

# Optimal Unemployment Insurance with Hidden Search Effort and Hidden Savings\*

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## Abstract

This paper considers optimal unemployment insurance (UI) with unobserved search effort and savings. Assuming linear search costs, it develops new variational arguments which identify a solution to the necessary conditions for optimality. The structure of the optimal UI program has unusual, yet highly intuitive, properties. A numerical example finds the optimal policy is well approximated by a lump sum severance payment, constant unemployment benefit payments while unemployed and an interest free loan. The lump sum layoff payment compensates the worker against his/her drop in permanent income though being laid-off, while the loan targets the assumed liquidity constraint - that unemployed workers cannot borrow against future earnings.

**Keywords:** Unemployment Insurance, Hidden Search Effort, Hidden Savings, Severance payments.

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# 1 Introduction

This paper is a central building block for the design of optimal unemployment insurance (UI) schemes - it considers the most basic moral hazard problem (search effort is not observed) for the most natural case (workers can save and consumption is not observed). It is perhaps surprising that such a seemingly straightforward problem has been so problematic to solve. Yet the identified solution here is both elegant and intuitive and yields new insights into optimal policy.

Shavell and Weiss (1979) introduced the optimal UI problem where the Planner insures workers against layoff risk but does not observe job search effort. That paper assumes workers cannot save and argues unemployment benefit payments should be frontloaded: early UI payments are relatively generous but UI payments at long durations are reduced to give the unemployed a stronger incentive to go out and look for work. A large policy literature has then asked how quickly should UI payments fall with duration?<sup>1</sup> But ruling out savings behaviour is not only unreasonable, Rogerson (1985) shows it also yields distorted policy prescriptions. Werning (2002) was the first to formally attack the optimal UI problem with hidden savings and used the first order approach to describe the optimal contract. But the essential difficulty is that the worker's search and consumption decision problem is not concave (see for example Lentz and Torbaes (2005)). As the first order approach cannot be applied, identifying a solution to the necessary conditions for optimality is a complex problem. Indeed Kocherlakota (2004) explains why standard recursive mechanism design arguments cannot identify the necessary conditions for optimality (but also see Abrahams and Pavoni (2008)). By restricting attention to the case that search costs are linear, I develop variational arguments which solve the optimal UI problem.

Unemployment insurance essentially contains two separate components. First there is layoff risk: job destruction shocks imply an employed worker faces a fall in permanent income through layoff. Clearly a risk averse employed worker would like to purchase insurance against such risk. But once unemployed, search frictions imply the worker also faces re-employment risk: it takes time to find work and re-employment is also a stochastic process. Unlike a job destruction shock, however, the re-employment shock yields an increase in permanent income. An optimal unemployment insurance program provides insurance against both types of risk, taking into account that job search effort is endogenous and workers can smooth consumption using a savings strategy.

A useful insight for what follows is that efficient insurance implies the marginal utility of consumption should be smooth over the entire "unemployment event". With additively separable preferences, as considered here, this is equivalent to consumption being smooth over the unemployment event (e.g. footnote 1). But when workers

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<sup>1</sup>Indeed with no savings, Coles (2008) establishes that optimality in a standard matching equilibrium implies an initial replacement rate of one. Thus early UI payments perfectly smooth consumption across the job destruction shock.

use savings strategies, optimal savings behaviour and smooth consumption would suggest permanent income must also be smooth. Although computing the optimal UI program is complex, a numerical example finds the optimal policy is extremely well approximated by an appropriate mix of

- (i) a lump sum severance payment which compensates the laid-off worker for the drop in permanent income through being laid-off,
- (ii) an interest free loan which is repaid when re-employed, and
- (iii) constant unemployment benefit while unemployed.

Indeed this policy approximation is so good, the resulting loss in efficiency lies within computing rounding error.

The severance layoff payment would seem to play two important roles. First by smoothing permanent income across the job destruction shock, the severance layoff payment and an optimal (dis)-saving strategy ensure consumption is smooth across that shock. But second, by frontloading UI payments to an initial lump sum, a severance payment minimises the distortion on continuing job search effort. Somewhat surprisingly, then, allowing savings reinforces the original Shavell and Weiss (1979) argument for frontloading UI payments.

The severance layoff payment of course targets layoff risk. The interest free loan instead targets the assumed borrowing constraint, that unemployed workers cannot borrow against future earnings. Finally the (constant) UI payments target re-employment risk. It is this partial insurance against re-employment risk which implies the UI program is less efficient than the full information benchmark. Nevertheless the numerical example finds that when workers use an optimal savings strategy and payments (i)-(iii) are appropriately chosen, the efficiency loss relative to the full information benchmark is very small.

Formally this paper develops variational arguments which identify a solution to the necessary conditions for optimality when search costs are linear and effort  $k$  is bounded,  $k \leq 1$ . Although Kocherlakota (2004) conjectures an optimal policy for this case, I show it is not optimal. Instead the improved policy identified here has rather unusual properties. For example along the optimal path, the worker strictly prefers to choose maximal search effort  $k = 1$  at all durations. Standard arguments would then suggest marginally increasing UI payments at positive durations to improve the quality of the insurance program noting that, along the optimal path, the worker would still prefer to choose maximal search effort  $k = 1$ . But this argument is fallacious. It turns out that, at the policy optimum, any policy variation which potentially improves insurance efficiency instead implies the laid-off worker switches to a completely different search and consumption strategy. The non-concavity of the worker's search and consumption decision problem implies standard marginal arguments do not apply.

Two particular types of search strategies constrain the optimal policy design. One is a retirement strategy - rather than actively seek work, the worker never searches for employment and instead chooses a low level of consumption consistent with the

permanent income which accrues from future UI receipts. The retirement strategy binds on the UI design as the optimal UI contract implements a loan scheme - the worker pays back part of the UI receipts consumed while unemployed through a higher income tax when re-employed. As the retirement strategy allows the worker to default on all such loans, the retirement option is binding on the optimal policy design.

The second constraining strategy is a “holiday” strategy: the worker does not search for durations less than some duration  $\tau^H > 0$  and switches to active search thereafter (always choose  $k = 1$ ). At the policy optimum, it turns out the laid-off worker is indifferent between the Planner’s preferred strategy - that the worker always searches- an infinity of holiday strategies  $\tau^H \in (0, \infty)$  and the retirement strategy (never search and default on all loans). If, for example, the Planner tries to improve the insurance properties of the UI scheme by increasing UI payments at duration  $\tau^H$ , the holiday strategy  $\tau^H$  then strictly dominates the active search strategy (always search). As the switch to a holiday strategy implies a discrete increase in the cost of the UI program, the identified policy satisfies the necessary conditions for optimality. Furthermore the numerical example finds that policy is well approximated by an appropriate combination of severance layoff payment, interest free loan and constant UI payments.

There are two important caveats to these insights. Hopenhayn and Nicolini (1997) demonstrated, with no savings, that re-employment taxes yield large welfare gains. But here with savings and an appropriate severance layoff payment, the central role of the re-employment tax is to implement a loans program. This result, however, begs the question why is the Planner willing to loan funds to unemployed workers when the private banking sector is assumed unwilling to do so? I shall return to this issue in the conclusion.

The second caveat is that firms might face productivity shocks rather than pure job destruction shocks (e.g. Mortensen and Pissarides (1994)). Lump sum severance payments would seem to exacerbate the temporary layoff problem. Feldstein (1976) argues that without experience rating, firms have an incentive to layoff workers in low productivity states and so use the UI program to subsidise wages. If the Planner in addition pays lump sum severance payments, the temporary layoff problem will clearly worsen. But in interesting work, Pissarides (2004) and Fella (2006) ignore the optimal UI problem considered here and instead assume the UI program simply pays constant  $b$ . Given  $b$ , those papers consider privately optimal contracts between risk neutral firms and risk averse employees. Both papers argue that severance layoff payments are privately optimal. As public insurance crowds out private insurance, the question then is who should insure employed workers against layoff risk? Temporary layoff issues would suggest firms should offer this insurance; i.e. the Planner only offers constant and relatively low UI payments which target re-employment risk. Privately optimal contracting between firms and employees would then imply an employment contract which specifies a severance payment in the event of a layoff. Of course risk

pooling across risk averse firms or other market failures (e.g. further moral hazard, adverse selection or bankruptcy issues) would suggest an additional insurance role for the Planner. An important contribution of this paper, however, is to establish a framework within which such policy tools might be chosen optimally.

There are several important related literatures. A large part of the matching literature has focussed on how UI payments distort equilibrium wage formation and unemployment. With Nash bargaining as considered in Millard and Mortensen (1997), Fredriksson and Holmlund (2001), Cahuc and Lehmann (2002) it is typically argued that UI payments might increase with duration: early UI payments raise the value of unemployment too high, so that it is better to keep those payments low and so reduce the distortion on equilibrium wages. With strategic bargaining Coles and Masters (2006) instead establish that UI payments around the one-year mark distort wages the most.<sup>2</sup> In contrast, Hosios (1990) considers how job creation subsidies and optimal UI can be used to generate efficient equilibrium outcomes. Coles (2008) extends that analysis to the case that workers are risk averse but cannot save.

A different approach instead assumes search effort is fixed but search is sequential and wage offers are drawn from an exogenous distribution. Again with no savings, Mortensen (1977), van den Berg (1990) consider how duration dependent UI distorts job seeker reservation wages and thus worker re-employment rates. Albrecht and Vroman (2005) consider an equilibrium where UI payments expire according to a Poisson process. A different approach assumes a constant UI program but notes that a worker might reject a low wage offer and continue search. With no UI, workers might have too low reservation wages and it is then efficient to subsidise search. Papers in this literature include Mortensen and Pissarides (1999), Marimon and Zilibotti (1999), Shimer and Werning (2005).

There is also a literature on optimal unemployment insurance with moral hazard when workers self-insure by accumulating savings while employed, and dissave while unemployed. Key papers include Hansen and Imrohoroglu (1992), Abdulkadiroglu et al (2002), Wang and Williamson (2002), Weitzblum (2003). These models are much too complex to solve analytically and so insights are obtained through simulations. The results of Abdulkadiroglu et al (2002), however, are particularly germane. That paper sets re-employment taxes to zero and finds, using a discrete time framework, that the optimal UI program has an extremely large first period payment, and low payments thereafter which (very slowly) increase with duration. This is highly resonant with the results identified here: that once re-employment taxes are set equal to zero, layoff severance payments become optimal.

A different literature focusses on the effect of layoff payments on market turnover; e.g. Lazear (1990), Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993), Millard and Mortensen (1997), Pissarides (2004), Fella (2006). Lazear (1990) argues that if workers are risk neutral and contracts are efficient, then a legislated firing

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<sup>2</sup>Coles and Masters (2007) further establish that via re-entitlement effects, duration dependent UI programs can stabilise employment over the cycle.

cost, paid by the firm to the worker on layoff, has no real effects - wage bargaining at the point of hire implies the negotiated wage falls one-for-one with the increased severance fee. But with risk averse workers and savings, lump sum layoff fees have desirable insurance properties: the laid-off worker can smooth consumption using a suitable dissavings strategy and, unlike UI payments, a severance fee does not distort search effort.

The paper is organised as follows. The next section describes the model and optimal worker behaviour when UI payments and re-employment taxes are duration independent. This section not only explains why workers might use “holiday strategies” it also identifies necessary conditions for optimal holiday strategies. Section 3 formally describes the optimal UI problem and section 4 identifies a solution to the necessary conditions for optimality. Section 5 uses a numerical example to illustrate the argument and section 6 discusses the policy implications. Section 7 concludes.

## 2 Model.

Time is continuous and has an infinite horizon. The worker (the agent) is strictly risk averse, infinitely lived and has subjective rate of time preference  $r$  which is also the market interest rate. The worker is initially unemployed and, being unable to borrow against future earnings, must hold assets  $A \geq 0$  at all times.

The Planner designs an unemployment insurance (UI) scheme to insure the laid-off worker against unemployment risk. Following Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Werning (2002), Kocherlakota (2004) I simplify by considering only a single unemployment spell; i.e. re-employment is an absorbing state. While unemployed there is a moral hazard problem - the job search effort of the worker is not observed by the Planner. This hidden action problem implies UI payments  $b(\cdot)$  cannot be conditioned on search effort. There are also hidden savings - the Planner observes neither consumption nor assets  $A$ .

At each unemployment duration the worker chooses search effort  $k$  and consumption  $x \geq 0$ . Search is a binary choice variable,  $k \in \{0, 1\}$ , and so the worker either does not search or searches for work. Typically it is instead assumed that  $k$  is a continuous choice variable with search cost  $c(k)$ . It is important to note, however, that the model is also consistent with  $k$  being a continuous choice variable  $k \in [0, 1]$  with linear search costs.<sup>3</sup> Thus the results obtained in Kocherlakota (2004) are pertinent.

Given effort  $k \in \{0, 1\}$ , the worker becomes employed according to a Poisson process with parameter  $\gamma k$ .  $\gamma$  describes how easy it is to find work and, in a matching equilibrium, it depends on labour market tightness. Coles (2008) provides a complete description of optimal unemployment policy for the case when  $\gamma$  is endogenously determined but workers cannot save. To abstract from those policy issues I assume

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<sup>3</sup>In continuous time, the worker can always convexify search effort by choosing  $k = 1$  for fraction  $\theta$  of any time period  $\Delta$ .

here that  $\gamma > 0$  is exogenous. The flow cost of search is zero if  $k = 0$  and is  $c > 0$  if  $k = 1$ . If the worker is indifferent between choosing  $k = 0$  or 1 assume the worker chooses  $k = 1$ . Once re-employed the worker earns exogenous wage  $w > 0$ .

Preferences are additively and time separable. If the worker consumes  $x \geq 0$  and searches with effort  $k \in \{0, 1\}$ , the worker obtains flow utility  $u(x) - ck$ .  $u(\cdot)$  describes the utility from consumption, is strictly increasing, strictly concave, satisfies  $\lim_{x \rightarrow 0^+} u'(x) = \infty$  and is twice differentiable for all  $x > 0$ .

The UI program is denoted  $B = \{B_0, b(\cdot), D(\cdot)\}$  and has three components:

(i)  $B_0 \geq 0$  is a lump sum layoff payment which an unemployed worker receives at duration  $\tau = 0$ ;

(ii)  $b(\tau) \geq 0$  describes the flow UI payment to job seekers at unemployment durations  $\tau > 0$  and

(iii)  $D(\tau)$  is a lump sum tax deduction on re-employment which depends on the length  $\tau$  of the completed unemployment spell. The tax is implemented by setting an income tax premium  $\hat{t}(\tau)$  on future wages such that  $\hat{t}w = rD(\tau)$ ; e.g. Hopenhayn and Nicolini (1997).

Given layoff payment  $B_0 \geq 0$  and assets  $A_0 \geq 0$  carried over from a previous employment spell, the laid-off worker has initial assets  $A(0) = A_0 + B_0$ . A strong assumption is that  $A_0$  is known by the Planner. In most of the paper I follow Kocherlakota (2004) and assume  $A_0 = 0$ , thus implying the laid-off worker is potentially liquidity constrained. While unemployed, the worker's assets evolve with duration according to

$$\frac{dA}{d\tau} = rA - x + b(\tau), \quad (1)$$

where the price of the consumption good is set to unity. If the job seeker becomes re-employed at duration  $\tau$  with assets  $A \geq 0$ , the optimal savings strategy implies consumption equals permanent income  $w + r(A - D(\tau))$  from then onwards. Thus given  $B$ , the expected lifetime value of becoming re-employed at duration  $\tau$  with assets  $A \geq 0$  is

$$W^E(A, \tau | B) = \frac{u(w + r(A - D(\tau))) - d}{r},$$

where  $d \geq 0$  is the disutility of labour. Given  $B$ , each unemployed worker chooses consumption and job search effort to maximise expected lifetime utility. Given the optimal consumption and search strategy of the worker, the Planner chooses  $B$  to maximise the worker's value of being unemployed at duration  $\tau = 0$ , subject to a budget constraint, that the expected cost of the program cannot exceed (exogenous)  $C_0 \geq 0$ .

## 2.1 Optimal Job Search and Consumption.

Conditional on being unemployed and the UI program  $B$ , let  $W^U(A, \tau | B)$  denote the worker's expected lifetime utility using an optimal savings and job search strategy

given current assets  $A \geq 0$  and unemployment duration  $\tau \geq 0$ . Over arbitrarily small time period  $\Delta > 0$ , the Bellman equation describing  $W^U$  is

$$W^U(A, \tau | B) = \max_{\substack{x \geq 0 \\ k \in \{0,1\}}} \left\{ \begin{array}{l} [u(x) - ck]\Delta \\ + e^{-r\Delta} [(1 - e^{-\gamma k\Delta})W^E(A', \tau + \Delta | B) + e^{-\gamma k\Delta}W^U(A', \tau + \Delta | B)] \end{array} \right\} \quad (2)$$

subject to

$$A' = e^{r\Delta}[A - x\Delta + b(\tau)\Delta] \geq 0.$$

The first term in (2) describes the flow payoff while unemployed. The second describes the expected continuation payoff where the worker has continuation assets  $A'$  and with probability  $(1 - e^{-\gamma k\Delta})$  finds employment over the next instant  $[\tau, \tau + \Delta)$  and so enjoys  $W^E(\cdot)$ , otherwise he remains unemployed and continues search. Note that  $W^E(\cdot)$  has already been determined. Thus in the limit as  $\Delta \rightarrow 0$ , the Bellman equation implies a pair of policy rules  $x = x^*(A, \tau | B), k = k^*(A, \tau | B)$  which describe optimal consumption and optimal search while unemployed.

It is useful to define actions and payoffs along the optimal path. Let  $A^U(\tau | A_0, B)$  denote the asset path  $A(\cdot)$  while unemployed when the worker uses the above optimal policy rules. Given that asset path, then the consumption and search effort paths while unemployed are denoted

$$\begin{aligned} x^U(\tau | A_0, B) &= x^*(A^U(\tau | \cdot), \tau | B), \\ k^U(\tau | A_0, B) &= k^*(A^U(\tau | \cdot), \tau | B). \end{aligned}$$

Also define

$$\begin{aligned} \mu(\tau | A_0, B) &= \frac{\partial W^U(A^U(\tau | \cdot), \tau | B)}{\partial A} \\ V^U(\tau | A_0, B) &= W^U(A^U(\tau | \cdot), \tau | B) \end{aligned}$$

so that  $\mu$  describes the marginal value of savings and  $V^U$  describes the value of being unemployed along the optimal path.

In the limit as  $\Delta \rightarrow 0$ , the Bellman equation implies  $k = 1$  is privately optimal if and only if

$$[W^E(A, \tau | B) - W^U(A, \tau | B)] \geq c/\gamma;$$

i.e. the surplus through re-employment must be sufficiently large. I shall say the **no-holiday constraint** is satisfied if and only if

$$W^E(A^U(\cdot), \tau | \cdot) - V^U(\tau | \cdot) \geq c/\gamma \text{ for all } \tau \geq 0; \quad (3)$$

i.e., the no-holiday constraint implies  $k^U = 1$  along the optimal path. Conversely the worker chooses  $k = 0$  whenever  $W^E(\cdot) - W^U(\cdot) < c/\gamma$ . Claim 1 now describes the worker's optimal savings strategy along the optimal path.

**Claim 1.** Optimal Consumption.

Given the optimal search effort path  $k^U(\cdot)$ , the optimal consumption and asset paths  $x^U, A^U$  satisfy the following conditions:

$$u'(x^U) = \mu \quad (4)$$

where  $\mu$  evolves according to

$$\mu = u'(b(\tau)) \text{ while } A \geq 0 \text{ is binding}$$

$$\gamma k^U \mu - \frac{d\mu}{d\tau} = \gamma k^U u'(w + r(A^U - D(\tau))) \text{ while } A \geq 0 \text{ is non-binding} \quad (5)$$

and  $A^U$  evolves according to:

$$rA^U - \frac{dA^U}{d\tau} = x^U - b(\tau) \quad (6)$$

subject to the initial condition  $A^U(0) = A_0 + B_0$ .

When the worker is liquidity constrained, he/she consumes  $x = b(\tau)$ . But when liquidity unconstrained, he/she consumes so that the marginal utility of consumption equals the marginal value of savings where, along the optimal path, today's marginal value of savings ( $\mu$ ) equals tomorrow's expected marginal value of savings. The proof of claim 1 shows  $\mu$  must then satisfy the differential equation (5). Note that whenever  $k^U = 0$ , (5) implies  $\mu(\cdot)$  is (locally) constant and so optimal consumption  $x^U(\cdot)$  is constant during such phases.

The search effort path  $k^U$  is not determined in claim 1. If  $B$  satisfies the no-holiday constraint then  $V^U$  evolves according to

$$(r + \gamma)V^U - \frac{dV^U}{d\tau} = u(x^U) - c + \gamma \frac{u(w + r(A^U - D)) - d}{r}.$$

## 2.2 An Illustrative Example

Anticipating the results and insights below, it is useful first to illustrate optimal worker behaviour when  $b(\cdot) = \underline{b}$  and  $D(\cdot) = D_0$  are duration independent. I show in the proof of Theorem 1 that there are two classes of optimal strategies. One is a "retirement strategy" where the unemployed worker never seeks employment. The second is a "holiday" strategy where the unemployed worker seeks employment at some future (finite) duration, including the case that he/she searches immediately (no holiday). The optimal strategy choice depends on the worker's initial asset position  $A(0)$ . Theorem 1 identifies the following asset partition:

(i) if  $A(0) \in [0, A^H]$ , the no holiday constraint is satisfied; i.e. along the optimal path the worker always chooses  $k = 1$ ;

(ii) if  $A(0) \in (A^H, A^R)$ , the worker uses a holiday strategy; i.e. along the optimal path the worker initially chooses  $k = 0$  and savings fall over time until duration  $\tau^H$ , where assets  $A(\tau^H) = A^H$ , at which point the worker switches to  $k = 1$  thereafter;

(iii) if  $A(0) \geq A^R$ , the worker uses a retirement strategy; i.e. along the optimal path the worker always chooses  $k = 0$ .

Of course these asset thresholds  $A^H, A^R$  depend on  $\underline{b}$  and  $D_0$ . In the next section these asset thresholds are duration dependent as  $b(\cdot)$  and  $D(\cdot)$  are duration dependent. Nevertheless to illustrate ideas, it is useful to start with the simpler duration independent case.

The characterisation of the optimal retirement strategy is simple: the unemployed worker always consumes permanent income  $\underline{b} + rA$ . Part III of Theorem 1 establishes that the retirement strategy is optimal for sufficiently high assets  $A \geq A^R$ . The proof of Theorem 1 now establishes the optimal holiday strategy for lower assets  $A < A^R$ .

**Theorem 1.** Optimal Job Search and Consumption

Given any UI scheme  $B = \{B_0, \underline{b}, D_0\}$  with  $\underline{b} > 0$  and satisfying  $u(\underline{b}) < u(w - rD_0) - d - rc/\gamma$ , then optimal job search and consumption is characterised by a pair of asset thresholds  $A^H(B), A^R(B) \geq 0$  such that:

(I) for  $A \leq A^H$ , the optimal policy rules are  $k^* = 1$  and  $x = x^*(A; B)$  is a continuous and strictly increasing function of  $A$  with  $x^* = \underline{b}$  at  $A = 0$  and  $x^* \in (\underline{b} + rA, w + r(A - D_0))$  for all  $A > 0$ ;

(II) for  $A \in (A^H, A^R)$ , the optimal policy rules are  $k^* = 0$  and  $x^* = x^H$ , where  $x^H = x^*(A^H; \cdot)$  and satisfies  $x^H > \underline{b} + rA$ ;

(III) for  $A \geq A^R$ , the optimal policy rules are  $k^* = 0$  and  $x^* = \underline{b} + rA$ .

The proof of Theorem 1 is in the Appendix.

With assets  $A \leq A^H$ , the worker chooses  $k^* = 1$  and consumes  $x^* \geq rA + \underline{b}$ . As assets fall with duration, consumption also falls smoothly with duration until the worker eventually becomes liquidity constrained and consumes  $x^* = \underline{b}$ . The no holiday constraint is thus satisfied for assets in this region.

For intermediate assets  $A \in (A^H, A^R)$  the worker takes a ‘‘holiday’’: the worker chooses  $k^* = 0$  and consumes a fixed amount  $x^H$  where  $x^H \equiv x^*(A^H)$ . But  $x^H > rA + \underline{b}$  in this region implies assets strictly fall with duration. At  $\tau^H$ , where  $A(\tau^H) = A^H$ , the worker switches to active job search. Optimality of the holiday strategy requires

$$u'(x^H) = \frac{\partial W^U(A^H|B)}{\partial A}$$

so that consumption is smooth across the switch to active search (i.e. holiday consumption  $x^H = x^*(A^H)$ ), and

$$\frac{u(w + r(A^H - D_0)) - d}{r} - W^U(A^H|B) = c/\gamma,$$

so that the switch to active search is optimal when  $A = A^H$ . These two conditions also describe optimal holiday strategies in the next section, but with the added complication that  $W^U$ , and thus  $A^H$ , depend on duration  $\tau$ .

### 3 The Optimal UI Problem.

From now on I restrict attention to the case  $A_0 = 0$ . Kocherlakota (2004) provides a formal definition of the optimal UI program with unobserved job search effort and hidden savings. As will be made clear below, the solution to the optimisation problem is not recursive. With linear search costs Kocherlakota (2004) makes the following conjecture:

A1 : in the optimal program “*the principal wants to (weakly) implement a sequence of effort choices  $k_t \in (0, 1)$  for all  $t$* ”,

and characterises the resulting optimal UI program. A simple numerical example, however, finds this conjectured policy is not optimal (see Table 3 below). Instead here I simplify by assuming  $C_0$  is sufficiently small that the optimal policy has the following property:

A2 : in the optimal program, the no holiday constraint is satisfied.

Of course if  $C_0$  were extremely generous, say equal to \$100 million, the optimal policy would be to pay  $C_0$  as a lump sum and allow the worker to retire. But it would seem politically unlikely that UI schemes are so well funded the Planner would wish to finance holiday strategies, let alone retirement strategies. Of course it needs to be verified, at the policy optimum, that policy perturbations which fail the no holiday constraint do not improve welfare.

I shall refer to a strategy where the worker always chooses  $k = 1$  as an active search strategy, where conjecture A2 implies an active search strategy must be privately optimal in the optimal program. Given  $B$  and an active search strategy, Claim 1 then describes the privately optimal consumption path  $x^U(t)$  and corresponding asset path  $A^U(\tau)$ . Should the worker find employment at duration  $\tau$  he/she enjoys re-employment consumption  $x^E(\tau) = w + r(A^U(\tau) - D(\tau))$  with corresponding payoff  $W^E = [u(x^E) - d]/r$ .

It is now mathematically convenient to adopt Kocherlakota’s policy normalisation. Suppose the optimal policy implies consumption plans  $x^U(\cdot), x^E(\cdot)$ . As  $x^U(\cdot)$  is consistent with a privately optimal savings strategy, Kocherlakota (2004) establishes these plans can be implemented by putting  $B_0 = 0$ , setting  $b(\cdot) = x^U(\cdot)$  so that assets  $A^U(\cdot)$  are zero along the optimal savings path, and setting the re-employment tax so that  $x^E(\tau) = w - rD(\tau)$ . In the discussion section I consider other policy normalisations.

Recall  $W^U(A, \tau|B)$  denotes the value of being unemployed at duration  $\tau$  with assets  $A$  using an optimal search and consumption strategy. Conjecture A2 and the above policy normalisation implies the following optimal UI program:

$$\begin{aligned}
& \min_{x^U, x^E \geq 0} \int_0^\infty e^{-(r+\gamma)\tau} \left[ x^U(\tau) + \gamma \frac{x^E(\tau) - w}{r} \right] dt \\
& \text{subject to } (i) W^U(0, 0|\cdot) = W^* \\
& \quad (ii) W^U(0, \tau|\cdot) \leq \frac{u(x^E(\tau)) - d}{r} - \frac{c}{\gamma} \text{ for all } \tau \geq 0 \\
& \quad (iii) \gamma\mu \geq \dot{\mu} + \gamma u'(x^E) \text{ with } \mu = u'(x^U).
\end{aligned}$$

The objective function is the cost of the UI program, where the worker receives benefit  $b = x^U(\tau)$  while unemployed and pays tax  $D = \frac{w - x^E(\tau)}{r}$  if re-employed at duration  $\tau$ . Constraint (i) requires the laid-off worker enjoys welfare payoff  $W^*$ , (ii) is the no holiday constraint (active search with  $A^U \equiv 0$  is always incentive compatible along the optimal path) and (iii) requires the consumption path  $x^U$  is consistent with an optimal savings strategy and so  $A^U = 0$  along the optimal path (constraint (iii) with strict inequality implies the worker is liquidity constrained). Note in this problem that  $W^U$  is also endogeneous and depends on the entire policy  $\{x^U(\cdot), x^E(\cdot)\}$ .

Kocherlakota (2004) explains why the usual recursive arguments cannot identify the policy optimum. Here instead I develop variational arguments which identify a solution to the necessary conditions for optimality. The arguments used are special in that they reflect this optimal UI program is not concave. But the solution identified below has not been pulled out of thin air. Rather it is the outcome of a trial and error approach.

A critical insight for the variational arguments which follow, is that the retirement strategy is a binding constraint on the optimal UI contract. This occurs as the optimal UI program sets up a loans facility (see Table 2 below): some of the UI payments received while unemployed are repaid when re-employed as a re-employment tax. As the retirement strategy allows the worker to default on all such loans, this default option constrains the optimal UI design. In what follows, any cost reducing policy variation must ensure the active search strategy continues to (weakly) dominate the retirement strategy.

To characterise the necessary conditions for optimality, I use two (local) perturbation arguments: one to characterise  $x^E(\cdot)$ , the other to characterise  $x^U$ . In both cases, the perturbation considered holds constant the value of the active search and retirement strategies (where both equal  $W^*$ ) and asks whether a variation potentially exists which can reduce the cost of the UI program.

**Policy Perturbation I.** Let  $B^*$  denote the optimal policy and consider an alternative policy  $B$  which leaves policies  $\{b^*(\cdot), D^*(\cdot)\}$  unchanged except at two places:

$$D(\tau) = D^*(\tau) + dD_0 \text{ for } \tau \in [\tau_0, \tau_0 + \Delta];$$

$$D(\tau) = D^*(\tau) + dD_1 \text{ for } \tau \in [\tau_1, \tau_1 + \Delta];$$

where  $\tau_0 \geq 0, \tau_1 \geq \tau_0 + \Delta$  and  $\Delta > 0$  (but arbitrarily small). This policy perturbation simply varies the re-employment tax at two separate durations. Of course a necessary condition for optimality is that there is no cost reducing policy perturbation I for all  $\tau_1 > \tau_0 \geq 0$ .

**Lemma I:** For any  $0 \leq \tau_0 < \tau_1$  and in the limit as  $\Delta \rightarrow 0$ , any policy perturbation I which holds the value of the active search and retirement strategies constant and continues to satisfy the no holiday constraint:

- (i) cannot reduce cost if  $x^E(\tau_0) = x^E(\tau_1)$ , but
- (ii) if  $x^E(\tau_0) > x^E(\tau_1)$ , a potentially cost reducing policy perturbation I exists and increases  $D_0$  while decreasing  $D_1$ ; i.e. it makes consumption  $x^E(\cdot)$  more equal across re-employment states.

Lemma I simply describes efficient insurance over re-employment states. To establish this lemma, note the Envelope Theorem implies the first order effect of this policy variation on the value of the active search strategy is:

$$dV^U(0) = - \left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right] dD_0 - \left[ \int_{\tau_1}^{\tau_1+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right] dD_1.$$

where the worker pays the additional re-employment taxes by finding employment over the corresponding time intervals. [Note: the change in re-employment consumption implies the worker also updates his overall consumption strategy, but optimality of the original consumption plan implies these latter utility gains are second order.]

The restriction

$$dD_0 = - \frac{\left[ \int_{\tau_1}^{\tau_1+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right]}{\left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right]} dD_1.$$

ensures this policy variation does not change  $V^U(0)$ . Clearly this variation does not affect the value of the retirement strategy. Assuming the active search strategy remains privately optimal, this variation changes the cost of the UI program by

$$dC_0 = - \left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} \gamma dt \right] dD_0 - \left[ \int_{\tau_1}^{\tau_1+\Delta} e^{-(r+\gamma)t} \gamma dt \right] dD_1.$$

Substituting out  $dD_0$  and taking the limit  $\Delta \rightarrow 0$  implies

$$dC_0 = \left[ \frac{u'(x^E(\tau_1))}{u'(x^E(\tau_0))} - 1 \right] \gamma e^{-(r+\gamma)\tau_1} \Delta dD_1$$

and so establishes the lemma.

#### Policy perturbation II.

Consider instead the policy variation

$$D(\tau) = D^*(\tau) + dD_0 \text{ for } \tau \in [\tau_0, \tau_0 + \Delta];$$

$$b(\tau) = b^*(\tau) + db_0 \text{ for } \tau \in [\tau_0, \tau_0 + \Delta];$$

$$b(\tau) = b^*(\tau) + db_1 \text{ for } \tau \in [\tau_1, \tau_1 + \Delta];$$

where  $\tau_1 > \tau_0$  and  $\Delta > 0$  (but small). This perturbation changes UI payments at two distinct durations  $\tau_0, \tau_1$  with an additional variation in  $D(\cdot)$  at  $\tau_0$ . I again restrict attention to perturbations which hold the value of the active search and retirement strategies constant and ask whether a perturbation II exists which reduces cost.

**Lemma II:** For any  $0 \leq \tau_0 < \tau_1$  and in the limit as  $\Delta \rightarrow 0$ , any policy perturbation II which holds the value of the active search and retirement strategies constant and continues to satisfy the no holiday constraint:

(i) cannot reduce cost if  $x^E(\cdot)$  is constant over  $[\tau_0, \tau_1]$  and the worker is not liquidity constrained in the active search strategy, but

(ii) if  $x^E(\cdot)$  is decreasing over  $[\tau_0, \tau_1]$  with  $x^E(\tau_0) > x^E(\tau_1)$ , a potentially cost reducing policy perturbation II exists which increases  $b_0$ , increases  $D_0$  and decreases  $b_1$ ; i.e. UI payments are frontloaded more and the corresponding surplus is extracted by an increase in the re-employment tax.

The argument used to establish Lemma I continues to apply. By the Envelope Theorem, the first order effects of this policy variation on  $V^U(0)$  is:

$$dV^U(0) = \left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} u'(x^U(t)) dt \right] db_0 + \left[ \int_{\tau_1}^{\tau_1+\Delta} e^{-(r+\gamma)t} u'(x^U(t)) dt \right] db_1 - \left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right] dD_0.$$

The change in the value of the retirement strategy is

$$dV^R = \left[ \int_{\tau_0}^{\tau_0+\Delta} u'(x^R) e^{-rt} dt \right] db_0 + \left[ \int_{\tau_1}^{\tau_1+\Delta} u'(x^R) e^{-rt} dt \right] db_1.$$

Variations which hold  $V^U(0)$  and  $V^R$  constant require

$$db_0 = - \frac{\int_{\tau_1}^{\tau_1+\Delta} e^{-rt} dt}{\int_{\tau_0}^{\tau_0+\Delta} e^{-rt} dt} db_1$$

$$\left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} \gamma u'(x^E(t)) dt \right] dD_0 = \left[ \int_{\tau_0}^{\tau_0+\Delta} e^{-(r+\gamma)t} u'(x^U(t)) dt \right] db_0 + \left[ \int_{\tau_1}^{\tau_1+\Delta} e^{-(r+\gamma)t} u'(x^U(t)) dt \right] db_1.$$

Letting  $\Delta \rightarrow 0^+$ , standard algebra establishes the total change in cost is

$$dC_0 = e^{-(r+\gamma)\tau_1} \Delta \left[ 1 - e^{\gamma(\tau_1-\tau_0)} + \frac{e^{\gamma(\tau_1-\tau_0)} u'(x^U(\tau_0)) - u'(x^U(\tau_1))}{u'(x^E(\tau_0))} \right] db_1.$$

If using the active search strategy the worker is not liquidity constrained over  $[\tau_0, \tau_1]$ , claim 1 implies:

$$u'(x^U(\tau_1)) = e^{\gamma(\tau_1-\tau_0)} u'(x^U(\tau_0)) - \int_{\tau_0}^{\tau_1} \gamma u'(x^E(t)) e^{-\gamma(t-\tau_1)} dt.$$

If in addition  $x^E$  is constant over  $[\tau_0, \tau_1]$  then  $dC_0/db_1 = 0$ ; i.e. there is no cost reducing perturbation.

If instead  $x^E$  is decreasing over  $[\tau_0, \tau_1]$  with  $x^E(\tau_0) > x^E(\tau_1)$ , then regardless of whether the worker is liquidity constrained or not; i.e.

$$u'(x^U(\tau_0)) \geq e^{-\gamma(\tau_1-\tau_0)}u'(x^U(\tau_1)) + \int_{\tau_0}^{\tau_1} \gamma u'(x^E(t))e^{-\gamma(t-\tau_0)}dt,$$

then  $dC_0/db_1 > 0$ ; i.e. a potentially cost reducing perturbation exists which increases  $b_0$ , decreases  $b_1$  and increases the re-employment tax at  $\tau_0$ .

## 4 A Solution to the Necessary Conditions for Optimality

Given the optimal policy  $x^U(\cdot), x^E(\cdot)$ , necessary conditions for optimality are that there are no cost reducing policy perturbations I and II for all  $\tau_1 > \tau_0 \geq 0$ . To provide some insight for the identified solution, I first (very quickly) describe two examples of failed “guesses” for the optimal UI program.

A natural guess would be to assume the first order approach applies (i.e. the marginal value of search always equals  $c$ ). Some work then shows optimality implies the unemployed worker is not liquidity constrained in the active search strategy. Claim 1 then implies  $x^U(\cdot)$  satisfies

$$\frac{dx^U}{d\tau} = \gamma \frac{u'(x^E) - u'(x^U)}{-u''(x^U)}. \quad (7)$$

But computing the resulting “optimal” policy, where supposedly the worker is everywhere indifferent between  $k = 0, 1$  along the optimal path, it is found the worker instead never searches and strictly prefers the retirement strategy. The first order approach thus does not identify the optimal policy.

The next obvious guess, then, is to assume the “retirement strategy” is a binding constraint on the optimal policy. Lemmas I(i) and II(i) then suggest the optimal UI policy is:  $x^E(\cdot) = \bar{x}^E$  for all  $\tau$  and also the worker is not liquidity constrained in the active search strategy; i.e. (7) again applies. The proofs of Lemmas I and II imply there is then no (local) cost reducing policy perturbation for all  $\tau_0, \tau_1$ . But a little work finds the no holiday constraint always fails: although by construction the laid-off worker is indifferent between the active search and retirement strategies, faced with this policy the worker deviates to a holiday strategy (and so contradicts A2).

The solution to the necessary conditions for optimality instead finds that the laid-off worker is not only indifferent between the active search strategy and the retirement strategy, but is also indifferent to a continuum of holiday strategies which I index by  $\tau^H \in (0, \infty)$ . An optimal holiday strategy  $\tau^H$  takes the following form:

- (i) the worker consumes constant  $x^H$  and chooses  $k = 0$  for durations  $\tau < \tau^H$ ;
- (ii) at duration  $\tau = \tau^H$ , with assets  $A^H(\tau^H)$ , the worker switches to the active search strategy.

For such a holiday strategy to be optimal, I require:

$$\int_0^{\tau^H} e^{r(\tau^H-t)} [x^U(t) - x^H] dt = A^H, \quad (8)$$

so that assets  $A = A^H$  at  $\tau^H$ ;

$$W^U(A^H, \tau^H|B) = \frac{u(x^E(\tau^H) + rA^H) - d}{r} - \frac{c}{\gamma}, \quad (9)$$

so that the switch to active search at  $\tau^H$  is optimal, and

$$u'(x^H) = \frac{\partial W^U(A^H, \tau^H|B)}{dA}, \quad (10)$$

so that holiday consumption  $x^H$  is consistent with an optimal savings strategy.

With no savings, the optimal UI contract increases UI payments at each duration  $\tau$  to the point where the worker is just indifferent between  $k = 0, 1$  (but chooses  $k = 1$  by convention). Indeed this is the solution identified by the first order approach (but then finds the retirement strategy dominates). With hidden savings, the structure of the optimal UI policy is quite different. Instead the optimal UI contract increases UI payments at each duration  $\tau$  to the point where the laid-off worker is just indifferent between the active search strategy and the holiday strategy  $\tau^H = \tau$ . In other words at the policy optimum, each of these holiday strategies is privately optimal; i.e.,

$$\int_0^{\tau^H} e^{-rt} u(x^H) dt + e^{-r\tau^H} W^U(A^H, \tau^H|B) = W^* \text{ for all } \tau^H \in [0, \infty). \quad (11)$$

Note then that any cost-reducing perturbations must ensure the worker not only (weakly) prefers the active search strategy to the retirement strategy, but also to all holiday strategies  $\tau^H > 0$ .

Note the dynamical system (7)-(11) jointly determines  $\{x^U, x^E, x^H, A^H\}$ , where (7) ensures the worker is not liquidity constrained in the active search strategy. I shall refer to a solution to these conditions as an improved policy. Of course computing an improved policy requires characterising  $W^U(\cdot)$  which itself depends on  $\{x^U, x^E\}$ . Identifying an improved policy is possible using backward induction from a suitable limiting condition (also described in detail below). To facilitate the discussion, however, I first describe the solution to this dynamical system for particular numerical values.

## 5 A Numerical Example.

Using a year as the reference unit of time, suppose annual discount rate  $r = 0.04$  and CRRA utility function  $u(x) = x^{1-\sigma}/(1-\sigma)$  with risk aversion parameter  $\sigma = 2.2$  (e.g. Lentz (2005)). Let  $w = 100$  and set  $d$  so that  $u(100) - d = u(75)$ ; thus a worker is indifferent to a 25% wage cut in return for full leisure. I set  $c = d$ , which implies active job search is as unpleasant as working, and Poisson arrival rate of job offers  $\gamma = 2$ , which implies the average unemployment spell of an active job seeker is 6 months (which corresponds to the average unemployment spell in the U.K.).

Under conjecture A1, Kocherlakota (2004) shows the “optimal” UI program is a re-employment bonus program in which UI payments are duration independent and  $B_0 = 0$  (no severance payments). Conjecture A1 requires the worker is everywhere indifferent between  $k = 0$  and 1. This requires the no holiday constraint is satisfied with equality:

$$\frac{u(b_0) - c + \gamma \frac{u(w-rD_0)-d}{r}}{r + \gamma} = \frac{u(w - rD_0) - d}{r} - \frac{c}{\gamma}. \quad (12)$$

As this condition implies  $b_0 < w - rD_0$ , the liquidity constraint also binds: the unemployed worker optimally consumes  $x = b_0$  at all durations. The policy  $(b_0, D_0)$  must also satisfy the budget constraint

$$\frac{b_0 - \gamma D_0}{r + \gamma} = C_0. \quad (13)$$

Thus (12)-(13) jointly determine the re-employment bonus program  $(b_0, D_0)$ .

I set  $C_0$  equal to the worker’s drop in permanent income through being laid-off. The above parameter values imply  $C_0 = 49$  which is (slightly less than) half a year’s salary (where 6 months is the expected duration of unemployment). The full information benchmark then implies perfect consumption smoothing:  $x^U = x^E = 100$  and the insurance contract specifies search effort  $k = 1$ . With hidden search effort and savings, however, and  $C_0 = 49$ , the re-employment bonus program sets  $b_0 = 74.9$  and re-employment bonus  $-D_0 = 12.5$  which is equivalent to 6.5 weeks salary. Given this scheme, the laid-off worker consumes  $x^U = b_0 = 74.9$  while unemployed and is everywhere indifferent between  $k = 0, 1$ . But as the choice  $k = 1$  minimises the expected cost of the UI budget, note the Planner strictly prefers the worker chooses  $k = 1$  which then contradicts conjecture A1.

Let  $W^K$  denote the welfare value of being unemployed given the re-employment bonus program. Setting welfare  $W^* = W^K$ , I now describe the improved policy  $\{x^U, x^E, x^H, A^H\}$  as determined by (7)-(11). The reduced cost of the UI program identifies the corresponding efficiency gain. Table 1 describes the corresponding consumption outcomes.

**Table 1: The Improved Policy**

Duration $\tau$	0	1	2	3	12	26	52	104
$x^U(\tau)$	99.81	99.78	99.76	99.73	99.49	99.13	98.46	97.15
$x^E(\tau)$	100.50	100.47	100.45	100.42	100.18	99.81	99.13	97.80

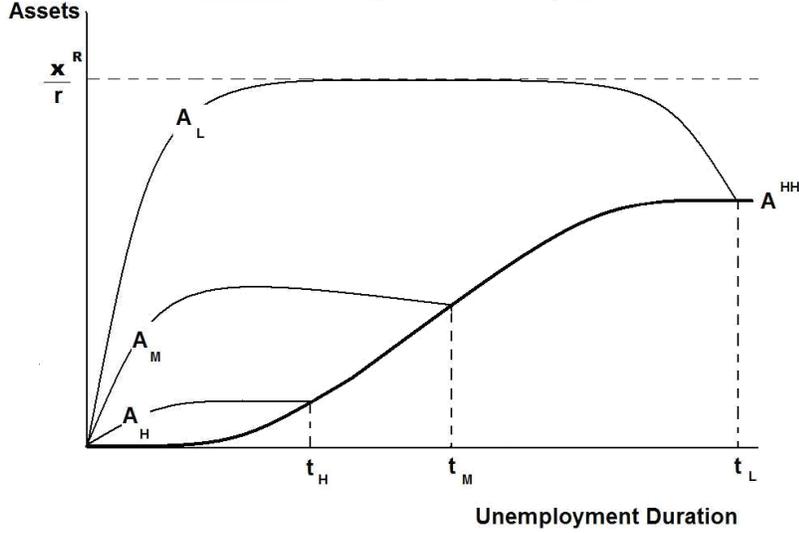
In this Table duration  $\tau$  is measured in weeks. In the improved policy, consumption while unemployed ( $x^U$ ) starts very close to  $w = 100$  and gradually falls thereafter. To incentivise job search effort, workers receive a re-employment bonus (and so consume  $x^E > 100$ ) if quickly re-employed, but that bonus is gradually reduced as the completed spell of unemployment increases. At week 22, the bonus becomes a re-employment tax.

The improved policy yields a 9% reduction in the cost of the UI program relative to the re-employment bonus program. The efficiency gain arises as consumption is much smoother over the unemployment spell. To see this, it is helpful to think of the worker as initially employed on wage  $w = 100$ , is laid off and then receives benefits according to the above Table. The improved policy finds that consumption drops across the layoff shock from  $w = 100$  to  $x^U(0) = 99.81$ . This is much smaller than in the re-employment bonus program where consumption drops to  $\underline{b} = 74.9$ . Thus consumption is much smoother across the layoff shock. But re-employment is also a stochastic event. Across the re-employment shock, the improved policy again yields a small increase in consumption  $x^E - x^U \simeq 0.7$  while the re-employment bonus program implies a consumption increase of 25.6. By yielding a much smoother consumption profile over the unemployment event, the improved policy is much more efficient than the re-employment bonus program.

As in Shavell and Weiss (1979), Hopenhayn and Nicolini (1997) with no savings, the improved policy uses duration to punish the worker for failing to find work: consumption decreases with the length of the unemployment spell. Thus although the consumption gain by finding employment is small, especially when compared to that in the re-employment bonus program, active search remains incentive compatible. In the next section I discuss in detail the policy insights of this improved policy. In the remainder of this section I explain why the improved policy satisfies the necessary conditions for optimality.

Associated with the improved policy is the holiday consumption rule  $x^H(\tau^H)$  and asset threshold  $A^H(\tau^H)$ . The improved policy finds  $x^H(\tau^H)$  is a strictly decreasing function where  $x^H(0) = x^U(0)$ . The active search strategy thus corresponds to the optimal holiday strategy  $\tau^H = 0$ , where  $x^U(\cdot)$  then describes the worker's optimal consumption strategy during the active search phase; i.e. for continuing durations  $\tau \geq \tau^H = 0$ . As  $\tau^H \rightarrow \infty$ , holiday consumption  $x^H$  converges to consumption  $x^R$  in the optimal retirement strategy. As the value of the retirement strategy  $u(x^R)/r$  yields  $W^*$  (the retirement strategy binds on the optimal UI design) this ensures the limiting holiday strategy also yields  $W^*$ .

**Figure 1 : asset paths and holiday spells**



As holiday consumption  $x^H < x^U(0)$  for all  $\tau^H > 0$ , all holiday strategies  $\tau^H > 0$  imply the laid-off worker initially accumulates assets. Figure 1 depicts the asset paths for three different holiday strategies with associated holiday consumption levels  $x_L, x_M, x_H \in (x^R, x^U(0))$  and  $x_L < x_M < x_H$ .  $A_i(\cdot)$  denotes the corresponding asset path associated with each holiday strategy  $i = L, M, H$ .

Obviously lower holiday consumption  $x^H$  yields a strictly higher asset path. Note that the worker switches to active search when the asset path  $A_i(\cdot)$  reaches the critical asset threshold  $A^H(\tau)$ . Thus holiday strategies with lower holiday consumption  $x^H$  correspond to higher asset paths and longer holiday spells  $\tau^H$ .

The topmost asset path in Figure 1 arises when  $x_L$  is very close to retirement consumption  $x^R$ . In the retirement strategy, the worker consumes permanent retirement income

$$x^R = r \int_0^{\infty} e^{-rt} x^U(t) dt$$

and so accumulates retirement assets  $A^R = x^R/r$  as  $\tau \rightarrow \infty$ . For any holiday consumption  $x_L$  close to but strictly greater than  $x^R$ , the corresponding asset path approaches  $x^R/r$  but can never converge to it. Eventually at very long durations, the asset path  $A(\cdot)$  begins to decrease with duration and the worker switches to active search when  $A(\cdot)$  reaches  $A^H$ .<sup>4</sup> Nevertheless as  $x_L \rightarrow x^R$ , the corresponding optimal holiday strategy implies an arbitrarily long holiday spell with consumption arbitrarily close to the retirement level. Thus the payoff of the limiting holiday strategy is the same as the retirement strategy payoff (and equals  $W^*$ ).

<sup>4</sup>For these numerical values  $\lim_{\tau \rightarrow \infty} A^H(\tau) = 1421.2$  while  $x^R/r = 1872.5$

I now establish there are no (local) cost reducing policy perturbations. Note the optimal UI contract is not constrained by marginal efficiency trade-offs along the optimal (active search) path: at strictly positive durations, the worker using the active search strategy strictly prefers  $k = 1$  to  $k = 0$ .<sup>5</sup> Nevertheless there are no efficiency gains as the contract is instead constrained by the worker's potential choice of holiday strategies when laid off. Specifically any cost reducing perturbations must ensure the worker not only (weakly) prefers the active search strategy to the retirement strategy but also to all holiday strategies  $\tau^H > 0$ .

Consider then any **Perturbation I**. As  $x^E(\cdot)$  decreases with duration in the improved policy,<sup>6</sup> a potentially cost reducing perturbation I is to pick any two durations  $0 \leq \tau_0 < \tau_1$ , increase the re-employment tax at  $\tau_0$  while reducing the re-employment tax at  $\tau_1$  by a compensating amount and thus improve insurance across these two re-employment states. By design Perturbation I leaves the value of the active search strategy and retirement strategy unchanged. This perturbation, however, strictly increases the value of any holiday strategy with  $\tau^H \in (\tau_0, \tau_1)$ : the increased re-employment tax at  $\tau_0$  does not affect this holiday strategy, while the reduced re-employment tax at  $\tau_1$  strictly increases its value. This perturbation thus leads the worker to switch to a holiday strategy  $\tau^H \in (\tau_0, \tau_1)$ . Thus for all  $0 \leq \tau_0 < \tau_1$ , there is no cost reducing perturbation I satisfying the no holiday constraint. [when the worker switches from active search to holiday strategy  $\tau^H > 0$ , the expected cost of the UI program increases by a discrete amount].

Similarly a cost reducing **Perturbation II** potentially exists. But again such a perturbation implies an increase in the re-employment tax  $D(\tau_0)$  and it is straightforward to show this perturbation strictly increases the value of holiday strategy  $\tau^H = \tau_0^+$ . Thus there is also no cost reducing perturbation II satisfying the no holiday constraint.

Finally I show how to solve for the improved policy. First I need to consider its limiting properties as  $\tau^H \rightarrow \infty$ . As the limiting holiday strategy implies the worker consumes  $x^H$  for an indefinitely long period of time, it is straightforward to argue

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<sup>5</sup>consuming  $x^U$  implies the active searcher has assets  $A^U = 0$ . But  $0 = A^U < A^H$  for  $\tau > 0$  implies the active searcher strictly prefers  $k = 1$ .

<sup>6</sup>If  $x^E$  is increasing, the argument given in the the text, that there is no cost reducing perturbation I, fails. For example suppose  $x^E$  is strictly increasing over small interval  $[\tau_0, \tau_1]$ . Then a cost reducing policy perturbation exists. The easiest way to see this is to set  $x^E$  equal to a constant over this region and, so improve re-employment insurance. Given the improved insurance, decrease  $b(0)$  so that  $V^U(0)$  is held constant. The reduction in  $b(0)$  can be interpreted as the price of the improved insurance. This variation lowers  $V^R$ . It also lowers the value of any holiday strategy as setting  $x^E$  constant over this interval does not provide perfect insurance (as assets  $A^U$  are non zero and change over time in the holiday strategy) while each holiday strategy receives lower  $b(0)$ . As such a policy perturbation satisfies the no holiday constraint and is cost reducing, it contradicts optimality of the improved policy.

$x^H \rightarrow x^R$  in this limit.<sup>7</sup> Ensuring the limiting holiday strategy yields  $W^*$  thus requires

$$\frac{u(x^R)}{r} = W^*.$$

Note that putting  $W^* = W^K$  implies retirement consumption  $x^R$  is now fully determined. Furthermore retirement consumption  $x^R$  is the same as  $b_0$  in the re-employment bonus program.

As  $\tau^H \rightarrow \infty$ , it must be that  $x^E$  converges to some limiting value  $\bar{x}^E \geq 0$  ( $x^E$  is positive and decreasing - see footnote 6). (7) in addition implies  $x^U$  converges to zero. As  $\tau^H \rightarrow \infty$ , the improved policy thus has limiting solution  $(x^U, x^E, x^H, A^H) = (0, \bar{x}^E, x^R, \bar{A}^H)$  where the definition of the improved policy requires  $\bar{x}^E, \bar{A}^H$  satisfy

$$W^U(\bar{A}^H, \infty|B) = \frac{u(\bar{x}^E + r\bar{A}^H) - d}{r} - \frac{c}{\gamma}, \quad (14)$$

$$u'(x^R) = \frac{\partial W^U(\bar{A}^H, \infty|B)}{dA}, \quad (15)$$

with  $B$  given by  $x^U(\cdot) = 0, x^E(\cdot) = \bar{x}^E$ . With duration independent payments,  $W^U(\cdot, \infty|B)$  is fully described by Theorem 1. Solving the above two equations for  $\bar{A}^H$  and  $\bar{x}^E$  is a trivial numerical exercise. The critical leap in logic, however, is to realise that although  $x^H \rightarrow x^R$  in this limit, it is not the case that  $A^H \rightarrow x^R/r$  (see Figure 1).

Given this boundary solution for  $(x^U, x^E, x^H, A^H)$ , I now iterate backwards the system (7)-(10). I need, however, to replace (8) describing  $A^H$ ,

$$A^H = \int_0^{\tau^H} e^{r(\tau^H-t)} [x^U(t) - x^H] dt,$$

as this condition requires knowing  $x^U$  at early durations. But retirement consumption must satisfy  $x^R = r \int_0^\infty e^{-rt} x^U(t) dt$ . Using this in (8) implies  $A^H$  can be rearranged as:

$$A^H = \frac{x^H}{r} - \left[ \frac{x^H}{r} - \frac{x^R}{r} \right] e^{r\tau^H} - \int_{\tau^H}^\infty e^{-r(t-\tau^H)} x^U(t) dt, \quad (16)$$

which now depends only on future (discounted) values of  $x^U(\cdot)$  and recall that  $x^R$  is known (given  $W^*$ )

It is now straightforward to use backward induction to identify the improved policy. Asymptotically the implied differential equations imply  $x^U$  declines exponentially at rate  $-\gamma/\sigma$  while  $x^E, A^H, x^H$  converge to their boundary values at rate

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<sup>7</sup>In this limit  $x^H > x^R$  is not financeable ( $x^R$  is the most that can be consumed given the expected flow of UI receipts).  $x^H < x^R$  implies the worker accumulates an unboundedly large amount of assets at arbitrarily long durations. A switch to active search, at an indefinitely long duration is then inconsistent with optimality (use Theorem 1 and note  $x^U \rightarrow 0, x^E \rightarrow \bar{x}^E > 0$  in this limit implies  $A^R(\tau)$  is finite).

$r$ . The algorithm fixes an  $x^E(T)$  slightly higher than  $\bar{x}^E$  and leaves  $x^U(T) = a_0$  as a free parameter. Given asymptotic approximations  $x^U(\tau) = a_0 e^{-\gamma(\tau-T)/\sigma}$   $x^E(t) = \bar{x}^E + (x^E(T) - \bar{x}^E)e^{-r(t-T)}$ , one can then solve numerically for  $W^U(\cdot)$  and thus solve (9)-(10) for  $x^H(T), A^H(T)$ . One also computes  $T$  as

$$\int_0^T e^{-rt} u(x^H(T)) dt + e^{-rT} \left[ \frac{u(x^E(T) + rA^H(T)) - d}{r} - \frac{c}{\gamma} \right] = W^*.$$

Given this asymptotic solution for  $x^U, x^E$  for  $\tau \geq T$ , it is now straightforward to iterate (7)-(10) and (16) backwards for  $\{x^U, x^E, x^H, A^H\}$  for all  $\tau \geq 0$ . It is important to note that this iteration guarantees each holiday strategy yields the same payoff  $W^*$ ; i.e. the requirement that each optimal holiday yields payoff  $W^*$  is not an additional constraint. The free parameter  $a_0$  is tied down by the boundary condition  $A^H = 0$  at  $t = 0$  which, by (16), ensures  $x^R = r \int_0^\infty e^{-rt} x^U(t) dt$ . Two useful checks for rounding error are (i)  $x^E(0)$  satisfies

$$\left[ \frac{u(x^E(0)) - d}{r} - \frac{c}{\gamma} \right] = W^*,$$

and (ii)  $x^H(0) = x^U(0)$  as the active search strategy is equivalent to the optimal holiday strategy  $\tau^H = 0$ . The numerical program<sup>8</sup> set  $x^E$  sufficiently close to  $\bar{x}^E$  that  $T = 98.4$  years and rounding error checks (i) and (ii) were hit with error no greater than  $10^{-6}$ . The resulting structure is then as described above.

If  $u(0)$  is finite (i.e. payoffs are bounded below), the Principle of Unimprovability and the above backward induction argument can be used to establish these search and consumption strategies are optimal. The argument is tiresome - it requires using the above structure to describe consumption and search strategies for all  $A, \tau$  - but is straightforward (see the proof of Theorem 1 for example).

## 6 Policy Implications: Severance Payments and Loans.

As the worker is not liquidity constrained in the improved policy other, more insightful policy normalisations exist. With perfect capital markets, the arguments in Malcomson and Spinnewyn (1989), Fudenberg et al (1990) imply any policy  $B$  satisfying

$$B_0 e^{rt} + \int_0^t e^{r(t-\tau)} b(\tau) d\tau - D(t) = \int_0^t e^{r(t-\tau)} x^U(\tau) d\tau + \frac{x^E(t) - w}{r}$$

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<sup>8</sup>the programs (which used Gauss) are available from the author: koch6 identifies  $\bar{x}^E$  and  $\bar{A}^H$ , and koch8 performs the backward induction.

is payoff equivalent. As in the proof of Ricardian equivalence, by adapting his/her savings strategy appropriately, the worker can implement the same (optimal) consumption stream for any such  $B$ . It is then tempting to “normalise”  $D = 0$  as in Werning (2002). The above condition and the improved policy payments in Table 1 would then imply a lump sum layoff payment equal to 6.5 weeks wages, initial UI payment  $b(0) = 65.1$  with further payments gradually increasing with duration. But this is not a valid normalisation as at week 22 the worker would become liquidity constrained.

A valid policy normalisation must also imply the unemployed worker has positive assets at all durations and so is never liquidity constrained. This latter restriction requires

$$B_0 e^{rt} + \int_0^t e^{r(t-\tau)} [b(\tau) - x^U(\tau)] dt \geq 0 \text{ for all } t \geq 0,$$

which is equivalent to  $D(t) \geq (w - x^E(t))/r$ . Thus Table 1 implies re-employment taxes have to be strictly positive after week 22.

Table 2 reveals the true underlying nature of the improved policy. It considers two policy normalisations. The first column describes the improved UI policy when  $b = x^U$  and the initial re-employment tax  $D(0)$  is set equal to zero. This implies the worker receives a lump sum layoff payment equal to 6.5 weeks wages. But note in column 1 that the re-employment tax increases linearly with the length of the completed unemployment spell. The improved policy essentially operates a loan scheme. In the first week, the unemployed worker enjoys consumption  $x^U(0) = 99.1$  but portion  $b^L = 34.0$  of that consumption is funded as a loan which is repaid when re-employed. The remaining portion,  $b^{UI}(0) = 65.1$  is not repaid and is instead funded out of the UI budget. It is striking that the re-employment tax is increasing at almost the same rate even after 2 years.

**Table 2: Improved UI Policy as a Loans Program.**

	Policy Normalisation 1	Policy Normalisation 2
duration	$b = x^U, B_0 = 12.5$	$b = 60, B_0 = 140$
weeks	$D(t)$	$D(t)$
0	0	127.46
1	0.68	127.47
2	1.35	127.48
3	2.03	127.49
12	8.10	127.55
26	17.49	127.60
52	34.72	127.46
104	68.47	126.49

Column 2 instead fixes UI payment  $b$  equal to a constant. These choices,  $b = 60$  and  $B_0 = 140$  are significant as the re-employment tax hardly changes with duration. Even after 2 years, the re-employment tax remains close to its initial value of 127.5.<sup>9</sup> The insight is that the improved policy is materially equivalent to a UI program with constant replacement rate equal to 60%, a lump sum layoff payment equal to 6.5 weeks wages and an interest free loan equal to 127.5 which is repaid when the worker is re-employed.

But UI programs comprising of a lump sum layoff payment and duration independent  $(\underline{b}, D_0)$  were fully characterised in the first section of this paper. An interesting exercise then is to ask how efficient is a constant UI program with a loans program? Table 3 reports the outcome to the following exercise. For given  $\underline{b}, D_0$ , Theorem 1 identifies the critical asset level  $A^H$  at which point a holiday strategy becomes optimal. I set layoff payment  $B_0 = A^H(\underline{b}, D_0)$  so that the no holiday constraint is exactly satisfied. I then search numerically for pairs  $(\underline{b}, D_0)$ , with  $B_0 = A^H(\underline{b}, D_0)$ , which then yield layoff value  $W^K$ . Table 3 reports 5 such policies.

Table 3: Layoff Severance Payments and a Loans Program.

Policy	$B_0$	$\underline{b}$	$D_0$	$Cost$	$x^U(0 B)$
i. RBP	0	74.9	-12.5	49.0	74.9
ii. No Loan	12.5	69.6	0	46.7	92.8
iii. Small Loan	31.9	66	19.4	45.3	97.5
iv. "Improved" Policy	140.0	60	127.5	44.5	99.81
v. Large Loan	389.9	50	377.4	44.5	99.81
Full Information	.	.	.	44.3	99.81

For each policy, Table 3 reports the associated cost of the UI program and consumption  $x^U(0|.)$  at the start of the layoff phase. An important insight is that in each reported policy the gap,  $B_0 - D_0 = 12.5$ , is the same. Thus each of these policies corresponds to a lump sum severance payment equal to 6.5 weeks salary and an interest free loan  $D_0$  which is repaid when re-employed. Moving down the rows corresponds to an increase in the interest free loan  $D_0$ , where UI payments  $\underline{b}$  are correspondingly reduced to ensure active search remains incentive compatible. Table 3 shows that increasing loans, by increasing worker liquidity, implies greater efficiency (achieves welfare  $W^K$  at lower cost).

Policy (i) describes the re-employment bonus program. Policy (ii) describes the case when the re-employment bonus  $D_0 = 0$ . With no re-employment bonus then, relative to row 1, the replacement rate  $\underline{b}$  is lowered from 74.9 to 69.6 to ensure active search remains incentive compatible. The Planner, however, compensates with a lump sum layoff payment equal to 6.5 weeks wages. The advantage of the lump sum layoff payment is the worker can then use a (dis)saving strategy to smooth consumption

<sup>9</sup>results obtained in Koher10f

over the unemployment spell and the re-employment shock. This policy lowers the cost of the program by 5% relative to the re-employment bonus program. However, as the worker is short of liquid funds, the liquidity constraint  $A \geq 0$  significantly distorts the worker's optimal (dis)saving strategy during the unemployment spell.

The policy in row 3 in addition offers a small loan equivalent to 10 weeks salary; i.e. the laid off worker receives a lump sum severance payment of 6.5 weeks salary, an interest free loan of 10 weeks salary which is repaid on re-employment, and unemployment benefit is reduced to  $\underline{b} = 66$  to ensure active search remains incentive compatible. The added liquidity yielded by the small loan further increases efficiency. The 4th row corresponds very closely to the improved policy described in Table 2. The fifth row considers what happens with very large loans and finds there is no further efficiency gain. Relative to the 4th row, the loan is increased by 250 and  $b$  is reduced by the interest earned by putting this "interest free" loan in the bank, where  $r \times 250 = 10$ . The added liquidity does not further reduce the cost of the program.<sup>10</sup> But nor does it increase cost. Indeed normalising  $b = 50$  and putting  $B_0 = 389.9$ , the improved policy finds  $D(\cdot)$  again hardly changes with duration and this time hovers around 377.4. The final row describes the full information benchmark when consumption and search effort is perfectly contractible and  $W^* = W^K$ . In that case the Planner sets  $x^U = x^E = 99.81$ ,  $k = 1$  and the implied cost of this UI program is 44.3. Thus the improved policy yields a payoff which is very close to the full information benchmark.

## 7 Conclusion.

The central contribution of this paper has been to characterise an explicit solution to the necessary conditions for optimality for the original Shavell and Weiss (1979) optimal UI problem but when workers can save and extended to re-employment taxes. Tractability was obtained by assuming linear search costs. The solution is both elegant and intuitive: the worker, when laid-off, is indifferent between always searching for work and a continuum of holiday strategies (including retirement). A (local) cost reducing variation on the improved policy does not exist - any potentially cost reducing variation finds the active search strategy is then dominated by one of the holiday strategies (where a deviating holiday strategy yields a discrete increase in the cost of the UI program).

Throughout I have been careful to describe the solution to the necessary conditions for optimality as an improved policy. The optimal UI problem has two central difficulties. First the worker's search and consumption decision problem is not concave and thus the first order approach cannot be applied. Simply identifying necessary conditions for optimality is a complex problem, but is feasible when search costs are linear. But the second difficulty is that there can be no proof that this solution to

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<sup>10</sup>Results in Table 1 obtained in gaussfile <Kocher3a>

the necessary conditions for optimality identifies the global optimum. For example the re-employment bonus program as identified in Kocherlakota (2004)) also satisfies the necessary conditions for optimality identified here. Indeed the structure of that policy is almost identical to the improved policy: both ensure the laid-off worker is indifferent between the active search strategy and an infinity of optimal holiday strategies (including retirement). The difference, of course, is that workers are not liquidity constrained in the improved policy. Rogerson (1985) provides the appropriate insight. When workers cannot save, Shavell and Weiss (1979) show UI payments are frontloaded to “punish” workers who do not quickly find employment. Rogerson (1985) shows, in such a scheme, that an unemployed job seeker would like to save out of early UI payments. Of course this is not incentive compatible when workers can save. Instead the strongest incentive compatible punishment when workers can save leaves the worker liquidity unconstrained. The re-employment bonus program would seemingly identify a local minimum.

It would seem historical accident that the optimal UI literature has focussed on re-employment taxes rather than severance payments. With no savings in Hopenhayn and Nicolini (1997), a lump sum layoff payment is undesirable as the worker would have to consume it immediately. With savings and an appropriate severance payment, however, it has been shown here that the central role of the re-employment tax is to implement a loans program. But this policy insight begs the question why is the Planner willing to loan funds against future earnings while the private banking sector is not? In a more standard turnover framework where workers face the risk of repeat unemployment spells, I consider elsewhere the efficacy of UI policies which do not offer loans. Numerical simulations with no loans find the efficiency of the UI program remains close to the full information benchmark. Lump sum layoff payments (to insure employed workers against the drop in permanent income through job destruction shocks) and constant UI payments (to insure partially against re-employment risk) remain valuable policy tools. But no loans imply workers while employed accumulate a small buffer stock of savings to self-insure against future binding liquidity constraints when unemployed. As this buffer stock is small (noting in addition that workers, when laid off, receive a lump sum severance payment) the efficiency loss through offering no loans is also small.

## 8 Appendix A

### Proof of Claim 1.

There are two cases depending on whether the liquidity constraint is binding or not.

#### (i) Unconstrained consumption ( $A' > 0$ ).

If the liquidity constraint  $A' \geq 0$  is not binding at duration  $\tau$ , standard arguments imply optimal consumption  $x$  is

$$u'(x) = \frac{\partial W^U(A, \tau | B)}{\partial A}; \quad (17)$$

i.e., the marginal utility of consumption equals the marginal value of savings. The Envelope Theorem implies over (arbitrarily small) time period  $\Delta > 0$ , the marginal value of savings evolves according to

$$\frac{\partial W^U(A, \tau | B)}{\partial A} = (1 - e^{-\gamma k^* \Delta}) \frac{\partial W^E(A', \tau + \Delta | B)}{\partial A'} + e^{-\gamma k^* \Delta} \frac{\partial W^U(A', \tau + \Delta | B)}{\partial A'} \quad (18)$$

where  $k^* \in \{0, 1\}$  is the optimal search effort choice and  $A' = e^{r\Delta}[A - x^*\Delta + b(\tau)\Delta]$  is the continuation asset level given the optimal consumption choice  $x^*(\cdot)$ . Thus an optimal savings strategy implies today's marginal value of savings equals tomorrow's expected marginal value of savings. Rearranging appropriately and letting  $\Delta \rightarrow 0$ , recalling that  $\mu(\cdot)$  is defined as  $\partial W^U / \partial A$  along the optimal path yields the differential equation (5) for  $\mu(\cdot)$ .

(ii) **Liquidity constrained consumption** ( $A' = 0$ ). If the liquidity constraint binds at  $\tau$ , then optimal consumption  $x^* = b(\tau)$  and the Kuhn-Tucker condition for optimality is

$$u'(b(\tau)) \geq \left[ \frac{\partial W^U(0, \tau | B)}{\partial A} \right] \quad (19)$$

so that the marginal utility of today's consumption exceeds the marginal value of savings. The Envelope theorem then implies

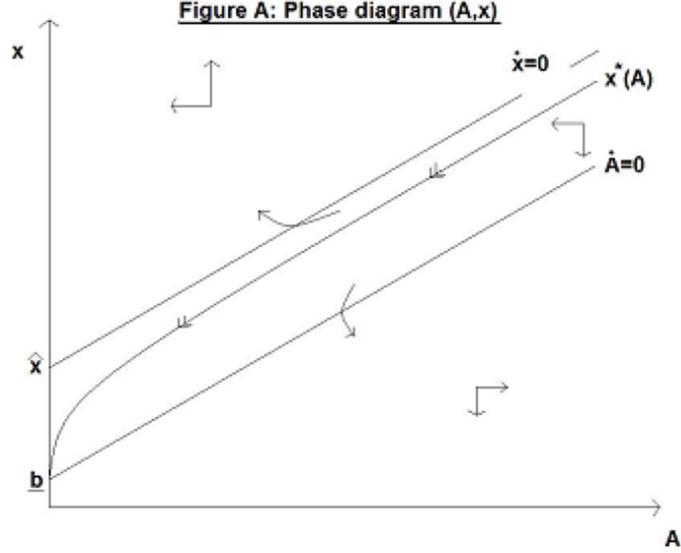
$$\frac{\partial W^U(0, \tau | B)}{\partial A} = u'(b(\tau)).$$

Thus  $\mu = u'(b(\tau))$  when the liquidity constraint binds. This completes the proof of the Claim.

### Proof of Theorem 1.

Let  $\hat{w} = w - rD_0$  denote the (effective) re-employment wage. As  $\underline{b}, D_0$  are fixed in this proof, I simplify notation by subsuming reference to  $B$ .

I prove Theorem 1 using the Principle of Unimprovability. Using forward induction from  $A = 0$ , the following first identifies the unimprovable holiday strategy for assets



$A > 0$  and so identifies the value of the optimal holiday strategy, which I denote  $W^H(A)$ . In the optimal retirement strategy, the worker instead consumes permanent income  $\underline{b} + rA$  and so enjoys retirement value  $W^R(A) = u(\underline{b} + rA)/r$ . I will establish a single crossing property, that  $W^H > W^R$  if and only if  $A < A^R$  where  $W^H(A^R) = W^R(A^R)$ . The Principle of Unimprovability [Proposition 4, page 813 in Kreps (1993)] then establishes the Theorem.

First consider  $A = 0$ . As the restriction  $u(\underline{b}) < u(\widehat{w}) - d - rc/\gamma$  implies  $\underline{b} < \widehat{w}$ , optimal consumption smoothing implies  $x^* = \underline{b}$  (the worker is liquidity constrained). Furthermore the restriction  $u(\underline{b}) < u(\widehat{w}) - d - rc/\gamma$  implies  $k = 1$  is strictly preferred to  $k = 0$ . Thus the optimal policy rules at  $A = 0$  are  $x^* = \underline{b}$  and  $k^* = 1$ .

Consider now  $A > 0$  but sufficiently small that  $k^* = 1$  remains optimal. The optimal consumption path is identified by solving (4),(5),(6) in Claim 1 for  $\{x, \mu, A\}$  with  $k = 1$ . As (4) implies  $u''(x)\dot{x} = \dot{\mu}$ , use this to solve out  $\mu$  in (5), and note that optimal  $(x, A)$  must then evolve according to the autonomous pair of equations:

$$[-u''(x)]\dot{x} = \gamma[u'(\widehat{w} + rA) - u'(x)] \quad (20)$$

$$\dot{A} = rA - x + \underline{b} \quad (21)$$

while  $x, A > 0$ . Figure A describes the corresponding phase diagram.

$\underline{b} < \widehat{w}$  implies the  $\dot{x} = 0$  locus lies above the  $\dot{A} = 0$  locus. Given  $k^* = 1$ , Claim 1 implies the optimal consumption path corresponds to the flow line which limits to  $(A, x) = (0, \underline{b})$ . Let  $x = x^*(A; \underline{b})$  denote that path. As  $u(\cdot)$  is twice differentiable for all  $x \geq \underline{b} > 0$ , it follows that  $x^*$  is continuous.  $x^*$  is also strictly increasing in  $A$ . As  $x^*$  cannot cross the  $\dot{x} = 0$  locus nor the  $\dot{A} = 0$  locus, it also follows that

$x^* \in (\underline{b} + rA, \widehat{w} + rA)$  and thus assets  $A$  are strictly decreasing with duration. As  $(A, x) = (0, \underline{b})$  is not a stationary point in this dynamic system,  $(A, x)$  reaches  $(0, \underline{b})$  in finite time. This consumption path, with  $k^* = 1$ , determines  $W^H(A)$  for small  $A$ .

Now define re-employment surplus

$$S(A) = \frac{u(\widehat{w} + rA) - d}{r} - W^H(A)$$

and note that  $k = 1$  is optimal only if  $S(A) \geq c/\gamma$ . The restriction  $u(\underline{b}) < u(\widehat{w}) - d - rc/\gamma$  implies  $S(0) > c/\gamma$ . Differentiating with respect to  $A$  implies:

$$\begin{aligned} \frac{dS}{dA} &= u'(\widehat{w} + rA) - \frac{\partial W^H}{\partial A} \\ &= u'(\widehat{w} + rA) - u'(x^*(A)). \end{aligned}$$

As  $x^*(A) < \widehat{w} + rA$ , we have  $dS/dA < 0$  and so an  $A^H > 0$  exists where  $S(A^H) = c/\gamma$ . Thus at  $A = A^H$ , the no holiday constraint fails.

For  $A > A^H$ , where  $S(A) < c/\gamma$ , optimal job search implies  $k^* = 0$ . (5) in Claim 1 then implies  $\mu$ , and hence consumption, is constant during this phase. Let  $x^H$  denote that consumption choice. An optimal holiday strategy exists only for  $A$  satisfying  $x^H > \underline{b} + rA$  so that assets strictly decrease with duration and so reach  $A^H$  at a finite duration. But optimal consumption smoothing then implies  $x^H = x^*(A^H)$  at  $A = A^H$ . Thus an optimal holiday strategy implies

$$\begin{aligned} k &= 1, x = x^*(.) \text{ for } A \leq A^H \\ k &= 0, x = x^H \text{ for } A > A^H \end{aligned}$$

with  $x^H = x^*(A^H)$ . Strictly declining assets during the holiday phase ( $A > A^H$ ) requires assets  $A < A^R$  where  $A^R$  is defined by  $x^H = \underline{b} + rA^R$ . As  $x^H = x^*(A^H) > \underline{b} + rA^H$  this implies  $A^R > A^H$ . Further note that as  $A \rightarrow (A^R)^-$ , the holiday phase becomes arbitrarily long in duration and  $W^H(A)$  converges in value to the retirement strategy (i.e. consume permanent income  $x^H = \underline{b} + rA^H$  and never search). Thus at  $A = A^R$ , I have  $W^H(A) = W^R(A)$ . I now compare  $W^H(A)$  with the value of the retirement strategy  $W^R(A) = u(\underline{b} + rA)/r$  for all  $A$ .

Note first that for  $A < A^R$ , the Envelope Theorem implies  $dW^H/dA$  equals the marginal utility of consumption given the optimal consumption rule. But the definition of  $A^R$  implies optimal consumption in the holiday strategy always exceeds  $\underline{b} + rA$  (for  $A < A^R$ ). As  $u(.)$  is strictly concave, the slope of  $W^H$  is therefore strictly less than the slope of  $W^R$  for all  $A \in (0, A^R)$ . As  $W^H = W^R$  at  $A = A^R$  then  $W^H > W^R$  for  $A < A^R$ . Thus the holiday strategy dominates the retirement strategy for all such  $A$ . Instead for  $A > A^R$ , consuming holiday consumption  $x^H$  indefinitely is strictly dominated by the retirement strategy where consumption instead equals permanent income  $\underline{b} + rA > x^H$ .

By construction, these strategies (as described in the Theorem) are unimprovable strategies. To establish that they describe an optimal strategy, I now use the Principle of Unimprovability. Note that in an optimal savings strategy, it is never optimal to consume  $x < \underline{b}$  as  $\underline{b}$  is a lower bound on future income. Hence there is no loss in generality by imposing the additional restriction  $x \geq \underline{b}$ . In this extended case, the restrictions  $x \geq \underline{b}$  and  $k \in \{0, 1\}$  imply the “one period” payoff function  $u(x) - ck$  is bounded below by  $u(\underline{b}) - c$ . Thus for  $\underline{b} > 0$ , the Principle of Unimprovability implies the above describes optimal strategies.

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