Inflation Targeting in a Small Open Economy: The Colombian Case

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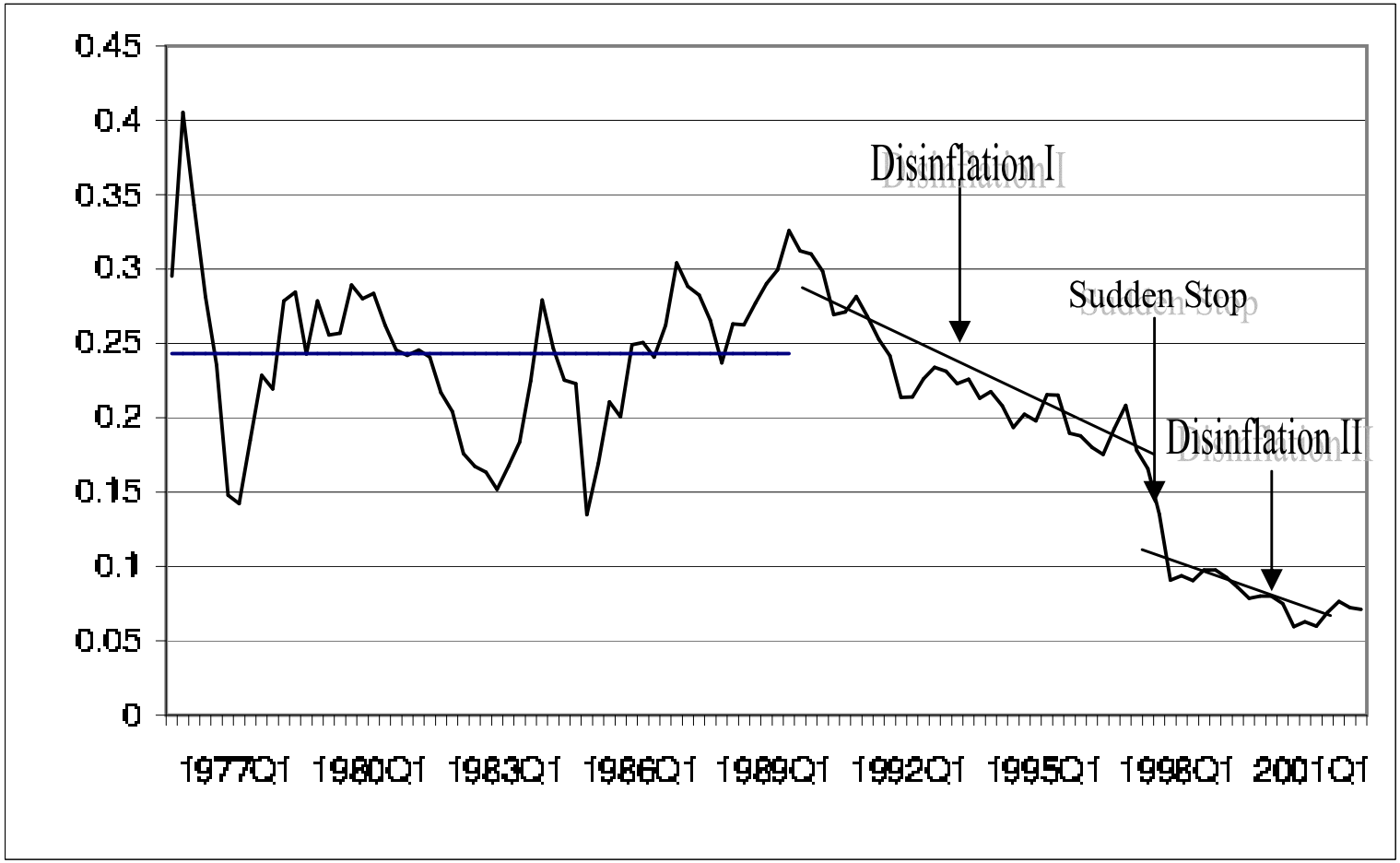
Monetary Policy in Colombia

- The Banco de la República conducts monetary policy using a "Flexible Forecast Inflation Targeting" strategy since September 1999.

- At the end of a given year, the Bank announces an inflation target for the next two years and publishes a Quarterly Inflation Report.

- The "logic" of the Quarterly Inflation's Report is built upon a "core" model and its forecast (and yes, the interest rate forecast is also public).

“Chapulín Colorado-Disinflation “



The Current Core Model

- A semi-structural model, commonly used in many Central Banks.

- Very useful (within the Bank) to discipline the economic discussion around the elements of a model.

- We are in the process of building a *structural* model that captures the main frictions that characterize Emerging Market Economies.

The GE Model for IT in Colombia

- We see the GE model as a tool for improving consistency of our "story".

- We have already used the GE model for policy analysis.

- Our aim is to "take the GE model bowling".

What Kind of Model do we Want?

- Theoretically rich, tractable and explicable.
- Useful for contrasting alternative economic stories.
- Able to match the main facts about the Colombian economy.
- Flexible to the *formal* imposition of judgement.

Our General Strategy

- Identify the basic facts about the dynamics of the relevant Colombian macroeconomic series.

- Calibrate and solve a basic small open economy model.

- Compare the DGP of the model against the data (we use spectral analysis).

- Forecast a subset of variables (inflation, output and interest rates) using Bayesian techniques.

Main Features:

1. Small open economy: agents can freely borrow and lend in international financial markets.

2. Two sector model:

   - producers in perfect competition

   - retailers in monopolistic competition

3. Nominal rigidities: a fraction of retailers set prices based on past inflation and their last period's price.

4. Money saves transaction costs (need money demand).

5. Imperfect pass-through.

6. Perfectly competitive labor market (Hansen, 1985).

7. Habit persistence to avoid the "price-consumption puzzle".

8. Government:

   - Monetary policy rule: set nominal interest rate to target inflation.

   - Taxes/transfers & unproductive expenditure.

The Representative Household's Problem:

$$\max_{\{c,h,x,k,m,H\}} E_t \left(\sum_{t=0}^{\infty} \beta^t u\left(c_t, H_t, h_t, \mu_t^u\right) \right)$$

subject to

$$\begin{aligned} c_t + \Phi + m_{t+1}^d + q_t x_t + b_{t+1} + q_t F_{t+1} \;=\; & \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + \\ & \frac{m_t^d}{(1+\pi_t^c)} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + \\ & F_t q_t (1+i_t^f) + \tau_t \end{aligned}$$

$$k_{t+1} - (1-\delta)k_t - f\left(\frac{x_t}{k_t}\right)k_t = 0$$

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0$$

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

and where

$$\left(1 + i_t^f\right) = (1 + i_t^*)\left(1 + \vartheta\left(\frac{F_t}{y_t}\right)\right)$$

The Producer's Problem:

$$\max_{\{k,h\}} \Pi_t = P_t A_t (k_t^d)^{\alpha} (h_t^d)^{(1-\alpha)} - R_t k_t^d - W_t h_t^d$$

The Retailer's Problem:

Each period retailer's profits are given by:

$$\Pi^R(z)_t = c(z)_t(p^c(z)_t - P_t)$$

If allowed to optimize:

$$\max_{\{p^c(z)_t)} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\,\varepsilon^j \Delta_{t+j} \frac{\Pi^R(z)_{t+j}}{P^c_{t+j}}$$

else:

$$p_t^{rule}(z) = p^c_{t-1}(z)(1+\pi^c_{t-1})$$

Consolidated Monetary and Fiscal Authority

The government budget constraint:

$$m_{t+1}^{s} - \frac{m_t^s}{1+\pi_t^c} + \Phi\left(c_t, m_{t+1}, I_t\right) = \tau_t + g_t$$

Monetary policy rule:

$$i_t = i + \zeta\left(\pi_t^c - \overline{\pi}^c\right) + \xi\left(y_t - y^{ss}\right)$$

Competitive Equilibrium

Definition: A price system is a positive sequence

$\{W_t, R_t, p_t^{rule}, p_t^{opt}, P_t^c, P_t, e_t, i_t, i_t^f\}_{t=0}^{\infty}$.

Definition: Given the exogenous sequence $\{A_t, \mu_t^{\vartheta}, \mu_t^u, \frac{g_t}{y_t}, P_t^*\}_{t=0}^{\infty}$ and m_0, k_0, b_0, F_0, $H_0 > 0$, an equilibrium is a price system, a sequence of quantities $\{c_t\}_{t=0}^{\infty}$, $\{x_t\}_{t=0}^{\infty}$, $\{k_t\}_{t=1}^{\infty}$, $\{h_t\}_{t=0}^{\infty}$, $\{H_t\}_{t=1}^{\infty}$, $\{b_t\}_{t=1}^{\infty}$, $\{F_t\}_{t=1}^{\infty}$ and a positive sequence of real money $\{m_t\}_{t=1}^{\infty}$in order that:

1. Given the prices and lump sum transfers, household's optimal control problem is solved with $\{m_t^d = m_t^s = m_t\}_{t=1}^{\infty}$,

$\{k_t^d = k_t^s = k_t\}_{t=1}^{\infty}$, $\{b_t = 0\}_{t=1}^{\infty}$, $\{h_t^d = h_t^s = h_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$ and a level of $\{F_t\}_{t=1}^{\infty}$ such that $(1 + i_t) = \left(1 + i_t^f\right)(1 + d_t)$

2. The governments budget constraint and policy rule are satisfied for all $t \geq 0$.

3. $Y_t = C_t + I_t + G_t + F_{t+1} - \left(1 + i_t^f\right) F_t$ for all t.

Nonlinear Dynamic System:

$$c_t + x_t + g_t + F_{t+1} - \left(1 + i^f_{t+1}\right) F_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

$$u_{c_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \eta_t \rho = \lambda_t 1 + \Phi_{c_t}\left(c_t, m_{t+1}, x_t\right)$$

$$u_{h_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \lambda_t q_t A_t (1-\alpha) k_t^{\alpha} h_t^{-\alpha} = 0$$

$$\beta E_t \left(\lambda_{t+1} q_{t+1} A_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \gamma_{t+1}\left(c_0 - \frac{c_2 x_{t+1}^2}{k_{t+1}^2}\right) + \gamma_{t+1}(1-\delta)\right) = \gamma_t$$

$$\beta E_t \left(\frac{\lambda_{t+1}}{\left(1+\pi^c_{t+1}\right)\left(1+\Phi_{m_{t+1}}\left(c_t, m_{t+1}, x_t\right)\right)} \right) = \lambda_t$$

$$\beta E_t \left(\frac{\lambda_{t+1}\left(1+i_{t+1}\right)}{\left(1+\pi^c_{t+1}\right)} \right) = \lambda_t$$

$$\beta E_t \left(\lambda_{t+1}\left(1+i^f_{t+1}\right) q_{t+1} \right) = \lambda_t q_t$$

$$\beta E_t \left(\eta_{t+1} + U_{H_{t+1}}\left(c_{t+1}, H_{t+1}, h_{t+1}, \mu^u_{t+1}\right) - \eta_{t+1}\rho \right) = \eta_t$$

$$\lambda_t \left(\Phi_{x_t} \left(c_t, m_{t+1}, x_t \right) + q_t \right) = \gamma_t \left(c_1 + \frac{2c_2 x_t}{k_t} \right)$$

$$k_{t+1} - (1-\delta)k_t - f\left(\frac{x_t}{k_t}\right) k_t = 0$$

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0$$

$$i_t = i + \zeta \left(\pi_t^c - \overline{\pi}^c \right) + \xi \left(y_t - \overline{y} \right)$$

$$\left(1+i_t^f\right) = (1+i_t^*)\left(1+\vartheta\left(\frac{F_t}{y_t}\right)\right)$$

$$P_t^c = \left[\varepsilon(p_t^{rule})^{1-\theta} + (1-\varepsilon)(p_t^{opt})^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$(1+\pi_t^c) = \left(\varepsilon(1+\pi_{t-1}^c)^{(1-\theta)} + (1-\varepsilon)\left(\frac{p_t^{opt}}{P_t^c}\right)^{(1-\theta)}(1+\pi_t^c)^{(1-\theta)}\right)^{\frac{1}{1-\theta}}$$

$$p_t^{rule} = p_{t-1}^c(1+\pi_{t-1}^c)$$

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1}\frac{\Theta_t}{\Psi_t}$$

$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^{\theta} \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^{\theta - 1} \Psi_{t+1} \right)$$

Solution

1. Calculate the steady state of the first order nonlinear dynamic system that characterizes the competitive equilibrium and log-linearize around the steady state.

2. Using KPR Method, the solution is of the form:

$$\mathbf{Y}_t = \mathbf{H}\mathbf{x}_t$$

$$\mathbf{x}_{t+1} = \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{\mathbf{t+1}}$$

where $\mathbf{Y}$ is a vector of control, co-state and flow variables, $\mathbf{x}$ is a vector of states, $\mathbf{H}$ is the linearized policy function and $\mathbf{M}$ the state transition matrix. η_{t+1} is an innovation vector.

Calibration

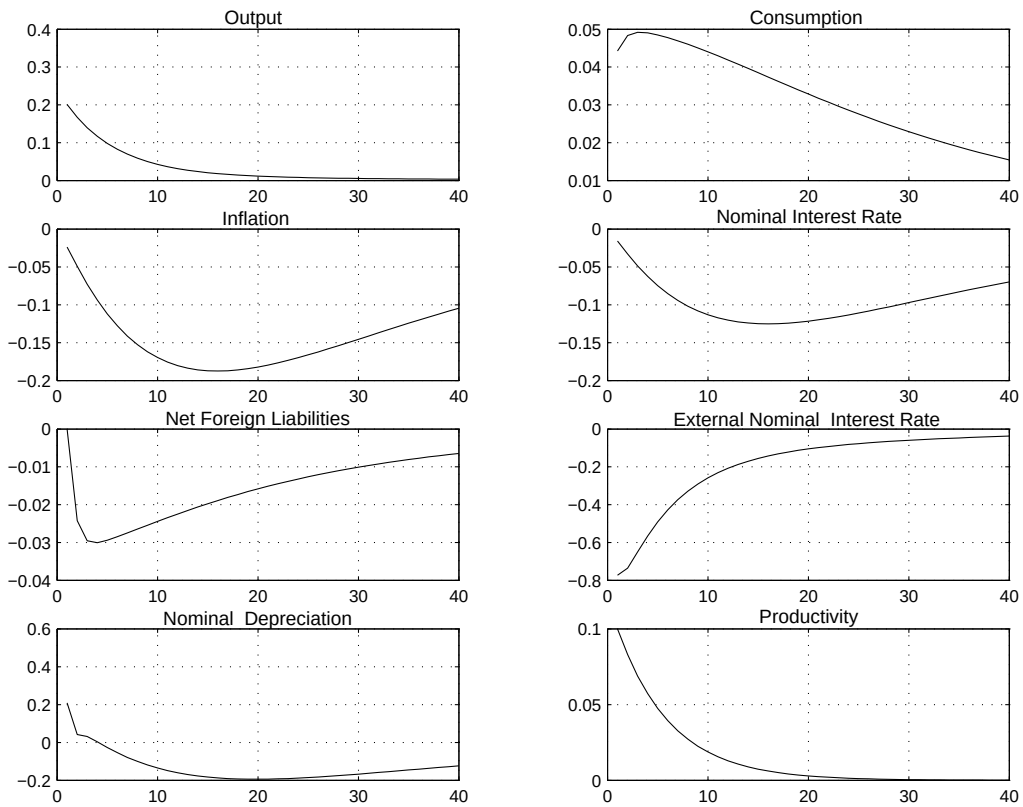
- The model was calibrated to match some long run values of observed macro time series

- Some micro and macro studies were used also

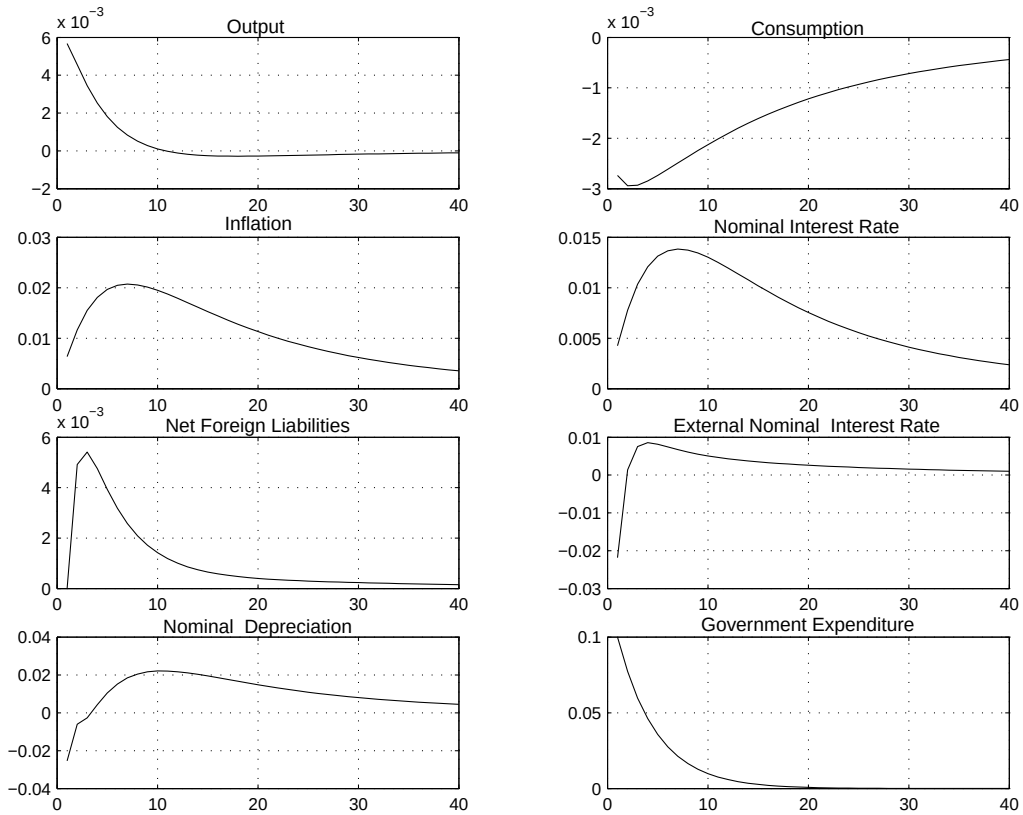
- Simulations were performed with only one source of shocks (productivity shocks)

Parameter	Value	Target
B	**1.06**	To have $h = 0.33$
α	**0.33**	Approximately corresponds to the capital share in income
δ	**0.012**	constructed capital time series
a	**1.86**	From the estimation of the demand for money
κ	**0.06**	From the estimation of the demand for money
ν	**0.025**	From the estimation of the demand for money
ρ_{tech}	**0.83**	Fitted time series of productivity
θ	**5**	Corresponds to a markup of 25%
ε	**0.75**	Prices changing every one year
r	**6.81(a)**	Estimated anual real interest rate for Colombia, Vasquez (2003)
β	**0.984(t)**	Consistent with r
$\overline{\pi}$	**5.5(a)**	Target set for this year by the Central Bank
i	**0.03(t)**	Fixed according to $\overline{\pi}$ and r
i^*	**0.03(a)**	International macro literature
φ	**0.5**	Observed consumption's volatility
ρ	**0.8**	Observed consumption's persistence

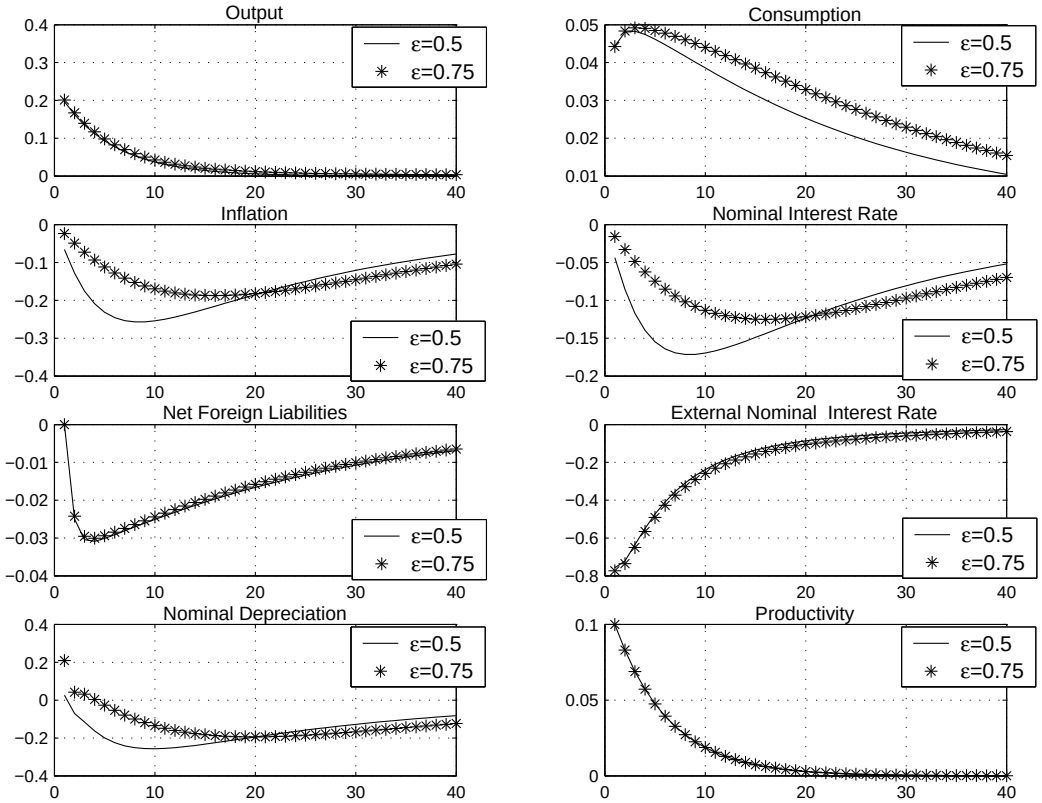
Impulse Response to a Productivity Shock



Impulse Response to a Government Expenditure Shock



Sensitivity Analysis: Productivity Shock when Prices are more Flexible



Assessing Population and Model Agreement

Diebold et al. Methodology

- Estimate Population Spectrum from Data
- Determine Salient Features of Data
- Assess Sampling Variability of Estimate
- Determine Model Theoretical Spectrum and Compare (test hypothesis)

The Hypothesis

When we compare sample and model spectrum and determine if last falls into confidence bands we are testing the null

$$H_0 : \mathbf{F}_{\mathbf{Y}}(\omega) = \mathbf{F}_M(\omega) \qquad \text{for} \qquad \omega \in [\omega_0, \omega_1]$$

which means that the true (but unknown) population spectrum equals model theoretical spectrum for frequencies between ω_0 and ω_1.

Estimating Population Spectrum

The estimated data (HP filtered) spectrum is:

$$\widehat{\mathbf{F}}^{*}_{\mathbf{Y}}\left(\omega_j\right) = \frac{1}{2\pi}\left\{\sum_{\tau=-(t-1)}^{t-1} \mathbf{\Lambda}\left(\tau\right)\widehat{\mathbf{\Gamma}}_{\tau}\exp\left(-iw_j\tau\right)\right\}$$

$\mathbf{\Lambda}\left(\tau\right)$ is a matrix of lag windows (kernel) that is:

- A Truncated, Symmetric and Positive weigthing function of lags. Truncation lag defines the window size. Outside window weights $= 0$.

- The window size generates a trade-off between the smoothness and consistency of the estimated spectra and the biasness of the sample.

Assesing Sampling Variability

- Difficult to obtain analitically for general processes (phase and coherence - non linear)

- Can do Cholesky factor bootstrapping:
  - Stack observed sample vectors in $\mathbf{Z} = \left[\mathbf{Y}_1^T, \mathbf{Y}_2^T, \cdots, \mathbf{Y}_t^T\right]^T$
  - For i-th iteration, randomly draw $\varepsilon^{(i)}$ from an NT dimensional standard distribution
  - Compute $\mathbf{z}^{(i)} = \overline{\mathbf{z}} + \mathbf{P}^* \varepsilon^{(i)} \sim \left(\mathbf{1}_T \otimes \mu, \mathbf{\Sigma}^* = \mathbf{P}^* \mathbf{P}^{*T}\right)$ where $\overline{\mathbf{z}} = \mathbf{1}_T \otimes \overline{\mathbf{Y}}$ and $\mathbf{\Sigma}^*$ is variance covariance matrix affected by $\mathbf{\Lambda}(\tau)$

- Compute $\widehat{\mathbf{F}}^{*(i)}\left(\omega_j\right)$
- Obtain variance of spectrum, co-sprectrum, coherence and phase functions.

Model Theoretical Spectrum

If model can be written in SSF

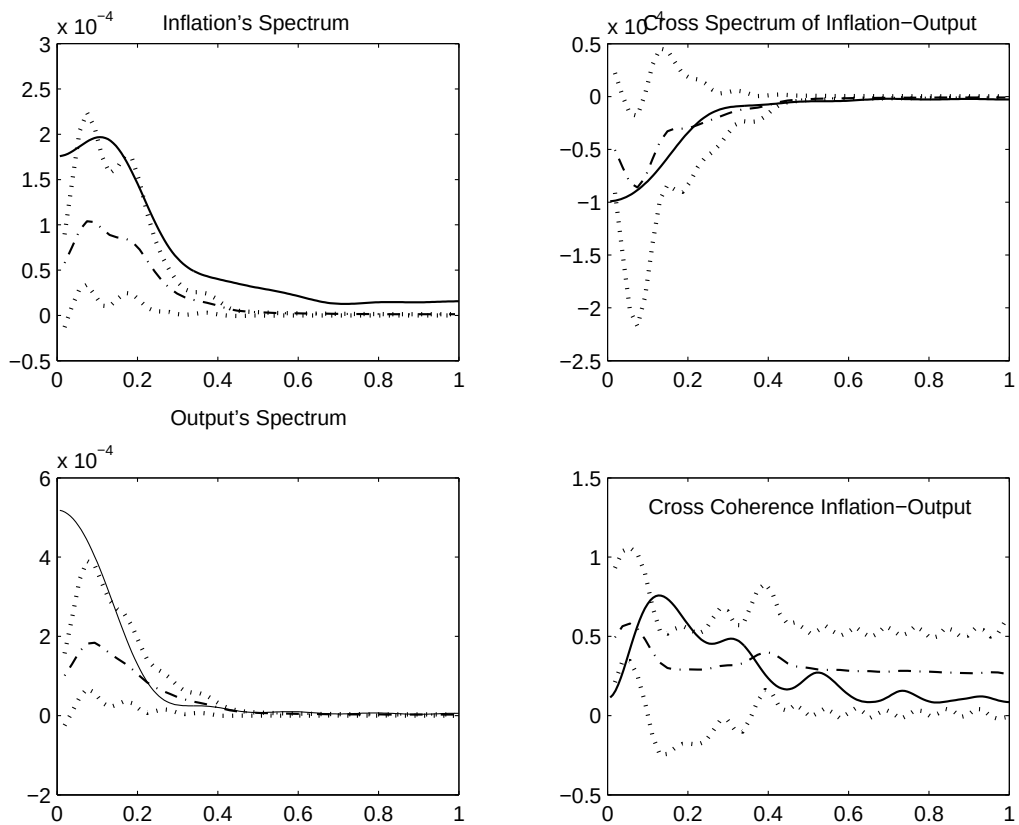
$$\mathbf{Y}_t = \mathbf{H}\alpha_t$$
$$\alpha_{t+1} = \mathbf{M}\alpha_{t-1} + \mathbf{R}\eta_{t+1}$$

where η_{t+1} is iid$(\mathbf{0}, \mathbf{\Omega})$

- We can use spectral arithmetic to compute model spectrum. Hamilton(1994, Ch. 6 and 10)

- The spectrum was calculated with just the productivity shock and applying the Hodrick and Prescott filtration.

Results


Inflation's Spectrum
x 10^−4
Cross Spectrum of Inflation−Output
x 10^−4
Output's Spectrum
x 10^−4
Cross Coherence Inflation−Output

Salient Features of Data

- Inflation and output gap are dominated by periodic movements between 2 and 25 quarters. No long movements (data is HP filtered). Peak between 10 and 12 quarters, some degree of stickiness or persistence.

- Co-spectrum and coherence show results in the same direction.

- Coherence does not show significant frequency dominance.

Model Spectrum

- Some persistence both in the univariate spectra as well as in the cross spectra.

- Monotone spectra for output gap and cross spectra.

- Inflation spectra peaks for periodic movements of between 9 and 10 quarters.

- Higher correlation for periodic movements around five years.

Comparison

- Theoretical spectra and cross spectra fall into uncertainty bands for periodic movements of inflation and comovements of inflation and output gap of up to 5 years, and for periodic movements of output gap of up to 10 quarters.

- For longer periods the spectra and cross spectra of the model are significantly different from the population ones.

- Coherence falls into the uncertainty bands for most of the frequencies but the ones surrounding the model coherence peak and the long run periodic movements.

Final Remarks

- This is our first step in developing a consistent framework for policy analysis and forecast in Colombia

- We hope this framework can improve the implementation of Inflation Targeting in terms of: consistency, transparency and accountability

- We have seen how spectral analysis can help us to identify the scope and limitations of our core model

The challenge

- Forecasting: we have performed some Bayesian forecasting exercises using the model as a prior. The results are promising (forthcomming).

- Theory: the model lacks several important properties of EMs
  - Liability dollarization
  - Sudden Stop of capital inflows
  - Financial / Housing sector frictions