Global Yield Curve Dynamics and Interactions: A Generalized Nelson-Siegel Approach

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Abstract: The popular Nelson-Siegel yield curve model is routinely fit to intra-country bond yields, facilitating extraction of latent level, slope and curvature components. In this paper we move to a global context, modeling a potentially large *set* of country yield curves via a generalized Nelson-Siegel dynamic factor model that allows for both global and country-specific yield factors. We extract the global factors from term structures of government bond yields for the U.S., Germany, Japan, and the U.K. The results indicate that global yield factors do indeed exist and are economically important, generally explaining significant fractions of country yield curve dynamics, with interesting differences across countries.

Key Words: Yield curve, term structure, interest rate, dynamic factor model, global yield, world yield

JEL Codes: G1, E4, C5

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1. Introduction

The yield curve is of great interest both to academics and market practitioners. Hence yield curve modeling has generated a huge literature spanning many decades, particularly as regards the term structure of government bond yields. Most of that diverse literature is nevertheless unified by the assumption that the yield curve is driven by a number of latent factors (e.g., Litterman and Scheinkman, 1991; Balduzzi, Das, Foresi and Sundaram, 1996; Bliss, 1997a, 1997b; Dai and Singleton, 2000). Moreover, in many cases the latent yield factors may be interpreted as level, slope and curvature (e.g., Andersen and Lund, 1997; Diebold and Li, 2006). The vast majority of the literature studies a single country's yield curve in isolation and relates domestic yields to domestic yield factors, and more recently, to domestic macroeconomic factors (e.g., Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2006).

Little is known, however, about whether common *global* yield factors are operative, and more generally, about the nature of dynamic cross-country bond yield interactions. One might naturally conjecture the existence and enhanced importance of global yield factors in recent decades, due to enhanced global bond market integration. Conversely, understanding global bond yield factors (if any) is surely crucial for understanding global bond market integration. Either way, the existence and nature of global bond yield factors are of great interest. Numerous questions arise. Do global yield factors exist? If so, what are their dynamic properties? How do country yield factors load on the global factors, and what are the implications for cross-country yield curve interactions? How much of country yield factor variation is explained by global factors, and how much by country-specific factors, and does the split vary across countries in an interpretable way? Has the importance of global yield factors increased in recent years, due for example to improvements in global financial market integration?

In this paper we begin to address such questions in the context of a powerful yet tractable yield curve modeling framework. Building on the classic work of Nelson and Siegel (1987) as extended by Diebold and Li (2006), we construct a hierarchical dynamic model for sets of country yield curves, in which country yields may depend on country factors, and country factors may depend on global factors. Using government bond yields from the U.S., Germany, Japan, and the U.K., we estimate the model and extract and examine the global yield curve factors.

Our generalized Nelson-Siegel approach is related to, but distinct from, existing work that

tends to focus on spreads between domestic bond yields and a "world rate" (e.g., Al Awad and Goodwin, 1998), implicit one-factor analyses based on the international CAPM (e.g., Solnik, 1974, 2000; Thomas and Wickens, 1993), multi-factor analyses of long bond spreads (e.g., Dungey, Martin and Pagan, 2000), and affine equilibrium analyses (e.g., Brennan and Xia, 2004).

We proceed as follows. In section 2 we describe our basic econometric modeling framework, and in section 3 we discuss the multi-country bond yield data. In section 4 we estimate the global yield curve model and discuss the full-sample results. In section 5 we re-estimate the model over various sub-samples and discuss the results. We conclude in section 6.

2. Econometric Framework

Diebold and Li (2006), Diebold, Rudebusch and Aruoba (2006) and Diebold Piazzesi and Rudebusch (2005) show that, in a U.S. closed-economy environment, a generalized Nelson-Siegel model accurately approximates yield curve dynamics and provides good forecasts. Here we extend that framework to a multi-country environment. We allow for both global and country-specific factors, in a way that parallels recent real-side advances in the international business cycle literature, such as Lumsdaine and Prasad (1997), Gregory and Head (1999), and Kose, Otrok and Whiteman (2003).

Single-Country

The Diebold-Li factorization of the Nelson-Siegel yield curve for a single country (at a particular and arbitrary point in time) is

$$y_i(\tau) = l_i + s_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau}\right) + c_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau}\right) + v_i(\tau), \tag{1}$$

where $y_i(\tau)$ denotes the continuously-compounded zero-coupon nominal yield on a τ-month bond, l_i, s_i, c_i and λ_i are parameters, and $v_i(\tau)$ is a disturbance with standard deviation $\sigma_i(\tau)$. Following Diebold and Li, we dynamize the model by allowing the parameters to vary over time,

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau} - e^{-\lambda_{it}\tau}\right) + v_{it}(\tau). \tag{2}$$

We can interpret l_{it}, s_{it}, and c_{it} as *latent factors*. In particular, they are level, slope and curvature factors, respectively, because their factor loadings are a constant, a decreasing function of τ, and a concave function of τ. (Hence the notation *l*, *s* and *c*.) As the yield factors vary over time, this generalized Nelson-Siegel model can generate a variety of time-varying yield curve shapes.

Henceforth we will assume constancy of the λ_{it} parameters over countries and time. Following Diebold and Li (2006), there is little loss of generality from doing so, because λ simply determines the maturity at which the curvature loading is maximized. Hence we write

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + v_{it}(\tau). \tag{3}$$

Assuming an autoregressive structure for the factor dynamics yields a state space system, as emphasized by Diebold, Rudebusch and Aruoba (2006). Effectively, the generalized Nelson-Siegel model does not need to be cast in state space form – it is *already* in state space form.

<u>Multi-Country</u>

We now move to an *N*-country framework. We allow global yields to be depend on "global factors,"

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + V_t(\tau), \tag{4}$$

where the $Y_t(\tau)$ are global yields and L_t, S_t and C_t are global yield factors. We allow the global yield factors to follow a first-order vector autoregression,

$$\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \\ U_t^c \end{pmatrix}, \tag{5}$$

where the U_t^n are disturbances such that $EU_t^n U_{t'}^{n'} = (\sigma^n)^2$ if $t=t'$ and $n=n'$, and 0 otherwise, $n = l, s, c$.

An important feature of our approach, as we will make clear, is that it does not require that we *observe* global yields or global yield factors. We substitute out the "global yields" via the factor structure (4), which eliminates the need to observe or even define them directly. We then handle the underlying factors L, S and C as latent variables in a state space / Kalman filtering framework.

Each country's yield curve remains characterized by (1), but we now allow the country common factors, l_{it}, s_{it}, and c_{it}, to load on the global common factors L_t, S_t and C_t, as well as country idiosyncratic factors:

$$l_{it} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{it}^l \tag{6a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{it}^s \tag{6b}$$

$$c_{it} = \alpha_i^c + \beta_i^c C_t + \varepsilon_{it}^c, \tag{6c}$$

where $\{\alpha_i^1, \alpha_i^s, \alpha_i^c\}$ are constant terms, $\{\beta_i^1, \beta_i^s, \beta_i^c\}$ are loadings on global common factors, and $\{\varepsilon_{it}^1, \varepsilon_{it}^s, \varepsilon_{it}^c\}$ are country idiosyncratic factors, $i = 1, \ldots, N$. Because we include constant terms in (6), with no loss of generality we assume that the country idiosyncratic factors have zero mean. As with the global common factors, we allow the county idiosyncratic factors to have first-order autoregressive dynamics,

$$\begin{pmatrix} \varepsilon_{it}^l \\ \varepsilon_{it}^s \\ \varepsilon_{it}^c \end{pmatrix} = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} & \phi_{i,13} \\ \phi_{i,21} & \phi_{i,22} & \phi_{i,23} \\ \phi_{i,31} & \phi_{i,32} & \phi_{i,33} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \\ \varepsilon_{i,t-1}^c \end{pmatrix} + \begin{pmatrix} u_{it}^l \\ u_{it}^s \\ u_{it}^c \end{pmatrix}, \tag{7}$$

where the u_{it}^n are disturbances such that $Eu_{it}^n u_{i't'}^{n'} = (\sigma_i^n)^2$ if $i=i'$, $t=t'$ and $n=n'$, and 0 otherwise, $n = l, s, c$.

Many variations, extensions and specializations of this basic model are of course possible.

For example, a useful specialization to facilitate tractable estimation would restrict the dynamic matrices in (5) and (7) to be diagonal. (We shall do this.) As another example, an interesting extension would include not only global factors, but also regional factors, in which case country factors could depend on regional factors, which in turn could depend on global factors. (We shall not do this.)

3. Data Construction, Data Description, and Preliminary Analysis

In this section, prior to fitting the full global yield model, we discuss and describe the data, and we perform some preliminary analyses that provide motivation and background, and that are of interest in their own right.

Data Construction

Our data, generously supplied by Michael Brennan and Yihong Xia for 1985.09-2002.05 and extended by us to 2005.08, consist of government bond prices, coupon rates, and coupon structures, as well as issue and redemption dates, in local currency terms for the U.S., Germany, Japan, and the U.K.

We calculate zero-coupon bond yields using the unsmoothed Fama-Bliss (1987) approach.[1] We measure the bond yields on the second day of each month. We also apply several data filters designed to enhance data quality and focus attention on maturities with good liquidity. First, we exclude floating rate bonds, callable bonds and bonds extended beyond the original redemption date. Second, we exclude outlying bond prices less than 50 or greater than 130 because their price premium/discounts are too high and imply thin trading, and we exclude yields that differ greatly from yields at nearby maturities. Finally, we use only bonds with maturity greater than one month and less than fifteen years, because other bonds are not actively traded. Indeed, to simplify our subsequent estimation, using linear interpolation we pool the bond yields into fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months, where a month is defined as 30.4375 days.

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[1] Our zero-coupon bond yields are highly correlated with those obtained by Brennan and Xia (2003), who use a cubic spline and maturities of 3, 6, 12, 24, 36, 60, 84, 96, 108 and 120 months.

Data Description

In Figure 1 we show the government bond yield curves over countries and time. It is apparent that all yield curve levels move substantially, in complicated ways. Cross-country comparison of the yield curves, moreover, reveals some commonality in level movements. Yield curve slopes and curvatures vary less, although they do of course vary, and they may also display cross-country commonality in movements, although the graphical analysis is not powerful enough to say for sure.

In Table 1 we report summary statistics for bond yields. The Japanese yields are lowest on average, approximately 2-3%. For the U.S., the yield curves are upward-sloping on average, whereas the curves for Germany, Japan and the U.K. are slightly hump-shaped. Yield volatility tends to decrease with maturity for all countries. In addition, all yields are highly persistent for all countries, with average first-order autocorrelation greater than 0.95, and long yields tend to be more persistent than short yields.

Preliminary Analysis

Our ultimate goal is to provide a flexible and coherent, yet tractable, global yield curve modeling framework. To facilitate that goal, we first conduct a preliminary estimation of the Nelson-Siegel factors separately for each country.[2] That is, we estimate the level, slope and curvature factors, $\{l_{it}, s_{it}, c_{it}\}$, $t = 1, \ldots, T$ and $i = 1, .., N$, via a series of ordinary least squares regressions for each country. Note that each country's estimated $\{l_{it}, s_{it}, c_{it}\}$ potentially reflect both global and country-specific influences.[3]

In Figure 2 we plot the estimated $\{l_{it}, s_{it}, c_{it}\}$ factors separately for each country, and in Figure 3 we group together all the level factors, all the slope factors, and all the curvature factors, to better reveal commonality in factor dynamics. It is clear that the estimated level and slope factors show strong commonality across countries, whereas there is less evidence of commonality in curvature.

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[2] Throughout, we set λ=0.0609, which maximizes the loadings on the curvature factor at 30 months, as explained in Diebold and Li (2006).

[3] In section 4 we will explicitly decompose the country factors into global and country-specific components.

We present descriptive statistics for the estimated factors in Table 2. The factor autocorrelations reveal that all factors display persistent dynamics, with the level most persistent, and then slope, and then curvature. Augmented Dickey-Fuller tests fail to reject unit autoregressive roots in the level factor and produce mixed results for slope and curvature. However, although the factor roots are not easily distinguished from unity using statistical tests, economic theory strongly suggests that the roots are less than one. That is, theory shows that nominal bond yields can not go negative, but they *would* eventually go negative (with probability one) if they contained unit roots.

Finally, as a preliminary exploration of the commonality of movements in country yield curves, we conduct a principal component analysis on the estimated level, slope and curvature factors, as reported in Table 3. The results suggest the existence of global level and slope factors. Specifically, the first principal component for levels explains more than eighty percent of variation, and the first principal component for slopes explains more than fifty percent of variation.

4. Multi-Country Model Estimation

In this section, we estimate the global yield curve factor model, exploiting its state-space structure for both parameter estimation and factor extraction.

<u>State Space Representation, Identification, and Estimation Strategy</u>

The multi-country yield curve model has a natural state-space representation. The measurement equations are:

$$\begin{pmatrix} y_{1t}(\tau_1) \\ y_{1t}(\tau_2) \\ \cdots \\ y_{Nt}(\tau_J) \end{pmatrix} = A \begin{pmatrix} \alpha_1^l \\ \alpha_1^s \\ \cdots \\ \alpha_N^c \end{pmatrix} + B \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + A \begin{pmatrix} \epsilon_{1t}^l \\ \epsilon_{1t}^s \\ \cdots \\ \epsilon_{Nt}^c \end{pmatrix} + \begin{pmatrix} v_{1t}(\tau_1) \\ v_{1t}(\tau_2) \\ \cdots \\ v_{Nt}(\tau_J) \end{pmatrix} \tag{8}$$

where

$$A = \begin{pmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}-e^{-\lambda\tau_1}\right) & 0 & \dots & 0 \\ 1 & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}\right) & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}-e^{-\lambda\tau_2}\right) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & \left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}\right) & \left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}-e^{-\lambda\tau_J}\right) \end{pmatrix} \tag{9}$$

$$B = \begin{pmatrix} \beta_1^l & \beta_1^s\left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & \beta_1^c\left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}-e^{-\lambda\tau_1}\right) \\ \beta_1^l & \beta_1^s\left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}\right) & \beta_1^c\left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}-e^{-\lambda\tau_2}\right) \\ \dots & \dots & \dots \\ \beta_N^l & \beta_N^s\left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}\right) & \beta_N^c\left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}-e^{-\lambda\tau_J}\right) \end{pmatrix} \tag{10}$$

The transition equations are the union of (5) and (7).

Note that the global common factors and factor loadings are not separately identified. To identify the signs of factors and factor loadings, we assume that the global common factor loadings for the U.S. are positive, that is, $\beta_{us}^n > 0$, $n = l, s, c$. To identify scales, we assume that innovations to global factors have unit standard deviation, that is, $\sigma^n = 1$, $n = l, s, c$.[4]

Fully-efficient Gaussian maximum likelihood estimates are readily obtained in principle via application of the Kalman filter to the model in state space form, as in the single-country framework of Diebold, Rudebusch and Aruoba (2006). In practice, however, one-step maximum

[4] This follows Sargent and Sims (1977) and Stock and Watson (1989).

likelihood is particularly difficult to implement in our multi-country environment, because of the large number of parameters. Hence we use a convenient multi-step estimation method, which still exploits the state space structure, in the tradition of Diebold and Li (2006). In the first step, we estimate model (2) to obtain level, slope and curvature factors, l_{it}, s_{it}, c_{it}, separately for each country. In the second step, using the Kalman filter we estimate a dynamic latent factor model composed of the country factor decomposition equation (6), the dynamic equation for the global factors (5), and the dynamic equation for the country idiosyncratic factors (7). In the second-step estimation, (6) are the measurement equations (where we treat first-step estimated country yield curve factors l_i, s_i, and c_i as observed), and (5) and (7) are the transition equations.

Estimated Parameters and Factors

Motivated by the results of single-country analyses, which indicated little cross-factor dynamic interaction, we assume that the VARs given by equations (5) and (7) have diagonal autoregressive coefficient matrices. This drastically simplifies the second-step estimation, because it implies that we can estimate the model factor-by-factor, splitting the second step into three simple sub-steps. For each factor, there are seventeen parameters to estimate: one autoregressive coefficient for the global factor, four intercepts, four loadings on the global factor, four autoregressive coefficients for the idiosyncratic factors, and four standard deviations for the innovations to the idiosyncratic factors.[5]

Given parameters, we use the Kalman filter to evaluate the likelihood function, which we maximize iterating between steps of the Marquart and Berndt-Hall-Hall-Hausman algorithms, using numerical derivatives, optimal stepsize and a convergence criterion of 0.0001. We initialize the Kalman filter using the unconditional covariance matrix of the state vector, and we choose the initial parameters using the results of least squares regressions of (5), (6) and (7), using the first principal components in place of the latent global yield factors.

The results appear in Table 4. The global yield factors are highly serially correlated, and all country level and slope yield factors load positively and significantly on the corresponding global yield factors. The country-specific factors are also highly serially correlated, although

[5] In what follows, we perform two-step estimation for the level and slope factors. We do not estimate the model for curvature because the estimated curvature factor is prohibitively noisy.

slightly less so than the global factors. Again, although the various global and idiosyncratic yield factors have large dominant roots not easily distinguished from unity using conventional unit root tests, economic theory strongly suggests that the roots are less than one.

In Figure 4 we plot the global level and slope factors extracted using the Kalman filter, along with two standard error bands. The narrow bands indicate that the factors are estimated with high precision. It is reassuring to note that the global factors and earlier-discussed principal components are highly correlated, as we show in Figure 5. It is also important to note, however, that although related, they are not at all identical.

The paths of the global level and slope factors reflect the major developments in the global countries over the past twenty years. First, the global level factor is closely related to the inflation rate in the industrial countries, reflecting the decline in inflation from the 1980s into the 1990s and onward. Indeed, the global level factor is presently at an all-time low.

Second, the global slope factor moves with the state of macroeconomic activity (or expected macroeconomic activity, but we will not pursue that distinction). In particular, the estimated global slope factor peaked in the early 1990s and early 2000s, the two recessionary episodes in our sample.[6]

Variance Decompositions

With orthogonal global and country-specific factors, via (6) we can write the variance of level and slope factors as:[7]

$$var(l_{it}) = (\beta_i^l)^2 var(L_t) + var(\varepsilon_{it}^l) \tag{11a}$$

$$var(s_{it}) = (\beta_i^s)^2 var(S_t) + var(\varepsilon_{it}^s), \tag{11b}$$

[6] Note that our slope factor actually proxies the *negative* of yield curve slope, as shown in Diebold and Li (2006).

[7] The global and country factors extracted via the Kalman filter may be correlated even if the global factors and country factors are truly uncorrelated. Hence we orthogonalize the extracted factors to ensure that the variance decomposition adds up correctly. Specifically, we regress the country factor on the global factor and update the global factor loading and country factor variance accordingly.

for $i = 1, \ldots, N$. Hence we can decompose the variation in country level and slope factors into components driven by global yield variation and country-specific variation.

As we report in Table 5, the global level factor L explains a large fraction of the variation in country level factors, for all countries. However, the variance decomposition results for slope are more diverse. The U.S. slope factor, in particular, loads little on the global slope factor, indicating independence of the U.S. business cycle.

5. Split-Sample Analysis

In this section we provide some evidence on the stability (or lack thereof) of the dynamics linking the four countries' yield curves. As mentioned earlier, one might naturally conjecture the enhanced importance of global yield factors in recent decades, due to enhanced global bond market integration. Here we provide a first assessment of the evidence.

We split our sample into two equal subsamples, 1985:9-1995:8 and 1995:9-2005:8, and we perform a likelihood-ratio test of structural stability. The results strongly reject stability, as reported in Table 6, and we seek to understand why. Toward that end, we report split-sample estimation results in Table 7 and split-sample variance decomposition results in Table 8. The results display interesting nuances and are certainly more involved than, for example, a simple uniform increase in importance of the global factor in the second sub-sample.

Consider first the level factors. We find that the importance of the global level factor for German and American country yield levels has increased, but that the opposite is true for Japan and the U.K. Because yield curve levels relate mostly to inflation, the shifts suggest that inflation rates in Germany and the U.S. have become more dependent on the global economy and hence more correlated (relative to those of Japan and the U.K.), due possibly to increased interdependence of monetary policy in the Eurozone and the U.S.

The situation is similar for the slope factors. The importance of the global slope factor for German and American country yield slopes has increased, but its importance has decreased for Japan and the U.K. Because yield curve slopes relate mostly to real economic activity, this suggests that economic activity in U.S. and Germany has become relatively more dependent, perhaps again due to increased interdependence of monetary policy in the Eurozone and the U.S.

6. Summary and Concluding Remarks

We extend the single-country yield curve model in Nelson-Siegel (1987) and Diebold-Li (2006) to a multi-country setting, and we employ the model to study the common factors in the yield curves of four developed countries: the U.S., Germany, Japan, and the U.K. We develop a hierarchical model in which country's bond yields may depend on global level, slope and curvature factors, as well as country-specific idiosyncratic level, slope and curvature factors. Using a monthly dataset of the four countries' government bond yields from 1985:9 to 2005:8, we extract global factors and country-specific factors for both the full sample and the 1985:9-1995:8 and 1995:9-2005:8 sub-samples. We find that highly-persistent global factors are indeed responsible for significant fractions of country yields, and that the nature of the relationship has changed in significant and nuanced ways from the first sub-sample to the second.

We believe that a promising direction for future work is moving from a classical to a Bayesian framework for estimation and inference, with a corresponding move from maximization of the likelihood using numerical methods to drawing from posterior distributions using Markov chain methods. The Markov chain methods are simple and stable, even with many more parameters than under our present restrictive assumptions; hence they have the potential to facilitate a much richer analysis in an asset market parallel to the real-side work of Kose, Otrok and Whiteman (2003).[8]

[8] The key development is Carter and Kohn (1994), as insightfully exposited, extended and applied in Kim and Nelson (1998).

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Table 1: Summary Statistics for Bond Yields

	U.S.				
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$
3	4.76	2.07	0.87	9.32	0.93
6	4.93	2.07	0.96	9.69	0.98
9	5.07	2.09	1.01	9.74	0.97
12	5.19	2.10	1.03	9.72	0.98
15	5.31	2.11	1.07	9.74	0.98
18	5.40	2.10	1.12	9.69	0.98
21	5.48	2.06	1.14	9.59	0.98
24	5.54	2.01	1.25	9.58	0.98
30	5.67	1.97	1.24	9.57	0.98
36	5.79	1.91	1.58	9.55	0.98
48	5.94	1.85	1.27	9.97	0.98
60	6.02	1.90	1.74	10.05	0.98
72	6.26	1.86	1.99	10.72	0.98
84	6.41	1.80	2.05	10.30	0.98
96	6.61	1.77	2.02	10.56	0.98
108	6.71	1.66	3.58	10.37	0.98
120	6.73	1.59	3.35	10.83	0.96

Table 1 (Continued): Summary Statistics for Bond Yields

Germany					
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$
3	3.38	1.61	1.98	7.98	0.87
6	4.11	2.04	0.83	10.08	0.81
9	5.04	2.22	1.46	9.88	0.86
12	5.08	2.06	1.07	9.59	0.98
15	5.14	2.08	1.45	9.49	0.98
18	5.18	2.05	1.82	9.63	0.98
21	5.17	2.03	1.70	9.39	0.98
24	5.18	1.98	1.65	9.47	0.98
30	5.32	1.96	1.76	9.54	0.98
36	5.38	1.90	1.73	9.25	0.98
48	5.58	1.78	2.46	9.35	0.98
60	5.56	1.79	2.20	9.32	0.98
72	5.72	1.67	2.70	9.39	0.98
84	5.85	1.57	2.93	9.39	0.97
96	5.94	1.46	3.06	9.38	0.96
108	5.82	1.26	3.18	8.44	0.96
120	5.57	1.17	3.18	8.42	0.96

Table 1 (Continued): Summary Statistics for Bond Yields

Japan					
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$
3	2.14	2.55	0.01	8.35	0.62
6	2.38	2.51	0.01	8.40	0.98
9	2.35	2.48	0.01	8.40	0.99
12	2.36	2.44	0.01	8.41	0.99
15	2.38	2.41	0.01	8.41	0.99
18	2.41	2.40	0.02	8.40	0.99
21	2.43	2.38	0.02	8.38	0.99
24	2.46	2.37	0.03	8.32	0.99
30	2.53	2.33	0.06	8.35	0.99
36	2.59	2.27	0.04	8.34	0.99
48	2.76	2.20	0.13	8.15	0.99
60	2.95	2.15	0.17	8.10	0.99
72	3.10	2.10	0.22	7.90	0.99
84	3.27	2.03	0.29	7.83	0.98
96	3.40	1.96	0.39	7.73	0.98
108	3.41	1.97	0.19	7.16	0.98
120	2.79	1.62	0.16	6.56	0.96

Table 1 (Continued): Summary Statistics for Bond Yields

U.K.

Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$
3	6.92	3.00	3.29	14.77	0.65
6	6.89	2.94	1.06	14.59	0.79
9	7.09	3.03	2.44	14.41	0.92
12	7.04	3.02	2.67	14.23	0.94
15	6.99	2.99	2.40	14.15	0.97
18	6.95	2.94	2.32	14.44	0.97
21	6.93	2.90	2.32	14.41	0.97
24	6.92	2.85	2.31	14.45	0.97
30	6.95	2.79	2.31	14.13	0.98
36	7.01	2.77	2.29	13.86	0.98
48	7.14	2.67	2.36	13.67	0.98
60	7.31	2.65	2.30	13.47	0.98
72	7.34	2.52	1.89	13.35	0.97
84	7.27	2.40	3.06	11.43	0.98
96	7.25	2.47	3.03	12.34	0.98
108	7.13	2.54	2.55	12.93	0.98
120	7.12	2.44	3.64	13.34	0.98

Notes to table: All yield data are monthly, 1985.09 through 2005.08. $\rho(\tau)$ denotes an autocorrelation at displacement τ.

Table 2: Descriptive Statistics for Estimated Country Level, Slope and Curvature Factors

U.S.								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	7.17	1.53	4.05	11.42	0.97	0.70	0.39	-1.14
$\hat{s}_{it}$	-2.41	1.62	-5.96	0.67	0.92	0.37	-0.08	-3.14*
$\hat{c}_{it}$	-1.35	2.45	-6.41	8.11	0.77	0.27	0.06	-3.77**

Germany								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	6.74	1.46	3.04	9.63	0.95	0.60	0.35	-1.28
$\hat{s}_{it}$	-1.38	2.21	-5.22	6.32	0.96	0.43	-0.18	-3.06*
$\hat{c}_{it}$	-2.80	3.43	-16.00	9.34	0.83	-0.05	0.02	-3.04*

Japan								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	4.07	1.80	0.68	7.56	0.98	0.79	0.56	-1.06
$\hat{s}_{it}$	-1.59	1.31	-4.07	1.71	0.96	0.63	0.20	-2.03
$\hat{c}_{it}$	-2.70	1.88	-6.91	5.70	0.83	0.36	0.10	-2.51

U.K.								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	7.36	2.61	2.37	12.04	0.96	0.79	0.58	-1.60
$\hat{s}_{it}$	-0.57	2.53	-9.32	5.67	0.90	0.27	0.11	-2.74
$\hat{c}_{it}$	0.22	6.72	-14.26	24.86	0.86	0.30	-0.13	-3.14*

Notes to table: All yield data are monthly, 1985.09 through 2005.08. $\rho(\tau)$ denotes an autocorrelation at displacement τ. ADF denotes an augmented Dickey-Fuller test statistic. Single and double asterisks denote statistical significance at the five and one percent levels, respectively.

Table 3: Principal Components Analysis for Estimated Country Level, Slope and Curvature Factors

Level Factors, $\hat{l}_{it}$, $i = US, GM, JP, UK, \quad t = 1, \ldots, T$

	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	3.406	0.271	0.227	0.962
Variance Prop.	0.851	0.068	0.057	0.024
Cumulative Prop.	0.851	0.919	0.976	1.000

Slope Factors, $\hat{s}_{it}$, $i = US, GM, JP, UK, \quad t = 1, \ldots, T$

	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	1.962	0.983	0.756	0.299
Variance Prop.	0.491	0.246	0.189	0.075
Cumulative Prop.	0.491	0.736	0.925	1.000

Curvature Factors, $\hat{c}_{it}$, $i = US, GM, JP, UK, \quad t = 1, \ldots, T$

	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	1.419	1.115	0.888	0.578
Variance Prop.	0.355	0.279	0.222	0.144
Cumulative Prop.	0.355	0.633	0.855	1.000

Notes to table: All yield data are monthly, 1985.09 through 2005.08. For each of the sets of estimated country level factors, slope factors and curvature factors, we report the eigenvalues, variance proportions and cumulative variance proportions associated with the five principal components.

Table 4: Estimates of Global Yield Curve Model

Global Level Factor

$$L_t = \underset{(0.01)}{0.99} L_{t-1} + U_t^l$$

Global Slope Factor

$$S_t = \underset{(0.02)}{0.97} S_{t-1} + U_t^s$$

Country Level Factors

$$l_{US,t} = \underset{(2.29)}{7.05} + \underset{(0.02)}{0.16} L_t + \varepsilon_{US,t}^l \qquad \varepsilon_{US,t}^l = \underset{(0.04)}{0.82} \varepsilon_{US,t-1}^l + \underset{(0.01)}{0.21} v_{US,t}^l$$

$$l_{GM,t} = \underset{(2.04)}{6.57} + \underset{(0.02)}{0.14} L_t + \varepsilon_{GM,t}^l \qquad \varepsilon_{GM,t}^l = \underset{(0.03)}{0.84} \varepsilon_{GM,t-1}^l + \underset{(0.01)}{0.15} v_{GM,t}^l$$

$$l_{JP,t} = \underset{(2.83)}{3.88} + \underset{(0.02)}{0.20} L_t + \varepsilon_{JP,t}^l \qquad \varepsilon_{JP,t}^l = \underset{(0.11)}{0.52} \varepsilon_{JP,t-1}^l + \underset{(0.01)}{0.06} v_{JP,t}^l$$

$$l_{UK,t} = \underset{(3.12)}{7.16} + \underset{(0.05)}{0.21} L_t + \varepsilon_{UK,t}^l \qquad \varepsilon_{UK,t}^l = \underset{(0.02)}{0.91} \varepsilon_{UK,t-1}^l + \underset{(0.03)}{0.33} v_{UK,t}^l$$

Country Slope Factors

$$s_{US,t} = \underset{(0.70)}{-2.44} + \underset{(0.07)}{0.17} S_t + \varepsilon_{US,t}^s \qquad \varepsilon_{US,t}^s = \underset{(0.02)}{0.94} \varepsilon_{US,t-1}^s + \underset{(0.03)}{0.32} v_{US,t}^s$$

$$s_{GM,t} = \underset{(1.10)}{-1.52} + \underset{(0.07)}{0.19} S_t + \varepsilon_{GM,t}^s \qquad \varepsilon_{GM,t}^s = \underset{(0.02)}{0.96} \varepsilon_{GM,t-1}^s + \underset{(0.03)}{0.31} v_{GM,t}^s$$

$$s_{JP,t} = \underset{(0.47)}{-1.48} + \underset{(0.05)}{0.15} S_t + \varepsilon_{JP,t}^s \qquad \varepsilon_{JP,t}^s = \underset{(0.03)}{0.93} \varepsilon_{JP,t-1}^s + \underset{(0.01)}{0.12} v_{JP,t}^s$$

$$s_{UK,t} = \underset{(1.10)}{-0.48} + \underset{(0.13)}{0.42} S_t + \varepsilon_{UK,t}^s \qquad \varepsilon_{UK,t}^s = \underset{(0.06)}{0.78} \varepsilon_{UK,t-1}^s + \underset{(0.09)}{0.97} v_{UK,t}^s$$

Notes to table: We report estimates of the parameters of the global model (6)-(9) that correspond to level and slope factors, obtained using the procedure discussed in the text.

Table 5: Variance Decompositions

Level Factors

	Germany	Japan	U.K.	U.S.
World	75.49	98.98	73.03	76.46
Country	24.51	1.02	26.97	23.53

Slope Factors

	Germany	Japan	U.K.	U.S.
World	27.77	70.59	80.13	5.83
Country	72.23	29.41	19.87	94.17

Notes to table: For each country we decompose its variation in level and slope factors into parts coming from world and country-specific factor variation. See text for details.

Table 6: Likelihood Ratio Tests of Structural Stability

	Level Factors		Slope Factors
		Log Likelihoods	
Full sample: 1985:9 - 2005:8	-589.675		-890.834
Subsample: 1985:9 - 1995:8	-329.819		-433.257
Subsample: 1995:9 - 2005:8	-236.963		-433.773
		Likelihood Ratio Statistics	
Likelihood Ratio Statistic	45.785		47.608
p-Value	0.000		0.000

Table 7a: Split-Sample Estimates of Global Yield Curve Model, Level Factors

1985.09-1995.08

Global Level Factor

$$L_t = \underset{(0.05)}{0.94} L_{t-1} + U_t^l$$

Country Level Factors

$$l_{US,t} = \underset{(0.49)}{8.41} + \underset{(0.07)}{0.29} L_t + \varepsilon_{US,t}^l \qquad \varepsilon_{US,t}^l = \underset{(0.06)}{0.85} \varepsilon_{US,t-1}^l + \underset{(0.03)}{0.16} v_{US,t}^l$$

$$l_{GM,t} = \underset{(0.27)}{7.80} + \underset{(0.05)}{0.13} L_t + \varepsilon_{GM,t}^l \qquad \varepsilon_{GM,t}^l = \underset{(0.05)}{0.74} \varepsilon_{GM,t-1}^l + \underset{(0.01)}{0.18} v_{GM,t}^l$$

$$l_{JP,t} = \underset{(0.36)}{5.65} + \underset{(0.07)}{0.21} L_t + \varepsilon_{JP,t}^l \qquad \varepsilon_{JP,t}^l = \underset{(0.15)}{0.57} \varepsilon_{JP,t-1}^l + \underset{(0.03)}{0.09} v_{JP,t}^l$$

$$l_{UK,t} = \underset{(0.41)}{9.67} + \underset{(0.10)}{0.15} L_t + \varepsilon_{UK,t}^l \qquad \varepsilon_{UK,t}^l = \underset{(0.05)}{0.78} \varepsilon_{UK,t-1}^l + \underset{(0.06)}{0.45} v_{UK,t}^l$$

1995.09-2005.08

Global Level Factor

$$L_t = \underset{(0.04)}{0.96} L_{t-1} + U_t^l$$

Country Level Factors

$$l_{US,t} = \underset{(0.58)}{5.93} + \underset{(0.05)}{0.24} L_t + \varepsilon_{US,t}^l \qquad \varepsilon_{US,t}^l = \underset{(0.16)}{0.36} \varepsilon_{US,t-1}^l + \underset{(0.01)}{0.16} v_{US,t}^l$$

$$l_{GM,t} = \underset{(0.78)}{5.82} + \underset{(0.08)}{0.23} L_t + \varepsilon_{GM,t}^l \qquad \varepsilon_{GM,t}^l = \underset{(0.05)}{0.94} \varepsilon_{GM,t-1}^l + \underset{(0.01)}{0.11} v_{GM,t}^l$$

$$l_{JP,t} = \underset{(0.64)}{2.62} + \underset{(0.04)}{0.12} L_t + \varepsilon_{JP,t}^l \qquad \varepsilon_{JP,t}^l = \underset{(0.03)}{0.96} \varepsilon_{JP,t-1}^l + \underset{(0.01)}{0.07} v_{JP,t}^l$$

$$l_{UK,t} = \underset{(0.90)}{5.41} + \underset{(0.10)}{0.12} L_t + \varepsilon_{UK,t}^l \qquad \varepsilon_{UK,t}^l = \underset{(0.04)}{0.94} \varepsilon_{UK,t-1}^l + \underset{(0.02)}{0.21} v_{UK,t}^l$$

Notes to table: We report estimates of the parameters of the global model (6)-(9) that correspond to level and slope factors, obtained using the procedure discussed in the text.

Table 7b: Split-Sample Estimates of Global Yield Curve Model, Slope Factors

1985.09-1995.08

Global Slope Factor

$$S_t = \underset{(0.03)}{0.96} S_{t-1} + U_t^s$$

Country Slope Factors

$$s_{US,t} = \underset{(1.19)}{-2.71} + \underset{(0.08)}{0.26} S_t + \varepsilon_{US,t}^s \qquad \varepsilon_{US,t}^s = \underset{(0.04)}{0.95} \varepsilon_{US,t-1}^s + \underset{(0.04)}{0.24} v_{US,t}^s$$

$$s_{GM,t} = \underset{(1.30)}{-1.67} + \underset{(0.10)}{0.14} S_t + \varepsilon_{GM,t}^s \qquad \varepsilon_{GM,t}^s = \underset{(0.03)}{0.96} \varepsilon_{GM,t-1}^s + \underset{(0.02)}{0.26} v_{GM,t}^s$$

$$s_{JP,t} = \underset{(0.65)}{-1.22} + \underset{(0.08)}{0.15} S_t + \varepsilon_{JP,t}^s \qquad \varepsilon_{JP,t}^s = \underset{(0.04)}{0.92} \varepsilon_{JP,t-1}^s + \underset{(0.02)}{0.18} v_{JP,t}^s$$

$$s_{UK,t} = \underset{(1.81)}{-0.31} + \underset{(0.14)}{0.66} S_t + \varepsilon_{UK,t}^s \qquad \varepsilon_{UK,t}^s = \underset{(0.23)}{0.16} \varepsilon_{UK,t-1}^s + \underset{(0.17)}{0.44} v_{UK,t}^s$$

1995.09-2005.08

Global Slope Factor

$$S_t = \underset{(0.03)}{0.95} S_{t-1} + U_t^s$$

Country Slope Factors

$$s_{US,t} = \underset{(1.07)}{-1.90} + \underset{(0.08)}{0.43} S_t + \varepsilon_{US,t}^s \qquad \varepsilon_{US,t}^s = \underset{(0.35)}{0.12} \varepsilon_{US,t-1}^s + \underset{(0.04)}{0.14} v_{US,t}^s$$

$$s_{GM,t} = \underset{(2.17)}{-1.92} + \underset{(0.11)}{0.11} S_t + \varepsilon_{GM,t}^s \qquad \varepsilon_{GM,t}^s = \underset{(0.02)}{0.96} \varepsilon_{GM,t-1}^s + \underset{(0.04)}{0.41} v_{GM,t}^s$$

$$s_{JP,t} = \underset{(0.85)}{-2.27} + \underset{(0.04)}{0.10} S_t + \varepsilon_{JP,t}^s \qquad \varepsilon_{JP,t}^s = \underset{(0.03)}{0.97} \varepsilon_{JP,t-1}^s + \underset{(0.01)}{0.07} v_{JP,t}^s$$

$$s_{UK,t} = \underset{(1.24)}{-1.12} + \underset{(0.26)}{0.05} S_t + \varepsilon_{UK,t}^s \qquad \varepsilon_{UK,t}^s = \underset{(0.05)}{0.89} \varepsilon_{UK,t-1}^s + \underset{(0.30)}{1.23} v_{UK,t}^s$$

Notes to table: We report estimates of the parameters of the global model (6)-(9) that correspond to level and slope factors, obtained using the procedure discussed in the text.

Table 8: Split-Sample Variance Decompositions

Level Factors (1985.09-1995.08)

	Germany	Japan	U.K.	U.S.
Global	26.14	90.27	8.30	58.58
Country	73.86	9.73	91.70	41.42

Level Factors (1995.09-2005.08)

	Germany	Japan	U.K.	U.S.
Global	55.37	50.45	0.00	88.97
Country	44.63	49.55	100.00	11.03

Slope Factors (1985.09-1995.08)

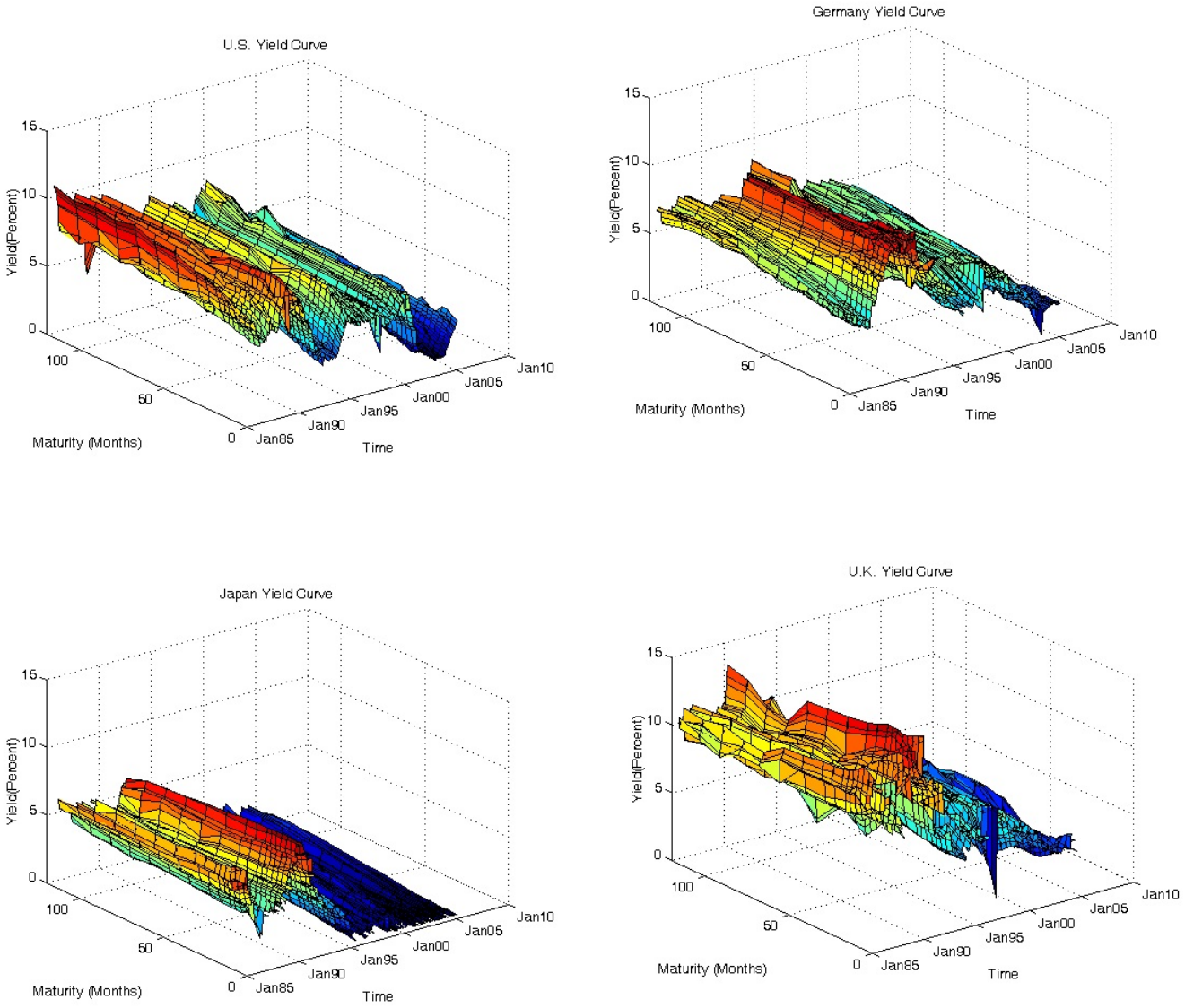
	Germany	Japan	U.K.	U.S.
Global	3.54	70.54	97.52	26.03
Country	96.46	29.46	2.48	73.97

Slope Factors (1995.09-2005.08)

	Germany	Japan	U.K.	U.S.
Global	9.13	5.56	2.66	98.05
Country	90.87	94.44	97.34	1.95

Notes to table: For each of Germany, Japan, U.K. and U.S. we decompose variation yield curve level and slope factors into parts coming from global and country-specific factor variation, separately for 1985.09-1995.08 and 1995.09-2005.08. See text for details.

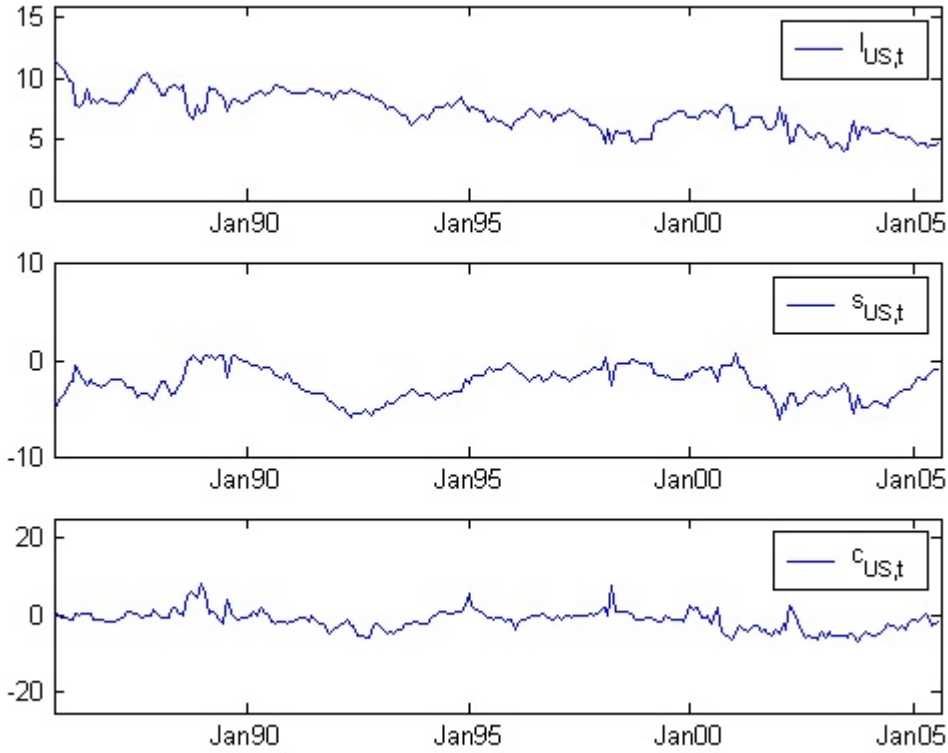
Figure 1: Yield Curves over Space and Time



Notes to Figure 1: All yield data are monthly, 1985.09 through 2005.08.

Figure 2: Estimated Country Level, Slope and Curvature Factors

U.S.



Germany

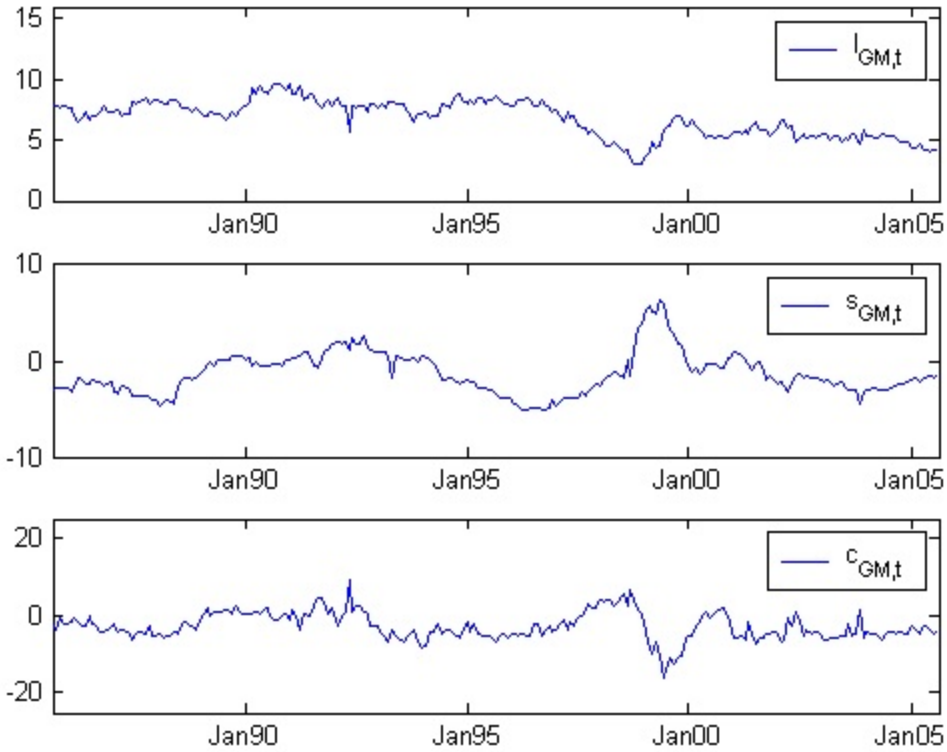
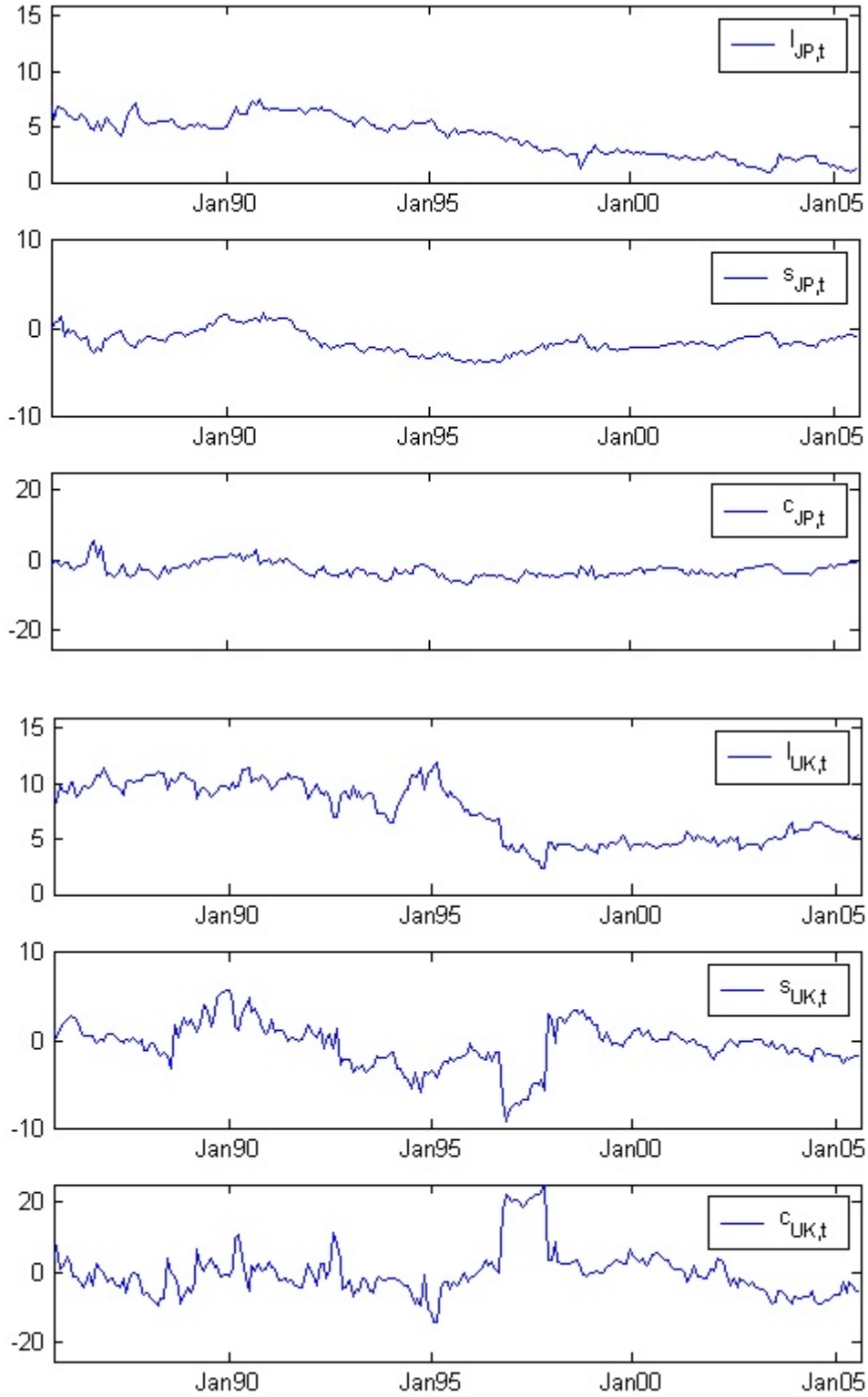


Figure 2 (Continued): Estimated Country Level, Slope and Curvature Factors

Japan



U.K.

Notes to Figure 2: All yield data are monthly, 1985.09 through 2005.08.

Figure 3a: Estimated Country Factors, Level

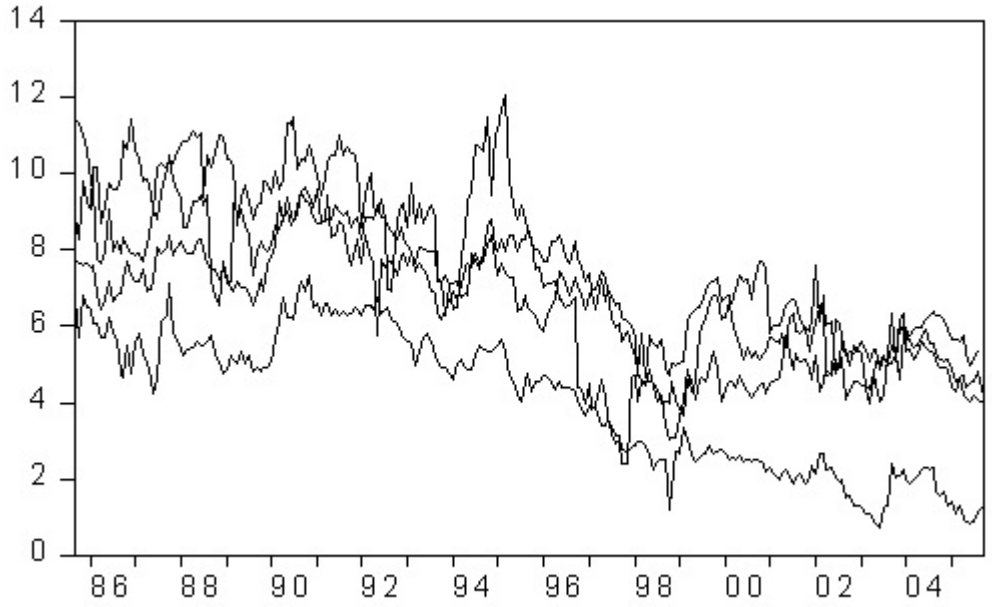


Figure 3b: Estimated Country Factors, Slope

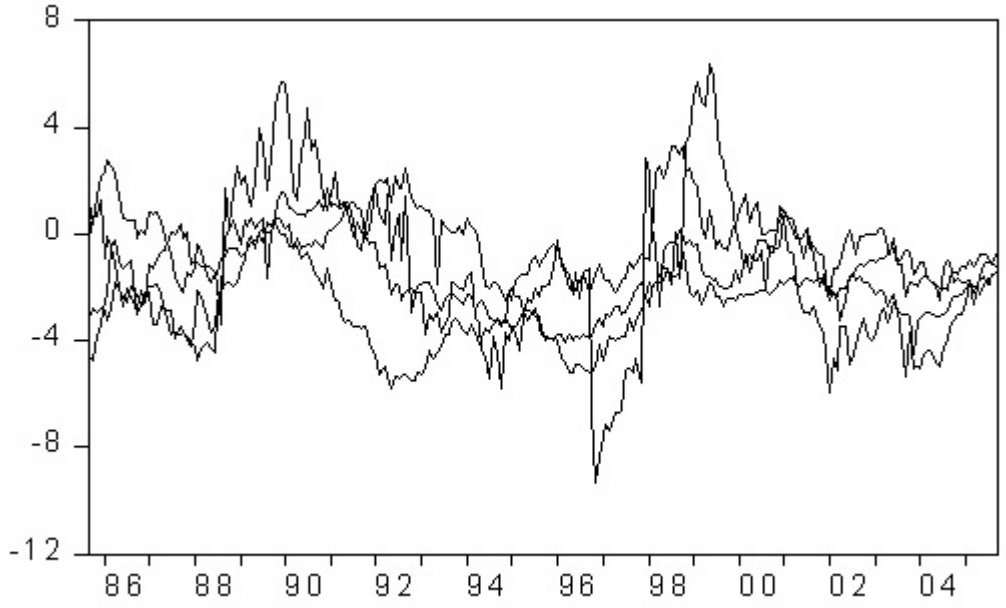


Figure 4a: Extracted Global Level Factor with Two Standard Error Band

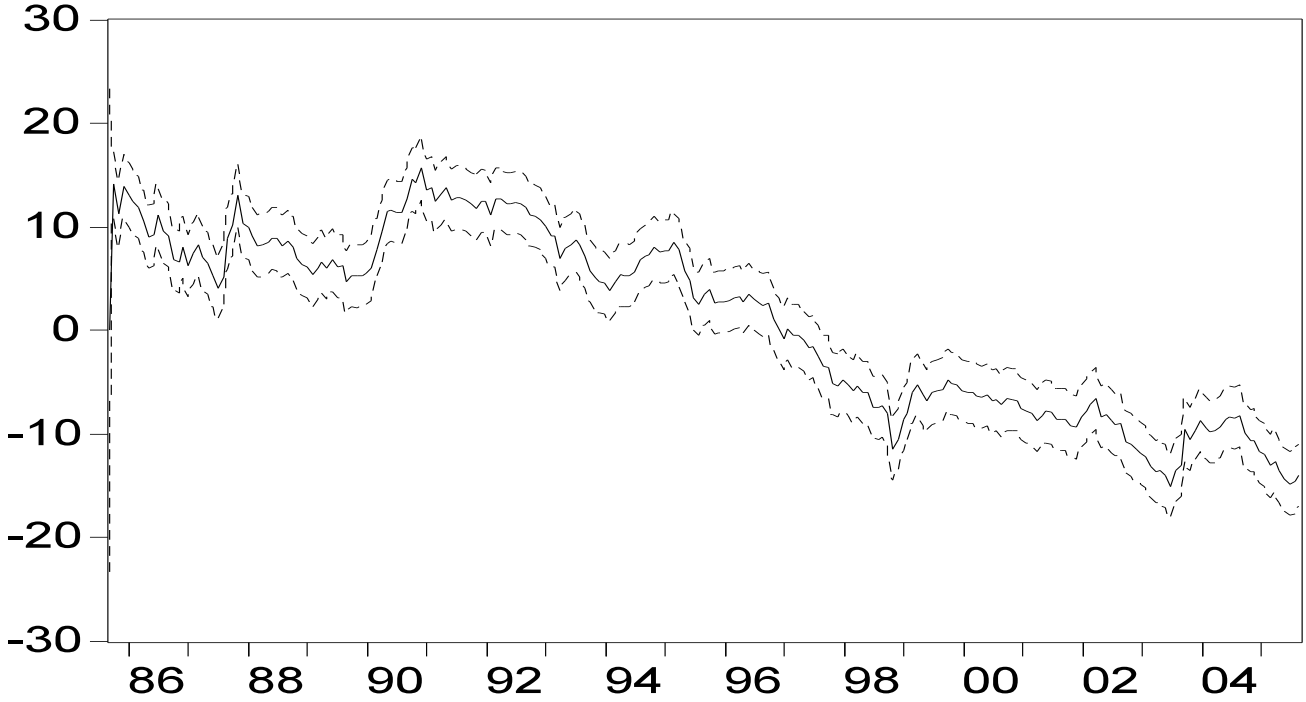


Figure 4b: Extracted Global Slope Factor with Two Standard Error Band

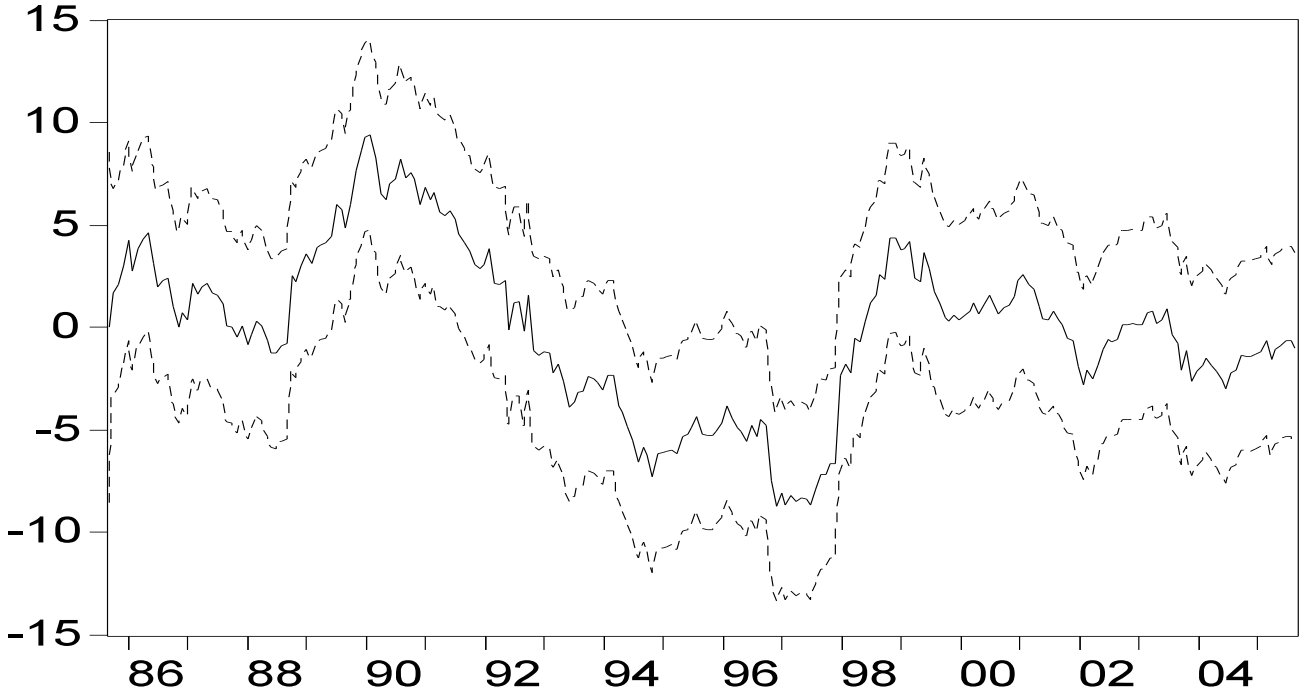


Figure 5a: Global Factors vs. First Principal Components, Level

Figure 5b: Global Factors vs. First Principal Components, Slope

Notes to figure: Solid lines are the estimated global factors. Dotted lines are principal components.