Do Funds-of-Funds Deserve Their Fees-on-Fees?*

Andrew Ang†
Columbia University and NBER

Matthew Rhodes-Kropf‡
Columbia University

Rui Zhao§
Columbia University

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†Columbia Business School, 3022 Broadway Uris 803, New York, NY 10027; ph: (212) 854-9154; fax: (212) 662-8474; email: aa610@columbia.edu; WWW: http://www.columbia. edu/∼aa610.

‡Columbia Business School, 3022 Broadway Uris 803, New York, NY 10027; ph: (212) 854-7480; fax: (212) 662-8474; email: mr554@columbia.edu; WWW: http://www.gsb.columbia.edu/faculty/mrhodeskropf

§Columbia Business School, 3022 Broadway Uris 311, New York, NY 10027; ph: (646) 335-3238; email: rz108@columbia.edu; WWW: http://www.columbia.edu/∼rz108.

Abstract

Since the after-fee returns in funds-of-funds are, on average, lower than hedge fund returns, it appears that funds-of-funds do not add value. However, we show that funds-of-funds should not be evaluated relative to hedge fund returns from reported databases. Instead, the correct fund-of-funds benchmark is the return an investor would achieve from direct hedge fund investments on her own without recourse to funds-of-funds. We use certainty equivalent concepts and revealed preference arguments to estimate attributes of the true, implied true fund-of-funds benchmark distribution. Since the benchmark characteristics seem reasonable, we conclude that, on average, funds-of-funds deserve their fees-on-fees.

1 Introduction

A fund-of-funds is a hedge fund that invests in other hedge funds. The funds-of-funds industry is large and growing remarkably fast. Funds-of-funds now constitute more than one quarter of the total assets under the direct management of hedge funds.[1] At first glance, the reasons for the increasing popularity in funds-of-funds are numerous. First, they allow investors to obtain exposure to hedge fund investments that are otherwise closed to individual investors. Second, funds-of-funds generally have much lower required investment minimums than those required by hedge funds. Third, they provide investors access to a diversified portfolio of hedge funds. Only individual investors with very large amounts of capital could replicate this degree of diversification. Finally, they provide good access to information and professional portfolio management that would otherwise be difficult and expensive to obtain.

However, investors in funds-of-funds pay a steep price for this convenience. A fund-of-funds passes onto investors all fees charged by the underlying hedge funds in the fund-of-funds' portfolio. In addition, investors in funds-of-funds must also pay an extra set of fees to compensate the funds-of-funds' managers. These fees-on-fees are not negligible. In the TASS database, the average management fee levied by funds-of-funds is 1.5% and the average fund-of-funds' incentive fee is over 9.2%. These fees are on top of an average management fee of 1.4% and an average incentive fee of 18.4% for hedge funds.

Hedge funds and funds-of-funds provide us with a unique platform to examine the value of access to alternative asset classes. Most asset classes are cheaply and easily accessible, but if a set of assets is difficult and costly to access, like venture capital or hedge funds, then we can use the returns of these assets and the fees paid to access these investments to obtain a glimpse of the perceived value investors place on these assets. In this paper, we infer the economic assumptions underlying the revealed preference of an investor who is indifferent between a fund-of-funds investment and a hedge fund investment which that investor could make without recourse to funds-of-funds. Characterizing these economic assumptions allows us to judge if the fees-on-fees of funds-of-funds are reasonable.

Previous work on the hedge fund industry compares hedge funds with funds-of-funds and finds that, on average, funds-of-funds under-perform hedge funds.[2] Many authors claim that

[1] Since 2000, funds-of-funds have received 35% of the new inflows into hedge funds, compared to receiving 11% percent of new flows in the early 1990s. For funds in the TASS database, the total value of assets under management for funds of funds is $70.1 billion compared to $282.4 billion for hedge funds as of September 2003. We compute the inflows from the TASS data.

[2] See, among others, Kat and Amin (2001), Amin and Kat (2002), Ackermann, McEnally and Ravenscraft (1999), Lhabitant and Learned (2002), Brown, Goetzmann and Liang (2004), Capocci and Hubner (2004), and Fung and Hsieh (2004).

the extra fees charged by funds-of-funds are too high and outweigh the efficiency gains of investments in funds-of-funds. In particular, Brown, Goetzmann and Liang (2004) claim that the extra fees do not provide an appropriate incentive for funds-of-funds managers. The general consensus is that the fund-of-funds industry offers poor value to investors.[3]

The first contribution of this paper is to show that the conventional analysis of comparing alphas across hedge funds and funds-of-funds does not adequately measure the true potential benefit of a fund-of-funds investment. Comparing returns across two asset classes is valid if both assets are easily accessible. For example, mutual funds can be compared to index funds since investors can invest in either without issue. However, a direct comparison of hedge fund and fund-of-funds returns misses the unique nature of the hedge fund industry. When an investor decides between a fund-of-funds and a direct hedge fund investment, she compares the fund-of-funds to the set of hedge funds that she can locate and invest in by herself without using a fund-of-funds. Hedge funds are hard to find, hard to evaluate, hard to monitor, have high minimums, and are often closed to new investors. Thus, an investor choosing between direct hedge fund investments and using funds-of-funds has to compare her own costs and skill of locating, evaluating and monitoring hedge funds which she could enter with the costs and skill of the fund-of-funds manager.

This discussion makes it clear that the evaluation of a fund-of-funds versus direct hedge fund investments is different for every investor. An investor with a large amount of capital who has expertise in (and a low cost structure for) finding and evaluating hedge funds would prefer to invest directly in hedge funds, rather than investing indirectly through funds-of-funds. But, for an investor with little or no expertise in the hedge fund industry, the probability of choosing an incompetent hedge fund manager, or a hedge fund following a poor investment strategy, may be very high. Indeed, there are large cross-sectional differences in the performance of individual hedge funds (see, for example, Li, Zhang and Zhao, 2005). Unskilled investors potentially face a large penalty for indiscriminately selecting hedge funds on their own, and thus many choose to invest through funds-of-funds instead.

As a result, the universe of hedge funds that we see in data are funded either by expert fund-of-funds managers or by investors with sufficient resources and skills that enable them to make direct hedge fund investments. In data, we do not observe the set of hedge funds that received no funding, but would have received funding if unskilled investors were forced to directly invest in hedge funds without investing through funds-of-funds. Hence, by construction, the observable set of hedge fund investments is biased and appears to be good relative to the set of after-fee

[3] A rare counterexample is Fung and Hsieh (2000), who argue that the high fees of funds-of-funds cover the costly management of a hedge fund portfolio and that funds-of-funds must hold cash balances to cover the addition and withdrawal of hedge funds, which lowers their returns relative to hedge funds.

returns of funds-of-funds. While the return characteristics of the funds-of-funds and hedge fund market tells us about the equilibrium when investors with different skills and costs sort themselves into users of funds-of-funds and direct hedge fund investors, these return differences do not answer the question if funds-of-funds deserve their high fees.

The second goal of this paper to show how we can gain insight into the value of the funds-of-funds industry. To evaluate funds-of-funds, the correct benchmark should be the full distribution of hedge funds that any particular investor can access, rather than a set of hedge funds observable in data. There is no direct way to compare funds-of-funds with their true benchmark because we do not observe the full distribution of hedge funds. However, we can use revealed preference to estimate the true underlying hedge fund distribution of the marginal investor. Specifically, we ask what the alternate, accessible hedge fund distribution would look like in order to make an investor indifferent between a direct hedge fund investment and a fund-of-funds investment. Since investors choose to invest in funds-of-funds, the true distribution of hedge funds these investors can access must be at least as bad as the distribution that makes them indifferent between the observed fund-of-funds investments and the true set of hedge funds to which the investors can access.

Using certainty equivalent concepts, we estimate the implied benchmark distribution for funds-of-funds. Then, we compare characteristics of the benchmark with the hedge fund returns observed in data. We find that the conditions where an investor chooses a fund-of-funds over a hedge fund are economically reasonable and plausible. This is particularly true for smaller and more risk-averse investors. Consider an investor holding a low-cost, benchmark portfolio of well-diversified domestic and international assets who cannot short more than -20% with a risk aversion of $\gamma = 8$. This investor finds that funds-of-funds add value if she believes that her own direct investments in hedge funds would result in an average return just 0.50% per annum lower than the median return of funded hedge funds in data. Alternatively, if her own direct hedge fund investments have returns that are at least 1.30% per annum more volatile than observable hedge fund returns, she would find that a fund-of-funds, rather than a direct hedge fund investment, would improve her utility. Thus, on average, funds-of-funds can provide sensible investment vehicles to obtain exposure to hedge fund investment strategies.

Investors' revealed preferences tell us what they must believe about their own ability in order to chose a fund-of-funds. If we had found that investors needed to believe that they were implausibly bad on their own in order to justify a fund-of-funds, we would have concluded that fund-of-funds were over-charging for their investment performance. However, our analysis shows that contrary to popular belief and past work, it is relatively easy to justify funds-of-funds' fees. While we apply revealed preference arguments to the funds-of-funds industry, our analysis can be used more broadly to examine the potential value of an asset class when markets are incomplete and to compute the economic value of access to broader diversification vehicles with limited access. Thus, we hope to change the way future analysis on alternative asset classes is conducted.

We comment that our analysis focuses only on characterizing the true benchmark for funds-of-funds and not on investigating the absolute performance of hedge funds or funds-of-funds relative to standard asset pricing models. Whether hedge funds have average returns in excess of their risk profiles is still an open question. Studies like Fung and Hsieh (2001) cannot reject that there is no average excess performance of hedge funds after factors with option-like payoffs are included. On the other hand, Bailey, Li and Zhang (2004) find evidence of the average out-performance of hedge funds under the null of no arbitrage, even when non-linear factor payoffs are considered.[4] Our work is silent on the absolute investment performance of hedge funds and funds-of-funds, and we focus only on what the expected relative performance of after-fee returns of funds-of-funds compared to hedge funds would have to be in order for investors to optimally choose to pay the added fees of funds-of-funds.[5]

The rest of this paper is organized as follows. Section 2 describes how the presence of skilled funds-of-funds' managers causes the observed hedge fund returns to not represent the true hedge fund universe. In Section 3, we formulate the asset allocation problem and show how to characterize the true fund-of-funds benchmark. In Section 4, we describe the hedge fund and fund-of-funds data and compute statistics that are robust to reporting lags and non-synchronous trading. We lay out our empirical results evaluating fund-of-funds performance in Section 5. Section 6 conducts a series of robustness checks. Finally, Section 7 concludes.

2 What is the Appropriate Fund-of-Funds Benchmark?

Comparing after-fee alphas of funds is a common portfolio evaluation tool used to gauge the performance of equity investments. However, investing in hedge funds is very different from investing in the stock market. First, the best hedge funds are closed (presumably filled with the money of the smart investors who recognized the superiority of these hedge funds at an early stage). Second, hedge funds require high minimum investments, with the top hedge funds

[4] Other authors computing alphas of funds-of-funds and hedge funds include Fung and Hsieh (1997, 2000, 2001), Ackermann, McEnally and Ravenscraft (1999), Liang (1999), Edwards and Caglayan (2001), Ben Dor and Jagannathan (2002), Agarwal and Naik (2000, 2004), and Brown and Goetzmann (2004), among many others.

[5] We also do not address the question of optimal fees for hedge funds or funds-of-funds. Recent studies focusing on the optimal fee structure of hedge funds or funds-of-funds include Anson (2001), Goetzmann, Ingersoll and Ross (2003), Hodder and Jackwerth (2004), Bhansali and Wise (2005), and Stavros and Westerfield (2005).

requiring investments in the millions, sometimes tens of millions, of dollars. Even if a wealthy investor meets minimum requirements, there is no guarantee that a successful hedge fund will take that investor as a client. Third, and most importantly, unlike listed stocks that must provide timely disclosure notices and accounting reports, hedge funds are often secretive with little or no obvious market presence. Thus, it is plausible to assume that fund-of-funds managers and individual investors have different abilities in evaluating hedge funds, either because fund-of-funds managers have expertise in picking good hedge funds, or because they gather superior information at a cost.

A simple comparison of the alphas of funds-of-funds to the alphas of hedge funds does not address the question of whether funds-of-funds add value to the investors who choose to use them. An investor with little skill is not choosing between the alpha of the universe of hedge funds and the fund-of-funds' alpha. Rather, she is choosing between the utility gain from an investment in a fund-of-funds and the utility gain from an investment in a hedge fund that she can find, meet the minimum requirements, and monitor. An investor may decide that the diversification benefits, access, and skills of a fund-of-funds manager easily outperforms her own opportunity set. Thus, to determine if funds-of-funds are adding value we need to compare the utility of an investor in a fund-of-funds to the utility she would achieve if funds-of-funds did not exist.[6]

We provide a simple model to show that funds-of-funds are a useful investment vehicle for unskilled investors because funds-of-funds offer them an opportunity to access the skill set of sophisticated, skilled investors. In the following model, we show that unskilled investors are willing to pay to enter an economy where everyone has access to superior skills to evaluate hedge funds. Thus, in equilibrium, unskilled investors invest through skilled funds-of-funds. Funds-of-funds perform the same as hedge funds on a pre-fee basis but investors in funds-of-funds receive lower returns on an after-fee basis. Funds-of-funds add value because the unskilled investor's alternative is to invest in hedge funds on her own and, consequently, earn lower average returns than skilled investors.

[6] By unskilled investors, we do not mean investors with little money or investors without any financial knowledge. By law, most hedge funds and funds-of-funds are organized under the qualified investor exemption in law and are limited to investors with a net worth of at least $5 million. By an unskilled investor, we mean an investor that does not have the same opportunity set to find hedge funds, or has inferior skills to evaluate and monitor hedge funds. In our model, we assume that fund-of-funds managers, on average, have such skills. To prevent poor hedge fund allocations, sophisticated investors spend significant resources to evaluate the skill of the managers. William H. Donaldson, Chairman of the U.S. Securities and Exchange Commission, notes in his May 2003 testimony to Congress that sophisticated hedge fund investors "perform extensive due diligence prior to investing, often taking months to research a hedge fund before making an investment." See http://www.sec.gov/news/testimony/052203tswhd.htm

The Model

Consider an economy with two types of hedge funds: good hedge funds (G) with per period after-fee returns $r_G \sim N(\mu_G, \sigma_G^2)$, and bad hedge funds ($B$) with per period after-fee returns $r_B \sim N(\mu_B, \sigma_B^2)$, where $\mu_G > \mu_B$ and $\sigma_G^2 < \sigma_B^2$. At each time period new hedge funds are born. A fraction φ of new funds are good and $(1 - \varphi)$ are bad. New funds either receive an investment of one unit of capital or they exit the market. Funded hedge funds produce a return for one period. At the end of that period their qualities are revealed. Investors would like to add money to funds revealed to be good, but these funds are closed to new investment. Investors withdraw money from bad funds which then exit the market. Good funds live one more period before retiring.

There are two types of investors in the economy. They are either skilled (S) or unskilled (U) at evaluating the quality of hedge funds. The probability that a skilled or unskilled investor evaluates the quality of a hedge fund correctly is θ_S and θ_U respectively, with $\theta_S > \theta_U \geq 0.5$. Thus, better skilled investors are more likely to know the true quality of the hedge funds. Let λ equal the fraction of new investors who are skilled at finding investments, and $(1 - \lambda)$ equal the fraction of new investors who are unskilled. At each time period investors evaluate hedge funds until they find one that they think is a good fund and invest. Therefore, conditional on their level of skill, investors will invest in good funds with probability ρ_i, where ρ_i is given by:

$$\rho_i = \varphi\theta_i / (\varphi\theta_i + (1 - \varphi)(1 - \theta_i)).$$

Unskilled investors can become better with time. We assume that after one period a fraction χ of unskilled investors become skilled. We solve the model for a steady state equilibrium that requires the assumption that $\chi = \lambda(\rho_G - \rho_B)/(1 - \rho_B)$. All results hold without this assumption but all solutions would be time dependent.

Investors invest for two periods and then consume. They have the same mean-variance utility over final wealth: $U = \mathrm{E}(r_p) - \frac{\gamma}{2}\,\mathrm{var}(r_p)$, where r_p is the two period return of the investor's portfolio of good and bad hedge funds, and γ is the investor's coefficient of risk aversion. There is also a riskless asset normalized to have a zero return.

In Figure 1, we pictorially represent the steady-state equilibrium of the model, where the universe of hedge funds includes good hedge funds that have survived, but are closed to new investment, and new good and bad hedge funds that have just arrived and have received funding. We are particularly interested in the expected return and variance of the average hedge fund in the market and the expected utility of the unskilled investors. We examine two cases: an economy where no funds-of-funds are available and unskilled investors must invest directly in hedge funds, and an economy with funds-of-funds through which the unskilled investors can channel their hedge fund investments.

Case of No Funds-of-Funds

Our first case is an economy where no funds-of-funds are available to unskilled investors. Thus, both skilled and unskilled investors directly invest in hedge funds based on their own ability θ_S and θ_U to select good funds. It is easy to show that, in steady state, the average hedge fund $(\overline{h})$ in the economy has an expected return and variance of:

$$\begin{aligned} \mathrm{E}(r_{\overline{h}}) &= 2[\lambda\rho_S + (1-\lambda)\rho_U]\,\mu_G + [1 - \lambda\rho_S - (1-\lambda)\rho_U]\,\mu_B \\ \mathrm{var}(r_{\overline{h}}) &= 2[\lambda\rho_S + (1-\lambda)\rho_U]\,\sigma_G^2 + [1 - \lambda\rho_S - (1-\lambda)\rho_U]\,\sigma_B^2, \end{aligned} \tag{1}$$

and the utility of the unskilled investor in each two-period economy is:

$$\begin{aligned} U_U &= \mathrm{E}(r_p^U) - \frac{\gamma}{2}\,\mathrm{var}(r_p^U) \\ &= 2\rho_U\left(\mu_G - \frac{\gamma}{2}\,\sigma_G^2\right) + (1-\rho_U)\left(\mu_B - \frac{\gamma}{2}\,\sigma_B^2\right), \end{aligned} \tag{2}$$

where r_p^U is the unskilled investor's return on direct hedge fund investments.

Case with Funds-of-Funds

In the second case, we introduce funds-of-funds that the unskilled investors can access. We assume that the funds-of-funds' managers have the same ability as the skilled investors, and thus evaluate fund quality correctly with probability θ_S. Naturally, this allows the previously unskilled investors with ability $\theta_U < \theta_S$ to now have the same ability as the skilled investors. In the steady-state equilibrium of this economy, the average hedge fund $(\overline{h})$ has an expected return and variance of:

$$\begin{aligned} \mathrm{E}(r_{\overline{h}}^*) &= 2\rho_S\,\mu_G + (1-\rho_S)\,\mu_B \\ \mathrm{var}(r_{\overline{h}}^*) &= 2\rho_S\,\sigma_G^2 + (1-\rho_S)\,\sigma_B^2, \end{aligned} \tag{3}$$

where we use the asterisk to denote the economy with funds-of-funds. Since by assumption $\mu_G > \mu_B$ and $\sigma_G^2 < \sigma_B^2$, it is straightforward to show that:

$$\mathrm{E}(r_{\overline{h}}^*) > \mathrm{E}(r_{\overline{h}}) \qquad \text{and} \qquad \mathrm{var}(r_{\overline{h}}^*) < \mathrm{var}(r_{\overline{h}}). \tag{4}$$

Thus, the existence of funds-of-funds alters the return distribution of funded hedge funds in the economy.

If we assume that a fund-of-funds manager charges a fixed fee f in percentage of capital gains, then the utility of unskilled investors in each two-period economy is:

$$\begin{aligned} U_U^* &= \mathrm{E}(r_p^{U^*}) - \frac{\gamma}{2}\,\mathrm{var}(r_p^{U^*}) \\ &= 2\rho_S \left[(1-f)\,\mu_G - \frac{\gamma}{2}\,(1-f)^2\sigma_G^2\right] \\ &\qquad +(1-\rho_S)\left[(1-f)\,\mu_B - \frac{\gamma}{2}\,(1-f)^2\sigma_B^2\right], \end{aligned} \tag{5}$$

where $r_p^{U^*}$ denotes the after-fee return of the unskilled investor that uses a fund-of-funds.

Discussion

While simple, this model illustrates three important points in comparing the returns of hedge funds and funds-of-funds:

Point 1 *The expected after-fee return of the average fund-of-funds investment is lower than the expected return of the average hedge fund, even though funds-of-funds' managers are skilled investors.*

In the second economy where funds-of-funds exist, all hedge fund investors in the economy are either skilled individual investors or fund-of-funds managers. They have the same ability to evaluate hedge funds, and thus earn the same expected returns before fees. But, the after fee-on-fees returns of funds-of-funds are lower. Thus, to evaluate funds-of-funds, it is not meaningful to directly compare the average after-fee returns of funds-of-funds to hedge funds, because by doing so, we are comparing the returns to unskilled investors (through funds-of-funds) with the returns that skilled investors could achieve. Instead, we must compare the gains from funds-of-funds with the gains that the same investors would have fared with direct hedge fund investments if funds-of-funds did not exist. That is, we need to compare the utility of the unskilled investors when no fund-of-funds exists (U_U) with their utility in the presence of funds-of-funds and their added fees (U_U^*).

Point 2 *Unskilled investors can potentially increase their utilities by investing through a fund-of-funds even though the average fund-of-funds do not outperform the average hedge fund.*

In the first economy where the unskilled investors directly invest in hedge funds without the benefit of a fund-of-funds intermediary, they receive a lower expected return and higher variance than the skilled investors, and thus an inferior utility. In the second economy where funds-of-funds exist, the unskilled investors earn the same before-fee expected return and utility as the skilled investors. Thus, as long as the fees of funds-of-funds are low enough, then unskilled investors increase their utility by using funds-of-funds, i.e., $U_U^* > U_U$. Hence, it is wrong to conclude that funds-of-funds are not adding value just by comparing after-fee average returns. In our example, funds-of-funds add value, but they produce lower after-fee returns than hedge funds.

Importantly, the presence of funds-of-funds also alters the distribution of funded hedge fund returns from equation (4). Observe that in the economy with funds-of-funds, the distribution of all funded hedge funds returns in the economy is better (with higher mean and lower variance) than the distribution of hedge fund returns when no fund-of-funds exists.[7]

Point 3 *In our theoretical model, we can directly compare the utility of less skilled investors in an economy with and without funds-of-funds. However, in reality we only see the data from the economy that includes funds-of-funds.*

In data, we only observe funded hedge funds that receive investments either through expert funds-of-funds or by sophisticated, skilled individuals. If individual investors all invested directly in hedge funds without funds-of-funds, then the set of funded hedge funds would be much worse than what we observe in data. This causes the observable returns of hedge funds to not represent the full, true distribution of hedge fund returns. It is plausible that in the hedge fund data that we observe, the left-hand tail of the true hedge fund distribution is truncated or alleviated, since many bad hedge funds are not funded! Thus, in reality we cannot do a direct comparison of investors' utilities with and without funds-of-funds.

This funding bias of existing hedge fund databases is very different from the survivorship or reporting bias discussed in the literature. Many successful hedge funds do not report to hedge fund databases (see Ackermann et al., 1999), making observed hedge fund returns biased downwards. On the other hand, Malkiel and Saha (2004) argue that many unsuccessful hedge funds, which ultimately fail, stop reporting to the hedge fund databases, which causes hedge fund returns to be biased upwards. The bias we discuss here is different from these survivorship biases because these biases still involve whether hedge funds that have received funding report, or do not report, to a database. Our bias is that we never observe the hedge funds that do not receive funding, but would have if funds-of-funds did not exist. It is this unobserved set of unfunded hedge funds, together with the observable set of funded hedge funds, that constitutes the true fund-of-funds benchmark.

To characterize the correct benchmark for funds-of-funds, we ask the following indirect question: What would investors have to believe about their own ability to invest in hedge funds

[7] It is interesting to note that good hedge funds should be strong supporters of funds-of-funds as it increases the probability that they can obtain capital, while bad hedge funds would not be in favor of the additional scrutiny of a fund-of-funds.

in order to make funds-of-funds a good idea? Answering this question also helps us to judge whether funds-of-funds add value.

The Value of Adding a Fund-of-Funds

In our model, if the discrepancy between the selection ability of the skilled and unskilled investors, θ_S and θ_U, is large enough, then the unskilled investors are better off investing through a fund-of-funds. The marginal θ_U for the unskilled investor to prefer a fund-of-funds is given by:

$$\theta_U^* = \frac{\rho_U^*(1-\varphi)}{(1-\rho_U^*)\varphi + \rho_U^*(1-\varphi)} \tag{6}$$

where

$$\rho_U^* = \frac{U_U^* - (\mu_B - \frac{\gamma}{2}\sigma_B^2)}{2(\mu_G - \frac{\gamma}{2}\sigma_G^2) - (\mu_B - \frac{\gamma}{2}\sigma_B^2)}$$

and U_U^* is given in equation (5).

In Table 1, we give an example to make our point clearer. We compute the break-even skill level, θ_U^*, where unskilled investors prefer funds-of-funds over direct investments in hedge funds. We assume that the mean and variance of the good and bad hedge funds are $\mu_G = 25\%$, $\sigma_G = 10\%$, and $\mu_B = 15\%$, $\sigma_B = 15\%$, respectively. We consider three different cases of the probability that a skilled investor evaluates hedge funds correctly, $\theta_S = 0.9, 0.8, 0.7$, and three different fee schedules f charged by the funds-of-funds, $f = 5\%, 10\%, 15\%$. We set the fraction of good hedge funds $\varphi = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and set the risk aversion of the unskilled investor at $\gamma = 4, 8, 12$. If the unskilled investor believes that she can correctly judge hedge funds with probability less than the θ_U^* reported in the table, then she is willing to use a fund-of-funds. Otherwise, she would prefer invest in a hedge fund directly. The larger is θ_U^*, the more likely the unskilled investor will choose a fund-of-funds, because it is easier for an unskilled investor to think she has a skill level lower than θ_U^*.

Table 1 shows that an unskilled investor only needs to think that she can do a little worse than fund-of-funds managers to prefer a fund-of-funds. For example, suppose that the number of good and bad hedge funds in the economy are equal ($\varphi = \frac{1}{2}$) and funds-of-funds charge a 10% fee ($f = 10\%$). Then, if $\theta_S = 90\%$, an unskilled investor with a risk aversion of $\gamma = 8$ only needs to believe that she will evaluate hedge funds correctly with probability $0.09\,(= 0.90 - 0.81)$ less than skilled investors to prefer a fund-of-funds. As γ increases, θ_U^* also increases. This means that unskilled investors with higher risk aversion levels are more likely to use funds-of-funds. As the fraction of good hedge funds in the economy declines, θ_U^* increases. This implies that unskilled investors are more willing to choose funds-of-funds when bad hedge funds abound. The intuition behind this result is that funds-of-funds become more useful tools for unskilled investors when there are more bad hedge funds in the world because their value in screening bad hedge funds increases.

Table 1 also emphasizes that the answer to the question, "Do funds-of-funds deserve their fees-on-fees?" cannot be answered with a universal "yes" or "no." Rather, the answer depends on who is asking the question. The more skilled an investor is, the less likely she will find funds-of-funds valuable. On the other hand, a less risk-averse individual is less likely to find value in a fund-of-funds. The question whether funds-of-funds add value is investor and time specific and depends on the investor's investment opportunity set, risk aversion, and the investor's belief about her own competence.

We can also draw on an analogy to the venture capital (VC) industry to emphasize this point (see Jones and Rhodes-Kropf, 2003). An investor would rather make an investment directly in a start-up that was funded by a top VC in order to avoid the fees paid to the VC intermediary. However, the average investor who tries to directly invest in start-up companies would make very poor choices because they lack the expertise to select and monitor start-ups. Thus, the set of start-ups in data are the start-ups that are funded by venture capitalists, which appear to have high alphas (see, for example, Gompers and Lerner, 1997). An investor deciding to enter a VC fund should not compare VC fund returns with the underlying returns from VC funded start-ups, but should compare the expected profit from a VC fund investment with the investments that she could make on her own.

In our simple model, it is easy to compute the value added by funds-of-funds because we directly model the whole universe of good and bad hedge funds. But, in data, funds-of-funds and hedge funds cannot be directly compared because the true set of hedge funds is not observable. The question we now ask is how to characterize the true, unobservable distribution of hedge funds available to an unskilled investor that is the correct fund-of-funds benchmark. Fortunately, we have the revealed preferences of investors who have already chosen to invest in funds-of-funds. This involves a portfolio allocation decision. We now show how a standard portfolio allocation framework can infer characteristics of the true, benchmark fund-of-funds distribution.

3 The Portfolio Allocation Problem

To characterize the true fund-of-funds benchmark, we employ certainty equivalent concepts from portfolio allocation theory. We assume that the investor has a standard mean-variance utility function:

$$U = \mathrm{E}(r_p) - \frac{1}{2}\gamma \, \mathrm{var}(r_p), \tag{7}$$

where r_p is the portfolio return, which is a function of portfolio weights in a risk-free asset and risky assets, and γ is the investor's coefficient of risk aversion. We choose mean-variance utility as it is the standard benchmark utility function and work with risk aversion levels of $\gamma = 4$, 8, and 12. Since it is well known that unconstrained mean-variance positions are sensitive to expected returns and can produce extreme portfolio positions (see, among others, Green and Hollifield, 1992; Black and Litterman, 1992), in addition to an unconstrained optimal portfolio, we also examine no short-sale constraints, as well as a shorting limit of -20%.

Hedge fund strategies typically generate option-like returns and have payoffs that depend on higher moments (see, for example, Fung and Hsieh, 2001). The mean-variance utility in equation (7) does not consider the effect of higher moments. Using utility functions where investors weight losses more than gains (like the first-order risk aversion utility function of Gul, 2001) would produce lower portfolio allocations in both hedge funds or funds-of-funds. While we do examine the proportion of the portfolio in hedge funds or funds-of-funds (see below), our focus is on the utility gain of adding a hedge fund compared to the utility gain from adding a fund-of-fund. Using more complex utility functions that depend on higher moments would only favor positions in funds-of-funds because by diversification, funds-of-funds are able to reduce the extreme movements of individual hedge funds and thus have lower volatility. Hence, using mean-variance utility to characterize the true benchmark for funds-of-funds is a conservative choice, and utility functions that take into account downside risk or higher moments would make funds-of-funds even more attractive relative to hedge funds.

The value of an investment in a fund-of-funds in utility terms depends on the current investment opportunity set. We specify different subsets of benchmark assets to include:

1. AC1: U.S. large and small stocks.

   Large and small stock returns are total returns from the Ibbotson S&P500 index and the Russell 2500 mid-to-small index, respectively.

2. AC2 = AC1 + U.S. Value and Growth Stocks.

   The value and growth returns are taken from the MSCI U.S. Large Cap Value Index and the Large Cap Growth Index, respectively.

3. AC3 = AC2 + U.S. Bonds.

   We use total returns on long-term government bonds, intermediate-term government bonds, and long-term corporate bonds, all from Ibbotson.

4. AC4 = AC3 + Commodities.

   Commodity returns are total returns on the Goldman Sachs commodity index.

5. AC5 = AC4 + Foreign Equity.

   For foreign equity, we take MSCI country returns for the U.K., Japan, Germany, and France, and the MSCI emerging market free index, expressed in U.S. dollar returns.

6. AC6 = AC5 + Foreign Bonds.

   Foreign bonds represent U.K., German and Japan 1-month Eurobond returns expressed in U.S. dollar returns.

All of the assets classes AC1-AC6 include a risk-free position in 1-month U.S. T-bills. Investors can invest in each of these benchmark assets using low-cost, index vehicles. We start by considering only an all equity position in only large and small capitalization stocks (AC1) and then progressively increase the set of assets. The full set of benchmark assets (AC6) consists of 16 risky asset positions in bonds and equities in both the U.S. and overseas markets, together with commodities.

We compute the diversification benefits, or utility gains, of adding a fund-of-funds vehicle to these different sets of basis assets. The asset allocation problem emphasizes that the optimal weight in funds-of-funds depends on the investor's risk aversion, the current investment opportunity set, and any portfolio constraints. The portfolio allocation perspective explicitly accounts for the diversification benefits of fund-of-fund investments through the variance-covariance matrix. Hence, our asset allocation approach is a natural way of evaluating funds-of-funds rather than simply computing and comparing alphas that may not result from an investable strategy by individuals.

To judge the economic magnitude of adding a hedge fund or a fund-of-funds position to a set of benchmark assets, we compute the percentage increase in the certainty equivalent (CE), similar to Kandel and Stambaugh (1996), Campbell and Viceira (1999), and Ang and Bekaert (2002), among others. The CE represents the certain amount of wealth required that is equivalent to holding the diversified position, from the perspective of the investor. That is, it is the sure monetary payment a risk-averse investor must receive in order to compensate the investor for not investing in the risky position. For example, if the CE is 9%, then 9% is the equivalent risk-free return that an investor must receive in order to compensate the investor not to hold the optimal portfolio. With mean-variance utility, the CE is simply the level of the utility function at the optimal portfolio weight. Thus, in standard mean-variance space with expected return on the y-axis and volatility or variance on the x-axis, the CE represents the point where the utility indifference curve crosses the y-axis. We use CE^* to represent the utility with the hedge fund or fund-of-funds position.

We examine the difference in CEs from adding hedge funds or funds-of-funds to a standard benchmark portfolio. If $CE^{\dagger}$ represents the optimal utility without a hedge fund or a fund-of-funds position, we can express the compensation required to forego the hedge fund or fund-of-fund position as a return in cents per dollar of wealth:

$$100 \times \left(\frac{1 + CE^{*}}{1 + CE^{\dagger}} - 1 \right), \tag{8}$$

The percentage increase in the CE is the cents per dollar amount which an individual must be compensated to give up the opportunity to invest in hedge funds or fund-of-funds.

The CE allows us to characterize the true, underlying distribution of hedge fund returns. For example, suppose that the true distribution of hedge funds has the same variance and correlations as the observed hedge fund returns in data (but different mean return). Then, we can compute the expected hedge fund return an investor would need to believe they could achieve on their own such that an investor would be indifferent between a direct hedge fund investment (with this return belief) and a fund-of-funds investment. Similarly, we can estimate the increase in risk that an investor would need to believe they face in choosing a hedge fund investment on her own, rather than using a fund-of-funds. This distribution is the true benchmark to which the marginal investor compares a fund-of-fund.

4 Hedge Fund and Fund-of-Funds Data

4.1 Description

We use the Tremont TASS database of hedge fund and fund-of-funds returns with a sample ending in September 2003. Although the first observation in the database is in February 1977, coverage of the funds in the TASS database is very thin prior to the 1990s. Hence, we focus on the period from June 1992 to September 2003, where the beginning of the sample is also the date where MSCI Growth and Value Indices become available.[8] Due to the short sample, we are also careful to conduct a series of robustness checks on the inputs to the asset allocation problem in Section 6.

[8] Prior to 1994, the TASS data backfills returns and does not include failed hedge funds. While some papers use hedge fund data post-1994 (see, for example, Fung and Hsieh, 2000; Agarwal and Naik, 2004), other papers use data prior to 1994, like Fung and Hsieh (1997, 2001), Brown and Goetzmann (2003), and Brown Goetzmann and Liang (2004). Getmansky, Lo and Makarov (2004) and Gupta and Liang (2005) use the full TASS sample. We do not directly compare hedge fund returns or funds-of-funds returns, or investigate the absolute level of hedge fund or fund-of-funds returns relative to performance benchmarks, where survivorship biases may create first-order effects. Nevertheless, all our results are tested with a series of robustness checks in Section 6.

TASS divides the funds into two groups: live and graveyard. At the end of September 2003, the database contains 4,131 hedge funds, of which 2,460 are live funds and 1,671 are graveyard funds. To be included in our sample, we require each fund to have at least 12 consecutive monthly net-of-fee returns, which removes 326 funds. We take those funds that TASS classifies as either a fund-of-funds or as a hedge fund. The hedge funds are further classified into one of nine primary categories. This removes another 110 funds. This process leaves us with 3,695 funds: 2,947 hedge funds and 748 fund-of-funds. All the returns are after-fee returns.

We do not rely on indices of hedge funds, or funds-of-funds, to estimate moments (although these are constructed by TASS). Instead, we use the entire cross-section of data to compute means, variances, and correlations. For example, to compute the representative volatility of hedge fund returns, we first compute the volatility of each individual hedge fund. Then, we report the median cross-sectional standard deviation, which serves as the volatility of a typical hedge fund return. Using all the cross-sectional data improves power and also permits us to examine an entire distribution of the inputs into the portfolio allocation problem.

Table 2 reports basic descriptive statistics of the hedge funds and funds-of-funds. We also list summary statistics of the hedge funds classified by investment styles defined by TASS, which are convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed income arbitrage, global macro, long/short equity hedge, and managed futures. A third of hedge funds (33%) follow long/short equity hedge investment strategies. Table 2 also reports details on the average incentive fee and management fee, along with the proportion of funds that have high watermark provisions. The average management fee for hedge funds (funds-of-funds) is 1.41% (1.54%). Funds-of-funds have average incentive fees approximately half the size of the average incentive fees for hedge funds, at 9.25% and 18.44% for funds-of-funds and hedge funds, respectively. Approximately a third (32%) of funds-of-funds have high watermarks, whereas the proportion of hedge funds having high watermarks is 43%.

In the last five columns of Table 2, we report various summary statistics of monthly after-fee excess returns of hedge funds and funds-of-funds. We use moments of after-fee returns in the portfolio allocation problem. For the standard deviation, skewness, and kurtosis statistics, we first compute these statistics for each individual fund, and then report the median across funds. We compute returns in excess of the 1-month U.S. T-bill risk-free rate. The median excess return for hedge funds is 0.54% per month, but only 0.32% per month for funds-of-funds. As Brown, Goetzmann and Liang (2004) comment, the 0.22% per month difference is mostly attributable to the double fee structure of funds-of-funds. Hedge fund excess returns are also more variable, at 3.88% per month, than returns on funds-of-funds, at 2.07% per month. Thus, funds-of-funds succeed in reducing the total volatility of an average hedge fund, but the overall Sharpe ratio of funds-of-funds is lower at 0.452 per annum than the Sharpe ratio of hedge funds at 0.518 per annum. Neither hedge funds nor funds-of-funds display excess skewness, and their kurtosis is below the kurtosis of a normal distribution. Thus, the tails of a normal distribution are a conservative approximation to reported hedge fund and fund-of-fund returns.

4.2 Estimating Moments of Returns

There are two important inputs of the moments of returns into an asset allocation problem with mean-variance utility: (i) the expected means and standard deviations of the asset returns and (ii) the expected correlations, or covariances, between the asset returns. Clearly, the true expectations of investors are unknown and unknowable. Thus, any portfolio allocation problem must make some guess at these expectations. Typically, this guess is based on the true historical information. In what follows, as a base case we will take historical means and medians as the true expectation. Later we will perform a sensitivity analysis on these assumptions. However, it is important to remember that the magnitude of the inputs is not very important to our study as we are interested in *comparing* funds-of-funds to direct hedge fund investments. Thus, the fact that many have argued that hedge funds outperformed expectations over our sample period is not relevant since funds-of-funds were impacted by the same surprise. Thus, our evaluation technique does not suffer from the 'error magnification' problems typical of optimal portfolio analysis.[9]

Means and Standard Deviations

As a baseline case, we take the median average excess return and the median standard deviation across all hedge funds or funds-of-funds, which represent the statistics of a typical hedge fund and a typical fund-of-funds. Later, we perform sensitivity analysis using the cross-sectional distribution of the expected returns and standard deviation statistics.

Asness, Krail and Liew (2001) demonstrate that the returns of hedge funds are affected by nonsynchronous price movements due to illiquid securities or lagged reporting. Lo and MacKinlay (1990) show that in their model of nonsynchronous trading, the means of asset returns are unaffected and the standard deviations of individual asset returns are biased upwards. This upward bias causes the mean-variance asset allocation problem to produce conservative estimates of the risky positions, particularly in hedge funds or funds-of-funds. Like the variances, the correlation between hedge fund or fund-of-funds returns with other asset classes

[9] The central way in which the inputs matter is if the expectations are such that no investor wishes to invest in either hedge funds or funds-of-funds. Under this scenario, we cannot compare the relative benefit of one non-optimal asset class to another non-optimal asset class.

is sensitive to nonsynchronous trading effects. However, the effect of the bias on the mean-variance optimization is ambiguous, so to produce estimates of correlations that are robust to nonsynchronous reporting lags, we introduce a methodology building on Dimson (1979).

Correlations

We compute correlations of hedge fund and fund-of-funds returns using Dimson (1979) betas. For each asset k and each fund i, we run a series of monthly excess returns on both contemporaneous and lagged asset returns (using up to $m = 1, \ldots, 6$ lags):

$$r_t^i = \alpha_k^i + \beta_{k,0}^i r_{k,t} + \beta_{k,1}^i r_{k,t-1} + \cdots + \beta_{k,m}^i r_{k,t-m}, \tag{9}$$

where r_t^i is the excess return of the ith fund and $r_{k,t}$ is the excess return of the kth benchmark asset. If non-synchronous trading exists, then the lagged betas $\beta_{k,m}^i$ are non-zero. The Dimson beta for the ith fund with respect to asset k, $\hat{\beta}_{k,m}^i$, is the sum of the $\beta_{k,m}$ betas across the lags:

$$\hat{\beta}_{k,m}^i = \beta_{k,0}^i + \beta_{k,1}^i + ... + \beta_{k,m}^i. \tag{10}$$

We compute the correlation implied by the Dimson beta, $\hat{\rho}_{k,m}^i$, as:

$$\hat{\rho}_{k,m}^i = \hat{\beta}_{k,m}^i \cdot \sigma_k / \sigma_i, \tag{11}$$

where σ_k is the standard deviation of asset k, and σ_i is the standard deviation of fund i. Since the beta estimates computed using the Dimson correction are closer to the true betas, the correlation estimates in equation (11) should provide more accurate estimates of the true correlations between fund returns and asset returns. We use the median cross-sectional correlation across all funds indexed by i in our portfolio allocation analysis.

Table 3 reports the Dimson-adjusted correlations of hedge fund and fund-of-funds returns with the benchmark assets. We report the results with three lag adjustments because the implied correlations are almost unchanged with more additional lags. There are several interesting results in Table 3. First, consistent with Asness, Krail and Liew (2001), the Dimson-adjusted correlations for both hedge funds and funds-of-funds increase in absolute value with the number of lags. For example, the correlation of fund-of-funds returns with U.S. large cap stocks more than doubles from 0.26 with no Dimson lags to 0.62 when three Dimson lags are included. Thus, not taking into account the Dimson lags over-states the diversification benefits of hedge funds or funds-of-funds.

Second, consistent with Capocci and Hubner (2004) and others, the signs of the correlations with the benchmark asset classes are the same for both hedge funds and funds-of-funds. For example, both hedge funds and funds-of-funds have, on average, positive correlations with both U.S. and foreign equities, but negative or zero correlations with U.S. and foreign bonds. However, the median absolute values of the correlations of funds-of-funds are always larger in magnitude than those of hedge funds. For example, accounting for the Dimson corrections, the correlation of hedge funds with U.S. value stocks (growth stocks) is 0.33 (0.35), while the correlation of fund-of-fund returns with U.S. value stocks (growth stocks) is 0.49 (0.57). Fung and Hsieh (2002) also report that individual hedge funds have lower correlations in absolute value to traditional asset classes than fund-of-funds returns. Thus, all else being equal, individuals should prefer to add a hedge fund, rather than a fund-of-funds, to their portfolio because of the lower correlations of hedge funds.

Baseline Case

In Table 4, we summarize the expected returns, standard deviations, and correlations for the baseline case of inputs for the asset allocation problem. The means and standard deviations of the benchmark assets are simple historical averages and volatilities over the June 1992 to September 2003 sample period. To obtain a total expected return for hedge funds and funds-of-funds, we combine the average excess returns reported in Table 2 with a risk-free rate assumption of 4.00%. The last two columns of the table repeat the correlations of hedge funds and funds-of-funds reported in Table 4 using three Dimson (1979) lags.

To give an idea of the diversification benefits available from adding a hedge fund or a fund-of-funds, Figure 2 plots the mean-variance frontiers generated by AC6 as well as adding a hedge fund or a fund-of-funds investment to AC6. When target expected returns are relatively low (below 0.75% per month), adding a fund-of-funds generates a lower volatility, so highly risk-averse investors already prefer funds-of-funds over an average hedge fund. For target expected returns above 0.75% per month, hedge funds generate a superior mean-variance frontier with lower volatility. Figure 2 shows that there are clearly diversification benefits in adding hedge funds or funds-of-funds to an already well-diversified portfolio. We now examine the actual hedge fund and fund-of-funds asset allocations implied by the mean-variance utility function (7) and characterize the true fund-of-funds benchmark from the certainty equivalents of the optimal asset allocation weights. We examine the sensitivity of our results to changes in the expected returns, volatilities, and correlations in Section 6.

5 Empirical Results

In Section 5.1, we start by showing that well-diversified portfolios contain significant holdings of hedge funds or funds-of-funds. Section 5.2 computes the utility gains of investing in hedge funds or funds-of-funds. In Section 5.3, we estimate characteristics of the unobserved, true fund-of-funds benchmark.

5.1 Optimal Holdings of Hedge Funds or Funds-of-Funds

We begin by reporting the optimal allocation to a median hedge fund from the mean-variance optimization problem in equation (7). Panel A of Table 5 reports the optimal asset allocations combining hedge funds with asset classes AC1, AC2, or AC6 for several levels of risk aversion. We report the allocations imposing no constraints, prohibiting short sales, and allowing short positions down to -20%. Panel B reports the asset positions when a typical fund-of-funds is added to the mix of benchmark assets.

The message of Table 5 is that there are non-negligible allocations in hedge funds and funds-of-funds in an investor's portfolio. In Panel A, the holdings of hedge funds are large for all the benchmark asset classes and the holdings of hedge funds are very similar across the asset classes. For example, for an investor only invested in U.S. equity in AC1, an investor with a risk aversion level of $\gamma = 8$ invests 30% of her portfolio in hedge funds. When we consider the full range of assets in AC6, the allocation to hedge funds is 45% for an unconstrained portfolio for a risk aversion of $\gamma = 8$. In this case, an investor holds many leveraged positions, particularly in U.S. intermediate-term bonds and U.S. equity. When short sales are prohibited, the allocation to hedge funds remains substantial, at 19% for a $\gamma = 8$ investor. When short positions down to -20% are permitted, the hedge fund allocations range from 43% to 22% for risk aversions $\gamma = 4$ to $\gamma = 12$, respectively.

Similarly, Panel B of Table 5 shows that investors optimally hold funds-of-funds. For the unconstrained portfolio allocation, the investor holds even larger positions in funds-of-funds compared to hedge funds in Panel A. For an unconstrained allocation over asset class AC6 and funds-of-funds, a $\gamma = 8$ investor places 247% of her portfolio into funds-of-funds. For the full AC6 position with no shorting permitted, an investor with a sufficiently high risk aversion holds funds-of-funds, with holdings of 8% and 24% for risk aversion levels of $\gamma = 8$ and $\gamma = 12$, respectively. Note that with no shorting, the $\gamma = 4$ investor with a base investment opportunity set of AC6 does not hold any fund-of-funds. Thus, any comparison of whether funds-of-funds add value relative to hedge funds to this investor cannot be answered. We construct a benchmark fund-of-funds distribution only for cases where investors optimally hold funds-of-funds.

While some of the asset positions from mean-variance utility are extreme and may not mirror actual portfolio decisions, Table 5 demonstrates that hedge funds or funds-of-funds play a role in an optimal portfolio. What is important is not the short, or leveraged, positions in other assets, but the fact that it is optimal to add hedge funds or funds-of-funds into a portfolio and that the hedge fund and funds-of-funds positions are positive. In using the asset allocation framework to infer the true, unobserved distribution of hedge fund returns, we concentrate on the allocations not allowing short sales and only permitting short sales down to -20%. Naturally, the gains of adding funds-of-funds in unconstrained portfolios are dramatically larger, so our estimates of the value-added benefits from funds-of-funds are conservative.

While we have used expectations that are the realized returns, it is important to remember that our study does a comparison using utilities of holding funds-of-funds. We emphasize that we are not arguing that the great realized performance of the hedge fund or fund-of-funds class was expected. Furthermore, we do not need such high expected returns to make our point. We only need inputs such that investors choose to allocate some of their portfolios to funds-of-funds. Thus, this section shows that with the historical averages as inputs, an optimal portfolio does contain funds-of-funds, and we can proceed with our comparison.

5.2 The Utility Gain in Adding Hedge Funds or Funds-of-Funds

In Table 6, we report the annualized CEs that a mean-variance investor could obtain by investing in benchmark asset classes (AC1 to AC6) alone, or adding a hedge fund or a fund-of-funds position to the asset classes (AC1 to AC6). Panel A reports the level of the CEs, while Panel B reports the percentage increases in certainty equivalents from adding a hedge fund or a fund-of-funds to the existing benchmark asset classes (see equation (8)). We focus our discussion on a risk aversion level of $\gamma = 8$, but we also report the cases for $\gamma = 4$ and 12 in the table.

The numbers reported in Panel A represent the risk-free return that is equivalent to holding the asset class, or the asset class combined with a hedge fund or a fund-of-funds. For example, the $\gamma = 8$ investor would be indifferent between an optimal position in U.S. large and small stocks (AC1) and a certain return of 5.63% per annum. To give up an optimal portfolio comprising large and small stocks together with hedge funds (funds-of-funds), the same investor would require a risk-free return of 6.15% (6.06%). Combining the full set of assets, AC6, with a hedge fund (fund-of-funds) position is equivalent to a risk-free return of 8.44% (8.21%). These are much higher than the risk-free rate assumption of 4%.

The increases in the CEs in Panel B represent the gain in utility that an investor realizes from moving from a low-cost benchmark portfolio (AC1-6) to adding hedge funds or funds-of-funds. For example, the utility gain in adding hedge funds (funds-of-funds) to AC1 is 0.49% (0.41%) for a $\gamma = 8$ investor. Since the hedge fund and fund-of-funds returns are after-fee returns, these represent risk-free gains of 0.49% and 0.41% per annum in opening up the investment opportunity set from U.S. large and small stocks to including hedge funds and funds-of-funds, respectively. In Panel B, the gains of adding hedge funds are generally larger than the gains of adding funds-of-funds. For example, starting from AC6, and allowing shorting down to -20%, a $\gamma = 8$ individual increases her utility by 0.47% when adding hedge funds, compared to 0.36% for adding funds-of-funds. When no short sales are permitted, the preference for hedge funds is even greater, with percentage utility gains of 0.23% (0.02%) for hedge funds (funds-of-funds).

However, the higher utility gains for adding hedge funds compared to adding funds-of-funds do not occur for all levels of risk aversion. In Panel B with AC6, a $\gamma = 12$ investor prefers to add a fund-of-funds rather than a hedge fund (with percentage CE gains of 0.39% and 0.46% for hedge funds and funds-of-funds, respectively). In Figure 3, we plot the certainty equivalents of AC6 in relation to adding hedge fund investments or funds-of-funds to AC6. In the top panel, we do not permit short sales and in the bottom panel we allow short positions down to -20%.

Figure 3 shows that for low levels of risk aversion, adding a hedge fund to AC6 dominates adding a fund-of-funds. This is consistent with the mean-variance frontiers in Figure 2. However, for risk aversion levels above 20 when shorting is not allowed, the utility gain of adding a fund-of-funds is higher than the utility gain of adding a hedge fund. When -20% short positions are permitted, funds-of-funds dominate hedge funds for risk aversion levels above 10. This is because funds-of-funds have lower expected returns and lower standard deviations than hedge funds. Consequently, less risk-averse investors prefer the higher returns of hedge funds and tolerant the additional volatility, while for more risk-averse investors, the lower volatility of funds-of-funds dominates.

Hence, sufficiently risk-averse investors prefer to hold funds-of-funds rather than directly invest in the hedge fund returns that we see in the data. Since the true distribution of hedge funds must be worse than the observed distribution of hedge fund returns, these high risk-averse individuals do not even need to think that they would do worse on their own direct hedge fund investments compared to a fund-of-funds in order to choose a fund-of-funds. Thus, from a portfolio perspective, funds-of-funds already provide preferred vehicles for individuals with very high risk aversion levels. To determine if low or moderate risk-averse investors should utilize funds-of-funds, we need to estimate the characteristics of the hedge funds these investors could access on their own. We now recover characteristics of the true, underlying hedge fund distribution using revealed preference.

5.3 Characterizing the Benchmark Fund-of-Funds Distribution

In Tables 7 to 9, we characterize moments of the true, benchmark fund-of-funds distribution which an unskilled investor could access. We cannot directly observe this distribution but we can characterize it using revealed preference arguments. We denote the mean of the benchmark distribution as μ_B and the standard deviation as σ_B.[10]

We know that an unskilled investor who is forced to directly invest in a hedge fund will face a direct hedge fund distribution at least as bad as the distribution of funded hedge funds in data (see equation (4) and Point 2 of the model). Hence, the benchmark distribution must have an expected return no greater than the hedge fund expected return in data, and the standard deviation of the benchmark distribution must be at least as volatile as the standard deviation of hedge fund returns in data. Since the observable median hedge fund average return and standard deviation are 0.873% and 3.876% per month, respectively, it must be the case that $\mu_B \leq 0.873\%$ and $\sigma_B \geq 3.876\%$.

We characterize the true universe of hedge funds that an individual may face by asking what would make an investor indifferent between investing in the benchmark distribution, which is unobserved, and investing in a typical fund-of-funds, which is observed. At the margin, the ex-ante utility of adding a hedge fund drawn from the true, benchmark fund-of-funds distribution must be the same as the ex-ante utility of making a fund-of-funds investment. Hence, we characterize the benchmark distribution by estimating moments which induce the same utility as adding a fund-of-funds.

Characterizing the Mean

In Table 7, we characterize the mean of the distribution of both funded and unfunded hedge funds (the true fund-of-funds benchmark distribution), μ_B, by making various assumptions on σ_B. Case 1, which assumes that $\sigma_B = 3.876\%$, is the most conservative case because we expect that the real σ_B is greater than the hedge fund volatility in data. In Cases 2 and 3, we assume that the true $\sigma_B = 1.1 \times 3.876\%$ and $\sigma_B = 1.2 \times 3.876\%$, respectively. The table reports the estimated μ_B such that an investor is indifferent between adding a hedge fund from the true, underlying hedge fund distribution and adding a typical fund-of-funds. Empty entries indicate that it is not optimal for an investor to hold any funds-of-funds, so constructing a benchmark distribution is not meaningful for these investors. Entries in italics indicate that an investor already prefers a fund-of-funds, rather than a direct hedge fund investment. These investors would have to think they would do substantially *better* than the average hedge fund if they invested on their own in order for them not to use a fund-of-funds.[11]

[10] Naturally, if the benchmark also has other important moments such as pronounced higher downside moments, it only makes funds-of-funds more attractive because diversification in funds-of-funds alleviates some of the effects of lower left-hand tail outliers.

[11] The repeated entries for the columns under the no short sale case occur because the assets in AC3 are the same ones held in AC4-AC6. For example, Panel B of Table 5 shows that there are zero holdings in commodities, international equity, and international bonds for AC6 for no short sales.

The entries in normal type in Table 7 report the highest possible μ_B that the investor could believe she would obtain on a direct hedge fund investment from the true hedge fund universe in order to choose a fund-of-funds. The median observable hedge fund return in data is 0.873% per month. A $\gamma = 8$ investor starting from AC6 facing short sale constraints has only to think that she would obtain 0.710% or less per month on average, or a reduction of 1.96% per annum compared to the observable hedge fund returns, on her own hedge fund investments for her to prefer to use a fund-of-funds. When the $\gamma = 8$ investor starting from AC6 can only short down to -20%, she only needs to believe that she would obtain investments $12 \times (0.873 - 0.837)\% = 0.43\%$ per annum worse on average by investing directly in hedge funds in order to prefer funds-of-funds. Note that these performance 'reductions' also include the costs of finding, allocating and overseeing the hedge fund investments if a fund-of-funds is not employed. Thus, investors who have a higher cost structure would find funds-of-funds even more attractive.

If the true hedge fund distribution is more disperse, like Cases 2 and 3, many more investors would already prefer to directly invest in funds-of-funds rather than hedge funds, as indicated by the greater number of cells in italic font. Hence, more realistic assumptions for true underlying σ_B only increase the preference for funds-of-funds. In Case 3, $\sigma_B = 1.2 \times 3.876\%$, an AC6 investor with $\gamma = 8$ has a benchmark funds-of-funds distribution with a mean of 0.752% when no short sales are permitted. Funds-of-funds should be benchmarked against this mean. Thus, to select a fund-of-funds, the investor must face an average return of only 1.02% per annum worse than the observable median hedge fund return. For a modest amount of -20% shorting, the same investor already prefers a fund-of-funds position. Thus, investors do not have to believe that they would perform very poorly on their own (or have very high costs) in order to prefer a fund-of-funds.

Characterizing Volatility

In Table 8, we infer the benchmark standard deviation (the standard deviation an investor would face if she invested on her own), σ_B, while making assumptions about μ_B. Similar to Table 7, entries in italics indicate the cases where investors already prefer to invest in funds-of-funds, so the inferred σ_B values are lower than the median volatility of hedge fund returns in data (3.876% per month). Case 1, where $\mu_B = 0.873\%$ per month, is the most conservative choice because the true μ_B must be less than or equal to the median 0.873% average return of hedge funds in data. For investors with $\gamma = 8$ who do not already prefer funds-of-funds, σ_B ranges from 3.924% to 6.839% per month. Hence, in order for a fund-of-funds to add value, these investors must believe that they would choose hedge funds that have volatilities up to 1.8 times higher than the hedge funds observed in data. Thus, funds-of-funds can substantially reduce the expected volatility of direct hedge fund investments for some investors.

In Case 3, we assume that $\mu_B = 0.765\%(= R_f + 0.8 \times 0.54\%)$ per month. With this assumption, most investors already prefer funds-of-funds over hedge funds. Nevertheless, if $\mu_B = 0.765\%$, the $\gamma = 8$ investor holding AC6 who is not permitted to short believes that her direct hedge fund investments have at least a monthly standard deviation of $4.890\% = 1.26 \times 3.876\%$ per month, whereas the typical hedge fund volatility is 3.876% per month. Thus, she would still assign high value to a fund-of-funds. When shorting down to -20% is permitted, the $\gamma = 8$ investor already prefers a fund-of-funds investment over a hedge fund.

Characterizing Left-Hand Tails

We can also view the value-added benefits of funds-of-funds as their ability to discriminate between hedge funds and filter out very bad hedge funds. Specifically, we ask what is the minimum fraction of the left-hand tail of the true hedge fund distribution that a smart fund-of-funds manager must be able to avoid to add value. To perform this analysis, we assume that the underlying distribution of both funded and unfunded hedge funds r_B is normally distributed, $r_B \sim N(\mu_B, \sigma_B^2)$, where σ_B is assumed to be $3.876\%, 1.1 \times 3.876\%$ and $1.2 \times 3.876\%$, respectively, and μ_B is the corresponding mean of the full hedge fund distribution that we calculated in Table 7. The assumption of normality is in the spirit of the mean-variance optimization problem. We assume that the fund-of-funds can truncate the left-hand tail of the distribution of the true universe of hedge funds, and we find the truncation point where an investor is indifferent between choosing a hedge fund from the truncated distribution and a fund-of-funds. In this analysis, we hold the correlations constant at the values in Table 4, but the truncation point alters the mean and variance of the hedge fund distribution open to funds-of-funds (which become the mean and variance of a truncated normal distribution).

Table 9 reports the results. In the conservative Case 1, where $\sigma_B = 3.876\%$ per month, for a $\gamma = 8$ investor starting from AC6 investing in a fund-of-funds is equivalent to removing at least 1.67% of the lower left-hand tail when short sales are not permitted or 0.30% of the lower left-hand tail when shorting down to -20% is allowed. An alternative description is that an investor only requires a belief that the typical fund-of-funds has the skill to remove at least the bottom 1.67% (0.30%) of the full distribution of underlying hedge funds in order for funds-of-funds to add value. It does not seem difficult to believe that funds-of-funds can screen out the worst 0.30% of an investor's hedge fund opportunity set. If the underlying hedge fund distribution has a volatility of $\sigma_B = 1.2 \times 3.876\%$, then when no shorting is permitted, the minimum left-hand tail that an average fund-of-funds can remove is just 1.18% in order for a $\gamma = 8$ investor to believe that funds-of-funds add value. Thus, investors need only believe that funds-of-funds have the skill to screen very small amounts of the left-hand tail of the true, underlying hedge fund distribution for funds-of-funds to add value.

6 Sensitivity Analysis

Tables 7 to 9 suggest that the set of assumptions needed for funds-of-funds to add value are reasonable and plausible. However, they are produced by considering expected return, volatility and correlation inputs equal to the realized return of the median hedge fund compared to the median fund-of-funds. In this section, we explore the robustness of our conclusions to several sensitivity tests. In Section 6.1, we investigate the effect of being able to add a portfolio of hedge funds and compare the portfolio to a typical fund-of-funds. Section 6.2 examines the effects of changing the assumptions of means, volatilities, and correlation.

6.1 Comparing Hedge Fund Portfolios with Funds-of-Funds

While individual investors may only decide between adding one hedge fund or one fund-of-funds to a portfolio, institutional investors who are able to directly invest in several hedge funds often choose to invest through fund-of-funds vehicles instead. When large institutional investors can themselves diversify underlying hedge fund investments, do funds-of-funds provide any value?

To address this question, we create samples of artificial funds-of-funds, each consisting of ten hedge funds. We then compare the effects of adding an artificial fund-of-funds versus an actual fund-of-funds to AC6, which is the asset universe most relevant to a large diversified, institutional investor. We construct the artificial funds-of-funds as follows. At the beginning of each year, we randomly select ten hedge funds from data to form an artificial fund-of-funds. We equally weight these ten randomly chosen hedge funds to form a portfolio and record the monthly returns of the portfolio over the next year. We rebalance the portfolio annually.[12] This process is repeated 748 times to match the number of funds-of-funds in our sample.

Panel A of Table 10 reports the summary statistics of monthly excess returns (%) of the 748 artificial funds-of-funds, compared to the 2947 hedge funds and 748 funds-of-funds in the data. By construction, we expect that the median excess return for artificial funds-of-funds, at 0.52% per month, to be almost the same as the median hedge fund return, at 0.54% per month. This is higher than the median fund-of-funds excess return of 0.32% because we do not

[12] If a hedge fund in the artificial fund-of-funds moves into the graveyard file, we assume that the money is evenly allocated to the remaining hedge funds. Thus, our artificial funds-of-funds are conservatively upward biased and represent the highest bound for the performance of an unskilled, institutional investor.

remove additional fees in the artificial funds-of-funds. Portfolio diversification in the artificial funds-of-funds reduces the median standard deviation of hedge funds from 3.88% per month to 2.53% per month. Funds-of-funds have an even lower median standard deviation, at 2.07% per month, indicating that fund-of-funds managers, on average, obtain better risk reductions than our artificial funds-of-funds.

In Panel B of Table 10, we conduct the same three exercises in Tables 7 to 9 to characterize the mean, volatility, and left-hand-tail of the benchmark fund-of-funds distribution, respectively. We characterize the true universe of hedge funds available to an institution by asking what would make an institutional investor indifferent between investing in a median artificial fund-of-funds (that is, ten randomly selected hedge funds) and a fund-of-funds. We assume the ex-ante utility of adding an artificial fund-of-funds from the true universe of hedge funds is equal to the ex-ante utility of making a typical fund-of-funds investment to characterize the benchmark fund-of-funds distribution. The results in Panel B show that even for institutional investors who are able to invest in diversified portfolios of hedge funds themselves, the assumptions for them to prefer a fund-of-funds over direct hedge fund investments are still economically plausible.[13]

In Case 1 of Panel B, we estimate the mean of the benchmark fund-of-funds distribution by assuming $\sigma_B = 2.53\%$, $\sigma_B = 1.1 \times 2.53\%$, and $\sigma_B = 1.2 \times 2.53\%$ per month, respectively. In data, the median artificial fund-of-funds return is $0.85\% (= R_f + 0.52\%)$ per month. If the true $\sigma_B = 2.53\%$ per month, then a $\gamma = 8$ investor who can short down to -20% prefers a fund-of-funds over her own investments in a portfolio of ten random hedge funds if she believes that she would obtain at best an expected return of 0.74% per month on average on her own.[14] This is just 0.11% per month (or 1.3% per annum) lower than the median return of randomly chosen hedge fund portfolios in data. In the case where $\sigma_B = 1.2 \times 2.53\%$, the same investor need only believe that she has to do worse than 0.80% per month to prefer a hedge fund. Put another way, a typical fund-of-fund adds at least 60 basis points of value per annum compared to an investors' own randomly selected portfolio of observable hedge funds. Or, pension fund managers who think they would spend 60 basis points allocating and overseeing hedge fund investments would prefer a fund-of-fund even if they expected the same performance as the after-fee returns of a fund-of-funds.

In Case 2, we estimate the volatility of the benchmark fund-of-funds distribution by making

[13] The entries for $\gamma = 4$ investors when short sale constraint binds are empty because in these cases, the optimal portfolio allocation assigns zero weight to funds-of-funds.

[14] Some may find it odd that we talk about risk aversion for large pension fund investors. We might think these institutions are risk neutral. However, these institutions are run by individuals. Further, these individuals typically receive very low powered incentives for good performance, but can be easily fired for poor performance. This type of contract leads to extremely risk averse behavior.

assumptions on μ_B. In the most conservative case where $\mu_B = 0.853\%(= 4\%/12 + 0.52\%)$ per month, a $\gamma = 8$ investor with a shorting constraint of -20% chooses a funds-of-funds if her own portfolio of hedge funds has a volatility greater than 3.48% per month. This compares to a median artificial fund-of-funds volatility of 2.53% per month, so funds-of-funds may be able to reduce volatility by at least 37%. In Case 3, we estimate the left-hand tail proportion from the true hedge fund distribution that skilled fund-of-funds managers must be able to screen. If $\sigma_B = 2.53\%$ per month, the $\gamma = 8$ investor with a -20% short sale constraint believes a fund-of-funds adds value if funds-of-funds' managers can remove, at minimum, the worst 1.77% of hedge funds from the left-hand tail of the true hedge fund distribution. Thus, in summary, funds-of-funds are still likely to add significant value even when an investor can invest in a portfolio of hedge funds on her own.

6.2 Robustness to Changing Moments

While we are careful to use robust measures for a typical hedge fund and fund-of-funds (particularly for computing correlations), it is still possible that the reported hedge fund and fund-of-funds returns themselves suffer from various survival bias and measurement errors (see Fung and Hsieh, 2000 and 2002; Malkiel and Saha, 2004). In this section, we conduct a series of robustness checks to determine how sensitive our results are to different inputs for the moments of fund-of-funds returns. So that our results can be easily interpreted, we concentrate on characterizing the value that funds-of-funds can add to the investor's average return, relative to the average return of an unskilled individual's investment in a hedge fund. Thus, we compare our robustness exercises on characterizing μ_B of the benchmark fund-of-funds distribution to Table 7 by altering inputs about the underlying hedge fund and fund-of-funds distributions. We purposely concentrate on the most conservative case where we assume that $\sigma_B = 3.876\%$ per month, which is the median hedge fund volatility in data.

Taking advantage of the cross-sectional distribution of the 2947 hedge funds and 748 funds-of-funds, we consider six robustness scenarios, two for each of the expected returns, correlations, and volatilities. We only report the results for allowing shorting down to -20%. In each panel of Table 11, we compute the benchmark fund-of-funds expected return, μ_B, when we take the 25%-tile and 75%-tile values from the cross-sectional distribution of the various moments. In each case, we hold other inputs fixed at their original level in Table 4. Entries in Table 11 with dashes indicate that it is not optimal for an investor to hold any funds-of-funds, and entries with plus signs denote that zero positions in both hedge funds and funds-of-funds. For these risk aversion levels, constructing a benchmark distribution is not meaningful. Entries in italics indicate that an investor already prefers a fund-of-funds, rather than a direct hedge fund investment, since by investing in a fund-of-funds, the investor gains access to a benchmark universe with a higher μ_B than the observed hedge funds in data.

In Panel A of Table 11, we examine robustness of our results to changing the expected return for the fund-of-funds return relative to the expected return of hedge funds. Previously, we used the median fund-of-funds return as the input for the expected return in the mean-variance optimization (as in Table 7) and compared the benchmark fund-of-funds distribution to the median hedge fund return. What is important for our analysis is the relative difference between the median hedge fund and fund-of-fund return. To check robustness, we use the cross-section of funds-of-funds to compute a distribution for the difference in returns between funds-of-funds and the median hedge fund return. We increase and decrease the median fund-of-funds return by 25% and 75% of the cross-sectional distribution, to 0.598% and 0.708% per month, respectively. Naturally, when fund-of-funds means are higher, most investors directly prefer funds-of-funds to hedge funds.

Panel A shows that for those investors that do prefer hedge funds, a $\gamma = 4$ investor starting from AC6 needs to believe that she would only earn 0.831% per month on her own. Thus, if this investor expects to perform just 0.50% per annum worse investing on her own (or has 0.50% per annum of costs) then she prefers a fund-of-funds. When fund-of-funds expected returns are set to 0.598% per month, a $\gamma = 8$ investor starting from AC6 must believe that she would only earn 0.721% per month on her own hedge fund investments, compared to the median hedge fund return of 0.873% per month in data. Note again, that these performance 'reductions' might come from the investors costs of direct hedge fund investing as well as actual poor investment performance.

In Panel B, we change the fund-of-funds volatility to its 25%-tile and 75%-tile level in the cross-sectional distribution. These levels are 1.124% and 3.363% per month, respectively. When the fund-of-funds volatilities is at the 25%-tile level, most investors already prefer funds-of-funds. A $\gamma = 4$ investor with AC6 only needs to think they would achieve returns $0.873\% - 0.837\% = 0.036\%$ per month, or 0.43% per annum, worse than the observable hedge fund returns in data in order to invest in a low volatility fund-of-funds. At the 75%-tile volatility level, a $\gamma = 8$ investor with AC6 would choose funds-of-funds if she believes that she would earn less than 0.671% per month on average (compared to the observable hedge fund median return of 0.873% per month) on her own.

Panel C examines the effect of changing all of the correlations of both hedge funds and fund-of-funds returns with the base assets. If all correlations decrease to the 25%-tile level, a $\gamma = 8$ investor starting from AC6 prefers funds-of-funds if she thinks that, by herself, she would earn at most 0.817% per month on average. This is 0.056% per month (or 0.67% per annum) less than the median average hedge fund returns in data. The lowest bound for the average return for which the investor must do worse to prefer funds-of-funds occurs for $\gamma = 4$ from AC3 (U.S. stocks and bonds), which is still only 0.629% per month, or 2.26% per annum. For the 75%-tile correlation levels, a $\gamma = 8$ investor already prefers funds-of-funds, for those cases where comparisons are meaningful.

In summary, our results that funds-of-funds are likely to add value under assumptions that are reasonable and plausible are robust to considering different inputs to means, volatilities and correlations across the cross-sectional distribution of fund-of-funds returns. Thus, although we used realized values as the expected inputs in our base case portfolio optimizations, this is no way drives our results or conclusions.

7 Conclusion

Funds-of-funds charge comparatively large fees-on-fees that are paid in addition to the fees paid to the underlying hedge funds and the after-fee alphas and average returns of funds-of-funds are lower than the after-fee alphas and average returns of hedge funds. It is tempting to conclude from this that funds-of-funds add little value. However, we argue that this comparison is not correct and funds-of-funds should not be evaluated relative to the set of hedge fund returns we observe in data. Thus, we need to reconsider how we evaluate alternative asset classes that may be difficult to access like hedge funds and funds-of-funds.

The hedge funds that we observe in data receive money either from sophisticated, skilled investors, or from skilled funds-of-funds. An investor with no skill in locating and monitoring hedge funds would, on average, choose hedge funds that are worse than the set of hedge funds observed in data, if she were forced to directly invest in hedge funds without recourse to funds-of-funds. The existence of the fund-of-funds industry helps investors gain access to a better skill set of finding, evaluating, selecting, and monitoring hedge funds. For unskilled investors, funds-of-funds add value, even if their after-fee returns are lower than the returns of hedge funds. Thus, the correct benchmark for a fund-of-funds investor is the universe of hedge funds that she would face on her own, rather than the set of observable hedge fund returns in data.

To characterize an appropriate benchmark for fund-of-funds investments, we use revealed preference to estimate the true benchmark distribution of funds-of-funds. Specifically, we characterize the true, underlying hedge fund distribution an investor faces, by assuming that an investor is indifferent between a direct hedge fund investment and a fund-of-funds investment. Using certainty equivalent concepts from optimal portfolio allocation theory, we estimate various moments of the fund-of-funds benchmark distribution. The set of assumptions required to believe that funds-of-funds add value on average is plausible and reasonable. For example, investors need only believe that they will earn slightly worse expected returns on their own direct hedge fund investments compared to the median return of hedge funds in data (around 1% per annum lower) for funds-of-funds to add value. Furthermore, we also find that funds-of-funds are likely to add value even for institutional investors, who are able to directly invest in a diversified portfolio of hedge funds.

Recently, there has been some debate regarding the regulation of hedge funds. Much of this debate focuses on the trade-off between the potential benefits from allowing broader access to hedge funds and the potential for the abuse of unskilled, individual investors. Our analysis suggests that allowing broader access to funds-of-funds, who report more information to investors, may be the appropriate solution. In particular, even though the average after-fee returns of funds-of-funds is lower than hedge fund returns in data, funds-of-funds need only offer slight improvements in raising expected returns, lowering volatilities, or screening the lower left-hand tail of the full underlying set of hedge funds for funds-of-funds to provide considerable value to unskilled investors.

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Table 1: Unskilled Investor's Willingness to Pay Funds-of-Funds

	$\varphi = \frac{1}{4}$			$\varphi = \frac{1}{2}$			$\varphi = \frac{3}{4}$		
Fund-of-Funds Fee	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Case 1: $\theta_S = 0.9$									
$f = 5\%$	0.88	0.88	0.89	0.85	0.86	0.87	0.76	0.79	0.82
$f = 10\%$	0.85	0.86	0.88	0.79	0.81	0.84	0.65	0.69	0.74
$f = 15\%$	0.82	0.84	0.86	0.74	0.77	0.80	0.56	0.61	0.66
Case 2: $\theta_S = 0.8$									
$f = 5\%$	0.77	0.78	0.79	0.75	0.76	0.77	0.69	0.71	0.73
$f = 10\%$	0.75	0.76	0.78	0.70	0.72	0.75	0.59	0.63	0.67
$f = 15\%$	0.72	0.74	0.77	0.65	0.68	0.72	0.51	0.55	0.60
Case 3: $\theta_S = 0.7$									
$f = 5\%$	0.67	0.68	0.69	0.66	0.67	0.68	0.61	0.63	0.65
$f = 10\%$	0.64	0.67	0.69	0.61	0.63	0.66	0.52	0.56	0.59
$f = 15\%$	0.61	0.65	0.68	0.57	0.60	0.63	0.45	0.50	0.54

This table reports the marginal skill in identifying good hedge funds θ_U^* for unskilled investors to prefer a fund-of-funds. We assume the mean and variance of the good and bad hedge funds are $\mu_G = 25\%$, $\sigma_G = 10\%$, and $\mu_B = 15\%$, $\sigma_B = 15\%$, respectively. The formula for θ_U^* is given in equation (6).

Table 2: Descriptive Statistics of Hedge Funds

		Number of Funds				Fee Structures			Monthly Excess Returns %				
	Category	Dead	Live	Total	Pct.%	Manag%	Incent%	High WM%	Mean	Median	S.D.	Skew.	Kurt.
Sorted by Type	Hedge Fund	1243	1704	2947	79.8	1.41	18.44	43.2	0.58	0.54	3.88	0.05	1.53
	Funds-of-Funds	243	505	748	20.2	1.54	9.25	32.2	0.27	0.32	2.07	-0.02	1.60
	Total	1486	2209	3695	100	1.43	16.6	41.0	0.51	0.48	3.39	0.03	1.54
Sorted by Primary Strategy	Convertible Arbitrage	38	124	162	4.4	1.27	18.32	53.1	0.66	0.60	1.36	-0.22	1.20
	Dedicated Short Bias	11	17	28	0.8	1.21	19.04	50.0	0.04	0.07	6.83	0.15	1.08
	Emerging Markets	122	104	226	6.1	1.45	16.42	19.9	0.59	0.50	6.66	-0.11	2.35
	Equity Market Neutral	68	131	199	5.4	1.29	19.35	53.8	0.44	0.26	2.13	-0.09	1.00
	Event Driven	106	217	323	8.7	1.26	18.74	49.5	0.60	0.51	1.86	-0.32	2.54
	Fixed Income Arb.	62	86	148	4.0	1.25	19.35	48.0	0.38	0.40	2.00	-0.80	3.76
	Global Macro	101	88	189	5.1	1.55	17.36	34.4	0.44	0.36	4.33	0.27	1.37
	L/S Equity Hedge	435	788	1223	33.1	1.17	18.83	52.9	0.75	0.72	4.62	0.17	1.37
	Managed Futures	300	149	449	12.2	2.23	17.89	17.6	0.26	0.35	5.11	0.25	1.07

This table reports summary statistics of hedge funds and fund-of-funds in the TASS database, subdivided by fund type and primary strategy. We include all funds with at least 12 monthly observations that are classified as either a fund-of-funds or as one of the 9 hedge fund primary categories defined by TASS. The table reports the number and proportion of dead, live, and total funds in each category. For the fee structures, we report the average annual management fee, incentive fee, and the percentage of funds with a high watermark provision within each category. We calculate the mean, standard deviation, skewness, and kurtosis of monthly excess returns for each fund and report the mean and median of average excess returns, together with the median of the standard deviation, skewness, and kurtosis across funds for each category. Our sample period is from June 1992 to September 2003.

Table 3: Dimson (1979) Adjusted Correlations of Hedge Fund and Fund-of-Funds Returns

		Hedge Funds				Funds-of-Funds			
Asset Class		No lags	1 Lag	2 Lags	3 Lags	No lags	1 Lag	2 Lags	3 Lags
U.S. Equities	Large Cap	0.207	0.316	0.407	0.423	0.256	0.417	0.564	0.618
	Small Cap	0.264	0.356	0.401	0.406	0.376	0.528	0.613	0.637
	Growth	0.191	0.293	0.354	0.350	0.237	0.406	0.533	0.569
	Value	0.162	0.244	0.296	0.329	0.189	0.331	0.418	0.492
U.S. Bonds	Long-term Gov.	-0.008	-0.052	-0.079	-0.087	0.053	-0.033	-0.144	-0.152
	Inter-term Gov.	-0.048	-0.106	-0.134	-0.112	-0.023	-0.153	-0.255	-0.225
	Long-term Corp.	0.037	-0.006	-0.016	-0.026	0.125	0.050	-0.005	-0.022
Commodities		0.077	0.073	0.090	0.093	0.137	0.144	0.171	0.193
Foreign Equities	U.K.	0.177	0.264	0.345	0.382	0.224	0.372	0.497	0.561
	Japan	0.149	0.170	0.206	0.182	0.205	0.252	0.306	0.276
	Germany	0.194	0.277	0.298	0.288	0.271	0.436	0.523	0.492
	France	0.201	0.293	0.321	0.306	0.278	0.459	0.554	0.552
	Emerging Markets	0.252	0.297	0.324	0.331	0.372	0.466	0.529	0.536
Foreign Bonds	U.K.	-0.039	-0.063	-0.166	-0.170	-0.032	-0.057	-0.212	-0.251
	Germany	-0.059	-0.099	-0.138	-0.167	-0.079	-0.118	-0.189	-0.220
	Japan	0.000	-0.002	0.016	0.036	0.002	-0.030	-0.004	0.042

The table lists Dimson (1979) adjusted correlations between the returns of hedge funds and funds-of-funds and 16 benchmark assets, computed from equations (9) to (11). We report the median Dimson-adjusted correlation across all funds. The sample period is from June 1992 to September 2003.

Table 4: Input Variables for the Asset Allocation Problem

	Asset Class	Total Expected Return %	Std Dev %	Corr w/HF	Corr w/FoF
U.S. Equities	Large Cap	0.915	4.301	0.423	0.618
	Small Cap	1.044	4.993	0.406	0.637
	Growth	0.830	5.729	0.350	0.569
	Value	0.946	3.896	0.329	0.492
U.S. Bonds	Long-term Government	0.798	2.621	-0.087	-0.152
	Intermediate-term Government	0.592	1.338	-0.112	-0.225
	Long-term Corporate	0.735	2.113	-0.026	-0.022
Commodities		0.493	5.315	0.093	0.193
Foreign Equities	U.K.	0.572	4.074	0.382	0.561
	Japan	0.176	6.417	0.182	0.276
	Germany	0.615	6.314	0.288	0.492
	France	0.686	5.445	0.306	0.552
	Emerging Markets	0.485	6.749	0.331	0.536
Foreign Bonds	U.K.	0.449	2.584	-0.170	-0.251
	Germany	0.371	2.949	-0.167	-0.220
	Japan	0.238	3.544	0.036	0.042
Hedge Funds		0.873	3.876		
Funds-of-Funds		0.653	2.067		

The table reports the means and standard deviations of the benchmark assets, hedge funds (HFs), and funds-of-funds (FoFs), together with the correlations of HF or FoF returns with the benchmark assets used to solve the asset allocation problem. The mean and standard deviation moments for HFs are computed by taking the cross-sectional median across all HFs and FoFs. The HF and FoF correlations are computed taking into account the Dimson correction outlined in Section 4.2. The sample period is June 1992 to December 2003. We assume that the annual risk-free rate is 4.00% and all returns are monthly.

Table 5: Asset Allocations with Hedge Funds or Funds-of-Funds

Panel A: Hedge Fund Holdings

		No Constraints			No Short Sales			Short down to -20%		
	Asset Class	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
	U.S. Large Cap	0.21	0.11	0.07	0.14	0.11	0.07	0.21	0.11	0.07
AC1	U.S. Small Cap	0.38	0.19	0.13	0.38	0.19	0.13	0.38	0.19	0.13
	Risk-free Asset	-0.19	0.40	0.60	0	0.40	0.60	-0.19	0.40	0.60
	Hedge Fund	0.60	0.30	0.02	0.49	0.30	0.20	0.60	0.30	0.20
	U.S. Large Cap	-2.37	-1.19	-0.79	0	0	0	-0.20	-0.20	-0.20
	U.S. Small Cap	0.52	0.26	0.17	0.20	0.10	0.06	0.37	0.25	0.17
AC2	U.S. Growth	0.45	0.23	0.15	0	0	0	-0.20	-0.18	-0.09
	U.S. Value	2.24	1.12	0.75	0.44	0.33	0.22	0.77	0.57	0.42
	Risk-free Asset	-0.61	0.20	0.46	0	0.28	0.53	-0.21	0.24	0.49
	Hedge Fund	0.77	0.38	0.26	0.36	0.29	0.19	0.47	0.32	0.21
	U.S. Large Cap	1.61	0.81	0.54	0	0	0	-0.20	0.18	0.28
	U.S. Small Cap	1.37	0.68	0.46	0.26	0.15	0.12	0.90	0.56	0.41
	U.S. Growth	-1.02	-0.51	-0.34	0	0	0	-0.20	-0.20	-0.20
	U.S. Value	0.40	0.20	0.13	0.14	0.14	0.12	0.75	0.34	0.18
	U.S. LT Gov Bonds	-1.09	-0.55	-0.36	0.45	0.52	0.42	1.18	0.64	0.28
	U.S. IT Gov Bonds	5.76	2.88	1.92	0	0	0.14	-0.20	0.19	0.65
	U.S. LT Corp Bonds	0.11	0.05	0.04	0	0	0	-0.20	-0.20	-0.20
	U.S. Commodities	-0.16	-0.08	-0.05	0	0	0	-0.17	-0.08	-0.05
AC6	U.K. Equities	-1.52	-0.76	-0.51	0	0	0	-0.20	-0.20	-0.20
	Japan Equities	-0.26	-0.13	-0.09	0	0	0	-0.20	-0.11	-0.08
	Germany Equities	-0.35	-0.18	-0.12	0	0	0	-0.20	-0.20	-0.16
	France Equities	0.67	0.33	0.22	0	0	0	0.03	0.13	0.14
	Emerging Markets	-0.37	-0.19	-0.12	0	0	0	-0.20	-0.20	-0.14
	U.K. Bonds	1.64	0.82	0.55	0	0	0.03	0.09	0.26	0.26
	Germany Bonds	-0.67	-0.34	-0.22	0	0	0	-0.20	-0.08	-0.08
	Japan Bonds	-0.17	-0.09	-0.06	0	0	0	-0.20	-0.16	-0.11
	Risk-free Asset	-5.84	-2.39	-1.29	0	0	0	-0.21	-0.17	-0.20
	Hedge Fund	0.89	0.45	0.30	0.15	0.19	0.18	0.43	0.30	0.22

Table 5 Continued

Panel B: Fund-of-Funds Holdings

	Asset Class	No Constraints γ=4	γ=8	γ=12	No Short Sales γ=4	γ=8	γ=12	Short down to -20% γ=4	γ=8	γ=12
	U.S. Large Cap	0.18	0.09	0.06	0.17	0.09	0.06	0.17	0.09	0.06
AC1	U.S. Small Cap	0.27	0.13	0.09	0.37	0.13	0.09	0.34	0.13	0.09
	Risk-free Asset	-0.68	0.17	0.44	0	0.17	0.44	-0.20	0.17	0.44
	Fund-of-Funds	1.23	0.61	0.41	0.46	0.61	0.41	0.69	0.61	0.41
	U.S. Large Cap	-0.72	-0.36	-0.24	0	0	0	-0.20	-0.20	-0.20
	U.S. Small Cap	0.39	0.19	0.13	0.25	0.04	0.03	0.39	0.19	0.13
AC2	U.S. Growth	-0.29	-0.15	-0.10	0	0	0	-0.20	-0.20	-0.11
	U.S. Value	1.32	0.66	0.44	0.52	0.32	0.21	0.78	0.57	0.42
	Risk-free Asset	-1.13	-0.06	0.29	0	0.06	0.37	-0.21	-0.05	0.29
	Fund-of-Funds	1.43	0.72	0.48	0.23	0.58	0.39	0.44	0.69	0.47
	U.S. Large Cap	-4.49	-2.24	-1.50	0	0	0	-0.07	0.20	0.27
	U.S. Small Cap	1.08	0.54	0.36	0.31	0.19	0.10	0.91	0.52	0.37
	U.S. Growth	1.36	0.68	0.45	0	0	0	-0.20	-0.20	-0.20
	U.S. Value	4.11	2.05	1.37	0.18	0.16	0.12	0.69	0.33	0.18
	U.S. LT Gov Bonds	0.75	0.38	0.25	0.51	0.56	0.44	1.21	0.75	0.40
	U.S. IT Gov Bonds	6.74	3.37	2.25	0	0	0.07	-0.20	-0.08	0.35
	U.S. LT Corp Bonds	-2.36	-1.18	-0.79	0	0	0	-0.20	-0.20	-0.20
	U.S. Commodities	-0.56	-0.28	-0.19	0	0	0	-0.16	-0.10	-0.08
AC6	U.K. Equities	-2.34	-1.17	-0.78	0	0	0	-0.20	-0.20	-0.20
	Japan Equities	-0.06	-0.03	-0.02	0	0	0	-0.20	-0.10	-0.06
	Germany Equities	-0.24	-0.12	-0.08	0	0	0	-0.20	-0.20	-0.16
	France Equities	0.35	0.17	0.12	0	0	0	0.01	0.09	0.09
	Emerging Markets	-0.23	-0.12	-0.08	0	0	0	-0.20	-0.20	-0.15
	U.K. Bonds	2.29	1.14	0.76	0	0	0.02	0.10	0.25	0.25
	Germany Bonds	-0.24	-0.12	-0.08	0	0	0	-0.20	-0.05	-0.04
	Japan Bonds	-0.32	-0.16	-0.11	0	0	0	-0.20	-0.17	-0.12
	Risk-free Asset	-9.78	-4.38	-2.58	0	0.01	0.01	-0.20	-0.20	-0.21
	Fund-of-Funds	4.94	2.47	1.65	0	0.08	0.24	0.31	0.56	0.51

Panel A reports the optimal mean-variance asset allocation of benchmark assets combined with a hedge fund position. We report results for three asset classes (AC1, AC2, AC6) where we impose no constraints on the portfolio positions, prohibit short sales, and allow shorting down to -20%. Panel B reports the optimal asset allocation of benchmark assets combined with a fund-of-funds position. The data and input variables are described in Table 4. We assume that the annual risk-free rate is 4% and that the coefficient of risk aversion, γ, equals 4, 8, or 12.

Table 6: Annualized Certainty Equivalents

Panel A: Annualized Certainty Equivalents (in Percentages)

Asset Class		No Short Sales			Short down to -20%		
		γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
AC1 = Lg Cap + Sm Cap		7.26	5.63	5.09	7.26	5.63	5.09
	+ HF	8.20	6.15	5.44	8.31	6.15	5.44
	+ FoF	7.69	6.06	5.37	7.91	6.06	5.37
AC2 = AC1 + Growth + Value		8.09	6.05	5.37	8.65	6.40	5.63
	+ HF	8.67	6.55	5.70	9.57	6.97	5.98
	+ FoF	8.21	6.45	5.63	9.06	6.94	5.96
AC3 = AC2 + U.S. Bonds		9.43	8.19	7.22	10.96	8.79	7.61
	+ HF	9.51	8.44	7.50	11.17	9.20	7.94
	+ FoF	9.43	8.21	7.35	10.96	8.97	7.86
AC4 = AC3 + Commodities		9.43	8.19	7.22	11.32	8.87	7.66
	+ HF	9.51	8.44	7.50	11.60	9.32	8.02
	+ FoF	9.43	8.21	7.35	11.32	9.11	7.97
AC5 = AC4 + Foreign Equities		9.43	8.19	7.22	14.06	10.43	8.71
	+ HF	9.51	8.44	7.50	14.54	10.93	9.11
	+ FoF	9.43	8.21	7.35	14.08	10.81	9.17
AC6 = AC5 + Foreign Bonds		9.43	8.19	7.22	14.47	10.66	8.97
	+ HF	9.51	8.44	7.51	15.07	11.18	9.40
	+ FoF	9.43	8.21	7.35	14.56	11.06	9.47

Table 6 Continued

Panel B: Increases in Certainty Equivalents (in Percentages)							
		No Short Sales			Short down to -20%		
Asset Class		γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
AC1 = Lg Cap + Sm Cap	+ HF	0.88	0.49	0.33	0.98	0.49	0.33
	+ FoF	0.40	0.41	0.27	0.61	0.41	0.27
AC2 = AC1 + Growth + Value	+ HF	0.54	0.47	0.31	0.85	0.54	0.33
	+ FoF	0.11	0.38	0.25	0.38	0.51	0.31
AC3 = AC2 + U.S. Bonds	+ HF	0.07	0.23	0.26	0.19	0.38	0.31
	+ FoF	0.00	0.02	0.12	0.00	0.17	0.23
AC4 = AC3 + Commodities	+ HF	0.07	0.23	0.26	0.25	0.41	0.33
	+ FoF	0.00	0.02	0.12	0.00	0.22	0.29
AC5 = AC4 + Foreign Equities	+ HF	0.07	0.23	0.26	0.42	0.45	0.37
	+ FoF	0.00	0.02	0.12	0.02	0.34	0.42
AC6 = AC5 + Foreign Bonds	+ HF	0.07	0.23	0.27	0.52	0.47	0.39
	+ FoF	0.00	0.02	0.12	0.08	0.36	0.46

Panel A reports the annualized certainty equivalents (CEs) in percentage terms that an investor with mean-variance utility could obtain by investing in different asset classes (AC1-AC6) alone, or combined with a hedge fund (HF) or a fund-of-funds (FoF). For each asset class, we report the CEs of investing in the three portfolios (i) benchmark assets only, (ii) benchmark assets and a FoF, and (iii) benchmark assets and a HF. In each case, we do not allow short sales or allow short positions down to -20%. Panel B reports the percentage increases in CEs (defined in equation (8)) that an investor could obtain by adding a hedge fund (HF) or a fund-of-funds (FoF) to her existing portfolio of benchmark assets. The input variables for the mean-variance asset allocation problem are listed in Table 4. We assume that the annual risk-free rate is 4% and the level of risk aversion γ equals 4, 8, or 12.

Table 7: Characterizing the Mean of the Benchmark Fund-of-Funds Distribution

	No Short Sales			Short down to -20%		
Asset Class	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Median Hedge Fund Return in Data = 0.873% per month						
Case 1: $\sigma_B = 3.876\%$ = Median Hedge Fund Volatility in Data						
AC1 + FoF	0.772	0.847	0.847	0.812	0.847	0.847
AC2 + FoF	0.730	0.842	0.842	0.766	0.866	0.866
AC3 + FoF	–	0.710	0.789	–	0.786	0.838
AC4 + FoF	–	0.710	0.789	0.633	0.802	0.855
AC5 + FoF	–	0.710	0.789	0.711	0.835	*0.896*
AC6 + FoF	–	0.710	0.789	0.731	0.837	*0.899*
Case 2: $\sigma_B = 1.1 \times 3.876\%$						
AC1 + FoF	0.813	*0.898*	*0.898*	0.860	*0.898*	*0.898*
AC2 + FoF	0.761	*0.893*	*0.893*	0.804	*0.920*	*0.919*
AC3 + FoF	–	0.731	0.825	–	0.820	*0.882*
AC4 + FoF	–	0.731	0.825	0.663	0.838	*0.900*
AC5 + FoF	–	0.731	0.825	0.726	*0.875*	*0.945*
AC6 + FoF	–	0.731	0.825	0.752	*0.876*	*0.947*
Case 3: $\sigma_B = 1.2 \times 3.876\%$						
AC1 + FoF	0.857	*0.949*	*0.949*	*0.907*	*0.949*	*0.949*
AC2 + FoF	0.791	*0.944*	*0.944*	*0.844*	*0.973*	*0.972*
AC3 + FoF	–	0.752	0.862	–	*0.854*	*0.925*
AC4 + FoF	–	0.752	0.862	0.693	*0.874*	*0.945*
AC5 + FoF	–	0.752	0.862	0.742	*0.914*	*0.995*
AC6 + FoF	–	0.752	0.861	0.773	*0.915*	*0.996*

The table reports the expected monthly return, μ_B (in percentages), which is constructed so that an investor is indifferent between adding a hedge fund with this expected return, or adding a fund-of-funds to an asset class defined in Section 3 (AC1-AC6). We assume that the fund-of-funds benchmark standard deviation is equal to $\sigma_B = 3.876\%$ per month in Case 1, which is the median standard deviation of hedge fund excess returns in data; $\sigma_B = 1.1 \times 3.876\%$ in Case 2; and $\sigma_B = 1.2 \times 3.876\%$ in Case 3. A hyphen in the table means that there is zero allocation to a fund-of-funds in that category. Entries in italics indicate that the mean is greater than the observed median hedge fund average return in data (0.873% per month) and consequently, in these cases, investors already prefer a fund-of-funds even if they can obtain somewhat higher returns on their own. This is because these investors are sufficiently risk-averse and favor the diversification benefits of funds-of-funds.

Table 8: Characterizing the Volatility of the Benchmark Fund-of-Funds Distribution

	No Short Sales			Short down to -20%		
Asset Class	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Median Volatility of Hedge Fund Returns in Data = 3.876% per month						
Case 1: $\mu_B = 0.873\%(= R_f + 0.54\%)$ = Median Hedge Fund Return in Data						
AC1 + FoF	4.799	4.076	4.076	4.374	4.076	4.076
AC2 + FoF	5.660	4.110	4.110	4.933	3.924	3.930
AC3 + FoF	–	6.839	4.775	–	4.872	4.187
AC4 + FoF	–	6.839	4.775	9.305	4.644	4.034
AC5 + FoF	–	6.839	4.775	7.864	4.251	*3.695*
AC6 + FoF	–	6.839	4.782	6.504	4.233	*3.665*
Case 2: $\mu_B = 0.819\%(= R_f + 0.9 \times 0.54\%)$						
AC1 + FoF	4.315	*3.669*	*3.669*	3.937	*3.669*	*3.669*
AC2 + FoF	5.000	*3.699*	*3.699*	4.410	*3.531*	*3.537*
AC3 + FoF	–	5.868	4.200	–	4.259	*3.702*
AC4 + FoF	–	5.868	4.200	7.542	4.059	*3.566*
AC5 + FoF	–	5.868	4.200	6.552	*3.720*	*3.267*
AC6 + FoF	–	5.868	4.204	5.511	*3.691*	*3.228*
Case 3: $\mu_B = 0.765\%(= R_f + 0.8 \times 0.54\%)$						
AC1 + FoF	*3.811*	*3.261*	*3.261*	*3.499*	*3.261*	*3.261*
AC2 + FoF	4.324	*3.288*	3.288	*3.866*	*3.139*	*3.144*
AC3 + FoF	–	4.890	*3.623*	–	*3.642*	*3.214*
AC4 + FoF	–	4.890	*3.623*	5.779	*3.471*	*3.095*
AC5 + FoF	–	4.890	*3.623*	5.231	*3.186*	*2.836*
AC6 + FoF	–	4.890	*3.624*	4.509	*3.145*	*2.787*

The table reports the monthly standard deviation, σ_B, (in percentage terms), which is constructed so that an investor is indifferent between adding a hedge fund with this standard deviation, or adding a fund-of-funds to an asset class defined in Section 3 (AC1-AC6). We assume that the fund-of-funds benchmark expected return is equal to $\mu_B = 0.873\% = R_f + 0.54\%$ per month in Case 1, which is the assumed risk-free rate of 4% per annum plus the median expected excess return of hedge funds in data; $\mu_B = R_f + 0.9 \times 0.54\%$ in Case 2; and $\mu_B = R_f + 0.8 \times 0.54\%$ in Case 3. A hyphen in the table means that there is zero allocation to funds-of-funds in that category. Entries in italics indicate that the benchmark fund-of-funds standard deviation is less than the observed median hedge fund average volatility in data (3.876% per month) and consequently, in these cases, investors prefer a fund-of-funds even if they can obtain a somewhat lower volatility on their own. This is because these investors are sufficiently risk-averse and favor the diversification benefits of funds-of-funds.

Table 9: Characterizing the Left-Hand Tail of the Benchmark Fund-of-Funds Distribution

	No Short Sales			Short down to -20%		
Asset Class	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Case 1: $\sigma_B = 3.876\%$						
AC1 + FoF	0.97	0.22	0.22	0.55	0.22	0.22
AC2 + FoF	1.44	0.26	0.26	1.03	0.05	0.06
AC3 + FoF	–	1.67	0.79	–	0.83	0.29
AC4 + FoF	–	1.67	0.79	2.60	0.65	0.15
AC5 + FoF	–	1.67	0.79	1.66	0.33	0.00
AC6 + FoF	–	1.67	0.79	1.42	0.30	0.00
Case 2: $\sigma_B = 1.1 \times 3.876\%$						
AC1 + FoF	0.54	0.00	0.00	0.10	0.00	0.00
AC2 + FoF	1.09	0.00	0.00	0.62	0.00	0.00
AC3 + FoF	–	0.47	0.00	–	1.18	0.09
AC4 + FoF	–	1.43	0.42	2.24	0.30	0.00
AC5 + FoF	–	1.43	0.42	1.48	0.00	0.00
AC6 + FoF	–	1.43	0.42	1.18	0.00	0.00
Case 3: $\sigma_B = 1.2 \times 3.876\%$						
AC1 + FoF	0.13	0.00	0.00	0.00	0.00	0.00
AC2 + FoF	0.77	0.00	0.00	0.24	0.00	0.00
AC3 + FoF	–	1.18	0.09	–	0.16	0.00
AC4 + FoF	–	1.18	0.09	1.88	0.00	0.00
AC5 + FoF	–	1.18	0.09	1.30	0.00	0.00
AC6 + FoF	–	1.18	0.09	0.96	0.00	0.00

The table reports the left-hand tail proportion in percentage terms of the normal distribution fitted to the observed median return and volatility of hedge fund returns in data. If the normal distribution were truncated at that left-hand tail proportion, the investor would be indifferent between adding the truncated hedge fund distribution and adding a median fund-of-funds to an asset class defined in Section 3 (AC1-AC6). We assume that the original hedge fund distribution is normally distributed with mean $\mu_B = 0.873\%$ per month, which is the median expected return of hedge funds in data, and monthly volatility σ_B. In Case 1, we assume that $\sigma_B = 3.876\%$ per month, which is the median standard deviation of hedge fund returns in data. In Cases 2 and 3, we assume $\sigma_B = 1.1 \times 3.876\%$ and $\sigma_B = 1.2 \times 3.876\%$, respectively. A hyphen in the table means that there is zero allocation to funds-of-funds in that category. Entries with zero indicate that an investor already prefers a fund-of-funds investment over a hedge fund investment.

Table 10: Artificial Funds-of-Funds

Panel A: Summary Statistics

	Number	Mean Ex Ret	Median Ex Ret	Median Std Dev
Artificial Fund-of-Funds	748	0.53	0.52	2.53
Hedge Fund	2947	0.58	0.54	3.88
Fund-of-Funds	748	0.27	0.32	2.07

Panel B: Characterizing the Benchmark Hedge Fund Distribution

	No Short Sales			Short down to -20%		
AC6 + FoF	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Case 1: Characterizing the Mean						
$\sigma_B = 2.53\%$	–	0.745	0.769	0.733	0.744	0.763
$\sigma_B = 1.1 \times 2.53\%$	–	0.769	0.803	0.754	0.774	0.798
$\sigma_B = 1.2 \times 2.53\%$	–	0.793	0.838	0.774	0.803	0.833
Case 2: Characterizing the Volatility						
$\mu_B = R_f + 0.52\%$	–	3.685	3.174	4.046	3.484	3.202
$\mu_B = R_f + 0.9 \times 0.52\%$	–	3.147	2.789	3.409	3.043	2.828
$\mu_B = R_f + 0.8 \times 0.52\%$	–	2.602	2.400	2.761	2.595	2.448
Case 3: Characterizing the Left-hand Tail						
$\sigma_B = 2.53\%$	–	1.77	1.33	1.97	1.77	1.44
$\sigma_B = 1.1 \times 2.53\%$	–	1.33	0.76	1.60	1.25	0.84
$\sigma_B = 1.2 \times 2.53\%$	–	0.92	0.23	1.25	0.76	0.30

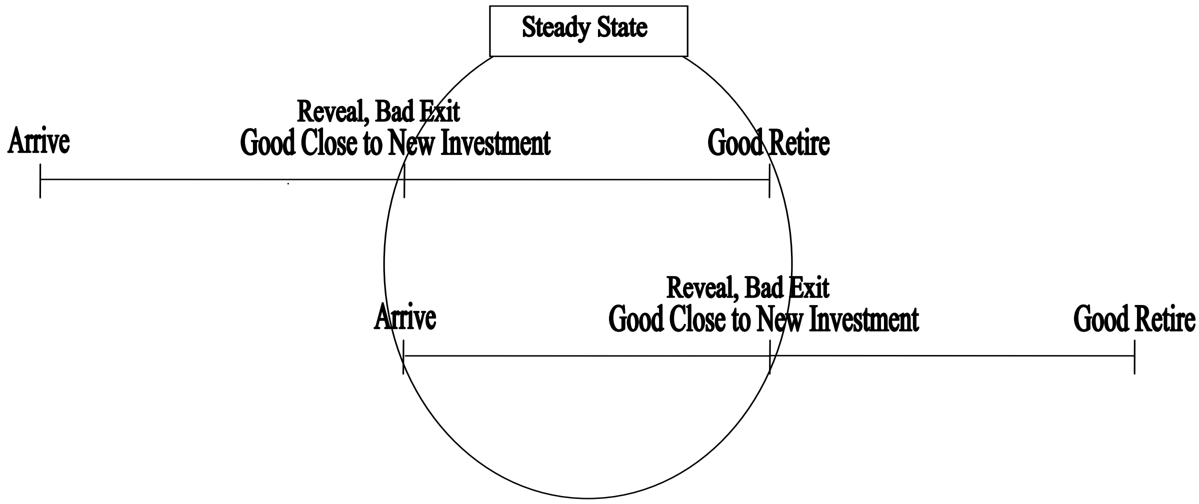
Panel A reports summary statistics of monthly excess returns in percentage terms of the 748 artificial funds-of-funds, compared to hedge funds and funds-of-funds in data. To construct the sample of artificial funds-of-funds, we randomly select 10 hedge funds from hedge fund data at the beginning of each year. We equally weight the hedge funds and record the monthly returns of the portfolio for the year. We rebalance the portfolio annually. This process is repeated 748 times to match the number of funds-of-funds in our sample. In Panel B, we characterize the expected monthly return, standard deviation, and left-hand tail proportion (in percentage terms) of the benchmark fund-of-funds distribution similar to Tables 7 to 9. We estimate the fund-of-funds benchmark by assuming that the institutional investor is indifferent between adding an artificial fund-of-funds and adding a fund-of-funds to Asset Class 6, AC6, defined in Section 3. A hyphen in a cell means that there is zero allocation to funds-of-funds in that category. In each case, we assume that the risk-free rate is $R_f = 4\%/12$ per month.

Table 11: Robustness Checks on Hedge Fund /Fund-of-Funds Moments

	Lower			Higher		
Asset Class	γ=4	γ=8	γ=12	γ=4	γ=8	γ=12
Median Hedge Fund Return in Data = 0.873% per month						
Panel A: Changing Expected Returns						
AC1 + FoF	0.721	0.722	0.722	*0.887*	*0.971*	*0.971*
AC2 + FoF	0.691	0.742	0.745	0.841	*0.989*	*0.989*
AC3 + FoF	–	0.675	0.734	0.735	*0.884*	*0.947*
AC4 + FoF	–	0.690	0.750	0.757	*0.907*	*0.966*
AC5 + FoF	–	0.720	0.780	0.805	*0.949*	*1.005*
AC6 + FoF	–	0.721	0.783	0.831	*0.952*	*1.011*
Panel B: Changing Volatilities						
AC1 + FoF	*0.935*	*1.180*	*1.316*	–	–	–
AC2 + FoF	*0.876*	*1.139*	*1.291*	0.580	0.588	0.591
AC3 + FoF	–	*0.940*	*1.098*	–	0.619	0.648
AC4 + FoF	0.737	*0.964*	*1.117*	0.633	0.634	0.663
AC5 + FoF	0.782	*1.018*	*1.153*	–	0.661	0.691
AC6 + FoF	0.837	*1.021*	*1.139*	–	0.671	0.707
Panel C: Changing Correlations						
AC1 + FoF	0.736	0.839	0.839	0.862	*0.885*	*0.885*
AC2 + FoF	0.697	0.822	0.821	0.838	*0.907*	*0.907*
AC3 + FoF	0.629	0.725	0.812	+	+	+
AC4 + FoF	0.653	0.749	0.829	+	+	+
AC5 + FoF	0.708	0.816	0.871	–	*0.890*	*1.154*
AC6 + FoF	0.733	0.817	0.851	–	*1.004*	*1.272*

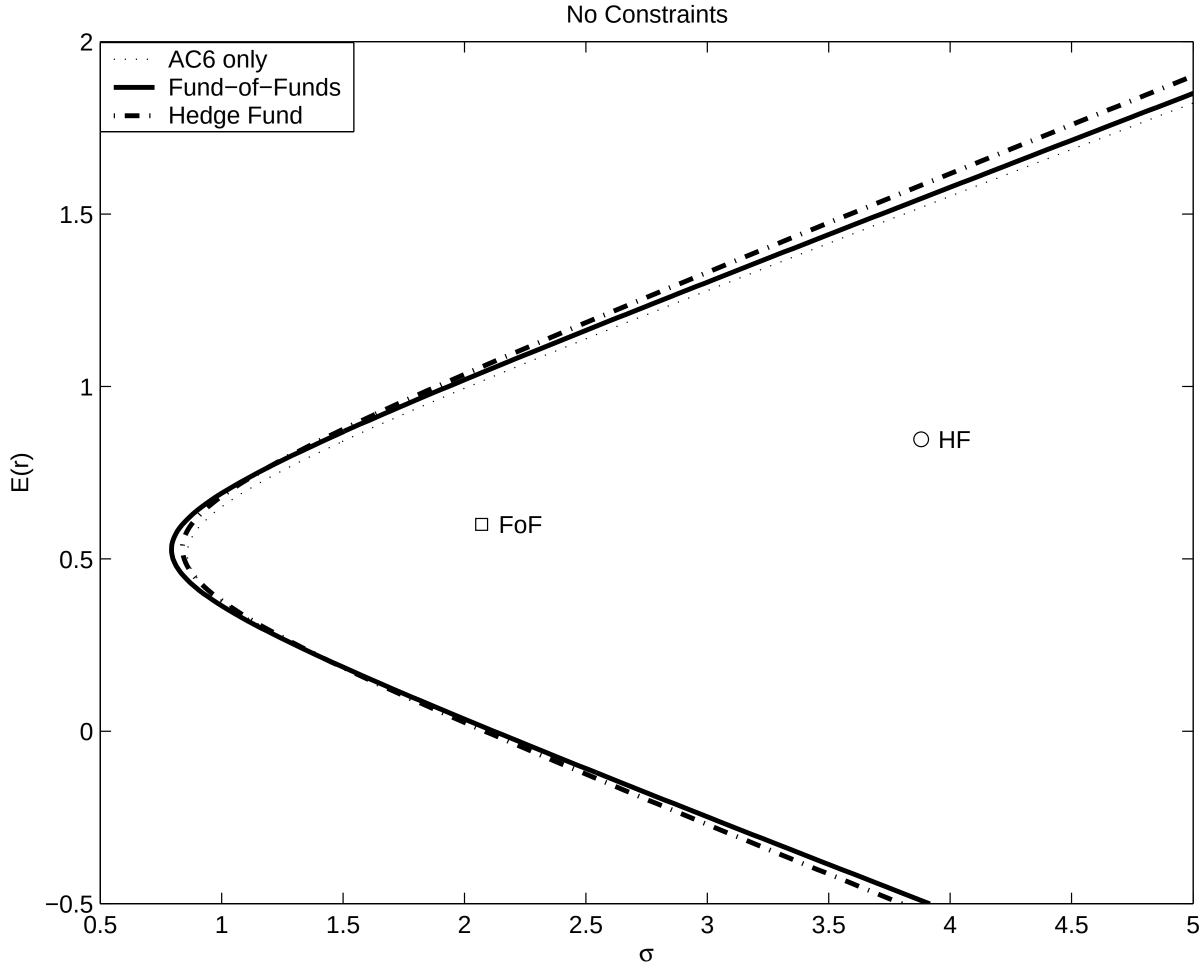
This table reports the benchmark fund-of-funds expected monthly return (in percentages), μ_B, under six robustness scenarios for allowing shorting down to -20%. In Panel A, we change the expected return of funds-of-funds from the median (0.653% per month) to 0.598% and 0.708% per month. These numbers represent changing the expected return of funds-of-funds from the median to the 25% and 75% values of the cross-sectional distribution of the difference between fund-of-funds returns and the median hedge fund return, holding other inputs fixed at their original level in Table 4. In Panel B, we change the fund-of-funds volatility from the median (2.067% per month) to the 25%-tile and 75%-tile values (1.124% and 3.363% per month), holding other inputs fixed. In Panel C, we change the correlations of both hedge funds and funds-of-funds returns with the base assets from the median to the 25%-tile and 75%-tile values, holding all other inputs fixed. Entries with plus signs means that there is zero allocations to both hedge fund and fund-of-funds, and entries with dashes means that it is not optimal for an investor to hold any fund-of-funds. In these cases, constructing a benchmark distribution is not meaningful. Entries in italics indicate that an investor already prefers a fund-of-funds over a hedge fund, as the fund-of-funds can provide access to a benchmark distribution with a higher μ_B than the median hedge fund return in data.

Figure 1: Steady-State Equilibrium of the Model



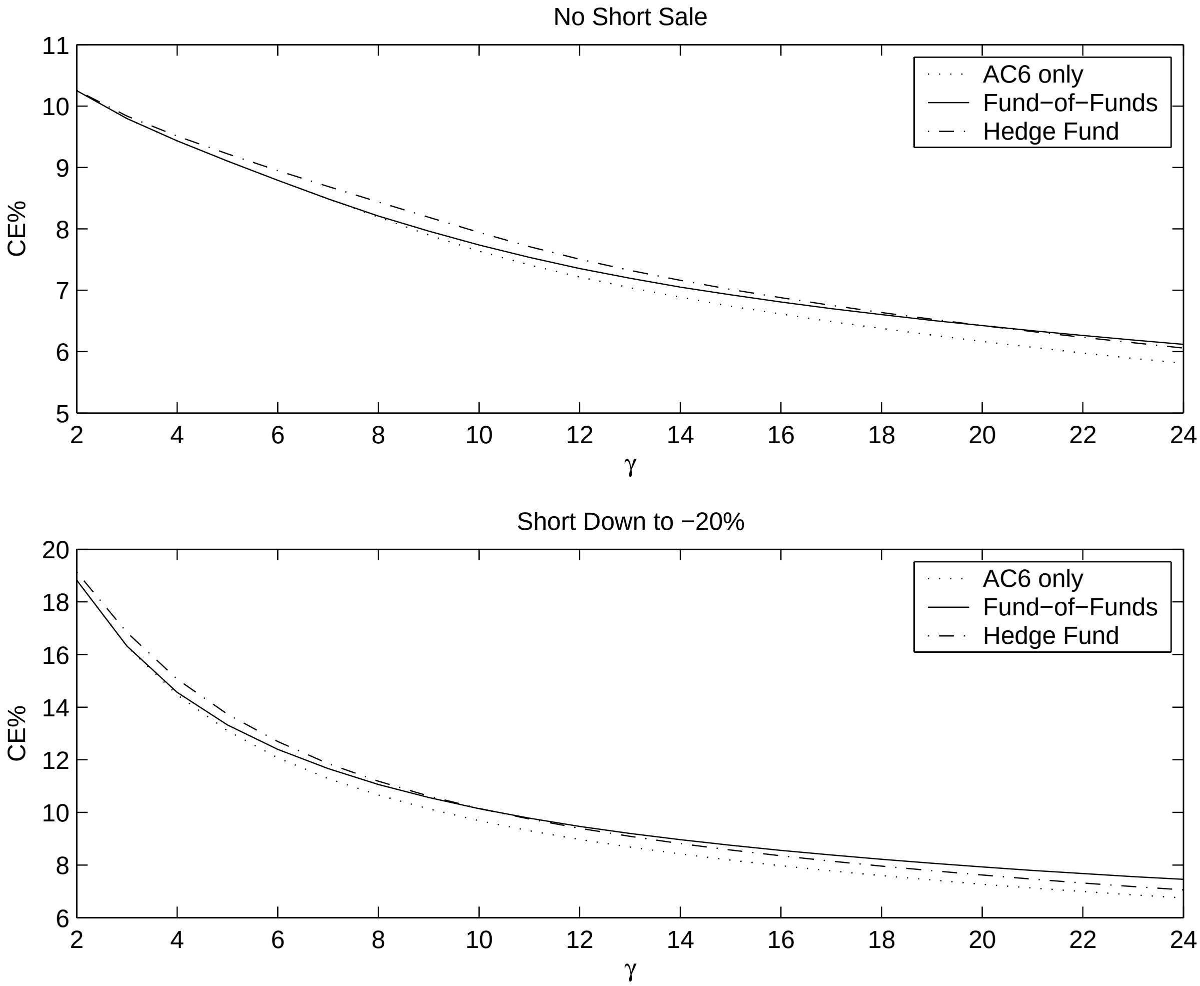
Each line represents the timeline of a new hedge fund that receives capital. At the end of period 1, quality is revealed. Bad hedge funds exit the market while good hedge funds close to new investment and live one more period. At any point in time, the universe of hedge funds includes good, seasoned hedge funds that have withstood the test of time, new good hedge funds, and new bad hedge funds.

Figure 2: Mean-Variance Frontiers



The plot shows the mean-standard deviation frontiers generated by AC6 only, AC6 and a typical hedge funds, and AC6 and a typical funds-of-funds. The mean-variance frontiers are produced using the moments in Table 4. We also show the individual position in mean-standard deviation space for hedge funds (HF) and funds-of-funds (FoF). The sample period is from June 1992 to September 2003.

Figure 3: Certainty Equivalents as a Function of Risk Aversion



We plot the annualized certainty equivalents for different levels of risk aversion, for $\gamma = 2$ to 24. In both panels, we plot the certainty equivalents of AC6 only; for including hedge funds to AC6; and for including funds-of-funds to AC6. In the top panel, we do not permit short sales, while in the bottom panel, short sales are allowed down to -20%.