Housing and the Macroeconomy: The Role of Implicit Guarantees for Government Sponsored Enterprises*

Karsten Jeske
Federal Reserve Bank of Atlanta

Dirk Krueger
University of Pennsylvania, CEPR and NBER

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Abstract

This paper studies the macroeconomic effects of implicit government bailout guarantees of debt obligations of Government-Sponsored Enterprises such as Fannie Mae and Freddy Mac. We construct a model with heterogenous infinitly lived households and competitive housing and mortgage markets where the government provides households with a mortgage interest rate subsidy. We use this model to evaluate the aggregate and distributional impacts of this government subsidy to home owners. We find that eliminating the interest rate subsidy leads to lower equilibrium housing investment, substantially lower mortgage default rates and higher aggregate welfare. The welfare effects of removing the subsidy, however, vary substantially across members of the population with different economic characteristics.

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*The authors can be reached at Karsten.Jeske@atl.frb.gov and dkrueger@econ.upenn.edu. We have benefited from helpful comments by seminar participants at the Wharton macro lunch, the Banco de Espana, The Housing Conference at the Bank of Canada and the Federal Reserve Bank of Atlanta. Part of this paper was completed while Krueger was a research visitor at the European Central Bank

1 Introduction

With close to 70% the United States displays one of the highest home ownership ratios in the world. Part of the attractiveness of owner-occupied housing stems from a variety of subsidies the government provides to homeowners. Apart from direct subsidies to low-income households via HUD (need to write this out or explain) programs, three important indirect subsidies exist. The first - and most well known - is the fact that mortgage interest payments (of mortgages up to $1 million) are tax-deductible. Second, the implicit income from housing capital (i.e. the imputed rental-equivalent) is not taxable, while other forms of capital income (e.g. interest, dividend and capital gains income) are being taxed. Gervais (2001) addresses the adverse effects of these two subsidies within a general equilibrium life-cycle model.

The third subsidy arises from the special structure of the US mortgage market. Essentially all home mortgages in the US are being sold from individual banks to so called Government Sponsored Enterprises (GSEs) who in turn refinance themselves via the bond market. A formidable summary of the institutional details surrounding GSEs can be found in Frame and Wall (2002a) and (2002b). The three most important GSE are the two privately owned and publicly traded companies Fannie Mae (Federal National Mortgage Association) and Freddie Mac (Federal Home Loan Mortgage Association), and the FHLB (Federal Home Loan Bank system), a public and non-profit organization. The close link of GSEs to the federal government creates the impression that the government provides a guarantee to GSEs shielding them from aggregate risks, most notably aggregate credit risk which lowers their refinancing cost to below what private institutions would have to pay. The purpose of this paper is to quantify the macroeconomic and distributional effects of this subsidy; our paper is - to our knowledge - the first attempt to do so within a structural dynamic general equilibrium model.

According to Frame and Wall, GSEs enjoy an array of government benefits, for example being exempt from state and federal income taxes, a line of credit with the Treasury Department and very importantly a special status of GSE-issued debt. In particular, GSE securities can serve as substitutes to government bonds for transactions between public entities that normally require to be done in Treasuries. The Federal Reserve System also accepts GSE debt as a substitute for Treasuries in their portfolio of repurchase agreements. While no written federal guarantee for GSE debt exists, market participants view the special status of GSE debt as an indication of an implicit guarantee making them almost as safe as Treasury bills. The perception of a federal guarantee is further fueled by the sheer size of the GSE mortgage portfolio amounting to about 3 trillion dollars, 2.4 trillion dollars of which coming from the larger two GSEs, Fannie Mae and Freddie Mac. Insolvency of any one or both of these companies, say, due to an adverse shock in the real estate market that increases aggregate mortgage delinquency, will cause major disruptions in the financial system, which is why market participants consider housing GSEs to be too large to fail. Finally, two previous government bailouts of housing GSEs - Fannie Mae in the early 1980s and one of the smaller housing GSEs in the late 1980s - are further evidence that a bailout is likely should housing GSEs get into financial trouble.

The implicit federal guarantee is more than mere perception; most importantly, it is reflected in interest rates GSEs pay when borrowing. GSEs can borrow at rates only marginally higher than the Treasury but about 40 basis points lower than private companies without a government guarantee, according to the Congressional Budget Office CBO (2001). This is despite the fact that GSEs are highly leveraged entities with an equity cushion of only about 3% of their obligations, much lower than the 8.45% in the thrift industry (figures taken from Frame and Wall (2002a)). To the extend that part of the interest advantage of GSEs is passed through to homeowners, there exists a subsidy from the federal government to homeowners.

In order to assess the macroeconomic and distributional effects of this subsidy we construct a heterogeneous agent general equilibrium model with incomplete markets in the tradition of Bewley (1986) and Aiyagari (1994). We add a real estate sector and mortgages, i.e., we allow households to borrow against their real estate wealth. We also explicitely model mortgage default. In the model we approximate the implicit bailout guarantee with a tax-financed direct subsidy to mortgage interest rates. Our economy would then corrspond to a world in which the government taxes income every period and either saves the proceeds in an effort to smooth out the spending shock of a potential insolvency of the GSEs, or alternatively, is able to buy insurance from the outside world via, say, a market for credit derivatives. Thus, in our model economy the government subsidizes home ownership by reducing effective mortgage interest rates.

Notice that it is not clear a priori that this subsidy has negative welfare consequences. This is due to a second-best argument because our economy has incomplete markets. Specifically, lowering the borrowing cost through a subsidy will reduce the severity of the borrowing constraint of agents with adverse idiosyncratic shocks.

Our results can be described as follows. First, the subsidy leads to an increase in investment in housing assets and an increase in the construction of real estate. Looking at the distribution of housing assets, the mortgage subsidy does not significantly change the share of households with positive holdings of real estate. This is because on the one hand the subsidy makes real estate ownership more attractive but on the other hand, through a general equilibrium effect, rental housing becomes cheaper, which discourages homeownership for low-income and low-asset households.

Using a steady state utilitarian social welfare functional we find that the aggregate welfare implications of the subsidy are negative, in the order of 0.32% of consumption equivalent variation. In addition to the adverse aggregate welfare effects the results also suggest that primarily households with low wealth prefer to live in an economy without subsidy while high wealth households benefit strongly from it, indicating adverse distributional effects of the reform.[1]

The remainder of the paper is organized as follows. Section 2 introduces the model and defines equilibrium in an economy with a housing and mortgage market. Section 3 characterizes equilibria. Section 4 describes the calibration of our economy. Section 5 details the numerical results comparing two steady states in economies with and without a mortgage interest subsidy. Section 6 concludes the paper.

2 The Model

The endowment economy is populated by a continuum of measure one of infinitely lived households, a continuum of competitive banks and a continuum of housing construction companies. Households face idiosyncratic endowment and housing depreciation shocks. In what follows we will immediately proceed to describing the economy recursively, thereby skipping the (standard) sequential formulation of the economy.

[1] Gruber and Martin (2003) also study the distributional effects of the inclusion of housing wealth in a general equilibrium model, but do not address the role of government housing subsidies for this question.

2.1 Households

Households have idiosyncratic endowment of the perishable consumption good given by $y \in Y$. These endowments follow a finite state Markov chain with transition probabilities $\pi(y'|y)$ and unique invariant distribution $\Pi(y)$.

Households derive period utility $U(c,h)$ from consumption and housing services h, which can be purchased at a price p_l (relative to the numeraire consumption good). In addition to consumption and housing services the household can purchase two types of assets, one period bonds b' and houses g'. The price of bonds is denoted by P_b and the price of houses by P_h. Whereas households cannot short-sell bonds, they can borrow against their real estate property. Let by m' denote the size of their mortgage, and by P_m the receipt of resources (the consumption good) for each unit of mortgage issued and to be repaid tomorrow. These receipts will be determined in equilibrium by competition of banks, and will depend on the characteristics of households as well as the size of the mortgage m' and size of the collateral g'. Houses depreciate stochastically; let $F(\delta')$ denote the cumulative distribution function of the depreciation rate δ' tomorrow, which has support $D = [\underline{\delta}, \bar{\delta}]$. Households possess the option of defaulting on their mortgages, at the cost of losing their housing collateral. They will choose to do so whenever

$$m' > P_h(1-\delta')g'.$$

If there is a government bailout guarantee, then the government obtains general tax revenues by levying proportional taxes τ on endowments. It will use the receipts from these taxes to bail out part of the mortgages that private households have defaulted on. Finally let a denote cash at hand, that is, after tax endowment plus receipts from all assets brought into the period.

The individual state of a household consists of $s = (a, y)$. Let the cross-sectional distribution over individual states be given by μ. Since we will restrict our analysis to stationary equilibria in which μ is constant over time, in what follows the dependence of aggregate prices and quantities on μ is left implicit.

The dynamic programming problem of a household reads as

$$v(s) = \max_{c,h,b',m',g' \geq 0} \left\{ U(c,h) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^{\bar{\delta}} v(s') dF(\delta') \right\} \tag{1}$$

s.t.

$$c + b'P_b + hP_l + g'P_h - m'P_m(g', m') = a + g'P_l$$

$$a'(\delta', y', m', g') = b' + \max\{0, P_h(1-\delta')g' - m')\} + (1-\tau)y'$$

Note that the budget constraint implies the timing convention that newly purchased real estate g' can immediately be rented out in the same period. The function T describes the aggregate law of motion for the cross-sectional distribution over households' characteristics.

2.2 The Real Estate Construction Sector

Firms in the real estate construction sector act competitively and face the linear technology

$$I = A_h C_h$$

where I is the output of houses of a representative firm, C_h is the input of the consumption good and A_h is a technological constant, measuring the amount of consumption goods required to build one house. For now we assume that this technology is reversible, that is, real estate companies can turn houses back into consumption goods using the same technology. Thus the problem of a representative firm reads as

$$\begin{aligned} & \max_{I,C_h} P_h I - C_h \\ & \text{s.t.} \\ I \ = \ & A_h C_h \end{aligned} \tag{2}$$

Thus the equilibrium house price necessarily satisfies

$$P_h = \frac{1}{A_h}.$$

2.3 The Banking Sector

Let r_b denote the risk free interest rate on one-period bonds, to be determined in general equilibrium. Competitive banks take the refinancing costs $P_b = \frac{1}{1+r_b}$ as given. In addition we assume that issuing mortgages is costly; let r_w be the percentage real resource cost, per unit of mortgage issued, to the bank. This cost captures screening costs, administrative costs as well as maintenance costs of the mortgage (such as preparing and mailing a quarterly mortgage balance). As a consequence, the effective net cost of the banking sector for financing one dollar of mortgage, equals $r_b + r_w$.

Mortgage receipts P_m for a mortgage of size m' against real estate of size g' are determined by perfect competition in the banking sector, which implies that banks make zero expected profits for *each mortgage* they issue (as in Chatterjee et al. (2005)). Banks take account of the fact that household may default on their mortgage, in which case the bank recovers the collateral value of the house, which we assume to be a fraction $\gamma \leq 1$ of the value of the real estate. For ease of exposition we assume that the cost of mortgage generation is paid not when the mortgage is issued, but when it repaid, which implies that households defaulting on their mortgage payments also default on paying for the cost of generating the mortgage. Since this cost is fully priced into the mortgage, this is equivalent to assuming that the resource cost of mortgage issue is due at the receipt of the mortgage, but makes notation less cumbersome.

In order to define a typical banks' problem we first have to characterize the optimal default choice of a household. The cut-off level of depreciation, above which a household defaults on her mortgage is given as follows.. Define as $\kappa' = \frac{m'}{g'}$ the leverage (for $g' > 0$) of a mortgage m' backed by real estate g'. Then if the default cut-off $\delta^*(m', g')$ is in the interior of $D = [\underline{\delta}, \bar{\delta}]$ it is given by

$$m' = (1 - \delta^*(m', g'))P_h g'$$

and thus explicitly

$$\delta^*(m', g') = \delta^*(\kappa') = \begin{cases} \underline{\delta} & \text{if } 1 - \frac{\kappa'}{P_h} < \underline{\delta} \\ 1 - \frac{m'}{g' P_h} = 1 - \frac{\kappa'}{P_h} & \text{if } 1 - \frac{\kappa'}{P_h} \in [\underline{\delta}, \bar{\delta}] \\ \bar{\delta} & \text{if } 1 - \frac{\kappa'}{P_h} > \bar{\delta} \end{cases}$$

Evidently a household that obtains a mortgage $m' > 0$ without collateral, i.e. with $g' = 0$ defaults for sure. The receipt for this mortgage thus necessarily has to equal 0 as well, i.e. $P_m(s, g' = 0, m') = 0$. For other types of mortgages (m', g') with $m' > 0$ and $g' > 0$, the banks' problem is to choose the price $P_m(g', m')$ to maximize

$$\begin{aligned} & \max_{P_m(g',m')} \left[-m' P_m(g', m') + \frac{1}{1 + r_b + r_w} \left\{ m' F(\delta^*(\kappa')) + \gamma P_h g' \int_{\delta^*(\kappa')}^{\bar{\delta}} (1 - \delta') dF(\delta') \right\} \right] \\ = & \; m' \max_{P_m(g',m')} \left[-P_m(g', m') + \left(\frac{1}{1 + r_b + r_w} \right) \left\{ F(\delta^*(\kappa')) + \frac{\gamma P_h}{\kappa'} \int_{\delta^*(\kappa')}^{\bar{\delta}} (1 - \delta') dF(\delta') \right\} \right] \end{aligned} \quad (3)$$

In the presence of a government bailout, the government effectively subsidizes mortgages, in forms to be specified below.

2.4 The Government

As stated above the government levies endowment taxes τ on households to subsidize mortgages. Subsidies take the form of direct interest rate subsidies.[2]

Define the interest rate on a mortgage with characteristics (m', g') as

$$r_m(g', m') = \frac{1}{P_m(g', m')} - 1$$

where $P_m(g', m')$ is the mortgage pricing function without subsidy. Define as $r_m(g', m')$ and $\hat{P}_m(s, g', m')$ the corresponding entities with subsidy. Since the subsidy is a mortgage interest rate subsidy we model it as

$$r_m(g', m') = r_m(g', m') - sub(g', m')$$

and thus

$$\hat{P}_m(s, g', m') = \frac{P_m(s, g', m')}{1 - sub(g', m') * P_m(s, g', m')} \geq P_m(s, g', m')$$

The total subsidy for a mortgage of characteristics (s, g', m') is thus

$$\begin{aligned} sub(g', m') &= m' \left(\hat{P}_m(s, g', m') - P_m(s, g', m') \right) \\ &= m' P_m(s, g', m') \left(\frac{sub(g', m') P_m(s, g', m')}{1 - sub(g', m') P_m(s, g', m')} \right) \end{aligned}$$

and the total economy-wide subsidy is

$$Aggsub = \int sub(g', m') d\mu$$

Thus taxes have to satisfy

$$\begin{aligned} \tau \int y d\mu &= Aggsub \\ \tau &= \frac{Aggsub}{\bar{y}} \end{aligned} \quad (4)$$

where $\bar{y}$ is average (aggregate) endowment in the economy.

[2] Other forms of mortgage subsidies can be easily mapped into these interest rate subsidies.

2.5 Equilibrium

We are now ready to define a stationary recursive Competitive Equilibrium. Let $S = R_+ \times Y$ denote the individual state space.

Definition 1 *Given a government subsidy policy sub* $: R_+ \times R_+ \rightarrow R$, *a* ***Stationary Recursive Competitive Equilibrium*** *are value and policy functions for the households,* $v, c, h, b', m', g' : S \rightarrow R$, *policies for the real estate construction sector* I, C_h, *prices functions* P_l, P_h, P_b, *mortgage pricing functions* $P_m, \hat{P}_m : R_+ \times R_+ \rightarrow R$, *a government tax rate* τ *and an invariant distribution* μ *such that*

1. *(Household Maximization) Given prices* $P_l, P_h, P_b, \hat{P}_m$ *and government policies the value function solves* (1) *and* c, h, b', m', g' *are the associated policy functions.*
2. *(Real Estate Construction Company Maximization) Given* P_h, *policies* I, C_h *solve* (2).
3. *(Bank Maximization) Given* P_h, P_b, *the function* P_m *solves* (3)
4. *(Government Budget Balance) The tax rate function* τ *satisfies* (4), *given the functions* $m', P_m, \hat{P}_m, sub$.
5. *(Market Clearing in Rental Market)*

$$\int g'(s)d\mu = \int h(s)d\mu$$

6. *(Market Clearing in the Bond Market)*

$$\int b'(s)d\mu = \int m'(s)d\mu$$

7. *(Invariance of Distribution* μ*). The distribution* μ *is invariant with respect to the Markov process induced by the exogenous Markov process* π *and the policy functions* m', g', b'.

When we derive the welfare consequences of removing the mortgage interest subsidy, we measure aggregate welfare via a Utilitarian social welfare function in the steady state, defined as

$$\mathcal{WEL} = \int v(s)\mu(ds)$$

where μ is the invariant distribution.

3 Theoretical Results

In this section we state theoretical properties of our model the use of which makes the computation of the model easier. These results consist of a characterization of the mortgage interest rate, a partial characterization of the solution to the household maximization problem and, finally, bounds on the equilibrium rental price P_l.

3.1 Mortgage Interest Rates

From equation (3) and the fact that competition requires profits for all mortgages issued in equilibrium to be zero we immediately obtain a characterization of equilibrium mortgage payoffs as

$$\begin{aligned} P_m\left(g', m'\right) &= \left(\frac{1}{1+r_b+r_w}\right)\left\{F(\delta^*(m', g')) + \frac{\gamma P_h}{\kappa'}\int_{\delta^*(m',g')}^{\bar{\delta}}(1-\delta')dF(\delta')\right\} \\ &= \left(\frac{1}{1+r_b+r_w}\right)\left\{F(\delta^*(\kappa')) + \frac{\gamma}{A_h\kappa'}\int_{\delta^*(\kappa')}^{\bar{\delta}}(1-\delta')dF(\delta')\right\} \\ &= P_m(\kappa') \end{aligned}$$

with implied interest rates

$$r_m(\kappa') = \frac{1}{P_m(\kappa')} - 1$$

We note the following facts:

1. Mortgages are priced exclusively based on leverage $\kappa' = \frac{m'}{g'}$, that is $P_m(m', g') = P_m(\kappa')$ where it is understood that the optimal choice of κ' is a function of household characteristics s.
2. $P_m(\kappa')$ is decreasing in κ', strictly so if the household defaults with positive probability. Thus mortgage interest rates are increasing in leverage κ'.
3. Households that repay their mortgage with probability one have $\delta^*(\kappa') = \bar{\delta}$ and thus $P_m(\kappa') = \left(\frac{1}{1+r_b+r_w}\right)$, i.e. they can borrow at the rate $r_b + r_w$.
4. Since for all $\delta' > \delta^*(\kappa')$ we have $\gamma P_h \kappa'(1-\delta') < 1$, households that do default with positive probability tomorrow receive $P_m(\kappa') < \left(\frac{1}{1+r_b+r_w}\right)$ today, that is, they borrow with a risk premium $r_m(\kappa') > r_b + r_w$.

3.2 Simplification of the Household Problem

In the household problem define as

$$\begin{aligned} u(c; P_l) &= \max_{\tilde{c},h\geq 0} U(\tilde{c}, h) \\ & \quad s.t. \\ \tilde{c} + P_l h &= c \end{aligned}$$

Then the above problem can be rewritten as

$$v(s) = \max_{c,b',m',g'\geq 0}\left\{u(c; P_l) + \beta\sum_{y'}\pi(y'|y)\int_{\underline{\delta}}^{\bar{\delta}} v(s')dF(\delta')\right\}$$

$$\begin{aligned} \text{s.t.} \qquad & c + b'P_b + g'\left[P_h - P_l\right] - m'P_m(\kappa') = a \\ a'(\delta', h', m', g') &= b' + \max\{0, P_h(1-\delta')g' - m')\} + (1-\tau)y' \end{aligned}$$

3.3 Endogenous Borrowing Limit

We now want to show that it is never strictly beneficial for a household to obtain a mortgage with higher leverage than that level which will lead to default for sure. We will carry out the discussion in the next two subsections for the case without government bailout policy; the analysis goes through unchanged with government policy, mutatis mutandis. Define the leverage that leads to certain default by the smallest number $\bar{\kappa}$ such that

$$\begin{aligned} \delta^*(\bar{\kappa}) &= \underline{\delta} \\ \bar{\kappa} &= (1-\bar{\delta})P_h = \frac{1-\underline{\delta}}{A_h} \end{aligned}$$

Now we rewrite the budget constraint as

$$\begin{aligned} c + b'P_b + g'\left[P_h - P_l - \frac{m'}{g'}P_m(\frac{m'}{g'})\right] &= a \text{ or} \\ c + b'P_b + g'\left[P_h - P_l - \kappa' P_m(\kappa')\right] &= a \text{ or} \\ c + b'P_b + g'P(\kappa') &= a \end{aligned}$$

where

$$P(\kappa') = P_h - P_l - \kappa' P_m(\kappa')$$

is the is down payment per unit of real estate purchased, net of rental income. With this definition the total down payment for a house of size g' is given by $g'P(\kappa')$

For all $\kappa' \geq \bar{\kappa}$ we have

$$\begin{aligned} \kappa' P_m(\kappa') &= \left(\frac{1}{1+r_b+r_w}\right)\left\{\kappa' F(\underline{\delta}) + \gamma P_h \int_{\underline{\delta}}^{\bar{\delta}} (1-\delta')dF(\delta')\right\} \\ &= \left(\frac{1}{1+r_b+r_w}\right)\gamma P_h \int_{\underline{\delta}}^{\bar{\delta}} (1-\delta')dF(\delta') \\ &= \left(\frac{1}{1+r_b+r_w}\right)\gamma P_h(1-E(\delta')) \\ &= \bar{\kappa} P_m(\bar{\kappa}) \end{aligned}$$

and thus leveraging further does not bring extra revenues today and does not change resources obtained tomorrow (since the household defaults for sure and thus loses all real estate).[3] That is, the household faces an endogenous effective borrowing constraint of the form

$$\begin{aligned} \kappa' &\leq \bar{\kappa} \text{ or} \\ m' &\leq \left[\frac{1-\underline{\delta}}{A_h}\right] g' \end{aligned}$$

One can interpret $1-\bar{\kappa}$ as the minimum down payment requirement in this economy.

[3]The household is obviously indifferent between choosing $\kappa' = \bar{\kappa}$ and $\kappa' > \bar{\kappa}$; from here on we resolve any indifference of the household by assuming that in this case he chooses $\kappa' = \bar{\kappa}$.

3.4 Bounds on the Equilibrium Rental Price of Housing

3.4.1 An Upper Bound

Evidently for all admissible choices of the household it has to be the case that $P(s, \kappa') \geq 0$, otherwise the household can obtain positive cash flow today by buying a house; the default option on the mortgage guarantees that the cash flow from the house tomorrow is non-negative. Thus, the absence of this arbitrage in equilibrium requires $P(\kappa') \geq 0$. Therefore in particular

$$P(\kappa' = \bar{\kappa}) = P_h - P_l - \bar{\kappa} P_m(\kappa' = \bar{\kappa}) \geq 0$$

But

$$\begin{aligned} P(\bar{\kappa}) &= P_h - P_l - \bar{\kappa} P_m(\bar{\kappa}) \\ &= P_h - P_l - \left(\frac{1}{1 + r_b + r_w}\right) \gamma P_h (1 - E(\delta')) \geq 0 \\ P_l &\leq P_h - \left(\frac{1}{1 + r_b + r_w}\right) \gamma P_h (1 - E(\delta')) \end{aligned}$$

which places an upper bound on the equilibrium rental price.

Thus

$$\begin{aligned} P_l &\leq P_h - \left(\frac{\gamma P_h}{1 + r_b + r_w}\right)(1 - E(\delta')) \\ &= P_h * \left[\frac{r_b + r_w + \gamma E(\delta') + 1 - \gamma}{1 + r_b + r_w}\right] \end{aligned}$$

If $\gamma = 1$, this condition simply states that the rental price P_l cannot be larger that the user cost of housing $\frac{r_b + r_w + E(\delta')}{1 + r_b + r_w} P_h$.

3.4.2 A Lower Bound

Housing is an inherently risky asset. Since households are risk averse, for them to purchase the housing asset the expected return of housing at zero leverage has to be at least as high as the risk free interest rate. This implies

$$\left(\frac{1}{1 + r_b + r_w}\right) P_h \int_{\underline{\delta}}^{\bar{\delta}} (1 - \delta') dF(\delta') \geq P_h - P_l$$

Remembering that $P_h = \frac{1}{A_h}$ yields

$$\begin{aligned} \left(\frac{1}{1 + r_b + r_w}\right) P_h (1 - E(\delta')) &\geq P_h - P_l \text{ or} \\ P_l &\geq P_h \left[\frac{r_b + r_w + E(\delta')}{1 + r_b + r_w}\right] \end{aligned}$$

which states that the rental price of housing cannot be smaller than the (expected) user cost of housing in equilibrium (otherwise nobody would invest in housing, which cannot be an equilibrium given strictly positive demand for housing services by consumers).[4]

In summary, what these theoretical results buy us, besides being interesting in its own right, is a simplified household problem, a concise characterization of the high-dimensional equilibrium mortgage interest rate function and bounds for the equilibrium rental price, one of the endogenous prices to be determined in our analysis.

4 Calibration

4.1 Technology

Table 1: Technology Parameters

Parameter	Interpretation	Value	Target
A_h	Technology Const. in Housing Constr.	1.0	none (normalized)
π	Transition Matrix for Income	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
y	Income States	see below	Tauchen $\rho = 0.98, \sigma_e = 0.30$
γ	Foreclosure Technology	0.78	Pennington and Cross (2004)
	Depreciation process	see below	BEA, OFHEO data

Housing Technology: We normalize the housing construction constant to $A_h = 1.0$, and thus the price of one unit of housing to unity.

Income process: For a continuous state $AR(1)$ process of the form

$$\begin{aligned} \log y' &= \rho \log y + (1-\rho^2)^{0.5}\varepsilon \qquad (5) \\ E(\varepsilon) &= 0 \\ E\left(\varepsilon^2\right) &= \sigma_e^2 \end{aligned}$$

we can calculate the unconditional standard deviation to be σ_e and the one-period autocorrelation (persistence) to be ρ. Estimates for ρ in the literature vary somewhat, but center around values close to, but lower than 1. Motivated by the analysis by Storesletten et al. (2004) we select $\rho = 0.98$. The estimates for the standard deviation range from 0.2 to 0.4 (see Aiyagari (1994) for a discussion), so we choose $\sigma_\varepsilon = 0.3$.

[4] With $\gamma = 1$ we thus immediately obtain that the rental price of housing P_l equals its user cost $P_h\left[\frac{r_b + r_w + E(\delta')}{1 + r_b + r_w}\right]$. In fact, what happens in this equilibrium is that households purchase houses, leverage such that they default for sure tomorrow and the houses end up in the hand of the banks. Since these are risk-neutral, default is fully priced into the mortgage and banks receive the full (depreciated) value of the house, banks rather than households (which are risk averse) should and will end up owning the real estate.

We approximate the continuous state AR(1) with a 5 state Markov chain using the procedure put forth by Tauchen and Hussey (1991). We get the five labor productivity shocks $y \in \{0.3586, 0.5626, 0.8449, 1.2689, 1.9909\}$ and the following transition matrix:

$$\Pi = \begin{bmatrix} 0.7629 & 0.2249 & 0.0121 & 0.0001 & 0.0000 \\ 0.2074 & 0.5566 & 0.2207 & 0.0152 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0152 & 0.2207 & 0.5566 & 0.2074 \\ 0.0000 & 0.0001 & 0.0121 & 0.2249 & 0.7629 \end{bmatrix}$$

which generates the stationary distribution $(0.1907, 0.2066, 0.2053, 0.2066, 0.1907)$ and average labor productivity of one.

Foreclosure technology: Pennington-Cross (2004) estimates the default loss parameter γ. This is done by looking at liquidation sales revenue from foreclosed houses and comparing it to a market price constructed via the OFHEO repeat sales index. He finds that on average the loss is 22%. The loss varies only slightly depending on the age of the loan, between 20% for loans 16-20 months old to 26% for loans up to 10 months old, so it is safe to assume that in the model $\gamma = 0.78$ for all loans.

The depreciation process: We calibrate the house value depreciation process to attain realistic levels of default in the model while at the same generating the statistical properties of the house price appreciation and depreciation observed in the data.

According to the Mortgage Banker Association (MBA (2006)), the quarterly foreclosure rate has been about 0.4 percent in between 2000 and 2006. Abstracting from the possibility that one house may go in and out of foreclosure multiple within one given year, this implies that on an annual basis, banks start foreclosure proceedings on about 1.6 percent of their mortgages. The ratio of mortgages in foreclosure that eventually end in liquidation was about 25 percent in 2005, according to MBA (2006). Most homeowners avoid liquidation by either selling their property, refinancing their mortgage or just paying off the arrears. Consequently, only about 0.4 percent of mortgages actually end in liquidation the way we model it here. Given the unusually strong home price appreciation over the past years, we view this figure as the lower bound on the foreclosure rate and thus target a default rate of 0.5 percent.

We target two data moments of depreciation, the mean and the standard deviation. The mean depreciation for residential housing according to the Bureau of Economic Analysis was 1.48% between 1960 and 2002 (standard deviation 0.05%), computed as consumption of fixed capital in the housing sector (Table 7.4.5) divided by the capital stock of residential housing. With regards to the standard deviation of depreciation we utilize data from the Office of Federal Housing Enterprise Oversight (OFHEO). It models house prices as a diffusion process and estimates within-state and within-region annual house price volatility. The technical details can be found in the paper by Calhoun (1996). The ballpark figure for the eight census regions is $9-10\%$ volatility in the years 1998-2004. We use the upper bound $\sigma_\delta = 0.10$ to account for the fact that nationwide volatility is slightly higher than the within-region volatility.

We found that using a log-normal distribution for the appreciation of real estate in our model, i.e., $\log(1-\delta) \sim N(-\mu_\delta, \sigma_\delta^2)$ with a mean and standard deviation above, does not generate a sufficient share of foreclosures. Apparently, the right tail of the distribution is too thin. In order to get more realistic levels of mortgage default we incorporate a fatter right tail in the distribution of δ. Specifically, we mix a log-normal distribution with a uniform distribution on $\delta \in [0, \delta^{\max}]$ and assign a weight ω_U to the Uniform part of the distribution. In other words, the probability distribution function for depreciation is

$$f(\delta) = (1-\omega_U)\,\sigma_\delta^{-1/2}\phi\left(\frac{\log(1-\delta)+\mu_\delta}{\sigma_\delta}\right) + \omega_U I_{\delta\in[0,\delta^{\max}]}\frac{1}{\delta^{\max}} \tag{6}$$

where ϕ is the pdf of a standard Normal distribution. We now have four parameters $\mu_\delta, \sigma_\delta^2, \delta^{\max}, \omega_U$ to pin down three moments, the mean depreciation, the standard deviation and the share of mortgages in default that the model generates. Because we have one degree of freedom we chose to fix $\sigma^2 = 0.08$. Setting the remaining three parameters to

$$\begin{aligned} \omega_U &= 0.0080 \\ \delta^{\max} &= 0.8000 \\ \mu_\delta &= 0.0152 \end{aligned}$$

we exactly pin down $VAR(\log(1-\delta)) = 0.10$, $E(\delta) = 0.0148$ and also attain a realistic share of foreclosures of 0.55 percent, slightly above the lower bound of 0.40 percent derived above.

4.2 Preferences

Table 2: Preferencs Parameters

Parameter	Interpretation	Value	Target
σ	Risk Aversion	2.0000	standard
β	Time Discount Factor	0.8870	Net Worth/Income
θ	Share Parameter on Nondur. Cons.	0.8590	Exp. Share in BEA Data

For the utility function we start with a CES functional form:

$$u(c,h) = (1-\beta)\frac{(\theta c^\nu + (1-\theta)h^\nu)^{\frac{1-\sigma}{\nu}} - 1}{1-\sigma}$$

Notice that the first order conditions in the intratemporal optimization problem yield the condition

$$\frac{h}{c} = \left(P_l\frac{\theta}{1-\theta}\right)^{\frac{1}{\nu-1}}$$

which implies that in steady state θ and ν cannot be pinned down separately. We therefore choose $\nu = 0$, which reduces the period utility function to

$$u(c,h) = \frac{c^{\theta(1-\sigma)}h^{(1-\theta)(1-\sigma)} - 1}{1-\sigma}$$

which allows us to easily calibrate θ to the share of housing vs. non-housing consumption. For the CRRA parameter we pick $\sigma = 2$ as is standard in the literature. See, for example, Attanasio (1999) and Gourinchas and Parker (2002).

The time discount parameter is calibrated to match targets in the data using the benchmark economy. We use data from the 2001 Survey of Consumer Finances and restrict our attention to only bonds and net real estate, i.e., real estate holdings net of mortgages. We then compute net worth to income ratio as a) the unrestricted mean over all households, b) the restricted mean of all households having a net worth smaller than 50 times median income[5] and c) the mean within the median net worth bin using 25 equally-sized bins along household net worth. The results are reported in Table 3.

Table 3: Survey of Consumer Finances: Household Portfolio Statistics

	unrestricted mean	restricted mean	median bin
Net worth / income	2.7733	2.2666	1.2137

One can see from this table that the net worth ratio is affected substantially by extremely high net worth households. Since our model we will have trouble matching the extreme skewness of the wealth distribution we decided to match the moments at the median household. Using $\sigma = 2.0$ and $\beta = 0.8870$ generates a net worth to income ratio of about 1.20, close to the value observed in the data.

The share of housing in total consumption θ is set to generate a realistic share of housing in total consumption which has been steady at 14.1% over the last 40 years with a standard deviation of only about 0.5% according to NIPA data. Hence, we set $\theta = 0.8590$.

4.3 Mortgage Parameters

Table 4: Mortgage Parameters

Parameter	Interpretation	Value	Target
sub	Implicit Interest Rate Subsidy	40 BP	CBO (2001)
r_w	Mortgage administration fee	20 BP	half the subsidy

On the interest rate subsidy we take the view that the pass-through is 100% to make the case for the GSEs as positive as possible. The subsidy is then chosen to match the estimated implicit interest rate differential of around 40 basis points. As a first guess, we pick a mortgage administration cost r_w equal to half the subsidy, which corresponds to an annual cost of $200 for a $100,000 mortgage.

[5] This would eliminate the top 0.93% of the wealth distribution.

5 Results

Table 5: Numerical Results: Consequences of Removing the Subsidy

Variable	Subsidy	No Subsidy	Difference
%Sub	40bp	0bp	-100%
$Sub/\bar{y}$	0.591%	0%	-100%
P_l	0.0699	0.0707	1.14%
r_b	5.59%	5.39%	-2.58%
H	2.2592	2.2430	-0.72%
M	1.5685	0.1402	-91.06%
Default share	0.55%	0.25%	-54.55%
Median Net Worth	1.2032	1.1330	-5.90%
Median Bond Portfolio Share	57.32%	0.0%	-100%
$\mu(g' > 0)$	90.09%	89.98%	-0.12%
$\mu(g' > h)$	43.51%	36.27%	-16.64%
Wealth Gini	0.4594	0.4625	0.67%
EV^{SS}	-2.5881	-2.5799	0.32[6]

In this[7] section we document results from our thought experiment, that is, we compare steady states of economies with and without a mortgage interest rates subsidy of 40 basis points. Table 5 summarizes the main macroeconomic aggregates. We see that a labor income tax of about 0.6% is required to finance the interest rate subsidy in general equilibrium.

The main economic impact on households from removing the subsidy is to make mortgages less attractive by increasing the effective interest rate. As a consequence aggregate mortgages taken out by households decline sharply, by 91%. The overall impact on investment into housing is substantially less severe. The stock of housing property declines by a small 0.72%. Consequently the main adjustment of households in response to the removal of the subsidy is to reduce the leverage they choose for their housing finance decision. This effect is clearly shown in figure 1, which plots leverage κ' (the fraction $\kappa' = m'/g'$ of the real estate purchase financed by mortgages) as a function of cash at hand and the current income shock.[8] We see that while with the subsidy those households that actually purchase real estate choose a leverage of about 73-77% (and declining with cash at hand), without the subsidy most households choose a substantially lower leverage, which is in addition declining rapidly as households become wealthier. The reason for this substantial change in household borrowing behavior lies in the following: at an administration cost of 20 basis points and an interest rate subsidy of 40 basis points borrowing to finance housing is cheaper than borrowing at the risk free rate, at least until leverage becomes so high that the substantial default probability at this high leverage drives up effective interest

[7] Computed as consumption equivalent variation, that is $(EV_{no\ subs}/EV_{subs})^{1/(1-\sigma)}$

[8] In each of the five panels representing the five income shocks, we restrict our attention to the lower 99 percent of the cash at hand distribution.

rates. Note that since the asset that is being leveraged against, housing, is risky, so even if households can borrow at rate lower than the risk free rate, there is no arbitrage opportunity for them. Without the subsidy taking out mortgages comes at an interest premium of 20 basis points over the risk-free lending rate because of the administrative costs for mortgages. Thus not surprisingly the propensity of households to borrow against the house declines substantially, relative to the subsidy.

Figure 2 plots the share of net worth a household holds in the risk-free asset (i.e. bonds), rather than the risky asset (housing). Bond portfolio shares are increasing in a household's cash at hand, which is especially pronounced with the subsidy. While this behavior of households may sound counterintuitive at first (wealth-poorer households putting a larger share of their wealth into the risky, rather than the safe asset), it is in line with recent work on portfolio choice behavior (see Cocco et al. (2005) or Haliassos and Michaelides (2001)). These authors have argued that it should be households with high cash at hand that hold a higher share of their portfolio in the save asset. Households with high net worth tend to be people with high financial relative to human wealth (the present discounted value of future labor income). As such, these households expect to finance their current and future consumption primarily with capital income, whereas low cash-at-hand people tend to rely mostly on their labor income. Thus it is relatively more important for the high cash at hand people not to be exposed to a lot of financial asset return risk. In fact, since idiosyncratic labor income shocks and house depreciation shocks are uncorrelated in our model, housing is a good asset for hedging labor income risk (of course the bond is even better in this regard, but has a lower expected return). In addition, bond portfolio shares are substantially lower without the subsidy that with the subsidy. This is plausible in light of the fact that the real interest rate on the bond is lower without than with the subsidy, but the expected return on the risky asset, housing, rises because of the increase in the rental rate of houses P_l. As a consequence of this general shift in households' portfolio composition the share of bonds in the median net worth households' portfolio declines substantially; whereas this household holds 57% of its net worth in bonds with the subsidy, this share drops to zero without the subsidy.

The behavioral changes induced by a change in the subsidy have significant general equilibrium price effects. The reduction in the attractiveness of mortgages is also reflected in a substantial decline in aggregate default rates which fall from 0.55% to 0.25% per year. Note that since default and foreclosure is costly in terms of real resources, this reduction will be a nontrivial factor in the welfare evaluation of the change of the government's subsidy policy. Since the supply of housing declines because of the increase in its financing cost, the equilibrium rental price of housing increases, by slightly more than one percent. The equilibrium risk-free interest rate declines by 20 basis points in response to the removal of the subsidy since the demand for loans to finance house purchases collapses. Thus while the reduction of the subsidy increases the effective interest rate on mortgages, holding leverage constant, by 40 basis points, half of that increase is offset by the general equilibrium effect on the interest rate that a reduction in the demand for loans has. Furthermore, in the aggregate the reduction in the attractiveness of mortgages is also reflected in a substantial decline in aggregate default rates which fall from 0.55% to 0.25% per year. Note that since default and foreclosure is costly in terms of real resources, this reduction will be a nontrivial factor in the welfare evaluation of the change of the government's subsidy policy.

Given that the subsidy only benefits home owners one would expect that removing it has important consequences for the distribution of home ownership, wealth and welfare. Since the only asset in positive net supply is real estate and we have already documented that the stock of houses declines by 0.7% due to the removal of the subsidy, so does total wealth in the economy. Median net worth falls to a much larger extent, about 6%. This rising gap between average and median wealth suggests that the distribution of wealth becomes more dispersed without the subsidy, which is confirmed by an increase in the Gini coefficient for (net) wealth. Figure 3 suggests that this is mainly due to a larger fraction of households at the borrowing constraint and a slightly fatter right tail of the wealth distribution in the scenario without the subsidy. Thus if wealth inequality is a direct concern of policy makers the removal of the subsidy is counterproductive along this dimension.

Another potential rationale for (indirectly) subsidizing mortgage interest rates on the part of the government is to increase home ownership rates in the economy. Table 1 shows that if this is indeed the ultimate goal of the government, it is successful, according to our model. The fraction of households that own some real estate is slightly higher with than without the interest rate subsidy. The fraction of households that own at least as much real estate as they use for their own housing services consumption increases more substantially, from 36% to 44% in the case of a 40 basis points subsidy. While in our model owning real estate is somewhat disassociated from using it as owner occupied housing, the fraction of households with $g' \geq h$ may serve as a good proxy of home ownership rates in our model.[9]

We now turn to a discussion of the welfare consequences of the reform. In terms of aggregate welfare, removing the subsidy increases steady state welfare, as measured by consumption equivalent variation, by a non-negligible 0.32%. That is, household consumption (of both non-durables and housing services) in the steady state without the subsidy has to be increased by this percentage in all states of the world and for all households, such that a household to be born into the steady state with the subsidy is indifferent to being born into the steady state without the subsidy. While steady state welfare comparisons are often problematic since they ignore the welfare consequences of the transition towards the new steady state, in this case an explicit consideration of the transition path is likely to reinforce our steady state welfare gains from removing the subsidy. In the new steady state without the subsidy the aggregate (housing) capital stock is lower than in the initial steady state with the subsidy. Therefore along the transition part of the housing capital stock is being consumed (or more precisely, part of housing depreciation is not replaced and the freed resources are being consumed). This, we conjecture, results in additional aggregate welfare gains from removing the subsidy and thus strengthens our normative conclusion for removing the interest rate subsidy of the government.

Figure 4 sheds some light on who (that is, households with which characteristics) benefits from the subsidy. The figure plots the consumption equivalent gain for households with different income and cash at hand. The same comments about ignoring the welfare effects along the transition apply, as before. Therefore this plot should only be understood as a thought experiment of asking the following question: in which economy would someone with state (a, y) prefer to start her life; an economy with or without subsidy. Our quantitative results suggest that households with low wealth and low income prefer to live in an economy without subsidy while households

[9] Note, however, that nothing links the housing stock a household owns to the housing services she consumes. This need not be the same physical house, although it is convenient for the interpretation of our results to make that association.

with high current income $y \in \{y_4, y_5\}$ and high wealth benefit from the subsidy. This is due to mainly two reasons: first, the subsidy keeps interest rates on wealthy households' assets high (because of the stronger mortgage demand), and second, it provides these households (which invest and leverage substantially in real estate) with a direct subsidy for this investment strategy. On the other hand, poorer households derive a larger share of their income from labor income which is subject to the tax that finances the mortgage rate subsidy. Thus these households benefit more strongly from removing the subsidy and the tax that comes with it, especially if their wealth falls into a region where debt-financed investment into real estate is suboptimal and thus the subsidy does not apply to these households.

6 Conclusions

We constructed a model with competitive housing and mortgage markets where the government provides banks with insurance against an aggregate shock to their solvency. We used this model to evaluate aggregate and distributional impacts of this implicit government subsidy to housing. Our main findings are that the subsidy policy leads to a higher housing stock and more mortgages with higher leverage leading to more mortgage delinquencies. The subsidy mostly benefits high income and mostly high wealth households. The aggregate welfare effect of the subsidy is negative despite the higher aggregate holdings of the housing asset.

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Figures

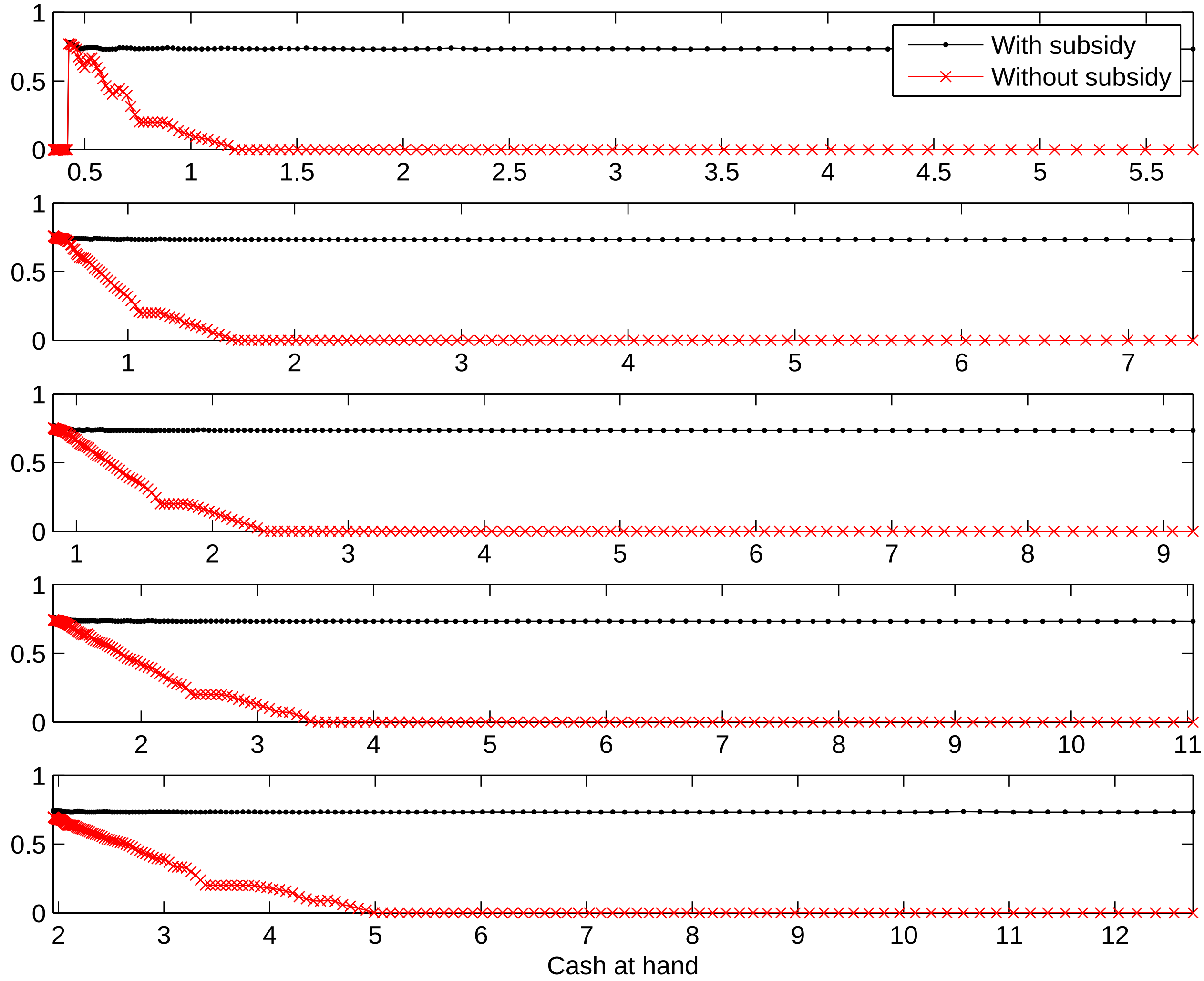

Figure 1: Leverage, as Function of Cash at Hand and Income

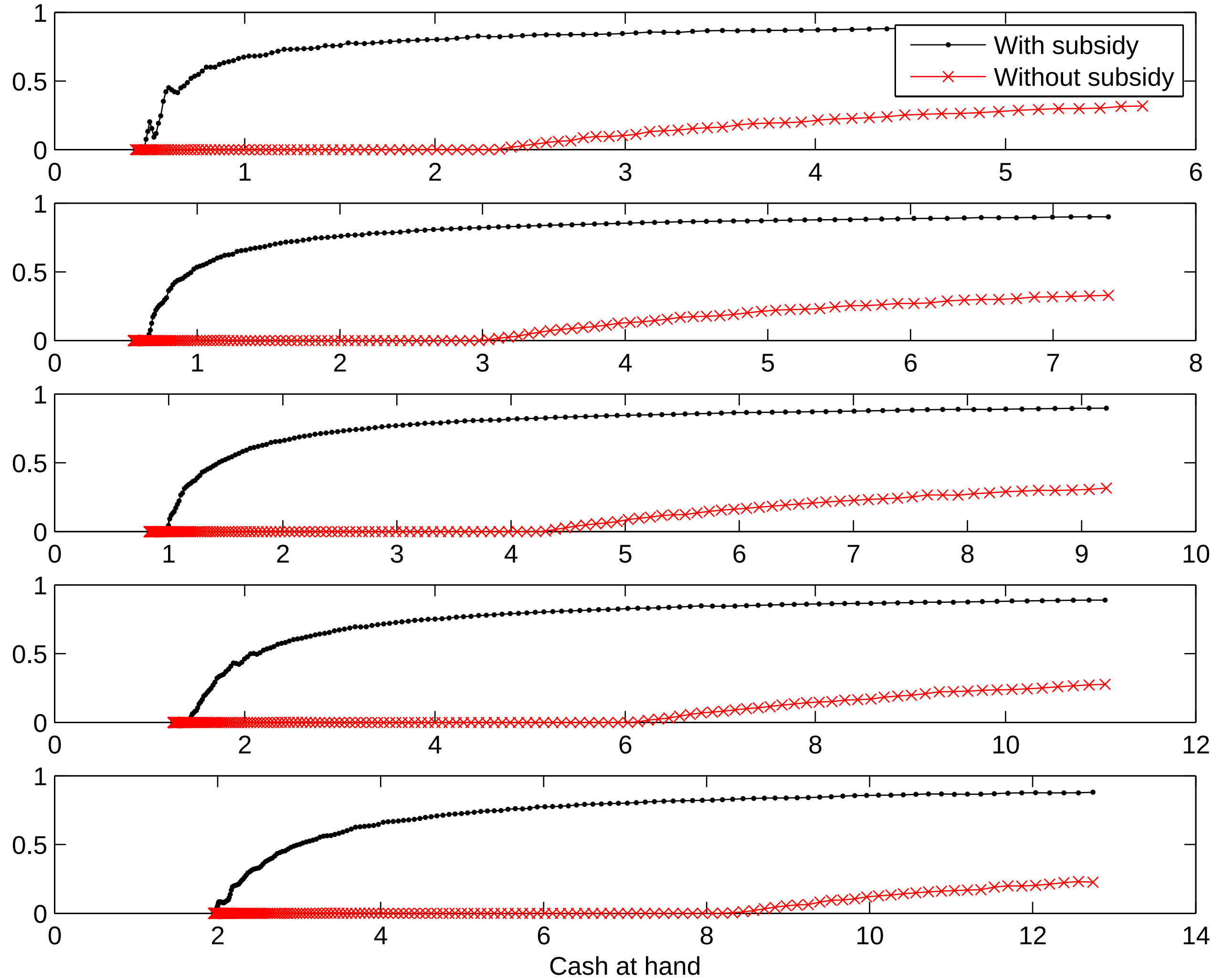

Figure 2: Bond Portfolio Share as Function of Cash at Hand and Income

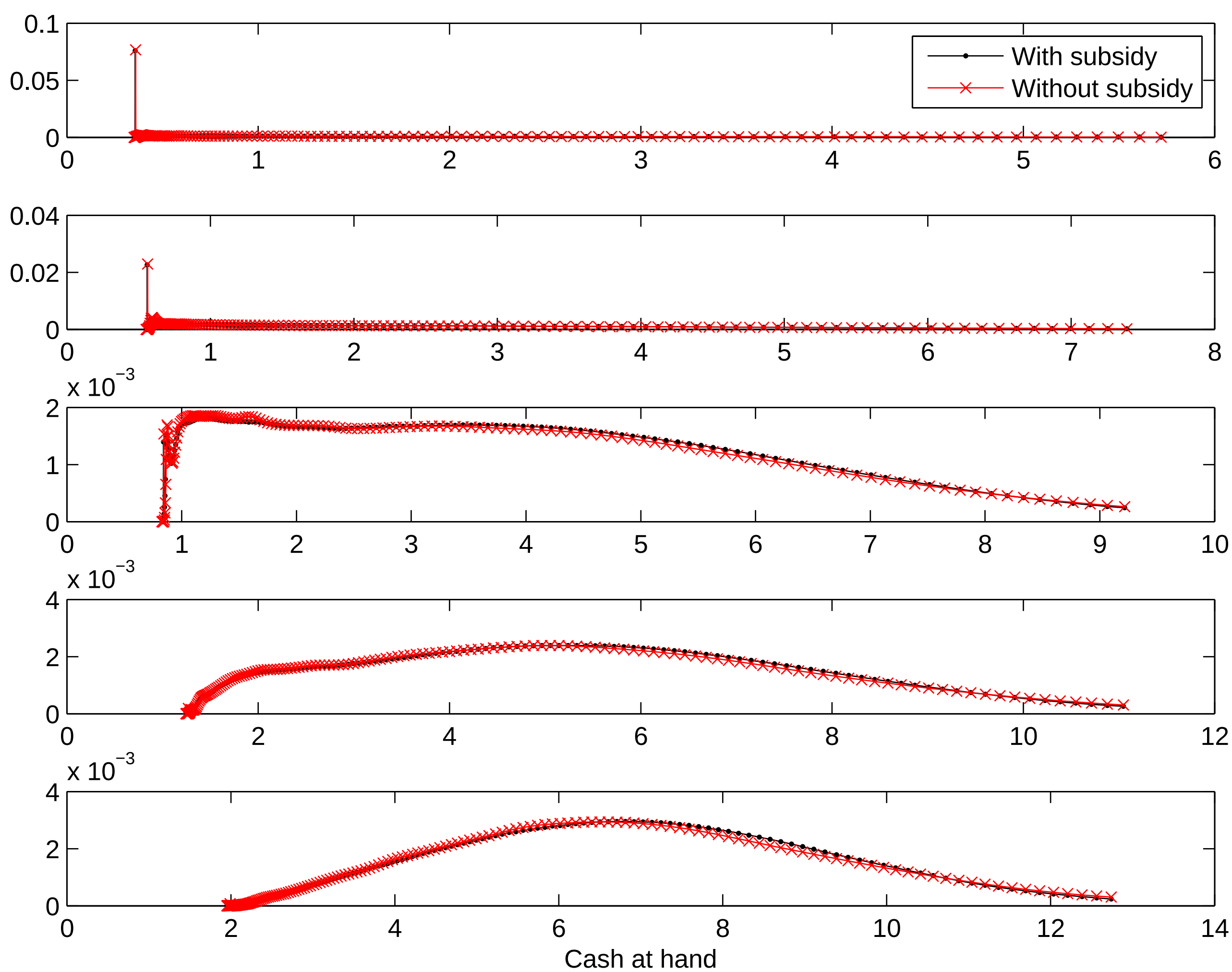

Figure 3: Invariant Distribution over Cash at Hand

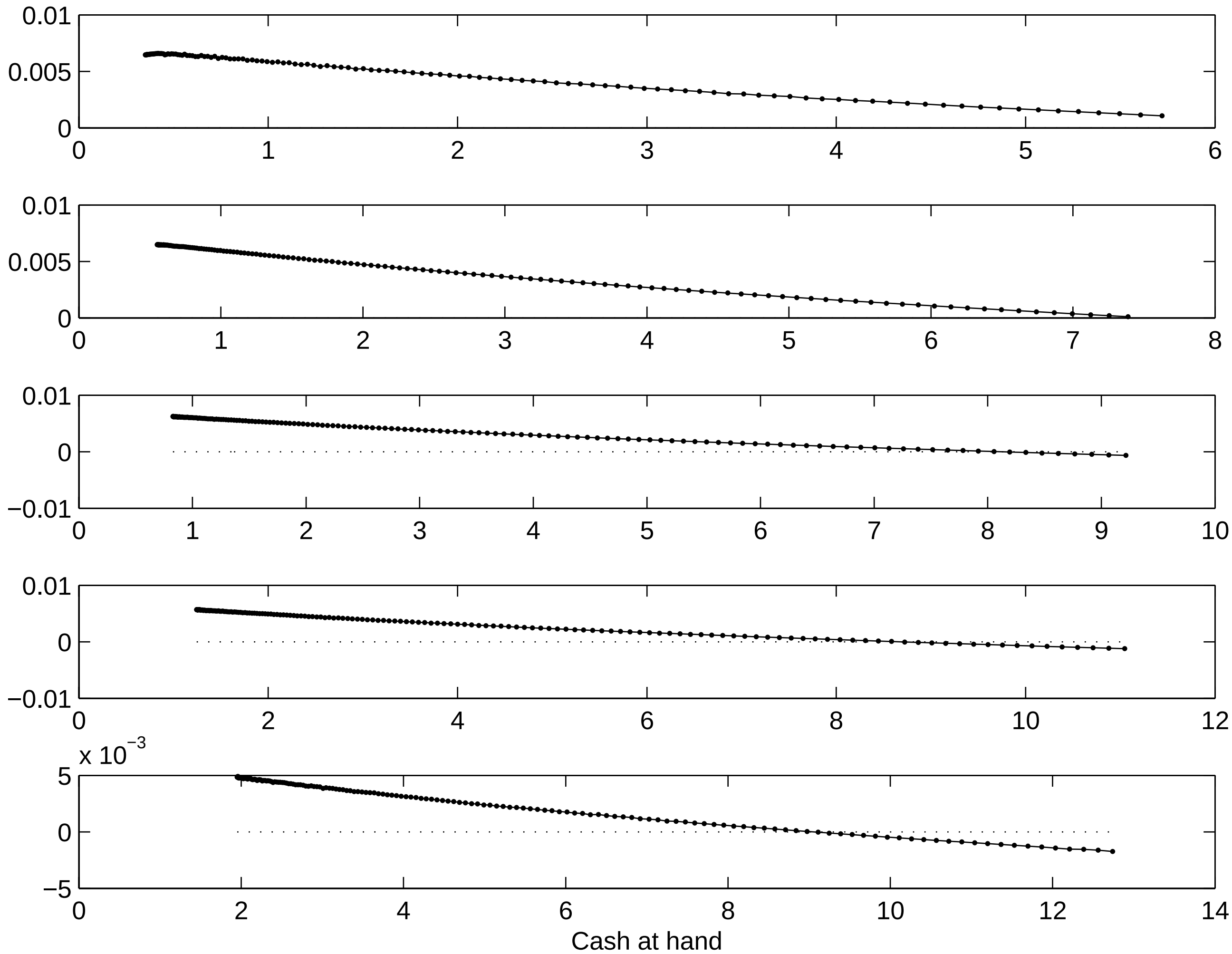

Figure 4: Steady State Welfare Consequences of Abolishing the Subsidy