Banking Competition, Credit Market Activity, and the Effects of Monetary Policy*

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Abstract

This paper studies the impact of strategic interactions among financial institutions in a model with money. We incorporate imperfectly competitive behavior by considering that banks compete in terms of the amount of loans they issue. As a benchmark, our results show that more competition leads to a higher volume of lending activity and lower interest rates. Interestingly, the type of strategic behavior and the degree of entry bear significant implications for the effects of monetary policy. Specifically, in the presence of non-cooperative behavior and limited entry, monetary policy only affects the rate of return to money. Therefore, it does not have any impact on the credit market. In contrast, under endogenous entry, as inflation affects the ability of banks to insure depositors against liquidity risk, monetary growth discourages bank entry and leads to higher costs of loans. We also examine the effects of monetary policy under collusive behavior. When banks have the ability to collude and implement monopoly pricing, higher inflation further aggravates credit market imperfections. We proceed by exploring how inflation-financed government debt leads to additional distortions in the presence of imperfect competition. In particular, we find that the interest rate on private sector loans will be higher than the interest rate on government bonds. This occurs even though both types of loans are risk-free in the model. Moreover, public sector debt crowds out funds and heightens pricing distortions from market power. In this manner, we introduce a number of significant transmission channels in which inflation obstructs financial market activity.

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1 Introduction

Numerous research demonstrates that banks provide valuable economic functions in financial markets. One significant factor that affects financial market activity is the degree of competition. Notably, the competitive structure of the banking industry varies across countries. As an example, the financial systems in Greece and Belgium have a relatively high degree of concentration. In contrast, the banking sectors in France and Germany have been much more competitive. However, in recent years, there has been a considerable increase in consolidation among banks in many countries. Furthermore, due to the global financial crisis, consolidation has advanced at an even more aggressive pace.

In response, policymakers have expressed concern about the resulting impact on interest rates and the degree of influence of monetary policy. Existing empirical studies indicate that these concerns are legitimate. To begin, Hannan (1991) and Corvoisier and Gropp (2002) argue that borrowers in markets with higher concentration ratios face higher costs for loans. Moreover, they may also experience more difficulty obtaining access to credit – Beck, Demirgüç-Kunt and Maksimovic (2003) document that credit rationing occurs more often in concentrated banking systems. Other studies emphasize that concentration affects deposit rates.

In addition to distortions from market power, there is a large body of research that stresses that inflation inhibits credit market activity. In particular, Boyd, Levine, and Smith (2001) point out that less lending takes place in countries with higher inflation rates. Therefore, monetary policy can contribute to financial market frictions.

This paper seeks to fill an important existing gap in the literature on financial market activity. Notably, very little research investigates how the transmission channels of monetary policy depend on the degree of competition in financial markets. That is, we view that the effects of monetary policy likely reflect strategic interactions among financial institutions. Therefore, the design of policy should account for the type of strategic behavior and barriers to entry.

In an attempt to address these issues, we construct a framework in which banks act as Cournot competitors in the credit market. In contrast, depositors obtain insurance against liquidity risk by depositing their funds in financial institutions. Furthermore, as in Schreft and Smith (1997), spatial separation and private information generate a transactions role for money.

As a benchmark, the paper begins by studying an economy in the presence of non-

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1 The Bank for International Settlements provides a comprehensive overview of consolidation patterns in the financial services sector during the 1990s. In particular, they caution that “Consolidation could reduce competition in these markets, increasing the cost of liquidity for some firms and impeding the arbitrage of interest rates between markets. In addition, consolidation could affect the performance of the market if the resulting large financial firms behave differently from their smaller predecessors.” Kwan (2004) describes the sources of change in the banking industry in the United States.

2 See Berger and Hannan (1989) and Neumark and Sharpe (1992).

3 Firms can use many instruments to compete in markets. However, price and quantity are primary strategic variables. Notably, Kreps and Scheinkman (1983) provide conditions in which Bertrand competition and Cournot competition yield the same outcomes.
cooperative behavior. To be expected, the results indicate that a higher degree of banking competition leads to an increase in loans and lower interest rates. However, the type of strategic interactions and the degree of entry bear significant implications for the effects of monetary policy. Specifically, under non-cooperative behavior and limited entry, monetary policy only affects the rate of return to money. Therefore, it does not have any impact on the credit market. In contrast, under endogenous entry, inflation lowers the return to money and interferes with the ability of banks to provide risk pooling services. Consequently, money growth discourages bank entry and leads to higher costs of loans. Due to the negative impact of inflation on credit market outcomes, the Friedman rule is the optimal monetary policy.

The analysis proceeds by exploring other possibilities for strategic behavior. To be specific, banks may have the ability to collude and implement monopoly pricing. In this setting, monetary growth induces banks to increase the liquidity of their portfolios in order to provide depositors with better insurance against liquidity risk. As a result, inflation further aggravates credit market imperfections since less funds will be available in the credit market. In this manner, it is important for central bankers to account for market distortions in setting monetary policy.

In addition to inefficiencies from the return to money, fiscal policy can interfere with credit market activity. For example, King and Levine (1993) discuss how public sector debt diverts funds away from the private sector. In order to study how the distortionary impact from crowding out depends on the degree of competition in the banking system, we extend the framework to include the possibility of inflation-financed government debt. In particular, the government not only issues fiat money, but also interest-bearing bonds. Interestingly, incorporating government debt into the framework leads to two additional transmission channels in which imperfect competition affects credit market outcomes. First, as government bonds provide banks with more investment opportunities, government debt introduces additional pricing distortions. Due to market power in the private credit market, the interest rate on private sector loans will be higher than the interest rate on government debt. This occurs even though both types of loans are risk-free in the model. Moreover, the reduced availability of funds to private borrowers heightens pricing distortions from imperfect competition. These transmission channels offer new mechanisms in which inflation obstructs credit market performance.

The remainder of the paper is organized as follows. Section 2 provides a survey of the literature on imperfect competition and financial market activity. Section 3 studies a benchmark model in which banks engage in non-cooperative behavior. Next, collusion is allowed and banks implement monopoly pricing across both the deposit and credit markets. Section 4 extends the model to examine the effect of government debt on credit market activity. Finally, Section 5 offers some concluding remarks. The proofs of major results are provided in the Appendix.
2 Related Literature

Previous research demonstrates that imperfect competition in the financial market can have a significant impact on economic activity. To begin, Smith (1998) constructs a model of Bertrand competition with spatial differentiation to discuss the propagation of business cycle fluctuations. Ghossoub, Laosuthi, and Reed (2012) study how the effects of monetary policy vary between monopolistic and competitive banking systems. They show that in a perfectly competitive banking system, higher rates of money growth generate a Tobin-type effect in which inflation lowers interest rates and simulates lending activity. However, in a price-distorted monopolistic banking system, inflation generates the opposite effect. Building on the structure of Ghossoub, Laosuthi, and Reed, Matsuoka (2011) compares optimal monetary policy in a monopolistic banking sector to a competitive banking system. Ghossoub (2012) demonstrates how the industrial organization of the banking system affects prices in capital markets.4

As an alternative to price competition, other papers investigate the impact of imperfectly competitive behavior arising from quantity competition. Notably, Williamson (1986) assumes that financial intermediaries compete in terms of the quantity of loans they issue. However, in his framework, money is only a store of value. In addition, the economic functions of banks are quite different – while Williamson discusses the role of banks to alleviate costs of private information, we emphasize the risk pooling role of financial institutions.

3 The Benchmark Model

We begin by considering an economy in the presence of non-cooperative behavior and limited banking entry. Following Schreft and Smith (1997) and Ghossoub, Laosuthi, and Reed (2012), banks compete by posting a schedule of prices in the deposit market.5 In contrast, competition in the credit market is characterized by quantity

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4 Ruckes (2004) applies Bertrand competition in the presence of asymmetric information to study the interactions between the level of screening by banks and prices in the credit market. In their work, both papers principally focus on the financial contracts offered by banks. However, they do not consider the effects of monetary policy. In contrast, Bagliano, Dalmazzo, and Marini (2000) propose that monetary policy can affect the ability of banks to collude in financial markets. While they demonstrate that the design of policy should account for strategic interactions among banks, depository institutions perform limited financial functions. In their framework, banks accept deposits and issue loans. That is, the primary function of banks is to promote intertemporal consumption smoothing. In contrast, following Diamond and Dybvig (1983), we incorporate that banks provide important risk pooling services to participants in financial markets.

5 Ghossoub, Laosuthi and Reed (2012) study financial market activity under price competition. Due to the Bertrand Paradox, they only compare economies with competitive banking systems to fully concentrated industries. This results in limited implications for credit market activity and the effects of monetary policy.
competition. In this manner, the degree of competition has a significant impact on the strategic behavior of banks.

3.1 Environment

The economy consists of an initial old generation and an infinite sequence of two-period-lived overlapping generations. At each date \( t = 0, 1, \ldots \), young agents are born to one of two separate islands. Each island contains a continuum of young agents with unit mass. Although the two locations are separated, there is a single consumption good available on both sites. The price level for one unit of goods is common across islands and is defined as \( p_t \) at time \( t \). For simplicity, the two islands are symmetric.

There are two types of ex-ante identical agents: depositors (which we also refer to as lenders) and borrowers. Young depositors are born with \( x > 0 \) units of the consumption good but do not receive endowments when old. They value consumption only in the last period \((c_2)\) with preferences given by: \( U(c_2) = \ln(c_2) \). In contrast, young borrowers are born without endowments but receive \( y > 0 \) units of the consumption good in their old-age. However, they derive utility from consumption in both periods of their lives. The discounted lifetime utility function of borrowers is expressed by: \( U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2) \).

In this economy, there are \( N \) identical financial intermediaries on each island. Each bank’s objective is to maximize profits in units of consumption goods. Banks have access to the financial system and can generate income by providing financial services to individuals in the economy. Specifically, banks offer a schedule of rates of return for each unit of deposits and charge prices for each unit of loans. With deposits received, banks can trade funds for two primary assets: fiat money and one-period, default-free loans. The return to fiat money depends on the total amount of currency in the economy. \( M_t \) denotes the per capita amount of the monetary base on each island and \( \sigma \) denotes the net growth rate of money. Therefore, the money supply for \( t > 0 \) evolves according to:

\[
M_{t+1} = (1 + \sigma)M_t
\]

(1)

Spatial separation and private information generate trade frictions in the economy. To be specific, there is no communication across islands. Consequently, private liabilities do not circulate between the two locations. Moreover, a relocation shock occurs on each island. In particular, a fraction of young depositors (\( \pi \)) must move to the other island. The probability of the relocation shock is exogenous, publicly known and the same in each site. Furthermore, fiat money is the only asset that agents can carry to the other location and exchange for goods. Therefore, the relocation shock plays the role of a liquidity preference shock in the Diamond-Dybvig model and fiat currency helps individuals avoid trading frictions. As a result, currency has an advantage over other assets in terms of liquidity. This allows fiat money to be dominated in rate of return.

The timing of actions is as follows. In each period, there are two stages of transactions. As shown in Figure 1, banks accept deposits and make portfolio choices in the first stage:
At the initial stage of date $t$, banks post the schedule of interest rates ($r_t^m$ for movers and $r_t^n$ for nonmovers) to be offered in the deposit market. In turn, young lenders deposit $d_t$ units of goods. Furthermore, banks receive currency reserves through money transfers from the monetary authority at the rate $\sigma M_{t-1}$. Given funds received, banks choose the amount of currency reserves ($m_t$). Additional money balances can be obtained by trading deposits for fiat money with relocated old agents. Banks use the remaining amount of deposits to issue loans ($l_t^s$) to young borrowers in the credit market.

In the second-stage, financial intermediaries pay non-movers with the income generated from the credit market. These transactions are illustrated in Figure 2:
After banks allocate deposits between money and loans, old borrowers receive their endowments. Due to their obligations in the credit market, borrowers must pay back their loans along with interest to the bank. Banks use these funds to finance payments to old non-movers. At the end of period $t$, the relocation shock occurs. Young relocated agents will go to the bank and withdraw currency. All old agents from the previous generation consume and die.

Next, we explain the behavior of each group of agents. We proceed by studying the impact of competition and the effect of monetary policy on credit market activity.

### 3.2 Depositors

Depositors are born with endowments but derive utility from consumption only in their old-age. Due to the relocation shock and limited communication across locations, these young individuals deposit all of their endowments in banks. The economy consists of young agents with unit mass and $N$ identical financial intermediaries. Therefore, the amount of deposits for each bank is given by:

$$d = \frac{x}{N}$$

### 3.3 Borrowers

Borrowers receive endowments when old and do not experience liquidity shocks. Furthermore, they value consumption during their youth and old-age. In order to smooth
consumption, they seek to obtain loans \((l^d_t)\) from the credit market. Given \(R_t\) as the real interest rate for each unit of loans, a borrower’s objective is:

\[
Max \ln(l^d_t) + \beta \ln(y - R_t l^d_t)
\]  
(2)

Therefore, individual loan demand is given by:

\[
l^d_t = \frac{y}{(1 + \beta)R_t}
\]  
(3)

Obviously, individual loan demand is inversely related to the price. When the endowment increases or the rate of time preference decreases, borrowers want to obtain more loans in order to smooth consumption. Furthermore, the total population of borrowers is equal to one in each location. Thus, the market loan demand \((L_t)\) is the same as the individual loan demand.

### 3.4 Banks

Banks compete by offering rates of return to deposits. Since banks post prices to be paid according to withdrawal dates, they act as Bertrand competitors in the deposit market. In contrast, banks compete in the credit market by offering quantities of loans. That is, each bank recognizes that its own decisions about the amount of loans will affect the market interest rate, but that its choices do not affect those of any other bank. Moreover, from the perspective of each borrower, the loans offered by each bank are the same. In this manner, the price for each unit of loans will be the same across banks and the credit market is characterized by Cournot competition.

Due to price competition in the deposit market, intermediaries are Nash competitors; that is, banks announce rates of return, taking the announced rates of return of other banks as given. In equilibrium, price competition among banks for depositors will force banks to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor. As a result, a bank’s objective is given by:

\[
Max \sum \pi \ln(r^m_t \frac{x}{N}) + (1 - \pi) \ln(r^n_t \frac{x}{N})
\]  
(4)

Banks use deposits to trade for currency reserves in the money market and issue loans in the credit market. Therefore, the bank’s balance sheet constraint requires that:

\[
m_t + l^d_t \leq \frac{x}{N}
\]  
(5)

Relocated agents cannot access their account in the other location due to limited communication. As a result, they must use fiat money to trade for goods. Thus, the payment to relocated individuals is given by the amount of reserves:

\[
\pi r^m_t \frac{x}{N} \leq m_t \frac{p_t}{p_{t+1}}
\]  
(6)
Since movers receive payments at the end of their youth, it is also convenient to refer to \( r_t^m \) as the short-term interest rate in the economy.

Fiat money is dominated in rate of return. Therefore, intermediaries will never choose to carry excess balances between periods. As a result, the return to nonrelocated agents is obtained by loan payments in the credit market:

\[
(1 - \pi)r_t^n \frac{\pi}{N} \leq R_t l_t^s
\]

Following the discussion for \( r_t^m \), \( r_t^n \) represents the long-term interest rate on deposits.

In addition, if the return of relocated agents is more than the return of non-relocated agents, all depositors would seek to withdraw their funds at the end of the current period. In order to prevent a liquidity crisis, the following self-selection constraint must also hold:

\[
r_t^m \leq r_t^n
\]

This constraint also reflects the requirement that money is dominated in rate of return.

**Proposition 1.** Suppose the borrower’s endowment is sufficiently high such that \( y \geq \frac{N(1-\pi)(1+\beta)x}{N-(1-\pi)(1+\sigma)} \). Under this condition, a steady-state equilibrium exists and is unique. Each bank competes by allocating funds to portfolios such that \( m = \left( \frac{N}{N-(1-\pi)} \right) \pi \frac{x}{N} \) and \( l^s = \left( \frac{N-1}{N-(1-\pi)} \right) (1 - \pi) \frac{x}{N} \).

As in the standard model with log preferences, the amount of money balances depends on the probability of the relocation shock and endowments, but is independent of the return to all types of assets. This occurs because the income and substitution effects associated with the return to money offset each other. Consequently, the supply of loans offered by each bank is also independent of monetary policy and the interest rate in the loan market.

When depositors are born with larger endowments, the total amount of deposits in the economy will be higher. Thus, each bank has more funds to allocate to cash reserves and loans. Furthermore, depositors are risk averse. If the fraction of movers is higher, individuals want to receive more insurance against liquidity risk. In order to maximize the expected utility of depositors, banks will hold more money balances and issue less loans.

Notably, each banks’ portfolio allocations depend on the degree of financial competition \( N \). The terms \( \left( \frac{N}{N-(1-\pi)} \right) \) and \( \left( \frac{N-1}{N-(1-\pi)} \right) \) reflect the fractions of deposits that are allocated to reserves and loans respectively. When the banking sector is more competitive, the level of distortions decreases. As a result, given deposits received, each bank is willing to increase the amount of loans but decrease the amount of currency reserves.

However, an increase in competition leads to a lower amount of deposits per bank \( \left( \frac{x}{N} \right) \). Thus, less funds are allocated to currency reserves and loans. Moreover, the effect of financial competition on the amount of deposits dominates the effect of competition from less market power. In this manner, a higher degree of competition
causes the amount of money balances and loans on each bank’s balance sheet to decrease.

At this point, we show that imperfectly competitive banks use deposits received to acquire more reserves and issue less loans compared to perfectly competitive banks. Therefore, a lower degree of competition leads to more insurance in the deposit market, but less consumption smoothing in the credit market. In a perfectly competitive banking economy, currency reserves and loans only depend on the probability of the relocation shock and deposits that each bank receives. The amount of consumption insurance is equal to the fraction \( \pi \) of deposits while the amount of loans is equal to the fraction \( 1 - \pi \) of deposits.

Based upon banks’ portfolio allocations and the assumption that all intermediaries are identical, the market price and the aggregate quantity of loans in the credit market are:

\[
R = \left( \frac{N - (1 - \pi)}{N - 1} \right) \frac{y}{(1 - \pi)(1 + \beta)x} \tag{9}
\]

\[
L = \left( \frac{N - 1}{N - (1 - \pi)} \right) (1 - \pi) x \tag{10}
\]

Interest rates and the amount of loans are independent of monetary policy. This occurs because each bank’s portfolio allocations do not depend upon the rate of return to money and borrowers are not subject to the relocation shock. However, the degree of tractability in the benchmark model yields clear insights about the impact of financial competition.

**Lemma 1.** An increase in competition leads to a higher aggregate volume of loans and lower interest rate in the credit market.

The effect of banking competition on the credit market can be seen by considering the terms \( \left( \frac{N - (1 - \pi)}{N - 1} \right) \) and \( \left( \frac{N - 1}{N - (1 - \pi)} \right) \) on the expressions of price and quantity respectively. As in the standard Cournot model, market supply is higher when there are more firms. Similarly, as there are more banks in operation, the amount of loans will increase. This results in a lower interest rate in the credit market.

The following corollary characterizes the interest rate and amount of lending in a perfectly competitive economy.

**Corollary 1.** In a perfectly competitive economy, the price and quantity of loans are given by \( R_{pc} = \frac{y}{(1 - \pi)(1 + \beta)x} \) and \( L_{pc} = (1 - \pi) x \) respectively.

Banks in an imperfectly competitive economy can exploit their market power by issuing less loans and holding more reserves. In contrast, when the economy is characterized by perfect competition, banks take prices as given. As a result, the amount of loans is high and the interest rate is low compared to an economy under imperfect competition in the financial sector.

Next, consider rates of return in the deposit market. Payments to movers are limited by the return to money, the probability of relocation shock, and the degree
of distortion of banks’ reserves. In steady-state equilibrium, it is easy to show that the short-term interest rate can be expressed as:

\[ r^m = \left( \frac{N}{N - (1 - \pi)} \right) \frac{1}{(1 + \sigma)} \]  

(11)

Money is dominated in rate of return but it is the only medium of exchange at the other location. Therefore, currency reserves are only used to pay movers. When the growth rate of money increases, the value of fiat money decreases. Consequently, the short-term deposit rate is lower. Furthermore, if the banking sector is more competitive, each bank issues more loans. This implies that banks hold less money balances. As a result, the return to relocated agents is lower when there is more financial competition.

Notably, the effect of monetary policy on the short-term interest rate depends upon the degree of financial competition. When the economy consists of more banks, each bank issues more loans and holds less currency reserves. Monetary policy only affects the rate of return to fiat money. Moreover, the effect of monetary policy on the short-term deposit rate is weaker in more competitive banking sectors.

In contrast to the return to movers, payments to nonmovers depend on the fraction of relocated agents, the amount of endowments, and the rate of time preference. The rate of return to nonrelocated agents in the steady-state is:

\[ r^n = \frac{y}{(1 - \pi)(1 + \beta)x} = R_{pc} \]  

(12)

Interestingly, the long-term interest rate is the same as the price of loans in a perfectly competitive economy. This occurs since the distortion in the credit market affects the market price and the amount of loans in banks’ portfolios in different ways. When the level of competition increases, the level of distortions decreases. As a result, the interest rate in the credit market is lower while the amount of loans is higher. However, the impact of competition on the market price of loans is offset by the the effect of competition on quantity. In this manner, the long-term interest rate on deposits is independent of the degree of financial competition.

Since monetary policy only affects the short-term interest rate on deposits in this setting, the primary role of the central bank is to encourage risk sharing. That is, monetary policy can affect deposit market performance, but it cannot influence credit market outcomes. Moreover, as mentioned above, central banks will have less influence on deposit market activity in highly competitive financial systems. Optimal monetary policy is the money growth rate in which movers and non-movers receive the same rate of return. As monetary policy has less impact in competitive banking systems, the money growth rate that achieves complete risk sharing must be lower in economies with a larger number of banks.

Under equilibrium entry, there is greater scope for monetary policy to affect financial market activity. We turn to this possibility immediately below.
3.5 Non-Cooperative Behavior and the Endogenous Degree of Banking Competition

To understand how the degree of banking competition can be endogenized, it is useful to continue to recognize that there are two different financial markets – a deposit market and a credit market. While the credit market is imperfectly competitive and banks compete in terms of the amount of loans they issue, banks compete in terms of prices in the deposit market. As a result, the deposit market effectively operates like a perfectly competitive market and all bank revenues from the loan market are transferred to depositors who do not move between locations. In this manner, depositors are the residual claimants of bank earnings. Alternatively, one could interpret that depositors are stockholders of banks which pay dividends.

Based upon this interpretation, one can determine the number of banks that are active in the credit market through an equilibrium entry condition. That is, under endogenous entry, depositors have incentives to establish more banks if they can obtain more surplus from participating in financial markets. In this setting, we posit that depositors have an alternative use of their funds which provides them with an income level equal to \( U \):

\[
\ln \left( \frac{r^m x}{N} \right) + (1 - \pi) \ln \left( \frac{r^n x}{N} \right) = \ln(U)
\] (13)

If the probability of a liquidity shock equals \( \frac{1}{2} \), the equilibrium number of banks is:

\[
N = \frac{1 + \sqrt{1 + \frac{32xy}{U^2(1+\beta)(1+\sigma)}}}{4}
\] (14)

Notably, monetary policy has a significant impact on the degree of competition. Thus, we offer the following observation:

**Lemma 2.** Higher inflation discourages bank entry. Consequently, monetary growth leads to a lower amount of loans and a higher interest rate in the credit market.

Interestingly, higher inflation not only decreases the return to money but exacerbates the degree of distortions in the credit market. This occurs because an increase in the growth rate of money results in a lower short-term interest rate in the deposit market. Since inflation interferes with the ability of banks to provide risk pooling services, a smaller number of banks will be established. Consequently, interest rates in the credit market will be higher and there will be a lower volume of lending activity.

At this point, we have established two different channels for monetary policy to affect financial market activity. In the model with fixed entry, inflation lowers the return to money. In addition, under equilibrium entry, inflation leads to less credit market activity since there will be less banks in operation.
4 Efficient Level of Banking Competition

Due to price competition in the deposit market, banks use all revenues to finance payments to depositors. As a result, social welfare depends on the expected utility of depositors and the lifetime utility of borrowers. Therefore, the efficient degree of banking competition is determined by:

$$\max_N N \ln(r^m_t x) + (1 - \pi) \ln(r^n_t x) + \ln(L) + \beta \ln(y - R_t L)$$

(15)

To maximize social welfare, the efficient number of banks is:

$$N = \frac{\pi(1 - \pi)}{(1 + \pi)^2 - 1}$$

(16)

When the degree of competition increases, the amount of loans is higher and the interest rate in the credit market is lower. Therefore, banks provide more consumption smoothing services to borrowers but offer less risk pooling services to lenders. In this manner, the efficient degree of competition will be such that the gain in utility among borrowers is equal to the loss of expected utility among lenders.

Notably, it is possible that there is excessive competition in the economy. If the number of banks is endogenous, banks continue entering the economy until the depositor’s expected utility is equal to the autarky level, $\ln(U)$. Thus, only borrowers benefit from an increase in the degree of competition. In contrast, if the financial industry is able to restrict the amount of entry, the expected utility of depositors can be higher.

4.1 Optimal Monetary Policy

In this section, we briefly discuss the determination of optimal monetary policy in the framework. In the benchmark model with fixed entry, monetary policy only affects the rate of return to money. As a result, social welfare is highest when depositors are fully insured against liquidity risk. That is, expected utility is maximized when the short-term interest rate in the economy is equal to the long-term interest rate. In this setting, money is costless to hold since agents are indifferent between realizations of the liquidity shock.

Under equilibrium entry, depositors will continue to establish banks until there is no excess surplus. Therefore, in stark contrast to the economy with a fixed number of banks, monetary policy only affects the welfare of borrowers. Although depositors’ expected utility is independent of the inflation rate when the number of banks is endogenous, borrowers will be affected since monetary policy influences credit market activity. As the inflation rate is higher, interest rates in the credit market will also be higher and borrowers will experience more difficulty smoothing their consumption over time. Consequently, borrowers obtain higher lifetime utility when the money growth rate is lower. Therefore, as in the economy with fixed entry, the optimal monetary policy is the Friedman Rule.
Finally, at the efficient level of banking competition, monetary policy only affects the rate of return to money. As a result, social welfare is maximized at the Friedman Rule. Thus, we obtain the following conclusion:

**Proposition 2.** The optimal monetary policy is the Friedman rule.

### 4.2 Credit Market Activity under Collusive Behavior

The deposit market is characterized by price competition. Banks post the schedule of deposit rates simultaneously and repeatedly. Since the deposit rates offered by all banks are freely observable, one may expect that banks might be able to maintain collusive arrangements in the deposit market. Therefore, as in Bagliano et. al. (2000), we continue by considering a setting in which banks collude and implement monopoly pricing in the deposit market.

When banks have the ability to collude, each bank’s objective is to choose portfolios \((m_t, l^s_t)\) and interest rates \((r^m_t, r^n_t)\) to maximize:

\[
\max_{m_t, l^s_t, r^m_t, r^n_t} R_t l^s_t + m_t \frac{p_t}{p_{t+1}} - \pi \frac{x}{N} r^m_t - (1 - \pi) \frac{x}{N} r^n_t
\]

In order to earn the highest level of profits, banks will offer deposit rates to extract all of the surplus from depositors. To motivate this idea clearly, we consider that depositors have alternatives for their funds outside of the banking system. For example, they could possess storage technologies which allow them to store their wealth. We refer to this state as the autarky state and let the associated level of expected utility to be equal to \(\ln(U)\). In this setting, banks will offer deposit contracts such that:

\[
\pi \ln\left(\frac{x}{N} r^m_t\right) + (1 - \pi) \ln\left(\frac{x}{N} r^n_t\right) = \ln(U)
\]

In turn, the profit-maximizing amount of loans offered by each bank is:

\[
l^s = \frac{x}{N} + \frac{(1 + \beta)(1 + \sigma)N^2U^2}{4y(N - 1)} - \sqrt{\frac{(1 + \beta)(1 + \sigma)U^2}{y(N - 1)} \left[ N x + \frac{(1 + \beta)(1 + \sigma)U^2}{y(N - 1)} \left( \frac{N^2}{4} \right)^2 \right]
\]

In contrast to an economy under non-cooperative behavior, the collusive arrangement involves banks adjusting portfolios to monetary policy. Lemma 3 offers the following observation:

**Lemma 3.** Under collusive behavior, a higher growth rate of money leads to a lower amount of loans and a higher interest rate in the credit market. Consequently, inflation further exacerbates the degree of distortions in the credit market.

The growth rate of money not only affects its rate of return, but also the interest rate in the credit market. Under higher inflation rates, the return to money falls.
Consequently, depositors would choose not to participate in the financial market. In order to retain its customers, each bank must provide individuals with insurance against the liquidity shock. Furthermore, agents are risk averse. In this manner, financial institutions must obtain higher levels of money balances. As a result, banks lend less funds to borrowers in the credit market and the interest rate will be higher.

Therefore, the analysis demonstrates that the effects of monetary policy depend on strategic interactions among banks. As a benchmark, under non-cooperative behavior and fixed entry, inflation only affects the rate of return to money. In contrast, if there is equilibrium entry of banks, money growth interferes with the ability of banks to provide risk pooling services. Thus, inflation discourages bank entry and allows financial institutions to exploit their market power. Furthermore, if banks have the ability to collude, inflation affects the availability of funds since banks distort the deposit market. This suggests that the design of monetary policy should account for strategic behavior among banks and barriers to entry in the financial system.

5 The Model with Government Bonds

The previous analysis demonstrates that market power creates distortions in credit markets. However, government policies can exacerbate these distortions. In addition to inefficiencies from inflation, fiscal policy can interfere with credit market activity. For example, King and Levine (1993) discuss how public sector debt diverts funds away from the private sector. In order to address this issue, we extend the framework to include the possibility of government debt. That is, the government not only issues fiat money, but also interest-bearing bonds.

Interestingly, incorporating government bonds into the model leads to two important transmission channels in which imperfect competition affects credit market activity. First, as government bonds provide banks with more investment opportunities, government debt introduces additional pricing distortions. Notably, in perfectly competitive financial markets, banks take prices as given. However, in the presence of market power, banks take into account that their supply of loans affects interest rates in the credit market. As a result, the standard no-arbitrage condition between private and public credit markets breaks down. To be specific, under perfect competition, the rate of return to private sector loans will be the same as the return to government bonds. In contrast, due to imperfectly competitive behavior in the credit market, the interest rate on private sector loans will be higher than the interest rate on government debt. This occurs even though both types of loans are risk-free in the model. Moreover, as mentioned above, government debt crowds out loans to the private sector. The reduced availability of funds exacerbates pricing distortions from imperfect competition.

As in the preceding section, the economy consists of two types of agents: depositors and borrowers. All agents have the same endowments and preferences. In contrast, there are three primary assets: fiat money, loans, and government bonds. All bonds are of one-period maturity and default-free. One unit of goods held in
bonds at $t$ constitutes a sure claim to $R_t^b$ units of goods at $t + 1$.\(^6\) The government does not have any direct expenditures and does not levy direct taxes at any date. Thus, the government need only manipulate the supply of its liabilities to guarantee that it can meet its interest obligations in each period. The government budget constraint is given by:

$$R_{t-1}^b b_{t-1} = \frac{(M_t - M_{t-1})}{p_t} + b_t$$  \hspace{1cm} (20)

Due to limited communication and spatial separation, agents cannot exchange privately issued claims across islands. Consequently, fiat money is the only asset that can be carried across locations.\(^7\) Therefore, currency has an advantage over loans and bonds in terms of liquidity. This allows fiat money to be dominated in rate of return.

Next, we briefly describe the timing of actions in the economy with government debt. Banks announce deposit-return schedules that depend on depositor-withdrawal dates and depositors deposit their funds at banks. Based upon deposits received, banks choose portfolios that consist of currency reserves, loans, and government bonds. Money balances are obtained by receiving transfers of fiat money from the monetary authority and by trading some of the deposits to relocated old agents. The rest of deposits will be invested in the credit market and the bond market. After bank portfolios for the current period are established, old borrowers receive their endowments and pay back their loans along with interest to the bank. Furthermore, intermediaries may receive additional income from the previous bond market if they are net lenders. However, if they are net borrowers, income from the previous loan market will be used to pay the principal and interest to the government. When the previous bond market is settled, banks use the funds to finance payments to non-relocated individuals. At the end of the period, the relocation shock occurs and old borrowers die.

### 5.1 Banks

As banks hold portfolios with a wider range of assets, the analysis retains tractability within the context of the benchmark framework. That is, banks engage in non-cooperative behavior and there is a fixed degree of entry in the financial industry. As in the previous analysis, banks compete in terms of the quantity of loans they offer to borrowers in the credit market. However, they also choose an amount of bonds, $b_t^d$, to hold. In the deposit market, banks compete by offering rates of return to deposits. As a result, the deposit market is effectively a perfectly competitive market. In this manner, banks choose rates of return and portfolios of assets ($r_t^m, r_t^n, m_t, l_t^p, b_t^d$) to

\(^6\)Alternatively, one unit of bonds held in period $t$ yields $I_t$ units of currency in period $t + 1$. Thus, we can express the real return to bonds as $R_t^b = I_t \frac{p_t}{p_{t+1}}$.

\(^7\)Government bonds cannot be used in interlocation exchange since they are issued in relatively large denominations.
maximize the expected utility of a representative depositor:

\[
Max_x \pi \ln(r^m_t \frac{x}{N}) + (1 - \pi) \ln(r^n_t \frac{x}{N})
\]  

(21)

Furthermore, the bank’s portfolio allocations must satisfy the balance sheet constraint:

\[ m_t + l^s_t + b^d_t \leq \frac{x}{N} \]  

(22)

Since currency is the only asset that can be transported across locations, the return to relocated agents is constrained by the amount of currency holdings and the inflation rate:

\[ \pi r^m_t \frac{x}{N} \leq m_t \frac{p_t}{p_{t+1}} \]  

(23)

In addition, money is dominated in rate of return. Therefore, intermediaries will not carry balances between periods. As a result, banks use the revenues from the credit market and the bond market to finance payments to nonrelocated agents:

\[ (1 - \pi) r^n_t \frac{x}{N} \leq R_t l^s_t + R^b_t b^d_t \]  

(24)

As in the previous section, banks can prevent liquidity crises by offering a schedule of deposit rates so that nonrelocated agents receive higher returns than relocated agents:

\[ r^m_t \leq r^n_t \]  

(25)

This constraint also reflects the requirement that money is dominated in rate of return.

In order to maximize the expected utility of a representative depositor, banks allocate funds to money balances, loans, and government bonds:

\[
m_t = \pi \frac{x}{N} + \left( \frac{\pi}{N(1 + \beta)R^b_t} \right) \left( \frac{y}{N} \right)
\]  

(26)

\[
l^s_t = \left( \frac{N - 1}{N} \right) \left( \frac{1}{(1 + \beta) R^b_t} \right) \left( \frac{y}{N} \right)
\]  

(27)

\[
b^d_t = (1 - \pi) \frac{x}{N} - \left[ \frac{N - (1 - \pi)}{N(1 + \beta)R^b_t} \right] \left( \frac{y}{N} \right)
\]  

(28)

From Lemma 1, in the absence of the bond market, the portfolio choices of banks are primarily determined by conditions in the deposit market. That is, the individual loan supply of each bank only depends on deposits received and the degree of concentration in the financial sector. By comparison, in the presence of government debt, the amount of funds lent by each bank depends on conditions in the private credit market and the interest rate on loans to the government. In this manner, the results reflect that banks regard the rate of return on government bonds as given, but they take into account how the amount of loans offered affects interest rates in the private credit
market. To begin, the term \( \frac{y}{N} \) is similar to the term \( \frac{x}{N} \) in the expression for loan supply in the benchmark model. Notably, \( y \) reflects the amount of funds demanded by borrowers. In addition, the loans provided by each bank are homogeneous products. Consequently, \( \frac{y}{N} \) represents that the amount of funds supplied by each bank will be lower as more banks are active in the credit market.\(^8\)

Moreover, the term \( \frac{N-1}{N} \) reflects the degree of pricing distortions arising from imperfectly competitive behavior. As the degree of competition in the credit market is higher, \( \frac{N-1}{N} \) goes to one and each bank lends relatively more funds to borrowers. In turn, banks demand less government bonds.

As in the benchmark model, money balances are increasing in the fraction of relocated agents. In contrast, the amount of loans is independent of the liquidity shock. Banks can take advantage of the bond market by borrowing or lending additional funds. Therefore, banks can issue loans that maximize profits in the credit market regardless of the probability of the relocation shock. *In this manner, government debt helps banks to exploit their market power in the credit market.*

In particular, currency reserves are negatively related to the rate of return to government debt. This occurs because the return in the bond market reflects the opportunity cost of holding money. It also represents the opportunity cost of loans to the private sector. If government bonds yield a higher rate of return, these costs increase. As a result, banks hold less money balances and issue less loans to the private credit market.

Based on banks’ portfolio allocations, the relationship between interest rates in the private credit market and the market for government bonds is:

\[
\left( \frac{N-1}{N} \right) R_t = R^b_t
\]

Interestingly, the rate of return in the private credit market is higher than the rate of return in the bond market. Each bank realizes that its own decisions about loans affect the market price. However, a bank’s decisions do not affect those of any other bank. Thus, banks issue loans such that the perceived marginal revenue in the private credit market \( \left( \frac{N-1}{N} R_t \right) \) is the same as the marginal revenue in the public debt market \( R^b_t \). That is, *distortions from imperfectly competitive behavior lead to higher interest rates on private sector loans than government debt.* As a result, *the standard no-arbitrage condition between private and public credit markets breaks down under imperfectly competitive behavior.*\(^9\) However, the wedge \( R_t - R^b_t \) is smaller as the number of banks is higher. In particular, under perfect competition, interest rates in both markets will be the same.

\(^8\)This result is analogous to the benchmark model. When there are more banks active in the credit market, the demand for funds at each bank falls since borrowers can obtain loans from more financial institutions. However, as we demonstrate, each bank allocates relatively more funds to loans when the market is more competitive. In the analysis below, the market supply of funds to the private sector will increase if there are more banks in the economy.

\(^9\)Recall the warning by Kwan (2004): “Consolidation could reduce competition in these markets, increasing the cost of liquidity by some firms and *impeding the arbitrage of interest rates* between markets.”

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Next, we examine interest rates offered to individuals in the deposit market. It is easy to demonstrate that the rate of return to relocated agents is:

\[ r_{t}^{m} = \left( 1 + \frac{1}{R_{t}^{b}(1 + \beta)x} \frac{y}{N} \right) \frac{p_{t}}{p_{t+1}} \]  

(30)

As in the benchmark framework, relocated agents can only carry fiat money to the other location. Thus, the short-term interest rate depends on the rate of inflation. In contrast to the benchmark model, the return to movers is independent of the probability of the relocation shock. This occurs because banks can receive or lend additional reserves through the bond market. For example, if the fraction of relocated agents is higher, banks acquire more money balances. However, money is dominated in rate of return. Therefore, banks do not hold excess reserves. As a result, the rate of return to movers is not affected by the liquidity shock.

In addition, the long-term interest rate on deposits can be expressed as:

\[ r_{t}^{n} = R_{t}^{b} + \frac{1}{(1 + \beta)x} \frac{y}{N} \]  

(31)

As in the preceding section, the long-term interest rate depends on agents’ endowments. In particular, when borrowers’ endowments are higher, the demand for loans increases. Thus, banks can generate more income from the credit market. Consequently, the rate of return to nonmovers is higher. In contrast to the benchmark model, at a given interest rate on private sector loans \( R_{t}^{b} \) the return to nonmovers is independent of the fraction of movers. This occurs because the loan supply of each bank is independent of the liquidity shock.

Next, we study outcomes in the steady-state:

5.2 Equilibrium

In equilibrium, three conditions must hold. First, the money market must clear. In particular, total money demand must be equal to money supply:

\[ Nm = M \]  

(32)

Second, the supply of bonds must satisfy the government budget constraint:

\[ R^{b}b = \frac{(M - M_{-1})}{p} + b \]  

(33)

Finally, interest rates between the private credit market and the market for government bonds require:

\[ \left( \frac{N - 1}{N} \right) R = R^{b} \]  

(34)

Given \( I \) as the gross nominal interest rate on government debt, the supply of bonds can be obtained by imposing steady-state on the system and substituting the money demand function into the government budget constraint:
\[ b = \frac{\sigma \pi x}{I - (1 + \sigma)} + \frac{\pi (1 + \sigma)}{[I - (1 + \sigma)] (1 + \beta)I} \left( \frac{y}{N} \right) \] (35)

Alternatively, one can apply the bank’s balance sheet constraint (22) and express (35) as:

\[ L = x - \pi x \left( \frac{I - 1}{I - (1 + \sigma)} \right) - \frac{\pi (1 + \sigma)}{(1 + \beta)I} \left( \frac{I - 1}{I - (1 + \sigma)} \right) \frac{y}{N} \] (36)

We refer to (36) as the bond market clearing condition. Notably, the last term represents distortions from market power and inflation. As the banking sector is more concentrated, each bank recognizes that its supply of loans has a more significant impact on interest rates in the private credit market. Moreover, inflation and public sector crowding out exacerbate distortions from imperfectly competitive behavior. At higher inflation rates, seigniorage revenues increase. This allows the government to issue more bonds. As a result, less funds are available for the private sector.

Furthermore, the inverse demand function for private sector loans (3) can be used to rewrite the relationship of interest rates between the credit market and the bond market from equation (34) as:

\[ L = \left( \frac{N - 1}{N} \right) \frac{(1 + \sigma)y}{(1 + \beta)I} \] (37)

This equation represents the no-arbitrage condition between private sector loans and government debt. In particular, as the financial sector is less competitive, the interest rate on private sector loans exceeds the real interest rate on government debt. Under perfect competition, the amount of funds available in the private credit market is equal to \( \frac{(1 + \sigma)y}{(1 + \beta)I} \).

In steady-state equilibrium, the bond market clearing condition and the no-arbitrage condition must be satisfied. We proceed by describing the properties of these two conditions. First, we analyze the bond market clearing condition. Figure 3 illustrates combinations of loans and interest rates that clear the market for government
In the Figure, \( BM_B \) and \( BM_L \) are the bond market curves when the government is a net borrower and a net lender in the bond market respectively. To characterize the locus described in Figure 3, we first consider the level of the nominal interest rate when the amount of loans is equal to zero. If borrowers are born with sufficiently high endowments, (36) yields two loci, \( I_0 < 1 \) and \( I_1 > 1 + \sigma \). However, money must be dominated in rate of return. Therefore, the locus \( I_0 \) is ignored.

We continue by studying the relationship between the nominal interest rate and the amount of loans. When \( I > 1 + \sigma \), the government is a net borrower. If the nominal interest rate is higher, required interest payments are higher. Therefore, the government will not be able to issue many bonds and satisfy its budget constraint. In order for the bond market to clear, the amount of loans must increase. That is, banks must be willing to issue more loans and hold less government debt.

Moreover, the relationship depends upon the degree of competition in the financial sector. It can be shown that the bond market curve is steeper if the financial industry is more competitive. That is, under higher degrees of competition in the banking system, the nominal interest rate on government debt must rise more in order for asset markets to clear. To gain deeper perspective, it is useful to refer to equation (35). Notably, the first term \( \frac{\sigma + \sigma}{T - (1 + \sigma)} \) reflects the amount of bond holdings if financial markets are perfectly competitive. Under higher nominal interest rates, the government must pay a higher rate of return and cannot issue as many bonds. As a result, the amount of loans to the private sector must increase. The second term \( \frac{\pi (1 + \sigma)}{T - (1 + \sigma) (1 + \sigma)^2} \left( \frac{N}{N} \right) \) represents the additional demand for government debt due to imperfectly competitive behavior. If there are more banks, the amount of bond holdings would be lower. Therefore, the nominal return to government debt must rise more if the banking sector is more competitive.
In addition, it is straightforward to demonstrate that \( \lim_{I \to \infty} L = (1 - \pi)x \). When the nominal interest rate approaches infinity, the government must pay a very high interest rate. Thus, the amount of bond supply must be very small. Moreover, from (29), we can observe that a higher interest rate in the bond market is associated with a higher interest rate in the private credit market. Therefore, banks are willing to issue more loans and hold less currency reserves. However, relocated agents need fiat money for transaction services. Furthermore, from (26), the smallest amount of money balances that banks hold is \( \pi x \). As a result, the amount of loan supply is close to \( (1 - \pi)x \) when the nominal interest rate approaches infinity.

Next, we consider the steady-state in which the government is a net lender \( (I < 1 + \sigma) \). First, when the nominal rate of return to bonds is equal to one, the amount of loans in the credit market is \( x \). Intuitively, if government bonds yield the same rate of return as fiat currency, the cost of borrowing from the government is very low. Thus, banks use all deposits to issue loans. In order to acquire money balances, banks borrow funds from the government and trade with old relocated depositors.

Furthermore, it is easy to show that \( \lim_{I \to \infty} I = 1 + \sigma \). When the amount of loans approaches infinity, the government lends a large amount of funds to the private sector. This can only occur if the nominal interest rate is close to \( 1 + \sigma \).

At this point, it is useful to compare our work on inflation-financed government debt with Schreft and Smith (1997). In a production economy, Schreft and Smith also incorporate inflation-financed government bonds. In their framework, if the government runs a budget deficit, crowding out from higher inflation rates also occur. In turn, the economy’s level of development suffers at higher rates of inflation since less investment and capital accumulation occur. There are two features that are distinct in our model. First, we model a private credit market which helps borrowers smooth consumption over time. Higher inflation rates and government debt reduce their ability to do so. Second, we show how this problem depends on the degree of competition in the economy’s banking system.

We proceed by analyzing the conditions in which banks are indifferent between lending funds to the government or the private sector. If the nominal interest rate falls, the real return to government bonds will be lower. This provides banks with greater incentive to lend in the private credit market. In contrast to Figure 3, Figure 4 represents the combination of private sector loans and interest rates in the public
We refer to the NA curve in Figure 4 as the no arbitrage curve. Moreover, if the financial sector is more competitive, banks have less market power. Consequently, the amount of lending activity responds more to the nominal return to government debt if there are more banks. As a result, the no-arbitrage curve will be more flat if the financial sector is more competitive. Furthermore, as $I \to \infty$, $L \to 0$ and as $L \to \infty$, $I \to 0$. When the nominal interest rate approaches infinity, banks incur a significantly high opportunity cost by lending in the private credit market. Therefore, except for funds dedicated to currency reserves, they invest all funds in government bonds. Finally, if the amount of loans is close to infinity, the cost of borrowing must be extremely low. This implies that the nominal interest rate must approach zero.

In order to determine the steady-state amount of loans and nominal interest rates, we consider the interactions between the bond market clearing condition and the no arbitrage condition. Thus, we utilize Figure 5 to demonstrate the resulting steady-
state equilibria:

Figure 5: Steady-State Equilibria

The no arbitrage condition is a continuous function. Therefore, it passes through $I = 1 + \sigma$. As a result, we establish the first steady-state equilibrium at point $A$. Furthermore, at $I = 1$, if borrowers receive sufficiently high endowments, the demand for loans will be relatively high. As result, the effect of the no arbitrage condition on the amount of loans will be stronger than the bond market clearing condition. That is, the amount of loans implied by the no arbitrage condition will be higher than the bond market clearing condition. Consequently, we establish the steady-state equilibrium at point $B$.

**Proposition 3:** Suppose borrowers receive sufficiently high endowments such that $y > \text{Max} \left( \frac{1+\sigma-\pi-2\pi}{\pi}, \frac{1}{N-1} \right) \frac{N(1+\beta)x}{1+\sigma}$. Under this condition, there are exactly two nontrivial steady-state equilibria with $I > 1$.

From Figure 5, one steady-state occurs at a high nominal interest rate and a low amount of loans. In contrast, the other steady-state has a low nominal interest rate and a large amount of loans. The possibility of multiple steady-state equilibria arises because the government can act differently in the credit market. Specifically, when the government is a net borrower, government debt competes with private loans in banks’ portfolios. Therefore, public sector crowding out diverts funds away from the private sector. Consequently, government debt exacerbates the distortions from imperfectly competitive behavior.

In contrast, if the government is a net lender, it transfers funds to banks. As a result, banks have more funds to invest in the credit market. *In this manner, the actions of the government can alleviate credit market distortions.* We proceed by
studying the interaction between monetary policy and the degree of competition in the financial sector.

5.3 The Effects of Banking Competition

To understand the impact of banking competition in the presence of government debt, we begin with a “partial equilibrium” perspective. In particular, we discuss the effects of banking competition through the different transmission channels introduced in the previous section. We consider that the initial analysis is partial equilibrium in that we study the effects of competition through the bond market clearing and no-arbitrage conditions individually.

We start by investigating the effects of competition through the bond market clearing condition. The impact is shown by differentiating (36):

$$\frac{\partial L}{\partial N} = \left( \frac{\pi(1 + \sigma)}{N(1 + \beta)I} \right) \left( \frac{I - 1}{I - (1 + \sigma)} \right) \left( \frac{y}{N} \right) > 0 \text{ if } I > (1 + \sigma) \quad (38)$$

As shown in Figure 6 below, the bond market clearing curve shifts out when the degree of competition is higher. From equation (38), at a fixed nominal interest rate on government debt, the amount of loans will be higher when there is more competition in the financial sector.

![Figure 6: The Effect of Banking Competition on the Bond Market Curve](image)

When there are more banks in the economy, each bank is willing to issue more loans since it perceives that it has less influence on interest rates in the private credit market. Therefore, the demand for government bonds decreases while the amount of private sector loans increases. In turn, the interest rate on government debt must fall. In this manner, increased competition implies that the crowding out effect is
less significant. However, if the banking sector is initially very competitive, increased entry in the financial sector will have less impact.

In contrast, when the government lends funds to banks, higher financial competition leads to the movement of the bond market curve from $BM_L^1$ to $BM_L^2$. *If the degree of competition is higher, pricing distortions are lower and banks will seek to borrow more from the government in order to lend to the private sector. Consequently, the costs of obtaining funds must rise in order for financial markets to clear. As a result, the total amount of funds devoted to the credit market actually falls when the sector is more competitive.*

Next, we examine how competition affects financial market behavior through the no arbitrage condition. The effect of financial competition can be expressed by:

$$\frac{\partial L}{\partial N} = \left( \frac{(1 + \sigma) y}{N(1 + \beta) I} \right) \left( \frac{y}{N} \right) > 0$$  \hspace{1cm} (39)

We illustrate this impact in Figure 7:

![Figure 7: The Effect of Banking Competition on the No Arbitrage Curve](image)

As shown in the Figure, when the degree of banking competition is higher, the no arbitrage curve moves from $NA^1$ to $NA^2$. This occurs because each bank perceives that the opportunity cost of issuing loans ($\left( \frac{N}{N-1} R^b \right)$) is lower in an economy with more banks. As a result, the volume of loans increases while the amount of government debt decreases. In turn, the nominal interest rate increases. Furthermore, as in the case of the bond market clearing condition, the effect on lending activity is less significant if the banking sector is more competitive. In this manner, the impact of competition on the credit market will be weaker if the credit market is initially competitive.
In summary, if the government runs a budget deficit, higher banking competition leads to a higher volume of private lending activity. Consequently, if the banking system is more competitive, the crowding out problem from inflation-financed government debt will be less severe. This occurs because the distortions from market power in the private credit market are not as strong. In contrast, when the government lends funds to banks, the net impact of banking competition is ambiguous. While the higher level of competition alleviates pricing distortions, it can also raise the costs of obtaining funds from the government. Nevertheless, we are able to demonstrate that:

**Proposition 4:** Suppose that both steady-state equilibria exist. Furthermore, let \( y \) satisfy \( \left\{ \frac{N(1+\beta)x}{(1+\sigma)} \left( \frac{1+\sigma-\pi}{\pi} \right), \frac{2(1+\sigma+\pi)}{1+\pi(N-2)} \right\} \) and \( N < 2 \left( \frac{1+\sigma+\pi+2\sigma}{1+\sigma+\sigma-\pi^2} \right) \). Under these conditions, a higher degree of financial competition leads to an increase in private sector loans.

We offer some interpretation for the conditions in the Proposition. To begin, an increase in borrowers’ endowments leads to higher demand for loans. This allows banks to exploit their market power in the credit market. Consequently, if banks lend funds to the government, the effect of the no arbitrage condition will be stronger than the effect of the bond market clearing condition. In contrast, if banks obtain funds from the government and borrowers receive very large amounts of endowments, the demand for loans will be relatively high. As a result, the effect of the bond market clearing condition may be stronger than the effect of the no arbitrage condition. That is, if more banks compete for funds from the government, higher levels of competition may actually lead to less credit market activity. This is due to the higher cost of funds from the government when the banking sector is more competitive. Yet, under the upper bound for borrowers’ endowments stated in the Proposition, competition leads to increased lending activity.

Furthermore, the number of banks cannot be too high. When the banking sector is highly competitive, there are few distortions from imperfectly competitive behavior. Since banks do not have the ability to exploit their market power, the effect of \( N \) through the no-arbitrage condition can be dominated by the effect through asset market clearing.

Next, we proceed by considering the impact of banking competition on the real interest rate in the credit market. By following Proposition 4 and applying (3), we obtain Proposition 5:

**Proposition 5:** Higher financial competition leads to a lower real interest rate in the private credit market.

Interestingly, the results demonstrate that the impact of competition on interest rates varies across financial markets. Consider the steady-state in which the government runs a budget deficit and crowds out funds from the private sector. In this scenario, both the no-arbitrage and bond market clearing conditions imply that increased competition in the financial sector will lead to an increase in the nominal
interest rate on loans to the government. Since this occurs at a fixed money growth
rate, the result implies that the real return to government debt will be higher. It is
natural to believe that interest rates in the private credit market would respond in the
same manner. However, this ignores the pricing distortions from imperfectly com-
petitive behavior in the private market for funds. For clarity, recall the no-arbitrage
condition implies that the interest rate on private loans exceeds the interest rate on
government debt: \( R = \left( \frac{N}{N-1} \right) R^b \). Although the real interest rate on government
debt is higher when the financial sector is more competitive, the degree of pricing
distortions falls (as reflected by the term \( \left( \frac{N}{N-1} \right) \)). As a result, economies with more
banks will have lower interest rates on private sector loans. Similar interpretation
applies to the steady-state in which the government runs a budget surplus.

At this point, we have shown that the impact of inflation-financed government
debt on private credit markets strongly depends on the degree of competition in an
economy’s banking system. Next, we examine how the impact of monetary policy
depends on the competitive structure of the financial system and the government’s
position in financial markets.

5.4 The Effects of Monetary Policy

In the benchmark analysis without government debt, we sought to identify different
mechanisms in which inflation leads to inefficiencies in financial markets. In the
model with noncooperative behavior and fixed entry, inflation only affects the rate of
return to money. Consequently, it only has an impact on the degree of risk pooling
among depositors. However, under equilibrium entry of banks, inflation discourages
bank entry. Since inflation reduces competition in the banking sector, interest rates
in the credit market rise and borrowers experience more difficulty smoothing their
consumption over time. Moreover, if banks have the ability to collude and implement
monopoly pricing, inflation further intensifies credit market frictions.

Notably, in the presence of government debt, we have identified two additional
transmission channels which affect the provision of funds to the private credit market.
The first reflects pricing distortions from imperfect competition – the interest rate on
private sector loans will be higher than the interest on government bonds. This occurs
even though both types of loans are risk-free in the model. The second channel is the
standard crowding-out problem – yet, here, the effect of fiscal policy clearly depends
on the incentives of financial institutions to transfer funds to private borrowers. This
exacerbates frictions from imperfectly competitive behavior. As demonstrated below,
these transmission channels offer new mechanisms in which inflation affects financial
market activity.

We begin by discussing the effects of inflation from the bond market clearing
condition. Upon differentiating (36):

\[
\frac{\partial L}{\partial \sigma} = -\left( \frac{\pi (I-1)}{[I-(1+\sigma)]^2} \right) \left( x + \frac{y}{N(1+\beta)} \right) < 0 \quad (40)
\]
As observed, higher inflation rates lend to less lending activity in the private loan market. In Figure 8, the bond market curves shift from $BM_1^B$ to $BM_2^B$. The reduced availability of funds to the private sector comes from two sources.

Notably, the first term, $-\left(\frac{\pi(I-1)}{[I-(1+\sigma)]^2}\right)x$, demonstrates how inflation affects the provision of funds under perfect competition. When the monetary authority adopts a higher rate of money growth, the real value of bonds decreases and the government can issue more debt. This leads to less lending activity in the private sector. The second term, $-\left(\frac{\pi(I-1)}{[I-(1+\sigma)]^2}\right)\left(\frac{y}{N(I+\beta)}\right)$ shows the additional distortionary effect of market power. As the government seeks to obtain more funds from banks, this provides banks with additional investment opportunities and allows them to exploit their market power in the private credit market. *This indicates that inflation-financed crowding out will have a stronger impact on the availability of funds to the private credit market if the banking system is more concentrated.*

Monetary policy also affects the no-arbitrage condition across financial markets. These interactions can be observed by differentiating (37):

$$\frac{\partial L}{\partial \sigma} = \left(\frac{N-1}{N}\right) \frac{y}{(1+\beta)I} \tag{41}$$

The impact of monetary expansion on the no arbitrage curve is illustrated in Figure
Figure 9: The Effect of Monetary Policy on the No Arbitrage Curve

As shown in the Figure, the no arbitrage curve moves from $NA^1$ to $NA^2$. Intuitively, when inflation is higher, the real return in the bond market falls. Thus, the opportunity cost of issuing loans is lower. From this perspective, inflation may enhance welfare by lowering the real return to government debt and providing banks with less opportunities to exploit their market power in the private credit market.

As a result, the effects of inflation on credit market outcomes appear to be ambiguous. While the accompanying crowding out problem can be severe, inflation may alleviate pricing distortions from imperfectly competitive behavior. In order to obtain more insights, we look at some numerical illustrations. As an example, consider the following set of parameters: $x = 1.5$, $y = 10$, $\beta = 0.9$, $\pi = 0.5$, $N = 2$. Table 1
presents the results:

<table>
<thead>
<tr>
<th>Growth Rate of Money</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Return to Money</td>
<td>0.952381</td>
<td>0.909091</td>
<td>0.869565</td>
<td>0.833333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equilibrium with Budget Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest rate in the Bond Market</td>
</tr>
<tr>
<td>Real Interest Rate in the Bond Market</td>
</tr>
<tr>
<td>Real Interest Rate in the Credit Market</td>
</tr>
<tr>
<td>Bonds</td>
</tr>
<tr>
<td>Loans</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equilibrium with Budget Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest rate in the Bond Market</td>
</tr>
<tr>
<td>Real Interest Rate in the Bond Market</td>
</tr>
<tr>
<td>Real Interest Rate in the Credit Market</td>
</tr>
<tr>
<td>Loans</td>
</tr>
</tbody>
</table>

Table 1: Effect of Inflation in the Presence of Government Debt

Moreover, the results are robust to different sets of parameters. As a result, we conclude:

If the government is a net borrower, inflation is negatively related to the amount of loans. In contrast, when the government is a net lender, higher inflation leads to a higher amount of loans.

Interestingly, these results relate to work by Boyd, Levine, and Smith (2001) who also find inflation is associated with a lower amount of credit market activity. They posit that the effects of inflation occur due to the presence of asymmetric information in the credit market. As inflation leads to lower real interest rates, both good and bad borrowers will seek to obtain funds. Consequently, banks respond by issuing less loans. By comparison, in our framework, inflation lowers the volume of lending activity because of pricing distortions from imperfect competition and public sector crowding-out effects.

How do the effects of monetary policy depend on the competitive structure of the banking system?

At this point, we have shown that both monetary policy and the degree of competition have a significant impact on the credit market. Moreover, monetary policy can exacerbate distortions from imperfectly competitive behavior. We conclude the analysis by studying how the effects of monetary policy depend upon the competitive structure on the financial industry. This is shown by differentiating (40) and (41) with respect to the number of banks:

\[
\frac{\partial L}{\partial \sigma \partial N} = \frac{(I - 1) \pi y}{(1 + \beta) N^2 [I - (1 + \sigma)]^2} > 0 \tag{42}
\]
As illustrated by (42) and (43), the effects of inflation depend on the competitive structure of the financial system. In order to gain deeper understanding, we first provide economic intuition for (42). As previously discussed, inflation will cause the bond market curve to shift back. This results in a lower availability of funds to the private sector. Nevertheless, when the degree of competition is higher, banks are willing to issue more loans. In this manner, the decrease in funds will be lower if the financial sector is more competitive. This suggests that monetary policy will have a stronger impact on credit market activity if the banking system is more concentrated.

We continue by explaining the interactions between the degree of competition and monetary policy through the no arbitrage condition. When inflation is higher, the real interest rate in the bond market decreases. Thus, the opportunity cost of issuing loans is lower. Furthermore, if the degree of competition increases, each bank perceives that the marginal cost for each unit of loans decreases even more. Therefore, an increase in the degree of competition results in a stronger impact of monetary policy through the no arbitrage condition.

As a result, monetary policy shifts the bond market curve and no arbitrage curves in different directions. Again, numerical examples offer some insight into the relationship between the competitive structure of the financial system and the impact of monetary policy. We use the same set of parameters from Table 1, but the impact of monetary policy is illustrated by an increase in the growth rate of money from 5% to 10%. Table 2 shows the relationship between monetary policy and the degree of financial competition:

<table>
<thead>
<tr>
<th>Number of Banks</th>
<th>2</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Change at Equilibrium with Budget Deficit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of Return in Credit Market</td>
<td>1.307706</td>
<td>0.718666</td>
<td>0.711842</td>
</tr>
<tr>
<td>Loans</td>
<td>-1.290826</td>
<td>-0.713538</td>
<td>-0.70681</td>
</tr>
<tr>
<td>% Change at Equilibrium with Budget Surplus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of Return in Credit Market</td>
<td>-2.738174</td>
<td>-0.735142</td>
<td>-0.70896</td>
</tr>
<tr>
<td>Loans</td>
<td>2.815261</td>
<td>0.740587</td>
<td>0.714023</td>
</tr>
</tbody>
</table>

Table 2: The Impact of Competition on Monetary Policy

From Table 2, we offer the following conjecture:

*The effect of monetary policy on the credit market is weaker when the financial sector is more competitive. That is, monetary policy will have a stronger impact on credit market activity if the banking system is more concentrated.*

Consequently, it appears that the impact of competition through the bond market clearing condition dominates the impact through the no arbitrage condition.

In the previous discussion, we showed that if the government runs a budget deficit, higher inflation leads to a lower amount of loans. However, when the degree of...
competition is higher, each bank wants to issue more loans. Therefore, the effects of inflation are weaker when the financial sector is more competitive.

In addition to the degree of banking competition, the impact of monetary policy may depend on the initial level of inflation. Huybens and Smith (1998) propose that the negative impact of inflation on long-run growth will become more pronounced at relatively high levels of inflation. Furthermore, Boyd, Levine and Smith (2001) and Boyd and Champ (2006) find evidence of threshold effects of inflation. In particular, they observe that the marginal effect of inflation decreases rapidly when inflation is above 15%.

While previous work investigates the impact of inflation on financial market outcomes, it emphasizes how the effects of monetary policy depend on the extent of information frictions in the credit market. In contrast, we aim to address this issue in a setting in which banks are engaged in strategic behavior due to market power. Interestingly, the numerical examples illustrate that the threshold effects from inflation may be determined by the competitive structure of the financial system. Table 3 presents some examples:

<table>
<thead>
<tr>
<th>Level of Inflation</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Change at Equilibrium with Budget Deficit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Interest Rate in the Credit Market</td>
<td>0.013077058</td>
<td>0.011654355</td>
<td>0.010457275</td>
</tr>
<tr>
<td>Loans</td>
<td>-0.012908256</td>
<td>-0.011520096</td>
<td>-0.010349052</td>
</tr>
<tr>
<td>% Change at Equilibrium with Budget Surplus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Interest Rate in the Credit Market</td>
<td>-0.027381742</td>
<td>-0.024950529</td>
<td>-0.022844645</td>
</tr>
<tr>
<td>Loans</td>
<td>0.02815261</td>
<td>0.025588988</td>
<td>0.023378723</td>
</tr>
</tbody>
</table>

Table 3: The Impact of Monetary Policy under Different Degrees of Inflation

The effects of monetary policy are weaker at higher inflation rates. Moreover, the inflation threshold is lower in economies that have less competitive banking systems.

The numerical examples in Table 3 demonstrate that inflation has a less significant impact on credit market activity at higher inflation rates. Moreover, the analysis in Table 2 suggests that monetary policy has a stronger effect on lending if the banking sector is less competitive. Taken together, this implies that the inflation thresholds identified in Boyd, Levine, and Smith and Boyd and Champ are lower in economies that have more concentrated banking systems.
6 Conclusions

A wide array of evidence illustrates that the competitive structure of the financial system varies across countries. This is likely to affect financial market activity. For example, Hannan (1991) and Corvoisier and Gropp (2002) identify that borrowers in more concentrated markets pay higher interest rates for loans. In addition to inefficiencies from market power, Boyd, Levine and Smith (2001) point out that inflation leads to a lower amount of credit market activity. Building on recent research which studies the interactions between monetary policy and the degree of competition in the banking industry, we construct a monetary model in which banks compete in terms of the amount of loans they issue. In particular, we demonstrate that the degree of price distortions depends on strategic interactions among banks and barriers to entry. Moreover, monetary policy can exacerbate imperfections arising from imperfectly competitive behavior in the credit market. Inflation-financed government debt can also contribute to these sources of inefficiency. Since the availability of government bonds provides banks with additional investment opportunities, banks can exploit their market power in the private credit market. As a result, the interest rate on private sector loans will be higher than the interest rate on government bonds even though both types of loans are risk-free in the model. Finally, the reduced availability of funds heightens pricing distortions from imperfect competition – numerical exercises indicate that monetary policy will have a stronger impact on credit market activity if the banking system is more concentrated. In this manner, the framework offers novel mechanisms in which inflation interferes with financial market performance.
7 Appendix

1. The profit-maximization choices of banks in the presence of non-cooperative behavior.

Due to price competition in the deposit market, banks allocate funds and set interest rates to maximize the expected utility of a representative depositor. The objective problem is:

$$\max_{r^m_t, r^n_t, m_t, l^e_t} \pi \ln(r^m_t \frac{x}{N}) + (1 - \pi) \ln(r^n_t \frac{x}{N})$$

subject to:

$$m_t + l^e_t \leq \frac{x}{N}$$

$$\pi r^m_t \frac{x}{N} \leq m_t \frac{p_t}{p_{t+1}}$$

$$(1 - \pi)r^n_t \frac{x}{N} \leq R_t l^e_t$$

Money is dominated in the rate of return. Furthermore, all constraints are binding. Therefore, we can solve for $r^m_t$ and $r^n_t$ as a function of money balances ($m_t$) and the amount of loans ($l^e_t$) respectively. As a result, the objective problem becomes:

$$\max_{m_t, l^e_t} \pi \ln\left(\frac{m_t}{N} \frac{p_t}{p_{t+1}}\right) + (1 - \pi) \ln\left(\frac{R_t l^e_t}{(1 - \pi)}\right)$$

By using the balance sheet constraint, $m_t = \frac{x}{N} - l^e_t$. Thus, we can write the profit function as:

$$\max_{l^e_t} \pi \ln\left(\frac{\frac{x}{N} - l^e_t}{\pi} \frac{p_t}{p_{t+1}}\right) + (1 - \pi) \ln\left(\frac{R_t l^e_t}{(1 - \pi)}\right)$$

Upon using the loan demand function and substituting the price of loans into the objective function of the bank:

$$\max_{l^e_t} \pi \ln\left(\frac{\frac{x}{N} - l^e_t}{\pi} \frac{p_t}{p_{t+1}}\right) + (1 - \pi) \ln\left(\frac{y l^e_t}{(1 - \pi)(1 + \beta)L}\right)$$

We take the first derivative with respect to $l^e_t$ and assume that all banks are symmetric. Therefore, each bank issues loans such that:

$$l^e_t = \left[\frac{(N - 1)}{N - (1 - \pi)}\right] (1 - \pi) \frac{x}{N}$$

This completes the proof of the profit-maximizing choices of banks in the presence of non-cooperative behavior.
2. The profit-maximization choices of the bank under collusive behavior.

Under collusive behavior, banks can generate profits. Thus, the bank’s objective problem is:

\[ \text{Max} \quad m_t l_t^s + m_t \frac{p_{t}}{p_{t+1}} - \pi \frac{x}{N} l_t^m - (1 - \pi) \frac{x}{N} r_t^n \]

Banks have the ability to collude in the deposit market. Therefore, they extract all surplus from depositors by offering rates of return such that:

\[ \pi \ln\left(\frac{r_t^m x}{N}\right) + (1 - \pi) \ln\left(\frac{r_t^n x}{N}\right) = \ln(U) \]

Upon the substitution of \( r_t^m \), the schedule of deposit rates is:

\[ \pi \ln\left(\frac{m_t}{\pi} \frac{p_t}{p_{t+1}}\right) + (1 - \pi) \ln\left(\frac{r_t^n x}{N}\right) = \ln(U) \]

This can be reduced to:

\[ \ln\left(\frac{m_t}{\pi} \frac{p_t}{p_{t+1}}\right)^\pi \left(\frac{x}{N} r_t^n (1 - \pi)\right) = \ln(U) \]

By using the balance sheet constraint, we can write \( r_t^n \) as a function of \( l_t^s \):

\[ r_t^n = \frac{1}{N} \left[ U \pi \pi \left(\frac{p_{t+1}}{p_t}\right) \pi \left(\frac{x}{N} - l_t^s\right)^\pi \right] \]

Therefore, substitute \( r_t^n \) into the bank’s profit function:

\[ \text{Max} \quad m_t l_t^s + m_t \frac{p_{t}}{p_{t+1}} - \pi \frac{x}{N} l_t^m - (1 - \pi) \left[U \pi \pi \left(\frac{p_{t+1}}{p_t}\right) \pi \left(\frac{x}{N} - l_t^s\right)^\pi \right]^{\frac{1}{(1-\pi)}} \]

Money is dominated in rate of return. As a result, the constraint on the return to movers is binding. In this manner, banks’ objective functions reduce to:

\[ \text{Max} \quad R_t l_t^s - (1 - \pi) \left[U \pi \pi \left(\frac{p_{t+1}}{p_t}\right) \pi \left(\frac{x}{N} - l_t^s\right)^\pi \right]^{\frac{1}{(1-\pi)}} \]

By substituting the inverse loan demand function, the maximization problem becomes:

\[ \text{Max} \quad \frac{y_t^s}{(1 + \beta) L} - (1 - \pi) \left[U \pi \pi \left(\frac{p_{t+1}}{p_t}\right) \pi \left(\frac{x}{N} - l_t^s\right)^\pi \right]^{\frac{1}{(1-\pi)}} \]
We take the first derivative with respect to \( l^s_t \). Furthermore, assume that all banks are symmetric and the probability of a relocation shock equals to \( \frac{1}{2} \). In this manner, individual loan supply is:

\[
l^s = \frac{x}{N} + \frac{(1 + \beta)(1 + \sigma)N^2U^2}{4y(N - 1)} - \sqrt{\frac{(1+\beta)(1+\sigma)U^2}{y(N-1)}} \left[ N \frac{x}{N} + \frac{(1+\beta)(1+\sigma)U^2}{y(N-1)} \left( \frac{N^2}{4} \right)^2 \right]
\]

This completes the proof of the profit-maximizing choices of banks under collusive behavior.

3. The profit-maximization choices of the bank in the model with government bonds.

Each bank maximizes the expected utility of a representative depositor by allocating funds to money, loans, and government bonds and setting the schedule of deposit rates:

\[
Max_{r^m_t, r^n_t, m_t, l^s_t, b_t} \pi \ln(r^m_t \frac{x}{N}) + (1 - \pi) \ln(r^n_t \frac{x}{N})
\]

subject to:

\[
m_t + l^s_t + b_t = \frac{x}{N} \]

\[
\pi r^m_t \frac{x}{N} = m_t \frac{p_t}{p_{t+1}}
\]

\[
(1 - \pi) r^n_t \frac{x}{N} = R_t l^s_t + R^b_t b_t
\]

By defining \( \gamma_t = \frac{N m_t}{X} \) and \( \phi_t = \frac{N l^s_t}{X} \) and substituting the values of \( r^m_t \) and \( r^n_t \), the maximization problem can be expressed as:

\[
Max_{\gamma_t, \phi_t} \pi \ln\left( \frac{\gamma_t}{\pi} \frac{p_t}{p_{t+1}} \frac{x}{N} \right) + (1 - \pi) \ln \left[ \frac{R_t \phi_t + (1 - \gamma_t - \phi_t) R^b_t}{(1 - \pi)} \frac{x}{N} \right]
\]

From the loan demand function, solve for \( R_t \) as a function of \( \phi_t \),

\[
R_t = \frac{N y}{(1 + \beta) x \sum \phi_{it}}.
\]

Then, upon the substitution of \( R_t \) and deleting the constant terms, the objective function is:

\[
Max_{\gamma_t, \phi_t} \pi \gamma_t + (1 - \pi) \ln\left( \frac{N y \phi_t}{(1 + \beta) x \sum \phi_{it}} + (1 - \gamma_t - \phi_t) R^b_t \right)
\]

By solving for \( \gamma_t \) and \( \phi_t \), the amount of money balances and loans is:

\[
m_t = \pi \frac{x}{N} + \frac{\pi y}{N^2(1 + \beta) R^b_t}
\]
l_t^* = \frac{(N - 1) y}{N^2 (1 + \beta) R_t^b}

This completes the proof of the profit-maximization choices of the bank in the model with government bonds.

In order to establish the existence of steady-state equilibrium, the following conditions must be satisfied:

1. The total money demand must be equal to money supply:

\[ N m_t = \frac{M_t}{p_t} \]

2. The amount of bond supply must satisfy the government budget constraint:

\[ R_{t-1}^b b_{t-1} = \frac{(M_t - M_{t-1})}{p_t} + b_t \]

3. Interest rates between the credit market and the bond market require:

\[ \left( \frac{N - 1}{N} \right) R_t = R_t^b \]

By imposing the steady-state, we substitute the money demand function into the government budget constraint and derive the bond market clearing condition:

\[ L = x - \left( \pi x + \frac{\pi (1 + \sigma) y}{N (1 + \beta) I} \right) \left( \frac{I - 1}{I - (1 + \sigma)} \right) \]

Furthermore, we can use the inverse demand function for loans and rewrite the relationship of interest rates between the credit market and the bond market as the no arbitrage condition:

\[ L = \left( \frac{N - 1}{N} \right) \frac{(1 + \sigma) y}{(1 + \beta) I} \]

In this manner, economic outcomes in the steady-state must satisfy the bond market clearing condition and the no arbitrage condition.

Next, consider the relationships between the amount of loans and nominal interest rate in the bond market that satisfy the bond market clearing condition. When the amount of loans is zero, there are two nominal interest rates, \( I_0 \) and \( I_1 \). It is easy to show that \( I_0 \) is less than one. In contrast, we impose the condition on the amount of depositor’s endowments such that \( I_1 > 1 + \sigma \):

\[ y > \frac{[(1 - \pi) + \sigma - 2\sigma \pi] N (1 + \beta) x}{\pi (1 + \sigma)} \]
Next, we find the condition such that the amount of loans and nominal interest rate in the bond market meet the requirements in both the bond market clearing condition and the no arbitrage condition. The no arbitrage curve is continuous. Thus, we establish the first steady-state in which the government is a net borrower. To establish the second steady-state, note that if the nominal interest rate in the bond market is equal to one, the amount of loans implied by the no arbitrage condition must be more than the amount of loans in the bond market clearing condition. This holds if:

\[ y > \left( \frac{N}{N-1} \right) \frac{1 + \beta}{1 + \sigma} \]

This completes the proof of Proposition 3.


We demonstrate the effect of banking competition on economic outcomes by differentiating the bond market clearing condition and the no arbitrage condition respectively:

\[
\frac{\partial L}{\partial N} = \left( 1 + \sigma \right) \frac{1 + \beta}{1 + \tau (1 + \pi)} \left( \frac{1}{(I - (1 + \sigma))} \right)
\]

The effect of financial competition on the amount of loans in the bond market clearing condition depends on the government’s net position in the bond market. In contrast, more financial institutions lead to an increase in the amount of loans from the no arbitrage condition. As a result, we begin by considering the economy in which the government is a net borrower in the bond market. In this economy, the effect of banking competition on the no arbitrage condition dominates the effect of banking competition on the bond market clearing condition if:

\[ y > \frac{N (1 + \beta) x (1 - \pi + \sigma)}{\pi (1 + \sigma)} \]

In contrast, when the government is a net lender in the bond market, the effect of banking competition on the no arbitrage condition is greater than the effect of banking competition on the bond market clearing condition if depositors receive endowments such that:

\[ y < \frac{2N(1 + \beta)x (1 + \pi + \sigma)}{(1 + \pi)(N - 2)(1 + \sigma)} \]

In this manner, we establish the lower bound and the upper bound for the amount of lender’s endowments. Thus, this interval is non-empty if:

\[ N < 2 \left[ \frac{1 + \sigma + \pi + 2\sigma\pi}{1 + \sigma + \sigma\pi - \pi^2} \right] \]

This completes the proof of Proposition 4.
References


