secant slopes

$$\frac{\psi(\alpha_1)}{\alpha_1} = \frac{1 - G(\underline{\sigma}_2|\xi(\alpha_1))}{1 - G(\underline{\sigma}_1|\xi(\alpha_1))} \quad \text{and} \quad \frac{1 - \psi(\alpha_1)}{1 - \alpha_1} = \frac{G(\underline{\sigma}_2|\xi(\alpha_1))}{G(\underline{\sigma}_1|\xi(\alpha_1))}$$

both strictly fall in α_1 , since $\underline{\sigma}_1 > \underline{\sigma}_2$.

(\Leftarrow) DERIVING A SIGNAL DISTRIBUTION. Conversely, fix a regular function h and a smoothly monotone onto function $\alpha_1(x)$. Also, put $\alpha_2(x) = h(\alpha_1(x))$, so that $\alpha_2(x) > \alpha_1(x)$. We must find a continuous signal density $g(\sigma|x)$ with the strict MLRP and thresholds $\sigma_1 > \sigma_2$ that rationalizes the h as the acceptance function consistent with these thresholds and signal distribution.

STEP 1: A DISCRETE SIGNAL DISTRIBUTION. Consider a discrete distribution with realizations in $\{-1, 0, 1\}$: $g_1(x) = \alpha_1(x)$, $g_0(x) = \alpha_2(x) - \alpha_1(x)$ and $g_{-1}(x) = 1 - \alpha_2(x)$. Indeed, for each caliber x , $g_i \geq 0$ and sum to 1. This obeys the strict MLRP because

$$\frac{g_0(x)}{g_1(x)} = \frac{\alpha_2(x) - \alpha_1(x)}{\alpha_1(x)} = \frac{h(\alpha_1(x))}{\alpha_1(x)} - 1$$

is strictly decreasing by the first secant property of h , and

$$\frac{g_0(x)}{g_{-1}(x)} = \frac{\alpha_2(x) - \alpha_1(x)}{1 - \alpha_2(x)} = -1 + \frac{1 - \alpha_1(x)}{1 - h(\alpha_1(x))}$$

is strictly increasing in x by the second secant property of h .

Let the college thresholds be $(\underline{\sigma}_1, \underline{\sigma}_2) = (0.5, -0.5)$. Then $G(\underline{\sigma}_1|x) = g_{-1}(x) + g_0(x) = 1 - \alpha_1(x)$ and $G(\underline{\sigma}_2|x) = g_{-1}(x) = 1 - \alpha_2(x)$. Rearranging yields $\alpha_1(x) = 1 - G(\underline{\sigma}_1|x)$ and $\alpha_2(x) = 1 - G(\underline{\sigma}_2|x)$. Inverting $\alpha_1(x)$ and recalling that $\alpha_2 = h(\alpha_1)$, we obtain $\alpha_2 = h(\alpha_1) = 1 - G(\underline{\sigma}_2|\xi(\underline{\sigma}_1, \alpha_1))$, thereby showing that h is the acceptance function consistent with this signal distribution and thresholds.

¹⁵A positive function $f(a, b)$ is strictly log-supermodular if $f(a', b')f(a, b) > f(a, b')f(a', b)$ for all $a' > a$ and $b' > b$. If f is twice differentiable, then this is equivalent to $f_{ab}f > f_a f_b$. This is the strict MLRP property for signal densities. It is well-known that this is preserved under partial integration.

STEP 2: A CONTINUOUS SIGNAL DENSITY. To create an atomless signal distribution, we smooth this example using the triangular kernel $k(s) = \max\{1 - s, 0\}$. Define $g(\sigma|x) = \beta \sum_{i=\{-1,0,1\}} g_i(x)k(\beta(\sigma - i))$. The strict MLRP implies that $g_i(x)$ is strictly log-supermodular in (i, x) . Also, the function $k(s)$ is concave in s , and thus log-concave in s too. This implies that $k(\beta(\sigma - i))$ is log-supermodular in (i, σ) (§1.5 in Karlin (1968)). Indeed, given twice differentiability, this follows from $k_{\sigma i}k - k_{\sigma}k_i > 0$ iff $-k''k + k'k' > 0$, which holds iff k is log concave. Thus, $g_i(x)k(\beta(\sigma - i))$ is log-supermodular in (i, x, σ) . Finally, partially summing out over $i = 1, 2, 3$ yields a log-supermodular function of (x, σ) (by Proposition 3.2 in Karlin and Rinott (1980)) — the MRLP property. Lastly, for small enough bandwidth $1/\beta > 0$, acceptance chances remain the same. \square

A.5 The Law of Demand: Proof of Theorem 2

It suffices to prove that the applicant pool shrinks at college 1 and expands at college 2 when $\underline{\sigma}_1$ rises. The other case is analogous and thus omitted.

STEP 1: THE APPLICANT POOL AT COLLEGE 1 SHRINKS. When $\underline{\sigma}_1$ increases, the acceptance relation shifts up by Theorem 2, and thus the above type sets change as well. Fix a caliber $x \in \mathcal{C}_2$ or $x \in \Phi$, so that $1 \notin S(x)$.¹⁶ We will show that x continues to apply either to college 2 only or nowhere, and thus the pool of applicants at college 1 shrinks. If $x \in \mathcal{C}_2$, then $\alpha_2(x)u - c \geq 0$ and $\alpha_2(x)u \geq \alpha_1(x)$, and this continues to hold after the increase in $\underline{\sigma}_1$, since $\alpha(x)$ falls while $\alpha_2(x)$ is constant. And if $x \in \Phi$, then clearly caliber x will continue to apply nowhere when $\underline{\sigma}_1$ increases.

STEP 2: THE APPLICANT POOL AT COLLEGE 2 EXPANDS. Fix a caliber $x \in \mathcal{C}_2$ or $x \in \mathcal{B}$, so that $2 \in S(x)$. It suffices to show that caliber x continues to apply to college 2 when the admission standard at college 1 increases. If $x \in \mathcal{C}_2$, then $\alpha_2(x)u - c \geq 0$ and $\alpha_2(x)u \geq \alpha_1(x)$; these inequalities continue to hold after $\underline{\sigma}_1$ rises, since $\alpha_1(x)$ falls while $\alpha_2(x)$ remains constant. And if $x \in \mathcal{B}$, then an increase in $\underline{\sigma}_1$ raises $MB_{21} = (1 - \alpha_1(x))\alpha_2(x)u$, thereby increasing the incentives of caliber x to apply to college 2. Thus, $x \notin \mathcal{C}_1 \cup \Phi$. Since x was arbitrary, it follows that the applicant pool at college 2, $\mathcal{B} \cup \mathcal{C}_2$, expands when $\underline{\sigma}_1$ increases. \square

¹⁶With a slight abuse of notation, we let Φ denote the set of calibers that apply nowhere. The same symbol was previously used to denote the analogous set in α -space.

A.6 Monotone Student Strategies: Proof of Lemma 2

The proof proceeds as follows. First, we show that $u \leq 0.5$ implies that if a caliber applies to college 1, any higher caliber applies as well. Second, we produce a sufficient condition that ensures that the admissions threshold at college 2 is sufficiently lower than that of college 1, so that if a caliber applies to college 2, then any lower caliber who applies to college sends an application to college 2, and calibers at the lower tail apply nowhere. From these two results, monotone student behavior ensues.

PROOF OF PART (a), STEP 1. We first show that the acceptance function $\alpha_2 = \psi(\alpha_1)$ crosses $\alpha_2 = 1/u(1 - c/\alpha_1)$ (i.e., $MB_{12} \equiv \alpha_1(1 - \alpha_2 u) = c$) only once when $u \leq 0.5$. Since (i) the acceptance function starts at $\alpha_1 = 0$ and ends at $\alpha_1 = 1$, (ii) $MB_{12} = c$ starts at $\alpha_1 = c$ and ends at $\alpha_1 = c/(1 - u)$, and (iii) both functions are continuous, there exists a crossing point. Clearly, they intersect when $\alpha_1(1 - \psi(\alpha_1)u) = c$. Now,

$$[(1 - \psi(\alpha_1)u)\alpha_1]' = 1 - u\psi(\alpha_1) - \alpha_1 u\psi'(\alpha_1) > 1 - u\psi(\alpha_1) - u\psi(\alpha_1) = 1 - 2u\psi(\alpha_1) \geq 1 - 2u > 0,$$

where the first inequality exploits $\psi(\alpha_1)/\alpha_1$ falling in α_1 (Lemma 1), i.e. $\psi'(\alpha_1) < \psi(\alpha_1)/\alpha_1$; the next two inequalities use $\psi(\alpha_1) \leq 1$ and $u \leq 0.5$. Since MB_{12} is rising in α_1 when the acceptance relation hits $\alpha_2 = (1 - c/\alpha_1)/u$, the intersection is unique.

PROOF OF PART (a), STEP 2. We now show that Step 1 implies the following single crossing property in terms of x : if caliber x applies to college 1 (i.e., if $1 \in S(x)$), then any caliber $y > x$ also applies to college 1 (i.e., $1 \in S(y)$). Suppose not; i.e., assume that either $S(y) = \emptyset$ or $S(y) = \{2\}$. If $S(y) = \emptyset$, then $S(x) = \emptyset$ as well, as $\alpha_1(x) < \alpha_1(y)$ and $\alpha_2(x) < \alpha_2(y)$, contradicting the hypothesis that $1 \in S(x)$. If $S(y) = \{2\}$, then there are two cases: $S(x) = \{1\}$ or $S(x) = \{1, 2\}$. The first cannot occur, for by Lemma 1 $\alpha_2(x)/\alpha_1(x) > \alpha_2(y)/\alpha_1(y)$, and thus $\alpha_2(y)u \geq \alpha_1(y)$ implies $\alpha_2(x)u > \alpha_1(x)$, contradicting $S(x) = \{1\}$. In turn, the second case is ruled out by the monotonicity of MB_{12} derived above, as caliber y has greater incentives than x to add college 1 to its portfolio, and thus $S(y) = \{2\}$ cannot be optimal.

PROOF OF PART (b), STEP 1. We first show that if the acceptance function passes above the point $(\bar{\alpha}_1, \bar{\alpha}_2) = \left(u(1 - \sqrt{1 - 4c/u})/2, (1 - \sqrt{1 - 4c/u})/2\right)$ — point P in the right panel of Figure 3 — then there is a unique crossing of the acceptance function and

$\alpha_2 = c/u(1 - \alpha_1)$, i.e. $MB_{21} = c$. Now, the acceptance function passes above $(\bar{\alpha}_1, \bar{\alpha}_2)$ if

$$\psi(\bar{\alpha}_1, \underline{\sigma}_1, \underline{\sigma}_2) \geq \bar{\alpha}_2. \quad (12)$$

This condition relates $\underline{\sigma}_1$ and $\underline{\sigma}_2$. Rewrite (12) using Lemma 1 as $\underline{\sigma}_2 \leq \eta(\underline{\sigma}_1) < \underline{\sigma}_1$, requiring a large enough “wedge” between the standards of the two colleges.

To show that (12) implies a unique crossing, consider the secant of $\alpha_2 = c/u(1 - \alpha_1)$ (the curve $MB_{21} = c$). It has an increasing secant if and only if $\alpha_1 \geq 1/2$. To see this, differentiate $\alpha_2/\alpha_1 = c/u\alpha_1(1 - \alpha_1)$ in α_1 . Notice also that $MB_{12} = c$ intersects the diagonal $\alpha_2 = \alpha_1$ at the points $(\alpha_1^\ell, \alpha_2^\ell) = (1/2 - \sqrt{1 - c/4u}/2, 1/2u - \sqrt{1 - c/4u}/2u)$ and $(\alpha_1^h, \alpha_2^h) = (1/2 + \sqrt{1 - c/4u}/2, 1/2u + \sqrt{1 - c/4u}/2u) > (1/2, 1/2u)$.

Condition (12) implies that $\psi(\alpha_1^\ell, \underline{\sigma}_1, \underline{\sigma}_2) > \alpha_2^\ell$. Since $\underline{\sigma}_2 < \underline{\sigma}_1$, we have $\psi(\alpha_1, \underline{\sigma}_1, \underline{\sigma}_2) \geq \alpha_2$ for all α_1 . Thus, the acceptance function crosses $MB_{21} = c$ at or above (α_1^h, α_2^h) . And since $\alpha_1^h > 1/2$, the secant of $MB_{21} = c$ must be increasing at any intersection with the acceptance function. Hence, there must be a single crossing point.

PROOF OF PART (b), STEP 2. We now show that this single crossing property in α implies another in x : If caliber x applies to college 2 (i.e., if $2 \in S(x)$), then any caliber $y < x$ that applies somewhere also applies to college 2 (i.e., $2 \in S(y)$ if $S(y) \neq \emptyset$). Suppose not; i.e., assume that $S(y) = \{1\}$. Then there are two cases: $S(x) = \{2\}$ or $S(x) = \{1, 2\}$. The first cannot occur, for by Lemma 1 $\alpha_2(x)/\alpha_1(x) < \alpha_2(y)/\alpha_1(y)$, and thus $\alpha_2(x)u \geq \alpha_1(x)$ implies $\alpha_2(y)u > \alpha_1(y)$, contradicting $S(x) = \{2\}$. The second case is ruled out by the monotonicity of MB_{21} given condition (12), as caliber y has greater incentives than x to apply to college 2, and thus $S(y) = \{1\}$ cannot be optimal.

Finally, notice that condition (12) also implies that $S(y) = \emptyset$ if $\alpha_2(y)u < c$, which happens for low calibers below a certain threshold. \square

A.7 Equilibrium Existence: Proof of Theorem 3

For definiteness, we now denote the infimum signal by $-\infty$, and the supremum signal by ∞ . Fix any $\kappa_2 \in (0, 1)$, and let $\underline{\sigma}_1^l(\kappa_2)$ be the unique solution to $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, -\infty)$, i.e., when college 2 accepts everybody. (Existence and uniqueness of $\underline{\sigma}_1^l(\kappa_2)$ follows from $\mathcal{E}_2(-\infty, -\infty) = 0$, $\mathcal{E}_2(\infty, -\infty) = 1$, and $\mathcal{E}_2(\underline{\sigma}_1, -\infty)$ increasing and continuous in $\underline{\sigma}_1$.)

Define $\bar{\kappa}_1(\kappa_2) = \mathcal{E}_1(\underline{\sigma}_1^l(\kappa_2), -\infty)$. Let $\kappa_1 \geq \bar{\kappa}_1(\kappa_2)$. We claim that there exists an

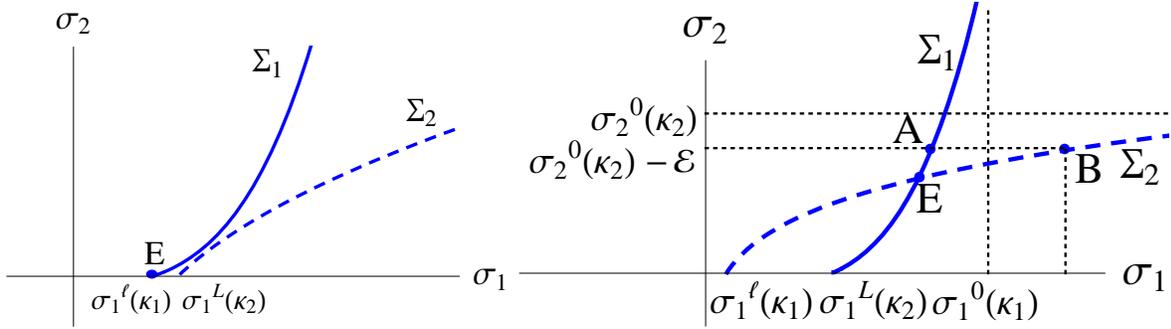


Figure 7: **Equilibrium Existence.** In the left panel, since $\kappa_1 > \bar{\kappa}_1(\kappa_2)$, the best response functions Σ_1 and Σ_2 do not intersect, and equilibrium is at E with $\underline{\sigma}_2 = 0$. The right panel depicts the proof of Theorem 3 for the case $\kappa_1 < \bar{\kappa}_1(\kappa_2)$.

equilibrium in which college 1 sets a threshold equal to $\underline{\sigma}_1^\ell(\kappa_1)$, which is the unique solution to $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, -\infty)$ and satisfies $\underline{\sigma}_1^\ell(\kappa_1) \leq \underline{\sigma}_1^L(\kappa_2)$, and college 2 accepts everybody. For given the lack of standard at college 2, $\underline{\sigma}_1^\ell(\kappa_1)$ fills college 1's capacity exactly, and given the standard of college 1, the enrollment at college 2 is $\mathcal{E}_2(\underline{\sigma}_1^\ell(\kappa_1), -\infty) \leq \kappa_2$ (as $\underline{\sigma}_1^\ell(\kappa_1) \leq \underline{\sigma}_1^L(\kappa_2)$ and $\mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$ is increasing in $\underline{\sigma}_1$), so by accepting everybody college 2 fills as much capacity as it can. This equilibrium is trivially stable, as Σ_2 is 'flat' at the crossing point (see Figure 7, left panel). Moreover, if $\kappa_1 > \bar{\kappa}_1(\kappa_2)$, then college 2 has excess capacity in this equilibrium.

Assume now that $\kappa_1 < \bar{\kappa}_1(\kappa_2)$. We will show that Σ_1 and Σ_2 must cross at least once (i.e., an equilibrium exists), and that the slope condition is satisfied (i.e., it is stable). First, note that in this case $\underline{\sigma}_1^L(\kappa_2) < \underline{\sigma}_1^\ell(\kappa_1)$ or, equivalently, $\Sigma_2^{-1}(-\infty, \kappa_2) < \Sigma_1(-\infty, \kappa_1)$. Second, as the standard of college 2 goes to infinity, college 1's threshold converges to $\underline{\sigma}_1^u(\kappa_1) < \infty$, the unique solution to $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \infty)$. This is the largest threshold that college 1 can set given κ_1 . Similarly, as the standard of college 1 goes to infinity, college 2's threshold converges to $\underline{\sigma}_2^u(\kappa_2) < \infty$, the unique solution to $\kappa_2 = \mathcal{E}_2(\infty, \underline{\sigma}_2)$, which is the largest threshold that college 2 can set given κ_2 . Third, for $\epsilon > 0$ small enough, the unique solution to $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2^u(\kappa_2) - \epsilon)$ is smaller than the unique solution to $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2^u(\kappa_2) - \epsilon)$. Equivalently, $\Sigma_2^{-1}(\underline{\sigma}_2^u(\kappa_2) - \epsilon, \kappa_2) > \Sigma_1(\underline{\sigma}_2^u(\kappa_2) - \epsilon, \kappa_1)$. Fourth, recall that Σ_1 and Σ_2 are continuous functions.

Since $\Sigma_2^{-1}(-\infty, \kappa_2) < \Sigma_1(-\infty, \kappa_1)$ and $\Sigma_2^{-1}(\underline{\sigma}_2^u(\kappa_2) - \epsilon, \kappa_2) > \Sigma_1(\underline{\sigma}_2^u(\kappa_2) - \epsilon, \kappa_1)$ (graphically, point A is to the left of point B in Figure 7), and Σ_1 and Σ_2 are continuous, it follows from the Intermediate Value Theorem that they must cross at least once with the slope condition being satisfied (see Figure 7, right panel). Thus, a stable equilibrium

exists when $\kappa_1 < \bar{\kappa}_1(\kappa_2)$. Moreover, in any equilibrium there is no excess capacity at either college, since $\Sigma_2^{-1}(-\infty, \kappa_2) < \Sigma_1(-\infty, \kappa_1)$.

Hence, a stable equilibrium exists for any $\kappa_2 \in (0, 1)$. Capacities are exactly filled when $\kappa_1 \leq \bar{\kappa}_1(\kappa_2)$, while there can be excess capacity at college 2 whenever $\kappa_1 > \bar{\kappa}_1(\kappa_2)$.

Using $\kappa_2 = \mathcal{E}_1(\underline{\sigma}_1^l(\kappa_2), -\infty)$, one can show that $\bar{\kappa}_1(\kappa_2)$ is equal to $1 - \kappa_2$ plus the mass of students who only applied to college 1 and were rejected by it. The latter goes to zero as c vanishes, for everybody applies to both colleges in the limit. Therefore, $\bar{\kappa}_1(\kappa_2)$ converges to $1 - \kappa_2$ as c goes to zero. \square

A.8 Sorting Equilibrium Implies Stochastic Dominance of Types

To justify our focus on this *ex-ante* notion, we observe that this induces *ex-post* sorting in enrollments: the best colleges stochastically get the best students.

Lemma 3 (Sorting and the Caliber Distribution) *In a sorting equilibrium, the caliber distribution at college 1 first-order stochastically dominates that at college 2.*

Proof: A monotone student strategy can be represented by the following partition of the set of types: $\Phi = [0, \xi_2)$, $\mathcal{C}_2 = [\xi_2, \xi_B)$, $\mathcal{B} = [\xi_B, \xi_1)$, $\mathcal{C}_1 = [\xi_1, \infty)$, where $\xi_2 < \xi_B < \xi_1$ are implicitly defined by the intersection of the acceptance function with c/u , $\alpha_2 = (1 - c/\alpha_1)/u$ (i.e., $MB_{12} = c$), and $\alpha_2 = c/[u(1 - \alpha_1)]$ (i.e., $MB_{21} = c$), respectively.

Let $f_1(x)$ and $f_2(x)$ be the densities of calibers accepted at colleges 1 and 2, respectively, where we have omitted $(\underline{\sigma}_1, \underline{\sigma}_2)$ to simplify the notation. Formally,

$$f_1(x) = \frac{\alpha_1(x)f(x)}{\int_{\xi_B}^{\infty} \alpha_1(t)f(t)dt} I_{[\xi_B, \infty)}(x) \quad (13)$$

$$f_2(x) = \frac{I_{[\xi_2, \xi_B]}(x)\alpha_2(x)f(x) + (1 - I_{[\xi_2, \xi_B]}(x))\alpha_2(x)(1 - \alpha_1(x))f(x)}{\int_{\xi_2}^{\xi_B} \alpha_2(s)f(s)ds + \int_{\xi_B}^{\xi_1} \alpha_2(s)(1 - \alpha_1(s))f(s)ds} I_{[\xi_2, \xi_1]}(x), \quad (14)$$

where I_A is the indicator function of the set A .

We shall show that, if $x_L, x_H \in [0, \infty)$, with $x_H > x_L$, then $f_1(x_H)f_2(x_L) \geq f_2(x_H)f_1(x_L)$; i.e., $f_i(x)$ is log-supermodular in $(-i, x)$, or it satisfies MLRP. The result follows as MLRP implies that the cdfs are ordered by first-order stochastic dominance.

Using (13) and (14), $f_1(x_H)f_2(x_L) \geq f_2(x_H)f_1(x_L)$ is equivalent to

$$\begin{aligned} & \alpha_{1H}I_{[\xi_B, \infty)}(x_H) \left(I_{[\xi_2, \xi_B]}(x_L)\alpha_{2L} + (1 - I_{[\xi_2, \xi_B]}(x_L))\alpha_{2L}(1 - \alpha_{1L}) \right) I_{[\xi_2, \xi_1]}(x_L) \geq \\ & \alpha_{1L}I_{[\xi_B, \infty)}(x_L) \left(I_{[\xi_2, \xi_B]}(x_H)\alpha_{2H} + (1 - I_{[\xi_2, \xi_B]}(x_H))\alpha_{2H}(1 - \alpha_{1H}) \right) I_{[\xi_2, \xi_1]}(x_H), \end{aligned} \quad (15)$$

where $\alpha_{ij} = \alpha_i(x_j)$, $i = 1, 2$, $j = L, H$. It is easy to show that the only nontrivial case is when $x_L, x_H \in [\xi_B, \xi_1]$ (in all the other cases, either both sides are zero, or only the right side is). If $x_L, x_H \in [\xi_B, \xi_1]$, then (15) becomes $\alpha_{1H}\alpha_{2L}(1 - \alpha_{1L}) \geq \alpha_{1L}\alpha_{2H}(1 - \alpha_{1H})$, or

$$\begin{aligned} & (1 - G(\underline{\sigma}_1 | x_H))(1 - G(\underline{\sigma}_2 | x_L))G(\underline{\sigma}_1 | x_L) \geq \\ & (1 - G(\underline{\sigma}_1 | x_L))(1 - G(\underline{\sigma}_2 | x_H))G(\underline{\sigma}_1 | x_H). \end{aligned} \quad (16)$$

Since $g(\sigma | x)$ satisfies MLRP, it follows that $G(\sigma | x)$ is decreasing in x , and hence $G(\underline{\sigma}_1 | x_L) \geq G(\underline{\sigma}_1 | x_H)$. Next, $1 - G(\sigma | x)$ is log-supermodular in (x, σ) , and hence

$$(1 - G(\underline{\sigma}_1 | x_H))(1 - G(\underline{\sigma}_2 | x_L)) \geq (1 - G(\underline{\sigma}_1 | x_L))(1 - G(\underline{\sigma}_2 | x_H)),$$

as $\underline{\sigma}_1 > \underline{\sigma}_2$ in a sorting equilibrium. Thus, (16) is satisfied, thereby proving that $f_i(x)$ is log-supermodular in $(-i, x)$, and so F_1 first-order stochastically dominates F_2 . \square

A.9 Conditions for Sorting to Fail: Proof of Theorem 4

PART (a): COLLEGE 2 IS TOO GOOD. Fix any κ_1, κ_2 , such that $\kappa_1 + \kappa_2 < 1$. We shall proceed in two steps. First, we show that since $u > 0.5$, we can use Theorem 1 to construct a non-sorting equilibrium in which colleges' behavior is monotone but students' is not. Second, we show that all equilibria induce the same type of behavior.

STEP 1: TOWARDS AN ACCEPTANCE FUNCTION. When $u > 0.5$, the secant from the origin to $MB_{12} = c$ falls as α_1 tends to $c/(1 - u)$ — as in the left panel of Figure 3. So for some $\underline{z} < c/(1 - u)$ s, a line from the origin to $(\underline{z}, 1)$ slices the MB_{12} curve twice. This would imply non-monotone student behavior if that line belonged to the acceptance function, such as: $h: [0, 1] \rightarrow [0, 1]$ by $h(\alpha) = \alpha/\underline{z}$ and on $[0, \underline{z})$, and $h(\alpha_1) = 1$ for $\alpha_1 \geq \underline{z}$.

STEP 2: A PIECEWISE-LINEAR ACCEPTANCE CHANCE α_1 . Choose $\underline{\xi}$ and $\bar{\xi}$ that uniquely solve $\kappa_1 = \int_{\underline{\xi}}^{\infty} f(x)dx$ and $\kappa_2 = \int_{\underline{\xi}}^{\bar{\xi}} f(x)dx$. Set $\alpha_1(x) = 0$ for $x < \underline{\xi}$. This function then jumps up to the rising line segment $\alpha_1(x) = \omega(x)\underline{z} + (1 - \omega(x))c/(1 - u)$ for $x \in [\underline{\xi}, \bar{\xi})$, where $\omega(x) \equiv (\bar{\xi} - x)/(\bar{\xi} - \underline{\xi})$. Lastly, α_1 jumps up $\alpha_1(x) = 1$ for $x > \bar{\xi}$.

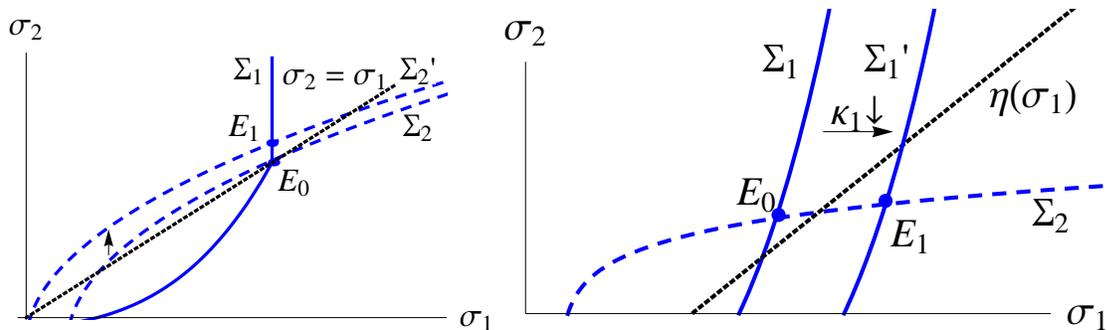


Figure 8: **Existence of Sorting and Non-Sorting Equilibria** In the left panel, we depict the non-sorting equilibrium constructed in the proof of Theorem 4. As κ_2 decreases, Σ_2 shifts up, leading to a non-sorting equilibrium at E_1 . The right panel illustrates the proof of Theorem 5. As κ_1 falls, the equilibrium standards at E_1 are guaranteed to satisfy $\underline{\sigma}_2 < \eta(\underline{\sigma}_1)$, thereby obtaining a sorting equilibrium.

STEP 3: STUDENT BEHAVIOR. Observe that $h(0) = 0$ and $h(1) = 1$, and that h is weakly increasing, with both $h(\alpha)/\alpha$ and $[1 - h(\alpha)]/[1 - \alpha]$ weakly decreasing. In this sense, h is a weakly regular function. This suggests that we set $\alpha_2(x) \equiv h(\alpha_1(x))$.

In this case, students $x \in [0, \underline{\xi})$ are accepted with zero chance at either college, and so apply nowhere. Next, because $h(\underline{z}) = 1$, any calibers $x \in [\underline{\xi}, \bar{\xi})$ are accepted with chance one at college 2, and with chance between \underline{z} and $c/(1 - u)$ at college 1. Further, any student $\bar{\xi}$ strictly prefers just to apply to college 2 (as in Figure 3). To see this, observe that $MB_{12} = (c/(1 - u))(1 - \alpha_2 u) > (c/(1 - u))(1 - u) = c$ when $\alpha_2 = c/(1 - u)$ and $\alpha_1 = 1$. Lastly, calibers $x > \bar{\xi}$ are always accepted at college 1 and only apply there.

STEP 4: SMOOTHING THE CONSTRUCTION. By smoothly bending the function h inside $(0, 1)$, an arbitrarily close function h^* is also regular. Next, we create a continuous and smooth acceptance chance $\tilde{\alpha}$. Any four small enough numbers, $\underline{\varepsilon}, \underline{\varepsilon}, \bar{\varepsilon}, \bar{\varepsilon} > 0$, yield a unique Bezier approximation $\tilde{\alpha}$ tangent to α at the four points $\underline{\xi} - \underline{\varepsilon}, \underline{\xi} + \underline{\varepsilon}, \bar{\xi} - \bar{\varepsilon}, \bar{\xi} + \bar{\varepsilon}$. Then $\tilde{\alpha}_1$ — and so the enrollment at college 1 — falls in $\bar{\varepsilon}$, and rises in $\bar{\varepsilon}$. Also, $\tilde{\alpha}_2 = h^*(\tilde{\alpha}_1)$ falls in $\bar{\varepsilon}$, and rises in $\bar{\varepsilon}$, and it also falls in $\underline{\varepsilon}$, and rises in $\underline{\varepsilon}$. Enrollment at college 2 shares this monotonicity, but the enrollment at college 1 is unaffected by $\underline{\varepsilon}$ and $\underline{\varepsilon}$.

Fix a small $\bar{\varepsilon} > 0$. Choose $\bar{\varepsilon} > 0$ so that college 1 still fills its capacity. WLOG, enrollment at college 2 has fallen. Then choose $\underline{\varepsilon} > 0$ large enough so that college 2 is over its capacity, then for some $\underline{\varepsilon} > 0$, the former enrollment at college 2 is restored.

Theorem 1 now yields a signal density $g(\sigma|x)$ and thresholds $\underline{\sigma}_1 > \underline{\sigma}_2$ such that h^* is the acceptance function. We have thus constructed a nonsorting equilibrium. \square

PART (b): COLLEGE 2 IS TOO SMALL. The proof is constructive, exploiting our graphical analysis. To begin, consider the point $(\alpha_1, \alpha_2) = (c, c/u)$ on the line $\alpha_2 = \alpha_1/u$. Then the acceptance function evaluated at $\alpha_1 = c$ is below c/u if and only if

$$\psi(c, \underline{\sigma}_1, \underline{\sigma}_2) < c/u. \quad (17)$$

We will restrict attention to pairs $(\underline{\sigma}_1, \underline{\sigma}_2)$ such that (17) holds. In this case, any student who applies to college starts by adding college 1 to his portfolio, and this happens as soon as $\alpha_1(x) \geq c$, or when $x \geq \xi(c, \underline{\sigma}_1)$. Then enrollment at college 1 is given by

$$\mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2) = \int_{\xi(c, \underline{\sigma}_1)}^{\infty} (1 - G(\underline{\sigma}_1|x))f(x)dx,$$

which is independent of $\underline{\sigma}_2$. Thus, for any capacity $\kappa_1 \in (0, 1)$, a unique threshold $\underline{\sigma}_1(\kappa_1)$ solves $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$. (Graphically, the Σ_1^{-1} function is “vertical” in the region in which (17) holds, since the applicant’s pool at college 1 does not depend on the admissions threshold set by college 2.)

The analysis above allows us to restrict attention to finding equilibria within the set of thresholds $(\underline{\sigma}_1, \underline{\sigma}_2)$ such that $\underline{\sigma}_1 = \underline{\sigma}_1(\kappa_1)$ and $\underline{\sigma}_2$ satisfies $\psi(c, \underline{\sigma}_1(\kappa_1), \underline{\sigma}_2) < c/u$.

Enrollment at college 2 is given by

$$\mathcal{E}_2(\underline{\sigma}_1(\kappa_1), \underline{\sigma}_2) = \int_B G(\underline{\sigma}_1(\kappa_1)|x)(1 - G(\underline{\sigma}_2|x))f(x)dx,$$

which is continuous, decreasing in $\underline{\sigma}_2$, and increasing in $\underline{\sigma}_1$ (see Theorem 2). Thus, $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1(\kappa_1), \underline{\sigma}_2)$ yields $\underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1(\kappa_1), \kappa_2)$, which is strictly decreasing in κ_2 .

Given κ_1 , let $\bar{\kappa}_2(\kappa_1) = E_2(\underline{\sigma}_1(\kappa_1), \underline{\sigma}_1(\kappa_1))$ be the level of college 2 capacity so that equilibrium ensues if both colleges set the same threshold.¹⁷ Since Σ_2 strictly falls in κ_2 , for any $\kappa_2 < \bar{\kappa}_2(\kappa_1)$, an equilibrium exists with $\underline{\sigma}_2 > \underline{\sigma}_1(\kappa_1)$. Then (a) for any $\kappa_1 \in (0, 1)$ and $\kappa_2 \in (0, \bar{\kappa}_2(\kappa_1)]$, there is a unique equilibrium with $\underline{\sigma}_1 = \underline{\sigma}_1(\kappa_1)$ and $\underline{\sigma}_2 \geq \underline{\sigma}_1(\kappa_1)$, having (b) non-monotone college and student behavior (Figure 8, left).¹⁸ \square

¹⁷It is not difficult to show that $\psi(c, \underline{\sigma}_1, \underline{\sigma}_2) < c/u$ is satisfied if $\underline{\sigma}_2 \geq \underline{\sigma}_1(\kappa_1)$.

¹⁸We are not ruling out the existence of another equilibrium that does not satisfy (17).

A.10 Conditions for Equilibrium Sorting: Proof of Theorem 5

Fix $\kappa_2 \in (0, 1)$. We first show that the stable equilibrium with no excess capacity whose existence was proven in Theorem 3 is also sorting when the capacity of college 1 is sufficiently small. More precisely, we will show that there is a threshold $\underline{\kappa}_1(\kappa_2)$, smaller than the bound $\bar{\kappa}_1(\kappa_2)$ defined in the proof of Theorem 3, such that for all $\kappa_1 \in (0, \underline{\kappa}_1(\kappa_2))$, there is a pair of admissions thresholds $(\underline{\sigma}_1, \underline{\sigma}_2)$ that satisfies $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$, $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$, and $\underline{\sigma}_2 < \eta(\underline{\sigma}_1)$ (i.e., a sorting equilibrium exists). Moreover, at this pair $\partial \Sigma_1 / \partial \underline{\sigma}_2 \partial \Sigma_2 / \partial \underline{\sigma}_1 < 1$ (i.e., the equilibrium is also stable).

The proof uses the following properties of $\underline{\sigma}_2 = \eta(\underline{\sigma}_1)$, which are not difficult to verify: (a) it is strictly increasing in $\underline{\sigma}_1$; (b) $\underline{\sigma}_2$ goes to infinity as $\underline{\sigma}_1$ goes to infinity; (c) $\underline{\sigma}_1$ goes to minus infinity as $\underline{\sigma}_2$ goes to minus infinity (take the inverse of $\eta(\cdot)$).

For any $\kappa_1 \in (0, \bar{\kappa}_1(\kappa_2))$, we know from Theorem 3 that there exists a pair $(\underline{\sigma}_1, \underline{\sigma}_2)$ that satisfies $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$ and $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$, with $\partial \Sigma_1 / \partial \underline{\sigma}_2 \partial \Sigma_2 / \partial \underline{\sigma}_1 < 1$. We will show that this pair is a sorting equilibrium if κ_1 is sufficiently small.

To prove it, let $M(\kappa_2) = \{(\underline{\sigma}_1, \underline{\sigma}_2) \mid \kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2) \text{ and } \underline{\sigma}_2 = \eta(\underline{\sigma}_1)\}$. Graphically, this is the set of all pairs at which $\underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1, \kappa_2)$ crosses $\underline{\sigma}_2 = \eta(\underline{\sigma}_1)$.

If $M(\kappa_2) = \emptyset$ we are done, for this implies that $\underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1, \kappa_2) < \eta(\underline{\sigma}_1)$ for all values of $\underline{\sigma}_1$, including those at which $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$ and $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$. To see this, note that (i) as $\underline{\sigma}_2$ goes to minus infinity $\underline{\sigma}_1 = \eta^{-1}(\underline{\sigma}_2)$ goes to minus infinity, while we proved in Theorem 3 that $\underline{\sigma}_1 = \Sigma_2^{-1}(\underline{\sigma}_2, \kappa_2)$ converges to $\underline{\sigma}_1^l(\kappa_2) > -\infty$. Also, (ii) $\underline{\sigma}_2 = \eta(\underline{\sigma}_1)$ goes to infinity as $\underline{\sigma}_1$ goes to infinity, while we proved in Theorem 3 that $\underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1, \kappa_2)$ converges to $\underline{\sigma}_2^u(\kappa_2) < \infty$. Properties (i) and (ii) reveal that if Σ_2 and η do not intersect, then Σ_2 is everywhere below η .

If $M(\kappa_2) \neq \emptyset$, let $(\underline{\sigma}_1^s(\kappa_2), \underline{\sigma}_2^s(\kappa_2)) = \sup M(\kappa_2)$, which is finite by property (b) of $\eta(\underline{\sigma}_1)$ and since $\underline{\sigma}_2 = \Sigma_2(\underline{\sigma}_1, \kappa_2)$ converges to $\underline{\sigma}_2^u(\kappa_2) < \infty$ as $\underline{\sigma}_1$ goes to infinity (see the proof of Theorem 3). Now, as κ_1 goes to zero, $\underline{\sigma}_1 = \Sigma_1(\underline{\sigma}_2, \kappa_1)$ goes to infinity for any value of $\underline{\sigma}_2$, for college 1 becomes increasingly more selective to fill its dwindling capacity. Since $\underline{\sigma}_2$ is bounded above by $\underline{\sigma}_2^u(\kappa_2)$, there exists a threshold $\underline{\kappa}_1(\kappa_2) \leq \bar{\kappa}_1(\kappa_2)$ such that, for all $\kappa_1 \in (0, \underline{\kappa}_1(\kappa_2))$, the aforementioned pair $(\underline{\sigma}_1, \underline{\sigma}_2)$ that satisfies $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1, \underline{\sigma}_2)$ and $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1, \underline{\sigma}_2)$ is strictly bigger than $(\underline{\sigma}_1^s(\kappa_2), \underline{\sigma}_2^s(\kappa_2))$, thereby showing that it also satisfies $\underline{\sigma}_2 < \eta(\underline{\sigma}_1)$. Hence, a sorting stable equilibrium exists for any κ_2 and $\kappa_1 \in (0, \underline{\kappa}_1(\kappa_2))$, with both colleges filling their capacities (see Figure 8, right panel).

To finish the proof, notice that, if there are multiple equilibria, both colleges fill their capacity in all of them (graphically, the conditions on capacities ensure that Σ_2 starts above Σ_1 for low values of $\underline{\sigma}_1$ and eventually ends below it). Moreover, adjusting the bound $\underline{\kappa}_1(\kappa_2)$ downward if needed, all equilibria are sorting (graphically, for κ_1 sufficiently small, the set of pairs at which Σ_1 and Σ_2 intersect are all below η). \square

A.11 Capacities and Standards: Proof of Theorem 6

Let $(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$ be stable equilibrium: i.e., $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$, $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$. By definition of stable equilibrium, $\Lambda \equiv \partial \mathcal{E}_1 / \partial \underline{\sigma}_1^e \times \partial \mathcal{E}_2 / \partial \underline{\sigma}_2^e - \partial \mathcal{E}_2 / \partial \underline{\sigma}_1^e \times \partial \mathcal{E}_1 / \partial \underline{\sigma}_2^e > 0$. Differentiating $\kappa_1 = \mathcal{E}_1(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$ and $\kappa_2 = \mathcal{E}_2(\underline{\sigma}_1^e, \underline{\sigma}_2^e)$ with respect to κ_i yields

$$\Lambda \frac{\partial \underline{\sigma}_i^e}{\partial \kappa_i} = \frac{\partial \mathcal{E}_j}{\partial \underline{\sigma}_j^e} < 0 \quad \text{and} \quad \Lambda \frac{\partial \underline{\sigma}_j^e}{\partial \kappa_i} = -\frac{\partial \mathcal{E}_j}{\partial \underline{\sigma}_i^e} < 0.$$

Thus, both admissions thresholds fall when the capacity of college $i = 1, 2$ increases. \square

A.12 Standards and Application Costs: Proof of Theorem 7

Let $c = c_1 = c_2$ be the initially equal applications costs. If c_i increases, the applicant pool at college i shrinks, and thus the Σ_i curve shifts down, while Σ_j , $j \neq i$, remains unchanged. It follows immediately that the functions now cross at a lower threshold pair, and consequently that $(\underline{\sigma}_1, \underline{\sigma}_2)$ both fall.

In a sorting equilibrium, the applicant pool at college 1 consists of calibers $x \in [\xi_B, \infty)$. From the last part, any cost increase depresses $\underline{\sigma}_1$ in equilibrium. It follows that ξ_B rises in equilibrium — since college 1 has the same capacity as before, if it is to have lower standards, it must also have fewer applicants. Let $(\xi_B^0, \underline{\sigma}_1^0)$ be the old equilibrium pair and $(\xi_B^1, \underline{\sigma}_1^1)$ the new one, with $\xi_B^0 < \xi_B^1$ and $\underline{\sigma}_1^0 > \underline{\sigma}_1^1$. Then the distribution function of enrolled students at college 1 under equilibrium $i = 0, 1$ is:

$$F_1^i(x) = \frac{\int_{\xi_B^i}^x (1 - G(\underline{\sigma}_1^i | t)) f(t) dt}{\int_{\xi_B^i}^{\infty} (1 - G(\underline{\sigma}_1^i | t)) f(t) dt}$$

We must show $F_1^1(x) \leq F_1^0(x)$ for all $x \in [\xi_B^1, \infty)$. For any x the denominators on both sides equal k_1 , so cancel them. Now notice that $0 = F_1^1(\xi_B^1) < F_1^0(\xi_B^1)$ and

$\lim_{x \rightarrow \infty} F_1^1(x) = \lim_{x \rightarrow \infty} F_1^0(x) = 1$. Since both functions are continuous in x , it will suffice to show that $\partial F_1^1 / \partial x > \partial F_1^0 / \partial x$ for all $x \in [\xi_B^1, \infty)$ to conclude that $F_1^1(x) < F_1^0(x)$ except in the limit. But this requires $(1 - G(\underline{\sigma}_1^1 | x)) f(x) > (1 - G(\underline{\sigma}_1^0 | x)) f(x)$, which is immediately true since $\underline{\sigma}_1^1 < \underline{\sigma}_1^0$. \square

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