Money and Banking in Search Equilibrium

Ping He

Lixin Huang

Randall Wright

“The theory of banking relates primarily to the operation of commercial banking. More especially it is chiefly concerned with the activities of banks as holders of deposit accounts against which cheques are drawn for the payment of goods and services. In Anglo-Saxon countries, and in other countries where economic life is highly developed, these cheques constitute the major part of circulating medium.”

Encyclopedia Britannica

Introduction

We present a new theory of money and banking based on an old story about money and banking.

The story is so well known it is in standard reference books:

“The direct ancestors of moderns banks were... the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money and so became the first English bank notes... The cheque came in at an early date, the first known to the Institute of Bankers being drawn in 1670, or so.”

Encyclopedia Britannica

More specialized economic sources echo this view:

1. Quinn (1997) “Goldsmith-Banking.” *Explorations in Economic History*.

“By the restoration of Charles II in 1660, London’s goldsmiths had emerged as a network of bankers... Some were little more than pawn-brokers while others were full service bankers. The story of their system, however, builds on the financial services goldsmiths offered as fractional reserve, note-issuing bankers. In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of moving, protecting and assaying specie.”

2. Joslin (1954) “London Private Bankers, 1720-1785.” *Economic History Review*.

“The crucial innovations in English banking history seem to have been mainly the work of the goldsmith bankers in the middle decades of the seventeenth century. They accepted deposits both on current and time accounts from merchants and landowners; they made loans and discounted bills; above all they learnt to issue promissory notes and made their deposits transferrable by ‘drawn note’ or cheque; so credit might be created either by note issue or by the creation of deposits, against which only a proportionate cash reserve was held.”

But a story is not a theory.

We want a **model** in which to study banks as institutions whose liabilities may be used as means of payment.

We also want to see whether bank liabilities are substitutes for or complements to money.

Clearly we need a model where there is a serious role for medium of exchange in the first place.

This is provided by the search-theoretic approach.

However, we need to relax the assumption in standard search models that all interaction takes place in markets with anonymous bilateral matching.

History tells us money was less than an ideal means of payment: coins were scarce, hard to transport, got worn or clipped, and could be lost or stolen.

While we could in principle focus on any one of these problems, in our model we assume cash is subject to *theft*.

This implies a role for the relatively *safe* demandable debt of banks.

To this extent, ours is not a model of bank notes but more of checking deposits.

Checks in the model, as in the real world, are designed with safety in mind.

Traveller's checks are the perfect example.

Outline

1. Introduction.

2. Money and exogenous theft.

3. Banking and exogenous theft.

4. Endogenous theft.

5. Endogenous prices.

6. Fractional reserves: the money multiplier.

7. Conclusions and extensions.

Basic Model

Continuum of infinitely-lived agents.

They prod. and cons. two types of goods:

general with $U(Q) = Q$ and $C(Q) = Q$

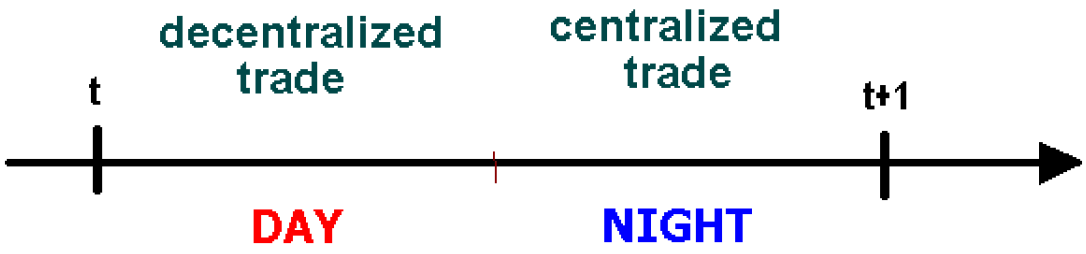
special with arbitrary $u(q)$ and $c(q)$

$M \in [0, 1]$ agents each w/ 1 unit of money

Assume general goods are nonstorable, while special goods and money are storable but indivisible, and agents can store only 1 object at a time.

One can relax indivisibility of special goods and money.

Each period has day and night subperiods, with general goods produced during the day and special goods at night.



Day: Centralized trade in general goods.

Night: Decentralized trade in special goods $\Rightarrow$ double coincidence problem.

Decentralized means anonymous bilateral matching, implies need for means of payment (Kocherlakota JET 98; Wallace IER 01).

This can be cash or check since latter is a commitment by the bank for cash.

New friction: money is unsafe – it can be stolen.

Theft is modeled as a simple special case of the Crime models in Burdett-Lagos-Wright (AER 03, IER 04)

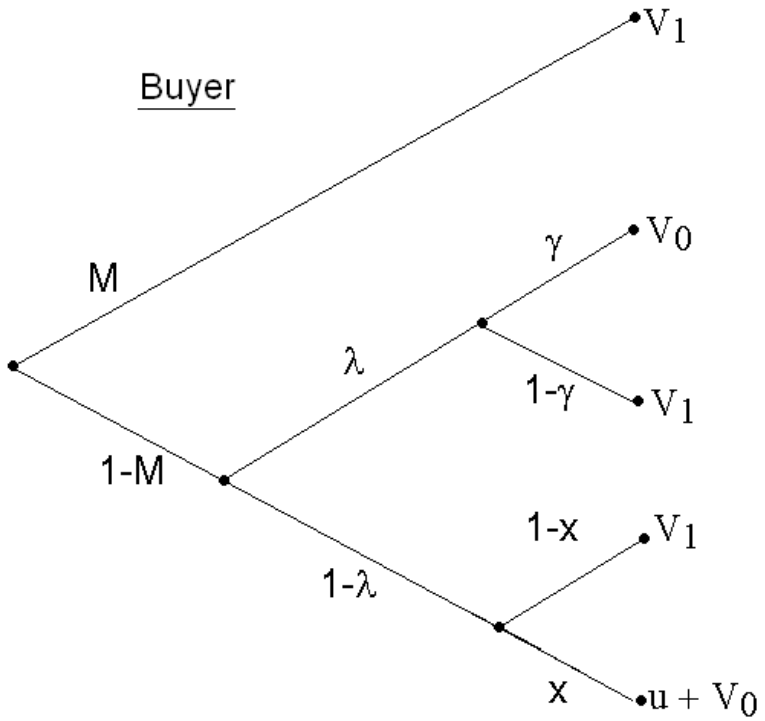
For simplicity goods cannot be stolen, and only individuals without money steal.

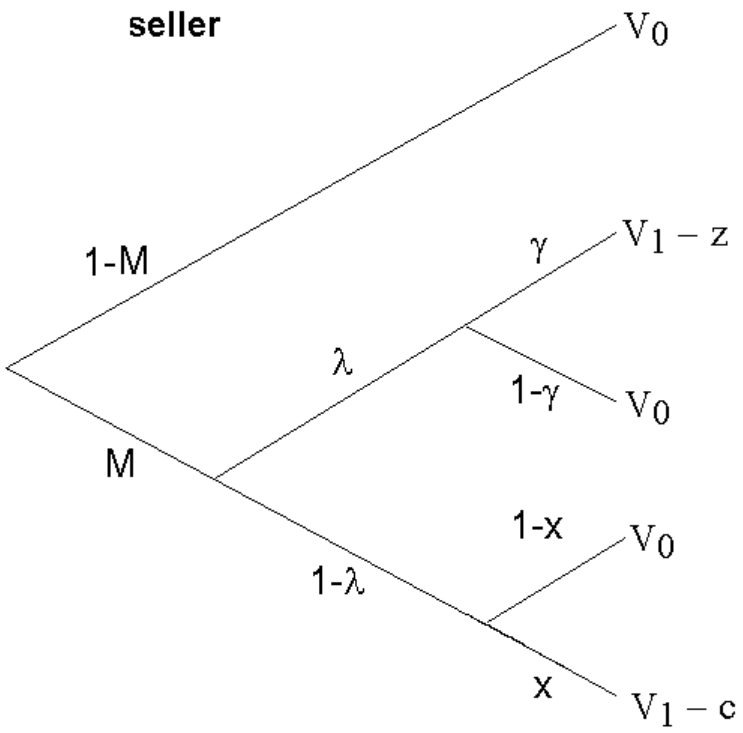
In decentralized market, if you are w/o cash and meet an agent w/ cash, you try to steal it with probability λ.

With probability γ you succeed, analogous to the single coincidence probability x

The cost of theft is z, analogous to the cost of honest production c.

Attraction of crime depends on $z - c$ and $\gamma - x$.


Buyer
M
1-M
λ
1-λ
γ
1-γ
1-x
x
V1
V0
V1
V1
u + V0


seller
1-M
M
λ
1-λ
γ
1-γ
1-x
x
V0
V1 − z
V0
V0
V1 − c

Exogenous λ, No Banks

$$rV_1 = (1-M)(1-\lambda)x(u+V_0-V_1) + (1-M)\lambda\gamma(V_0-V_1)$$

$$rV_0 = M(1-\lambda)x(V_1-V_0-c) + M\lambda\gamma(V_1-V_0-z)$$

IC for sellers: $V_1 - V_0 - c \geq 0$ (IC for buyers not binding, and for now no IC for thieves).

IR for sellers $V_0 \geq 0$ (IR for buyers not binding).

IC $\Leftrightarrow$ $c \leq C_M$ and IR $\Leftrightarrow$ $c \leq C_A$ where

$$C_M = \frac{(1-M)(1-\lambda)xu+M\lambda\gamma z}{r+(1-M)(1-\lambda)x+\lambda\gamma}$$

$$C_A = \frac{(1-M)[\lambda\gamma+(1-\lambda)x]u}{r+(1-M)[\lambda\gamma+(1-\lambda)x]} - \frac{\lambda\gamma z}{(1-\lambda)x}.$$

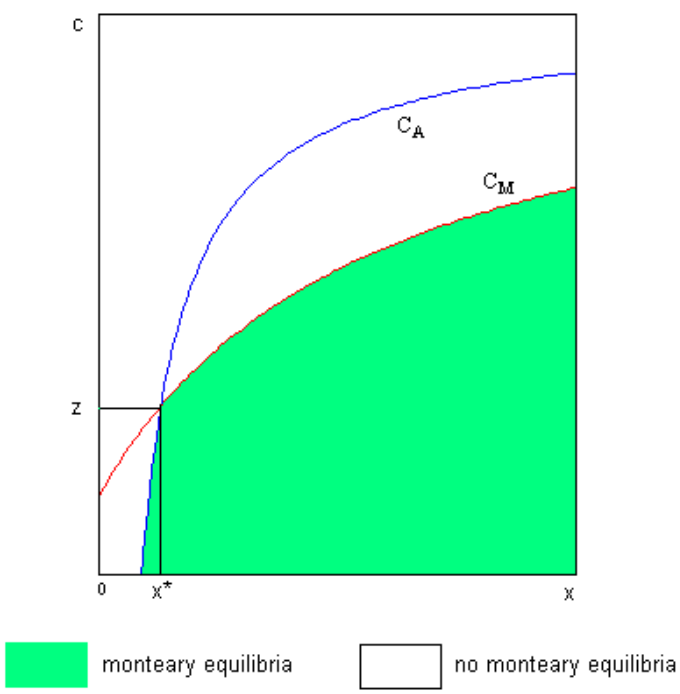
Prop: ME exists iff $c \leq \min\{C_M, C_A\}$.

ME is more likely when c is lower or x bigger.

ME is more likely when λ is lower iff $z \leq \hat{z}$.

Welfare is decreasing in λ and γ.

If $\lambda = 0$ we have standard KW model.



Exogenous λ, Banks

During the day, buyers can deposit coins in bankers' vaults for a fee paid in general goods, denoted ϕ.

They can write checks on these deposits, which are accepted by all sellers because...

Checks (i.e. deposits) cannot be stolen.

Resource cost a to manage each deposit.

Assume 100% reserves for now, so that competition implies $\phi = a$.

Let $\theta =$ prob buyer deposits money in bank.

Let $M_0 = M(1-\theta)$, $M_1 = M_0 + M\theta = M$.

Bellman eqns:

$$
\begin{aligned}
rV_m &= (1-M)(1-\lambda)x(u+V_0-V_1) \\
&\quad + (1-M)\lambda\gamma(V_0-V_1)+V_1-V_m
\end{aligned}
$$

$$
rV_d = (1-M)(1-\lambda)x(u+V_0-V_1)+V_1-V_d
$$

$$
rV_0 = (1-\lambda)Mx(V_1-V_0-c)+\lambda M_0\gamma(V_1-V_0-z)
$$

$V_1 = \max\{V_m, V_d\}$ implies BR-θ condition:

$$
\theta = \begin{cases} 0 & \text{if } V_d - a < V_m \\ [0,1] & \text{if } V_d - a = V_m \\ 1 & \text{if } V_d - a > V_m \end{cases}
$$

ME requires BR-θ, IC and IR.

Proposition:

(a) $\theta = 0$ is a ME iff $c \leq \min(C_M, C_A, C_1)$;

(b) $\theta = 1$ is a ME iff $C_3 \leq c \leq C_2$;

(c) $\theta \in (0,1)$ is a ME iff

(c1) $z < C_4$, $c \in [C_3, C_1]$ and $c \leq C_4$ or

(c2) $z > C_4$, $c \in [C_1, C_3]$ and $x \geq \tilde{x}$.

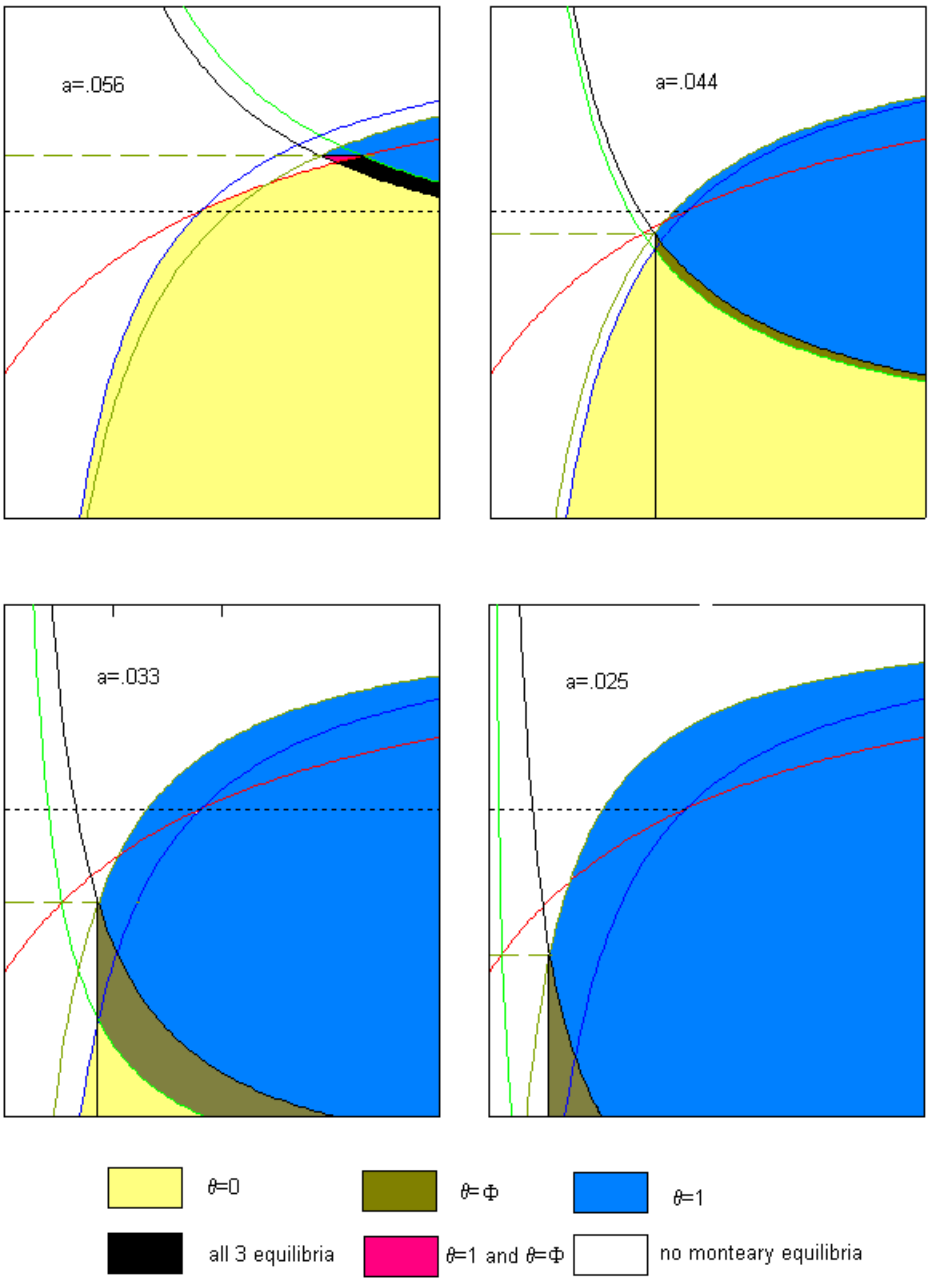
Remarks:

1. We may have unique or multiple ME.

2. $\theta \in (0,1) \Rightarrow$ *conurrent circulation*.

3. *Money may need banking*: ME may exist w/ banks but not w/o banks.

4. Fig shows ME in (x, c) space as $\hat{a}$ or M falls.


a=.056
a=.044
a=.033
a=.025
θ=0
θ=Φ
θ=1
all 3 equilibria
θ=1 and θ=Φ
no monteary equilibria

Endogenous λ, No Banks

BE for producers and thieves

$$rV_p = Mx(V_1 - V_p - c) + (1 - Mx)(V_0 - V_p)$$

$$rV_t = M\gamma(V_1 - V_t - z) + (1 - M\gamma)(V_0 - V_t),$$

$V_0 = \max\{V_t, V_p\}$ implies BR-λ

$$\lambda = \begin{cases} 0 & \text{if } V_t < V_p \\ [0,1] & \text{if } V_t = V_p \\ 1 & \text{if } V_t > V_p \end{cases}$$

Remark: ME must have $\lambda < 1$.

ME requires BR-λ and IC (IR not binding).

Proposition:

(a) $\lambda = 0$ is a ME iff

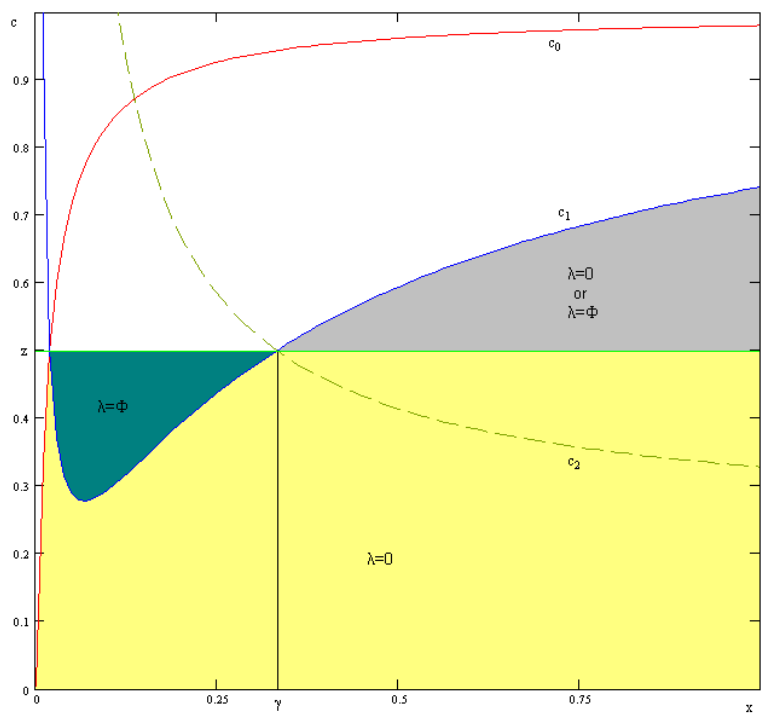
(a1) $x < x^*$ and $c \in [0, c_0]$ or

(a2) $x > x^*$ and $c \in [0, c_1]$;

(b) $\lambda \in (0, 1)$ is an equilibrium iff

(b1) $x > \gamma$ and $c \in [z, c_1)$ or

(b2) $x < \gamma$ and $c \in (c_1, z]$.



Endogenous λ, Banks

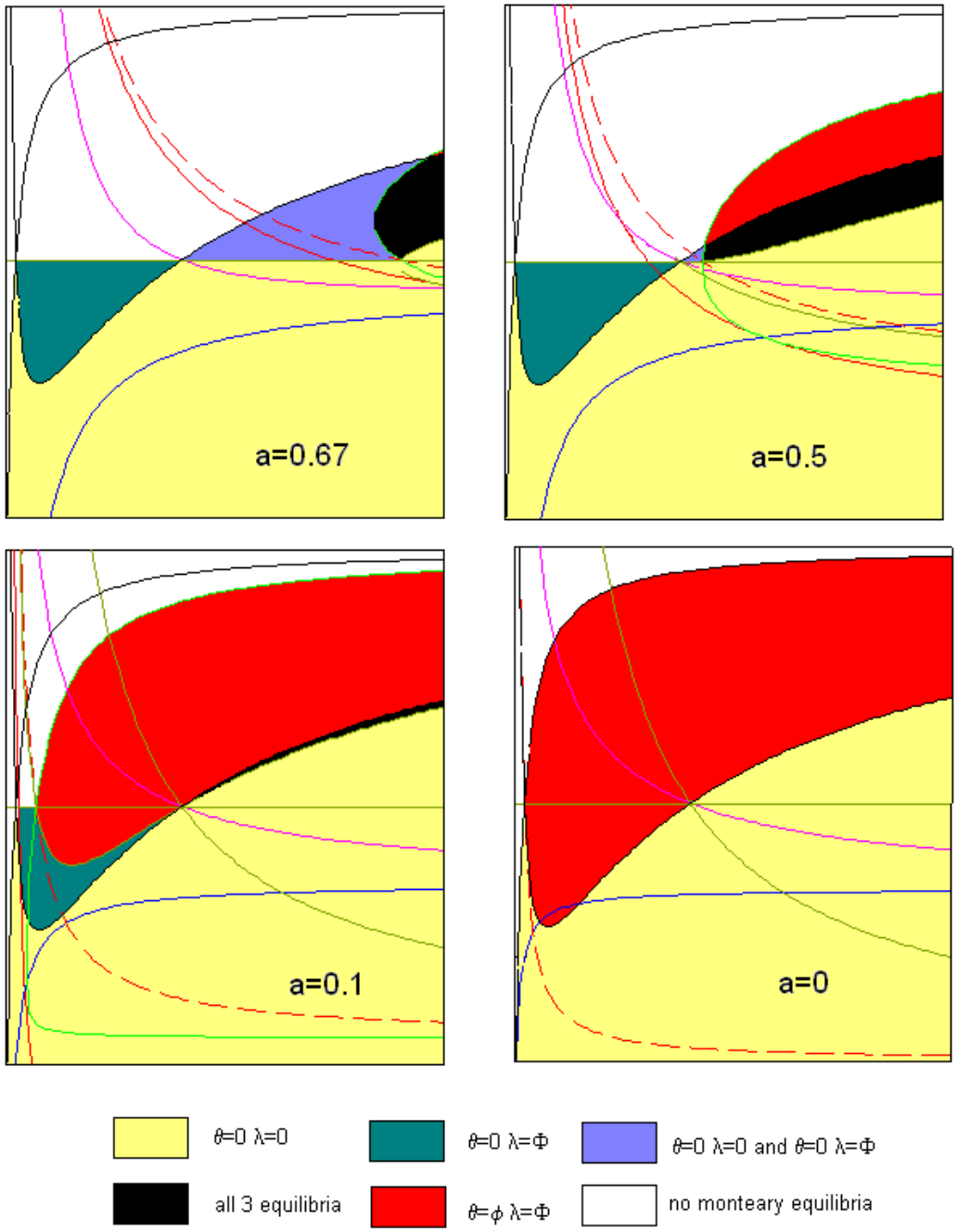
We now have IC and two BR conditions,

$$\theta = \begin{cases} 0 & \text{if } V_d - a < V_m \\ [0,1] & \text{if } V_d - a = V_m \\ 1 & \text{if } V_d - a > V_m \end{cases}$$

$$\lambda = \begin{cases} 0 & \text{if } V_t < V_p \\ [0,1] & \text{if } V_t = V_p \\ 1 & \text{if } V_t > V_p \end{cases}$$

		θ		
		0	$(0,1)$	1
	0			
λ	$(0,1)$			
	1			

Big $\hat{a}$ or $M \Rightarrow \theta = 0$. As $\hat{a}$ or M falls, $\exists$ ME for more parameters, for *two* reasons.



Prices, No Banks

Bellman equations

$$rV_1 = (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + (1-M)\lambda\gamma($$

$$rV_0 = M(1-\lambda)x[V_1 - V_0 - c(q)] + M\lambda\gamma(V_1 - V_0 - z$$

BS: take-it-or-leave it (OK if λ exogenous).

Implies $q = C_M(q)$ – same as C_M in model with indivisible goods, except $u(q)$ replaces u.

Also $V_0 \geq 0$ holds iff $q \leq C_A(q)$ – same as C_A in model with indivisible goods, except $u(q)$ replaces u.

ME exists iff solution to $q = C_M(q)$ satisfies $q \leq C_A(q)$, or equivalently $q \geq z$.

Prices, Banks

Bellman equations

$$rV_m = (1-M)(1-\lambda)x[u(q)+V_0-V_1] + (1-M)\lambda\gamma($$
$$rV_d = (1-M)(1-\lambda)x[u(q)+V_0-V_1] + V_1 - V_m$$
$$rV_0 = M(1-\theta)\lambda\gamma(V_1 - V_0 - z)$$

where $V_1 = \max\{V_d - a, V_m\}$.

$\theta = 0$ is ME iff $q = C_M(q)$, $q \geq z$, and $V_m \geq V_d - a$. Latter holds iff $\hat{a} \geq (1-M)\lambda\gamma q_0$.

$\theta = 1$ is ME iff $q = C_2(q)$ and $V_m \leq V_d - a$. Latter holds iff $\hat{a} \leq (1-M)\lambda\gamma q_1$

$\theta = \Phi$ is ME iff $q = \hat{a}/(1-M)\lambda\gamma > z$ and $C_3(C_4) \leq C_4 \leq C_1(C_4)$.

Fractional Reserves

Go back to λ fixed and indivisible goods.

Assume banks make loans at cost ρ.

Required reserve ratio α (will bind).

Equil condition: $\rho \geq V_1 - V_0$ with $=$ if $L > 0$.

Zero profit: $a = (1 - \alpha)r\rho + \phi$.

One solves for M_0, M_1, etc. in the usual way.

The Money Multiplier

$$
\begin{aligned}
M_1 &= M + \theta(1-\alpha)M + \theta^2(1-\alpha)^2 M + \ldots \\
&= \frac{M}{1-\theta(1-\alpha)}.
\end{aligned}
$$

Bellman's equations:

$$
\begin{aligned}
rV_m &= (1-\lambda)(1-M_1)x(u+V_0-V_1) \\
&\quad + \lambda(1-M_1)\gamma(V_0-V_1) + V_1 - V_m \\
rV_d &= (1-\lambda)(1-M_1)x(u+V_0-V_1) \\
&\quad + V_1 - V_d, \\
rV_0 &= (1-\lambda)M_1 x(V_1-V_0-c) \\
&\quad + \lambda M_0 \gamma(V_1-V_0-z),
\end{aligned}
$$

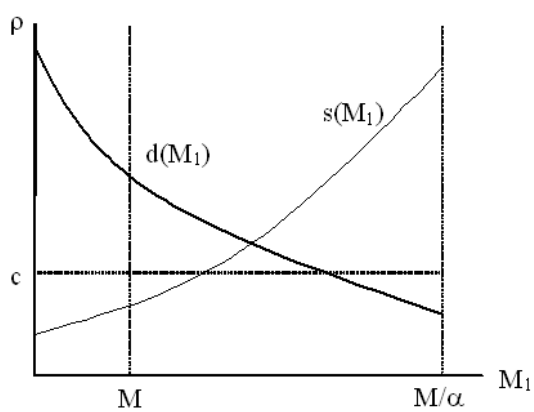
Lemma *The only possible equilibria are:* $\theta = 0$ *and* $\chi = 0$*;* $\theta \in (0,1)$ *and* $\chi \in (0,1)$*; and* $\theta = 1$ *and* $\chi \in (0,1)$.

Equilibria

Proposition *A unique equilibrium with* $\theta = \chi = 0$ *exists iff* $c \leq \min\{C_M, C_A\}$ *and* $\hat{a} \geq \hat{A}_1$.

Proposition *A unique equilibrium with* $\theta = 1$ *and* $\chi \in (0,1)$ *exists iff* $\hat{a} \leq \min\{\hat{A}_2, \hat{A}_3\}$.

Proposition *Assume* $c > max\{z, c^*\}$. *A unique equilibrium with* $\theta \in (0,1)$ *and* $\chi \in (0,1)$ *exists iff* $\hat{A}_3 < \hat{a} < \hat{A}_1$ *and* $\hat{a} \geq \hat{A}_4$.



Loan Market