Money and Prices in Models of Bounded Rationality*

Albert Marcet, Universitat Pompeu Fabra, CEPR and CREI.
Juan Pablo Nicolini, Universidad Torcuato Di Tella.

August 5, 2003

Abstract

In this paper we explore the ability of simple monetary models with bounded rationality to account for the joint distribution of money and prices. We impose restrictions on the size of the mistakes agents can make in equilibrium and argue that countries with high inflation are likely to satisfy these restrictions. Our computations show that the model with bounded rationality does neither improve nor deteriorates the ability of the model to match the data.

Keywords: Inflation, Money demand, Quasi-rationality.
JEL classification: D83, E17, E31.

1 Introduction

The purpose of this paper is to explore the empirical implications regarding the behavior of the joint distribution of money and prices of departing from rational expectations in very simple money demand models. There has long been recognized that standard rational expectations monetary models imply a correlation between money and prices that is way too high relative to the data. The reason is that velocity (the inverse of real money demand) fluctuates too little in the models relative to the data and so does the ratio of money to prices. Attempts to reconcile this feature have explored models with price stickiness or with segmented markets. Ours, is an attempt to explain the potential role of sticky expectations in driving the high frequency movements of money and prices.

Recent monetary models of bounded rationality specify processes that imply more sluggish adjustment of inflation expectations than the rational expectations versions. Thus, as long as velocity depends on expected inflation, it will

*Prepared for the conference on Monetary Policy and Learning sponsored by the Federal Reserve Bank of Atlanta. We want to thank Rodi Manuelli, Andy Neumeyer and Mike Woodford for comments, and Demian Pouzo for excellent research assistance.

also exhibit more sluggish movements than the rational expectations version. Therefore, the ratio of money and prices will also be more sluggish, so the implications for the co-movement between money and prices may be different from the rational expectations version and, hopefully, closer to the data. This paper provides a quantitative exercise to clarify this issue.

The bounded rationality literature main obstacle is that there are too many ways of being irrational, leaving room for too many degrees of freedom. A common methodological device to face this "free-parameters" problem has been to allow only for expectation-formation mechanisms that depart from rational expectations by a small distance, in some sense. In Marcet and Nicolini (2002) (MN from now on), we propose three different bounds and show that in a model similar to the one we use in this paper they become operative in determining the equilibrium values of the model parameters, therefore solving the free parameters problem. A key feature of the bounds, is that agents are "almost" rational in the sense that the expected value of the difference between expected or perceived inflation and the true value for inflation is very close to zero[1]. In this paper, we follow that methodological assumption and impose the bounds. This assumption, however, raises an important issue regarding the ability to distinguish, from the data, the fully rational from the almost rational hypothesis. Indeed, under some circumstances, the imposition of these bounds make the model with learning observationally equivalent to the rational expectations version. But this is not always the case as we show in our previous paper, in which we solve a simple seiniorage-driven model of hyperinflations. The learning mechanism we use combines tracking with least squares, two of the most common mechanisms used in the literature. The advantage of our combined mechanism is that it works well both in stable as well as in changing environments, so it can be applied to countries with low and stable inflation and countries with high average inflation that experience, from time to time, recurrent burst of hyperinflations. We show in that paper that the equilibrium outcome can be very different from the rational expectations equilibria only when the government follows a high average money growth rate policy. If the average money growth rate is low, the only equilibrium with bounded rationality is the rational expectations equilibrium. So, the imposition of bounds on the mistakes agents can make in equilibrium, implies, according to the model of our previous paper, that the rational expectations and bounded rationality hypothesis are observationally equivalent, except in countries where inflation is high on average. The reason for this result is that countries with low inflation also exhibit quite stable inflation. Thus, in these stable environments, the bounds imply that agents learn fast the true structure of the model and the outcome converges to the rational expectations outcome very quickly. However, countries with high average inflation also exhibit very unstable environments, as the econometric estimates we provide in the appendix of this paper clearly testify. In these changing environments, it is much harder - although not impossible, since the equilibrium must satisfy the bounds -to learn the true structure of

[1]Rational expectations implies that difference to be exactly equal to zero.

the model with simple backward looking schemes, and therefore the equilibrium outcome can be very different from the rational expectations equilibrium for a very long transition, characterized by recurrent bursts in inflation rates. We use this property of the model with learning in changing environments to compare the empirical implications of the bounded rationality hypothesis relative to the rational expectations version.

Our paper contributes to the literature along two dimensions. First, it contributes to the literature that analyzes high frequency movements between money and prices. Ours, is an attempt to explain the potential role of sticky expectations in driving the high frequency movements of money and prices in high inflation countries. Second, we identify a case in which even though the learning equilibria satisfies the bounds and is close to rational expectation, the equilibrium outcomes are different, so the models are not observationally equivalent. Thus, we can evaluate the empirical performance of the learning model relative to the rational expectations version, while not being subject to the free parameters criticism.

The model we discuss in this paper is a single real demand for money equation that is decreasing with expected inflation, and were the exogenous driving force is the money supply. We then fit a Markov switching regime statistical model for the money growth rate for five high inflation countries and solve both the bounded rationality and rational expectations versions of the model. Finally, we compare moments of the joint distributions of money and prices from both models and the data. We conclude that both versions of the model deliver the same quantitative implications and fit the data well. Thus, the high frequency behavior of money and prices provides evidence neither in favor, nor against the bounded rationality hypothesis.

2 The Model

The model consist in a demand for cash balances given by

$$P_t = \frac{1}{\phi} M_t + \gamma P^e_{t+1}, \tag{1}$$

where ϕ, γ are positive parameters. P_t, M_t are the nominal price level and the demand for money; and P^e_{t+1} is the forecast of the price level for next period. The driving force of the model is given by the stochastic process followed by the growth rate of money. This well known money demand equation is consistent with utility maximization in general equilibrium in a context of overlapping generations model.

To complete the model, one needs to specify the way agents forecast the future price level. In what follows, we compare the rational expectations version with a version in which agents use an ad-hoc algorithm that depends on past information. There are several ways in which one can restrict the bounded rationality or learning, version of the model to be "close" to rational expectations.

For example, it has been common to study the conditions under which the equilibrium outcome of the learning model converges to the outcome of the rational expectations model. To be more specific, consider, in the context of the money demand equation above, two alternative expectations formation mechanisms

$$\begin{aligned} P_{t+1}^{e} &= E_t\left[P_{t+1}\right] \\ P_{t+1}^{e} &= L\left(P_{t-1}, P_{t-2}, ...\right). \end{aligned}$$

We then obtain (possible many) solutions $\{P_t^{RE}, P_t^{L}\}_{t=0}^{\infty}$. For the sake of the argument, assume that the rational expectations version has a unique solution. Then, if one is concerned about outcomes, it is natural to use a notion of distance that depends on the equilibrium values of the models

$$\rho(\{P_t^{RE}\}_{t=0}^{\infty}, \{P_t^{L}\}_{t=0}^{\infty})$$

For example, if the model is deterministic, convergence to rational expectations can be written as

$$\lim_{t\to\infty}(P_t^{RE} - P_t^{L}) = 0$$

Thus, the equilibrium outcome has to "look like" a rational expectations equilibrium in the limit. Then, in systems that settle down[2], models with learning that are restricted to satisfy this long run property are observationally equivalent to rational expectations equilibria. Only the transition, if long enough, is then potentially suitable to compare the relative performance of both models. But the transition is not restricted by the bound, so we either have observational equivalence or unrestricted learning outcomes.

Alternatively, one could impose restrictions on the size of the systematic mistakes the agents make in equilibrium (which are zero in the rational expectations version). This amounts to define a metric

$$\rho(\{P_t^{L} - L\left(P_{t-1}, P_{t-2}, ...\right)\}_{t=0}^{\infty})$$

For example, if we impose

$$E_{t-1}[P_{t+1}^{L} - L\left(y_{t-1}, y_{t-2}, ...\right)] = 0, \text{ for all } t$$

then we have rational expectations. If instead, we impose that the mistakes need not be zero, but very small, we impose no restriction on the relationship between P_t^{RE} and P_t^{L}. If, as it is sometimes the case, P_{t+1}^{RE} can be written as

[2]Recall that in complex dynamic models where one only focuses on invariant distributions, it is assumed that the system "settled down".

a function L of past values, then the rational expectations equilibrium is also a learning equilibrium. But there may be other learning equilibria that behave differently. In this case, the empirical implications of the rational expectations version and the "small mistakes" version may be different, while the learning equilibrium is till close to the rational, in the proper metric.

In MN, we impose three bounds, including versions of the two discussed above. As we mentioned in the introduction, if inflation is low, then the only learning equilibrium that satisfies the bounds is the rational expectations equilibrium. However, when inflation is high, there are learning equilibrium outcomes that look different from the rational expectations one. Thus, for our empirical investigation, we focus the analysis on high inflation countries only.

2.1 The behavior of money and prices during hyperinflations

In order to fit into the two versions of the model a process for the nominal money supply, we first look at the evidence for five Latin-American countries that experienced high average inflation and very high volatility in the last decades: Argentina, Bolivia, Brazil, Mexico and Peru. We use data for M1 and consumer prices from the International Financial Statistics published by the International Monetary Fund. To compute growth rates, we take the logarithm[3] of the time $t+1$ variable divided by the time t value of the same variable.

While the high inflation years are mostly concentrated between 1975 and 1995 for most countries, the periods do not match exactly. Thus, we chose, for each country, a sub-period that roughly corresponds to its own unstable years. Figures 1.e to 1.e plot quarterly data on nominal money growth and inflation for the relevant periods in each case.

As it can be seen from the figures, for all those countries, average inflation was high, but there are some relatively short periods of bursts - the shaded areas - in both money growth and inflation rates, followed, again, by periods of stable but high inflation rates[4]. Note also that the behavior of the money growth rate, the driving force of the model, follows a very similar pattern. Thus, we propose to fit a Markov switching process for the rate of money growth[5].

[3]This measure underestimates growth rates when they becomes very high. With this scale, it is easier to see the movements of inflation rates in the - relatively - traquile periods in all the graphs. In what follows, we make the case that the data is best described as a two regime process, so this measure biases the result against us.

[4]This feature of the data has long been recognized in case studies of hyperinflations (see Bruno, Di Tella, Dornbusch and Fisher (1988) and Bruno, Fisher, Helpman and Liviatan (1991)).

[5]Actually, the data suggest that during the periods of hyperinflations, both the inflation rate and the rate of growth of the money supply are increasing over time, a fact that is consistent with the model in MN. The statistical model we fit assumes, for simplicity, that during the hyperinflations both have a constant mean. This assumption biases the results against the rational expectations model, since in the learning version, agents do not take into account the money supply rule, while we are providing the rational agents with a misspecified process for the money supply.

2.2 A Regime Switching Model for Money Growth

Our empirical analysis is based on quarterly data, since we are interested in high frequency movements in money and prices. Our eyeball inspection of Figures 1.a to 1.e suggests the existence of structural breaks. This is confirmed by the break-point Chow Test that we present in the appendix for the five countries, so we model $\log(M_t) - \log(M_{t-1})$ as a discrete time Markov switching regime. Thus, $\Delta \log(M_t) = \log(M_t) - \log(M_{t-1})$ is assumed to be distributed $N(\alpha_{s_t}, \sigma^2_{s_t})$; where $s_t \in \{0, 1\}$. The state s_t is assumed to follow a first order homogeneous Markov process with $\Pr(s_t = 1|s_{t-1} = 1) = q$ and $\Pr(s_t = 0|s_{t-1} = 0) = p$. The evolution of the first difference of the logarithm of the money supply can therefore be written as

$$\Delta \log(M_t) = \mu_o(1 - s_t) + \mu_1 s_t + \left[\sigma^2_o(1 - s_t) + \sigma^2_1 s_t\right] \varepsilon_t$$

where ε_t is assumed to be i.i.d. All empirical results regarding the modelling of the money supply are also reported in Appendix 1.

For all the countries, one state is always characterized by higher mean and higher volatility of $\Delta \log(M_t)$. Both pairs of μ_i and σ_i are statistically significant, as well as the transition probabilities, p and q, and in all cases, both states are highly persistent. The results represent very clear evidence of the existence of two states. The rate of money growth in the high mean-high volatility state ranges from three times the rate of money growth in the low mean-low volatility state for Argentina to nine times for Bolivia while the volatility of the high mean-high volatility ranges from one and a half times the volatility of the low mean-low volatility state for Mexico to eight times for Brazil. The differences are gigantic. The high-mean and high-volatility state is always consistent with the existence of high peaks of inflation in each of the countries. These periods are represented as the shaded areas in Figures 1.a to 1.e.

These results clearly demonstrate that the economic environment can be characterized by one in which there are changes in the monetary policy regime. That is the reason why the learning mechanism we use produces reasonably low prediction errors.

2.3 Rational Expectations Equilibria

With rational expectations (RE) we know that

$$P^e_{t+1} = E\left(P_{t+1}|I_t\right)$$

for all t, where I_t is the information set up to time t. Agents know the past observations of both $\frac{M_t}{M_{t-1}}$ and s_t where s_t characterize the state of $\frac{M_t}{M_{t-1}}$. Therefore it must be true that

$$I_t = \left(s_t, \left\{\frac{M_j}{M_{j-1}}\right\}_{j=1}^{t}\right)$$

since, due to the Markov property of the driving force, all the past information of $\{s_t\}$ is contained in s_t.

Given the structure of the money growth process, $E\left(P_{t+1}|I_t\right)$,will be linear in P_t, i.e.,

$$E\left(P_{t+1}|I_t\right) = \beta_s P_t$$

with β_s being state dependent, and must satisfy

$$\beta_s = \beta_o(1-s_t) + \beta_1 s_t.$$

To solve for an equilibrium, we must first find (β_o, β_1).

Let $\kappa_j = E\left(\frac{M_{t+1}}{M_t}|s_{t+1}=j\right)$, as we know that $\log\left(\frac{M_{t+1}}{M_t}\right) \sim N\left(\mu_j, \sigma_j\right)$, then

$$\begin{aligned}
\kappa_j &= E\left(\exp\left(\log\left(\frac{M_{t+1}}{M_t}\right)\right)|s_{t+1}=j\right) \\
&= \exp\left(\mu_j + \frac{1}{2}\sigma_j^2\right).
\end{aligned}$$

Now

$$\begin{aligned}
E\left(P_{t+1}|s_t=0\right) &= pE\left(\frac{M_{t+1}}{\phi\left(1-\gamma\beta_0\right)}|s_{t+1}=0, s_t=0\right) + (1-p)\,E\left(\frac{M_{t+1}}{\phi\left(1-\gamma\beta_1\right)}|s_{t+1}=1, s_t=0\right), \\
&= \frac{p}{\phi\left(1-\gamma\beta_0\right)}E\left(M_{t+1}|s_{t+1}=0, s_t=0\right) + \frac{(1-p)}{\phi\left(1-\gamma\beta_1\right)}E\left(M_{t+1}|s_{t+1}=1, s_t=0\right), \\
&= \frac{p\kappa_0}{\phi\left(1-\gamma\beta_0\right)}M_t + \frac{(1-p)\,\kappa_1}{\phi\left(1-\gamma\beta_1\right)}M_t, \\
&= p\kappa_0 P_t + (1-p)\,\kappa_1\frac{(1-\gamma\beta_0)}{(1-\gamma\beta_1)}P_t, \\
&= P_t\left(p\kappa_0 + (1-p)\,\kappa_1\frac{(1-\gamma\beta_0)}{(1-\gamma\beta_1)}\right).
\end{aligned}$$

Where the first equality arise from equation (1) and the fact that $P_{t+1}^e = E\left(P_{t+1}|s_t=0,...\right) = \beta_o P_t$. The third equality comes from the definition of κ, and the fourth one comes from equation (1). Combining this expression with $P_{t+1}^e = E\left(P_{t+1}|s_t=0,...\right) = \beta_0 P_t$ we get

$$\beta_0 = \left(p\kappa_0 + (1-p)\,\kappa_1\frac{(1-\gamma\beta_0)}{(1-\gamma\beta_1)}\right). \tag{2}$$

For an analogous derivation for $s_t = 1$ we get

$$\beta_1 = \left(q\kappa_1 + (1-q)\,\kappa_0\frac{(1-\gamma\beta_1)}{(1-\gamma\beta_0)}\right). \tag{3}$$

We must solve this system of equations, for the unknowns (β_0, β_1). The solution of the system is given by

$$\beta_1 = \frac{\kappa_0\left(1-q\right) + \kappa_1 q + \kappa_0\kappa_1\gamma\left(1-q-p\right)}{1+\gamma\kappa_0\left(1-p-q\right)}$$

and plugging this expression into (2) we obtain the solution for β_0.

2.3.1 Money-Inflation relationship under RE

Under RE and from

$$P_t = \frac{M_t}{\phi\left(1-\gamma\beta_{st}\right)},$$

the inflation rate must satisfy

$$\frac{P_t}{P_{t-1}} = \frac{\left(1-\gamma\beta_{s(t-1)}\right)}{\left(1-\gamma\beta_{st}\right)}\frac{M_t}{M_{t-1}},$$

where $\beta_{s(t-1)}$ is known in t, and β_{st} will be equal to β_o if $\frac{M_t}{M_{t-1}}\tilde{}N(\mu_o,\sigma_o)$ and equal to β_1 if $\frac{M_t}{M_{t-1}}\tilde{}N(\mu_1,\sigma_1)$.

2.4 Learning Mechanism Equilibrium

The learning mechanism we use is the same one we used in MN. Let

$$P^e_{t+1} = \beta_s P_t$$

were

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t}\left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right), \tag{4}$$

were α_t is called the gain of the algorithm and affects the sensitivity of expectations to current information. Two of the most common specifications for the gain sequence are tracking ($\alpha_t = \alpha$ for all t), which performs well in environments that change every period, and least squares ($\alpha_t = \alpha_{t-1}+1$) that performs well in environments that settle down. As the money supply process, the driving force of the model, is well approximated by a Markov Switching Regime with substantial persistence, a scheme that combines both mechanisms performs well, as we show in MN. The gain is assumed to follow

$$\begin{aligned} \alpha_t &= \alpha_{t-1}+1 \quad if \quad \left|\frac{\frac{P_{t-1}}{P_{t-2}}-\beta_{t-1}}{\beta_{t-1}}\right| \geq \nu \\ &= \overline{\alpha} \qquad\qquad\qquad otherwise \end{aligned} . \tag{5}$$

Where $\overline{\alpha}, \nu$ are the learning parameters. Thus, if errors are small, the gain follows a least squares rule, such that as long as the regime does not switch, agents soon learn the parameters of the money supply rule. However, once a big error is detected, the rule switches to a constant gain algorithm, so agents can learn the new parameters of the money supply rule.

2.4.1 Money-Inflation Relationship under Learning

The solution $\{\frac{P_t}{P_{t-1}}, \beta_t, \alpha_t\}$ must satisfy (4), (5) and

$$\frac{P_t}{P_{t-1}} = \left[\frac{1-\gamma\beta_{t-1}}{(1-\gamma\beta_t)}\right] \frac{M_t}{M_{t-1}}.$$

For each t we solve this stochastic system, taking $(\beta_{t-1}, \alpha_{t-1})$ as given (from the $t-1$ iteration).

2.5 Calibration

2.5.1 The rational expectations model

The money demand

We borrow the parameter values from our previous paper, in which we use observations from empirical Laffer curves to calibrate them. This is a reasonable choice, since as one empirical implication of the original model is that recurrent hyperinflations characterized by two regimes occur when average inflation - driven by average seiniorage - is "high", we need to have a benchmark to discuss what high means. We use quarterly data on inflation rates and seignorage as a share of GNP for Argentina [6] from 1980 to 1990 from Ahumada, Canavese, Sanguinetti and Sosa (1993) to fit an empirical Laffer curve. While there is a lot of dispersion, the maximum feasible seignorage is around 5% of GNP, and the inflation rate that maximizes seignorage is close to 60%. These figures are roughly consistent with the findings in Kiguel and Neumeyer (1992) and other studies. The parameters of the money demand γ and ϕ, are uniquely determined by the two numbers above. Note that the money demand function implies a stationary Laffer curve equal to

$$\frac{\pi}{1+\pi} m = \frac{\pi}{1+\pi} \phi \left(1 - \gamma(1+\pi)\right) \tag{6}$$

where m is the real quantity of money and π is the inflation rate. Thus, the inflation rate that maximizes seignorage is

$$\pi* = \sqrt{\frac{1}{\gamma}} - 1$$

which, setting $\pi* = 60\%$, implies $\gamma = 0.4$. Using this figure in (6), and making the maximum revenue equal to 0.05, we obtain $\phi = 0.37$. For simplicity, we use these numbers for all the countries.

The money supply

For the money supply, we use the estimated Markov switching models we discussed above. The results of the estimation are reported in Appendix 1. While the money demand parameters are assumed the same for each country, the money supply process is estimated using data from each country.

[6] The choice of country is arbitrary. We chose Argentina because we were more familiar with the data.

2.5.2 The learning model

The parameters described above are sufficient to solve the rational expectations model. However, we still need to be specific regarding our choice of the (still free!) parameters of the learning process, $\overline{\alpha}, \nu$.

In MN, we provide an operational definition of a bound of the type described above. Intuitively, we search for values of the parameter $\overline{\alpha}$ that satisfy a rational expectations-like fixed point problem. We look for values of the parameter such that in equilibrium, agents make almost zero systematic mistakes. Recall that the learning mechanism we propose is well suited to deal with changing environments, a result we show formally in MN. For our simulations, we use the equilibrium values we obtained in MN ($\overline{\alpha}$ between 2 and 4). For the value of ν,we also follow MN, where we used a value that was roughly equal to two standard deviations of the shock[7].

3 Evaluating the models

As we already mentioned, the main goal of this paper is to investigate the ability of the simple money demand equation to replicate the short run relationship between the inflation rate and the growth rate of money using both the rational expectations model and the "almost" rational version. Following the RBC tradition, we characterize the data using empirical moments of the joint distribution of money and prices. Table 1 presents moments of their partial distributions. As one should expect, average inflation is very similar to average money growth. There are slightly larger differences between the volatilities of inflation and money growth rates, without a clear pattern emerging from the table. For the analysis of the joint distribution of money and prices we will focus on the cross-correlogram. If velocity were constant, then the contemporaneous correlation between money and prices ought to be one, and the leads and lags should be equal to the auto-correlogram of the money growth process. Figure 2.a and Figure 2.b present the leads and lags for money and prices for the five countries. It is interesting to point out that, as it has been documented before, money and prices are highly correlated contemporaneously, contrary to the case of middle and low inflation countries[8]. In particular, the contemporaneous correlation for Mexico, the country in the sample with lower average inflation, is substantially lower than for the other countries. The results of the simulations are shown in Tables 2.a to 2.e. The columns show the moments for the inflation generated by the model under rational expectations, and under LM. Under LM we have two columns, one for each of the two possible values

[7] We also solved the learning model with an alternative specification for v, given the Markov structure of the money growth process: we replace v for v_{st}, given by $v_{\mathsf{st}} = v_{\mathsf{l}} = v \; if \quad s_{\mathsf{t}} = low \; state$, while $v_{\mathsf{st}} = v_{\mathsf{l}} = (\sigma_{\mathsf{h}}/\sigma_{\mathsf{l}})v \; if \quad s_{\mathsf{t}} = high \; state$. With this alternative specification we introduce the switching regime information into the LM, but it did not make any difference in the results, reported in the WP version.

[8] See Alvarez, Atkenson and Edmond (2001) and referneces therein for theoretical work that aims at matching these correlations for low inflation countries.

for $\overline{\alpha} = \{10/4, 10/3\}$.

As we can see, the three simulations (the one under RE and the two under LM) give very similar results for each of the countries. In fact, for Argentina, neither the mean nor the standard deviation of the RE model are statistically different from the ones generated by the two different versions of LM. Most importantly, none of these moments are statistically different from the actual moments of inflation. The same is true for Peru, Bolivia and Mexico. Similar results arise for Brazil, with the exception that the volatility of the inflation rate under learning overestimates the true volatility . It is interesting to point out that while the simulations for Argentina, Bolivia and Peru generate volatilities that underestimate the actual ones; for Brazil and Mexico the generated volatility overestimates the actual one.

Figures 3.a to 3.e and 4.a to 4.e present the leads and lags of the cross-correlogram between $\log(M_t/M_{t-1})$ and the inflation rates generated by RE, LM and the actual one for each of the five countries. We also include an approximation for the confidence band, $(\pm 2/\sqrt{T})$ for the cross-correlogram of the actual series (the dotted lines).

For each country, the leads and lags graphs generated with the three simulated series are very similar. Also, with the exception of Mexico, none of the cross-correlograms generated by either model is significantly different from the actual one. Both mechanism perform equally good in approximating the actual cross-correlogram. A noticeable fact is that in every country the contemporaneous correlation is lower in the simulated series than in the actual ones, except in Mexico, the country with the lowest average inflation. Furthermore, Mexico is the only country which presents significant differences between the actual and the simulated cross-correlogram. This is due to the fact that Mexico's actual inflation is not so correlated to $\log(M_t/M_{t-1})$ as the other variables are (shown in Figures 1.a and 1.b), and as the simulated inflation are highly correlated to $\log(M_t/M_{t-1})$ they perform worse for this particular case. But even in the case of Mexico the prediction of the money demand model is not very sensitive to the expectation formation mechanism.

The most important conclusion of the paper is that, although from the theoretical point of view, sticky expectations was a promising avenue to explain the short run behavior of money and prices, quantitatively, the models are empirically equivalent. Both models do imply different behavior for expected inflation. Indeed, Figure 5 plots expected inflation for both models for the case of Argentina[9]. The definitions are the same as the ones stated previously in Tables 2.a-2.e, i.e., "$BETA_LMi$", for $i = 0, 1$ corresponds to the expected inflation, β generated by the LM labeled as "$\frac{10}{3}, \frac{10}{4}$", respectively in Tables 2.a-2.e, while "$BETA_RE$" stands for the expected inflation generated by the RE model. As it can be seen in the figure, expected inflation under learning exhibits more high frequency movements. Thus implies that real money demand also moves more under learning. However, the impact on the behavior of the cross correlogram is quantitatively very small. This suggests that for the calibrated values

[9]The same happens in the other five countries.

of the money demand elasticity, the role that high frequency fluctuations on expectations have on the short run dynamics of money and prices is negligible.

4 Conclusion

The purpose of this paper is to explore the potential role of sluggish "almost-rational" expectations in explaining the high frequency movements between money and prices. There is evidence that shows a sluggish response of prices to money, so sluggish expectations imply movements on velocity that could potentially explain the data better than the rationalexpectations alternative. We impose the methodological restriction that the learning mechanism must produce very good forecasts within the model in a way that resembles rational expectations. We argue that the learning model we propose satisfies the methodological restriction in countries in which monetary policy exhibits frequent and substantial changes of regime and we argue that, form the point of view of the theory, the models are not necessarily observationally equivalent in that case. We fit a Markov switching process for the exogenous driving force - the money growth rate - in five Latin-American countries. There is ample evidence in favor of the regime switching structure. We quantitatively solve a calibrated money demand equation under the assumptions of both rational expectations and learning with the methodological restriction. The behavoir of expected inflation is indeed different in both models. However, we find that for the calibrated value of the elasticity of the money demand both models generate very similar empirical implications that match the facts in almost every dimension. Thus, we conclude, the short run behavior of money and prices provides evidence neither in favor nor against the bounded rationality hypothesis in expectations formation if agents are not allowed to make large mistakes in equilibrium.

References

[1] Ahumada, H., A. Canavese, P. Sanguinetti and W. Sosa Escudero, "Efectos distributivos del impuesto inflacionario: una estimación del caso argentino" Economía Mexicana, pp. 329-385, (1993)

[2] Alvarez, F., A. Atkeson, A. and C. Edmond, "On the sluggish response of prices to money in an inventory theoretic model of money demand", Mimeo, University of Chicago, (2002).

[3] Bruno, M., G. Di Tella, R. Dornbusch and S. Fisher. Inflation Stabilization. The experience of Israel, Argentina, Brasil and Mexico MIT Press, (1988)

[4] Bruno, M., S. Fisher, E. Helpman and N. Liviatan. Lessons of Economic Stabilization and its Aftermath, MIT Press, (1991).

[5] Chung, Hee-Taik. "Did Policy Makers Really Believe in the Phillips Curve?. An Econometric Test". Ph. D. dissertation, University of Minnesota, (1990).

[6] Eckstein, Z. and L. Leiderman, "Seignorage and the Welfare Cost of Inflation", Journal of Monetary Economics, no. 29, pp. 389-410, (1992).

[7] Evans, G.W. and S. Honkapohja. "Adaptive Forecasts, Hysteresis, and Endogenous Fluctuations", Economic Review, 1, Federal Reserve Bank of San Francisco, (1993).

[8] Evans, G.W. and S. Honkapohja. Learning and Expectations in Macroeconomics, Princeton University Press (2001).

[9] Kiguel, M. and P. A. Neumeyer, "Inflation and Seignorage: The Case of Argentina," Journal of Money Credit and Banking, 27:3, 672-82, August (1995).

[10] Ljung, L. "Analysis of Recursive Stochastic Algorithms", IEEE Trans. Automat. Control AC-22, pp 551-557 (1977).

[11] Marcet, A. and J.P. Nicolini, "Recurrent Hyperinflations and Learning", American Economic Review, forthcoming, (2002).

[12] Marcet, A. and T.J. Sargent. "Convergence of Least Squares Learning Mechanisms in Self-Referential Linear Stochastic Models", Journal of Economic Theory, no. 48, pp. 337-368, (1989a).

[13] Sargent, T.J. Bounded Rationality in Macroeconomics, Oxford University Press, (1993).

A Appendix 1

A.1 Chow Test for Structural Breaks

The Chow Test for the corresponding sub-samples generates the following results

Argentina (1975:01 1992:04)

Sub-samples 1975:01-1988:04, 1989:01-1990:01, 1990:02-1992:04

Chow Breakpoint Test:	1989:1 1990:1	Probability
F-statistic	3.775243	0.002964
Log likelihood ratio	22.09987	0.001161

Bolivia (1975:01 1995:04)

Sub-samples 1975:01-1983:03, 1983:04-1986:04, 1987:01-1995:04

Chow Breakpoint Test:	1983:4 1986:4	Probability
F-statistic	4.205059	0.003498
Log likelihood ratio	16.47067	0.002448

Brazil (1980:01 1995:04)

Sub-samples 1980:01-1987:04, 1988:01-1991:01, 1991:02-1995:04

Chow Breakpoint Test:	1988:1 1991:1	Probability
F-statistic	4.666468	0.001733
Log likelihood ratio	18.12746	0.001165

Mexico (1975:01 1995:04)

Sub-samples 1975:01-1989:04, 1990:01-1992:03, 1992:04-1995:04

Chow Breakpoint Test:	1990:1 1992:3	Probability
F-statistic	4.251409	0.003259
Log likelihood ratio	16.63831	0.002272

Peru (1975:01 1995:04)

Sub-samples 1975:01-1989:04, 1990:01-1991:01, 1991:02-1995:04

Chow Breakpoint Test:	1990:1 1991:1	Probability
F-statistic	16.67572	0.000000
Log likelihood ratio	53.90424	0.000000

A.2 Markov Switching Regime Estimation Results

In this sub-section we present the results of the Markov Switching Regimes estimation. Let $p = \Pr(s_t = 0|s_{t-1} = 0)$, $q = \Pr(s_t = 1|s_{t-1} = 1)$, and let μ_i and σ_i be the mean and the standard deviation of the growth rate of money in state i. The following tables summarize the results of the estimation.

Argentina (1975:01 1992:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.175533792	0.018297495	9.593323735
μ_1	0.454399941	0.098682095	4.604684802
q	0.915099885	0.066209340	13.82131101
p	0.919361013	0.074032079	12.41841415
σ_0	0.072853206	0.011943142	6.100003477
σ_1	0.303486106	0.079758969	3.805040466

Bolivia (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.059627604	0.009754163	6.113041314
μ_1	0.536370837	0.420877046	1.274412187
q	0.935126471	0.066531539	14.05538607
p	0.986617102	0.070142749	14.06584602
σ_0	0.050541181	0.003716448	13.59932350
σ_1	0.373992612	0.159586583	2.343509119

Brazil (1980:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.157931616	0.027645929	5.712653628
μ_1	0.517317340	0.128746245	4.018115930
q	0.926016849	0.125106881	7.401805917
p	0.957205334	0.068582989	13.95689156
σ_0	0.045445418	0.019317040	2.352607738
σ_1	0.389689368	0.089648053	4.346880447

Mexico (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.064677791	0.010181050	6.352762507
μ_1	0.206343338	0.021967776	9.393000814
q	0.733328049	0.174816708	4.194839597
p	0.951845883	0.060318779	15.780257828
σ_0	0.047614481	0.011171478	4.262147003
σ_1	0.061059573	0.025394331	2.404456906

Peru (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.119329114	0.015084511	7.910632032
μ_1	0.744060948	0.190436648	3.985303268
q	0.830803948	0.126355006	6.567003388
p	0.971727572	0.048361910	19.86071300
σ_0	0.082091249	0.008656942	10.06842486
σ_1	0.383326249	0.134969389	2.847524436

B Simulation Results

Table 1 shows the first and second moments of inflation and money growth for every country. Tables 2.a to 2.e show the first and second moments of the simulations.

			Table 1			
			Inflation ($\Delta \log P_t$)		Money Growth ($\Delta \log M_t$)	
Sample	Country		$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$
1975:01-1992:04	Argentina		0.146870	0.132087	0.138898	0.117783
1975:01-1995:04	Bolivia		0.069557	0.138701	0.072275	0.117080
1980:01-1995:04	Brazil		0.177091	0.131648	0.174454	0.158965
1975:01-1995:04	Mexico		0.036109	0.029010	0.037837	0.031375
1975:01-1995:04	Peru		0.104011	0.140325	0.095859	0.123695

		Table 2.a		
		Argentina		
Sample:	1975:01-1992:04			
			μ	σ
	$True$		0.146870	0.132087
	$\overline{\alpha} = \frac{10}{3}$		0.142280	0.137145
	$\overline{\alpha} = \frac{10}{4}$		0.141884	0.152390
	RE		0.144074	0.123287

		Table 2.b		
		Bolivia		
Sample:	1975:01-1995:04			
			μ	σ
	$True$		0.069557	0.138701
	$\overline{\alpha} = \frac{10}{3}$		0.072237	0.146258
	$\overline{\alpha} = \frac{10}{4}$		0.072189	0.161556
	RE		0.072755	0.127636

		Table 2.c		
		Brazil		
Sample:	1980:01-1995:04			
			μ	σ
	$True$		0.177091	0.131648
	$\overline{\alpha} = \frac{10}{3}$		0.164544	0.217368
	$\overline{\alpha} = \frac{10}{4}$		0.163226	0.295389
	RE		0.180857	0.168293

		Table 2.d		
		Mexico		
Sample:	1975:01-1995:04			
			μ	σ
	$True$		0.036109	0.029010
	$\overline{\alpha} = \frac{10}{3}$		0.037844	0.032996
	$\overline{\alpha} = \frac{10}{4}$		0.037805	0.033354
	RE		0.038116	0.033121

		Table 2.e		
		Peru		
Sample:	1975:01-1995:04			
			μ	σ
	$True$		0.104011	0.140325
	$\overline{\alpha} = \frac{10}{3}$		0.094706	0.146434
	$\overline{\alpha} = \frac{10}{4}$		0.094602	0.161485
	RE		0.096507	0.174697