The Life-Cycle Effects of House Price Changes

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ABSTRACT

We develop a life-cycle model to study the effects of house price changes on household consumption and welfare. The model explicitly incorporates the dual feature of housing as both a consumption good and an investment asset and allows for costly adjustments in housing and mortgage positions. Our analysis indicates that although house price changes have small aggregate effects, their consumption and welfare consequences on individual households vary significantly. In particular, the non-housing consumption of young and old homeowners is much more sensitive to house price changes than that of middle-aged homeowners. More importantly, while house price appreciation increases the net worth and consumption of all homeowners, it only improves the welfare of middle-aged and old homeowners. Young homeowners and renters are worse off due to higher life-cycle housing consumption costs.

Key Words: Life-cycle Model, Consumption, Savings, Housing, Mortgage
JEL Classification Codes: E21, R21

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ABSTRACT

We develop a life-cycle model to study the effects of house price changes on household consumption and welfare. The model explicitly incorporates the dual feature of housing as both a consumption good and an investment asset and allows for costly adjustments in housing and mortgage positions. Our analysis indicates that although house price changes have small aggregate effects, their consumption and welfare consequences on individual households vary significantly. In particular, the non-housing consumption of young and old homeowners is much more sensitive to house price changes than that of middle-aged homeowners. More importantly, while house price appreciation increases the net worth and consumption of all homeowners, it only improves the welfare of middle-aged and old homeowners. Young homeowners and renters are worse off due to higher life-cycle housing consumption costs.

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1. Introduction

The economics of housing is a subject of increasing interest to economists as well as policy makers. For a typical household in the U.S., housing is not only the single most important consumption good but also the dominant component of wealth. Recent research has focused on the link between house price changes and consumption allocations. This literature, however, has been mostly empirical and cannot address the welfare consequences of house price changes for individual households.[1]

When markets are complete, households can fully insure against their intertemporal consumption and income risks. House price changes will not affect their consumption and welfare. In reality, however, lacking proper financial products to generate full risk-sharing, households are exposed to house price uncertainties. Owning a home can alleviate the problem by purchasing future housing services at today's price. The hedging, however, is imperfect. Institutional and borrowing constraints frequently prevent young households with low levels of cash in hand from purchasing a house that matches their lifetime consumption need. Senior homeowners, in the meantime, are often forced to hold an equity position in their houses that lasts longer than their expected length of occupancy. This mismatch between life-cycle housing consumption need and housing investment position is worsened by the presence of lumpy housing adjustment costs.

In this paper, we investigate the effects of house price changes on household consumption and welfare both at the aggregate level and over the life cycle. We show that although house price changes have limited aggregate effects, the consumption and welfare consequences vary substantially at the individual household level, and depend crucially on a household's age and housing position. Specifically, the non-housing consumption of a young or old homeowner is more sensitive to house price changes than that of a middle-aged homeowner. More importantly, although house price appreciation increases the net worth and consumption of all homeowners, it only improves the welfare of middle-aged and old homeowners. Young homeowners and renters are worse off.

[1] See Case, Quigley, and Shiller (2003), Benjamin, Chinloy, and Jud (2004), and Campbell and Cocco (2004). Bajari, Benkard, and Krainer (2004) investigate aggregate welfare implications of house price changes in a complete market setting with perfect information.

These results stem from two key features of the model: the households' inability to insure against their lifetime income risks, and their inability to separate the dual role of housing as both a consumption good and an investment asset. A young homeowner is often liquidity-constrained because of his steep income profile and lack of access to credit. He is therefore more likely to take advantage of the relaxed collateral borrowing constraint afforded by house price appreciation and increase his non-housing consumption. An old homeowner has a short expected life horizon. Hence, he is more likely to capture the house wealth gains and increase his non-housing consumption accordingly. By contrast, a middle-aged homeowner has accumulated enough liquid savings to overcome the liquidity constraint and faces a relatively long expected life horizon. His consumption is thus least responsive to changes in house prices.

From the perspective of household welfare, house price appreciation does not lead to welfare improvement for all households. Young homeowners expect to upgrade their housing services as their income increases and their families expand. A positive house price shock, therefore, incurs net welfare losses for them, since the rise in the value of their existing homes is not large enough to compensate them for the rise in their lifetime housing costs. House price appreciation also lowers a renter's lifetime welfare, since he suffers from higher costs in acquiring housing services and yet does not receive any housing wealth gains. Only old homeowners receive net welfare gains.

Our paper extends the life-cycle consumption and savings literature that consists of, among many others, Zeldes (1989), Deaton (1991), and Carroll (1997), by explicitly introducing housing. The modeling strategy follows most closely that of Campbell and Cocco (2003), Cocco (2005), Fernandez-Villaverde and Krueger (2002), and Yao and Zhang (2005). While Campbell and Cocco (2003) examine a household's mortgage choice between a fixed rate loan and an adjustable rate loan, Cocco (2005) and Yao and Zhang (2005) study the effects of housing positions and house price risk on the portfolio allocation of liquid assets between stocks and bonds. Fernandez-Villaverde and Krueger (2002) investigate the role of durable goods in households' consumption and savings decisions in steady state.

Our paper also complements the recent empirical work devoted to the study of the effects of house price changes on consumption changes by explicitly modeling their theoretical relationships at both the aggregate and household levels in a life-cycle economy. While confirming

the positive effects of housing wealth gains on aggregate consumption found in the literature, we demonstrate that these positive net worth and consumption gains vary substantially across households and have large heterogeneous welfare consequences.[2]

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 characterizes households' consumption, housing, and mortgage decisions. Section 4 analyzes the effects of a permanent house price shock on household consumption and welfare and contrasts the results with those derived from liquid wealth gains. Section 5 concludes.

2. The Model Economy

2.1. Preferences and Endowments

We consider an economy where a household lives at most for the length of time T ($T > 0$). The probability that the household lives up to period t is given by the following survival function,

$$F(t) = \prod_{j=0}^{t} \lambda_j, \quad 0 \leq t \leq T, \tag{1}$$

where λ_j is the probability that the household is alive at time j conditional on being alive at time $j-1$, $j = 0, ..., T$. We set $\lambda_0 = 1$, $\lambda_T = 0$, and $0 < \lambda_j < 1$ for all $0 < j < T$.

The household derives utility from consuming a numeraire good C_t and housing services H_t, as well as from bequeathing wealth Q_t. The within-period utility takes the following modified Cobb-Douglas functional form,

$$U(C_t, H_t; N_t) = N_t \frac{[(\frac{C_t}{N_t})^{1-\omega}(\frac{H_t}{N_t})^{\omega}]^{1-\gamma}}{1-\gamma} = N_t^{\gamma} \frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma}, \tag{2}$$

where N_t denotes the exogenously given effective family size, which captures the economies of scale in household consumption as argued in Lazear and Michael (1980). We denote the bequest function as $B(Q_t)$.

[2]Other related recent papers include Flavin and Yamashita (2002), Gervais (2002), Ortalo-Magne and Rady (2003), Chambers, Garriga, and Schlagenhauf (2004), Davis and Heathcote (2003), Hurst and Stafford (2004), and Sinai and Souleles (2005).

In each period, the household receives income Y_t. Prior to the retirement age, which is set exogenously at $t = J$ $(0 < J < T)$, Y_t represents labor income and is given by

$$Y_t = P_t^Y \varepsilon_t, \tag{3}$$

where

$$P_t^Y = \exp\{f(t, Z_t)\} P_{t-1}^Y \nu_t \tag{4}$$

is the permanent labor income at time t. P_t^Y has a deterministic component $f(t, Z_t)$, which is a function of age and household characteristics Z_t. ν_t represents the shock to permanent labor income. ε_t is the transitory shock to Y_t. We assume that $\{\ln \varepsilon_t, \ln \nu_t\}$ are independently and identically normally distributed with mean $\{-0.5\sigma_\varepsilon^2, -0.5\sigma_\nu^2\}$, and variance $\{\sigma_\varepsilon^2, \sigma_\nu^2\}$, respectively. Thus, $\ln P_t^Y$ follows a random walk with a deterministic drift $f(t, Z_t)$.

After retirement, the household receives an income which constitutes a constant fraction θ $(0 < \theta < 1)$ of its preretirement permanent labor income,

$$Y_t = \theta P_J^Y, \quad \text{for } t = J, ..., T. \tag{5}$$

2.2. Housing and Mortgage Contracts

A household can acquire housing services through either renting or owning. A renter has a house tenure $D_t^o = 0$, and a homeowner has a house tenure $D_t^o = 1$. To rent, the household pays a fraction α $(0 < \alpha < 1)$ of the market value of the rental house. To become a homeowner, the household pays a portion ρ $(0 < \rho < 1)$ of the house value as closing costs to secure the title and mortgage. The house price appreciation rate $\tilde{r}_t^H$ follows an i.i.d. normal process with mean μ_H and variance σ_H^2. The shock to house prices is thus permanent and exogenous.[3]

[3]Flavin and Yamashita (2002), Campbell and Cocco (2003), and Yao and Zhang (2005) also assume that house price shocks are i.i.d. and permanent. Case, Quigley, and Shiller (2003) explore home price dynamics using data between 1982 and 2003. They find that home buyers' expectations are substantially affected by recent experience. Even after a long boom, home buyers typically have expectations that prices over the next 10 years will show double-digit annual price growth.

A household can finance home purchases with a mortgage. We assume that a mortgage loan initiated at time t matures at T.[4] The mortgage balance denoted by M_t needs to satisfy the following collateral constraint,

$$0 \leq M_t \leq (1-\delta)P_t^H H_t, \tag{6}$$

where $0 \leq \delta \leq 1$.[5] The borrowing rate r is time-invariant and the same as lending rate. A homeowner is required to spend a fraction ψ $(0 \leq \psi \leq 1)$ of the house value on repair and maintenance in order to keep the house quality constant.

At the beginning of each period, the household receives a moving shock, D_t^m, that takes a value of 1 if the household has to move for reasons that are not modeled here, and 0 otherwise. The moving shock does not affect a renter's housing choice since moving does not incur any cost for him. When a homeowner receives a moving shock ($D_t^m = 1$), he is forced to sell his house.[6] A homeowner who does not have to move for exogenous reasons can choose to liquidate his house voluntarily. The selling decision, D_t^s, is 1 if the homeowner sells and 0 otherwise. Selling a house incurs a transaction cost that is a fraction ϕ $(0 \leq \phi \leq 1)$ of the market value of the existing house. Additionally, the full mortgage balance becomes due upon the sale of the home. Following a home sale, a homeowner faces the same decisions as a renter coming into period t.

If the homeowner does not have to move for exogenous reasons and chooses to stay in the house, he has the option to convert some home equity to liquid wealth through a "cash-out" mortgage refinancing. D_t^r denotes the refinancing decision by the homeowner that takes a value of 1 if the homeowner refinances his mortgage, and 0 otherwise. Refinancing requires a cost that is a fraction τ $(0 \leq \tau \leq 1)$ of the house value. If the household decides not

[4]This specification of mortgage loan term follows Campbell and Cocco (2003). It eliminates time-to-maturity as a separable state variable and considerably simplifies the problem.

[5]By applying collateral constraints to both newly initiated mortgages and ongoing loans, we effectively rule out default. Default on mortgages is relatively rare in reality. According to the Mortgage Bankers Association, the seasonally adjusted three-month default rate for a prime fixed-rate mortgage loans is around 2 percent.

[6]We assume that house prices in the old and new locations are the same. In practice, however, house prices can differ across locations as in Sinai and Souleles (2005).

to refinance, it needs to pay down its mortgage balance according to either the fixed-rate mortgage amortization schedule set at the mortgage initiation,[7]

$$M_t = M_{t-1}(1+r) - \frac{M_{t-1}}{\sum_{j=t}^{T}(1+r)^{t-j-1}} = \frac{1-(1+r)^{t-T}}{1-(1+r)^{t-T-1}} M_{t-1}, \tag{7}$$

or the collateral borrowing constraint (equation 6). We use $\bar{l}_t = \frac{M_{t-1}(1+r)}{P_t^H H_{t-1}}$ to denote the household's beginning-of-the-period mortgage loan-to-value ratio, and $l_t = \frac{M_t}{P_t^H H_t}$ to denote the mortgage loan-to-value ratio upon mortgage initiation, mortgage payment, or refinancing.

2.3. Liquid Assets

In addition to holding home equity, a household can save in liquid assets which earn the same constant riskfree rate r as the borrowing rate. As a result, all mortgage refinances in our model are for consumption purposes only. We denote the liquid savings as S_t and assume that households cannot borrow non-collateralized debt, i.e.,

$$S_t \geq 0, \quad \text{for } t = 0, ..., T. \tag{8}$$

2.4. Wealth Accumulation and Budget Constraints

We denote the household's spendable resources or "wealth" upon home sale by Q_t.[8] It follows that for a renter ($D_{t-1}^o = 0$),

$$S_{t-1}(1+r) + P_{t-1}^Y \exp\{f(t, Z_t)\}\nu_t \varepsilon_t = Q_t, \tag{9}$$

and for a homeowner ($D_{t-1}^o = 1$),

$$S_{t-1}(1+r) + P_{t-1}^Y \exp\{f(t, Z_t)\}\nu_t \varepsilon_t + P_{t-1}^H H_{t-1}(1+\tilde{r}_t^H)(1-\phi) - M_{t-1}(1+r) = Q_t. \tag{10}$$

The intertemporal budget constraint, therefore, can be written as follows:

[7] Under an equal lending and borrowing rate, when refinancing is costly, a household always wishes to carry the maximum mortgage balance.

[8] Under this definition, conditional on selling his house, a homeowner's problem is identical to that of the renter and depends only on his age t, permanent income P_t^Y, and liquidated wealth Q_t.

(1) For a renter or a homeowner who decides to sell his house, if he chooses to rent in the current period ($D^o_{t-1} = D^o_t = 0$, or $D^o_{t-1} = D^s_t = 1$ and $D^o_t = 0$):

$$Q_t = C_t + S_t + \alpha P^H_t H_t. \tag{11}$$

(2) For a renter or a homeowner who decides to sell his house, if he chooses to buy a home in the current period ($D^o_{t-1} = 0$ and $D^o_t = 1$, or $D^o_{t-1} = D^s_t = D^o_t = 1$):

$$Q_t = C_t + S_t + (1 - l_t + \psi + \rho)P^H_t H_t. \tag{12}$$

(3) For a homeowner who decides to stay in the existing house without refinancing his mortgage in the current period ($D^o_{t-1} = D^o_t = 1$ and $D^s_t = D^r_t = 0$),

$$Q_t = C_t + S_t + (1 - l_t + \psi - \phi)P^H_t H_{t-1}. \tag{13}$$

(4) For a homeowner who decides to stay in the existing house and refinance his mortgage in the current period ($D^o_{t-1} = D^o_t = D^r_t = 1$ and $D^s_t = 0$),

$$Q_t = C_t + S_t + (1 - l_t + \psi + \tau - \phi)P^H_t H_{t-1}. \tag{14}$$

2.5. The Optimization Problem

We assume that upon death, a household distributes its spendable resources Q_t among "L" beneficiaries to finance their numeraire good consumption and housing services through renting for one period. Parameter "L" thus controls the strength of bequest motives. Under Cobb-Douglas utility, this assumption results in the beneficiary's expenditure on numeraire good and housing service consumption at a fixed proportion ($\frac{1-\omega}{\omega}$). Then the bequest function is defined by

$$B(Q_t) = L^\gamma \frac{[Q_t(\omega/\alpha P^H_t)^\omega (1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\gamma)}. \tag{15}$$

The household solves the following optimization problem at time $t = 0$, given its house tenure status (D^o_{-1}), after-labor income wealth (Q_0), permanent labor income (P^Y_0), house price (P^H_0), housing stock (H_{-1}), and mortgage balance ($M_{-1}(1+r)$):

$$\max_{\{C_t, H_t, S_t, D^o_t, D^s_t, D^r_t\}} E \sum_{t=0}^{T} \beta^t \Big\{ F(t)\, U(C_t, H_t; N_t) + [F(t-1) - F(t)]\, B(Q_t) \Big\}, \tag{16}$$

subject to the mortgage collateral borrowing constraint (equation 6), the mortgage amortization schedule (equation 7), the borrowing constraint on liquid asset (equation 8), wealth processes (equation 9 and 10), and the intertemporal budget constraints (equations 11 to 14). β is the time discount factor.

3. Model Calibration

In this section, we first calibrate the model parameters according to the U.S. economy. We then discuss the optimal decision rules for renters and homeowners, followed by the simulated life-cycle profiles of household consumption and saving.

3.1. Model Parameterization

The decision frequency is annual. A household enters the economy at age 20 ($t = 0$), and lives to a maximum of age 80 ($T = 60$). The mandatory retirement age is 65 ($J = 45$). The conditional survival rates are taken from the 1998 life tables of the U.S. National Center for Health Statistics (Anderson 2001). We use the 1995-2001 Survey of Consumer Finances (SCF) to calibrate the effective household size at each age (N_t). Specifically, we first calculate the average effective household size by the age of household head using the equivalence scale from the U.S. Department of Health and Human Services (Federal Register 2001). We then obtain a life-cycle profile of effective family size, using the synthetic cohort technique as described in Appendix B. Moving probabilities are calibrated to the average migration rates for non-housing related reasons between March 2001 and March 2002 in the Current Population Survey (CPS), as reported by the U.S. Census Bureau (2004).

For preferences, we set the relative risk aversion γ at 2. The housing preference parameter ω is set at 0.20, the average share of household housing expenditures found in the 2001 Consumer Expenditure Survey. We use the parameters for income process for a high school graduate as reported in Cocco, Gomes, and Maenhout (2005).[9] In particular, we choose values of 0.1 for the standard deviation of the permanent shock σ_ν and 0.27 for the standard deviation of the transitory shock σ_ε prior to retirement. Income replacement ratio at retirement is set at 0.68. Storesletten, Telmer, and Yaron (2004) report similar estimates for labor income processes.

The riskfree rate r is set at 0.03, approximately the average annualized post-WWII real return available on T-bills. For parameters that capture institutional features of the housing market, we set the annual rental cost α at 6 percent of the current house value. The annual maintenance and depreciation cost ψ is set at 1.5 percent of the house value, while the selling cost of a house ϕ is 6 percent of the market value of the house, the conventional fee charged by real estate agents. The mortgage collateral constraint is set at 80 percent.[10] Our housing purchase cost ρ is 1.0 percent of house value.[11] The refinancing cost τ is set at a relatively low 0.5 percent of the house value to implicitly allow for home equity access through home equity loans or home equity lines of credit in addition to mortgage refinances.

We assume that the housing appreciation rate $\tilde{r}_t^H$ is serially uncorrelated and has a mean of zero, which fell within the empirical range estimated by Goetzmann and Spiegel (2000).[12] The housing return volatility σ_H is set at 0.115, similar to estimates in Campbell and Cocco (2003) and Flavin and Yamashite (2002). We further assume that there is no correlation between housing returns and shocks to labor income in order to isolate the effects of house price changes.

[9] The measurement of labor income used here is broadly defined to include unemployment compensation, welfare, and transfers.

[10] Using the 1995 American Housing Survey, Chambers, Garriga, and Schlagenhauf (2004) calculate that the down payment fraction for first time home purchases is 0.1979 while the fraction for households that previously owned a home is 0.2462.

[11] Benett, Peach, and Peristiani (2001) report that an industry standard for the transaction cost for a new mortgage, excluding any up front points paid to the lender, is between 1 percent and 1.5 percent of the mortgage amount, or between 0.8 percent and 1.2 percent of the house value, assuming a 80 percent mortgage loan-to-value ratio.

[12] Based on 80 quarters of housing index data between March 1980 and March 1999, Goetzmann and Spiegel (2000) estimate that the real housing returns for the 12 largest Metropolitan Statistical Areas (MSAs) vary from -1.0 percent to 3.46 percent.

Finally, we choose the discount rate β, and the bequest strength parameter L to match the average wealth-labor income ratios and home ownership rates over the representative household's life cycle as observed in the U.S. economy. Table 1 summarizes our model parameterization. Details on obtaining a numerical solution are provided in Appendix A. To gain further insights of the model, we now turn to households' optimal decision rules, followed by simulated life-cycle consumption and saving profiles.

3.2. Optimal Housing and Consumption Decision Rules

3.2.1. A Renter's Optimal Decisions

A household entering the current period as a renter is described by its age (t) and wealth-permanent labor income ratio ($\frac{Q_t}{P_t^Y}$). Figure 1 presents the renter's optimal house tenure choice. The solid line represents the wealth-labor income ratio at which the household is indifferent between renting and owning. The household buys a home when its wealth-labor income ratio is above this line, and continues to rent otherwise. Under our parameterization, on average, renting costs more per period than owning the same house, i.e. $\alpha > r + \psi$. However, due to house purchasing and selling costs, a household prefers to own a house that matches its life-cycle income and wealth profiles so that the expected tenure in the house is sufficiently long. A household with a large amount of wealth on hand can afford the down payment for a house of desired value and therefore benefits more from home ownership.

The wealth-income ratio that triggers home ownership initially decreases with the household's age. This result is driven by the household's life-cycle income and mobility profiles. Since a young household faces high income growth rates, its desired house is large relative to its *current* income. A higher wealth-labor income ratio is needed to satisfy house down payment requirement to trigger home ownership. Additionally, young households have higher exogenous mobility rates which also raise the cost of owning. As the household approaches the terminal period, the threshold wealth-income ratio for home ownership moves up sharply reflecting the increasing importance of bequest motive, which is defined as a function of the bequeathed wealth net of house liquidation cost.

A renter's consumption and savings functions are similar to those identified in the precautionary savings literature with liquidity constraints (figures 2 and 3). At low wealth levels, a renter continues to rent and spends all his wealth on numeraire goods and rent payments. At slightly higher wealth levels, a renter saves a fraction of the wealth in liquid assets for intertemporal consumption smoothing and housing down payment. Note upon making a down payment toward purchasing a home, the household's liquid savings drop substantially.

3.2.2. A Homeowner's Optimal Decisions

A household entering the current period as a homeowner is characterized by its age (t), wealth-income ratio ($\frac{Q_t}{P_t^Y}$), house value-income ratio ($\frac{P_t^H H_{t-1}}{P_t^Y}$), and mortgage leverage ratio ($\frac{M_{t-1}(1+r)}{P_t^H H_{t-1}}$). Figure 4 plots a homeowner's endogenous house liquidation and mortgage refinancing decisions as a function of the household's beginning-of-the-period mortgage loan-to-value ratio and house value-income ratio, while holding his wealth-income ratio constant.[13] There are four regions of (in)actions: (1) the non-admissible region (N.A.) – the homeowner's mortgage loan-to-value ratio and house value-income ratio cannot take combinations in this region; (2) the stay region (STAY) – the homeowner stays in his existing house without mortgage refinancing; (3) the stay and refinance region (REFI) – the homeowner stays in his house and refinances his mortgage; and (4) the sell region (SELL) – the homeowner sells his house.[14]

Since a homeowner cannot take on unsecured debt, the value of his home equity cannot exceed his total wealth. The boundary of the non-admissible region is defined by $(1 - \bar{l}_t - \phi)P_t^H H_{t-1} = Q_t$. The homeowner stays in the house when his house value-labor income ratio is not too far from the optimal level he would have chosen as a renter. If he stays in the house, the homeowner can convert some home equity into liquid form through refinancing. This occurs when the homeowner's home equity is a large fraction of his total wealth, i.e., when his mortgage loan-to-value ratio is low or when his house value-wealth ratio is high.

For a homeowner who stays in his house, the composition of his wealth affects his non-housing consumption. More precisely, for a given house value-income ratio, as his leverage ratio decreases, the homeowner's liquid savings drop (figure 6), which in turn reduces his

[13] A homeowner that received an exogenous positive moving shock ($D_t^m = 1$) has to sell the house and his subsequent consumption and housing decisions are identical to those of a renter.

[14] For figures 4, 5, and 6, we hold the household age at 50 and the wealth-income ratio at 2.0.

non-housing consumption (figure 5). When the level of liquid assets becomes too low, the homeowner refinances his mortgage to gain access to illiquid home equity. The additional "cash" leads to immediate increases in both non-housing consumption and liquid savings.

3.3. Simulated Life-cycle Housing and Consumption Choices

We now examine a household's average life-cycle consumption and wealth accumulation through simulation. To do so, we first simulate permanent and transitory labor incomes, house prices, and moving shocks according to their respective governing stochastic processes. Then, we update state variables each period according to the optimal decision rules. For all simulated paths, households start at age 20 without housing or liquid wealth. We generate the time-series profiles of the optimal decisions by taking the average of 2 million simulations from $t = 0$ (age 20) to $t = 60$ (age 80).

The life-cycle profiles generated in our calibrated economy (figure 7) are similar to those found in the data (figure 8).[15] Specifically, home ownership rate is hump-shaped over age (figure 7a), while mortgage leverage decreases steadily with age (figure 7b).[16] Simulated housing consumption demonstrates a hump shape over the life cycle, matching that obtained in the data (figure 7c). As in the consumption literature with liquidity constraint and precautionary savings motives, non-housing consumption exhibits a hump shape (figure not shown). Due to significant selling costs, the housing consumption does not drop as quickly as non-housing consumption after peaking in the household's early 50s.

The proportion of net worth tied up in home equity exhibits a U-shaped pattern over the life cycle, consistent with empirical evidence in figure 7d, as well as in Flavin and Yamashita (2002). Intuitively, when the household is young, most of its wealth is committed to its house. As the household ages, liquid assets gradually surpass home equity as a primary vehicle of saving. After retirement, the household draws down its liquid assets first to supplement

[15] Appendix B provides details on the empirical estimations of the life-cycle profiles of home ownership rate, mortgage leverage ratio, house value, and home equity-net worth ratio using a pseudo-panel constructed from the 1995-2001 Survey of Consumer Finances.

[16] Compared to the data, the home ownership rate in our simulated economy increases more rapidly among young households. In addition, the average mortgage loan-to-value ratio generated by our model decreases more slowly prior to retirement than that in the data. These differences arise mainly due to the long amortization schedule assumed for the mortgage contract in the model, which substantially reduces the mortgage payments for young mortgage borrowers and makes housing more affordable.

retirement income in order to defer mortgage refinancing and house selling charges. Eventually, as a last resort, the household accesses housing wealth through mortgage refinancing or home sales to finance its retirement consumption.

The refinancing rate (figure not shown) demonstrates a bimodal pattern with young and old homeowners more likely to refinance than middled-aged homeowners. With equal lending and borrowing rates, a household refinances for consumption-smoothing purposes only. A young household does not have significant financial wealth and is more likely to be liquidity-constrained. Therefore, it benefits most from mortgage refinancing after a period of strong house appreciation. When an old homeowner has depleted his liquid savings, "cash-out" mortgage refinancing helps him further defer house selling costs and avoid the more expensive alternative means of acquiring housing services through renting.[17] By the terminal period, nearly 20 percent of all households in our simulation have sold their houses and switched back to renting, a number comparable to that reported by Venti and Wise (2000). The exit from home ownership is usually triggered by exogenous moving events.

Summary aggregate statistics for the benchmark model economy and their data counterparts are reported in Table 2. Our model generates statistics that replicate the targeted numbers reasonably well. However, the average net worth-labor income ratios are somewhat lower in the model than in the data for homeowners and renters respectively, while the home ownership rate is slightly higher.

4. Results

We now investigate the effects of house price changes on household consumption and welfare at both the aggregate levels and across individual households at different stage of the life cycle, using our benchmark economy. Then we explore the role of housing and mortgage adjustment costs. Lastly, we compare the effects of housing wealth gains to those from liquid asset.

[17] Hurst and Stafford (2004) find evidence that households use home equity to smooth consumption. Recent studies also suggest that seniors take money out of their homes through reduced expenditures on routine maintenance, alterations, and repairs (see Gyourko and Tracy 2003, and Davidoff 2004) instead of refinances or reverse mortgages (Feinstein and McFadden 1989).

To obtain the average effects of a permanent house price change on a household's consumption and welfare, for each age t, we separate households in our simulated economy into two groups: those who experienced a permanent house price appreciation and those who experienced a permanent house price depreciation. The two groups so constructed only differ in the house price shocks they receive at age t. Effectively, one can view the exercise as comparing the behavior of ex ante identical households in two different economic environments, one receiving a positive house price shock and the other receiving a negative house price shock.[18]

We focus on three economic variables: the average home ownership rate, the Marginal Propensity to Consume (MPC) out of housing wealth, and the household welfare. The MPC is calculated as the ratio of the mean consumption difference between households in the two different economic environments to the mean housing wealth difference.

Our welfare metric is defined as the necessary compensation to the households experiencing negative housing shocks that can bring their lifetime utility to the mean utility of households experiencing a positive house price shock. The compensation is in the form of a proportional increase in durable and non-durable consumptions for the remaining life span, as well as the bequeathed wealth upon death. Specifically, we first calculate by age the sum of value functions for the households experiencing a positive shock and a negative shock:

$$\overline{V}_t^j = \sum_{i=1}^{K_t^j} V_t^{ij} = \sum_{i=1}^{K_t^j} \left[v_t^{ij} \left(P_t^{Y\ ij} / (P_t^{H\ ij})^{\omega} \right)^{1-\gamma} \right], \quad t = 0, ..., T, \quad \text{and} \quad j = up,\ dn, \tag{17}$$

where j is the index for the state of housing returns and i is the index for the heterogenous agents in state j. K_t^j is the total number of agents at time t that fall in the j-th state of housing returns. Our utility cost measure can then be calculated as:[19]

$$\Omega_t = \left(\frac{\overline{V}_t^{up}}{\overline{V}_t^{dn}} \right)^{\frac{1}{1-\gamma}} - 1. \tag{18}$$

[18]Recall that in each period, the stochastic shocks to moving, housing returns, permanent and temporary components of labor incomes are approximated by a sixteen-state Markov chain. The shocks are assumed to be independent of each other and uncorrelated over time. Therefore, with a large number of simulations, the *ex ante* distribution of the state variables – home ownership status, wealth-income ratio, house value-income ratio, and mortgage loan-to-value ratios – should be identical for households experiencing either positive or negative house price shocks *ex post*.

[19]Aiyagari and McGrattan (1998) adopt a similar measure of welfare in an infinite horizon economy.

4.1. The Effects of House Price Appreciation on Consumption and Welfare

Table 3 reports the effects of house price changes on aggregate consumption and welfare.[20] A permanent two standard-deviation change in house prices has a rather limited effect on aggregate home ownership rate and total household welfare, with the former increasing 0.44 percent and the latter decreasing 0.98 percent. The aggregate MPC, at 4.06 percent, is within the range of empirical estimates,[21] albeit at the lower end.

The effects of permanent house price changes on individual households vary significantly as depicted in figure 9. Here we examine, by age, changes in housing positions along both the extensive margin and the intensive margin, and changes in non-housing consumption, net worth, and total welfare.

Figure 9a presents home ownership transitions upon the realization of the house price shock for homeowners at the beginning of the period, and figure 9b presents housing positions conditional on a household being a homeowner both before ("in") and after ("out") housing adjustments for the current period. In our simulation, young homeowners are more likely to exit home ownership after a negative house price shock than after a positive price shock. This is because if the household experiencing a negative house price shock is forced to move and sell the house for exogenous reasons, its wealth-income ratio is more likely to fall below the triggering level for home ownership. By contrast, for middle-aged homeowners, the proportion of households exiting home ownerships is not sensitive to house price changes. These households have accumulated significant wealth and can sustain home ownership despite changes in house prices. The home ownership exit patterns for senior homeowners are very similar across too groups, and are largely caused by the exogenous mobility shocks.

Young homeowners who choose to stay as homeowners tend to upgrade to bigger houses after a negative house price shock and do not actively adjust their house sizes after a positive house price shock. Middle-aged and old homeowners, on the other hand, tend to downgrade to smaller houses after a positive house price shock and do not change housing sizes after

[20] The statistics reported take into account the survival probability of households at different ages.

[21] Case, Quigley, and Shiller (2003) find that an additional dollar of house wealth increases household consumption by 3 to 15 cents. Benjamin, Chinloy, and Judd (2004) find the effect of housing wealth on household consumption of similar magnitude – 8 cents out of a dollar.

a negative house price shock. This asymmetry is primarily driven by the hump-shaped life-cycle housing consumption profile. A house price appreciation substitutes for active house up-sizing for young homeowners and accelerates down-sizing for old homeowners. A house price depreciation, in comparison, substitutes for active house downsizing for old homeowners and accelerates up-sizing for young homeowners.

Figures 9c and 9d depict the impact of house price shocks on homeowners' non-housing consumption and net worth.[22] The hump-shaped housing consumption profile over the life-cycle leads to a hump-shaped distribution of housing wealth gains. Not surprisingly, across all ages, those who experienced a permanent house price appreciation spend more on non-housing consumption than those who experienced a permanent house price depreciation. What is interesting, however, is that the non-housing consumption of young and old homeowners is more sensitive to house price changes than that of the middle-aged (figure 9e). As discussed earlier, young households are more likely to be liquidity-constrained. Housing appreciation, by increasing the collateral value, helps relax young homeowners' borrowing constraints and increase their non-housing consumption. Old homeowners have a short life horizon. They thus are more likely to capture the gains and increase their non-housing consumption and bequest accordingly. By contrast, middle-aged homeowners have accumulated enough liquid savings to overcome liquidity constraints. They also face a relatively long expected life span. Their consumption is, therefore, least responsive to house price changes.

Figure 9f presents the welfare consequences of house price changes for renters, homeowners, and households as a whole. Observe that house price appreciation unambiguously lowers renters' welfare since they have to bear the higher cost of acquiring lifetime housing services without receiving any housing wealth gains. According to our calculation, a positive house price shock of 11.5 percent leads to a welfare loss of around 4.5 percent, relative to the case of a negative house price shock of the same magnitude.

Surprisingly, although house price appreciation raises the non-housing consumption and networth positions for all homeowners (figures 9c and 9d), these consumption increases do not translate into welfare gains for all homeowners. In particular, a positive housing shock

[22] Under the assumption of Cobb-Douglas utility function, a renter responds to house price shocks by adjusting the level of housing service flows (H_t) while keeping housing expenditure ($\alpha P_t^H H_t$) unchanged.

incurs about a 2 percent utility loss for young homeowners in their late 20s to mid 30s. This result arises because young homeowners face a long horizon of future housing consumption, and on average, are expecting to move up in the housing ladder. Thus, their investment gains from existing housing positions are not sufficient to compensate them for the increase in their lifetime housing consumption costs. In our simulation, the break-even age for welfare is reached around age 50. Only households beyond the age of 65 receive a welfare gain exceeding 2 percent.

In summary, our analysis suggests although house price fluctuations have small aggregate effects, as argued in Sinai and Souleles (2005) and Bajari, Benkard, and Krainer (2004), they can create large distributional effects and these effects depend crucially on households' age and housing positions.

4.2. The Effects of Adjustment Costs

Housing market features large adjustment costs. To explore the quantitative impact of this adjustment cost, we now set the costs of house purchasing and selling, as well as mortgage refinancing, to zero. The new economy thus resembles that of Fernandez-Villaerde and Krueger (2002). The results are presented in figure 10.

In the absence of adjustment costs, the aggregate effects of a permanent house price appreciation remain small, as reported in table 3. At 0.53 percent, the increase in the average home ownership rate relative to the case of a negative house price appreciation is slightly higher than the benchmark case. Interestingly, the total welfare change is now positive. In our economy, house price appreciation affects a homeowner's welfare through three channels simultaneously. First it increases the household net worth position. Second it raises future housing consumption costs by (1) increasing the unit price of housing service flows; (2) increasing house selling costs and mortgage refinancing charges; and (3) increasing house maintenance costs. While a homeowner's wealth gains exactly offset the high unit costs of housing service flows for the existing house, higher adjustment and maintenance costs represent a deadweight loss in the economy. Yet, facing a new price vector, a household can reallocate its housing and non-housing expenditures. When housing adjustment costs are absent, the household can more

easily re-optimize over their consumption bundle, which leads to positive aggregate welfare effects.

The individual effects are still large and there are noticeable differences from the benchmark economy. Without housing adjustment costs, young households become homeowners much earlier, but they are also much more likely to switch back to renting after experiencing a negative house price shock (figure 10a). Old households never switch back to renting, even after receiving exogenous moving shocks, since house liquidation upon death is now costless. As seen in figure 10b and figure 10c, the life-cycle profile of housing consumption now follows more closely that of non-housing consumption, and demonstrates a pronounced hump. In addition, homeowners' non-housing consumption is much more responsive to changes in their housing wealth compared to the benchmark case. The MPCs out of housing wealth are much higher, and range from 18 percent for the very young to 6 percent for households in their 50s.

In terms of welfare, renters and young homeowners remain worse off by the house price appreciation. The welfare losses, however, are smaller and homeowners on average break even at a much younger age than the benchmark case, since earlier home ownership affords more households an opportunity to at least partially hedge house price risks. These results are intuitive. Without adjustment cost, households can freely reallocate expenditures between two consumption goods. This flexibility mitigates the adverse consequences of permanently higher house prices, since households can easily "re-balance." To summarize, the effects of housing adjustment costs on household consumption and welfare are quantitatively large and important.

4.3. Comparison with the Effects of Liquid Asset Gains

To investigate the role of the dual purpose of housing as both a consumption good and an investment asset, we examine the effects of wealth gains from a liquid asset as a proxy. The only liquid asset in our model is a riskless bond with a constant rate of return. We, therefore, study the effects of gains in liquid asset through temporary income shock since a household is

indifferent between a one-dollar gain from liquid asset and a one-dollar gain from transitory income in our economy.[23] The results are reported in figure 11.

Wealth gains from the liquid asset always lead to gains in both housing and non-housing consumptions. The MPCs out of liquid wealth range from 12 percent for young homeowners to around 6 percent for homeowners approaching retirement, much higher than the average MPC out of housing wealth gains in our benchmark economy, yet closer to the average MPC out of housing wealth gains without adjustment costs. The MPCs over the life cycle remain U-shaped reflecting the importance of liquidity and finite life horizon. The most interesting result concerns the welfare consequences. The wealth gains in liquid assets now lead to welfare improvements for all households. The reason is obvious. Unlike housing wealth gains, liquid asset gains are not accompanied by an increase in housing consumption costs.

5. Conclusions

In this paper, we developed a life-cycle model to study the effects of house price changes on household consumption and welfare. Several key features distinguish the model from the existing literature. First, we model housing choices along both the extensive margin of owning versus renting and the intensive margin of house value. Second, we introduce a long-term fixed-rate mortgage contract with a collateral requirement for financing house purchases. Third, we explicitly distinguish between liquid savings and illiquid home equity by accounting for house liquidation and mortgage refinancing costs.

Our analysis indicates that although the aggregate consequences of a permanent house price increase on a household's consumption and welfare are small, its effects at the individual household level vary significantly, depending on a household's age and home ownership status. Specifically, the non-housing consumption of young and old homeowners are more responsive to house price changes than that of middle-aged homeowners. More importantly, while middle-aged and old homeowners benefit from house price appreciation, renters and young homeowners are worse off.

[23] Since retired households no longer face any income risk, we restrict our discussion to households below age 65.

Our analysis also points out that housing adjustment costs are important quantitatively in explaining the large distributional effects. A complete elimination of the distributional effects, however, requires innovative financial products that separate the dual role of housing as both a consumption good and an investment asset.

References

Aiyagari, R. S., and Ellen McGrattan, 1998, The Optimal Quantity of Debt, *Journal of Monetary Economics*, 42(3), 847-69.

Anderson, Robert N., 2001, United State Life Tables 1998, *National Vital Statistics Reports*, 48, 1-40.

Bajari, Patrick, C. Lanier Benkard, and John Krainer, 2004, House Prices and Consumer Welfare, NBER working paper No. W9782.

Benett, Paul, Richard Peach, and Stavros Peristiani, 2001, Structual Change in the Mortgage Market and the Propensity to Refinance, *Journal of Money, Credit and Banking*, 33(4), 955-75.

Benjamin, John, Peter Chinloy, and Donald G. Jud, 2004, Real Estate Versus Financial Wealth in Consumption, *Journal of Real Estate Finance and Economics*, 29(3), 341-54.

Campbell, John, and Joao F. Cocco, 2003, Household Risk Management and Optimal Mortgage Choice, *Quarterly Journal of Economics*, 118, 1149-1194.

Campbell, John, and Joao F. Cocco, 2004, How Do House Prices Affect Consumption? Evidence from Micro Data, working paper, Harvard University and London Business School.

Carroll, Christopher D., 1997, Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis, *Quarterly Journal of Economics*, 112(1), 1-55.

Case, Karl E., John M. Quigley, and Robert T. Shiller, 2003, Comparing Wealth Effects: The Stock Market versus the Housing Market, working paper, University of California, Berkeley.

Chambers, Matthew, Carlos Garriga, and Don E. Schlagenhauf, 2004, Accounting for Changes in the Homeownership Rate, working paper, Florida State University.

Cocco, Joao F., 2005, Portfolio Choice in the Presence of Housing, *Review of Financial Studies*, 18, 535-567.

Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout, 2005, Consumption and Portfolio Choice over the Life-Cycle, *Review of Financial Studies*, 18, 491-533.

Davidoff, Thomas, 2004, Maintenance and the Home Equity of the Elderly, working paper, University of California at Berkeley.

Davis, Morris, and Jonathan Heathcote, 2003, Housing and the Business Cycle, *International Economic Review*, forthcoming.

Deaton, Angus, 1991, Saving and Liquidity Constraints, *Econometrica*, 59 (5), 1221-48.

Deaton, Angus, 1997, The Analysis of Households Surveys: A Microeconomic Approach to Development Policy. Baltimore and London: Johns Hopkins University Press for the World Bank.

Feinstein, Jonathan, and Daniel McFadden, 1989, The Dynamics of Housing Demand by the Elderly: Wealth, Cash Flow, and Demographic Effects, The Economics of Aging, National Bureau of Economic Research Project Report series Chicago and London: University of Chicago Press, 55-86.

Fernandez-Villaverde, Jesus, and Dirk Krueger, 2002, Consumption and Saving over the Life Cycle: How Important Are Consumer Durables? *Proceedings of the 2002 North American Summer Meetings of the Econometeric Society.*

Flavin, Marjorie, and Takashi Yamashita, 2002, Owner-Occupied Housing and the Composition of the Household Portfolio over the Life-cycle, *American Economic Review*, 92, 345-362.

Gervais, Martin, 2002, Housing Taxation and Capital Accumulation, *Journal of Monetary Economics*, 49(7), 1461-89.

Goetzmann, William N., and Matthew Spiegel, 2000, The Policy Implication of Portfolio Choice in Underserved Mortgage Markets, working paper, Yale University.

Gyourko, Joseph, and Joseph Tracey, 2003, Using Home Maintenance and Repairs to Smooth Consumption, working paper, Wharton School, University of Pennsylvania.

Hurst, Erik, and Frank Stafford, 2004, Home is Where the Equity is: Liquidity Constraints, Refinancing, and Consumption, *Journal of Money, Credit and Banking*, 36(6), 986-1004.

Lazear, Edward P., and Robert T. Michael, 1980, Family Size and the Distribution of Real Per Capita Income, *American Economic Review*, 70(1), 91-107.

Ortalo-Magné, François, and Steven Rady, 2003, Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints, working paper, University of Wisonsin-Madison.

Sinai, Todd, and Nicholas S. Souleles, 2005, Owner-Occupied Housing as a Hedge Against Rent Risk, *Quarterly Journal of Economics*, forthcoming.

Speckman, P., 1988, Kernel Smoothing in Partial Linear Models, *Journal of the Royal Statistical Society*, 50, 413-36.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron, 2004, Consumption and Risk Sharing over the Life Cycle, *Journal of Monetary Economics*, 51, 609-633.

Tauchen, George, and Robert Hussey, 1991, Quadrature-Based Methods for Obtaining Approximate Solutions for Nonlinear Asset Pricing Models, *Econometrica*, 59, 371-396.

U.S. Office of the Federal Register, National Archives and Records Administration, 2001 Federal Register.

U.S. Department of Labor, Bureau of Labor Statistics, 2003, Consumer Expenditures in 2001, Report 966.

U.S. Census Bureau, Population Division, 2004, Geographic Mobility: March 2001 to March 2002.

U.S. Census Bureau, Housing and Household Economic Statistics Division, 1992, 1991 Residential Finance Survey.

Venti, Steven, and David Wise, 2000, Aging and Housing Equity, NBER working paper 7882.

Yao, Rui, and Harold H. Zhang, 2005, Optimal Consumption and Portfolio Choices with Risky Labor Income and Borrowing Constraints, *Review of Financial Studies*, 18, 197–239.

Zeldes, Stephen P., 1989, Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence, *Quarterly Journal of Economics*, 104(2), 275-98.

Appendix A: Model Simplifications and Numerical Solutions

An analytical solution for our problem does not exist. We thus derive numerical solutions through value function iterations. Given the recursive nature of the problem, we can rewrite the intertemporal consumption and investment problem as follows:

$$V_t(X_t) = \max_{A_t} \left\{ \lambda_t \left[N_t^{\gamma} \frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma} + \beta E_t[V_{t+1}(X_{t+1})] \right] + (1-\lambda_t) B(Q_t) \right\}, \tag{19}$$

where $X_t = \{D_{t-1}^o, P_t^Y, P_t^H, H_{t-1}, M_{t-1}, Q_t\}$ is the vector of endogenous state variables, and $A_t = \{C_t, H_t, S_t, D_t^o, D_t^s, D_t^r\}$ is the vector of choice variables.

We simplify the household's optimization problem by exploiting the scale-independence of the problem and normalize the household's continuous state and choice variables by its permanent income P_t^Y or house value $P_t^H H_t$. The vector of endogenous state variables is transformed to $x_t = \{D_{t-1}^o, q_t, \overline{h}_t, \bar{l}_t\}$, where $q_t = \frac{Q_t}{P_t^Y}$ is the household's wealth-permanent labor income ratio, $\overline{h}_t = \frac{P_t^H H_{t-1}}{P_t^Y}$ is the beginning-of-period house value to permanent income ratio, and $\bar{l}_t = \frac{M_{t-1}(1+r)}{P_t^H H_{t-1}}$ is the beginning-of-period mortgage loan-to-value ratio. Let $c_t = \frac{C_t}{P_t^Y}$ be the consumption-permanent income ratio, $h_t = \frac{P_t^H H_t}{P_t^Y}$ be the house value-permanent income ratio, $s_t = \frac{S_t}{P_t^Y}$ be the liquid asset-permanent income ratio, and $l_t = \frac{M_t}{P_t^H H_t}$be the mortgage loan-to-value ratio. The evolution of normalized endogenous state variables is then governed by:

$$q_{t+1} = \frac{s_t(1+r) + D_t^o h_t (1+\tilde{r}_{t+1}^H)[1 - l_t(1+r)/(1+\tilde{r}_{t+1}^H) - \phi]}{\exp\{f(t+1, Z_{t+1})\}\nu_{t+1}} + \varepsilon_{t+1}. \tag{20}$$

$$\overline{h}_{t+1} = D_t^o h_t \left[\frac{1+\tilde{r}_{t+1}^H}{\exp\{f(t+1, Z_{t+1})\}\nu_{t+1}} \right] \tag{21}$$

$$\bar{l}_{t+1} = l_t \left[\frac{1+r}{1+\tilde{r}_{t+1}^H} \right] \tag{22}$$

The household's budget constraints (11) to (14) can then be written as

$$q_t = c_t + s_t + \alpha h_t, \tag{23}$$

$$q_t = c_t + s_t + (1 - l_t + \psi + \rho) h_t, \tag{24}$$

$$q_t = c_t + s_t + (1 - l_t + \psi - \phi) \overline{h}_t, \tag{25}$$

$$q_t = c_t + s_t + (1 - l_t + \psi + \tau - \phi) \overline{h}_t. \tag{26}$$

Define $v_t(x_t) = \frac{V_t(X_t)}{[P_t^Y/(P_t^H)^\omega]^{1-\gamma}}$ to be the normalized value function, then the recursive optimization problem (19) can be rewritten as,

$$v_t(x_t) = \max_{a_t} \Bigg\{ \lambda_t \left[N_t^\gamma \frac{(c_t^{1-\omega} h_t^\omega)^{1-\gamma}}{1-\gamma} + \beta E_t[v_{t+1}(x_{t+1})(\exp\{f(t+1, Z_{t+1})\}\nu_{t+1}/(1+\tilde{r}_{t+1}^H)^\omega)^{1-\gamma}] \right] + (1-\lambda_t) L^\gamma \frac{[q_t(\omega/\alpha)^\omega(1-\omega)^{1-\omega}]^{1-\gamma}}{1-\gamma} \Bigg\},$$

subject to

$$c_t > 0, \quad h_t > 0, \quad s_t \geq 0, \quad l_t \leq 1-\delta,$$

and equations (21) to (26), where $a_t = \{c_t, h_t, l_t, s_t, D_t^o, D_t^s, D_t^r\}$ is the normalized vector of choice variables. Hence the normalization reduces the number of continuous state variables to three with P_t^Y no longer serving as a state variable and P_t^H and H_{t-1} combining into $P_t^H H_{t-1}$.

We discretize the wealth–labor-income ratio (q_t) into 320 grids equally-spaced in the logarithm of the ratio, and the house value-labor income ratio ($\overline{h}_t$) and the mortgage loan-to-value ratio ($\bar{l}_t$) into equally-spaced grids of 160. Due to the nonnegativity constraint for the holdings of the liquid asset, the state space is not a cube for a homeowner since the liquidated home equity value can not be larger than the value of the total wealth. I.e. only states that satisfy $\overline{h}_t(1-\bar{l}_t-\phi) \leq q_t$ are admissible. The boundaries for the grids are chosen to be wide enough so that our simulated time series path always falls within the defined state space.

Under the assumption that only liquidated wealth will be passed along to beneficiaries, the household's house tenure status and housing and mortgage positions do not enter the bequest function. At the terminal date T, $\lambda_T = 0$, and the household's value function coincides with the bequest function,

$$v_T(x_T) = L^\gamma \frac{[q_T(\omega/\alpha)^\omega(1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\gamma)}.$$

The value function at date T is then used to solve for the optimal decision rules for all admissible points on the state space at date $T-1$.

For a household coming into period t as a renter ($D_{t-1} = 0$), we perform two separate optimizations conditional on house tenure decision – renting or owning – for the current period. A renter's optimal house tenure choice for the current period is then determined by comparing the contingent value functions of renting and owning. If a renter keeps renting,

he optimizes over only one choice variable c_t, since $h_t = \frac{c_t \omega}{\alpha(1-\omega)}$. If a renter initiates home ownership in the current period, he needs to choose the optimal c_t and h_t simultaneously. To calculate the expected next period's value function, we use two discrete states to approximate the realizations of each of the three continuous exogenous state variables ($\ln \varepsilon, \ln \nu$, and $\tilde{r}_t^H$) by Gaussian quadrature (Taughen and Hussey 1991). Together with two states for the realizations of moving shocks, the procedure results in sixteen discrete exogenous states for numerical integration. For points that lie between grid points in the state space, depending on the household's current period house tenure choice, we use either a one-dimension or three-dimension cubic spline interpolation to approximate the value function.

For a household coming into period t as a homeowner, we perform two separate optimizations conditional on its refinancing decision for the current period. In both cases, the household cannot adjust its house value-income ratio, i.e. $h_t = \overline{h}_t$, but can adjust its numeraire consumption. We take the higher value of the two optimized value functions as the value function contingent on staying. The value function contingent on moving – either endogenously or exogenously – is the same as the value function of a renter who is endowed with the same wealth-income ratio (q_t) because the entire mortgage balance is due upon home sales and q_t is defined as net of house selling costs. We compare the value functions contingent on moving and staying to determine the optimal house liquidation decision. A homeowner who cannot afford the minimum mortgage payment and house maintenance cost has to sell his home. Under our assumption and parameterization, a homeowner always has positive amount of equity in his house after home sales and thus has no incentive to default. This procedure is repeated recursively for each period until the solution for date $t = 0$ is found.

Appendix B: Empirical Analysis

This appendix describes data sources and explains the nonparametric regressions used to construct our empirical regularities summarized in Figure 8.

Our data comes from the Survey of Consumer Finances (SCF), collected by the Federal Reserve Board. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of US families. Our sample years include 1995, 1998, and 2001. The term "household" used in the paper corresponds to the term "family" used in the SCF.[24] The term "household age" used in the paper corresponds to the age of the family head in the SCF.

Following Fernandez-Villaverde and Krueger (2002), we exploit the repeated nature of the survey to build a pseudo panel. New households entering the survey are a large randomly-chosen sample of the U.S. population and, consequently, they contain information about the means (home ownership, mortgage loan-to-value ratio, etc.) of the groups they belong to. This information can be exploited by interpreting the observed group means as a panel for estimation purposes. This method is known as the pseudo panel or synthetic cohort technique. We define 55 cohorts according to the birth year of the household (1915 to 1970) and follow them through the sample, generating a balanced panel. The average size of cells for all households is 334, and the average size of cells for homeowners is 240.

To relate age and household housing decisions, we estimate the partial linear model

$$y_{it} = constant + \beta_i cohort_i + \beta_t \gamma_t + m(age_{it}) + \epsilon_{it}, \tag{27}$$

where $cohort_i$ is a dummy for each cohort except the youngest one, and γ_t a dummy for each survey year except 1995, $m(age_{it}) = E(y_{it}|age_{it})$ is a smooth nonparametric function of age_{it}.

To identify the separate effects of time, age, and cohort effects, we assume that time effects are orthogonal to a time trend and that their sum is normalized to zero, i.e., we attribute linear trends in the data to a combination of age and cohort effects (Deaton 1997).

[24] "Household" is reserved by the SCF to denote the set of the "family" (technically known as the "primary economic unit") and any other individual that lives in the same household but it is economically independent.

The partial linear model is estimated using the two-step estimator proposed by Speckman (1988). The nonlinear part is estimated using the Gaussian Kernel with a bandwidth of 5. We regress y_{it} on $m(age_{it})$ to obtain residuals and then we project the residuals on the time and cohort effects. The constructed new adjusted values for y_{it} are nonparametrically regressed on age.

Table 1
Baseline Model Parameters

Parameter	Symbol	Value
Demographics		
Maximum life-cycle period	T	60
Mandatory retirement period	J	45
Preferences		
Relative risk aversion	γ	2
Bequest strength	L	6
Discount rate	β	0.930
Housing preference	ω	0.200
Labor Income and House Price Processes		
Standard deviation of permanent income shock	σ_υ	0.103
Standard deviation of temporary income shock	σ_ε	0.272
Income replacement ratio after retirement	θ	0.682
Mean real housing return	μ_H	0.000
Standard deviation of housing return	σ_H	0.115
Liquid Savings		
Risk-free interest rate	r	0.030
Housing and Mortgage		
Rental cost	α	0.060
Down payment requirement	δ	0.200
House selling cost	ϕ	0.060
Maintenance and depreciation cost	ψ	0.015
House purchasing cost	ρ	0.010
Mortgage refinancing cost	τ	0.005

Table 2
The Baseline Model Economy

Statistics	U.S. Data	Model
Average Home Ownership Rate	0.67	0.72
Average Networth-Labor Income Ratio	3.10	3.20
Homeowner	4.29	3.82
Renter	0.76	0.26
Average House Value-Labor Income Ratio	2.77	3.39

Note: The average statistics for the U.S. data are calculated for a representative household over its life cycle using the synthetic cohort technique outlined in Appendix B. Data source: Survey of Consumer Finances, 1995-2001.

Table 3
The Effects of House Price Changes on Consumption and Welfare

Statistics	Benchmark	No adjustment cost
Changes in Home Ownership Rate (%)	0.44	0.53
Marginal Propensity to Consume (%)	4.06	6.94
Changes in Total Welfare (%)	-0.98	0.16

Note: With the exception of Marginal Propensity to Consume, the statistics are calculated as percentage changes in home ownership rate and consumer welfare in the economy experiencing a positive house price shock relative to the economy experiencing a negative house price shock.

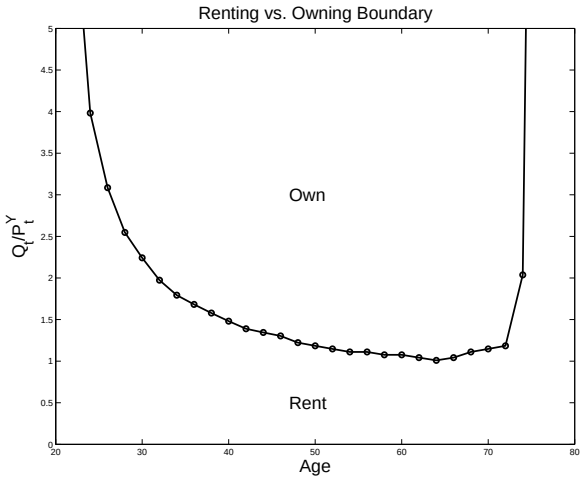

Figure 1. A Renter's House Tenure Decision as a Function of His Age and Net Worth-Permanent Labor Income Ratio

Figure 2. A Renter's Optimal Housing Consumption

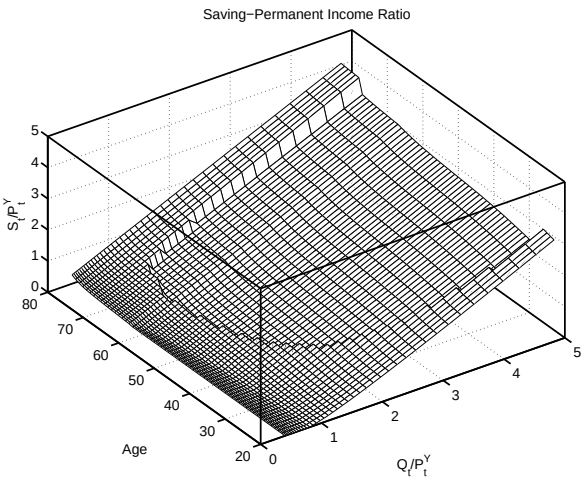

Figure 3. A Renter's Optimal Liquid Saving Decision

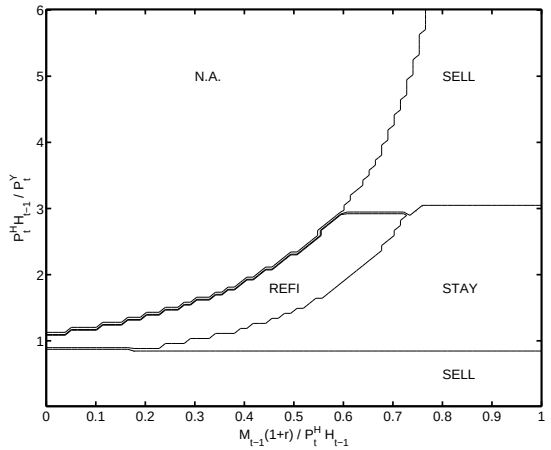

Figure 4. A Homeowner's Optimal House Tenure Choice

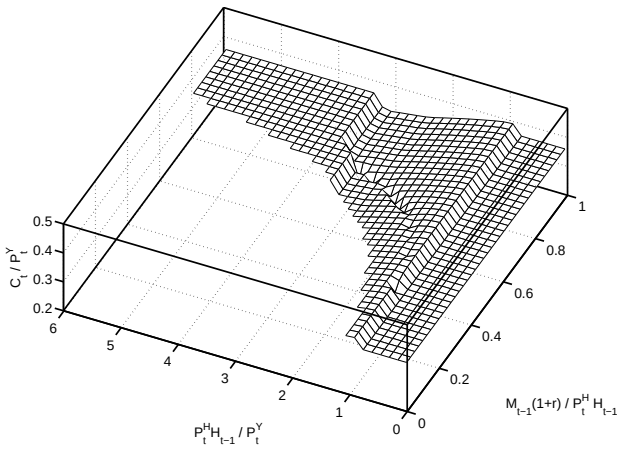

Figure 5. A Homeowner's Optimal Numeraire Good Consumption

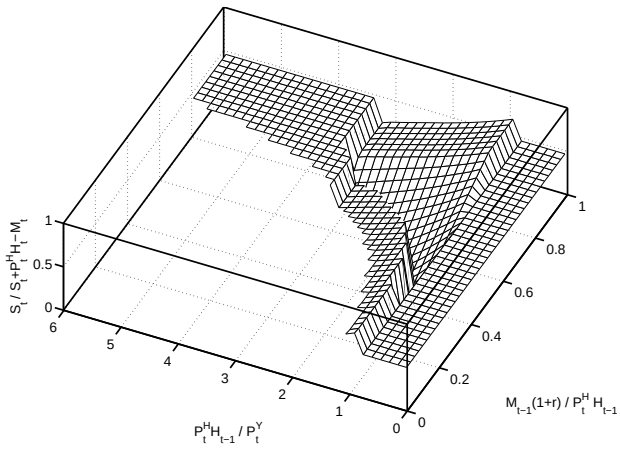

Figure 6. A Homeowner's Liquid Savings

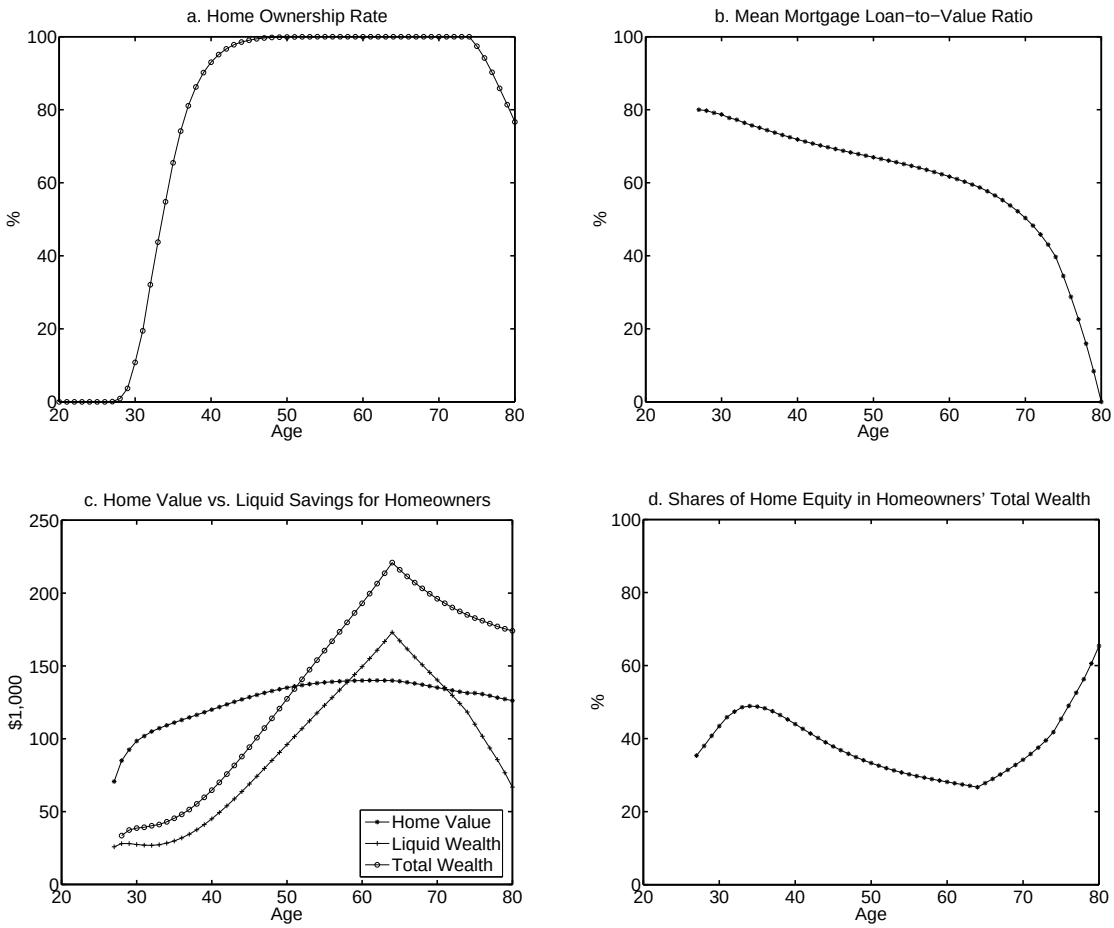

Figure 7. Optimal Life-cycle Housing and Consumption Decisions—Baseline Case

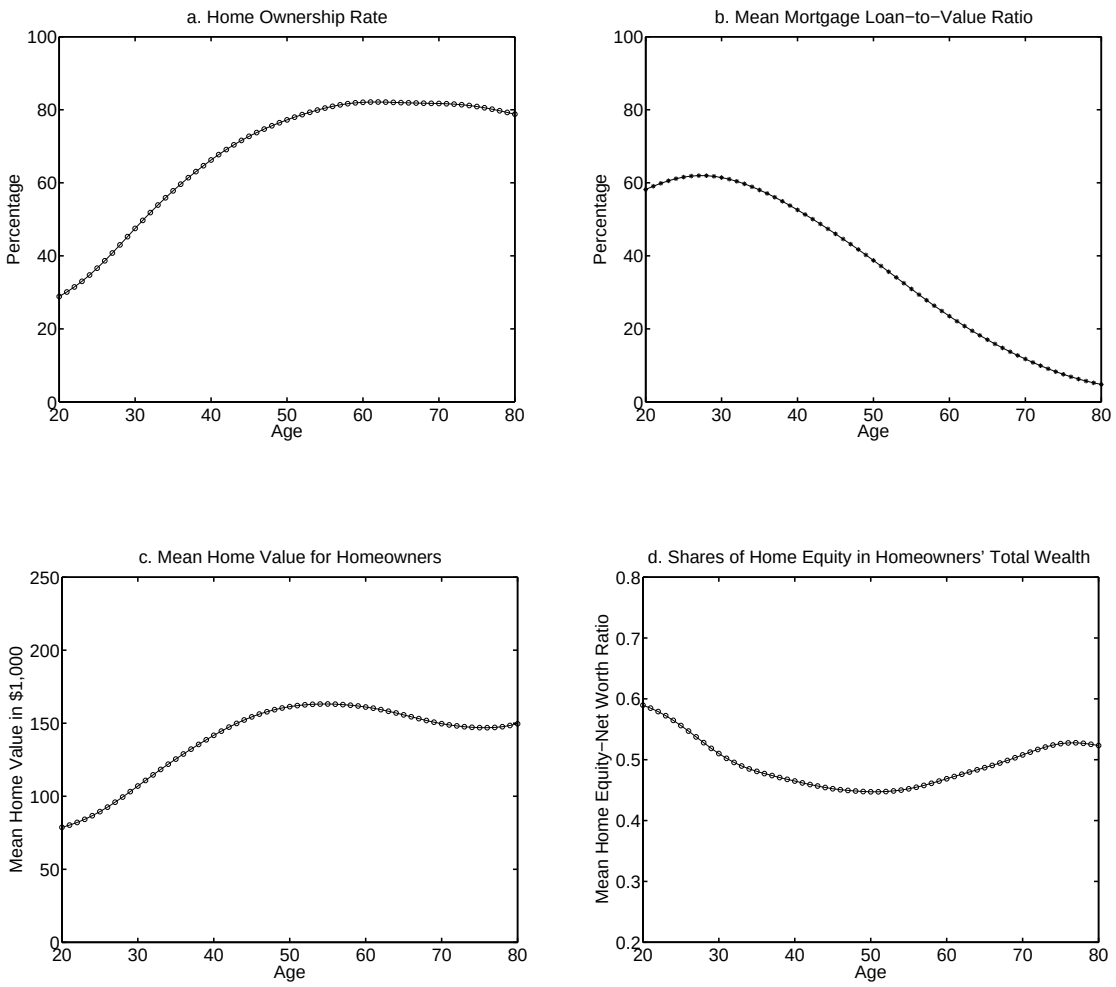

Figure 8. Empirical Life-Cycle Housing and Mortgage Choices Based on SCF Data from 1995 to 2001

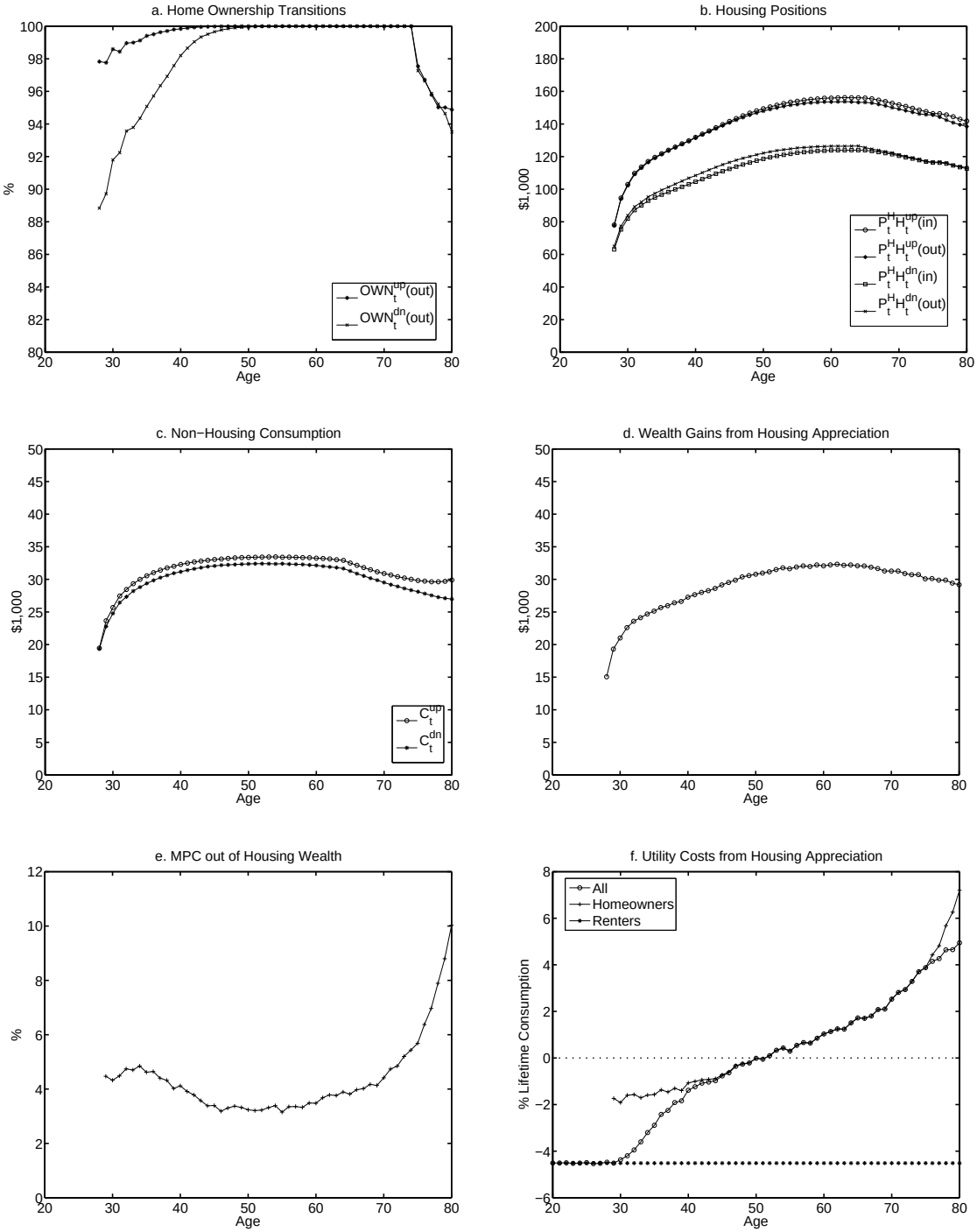

Figure 9. The Consumption and Welfare Effects of House Price Shocks under Baseline Parameterizations

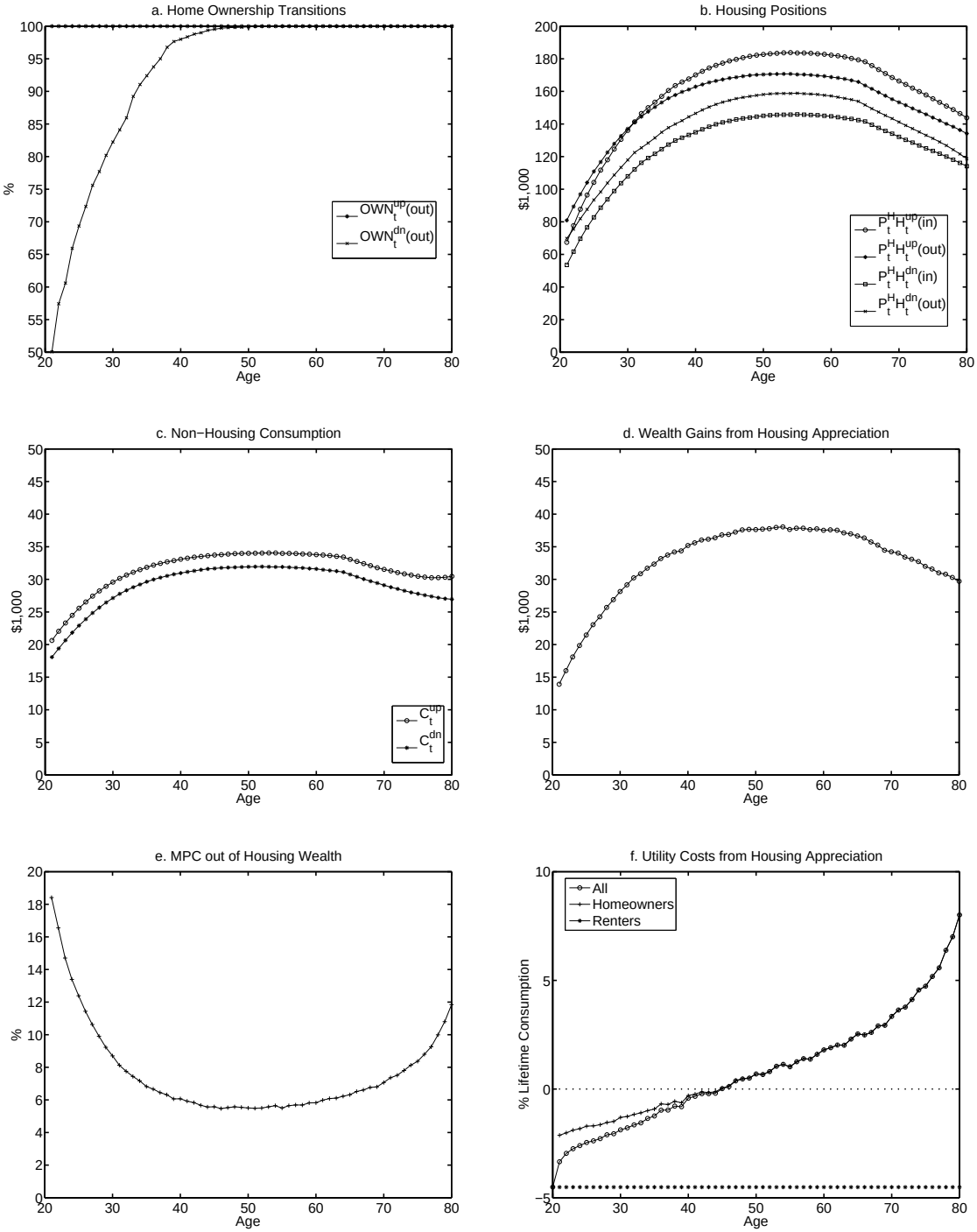

Figure 10. The Consumption and Welfare Effects of House Price Shocks with Zero Adjustment Costs

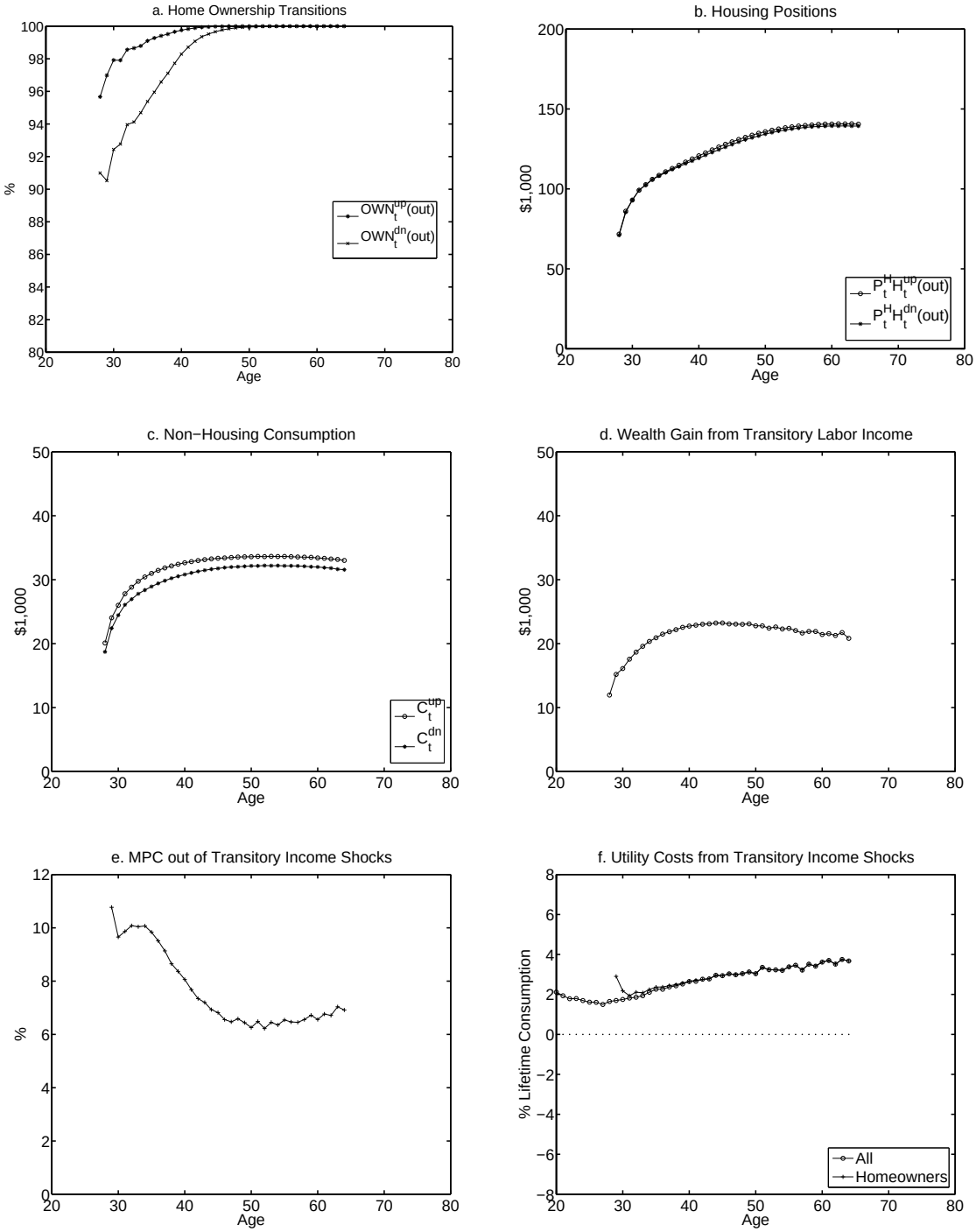

Figure 11. The Consumption and Welfare Effects of Transitory Income Shocks under Baseline Parameterizations