The Beveridge-Nelson Decomposition and Impulse-Response Analysis in the Presence of Markov-Switching: Has the Persistence of Real GDP Changed Since the Mid 1980's?

by

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Abstract

We show that, for a class of linear and multivariate Markov-switching models, exact calculation of the Beveridge-Nelson (BN) trend/cycle components is possible. The key to exact BN trend/cycle decomposition is to recognize that the latent first-order Markov-switching process in the model has an AR(1) representation, and that the model can be cast into a state-space form. Given the state-space representation, we also show that impulse-response analyses can be done with respect to an asymmetric discrete shock as well as to a symmetric continuous shock. The methodologies developed are applied to Kim, Morley, Piger's (2005) univariate Markov-switching model of real GDP with a post-recession 'bounce-back' effect and Cochrane's (1994) vector error correction model of real GDP and real consumption extended to incorporate Markov switching. Based on the parameter estimates, the calculated BN trend/cycle components, and the impulse-response analyses for each of these empirical models, we raise the possibility that the persistence of real GDP might have increased since the mid-1980's in the U.S.

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1. Introduction

The Beveridge-Nelson (BN) decomposition, first introduced by Beveridge and Nelson (1981) within the linear univariate context and then generalized by Stock and Watson (1987) to the multivariate case, is a way of decomposing an integrated economic time series into the sum of unobservable trend and cyclical components. Since its first introduction, it has proven its usefulness for both theoretical and empirical reasons. The BN trend component is defined as the long-horizon forecast of the level of the time series (adjusted for the drift term), [1] and various efficient computational methods for the BN trend/cycle decomposition have been provided. [See, for examples, Cuddington and Winters (1987), Miller (1988), Newbold (1990), and Arino and Newbold (1998).] In particular, Morley (2002), by employing the state-space approach, provides a general unified framework for exact calculation of the BN trend/cycle components for both univariate and multivariate linear processes.

More recently, Clarida and Taylor (2003) provide an extension of the BN decomposition to a particular class of univariate and multivariate nonlinear processes. In the nonlinear case, generating long-horizon forecasts is non-trivial, and they suggest a tractable method of doing this using Monte Carlo Integration.

In this paper, we further extend the BN decomposition to univariate and multivariate Markov-switching processes first introduced by Hamilton (1989). We show that, for a particular class of Markov-switching processes, exact calculation of the BN trend/cycle is possible, based on a state-space representation of the processes under consideration. The key to this BN decomposition is to note that the latent Markov-switching variable has an autoregressive representation. We also provide illustrative applications of the proposed methodology in both the univariate and multivariate processes, with particular attention to a univariate model of real GDP with Markov-switching and to a version of Cochrane's (1994) vector error correction model (VECM) of real GDP and real consumption extended to incorporate Markov-switching.

[1] Morley, Nelson, and Zivot (2003) show that the BN decomposition is identical to the decomposition based on unobserved components (UC) models introduced by Harvey (1985) and Clark (1987), once the restrictions imposed in the UC models are relaxed.

The fact that a Markov-switching model has a state-space representation allows us to perform interesting impulse-response analyses. Within the univariate Markov-switching framework, for example, there exist two independent shocks: a symmetric continuous shock and an asymmetric discrete shock to the latent Markov-switching variable in the model. We examine how the responses of real GDP to these two different types of shocks has changed since 1984Q2, which was identified by Kim and Nelson (1999) and McConnell and Quiros (2000) as the quarter of structural break in the dynamics of real GDP. Major issues to be investigated based on the impulse response analyses are: i) Has the long-run dynamics of real GDP changed? ii) If so, what would be the source of the change in the long-run dynamics of real GDP?

Section 2 provides a review of the BN decomposition for a linear process. Section 3 then provides a basic idea behind the BN decomposition of a Markov-switching process. In Section 4, we present an application of the methodology introduced in Section 3 to the univariate Markov-switching model of real GDP with a post-recession 'bounce-back' effect as proposed by Kim, Morley, and Piger (2005). In Section 5, the methodology is extended to the case of a vector error-correction model of real GDP and real consumption with Markov switching. Empirical results are discussed in Section 6, and, finally, a brief summary and a suggestion for further research are provided in Section 7.

2. Beveridge-Nelson Decomposition and Impulse-Response Analysis in the Absence of Markov Switching: Review

Assume that $y_t \sim I(1)$ and that its first difference follows a stationary AR(p) process with an intercept term μ, we have :

$$(1 - \phi(L))\Delta y_t = \mu + e_t, \quad e_t \sim i.i.d.N(0, \sigma^2), \tag{1}$$

where $\phi(L) = \phi_1 L + \phi_2 L^2 - ... + \phi_k L^k$ and all the roots of $1 - \phi(L) = 0$ lie outside the complex unit circle.

The minimum mean squared error predictor of $y_{t+\tau}$ conditional on information up to time t (I_t) is given by:

$$
\begin{aligned}
E[y_{t+\tau}|I_t] &= y_t + \sum_{j=1}^{\tau} E[\Delta y_{t+j}|I_t] \\
&= y_t + \sum_{j=1}^{\tau} E[\Delta y_{t+j} - E(\Delta y_{t+j})|I_t] + \tau E(\Delta y_{t+j}) \\
&= y_t + \sum_{j=1}^{\tau} E[\Delta y^*_{t+j}|I_t] + \tau E(\Delta y_{t+j}),
\end{aligned} \tag{2}
$$

where $\Delta y^*_{t+j} = \Delta y_{t+j} - E(\Delta y_{t+j})$ and where $E(\Delta y_{t+j}) = (1-\phi(1))^{-1}\mu$.

This suggests that the forecast profile is asymptotic to a linear function of the forecast horizon τ with slope equal to μ and a level $(y_t + \sum_{j=1}^{\tau} E[\Delta y^*_{t+j}|I_t])$ which itself is a stochastic process. Beveridge and Nelson (1981) interpret this level (when $\tau \to \infty$) as the permanent or trend component of y_t:

$$
\begin{aligned}
y_t^{TR} &= y_t + \sum_{j=1}^{\infty} E[\Delta y^*_{t+j}|I_t] \\
&= y_t - y_t^c,
\end{aligned} \tag{3}
$$

where y_t^c is the cyclical component.

In practice, calculation of the y_t^{TR} term is complicated by the presence of infinite sums of the forecasts. Various ways of evaluating the infinite sums of the forecasts have been proposed by Cuddington and Winters (1987), Miller (1988), Newbold (1990), and Arino and Newbold (1998). However, as shown in Morley (2001), the state-space approach provides a general unified framework for calculating the y_t^{TR} term. By noting that $\Delta y_t^* = \Delta y_t - E(\Delta y_t) = \Delta y_t - (1-\phi(1))^{-1}\mu$, we can rewrite equation (1) in the following state-space form:

$$
\begin{bmatrix} \Delta y_t^* \\ \Delta y_{t-1}^* \\ \vdots \\ \Delta y_{t-p+1}^* \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^* \\ \Delta y_{t-2}^* \\ \vdots \\ \Delta y_{t-p}^* \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{4}
$$

$$
(\tilde{y}_t^* = F\tilde{y}_{t-1}^* + \tilde{e}_t).
$$

Given the above state-space form, as $E[\Delta y^*_{t+j}|I_t]$ is the $(1,1)$ element of $F^j\tilde{y}_t^*$, the BN trend component of y_t in (3) has the following solution:

$$
\begin{aligned}
y_t^{TR} &= y_t + \sum_{j=1}^{\infty} E[\Delta y_{t+j}^* | I_t] \\
&= y_t + (1,1) \ element \ of \ (F + F^2 + F^3 + \ldots)\tilde{y}_t^* \\
&= y_t + (1,1) \ element \ of \ F(I_p - F)^{-1}\tilde{y}_t^*
\end{aligned} \tag{5}
$$

Given the state-space representation in equation (4), the impulse-response coefficient is given by:

$$
\frac{\partial y_{t+j}}{\partial e_t} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+j}}{\partial e_t} = (1,1) \ element \ of \sum_{i=0}^{j} F^i. \tag{6}
$$

3. Beveridge-Nelson Decomposition and Impulse-Response Analysis in the Presence of Markov Switching: Basic Framework

Consider the following version of Hamilton's Markov-switching Model (1989), in which we have an AR(k) process with a Markov-switching intercept term: [2]

$$
(1 - \phi(L))\Delta y_t = \mu_0 + \mu_1 S_t + e_t, \quad e_t \sim i.i.d.N(0, \sigma^2) \tag{7}
$$

where S_t is a discrete, first-order Markov-switching process which evolves according to the following transition probabilities:

$$
Pr[S_t = 1 | S_{t-1} = 1] = p, \quad Pr[S_t = 0 | S_{t-1} = 0] = q. \tag{8}
$$

Taking unconditional expectations on both sides of equation (7), we have:

$$
(1 - \phi(1))E(\Delta y_t) = \mu_0 + \mu_1 \pi, \tag{9}
$$

where $\pi = E(S_t) = \frac{(1-q)}{2-p-q}$. Subtracting equation (9) from equation (7) results in:

$$
\Delta y_t^* = \phi(L)\Delta y_t^* + \mu_1(S_t - \pi) + e_t, \tag{10}
$$

where $\Delta y_t^* = \Delta y_t - E(\Delta y_t)$.

As in the case of linear AR(k) process, the BN trend is given by:

$$
y_t^{TR} = y_t + \sum_{j=1}^{\infty} E[\Delta y_{t+j}^* | I_t], \tag{11}
$$

[2] It is straightforward to extend the model to incorporate a Markov-switching mean.

and the term $\sum_{j=1}^{\infty} E[\Delta y^*_{t+j}|I_t]$ is obtained from a state-space representation of equation (10). Writing equation (10) in state space-form is straightforward once we recognize that the latent Markov-switching process S_t has the following AR(1) representation as in Hamilton (1989):

$$S_t = \lambda_0 + \lambda_1 S_{t-1} + v_t, \quad v_t \sim (0, (1-\lambda_1^2)\pi(1-\pi)) \tag{12}$$

where $\lambda_0 = 1-q$, $\lambda_1 = p+q-1$, $E(v_t) = 0$ and $var(v_t) = (1-\lambda_1^2)\pi(1-\pi)$, and where $\pi = E(S_t)$. Alternatively, it can be written as:

$$(S_t - \pi) = \lambda_1(S_{t-1} - \pi) + v_t. \tag{12'}$$

Using equation (12'), we have the following state-space representation of equation (10):

$$\begin{bmatrix} \Delta y^*_t \\ \Delta y^*_{t-1} \\ \vdots \\ \Delta y^*_{t-k+1} \\ S_{t+1} - \pi \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_k & \mu_1 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & \lambda_1 \end{bmatrix} \begin{bmatrix} \Delta y^*_{t-1} \\ \Delta y^*_{t-2} \\ \vdots \\ \Delta y^*_{t-k} \\ S_t - \pi \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \\ \vdots \\ 0 \\ v_t \end{bmatrix}. \tag{13}$$

$$(\tilde{X}_t = F\tilde{X}_{t-1} + \tilde{e}_t)$$

Thus, the BN trend in the presence of Markov switching is given by:

$$\begin{aligned} y_t^{TR} &= y_t + \sum_{j=1}^{\infty} E[\Delta y^*_{t+j}|I_t] \\ &= y_t + (1,1)\ element\ of\ (F + F^2 + F^3 + \dots)E[\tilde{X}_t|I_t] \\ &= y_t + (1,1)\ element\ of\ F(I_p - F)^{-1}E[\tilde{X}_t|I_t], \end{aligned} \tag{14}$$

where $E[\tilde{X}_t|I_t] = [\,\Delta y^*_t \quad \Delta y^*_{t-1} \quad \dots \quad \Delta y^*_{t-k+1} \quad (E[S_{t+1}|I_t] - \pi)\,]'$

In a Markov-switching model, there exist two independent shocks: a symmetric continuous shock, e_t, and an asymmetric discrete shock, v_t. Given the state space form in (13), it is straightforward to perform impulse-response analysis with respect to these two alternative shocks:

$$\frac{\partial y_{t+j}}{\partial e_t} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial e_t} = (1,1) \ element \ of \ \sum_{i=0}^{j} F^i. \tag{15}$$

$$\frac{\partial y_{t+j}}{\partial v_t} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial v_t} = (1,k+1) \ element \ of \ \sum_{i=0}^{j} F^i. \tag{16}$$

Lastly, we show that, as in Beveridge and Nelson (1981), the BN trend component derived above can be reduced to a linear random walk process. To see this, we note that equation (7) can be decomposed into two components: a linear component and a Markov-switching component:

$$\Delta y_t = \Delta y_{1t} + \Delta y_{2t}, \tag{17}$$

where

$$\phi(L)\Delta y_{1t} = e_t \tag{18}$$

and

$$\phi(L)\Delta y_{2t} = \mu_0 + \mu_1 S_t. \tag{19}$$

Writing equation (19) in a deviation-from-mean form and by using $(1-\lambda_1 L)(S_t - \pi) = v_t$ from equation (12'), equation (19) can be rewritten as:

$$\phi(L)(1-\lambda_1 L)(\Delta y_{2t} - \pi^*) = \mu_1 v_t, \tag{19'}$$

where L is a lag operator and $\pi^* = E(\Delta y_{2t}) = \phi(1)^{-1}(\mu_0 + \mu_1 \pi)$, and where $\pi = E(S_t)$.

Then, based on equations (18) and (19'), it is easy to show that the trend components of y_{1t} and y_{2t} are given by the following random walks: [3]

[3] For example, the trend components of $y_{2,t+1}$ and y_{2t} are given by:

$$y_{2,t}^{TR} = y_{2t} + \sum_{j=2}^{\infty} (E(\Delta y_{2,t+j}|I_{t+1}) - \pi^*) + (\Delta y_{2,t+1} - \pi^*) + \pi^*$$

$$y_{2,t}^{TR} = y_{2t} + \sum_{j=1}^{\infty} (E(\Delta y_{2,t+j}|I_t) - \pi^*)$$

Subtracting y_{2t}^{TR} from $y_{2,t+1}^{TR}$, we have:

$$\Delta y_{2,t+1}^{TR} = \sum_{j=1}^{\infty} (E(\Delta y_{2,t+j} - \pi^*|I_{t+1}) - E(\Delta y_{2,t+j} - \pi^*|I_t)) + \pi^*$$

$$\begin{aligned}\Delta y_{1,t+1}^{TR} &= \sum_{j=1}^{\infty}(E(\Delta y_{1,t+j}|I_{t+1}) - E(\Delta y_{1,t+j}|I_t)) \\ &= \phi(1)^{-1}e_{t+1}\end{aligned} \tag{20}$$

$$\begin{aligned}\Delta y_{2,t+1}^{TR} &= \sum_{j=1}^{\infty}(E(\Delta y_{2,t+j} - \pi^*|I_{t+1}) - E(\Delta y_{2,t+j} - \pi^*|I_t)) + \pi^* \\ &= \phi(1)^{-1}(1-\lambda_1)^{-1}\mu_1 v_{t+1} + \pi^*\end{aligned} \tag{21}$$

As the first difference of the BN trend component of y_t is the sum the two terms in equations (20) and (21), we have the following random walk representation for the trend component of y_t:

$$\Delta y_t^{TR} = \pi^* + \epsilon_{t+1}, \tag{22}$$

where $\epsilon_t = \phi(1)^{-1}e_{t+1} + \phi(1)^{-1}(1-\lambda_1)^{-1}\mu_1 v_{t+1}$ with $E(\epsilon_t) = 0$ and $var(\epsilon_t) = \phi(1)^{-2}\sigma^2 + \phi(1)^{-2}(1-\lambda_1)^{-2}(1-\lambda_1^2)\mu_1^2\pi(1-\pi)$.

4. Application #1: Kim, Morley, Piger's (2005) Markov-Switching Model of Real GDP Growth with a Post-Recession 'Bounce Back' Effect

While Hamilton's (1989) model of asymmetric business cycles portrays the short, violent nature of recessions relative to booms, it fails to capture a very important feature of the post-war U.S. business cycles discussed in the literature: recessions typically are followed by high-growth recovery phases that push output back to its pre-recession level [Sichel (1994)]. That is, Hamilton's model fails to account for the fact that recessions may be

As the Wold representation of equation (19') can be written as:

$$\Delta y_{2t} - \pi^* = \psi^*(L)v_t,$$

where $\psi^*(L) = \phi(L)^{-1}(1-\lambda_1 L)^{-1}\mu_1 = \sum_{j=0}^{\infty}\psi_j^* L^j$, we have:

$$\begin{aligned}\Delta y_{2,t+1}^{TR} &= \sum_{j=1}^{\infty}(E(\Delta y_{2,t+j} - \pi^*|I_{t+1}) - E(\Delta y_{2,t+j} - \pi^*|I_t)) + \pi^* \\ &= \sum_{j=1}^{\infty}\psi_j^* v_{t+1} + \pi^* = \psi^*(1)v_{t+1} + \pi^* = \phi(1)^{-1}(1-\lambda_1)^{-1}\mu_1 v_{t+1} + \pi^*\end{aligned}$$

at least partially transitory. In order to capture such bounce back effects after recessions within a linear ARMA model, Beaudry and Koop (1993) allow the growth dynamics of real GDP to depend on an exogenous variable, named, 'current depth of recession': the gap between the current level of real GDP and the economy's historical maximum level.

Recently, Kim, Morley, and Piger (2005) extended Hamilton's Markov-switching model of asymmetric business cycles to allow for a post-recession bounce-back effect in the level of real GDP, while maintaining endogenously estimated business cycle regimes. The following describes their model:

$$(1-\phi(L))\Delta y_t = \mu_0 + \mu_1 S_t + \delta \sum_{j=1}^{m} S_{t-j} + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0,\sigma^2), \tag{23}$$

where Δy_t is the growth rate of real GDP and S_t is a latent, first-order Markov-switching process as defined earlier. [4] In the above model, μ_1 is constrained to be negative, so that $S_t = 1$ corresponds to a recessionary regime. When $\delta = 0$, the model collapses to the original Markov-switching model of Hamilton (1989). When $\delta > 0$, the summation term $\sum_{j=1}^{m} S_{t-j}$ implies that real GDP growth would be above average for some period after a recessionary regime, indicating the existence of a post-recession bounce-back effect. The existence of a post-recession 'bounce-back' effect implies that a recessionary shock is much less persistent, if not completely transitory, compared an expansionary shock.

A successful BN decomposition of y_t relies on an appropriate state-space representation of the above model in deviation from mean. In order to write equation (23) in deviation-from-mean, we first consider taking expectations on both sides of equation (23) to get:

$$(1-\phi(1))E(\Delta y_t) = \mu_0 + \mu_1 \pi + \delta \sum_{j=1}^{m} \pi \tag{24}$$

We then subtract equation (24) from equation (23) to get the model in deviation from mean, and rearranging terms, we obtain:

[4] For simplicity of exposition, we assume that the parameters of the model are constant. However, in actual application, we are going to drop this assumption and we allow for the possibility that they have changed since 1984Q4, the quarter identified by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) as the date of structural break in the dynamics of real GDP growth.

$$\Delta y_t^* = \phi(L)\Delta y_t^* + \mu_1(S_t - \pi) + \delta \sum_{j=1}^{m}(S_{t-j} - \pi) + \epsilon_t. \tag{25}$$

As before, based on equation (12'), the state-space representation of the model is then given by:

$$\begin{bmatrix} \Delta y_t^* \\ \Delta y_{t-1}^* \\ \vdots \\ \Delta y_{t-k+1}^* \\ S_{t+1} - \pi \\ S_t - \pi \\ \vdots \\ S_{t-m+1} - \pi \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_k & \mu_1 & \delta & \dots & \delta \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \lambda_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^* \\ \Delta y_{t-2}^* \\ \vdots \\ \Delta y_{t-k+1}^* \\ S_t - \pi \\ S_{t-1} - \pi \\ \vdots \\ S_{t-m} - \pi \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \\ \vdots \\ 0 \\ v_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{26}$$

$$(\tilde{X}_t = F\tilde{X}_{t-1} + \tilde{e}_t)$$

Thus, the BN trend for the process in equation (23) is given by:

$$\begin{aligned} y_t^{TR} &= y_t + \sum_{j=1}^{\infty} E(\Delta y_{t+j}^* | I_t) \\ &= y_t + (1,1) \ element \ of \ (F + F^2 + F^3 + \dots)E(\tilde{X}_t | I_t) \\ &= y_t + (1,1) \ element \ of \ F(I_p - F)^{-1}E(\tilde{X}_t | I_t), \end{aligned} \tag{27}$$

where $E(\tilde{X}_t|I_t) = [\, \Delta y_t^* \quad \Delta y_{t-1}^* \quad \dots \quad \Delta y_{t-k+1}^* \quad (E[S_{t+1}|I_t] - \pi) \dots \quad (E[S_{t-m+1}|I_t] - \pi)\,]'$

The impulse-response analysis with respect to symmetric (continuous) and asymmetric (discrete) shocks can be done using:

$$\frac{\partial y_{t+j}}{\partial e_t} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial e_t} = (1,1) \ element \ of \ \sum_{i=0}^{j} F^i. \tag{28}$$

$$\frac{\partial y_{t+j}}{\partial v_t} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial v_t} = (1,k+1) \ element \ of \ \sum_{i=0}^{j} F^i. \tag{29}$$

5. Application #2: Markov-Switching Vector Error Correction Model of Consumption and Real GDP

Consider the following version of Cochrane's (1994) vector error correction model of real GDP and real consumption extended to incorporate Markov-switching intercept terms:

$$\begin{bmatrix} 1-\phi_{yy}(L) & -\phi_{yc}(L) \\ -\phi_{cy}(L) & 1-\phi_{cc}(L) \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} \mu_{y,0}+\mu_{y,1}S_t \\ \mu_{c,0}+\mu_{c,1}S_t \end{bmatrix} + \begin{bmatrix} \alpha_y \\ \alpha_c \end{bmatrix} (z_{t-1}-\beta) + \begin{bmatrix} e_{y,t} \\ e_{c,t} \end{bmatrix}, \quad (30)$$

$$\begin{bmatrix} e_{y,t} \\ e_{c,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \rho\sigma_y\sigma_c \\ \rho\sigma_y\sigma_c & \sigma_c^2 \end{bmatrix} \right),$$

where y_t is the log of real GDP; c_t is the log of real consumption on nondurable goods and services; $z_t = y_t - c_t$; $\phi_{ij}(L) = \phi_{ij,1}L - \ldots + \phi_{ij,k}L^k$, $(i = c, y,\ j = c, y)$; and S_t is a latent first-order Markov-switching process with the transition probabilities defined before. Focusing on the real GDP equation (Δy_t) in equation (30), if we assume that $\phi_{yc}(L) \approx 0$, we can see that the last period's log of GDP/consumption ratio ($z_{t-1} = y_{t-1} - c_{t-1}$) replaces the term that captures the post-recession 'bounce-back' effect ($\sum_{j=1}^{m} S_t$) in equation (23).

Taking expectations on both sides of equation (30), we have:

$$\begin{bmatrix} 1-\phi_{yy}(1) & -\phi_{yc}(1) \\ -\phi_{cy}(1) & 1-\phi_{cc}(1) \end{bmatrix} \begin{bmatrix} E[\Delta y_t] \\ E[\Delta c_t] \end{bmatrix} = \begin{bmatrix} \mu_{y,0}+\mu_{y,1}\pi \\ \mu_{c,0}+\mu_{c,1}\pi \end{bmatrix} + \begin{bmatrix} \alpha_y \\ \alpha_c \end{bmatrix} (E(z_{t-1})-\beta) \quad (31)$$

Then, by subtracting equation (31) from equation (30), and by rearranging terms, we have the following set of two equations:

$$\Delta y_t^* = \phi_{yy}(L)\Delta y_t^* + \phi_{yc}(L)\Delta c_t^* + \mu_{y,1}(S_t - \pi) + \alpha_y z_{t-1}^* + e_{y,t}$$

$$\Delta c_t^* = \phi_{cy}(L)\Delta y_t^* + \phi_{cc}(L)\Delta c_t^* + \mu_{c,1}(S_t - \pi) + \alpha_c z_{t-1}^* + e_{c,t}, \quad (32)$$

where $\Delta c_t^* = \Delta c_t - E(\Delta c_t)$, $\Delta y_t^* = \Delta y_t - E(\Delta y_t)$ and $z_{t-1}^* = z_{t-1} - E(z_{t-1})$.

By assuming $k = 2$, for example, the following describes a state-space representation of the two equations in equation (32), which serves as the basis for exact calculation of the BN trend/cycle decomposition for the two variables:

$$
\begin{bmatrix} \Delta y_t^* \\ \Delta y_{t-1}^* \\ \Delta y_{t-2}^* \\ \Delta y_{t-3}^* \\ \Delta c_t^* \\ \Delta c_{t-1}^* \\ \Delta c_{t-2}^* \\ \Delta c_{t-3}^* \\ S_{t+1}-\pi \\ z_t^* \end{bmatrix}
=
\begin{bmatrix}
 & \tilde{\Phi}_{yy} & & & & \tilde{\Phi}_{yc} & & & \mu_{y,1} & \alpha_y \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \tilde{\Phi}_{cy} & & & & \tilde{\Phi}_{cc} & & & \mu_{c,1} & \alpha_c \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 \\
 & \tilde{A}_1 & & & & \tilde{A}_2 & & & \tilde{A}_3 & \tilde{A}_4
\end{bmatrix}
\begin{bmatrix} \Delta y_{t-1}^* \\ \Delta y_{t-2}^* \\ \Delta y_{t-3}^* \\ \Delta y_{t-4}^* \\ \Delta c_{t-1}^* \\ \Delta c_{t-2}^* \\ \Delta c_{t-3}^* \\ \Delta c_{t-4}^* \\ S_t-\pi \\ z_{t-1}^* \end{bmatrix}
+
\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} e_{c,t} \\ e_{y,t} \\ v_t \end{bmatrix}, \tag{33}
$$

$$
\left(\tilde{X}_t = F\tilde{X}_{t-1} + R\tilde{e}_t\right)
$$

where $\tilde{\Phi}_{ij} = [\,\phi_{ij,1} \quad \phi_{ij,2} \quad \phi_{ij,3} \quad \phi_{ij,4}\,]$, $i = c, y$ and $j = c, y$; $\tilde{A}_1 = \tilde{\Phi}_{yy} - \tilde{\Phi}_{cy}$; $\tilde{A}_2 = \tilde{\Phi}_{yc} - \tilde{\Phi}_{cc}$; $\tilde{A}_3 = \mu_{y,1} - \mu_{c,1}$; and $\tilde{A}_4 = 1 + \alpha_y - \alpha_c$. Thus, following the logic in the previous sections, the BN trend components of the log of real GDP and that of the log of real consumption are given by:

$$
\begin{aligned}
y_t^{TR} &= y_t + \sum_{j=1}^{\infty} E(\Delta y_{t+j}^* | I_t) \\
&= y_t + \ (1,1) \ element \ of \ (F + F^2 + F^3 + \ldots) E(\tilde{X}_t | I_t)\,, \\
&= y_t \ (1,1) \ element \ of \ F(I_p - F)^{-1} E(\tilde{X}_t | I_t)
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
c_t^{TR} &= c_t + \sum_{j=1}^{\infty} E(\Delta c_{t+j}^* | I_t) \\
&= y_t + \ (5,1) \ element \ of \ (F + F^2 + F^3 + \ldots) E(\tilde{X}_t | I_t)\,, \\
&= y_t + \ (5,1) \ element \ of \ F(I_p - F)^{-1} E(\tilde{X}_t | I_t)
\end{aligned} \tag{35}
$$

where $E(\tilde{X}_t|I_t) = [\,\Delta y_t^* \quad \ldots \quad \Delta y_{t-3}^* \quad \Delta c_t^* \quad \ldots \quad \Delta c_{t-3}^* \quad (E[S_{t+1}|I_t] - \pi) \quad z_t^*\,]'$.

In order to perform impulse-response analysis, the VAR errors e_{ct} and e_{yt} are orthogonalized so that consumption does not respond contemporaneously to a GNP shock. Equivalently, current consumption growth is included in the GNP growth regression. By denoting

$\begin{bmatrix} e_{ct} \\ e_{yt} \end{bmatrix} \sim i.i.d.N(0, \Omega)$, the Cholesky decomposition of the variance-covariance matrix Ω is given by the lower trangular matrix $B = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix}$. Then the state-space form in equation (33) can be re-written as:

$$\tilde{X}_t = F\tilde{X}_{t-1} + RB^*\tilde{e}_t^*, \tag{33'}$$

where $B^* = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\tilde{e}_t^* = \begin{bmatrix} e_{ct}^* \\ e_{yt}^* \\ v_t \end{bmatrix}$, and where e_{ct}^* and e_{yt}^* are uncorrelated and standardized symmetric shocks and v_t is a discrete shock. Thus, for example, the responses of real GDP to one-standard-deviation symmetric continuous shocks (e_{ct}^* and e_{yt}^*) and those to one unit asymmetric discrete shock v_t are given by:

$$\frac{\partial y_{t+j}}{\partial e_{ct}} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial e_{ct}} = (1,1) \ element \ of \ \sum_{i=0}^{j} F^i RB^* \tag{36}$$

$$\frac{\partial y_{t+j}}{\partial e_{ct}} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial e_{ct}} = (1,2) \ element \ of \ \sum_{i=0}^{j} F^i RB^* \tag{37}$$

$$\frac{\partial y_{t+j}}{\partial v} = \sum_{i=0}^{j} \frac{\partial \Delta y_{t+i}}{\partial v_t} = (1,3) \ element \ of \ \sum_{i=0}^{j} F^i RB^* \tag{38}$$

6. Empirical Results

In this section, we first estimate three alternative empirical models: i) a univariate linear model for the log of real GDP; ii) a univariate Markov-switching model for the log of real GDP with a post-recession 'bounce-back' effect introduced in Section 4; and iii) a Markov-switching vector error correction model of the logs of real GDP and real consumption introduced in Section 5 (MS-VECM Model). For each of these three models, we allow for a structural break in the parameters in 1984Q4, following Kim and Nelson (1999) and McConnell and Quiros (2000). These empirical models are then used to perform the BN trend/cycle decomposition and the impulse response analyses. The three models estimated with structural breaks are given by:

Univariate Linear Model

$$(1-\sum_{i=1}^{4}\phi_{i,D_t}L^i)\Delta y_t^* = e_t, \quad e_t \sim N(0,\sigma_{D_t}^2), \tag{1'}$$

Univariate MS model with 'bounce-back' effect

$$(1-\sum_{i=1}^{4}\phi_{i,D_t}L^i)\Delta y_t^* = \mu_{1,D_t}(S_t-\pi)+\delta_{D_t}\sum_{j=1}^{m}(S_{t-j}-\pi)+e_t, \quad e_t \sim N(0,\sigma_{D_t}^2) \tag{25'}$$

$$Pr[S_t=1|S_{t-1}=1]=p, \quad Pr[S_t=0|S_{t-1}=0]=q,$$

MS-VECM Model

$$\Delta y_t^* = \phi_{yy,D_t}(L)\Delta y_t^* + \phi_{yc,D_t}(L)\Delta c_t^* + \mu_{y,1,D_t}(S_t-\pi) + \alpha_{y,D_t}z_{t-1}^* + e_{y,t} \tag{32'}$$

$$\Delta c_t^* = \phi_{cy,D_t}(L)\Delta y_t^* + \phi_{cc,D_t}(L)\Delta c_t^* + \mu_{y,1,D_t}(S_t-\pi) + \alpha_{c,D_t}z_{t-1}^* + e_{c,t},$$

$$\begin{bmatrix} e_{y,t} \\ e_{c,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{y,D_t}^2 & \rho\sigma_{y,D_t}\sigma_{c,D_t} \\ \rho_{D_t}\sigma_{y,D_t}\sigma_{c,D_t} & \sigma_{c,D_t}^2 \end{bmatrix}\right)$$

$$Pr[S_t=1|S_{t-1}=1]=p, \quad Pr[S_t=0|S_{t-1}=0]=q.$$

$$\phi_{ij,D_t}(L) = \phi_{ij,1,D_t}L + \phi_{ij,2,D_t}L^2 + \phi_{ij,3,D_t}L^3 + \phi_{ij,4,D_t}L^4, \quad (i=y,c; j=y,c),$$

where $D_t = 0$, $before$ 1984$Q4$; $D_t = 1$ $since$ 1984$Q4$; $\pi = ([S_t) = (1-q)/(2-p-q)$; Δy_t^*, Δc_t^* and z_t^* are demeaned real GDP growth, demeaned real consumption growth, and demeaned log of GDP/consumption ratio, respectively. Data covers the sample period of 1948Q2 - 2005Q3. All the data are collected from the FRED at St. Louis Federal Reserve Bank. Before estimating the models, all the data including the log of GDP/consumption ratio are demeaned separately for the following sub-sample periods: 1948Q2 - 1973Q3, 1973Q4 - 1994Q4, and 1995Q1 - 2005Q3. [5]

6.1. Empirical Results on Univariate Linear Model

[5] The purpose is to consider potential structural breaks due to the productivity slowdown since the first Oil shock and the surge of productivity due to the 'new economy' since the mid-1990's. See Cogley (2005), for example.

Figure 1A depicts the log of real GDP and its BN trend component obtained from the univariate linear model, and the resulting cyclical component is shown in Figure 1B. The shaded areas represent the NBER recessions. The scale in all figures is log times 100 and so the cyclical component may be read as the percentage deviation from trend. The estimated BN cycle is small in amplitude, and it is not easy to get very meaningful business cycle implications from it. Table 1 reports the maximum likelihood estimates for the parameters and their standard errors for the univariate linear model. Since 1984Q4, the standard error the shock decreased considerably, and the sum of the AR coefficients increased (from 0.1932 to 0.4471), suggesting that the persistence of the shock might have increased. Figure 1C depicts the responses of real GDP over time to a unit shock ($\frac{\partial y_{t+j}}{\partial e_t}$, $j = 1, 2, ..., 50$). Before 1984Q4, a 1% increase in the shock increases real GDP by about 1.2%, while, since 1984Q4, a 1% increase in the shock increases real GDP by as much as 1.8%. Based on these observations, we raise the possibility that real GDP has become more persistent since the mid-1980's.

6.2. Empirical Results on Univariate MS model with Post-Recession 'Bounce-Back' Effect

Figures 2A and 2B depict the filtered and smoothed probabilities of a recession implied by the univariate model with a post-recession 'bounce-back' effect. The recession probabilities are largely in agreement of the NBER reference cycles. Figure 2C depicts the log of real GDP and its BN trend component implied as by the model. The resulting BN cycle depicted in Figure 2D is large in amplitude and persistent. Also, unlike the case of the linear model, the asymmetric nature of the cyclical component of business cycles is evident. The declines in the cycle agree very reasonably with the NBER dating of recessions (shaded areas) until the mid-1980's. In particular, the cyclical component is negative during the recessions and is close to zero during expansions, a result which are consistent with Friedman's (1993) 'plucking' model of business cycles as estimated by Kim and Nelson (1999).

Since the mid-1980'a, however, the BN cycle component is negative during expansions and positive during recessions (the 1991 and 2000 recessions), suggesting a potential struc-

tural change in the business cycle dynamics. The maximum likelihood estimates for the parameters and their standard errors for the model explain the key nature of the structural change. Focusing on the δ coefficient designed to capture the post-recession 'bounce-back' effect, we note that it is positive and significant before 1984Q4, while it is small and statistically insignificant since 1984Q4. The existence of a post-recession 'bounce-back' effect before 1984Q4 suggests that output losses during recessions were to a large extent temporary, creating negative cyclical components of output during the recessions. On the contrary, the disappearance of such a 'bounce-back' effect since 1989Q4 indicates that output losses during recessions became permanent. Note that our Markov-switching variable S_t has an AR(1) dynamics with positive autocorrelation. Thus, in the absence of the 'bounce-back' effect, a recessionary positive shock to S_t in any given quarter results in a further increase in the expectation of S_{t+j}, $j > 1$, in subsequent quarters. This implies that, with a recessionary positive shock to S_t, the long-horizon forecast of real GDP (the BN trend component) is even lower than the actual negative change in output, resulting in a positive cyclical component of real GDP. [6]

In this model we have two distinct types of shocks: a symmetric continouous shock (e_t) and an asymmetric discrete shock (v_t). Figure 2E shows the impulse-response analysis for a symmetric continuous shock ($\frac{\partial y_{t+j}}{\partial e_t}$, $j = 1, 2, ..., 50$). Before 1984Q4, a 1% increase in the shock increases real GDP by slightly more than 1% (1.05%) in the long run, while, since 1984Q4, a 1% increase in the shock increases real GDP by less than 1% (0.86%), suggesting that the symmetric continuous shock has become somewhat less persistent. This observation is consistent with the change in the sum of the autoregressive parameters from 0.0504 to -0.1653 as in Table 2. However, the impulse-response analysis for an asymmetric discrete shock ($\frac{\partial y_{t+j}}{\partial v_t}$, $j = 1, 2, ..., 50$) depicted in Figure 2F suggests that the persistence of the shock has become much higher. Before 1984Q4, a 1% increase in the asymmetric discrete shock (v_t) to the Markov-switching variable S_t, which endogenously determine the business cycle regime, initially decreases real GDP by 1.63% and decreases real GDP by -0.80% in the long run; since 1984Q4, the same increase in v_t initially decreases real GDP

[6] In this case, the variance of the change in the BN trend component is larger than that of the log of actual real GDP.

by 0.86% and decreases real GDP by as much as -2.43% in the long run. Such considerable increase in the persistence of the asymmetric discrete shock to S_t explains a large increase in the persistence of the shock observed in the univariate linear model. Clearly, we may argue that the disappearance of the post-recession 'bounce-back' effect has increased the persistence of real GDP considerably.

6.3. Empirical Results on Markov-Switching Vector Error Correction Model (MS-VECM)

Figures 3A and 3B depict the filtered and smoothed probabilities of a recession estimated from the MS-VECM model. As in the case of the univariate MS model with a post-recession 'bounce-back' effect, the recession probabilities are largely in agreement of the NBER reference cycles. In this model, the lagged log of the GDP/consumption ratio (z_{t-1}) term in the real GDP growth equation plays the role of the 'bounce-back' effect in the univariate MS model.

In Figures 3C and 3E, the log of real GDP and the log of real consumption are depicted along with their BN trend components. Figures 3D and 3F show the resulting BN cyclical components. As one might expect, the cyclical component of real consumption is much smaller in amplitude than that of real GDP. Focusing our attention on the cyclical component of real GDP, most of the down-turns are in close agreement with the shaded NBER recessions. Besides, the variance of the cyclical component of real GDP has decreased considerably since the mid-1980's. Table 3 reports the maximum likelihood estimates for the parameters and their standard errors for the MS-VECM model. However, with so many parameter estimates, it would not be easy to gain insights into how the dynamics of real GDP has changed. That is, it is hard to fathom whether or not real GDP has become more persistent since the mid-1980's based on the parameter estimates alone. We thus resort to the impulse-response analysis in order to examine a potential change in the persistence of real GDP.

In this model, we have two response variables (real GDP(y_t) and real consumption (c_t)) and three impulse shocks, two of which are symmetric continuous shocks (GDP shock (e_{yt}) and consumption shock (e_{c_t})) and one of which is an asymmetric discrete shock (v_t) to

the latent Markov-switching variable S_t that endogenously determines the business cycle regime. One of the important features of the impulse response functions is that the eventual response of the two variables (y_t and c_t) to each of the three shocks (e_{yt}, e_{ct}, and v_t) should be the same, as in Cochrane's (1994) linear VECM model without regime switching. If the two responses did not end up at the same value in the long run, the GDP/consumption ratio would not be restored following a shock. This feature results from the inclusion of the GDP/consumption ratio on the right-hand sides of both the real GDP and real consumption growth equations.

Figure 3G depicts the responses of real GDP and real consumption to one standard-error consumption shock (e_{ct}) before and after the mid-1980. We see that the long run responses of the two variables somewhat decreased (from 0.33 to 0.24). From Figure 3F, on the contrary, we note that the long run responses of the two variables to one standard-error GDP shock (e_{yt}) has increased considerably from close to zero to 0.18, suggesting that the GDP shock might have become more persistent since the mid-1980's. Figure 3I depicts the responses of the two variables to an asymmetric discrete shock (v_t). It suggests that the persistence of the asymmetric discrete shock (v_t) has increased since the mid-1980's: the long-run responses to a unit discrete shock decreases from -1.39 to -1.77. In summary, while the continuous consumption shock (e_{ct}) has a relatively smaller long-run impact on both consumption and GDP since the mid-1980's, both the continuous GDP shock (e_{yt})and the discrete shock (v_t) have a larger long-run impact on these variables since the mid-1980's. These observations lead us to raise the possibility that the dynamics of real GDP has become more persistent.

7. Summary and Suggestion for Further Studies

The purpose of this paper is multi-fold. First, we present a method for exact calculation of BN trend/cycle components for a class of linear and multivariate Markov-switching models. The key to the solution is to recognize that the latent first-order Markov-switching process has an AR(1) representation, and that we have a state-space representation of the model. Second, we show that, using the state-space representation of the model, impulse-

response analyses can be performed with respect to a symmetric continuous shock and to an asymmetric discrete shock. Third, we apply the methodologies developed to i) univariate linear model of real GDP; ii) Kim, Morley, Piger's (2005) univariate Markov-switching model of real GDP with a post-recession 'bounce-back' effect; and iii) Cochrane's (1994) vector error correction model of real GDP and real consumptions extended to incorporate Markov-switching.

Based on the parameter estimates, the calculated BN trend/cycle components, and the impulse-response analyses for each of these empirical model, we raise the possibility that the persistence of real GDP might have increased since the mid-1980's in the U.S. In particular, we argue that the disappearance of the post-recession 'bounce-back' effect since the mid-1980's might have increased the persistence of real GDP considerably. Existing literature reports a marked decline in the volatility of real activity and inflation since the mid-1980's and a reduction in the persistence of inflation over time. [See, Kim and Nelson (1999), Blanchard and Simon (2000), McConnell and Perez Quiros (2001), and Sargent and Cogley (2002), Kim, Piger, and Nelson (2004), for example.] However, little attention has been paid to a potential structural change in the persistence of real economic activity in the U.S. The preliminary empirical results in this paper would certainly warrant a careful further investigation of this issue.

References

[1] Fatas, A., 2000, "Endogenous Growth and Stochastic Trends," *Journal of Monetary Economics,* 45, 107-128.

[2] Arino, M.A.and P. Newbold, 1998, "Computation of the Beveridge-Nelson Decomposition for Multivariate Economic Time Series," *Economics Letters,* 61, 37-42.

[3] Beveridge, S. and C.R. Nelson, 1981, "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics, 7, 151-174.*

[4] Beaudry, P. and G. Koop, 1993, "Do Recessions Permanently Change Output?" *Journal of Monetary Economics,* 31, 149-163.

[5] Blanchard, O. J. and J. Simon, 2000, "The Long and Large Decline in the U.S. Output Volatility," *Brookings papers on Economic Activity,* ', 135-147.

[6] Clarida, R.H. and M.P. Taylor, 2003,"Nonlinear Permanent-Temporary Decompositions in Macroeconomics and Finance," *The Economic Journal,* 113, C125-C139.

[7] Clark, P. K., 1987, "The Cyclical Component of U.S. Economic Activity," *The Quarterly Journal of Economics,* 102, 797-814.

[8] Cochrane, J. H., 1994, "Permanent and Transitory Components of GNP and Stock Prices," *Quarterly Journal of Economics,* 109, 241-263.

[9] Cogley, T. and T. J. Sargent, 2001 "Evolving Post World War II U.S. Inflation Dynamics," *NBER Macroeconomics Annual,* 16.

[10] Cogley, T., 2005, "How Fast Can the New Economy Grow? A Bayesian Analysis of the Evolution of Trend Growth," *Journal of Macroeconomics,* 27, 179-207.

[11] Currington, J.T. and L. A. Winters, "The Beveridge Nelson Decomposition of Economic Time Series: A Quick Computational Method," *Journal of Monetary Economics,* 19, 125-127.

[12] Friedman, M, 1993, "The 'Plucking Model' of Business Fluctuations revisited," *Economic Inquiry,* 31, 171-177.

[13] Hamilton, J. D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica,* March, 57, 357-84.

[14] Harvey, A. C., 1985, "Trends and Cycles in Macroeconomic Time Series," *Journal of Business and Economic Statistics,* 3, 216-227.

[15] Kim, C.-J., J. Morley, and J. Piger, 2005, "Nonlinearity and the Permanent Effects of Recessions," *Journal of Applied Econometrics,* 20, 291-309.

[16] Kim, C.-J. and C.R. Nelson, 1999a, ''Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle", *Review of Economics and Statistics,* 81, 608-616.

[17] Kim, C.-J. and C.R. Nelson, 1999b, "Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components," *Journal of Mone, Credit, and Banking,* 31, 317-334.

[18] Kim, C.-J. and C.R. Nelson, 2004, "The Less Volatile U.S. Economy: A Bayesian Investigation of Timing, Breadth, and Potential Explanations," *Journal of Business and Economic Statistics,* Vol. 22, No. 1, 80-93.

[19] McConnell, M.M. and G.P. Quiros, 2001, ''Output fluctuations in the United States: What has changed since the early 1980's?', *American Economic Review,* Vol. 90, No. 5, 1464-1476.

[20] Miller, S. M., 1988, "The Beveridge Nelson Decomposition of Economic Time Series: Another Economical Computation Method," *Journal of Monetary Economics,* 21, 141-142.

[21] Morley, J., 2002, "A State-Space Approach to Calculating the Beveridge-Nelson Decomposition," *Economics Letters,* 75, 123-127.

[22] Morley, J. and J. Piger, 2005, "A Steady-State Approach to Trend/Cycle Decomposition of Regime-Switching Processes," Working Paper, Washington University and Federal Reserve Bank of St. Louis.

[23] Newbold, P., "Precise and Efficient Computation of the Beveridge-Nelson Decomposition of Economic Time Series," *Journal of Monetary Economics,* 26, 453-457.

[24] Sichel, D. E., 1994, "Inventories and the Three Phases of the Business Cycle," *Journal of Business & Economic Statistics," Vol. 12, No. 2, 269-277.*

[25] Stock, J. H. and M.W. Watson, 1987, "Testing for Common Trends,' *Journal of the American Statistical Association,* vol. 83, 1097-1107.

Table 1. Estimates for Univariate Linear Model of Real GDP Growth

$$(1 - \sum_{i=1}^{4} \phi_{i,D_t} L^i)\Delta y_t^* = e_t, \quad e_t \sim N(0, \sigma_{D_t}^2), \tag{1'}$$

$$D_t = 0, \ before \ 1984Q4; \quad D_t = 1 \ since \ 1984Q4,$$

where Δy_t^* is de-meaned real GDP growth.

Parameters	Estimates (S.E.)
	Before 1984Q4
$\phi_{1,0}$	0.3004 (0.0820)
$\phi_{2,0}$	0.1026 (0.0852)
$\phi_{3,0}$	-0.0734 (0.0854)
$\phi_{4,0}$	-0.1364 (0.0820)
σ_0	1.0953 (0.0641)
	Since 1984Q4
$\phi_{1,1}$	0.2183 (0.1089)
$\phi_{2,1}$	0.3572 (0.1097)
$\phi_{3,1}$	-0.1616 (0.1070)
$\phi_{4,1}$	0.0308 (0.1044)
σ_1	0.4391 (0.0339)
Log Likelihood	-270.5193

Table 2. Estimates for Univariate Markov-Switching Model of Real GDP Growth with Post-Recession Bounce Back Effect

$$(1-\sum_{i=1}^{4}\phi_{i,D_t}L^i)\Delta y_t^* = \mu_{1,D_t}(S_t-\pi)+\delta_{D_t}\sum_{j=1}^{m}(S_{t-j}-\pi)+e_t, \quad e_t \sim N(0,\sigma_{D_t}^2) \qquad (22')$$

$$Pr[S_t=1|S_{t-1}=1]=p, \quad Pr[S_t=0|S_{t-1}=0]=q.$$

$$D_t=0,\ before\ 1984Q4; \quad D_t=1\ since\ 1984Q4,$$

where Δy_t^* is de-meaned real GDP growth and $\pi = E[S_t] = (1-q)/(2-p-q)$.

Parameters	Estimates (S.E.)
p	0.7921 (0.0720)
q	0.9328 (0.0247)
	Before 1984Q4
$\phi_{1,0}$	0.1209 (0.0911)
$\phi_{2,0}$	0.0413 (0.0844)
$\phi_{3,0}$	-0.0445 (0.0786)
$\phi_{4,0}$	-0.0673 (0.0858)
$\mu_{1,0}$	-1.6329 (0.3210)
δ_0	0.2375 (0.0989)
σ_0	0.9143 (0.0773)
	Since 1984Q4
$\phi_{1,1}$	-0.0519 (0.1127)
$\phi_{2,1}$	0.1670 (0.1067)
$\phi_{3,1}$	-0.2508 (0.1067)
$\phi_{4,1}$	-0.0296 (0.1076)
$\mu_{1,0}$	-0.8616 (0.1518)
δ_0	0.0136 (0.0351)
σ_1	0.3458 (0.0318)
Log Likelihood	-263.0802

Table 3. Estimates for Markov-Switching Error-Correction VAR(4) Model of Real GDP Growth and Real Consumption Growth

$$\Delta y_t^* = \phi_{yy,D_t}(L)\Delta y_t^* + \phi_{yc,D_t}(L)\Delta c_t^* + \mu_{y,1,D_t}(S_t - \pi) + \alpha_{y,D_t} z_{t-1}^* + e_{y,t} \qquad (27')$$

$$\Delta c_t^* = \phi_{cy,D_t}(L)\Delta y_t^* + \phi_{cc,D_t}(L)\Delta c_t^* + \mu_{y,1,D_t}(S_t - \pi) + \alpha_{c,D_t} z_{t-1}^* + e_{c,t},$$

$$\begin{bmatrix} e_{y,t} \\ e_{c,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{y,D_t}^2 & \rho\sigma_{y,D_t}\sigma_{c,D_t} \\ \rho_{D_t}\sigma_{y,D_t}\sigma_{c,D_t} & \sigma_{c,D_t}^2 \end{bmatrix} \right)$$

$$Pr[S_t = 1 | S_{t-1} = 1] = p, \quad Pr[S_t = 0 | S_{t-1} = 0] = q.$$

$$D_t = 0, \ before \ 1984Q4; \quad D_t = 1 \ since \ 1984Q4,$$

$$\phi_{ij,D_t}(L) = \phi_{ij,1,D_t}L + \phi_{ij,2,D_t}L^2 + \phi_{ij,3,D_t}L^3 + \phi_{ij,4,D_t}L^4, \quad (i = y, c; j = y, c)$$

where $\pi = E[S_t] = (1-q)/(2-p-q)$; Δy_t^*, Δc_t^* and z_t^* are demeaned real GDP growth, demeaned real consumption growth, and demeaned equilibrium error, respectively.

Before 1984Q4		Since 1984Q4	
Parameters	Estimates (S.E.)	Parameters	Estimates (S.E.)
	Real GDP Growth		
$\phi_{yy,1,0}$	0.1554 (0.0850)	$\phi_{yy,1,1}$	-0.1061 (0.1179)
$\phi_{yy,2,0}$	0.1095 (0.0844)	$\phi_{yy,2,1}$	0.1869 (0.1127)
$\phi_{yy,3,0}$	-0.0083 (0.1123)	$\phi_{yy,3,1}$	-0.2126 (0.1416)
$\phi_{yy,4,0}$	-0.0834 (0.0796)	$\phi_{yy,4,1}$	-0.0433 (0.1059)
$\phi_{yc,1,0}$	0.4775 (0.1937)	$\phi_{yc,1,1}$	0.3053 (0.1641)
$\phi_{yc,2,0}$	0.1364 (0.1791)	$\phi_{yc,2,1}$	0.2860 (0.1582)
$\phi_{yc,3,0}$	-0.1266 (0.1855)	$\phi_{yc,3,1}$	0.1655 (0.1787)
$\phi_{yc,4,0}$	0.0713 (0.1803)	$\phi_{yc,4,1}$	-0.1783 (0.1599)
$\mu_{y,1,0}$	-0.9543 (0.2685)	$\mu_{y,1,1}$	-0.7751 (0.1358)
$\alpha_{y,0}$	-0.1434 (0.0430)	$\alpha_{y,1}$	-0.0958 (0.0888)
$\sigma_{y,0}$	0.9083 (0.0598)	$\sigma_{y,1}$	0.3162 (0.0348)
	Real Consumption Growth		
$\phi_{cy,1,0}$	0.0859 (0.0435)	$\phi_{cy,1,1}$	0.0552 (0.0843)
$\phi_{cy,2,0}$	0.0420 (0.0458)	$\phi_{cy,2,1}$	0.0613 (0.0771)
$\phi_{cy,3,0}$	0.0081 (0.0585)	$\phi_{cy,3,1}$	0.0004 (0.0984)
$\phi_{cy,4,0}$	-0.0029 (0.0399)	$\phi_{cy,4,1}$	0.0514 (0.0762)
$\phi_{cc,1,0}$	-0.1254 (0.0980)	$\phi_{cc,1,1}$	0.0510 (0.1128)
$\phi_{cc,2,0}$	-0.0456 (0.0954)	$\phi_{cc,2,1}$	-0.1053 (0.1224)
$\phi_{cc,3,0}$	-0.0602 (0.1043)	$\phi_{cc,3,1}$	0.1763 (0.1125)
$\phi_{cc,4,0}$	-0.0905 (0.0953)	$\phi_{cc,4,1}$	-0.1690 (0.1087)
$\mu_{c,1,0}$	-0.7794 (0.1661)	$\mu_{c,1,1}$	-0.2939 (0.1015)
$\alpha_{c,0}$	-0.0270 (0.0232)	$\alpha_{c,1}$	0.0756 (0.0537)
$\sigma_{c,0}$	0.4217 (0.0378)	$\sigma_{c,1}$	0.2521 (0.0209)
	Other Parameters		
ρ_0	0.2657 (0.0922)	ρ_1	0.2961 (0.1187)
p	0.7076 (0.0891)	—	—
q	0.9125 (0.0280)	—	—
	Log Likelihood	-339.6898	

Figure 1A. Lof of Real GDP and its BN Trend Compont from Univariate Linear Model

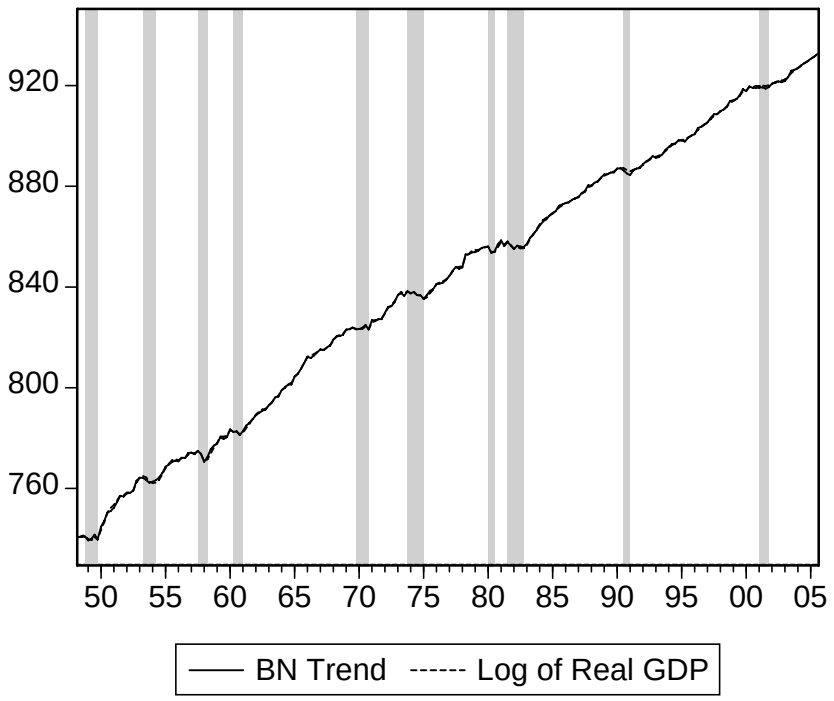

Figure 1B. Cyclical Component of Real GDP from Univariate Linear Model

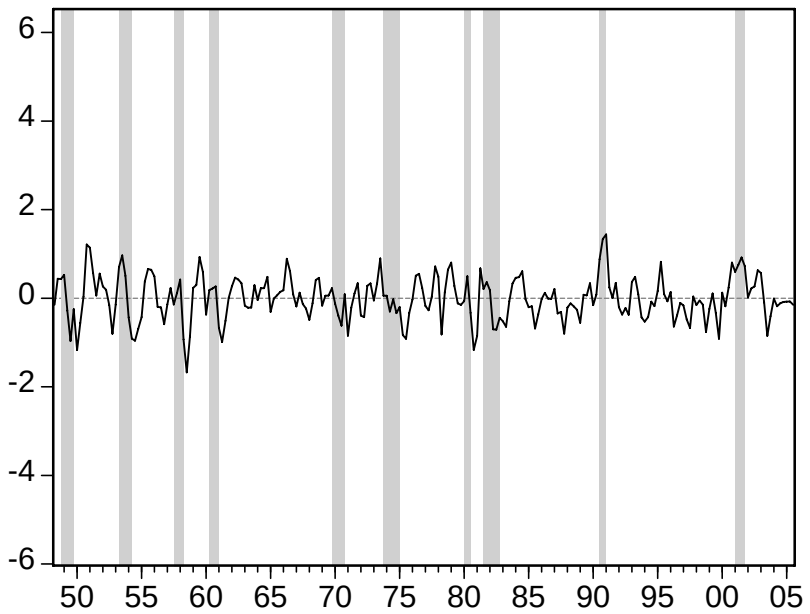

Figure 1C. Response of Real GDP to Its Own Shock: Univariate Linear Model

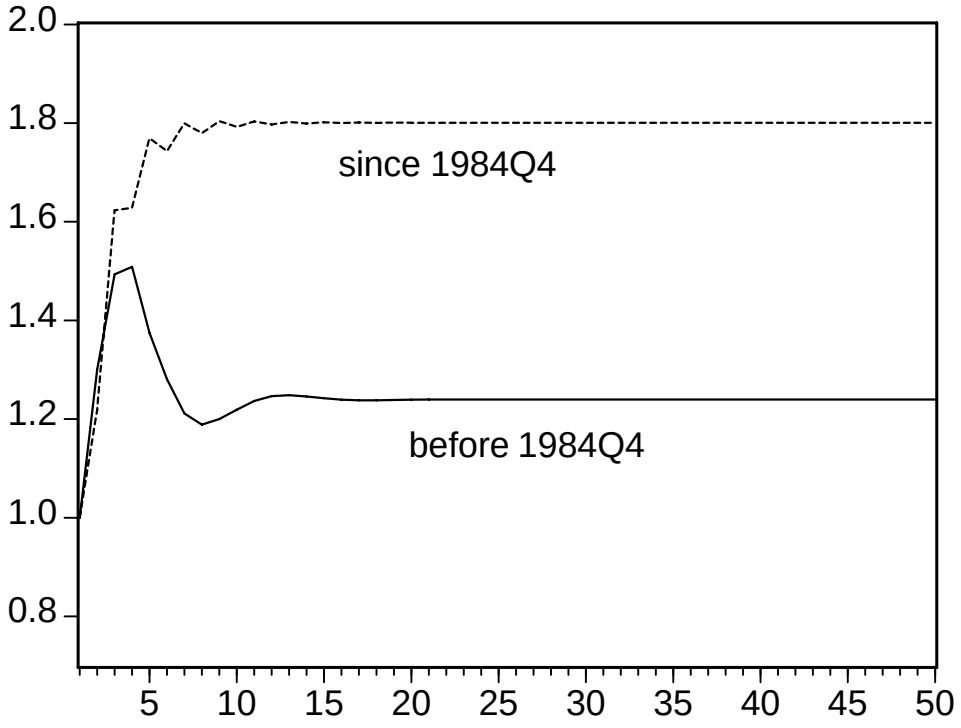

Figure 2A. Filtered Probability of Recession from Univariate Markov-Switching Model with Post-Recession Bounce Back Effect

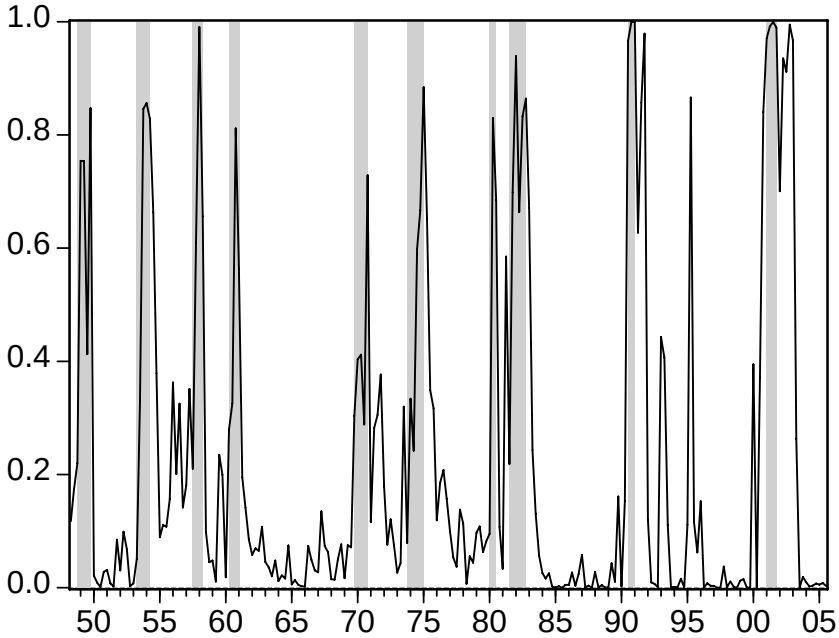

Figure 2B. Smoothed Probability of Recession from Univariate Markov-Switching Model with Post-Recession Bounce Back Effect

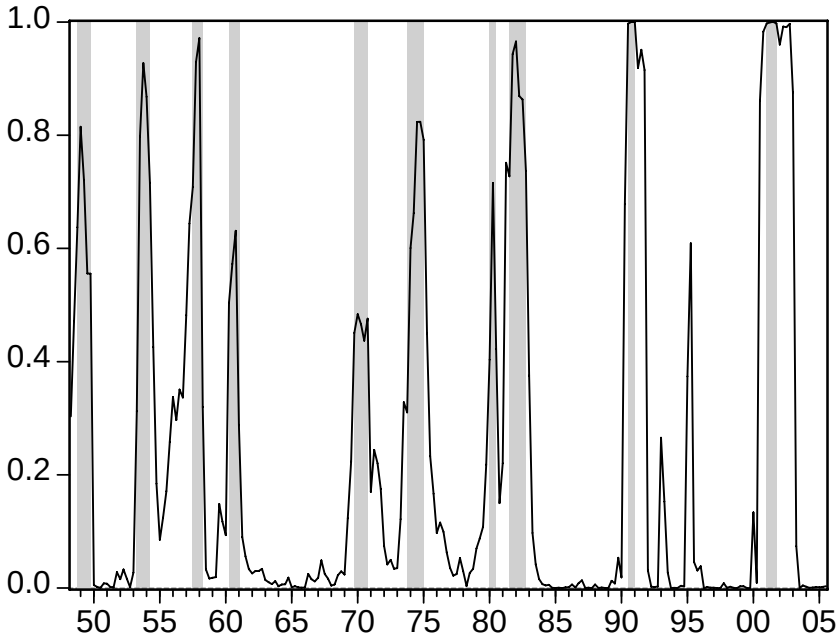

Figure 2C. Real GDP and its BN Trend Compont from Univariate Markov-Switching Model with Post Recession Bounce Back Effect

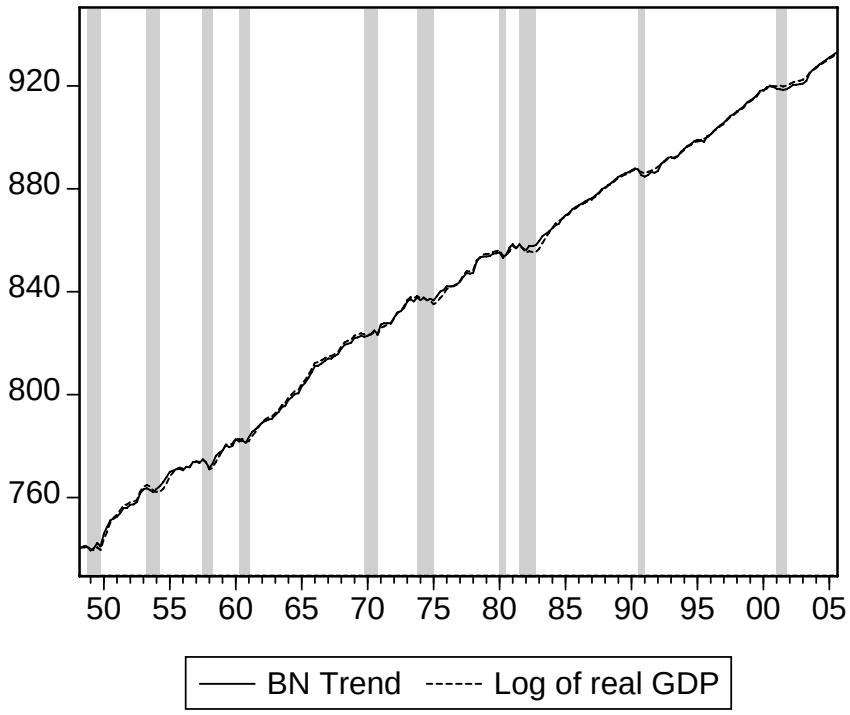

Figure 2D. Cyclical Component of Real GDP from Univariate Markov Switching Model with Post-Recession Bounce Back Effect

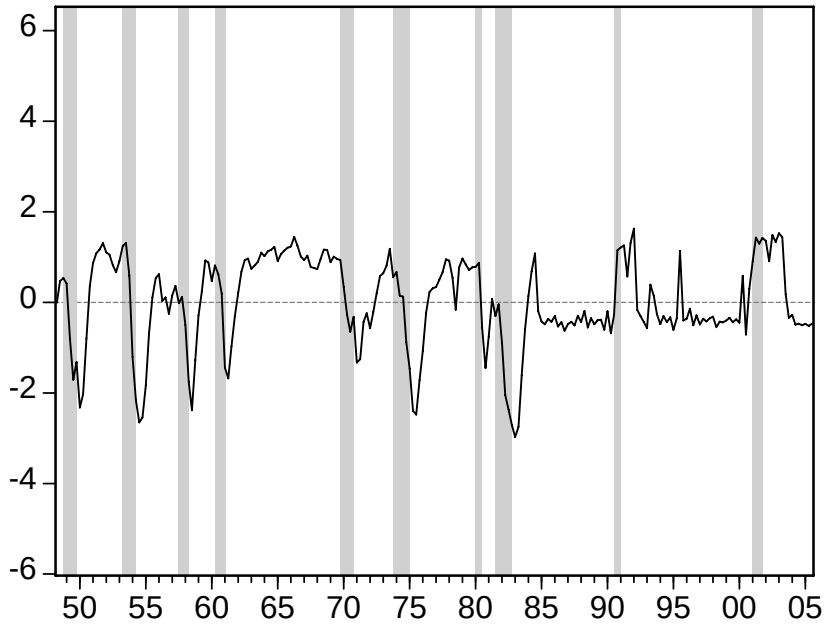

Figure 2E. Response of Real GDP to Continuous Shock: Univariate MS Model

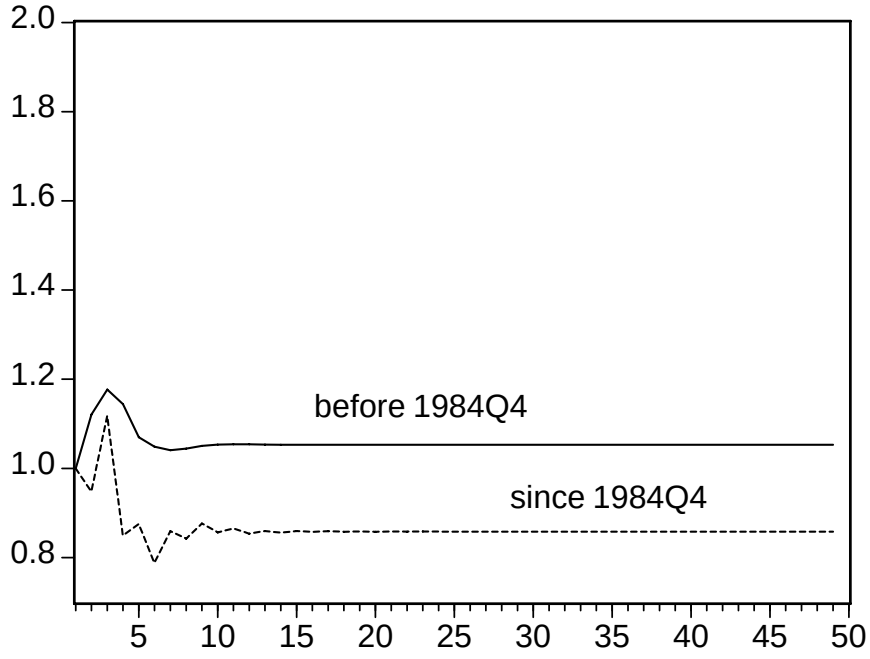

Figure 2F. Response of Real GDP to Discrete Shock: Univariate MS Model

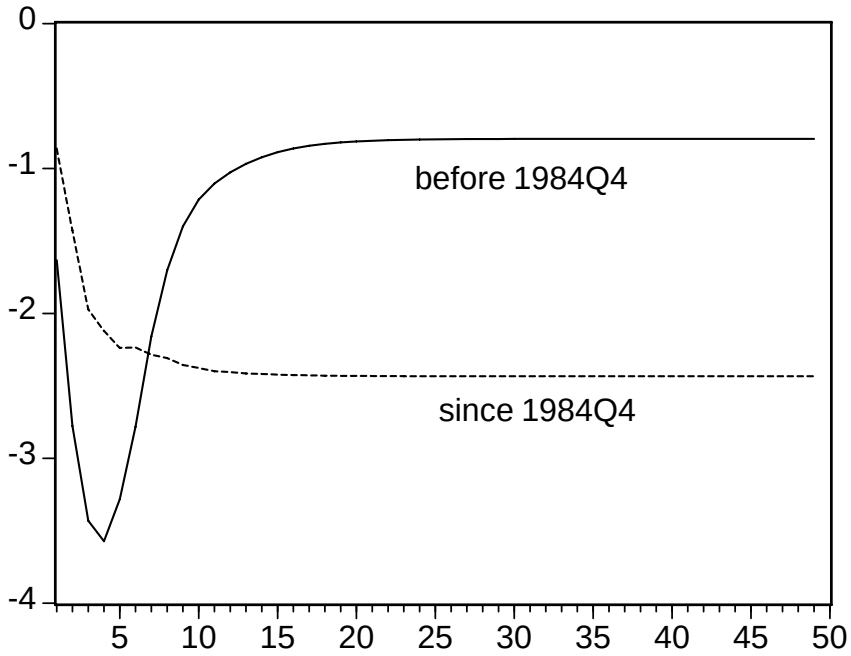

Figure 3A. Filtered Probability of Recession from Markov-Switching Error-Correction VAR(4) Model

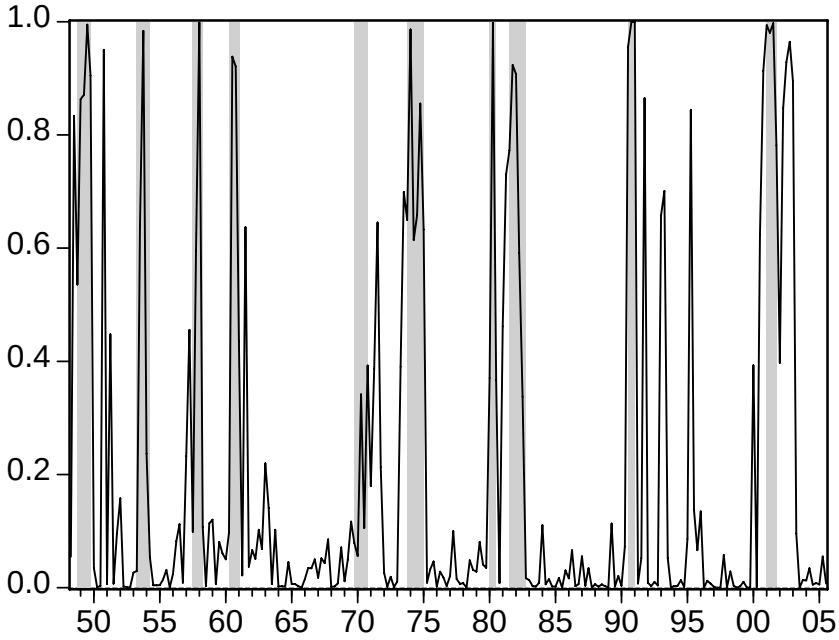

Figure 3B. Smoothed Probability of Recession from Markov-Switching Error-Correction VAR(4) Model

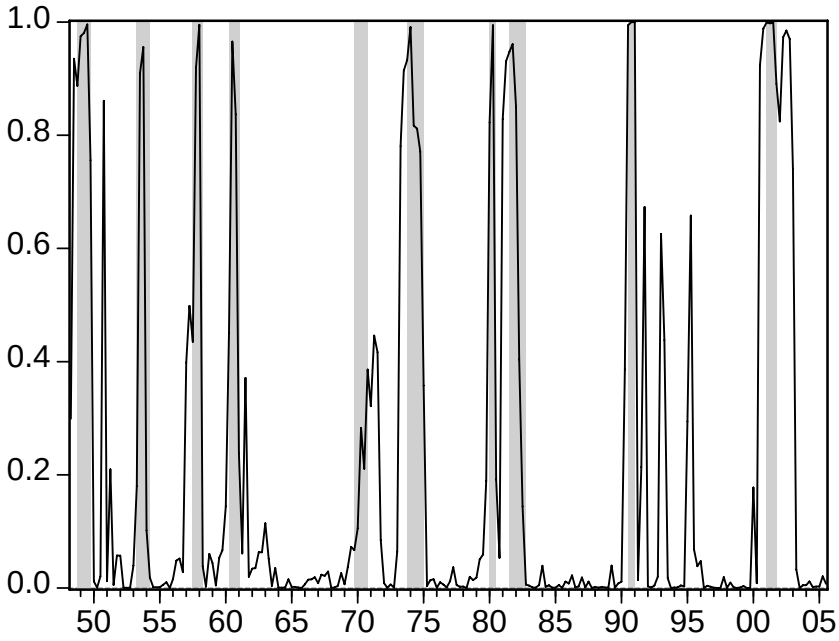

Figure 3C. Real GDP its BN Trend Component from Markov-Switching Error-Correction VAR(4) Model

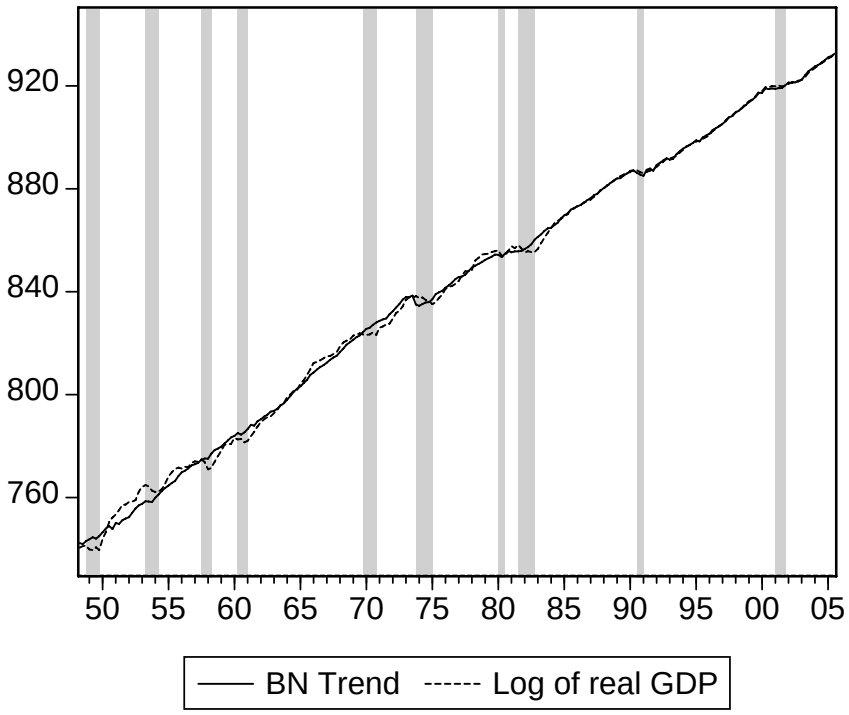

Figure 3D. BN Cyclical Component of Real GDP from Markov-Switching Error-Correction VAR(4) Model

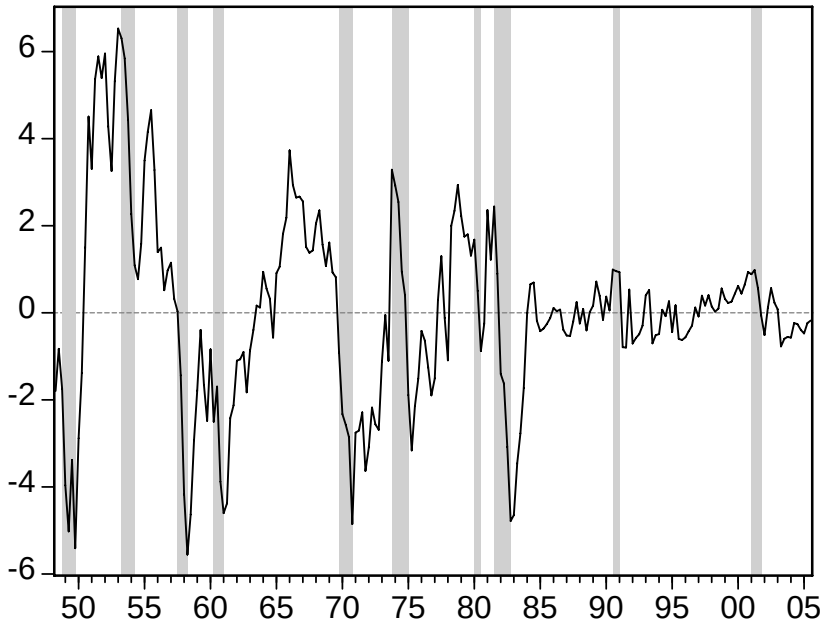

Figure 3E. Real Consumption Its BN Trend Component from Markov-Switching Error-Correction VAR(4) Model

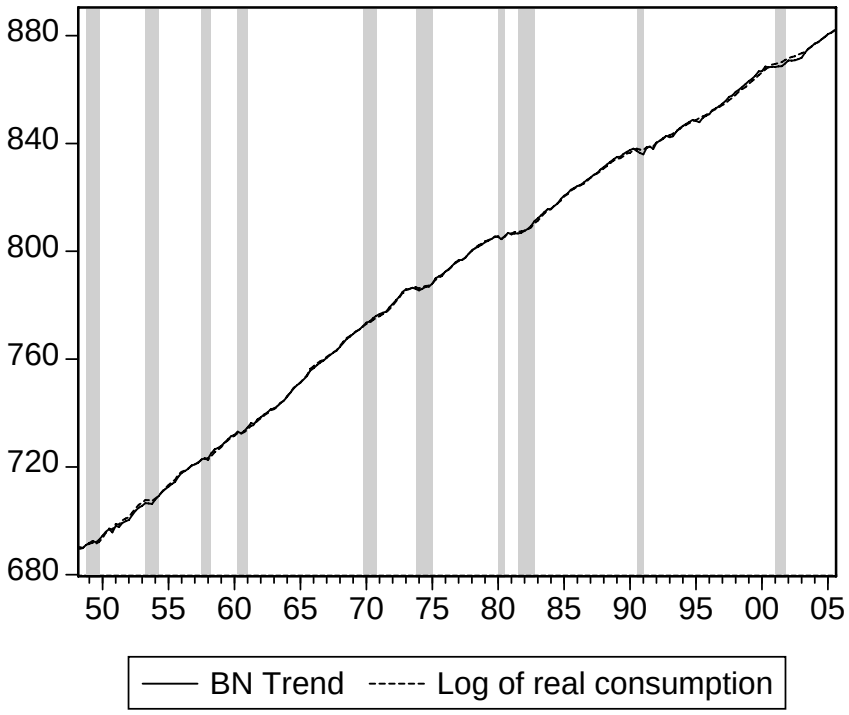

Figure 3F. Cyclical Component of Real Consumption from Markov-Switching Error-Correction VAR(4) Model

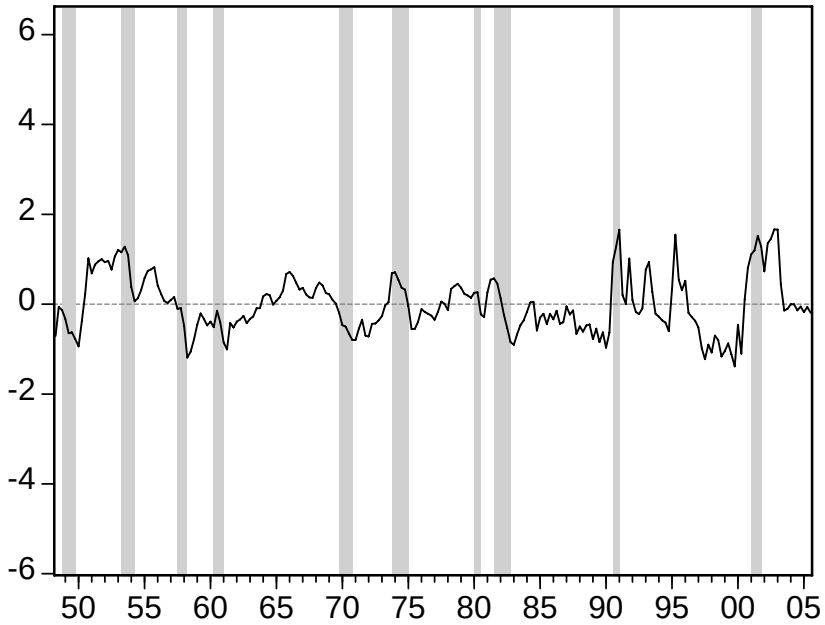

Figure 3G. Responses to Real Consumption Shock: MS-VECM Model

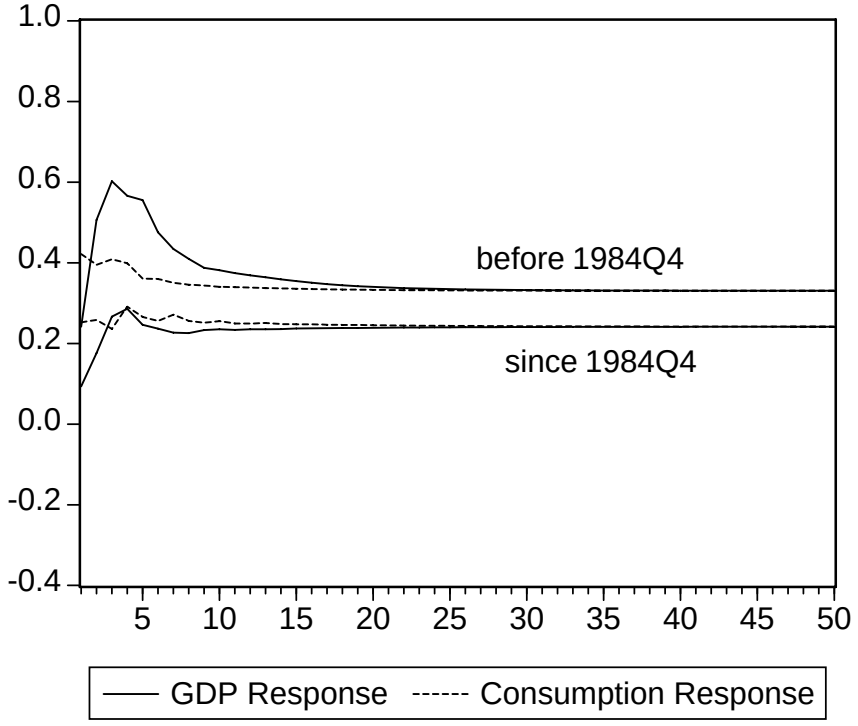

Figure 3H. Responses to Real GDP Shock: MS-VECM Model

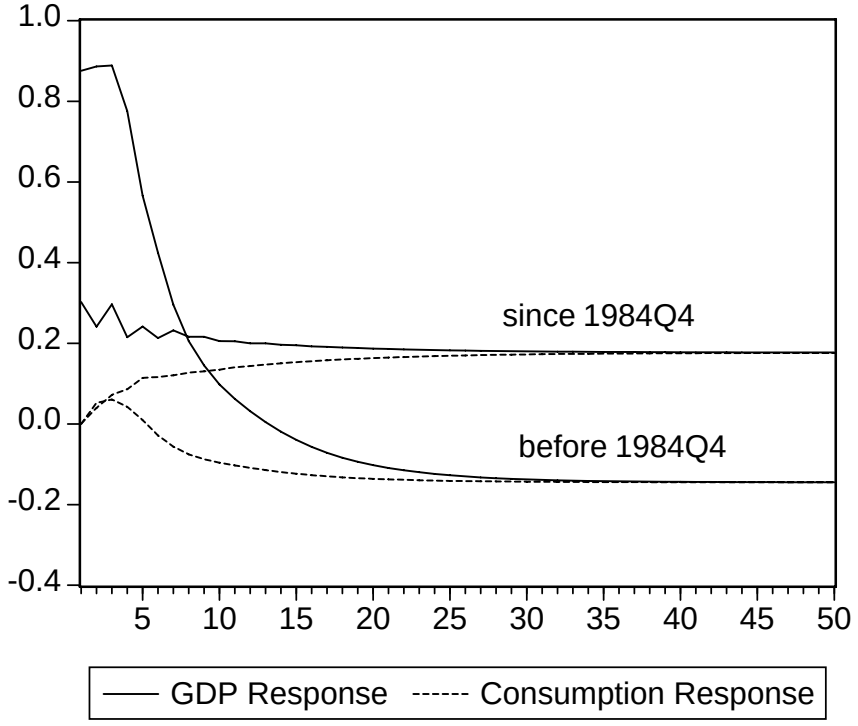
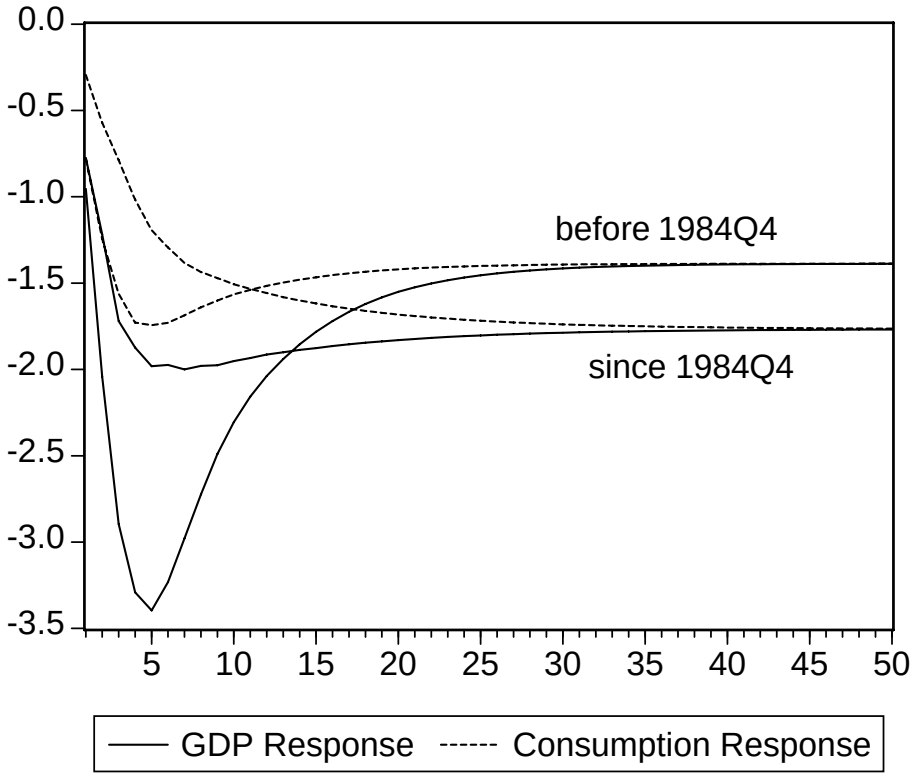
Figure 3I. Responses to Discrete Shock: MS-VECM Model