Optimal Public Debt Redux

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Abstract

We examine the role played by government investment in infrastructure in determining the optimal quantity of public debt in a heterogeneous agent economy with incomplete insurance markets. Calibrating our model to the key aggregate and distributional moments of the U.S. economy, we show that, (i) the inclusion of infrastructure, and (ii) transitional dynamics between stationary states critically affects the characterization of the optimal level of public debt. Welfare comparisons between stationary equilibria indicate that it is optimal for the government to accumulate assets (public surplus). However, once transitional dynamics are accounted for, the optimal share of public debt turns out to be positive and close to the current level of public debt in the U.S. These contrasting results underscore a previously ignored channel through which public investment and tax policies can generate differential trade-offs for the precautionary savings motive for households in the short run and long run. Our results also indicate that the inclusion of public infrastructure in the model specification implies a lower level of optimal debt relative to the model without infrastructure, both when comparing steady states as well as accounting for transitional dynamics.

Keywords: Infrastructure, public investment, heterogenous agents, public debt, welfare, transitional dynamics.

JEL Classification: E2, E6, H3, H4, H6

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1 Introduction

What is the optimal amount of public debt? This is an important question that has received a lot of attention recently, especially after the global financial crisis of 2008-2009. In a traditional representative agent macro model, the quantity of public debt is irrelevant for private decision making as long as the intertemporal budget constraint of the government satisfies the transversality condition, thereby ensuring that the government does not run a Ponzi scheme against the private sector. However, in a context where households receive idiosyncratic shocks that cannot be perfectly insured, public debt can have important consequences for agents’ decisions. Higher levels of public debt can be detrimental to welfare by crowding out private investment, which leads to lower wages, output and consumption in equilibrium. At the same time, public debt can also relax borrowing constraints for households by increasing liquidity in the economy which facilitates consumption smoothing, thereby improving aggregate welfare. A priori, it is therefore not clear whether it is optimal for the government to accumulate debt or assets (surplus) in equilibrium. This paper analyzes this important public policy question by embedding two key features in a calibrated heterogeneous agent framework: (i) the government’s provision of productive public goods such as infrastructure, and (ii) transitional dynamics of the economy when the government’s debt policy changes. Indeed, as we will subsequently demonstrate, taken together both of these features fundamentally alter the characterization of the optimal level of public debt.

In their seminal paper studying public debt in the United States, Aiyagari and McGrattan (1998) show that the optimal level of debt is positive and approximately two-thirds of GDP which, in fact, was very close to the actual share of public debt in the United States in the late 1990s. Additionally, they found that the welfare profile was flat near the optimum, suggesting small welfare losses from deviating from this level. Three critical issues that Aiyagari and McGrattan (1998) abstracted from were: (i) matching the wealth and
earnings distribution for U.S. households, (ii) accounting for transitional dynamics between debt policies when computing welfare effects, and (iii) accounting for the composition of government spending, specifically on productive public goods such as infrastructure. Using a labor productivity shock process that generates endogenous income and wealth distributions that more closely match the U.S. data, Rohrs and Winter (2016) find that the optimal level of public debt is actually a surplus when welfare effects are computed by comparing stationary equilibria. On the other hand, Desbonnet and Weitzenblum (2012) find that once the transition path is accounted for, there may exist significant welfare gains from increasing the level of public debt, though they do not characterize what the new optimum should be. Further, much of the existing literature on optimal public debt has only taken into account government consumption and transfers when characterizing government spending. Both of these components are modeled as being wasteful, with no consequences for the economy’s productive capacity. By contrast, public infrastructure, as embodied in an economy’s stock of roads, transportation networks, ports, power and electricity generation, etc., has important consequences for the productivity of private factors such as capital and labor These productivity effects are especially important in models where agents face idiosyncratic shocks and lack access to complete insurance markets, as changes in the factor prices faced by agents distort their savings, consumption and labor supply decisions, which in turn impacts welfare in the economy. Moreover, since the benefits of government investment in infrastructure to the private factors of production accrue only gradually over time (as its stock accumulates),

1 See Floden (2001) and Peterman and Sager (2016) for an assessment of the role of optimal transfers and life-cycle effects on optimal debt, respectively. There are also a few recent papers that examine optimal tax policy that relate to this literature (see Acikgoz (2013), Dyrda and Pedroni (2014), and Bakis et al. (2015)). However, unlike our work, these papers do not consider the simultaneous inclusion of public infrastructure and transitional dynamics when computing welfare effects in a model that is calibrated to replicate the degree of inequality in the U.S.

2 There is a large literature that investigates the growth enhancing features of infrastructure (Aschauer, 1989; Barro, 1990; Glomm and Ravikumar, 1994). The distributional effects of infrastructure investment have also been studied more recently (Chatterjee and Turnovsky, 2012; Gibson and Rioja 2016a, 2016b, and Klenert et al., 2014). To the best of our knowledge, the present paper is the first to incorporate both infrastructure investment and public debt into a model with incomplete markets.
the consequences of this type of spending for the optimal level of public debt cannot be correctly analyzed until the transition path between steady states is fully internalized.

The contribution of our paper is two-fold. First, we show that the introduction of public infrastructure into the aggregate production structure of a heterogenous agent economy that is calibrated to match U.S. wealth and earnings inequality fundamentally alters the calculation of the optimal quantity of public debt. Second, we show that there are stark differences between the short-run and long-run welfare consequences of a change in a country’s debt policy, which in turn leads to dramatically different conclusions for the optimal share of public debt once the transition path between stationary states is internalized. These issues have not been addressed simultaneously in the existing literature on optimal public debt, and our paper thus attempts to fill this important gap. We highlight the value-added of our paper by comparing the results of our analysis with an alternative model specification that is more standard in the literature, i.e., one that does not include productive government spending on infrastructure.

The starting point of our analysis is an economy where agents are ex-post heterogeneous due to the presence of idiosyncratic shocks to their labor productivity and imperfect insurance markets. Within this framework, we introduce a government that provides a stock of public infrastructure which, in turn, enters the aggregate production function as an input that generates positive spillovers for private capital and labor. The government also spends on public consumption goods and transfer payments, while collecting income tax revenue from the private sector. Consequently, the stock of public debt reflects the intertemporal effects of government borrowing to finance any excess of spending over revenues. We calibrate this economy to match the key aggregate and distributional characteristics of the U.S. economy. Starting with a comparison of stationary states, we show that the optimal share of public debt is actually a surplus. Comparing the model that includes public infrastructure to the one without, we find that the optimal surplus is substantially larger, with sizable welfare
effects around the optimum.

One caveat to the above result is that this calculation of the optimal level of public debt or surplus incorporates only the long-run welfare change between stationary equilibria. In other words, the underlying assumption is that any change in the level of debt is accompanied by an instantaneous “switch” to the new stationary equilibrium. While an understanding of these long run costs and benefits of increasing or reducing public debt is no doubt important, it is implausible to assume that the economy adjusts instantaneously to its new long-run equilibrium allocation. In fact, a change in debt policy will indeed lead to a gradual transition over time, as macroeconomic aggregates such as capital, consumption, labor supply, and output adjust to the change in government policy (and the accompanying changes in other parts of the government’s budget). This can fundamentally affect the calculation of welfare, in both the short run and long run. Consequently, our next step is to characterize the optimal level of public debt when the welfare consequences along the transition path are fully internalized. Here, we find that the long-run result of an optimal surplus no longer holds. Once the transition path is accounted for, the optimal level of public debt is positive, implying that the government should now be a net borrower in equilibrium. Furthermore, in the model that includes public infrastructure, we find that optimal level of debt is about 105% of GDP, which is very close to the recent debt level of the United States. In that sense, our results indicate that the qualitative result of Aiyagari and McGrattan (1998), when applied to today’s U.S. economy, is fairly robust.

To understand better the sharp contrasts in the calculation of optimal public debt, we need to focus on both the role of public infrastructure and the consequences of the changing composition of the government’s budget as the level of public debt is altered. Consider first the long run welfare effects. A reduction in public debt means that private capital will be crowded in. Now, the presence of public infrastructure, given its positive spillover for both private factors, will further boost the productivity of capital and labor. This
will tend to reduce the precautionary savings motive for households, as they can now rely more on labor income. Consequently, this leads households to increase consumption and reduce asset holdings, thereby increasing welfare. On the other hand, the reduction in government debt also tends to reduce liquidity in the economy and tightens constraints for credit-constrained households, which can reduce consumption and welfare in the economy. Overall, the welfare increase from the reduced precautionary savings motive dominates, with the presence of public infrastructure further strengthening this effect. This leads the government to accumulate assets (a public surplus) at the optimum.

By contrast, the short run welfare effects work very differently. While reducing public debt leads to a modest reduction in the tax rate in the long run, it requires a substantial increase in the tax rate in the short run in order to facilitate the debt reduction. This tax increase worsens welfare as it reduces the after-tax return on capital and labor income of agents. Therefore, increasing public debt is welfare enhancing in the short run: this allows the government to temporarily reduce taxes, which increases after-tax returns on capital and labor. Over time, this leads to a more indebted government at the optimum, in sharp contrast to the case where an economy can instantaneously “switch” from one stationary equilibria to another where a surplus is optimal. The presence of public infrastructure does, however, tend to temper the increase in public debt by providing an additional channel through which households can increase their flow of income from capital and labor. This in turn raises tax revenues for the government, thereby slowing down the accumulation of debt. The inclusion of public infrastructure in the model specification therefore implies a lower level of optimal debt relative to the standard model without infrastructure, both when comparing steady states as well as accounting for transitional dynamics. This underscores the importance of the role played by public investment in the characterization of the optimal level of public debt in the United States.

The rest of the paper is organized as follows. Section 2 describes the analytical framework.
Section 3 describes the calibration and computational procedure. Section 4 discusses the stationary equilibrium, while Section 5 deals with the optimal level of public debt. Section 6 considers transitional dynamics related to a counterfactual government policy that changes the stock of public debt and, finally, Section 7 concludes.

2 Analytical Framework

We consider an economy populated by a continuum of infinitely-lived households. The key feature of these households is that they face incomplete insurance markets, as in Aiyagari (1994), i.e., they are unable to purchase perfect insurance against the realization of an idiosyncratic labor productivity shock. Therefore, though all households are identical ex-ante, their inability to insure against the labor productivity shock makes them heterogenous ex-post. There is a government in the economy which spends tax revenues on two types of public goods - a wasteful public consumption good and the economy’s stock of public infrastructure which, in turn, generates productivity spillovers for private factors. The government can also sell or purchase bonds, resulting in a public debt or surplus.

2.1 Households

Households in this economy choose their rate of consumption, $c$, and time allocation between labor and leisure to maximize a per-period utility function given by:

$$U(c, l) = \frac{c^\eta (1 - l)^{1-\eta}}{1 - \sigma}$$

(1)

where $l$ denotes the allocation of the household’s unit time endowment to labor supply. Households are identical ex-ante, but receive idiosyncratic shocks to their labor productivity, $\epsilon$, at the beginning of each period. While agents cannot perfectly insure against these
fluctuations, they can partially insure against them by accumulating a stock of assets, $a$, that pay out a market-determined interest rate, $r$. These assets are comprised of private capital, which the households rent to the representative firm in the economy, and holdings of government bonds, $b$, such that a household’s portfolio is given by $a = k + b$. Therefore, the presence of incomplete markets generates a precautionary motive for savings, causing households to accumulate wealth during periods of high productivity, in order to compensate for periods where adverse productivity shocks are realized. Over time, this mechanism leads to an endogenous distribution of wealth across households. In maximizing the per-period utility (1), households are constrained by an intertemporal budget constraint

$$c + a' \leq [1 + (1 - \tau)r + (1 - \tau)w]e + T$$

(2)

where $a'$ denotes the household’s stock of wealth in the next period, $\tau$ represents the income tax rate, $w$ is the real wage rate, and $T$ represents a lumpsum transfer received from the government. We assume that the household-specific productivity shock, $\epsilon$, follows a Markov process with a transition matrix given by $\pi(\epsilon'|\epsilon)$. The household’s maximization problem can then be written as:

$$V(a, \epsilon) = \max_{c, l, a'} \left[ U(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(a', \epsilon') \right]$$

(3)

subject to the intertemporal budget constraint (2), along with the restriction that $a' \geq a$, where $a$ denotes the borrowing constraint faced by households.

### 2.2 Firms

The representative firm in this economy produces a flow of final output using a standard neoclassical technology and three inputs, namely aggregate capital, $K$, aggregate labor, $L$, and
and the stock of public infrastructure, $K_G$:

$$Y = K^\phi_G K^\alpha L^{1-\alpha}, \quad \phi \in (0, 1) \text{ and } \alpha \in (0, 1) \quad (4)$$

In the production function (4), the stock of public infrastructure, $K_G$, generates positive spillovers for the firm’s production, with an output elasticity of $\phi$. The representative firm is competitive and, in maximizing its flow of profits, takes all market prices and the stock of public infrastructure as exogenously given. The firm’s problem can be written as:

$$\max_{K,L} K_G^\phi K^\alpha L^{1-\alpha} - wL - (r + \delta_K)K \quad (5)$$

where $\delta_K$ is the rate of depreciation of private capital. The optimality conditions for the firm’s problem pins down the equilibrium real wage and return on capital:

$$w = (1 - \alpha)K^\phi_G \left( \frac{K}{L} \right)^\alpha \quad (5a)$$

$$r = \alpha K^\phi_G \left( \frac{K}{L} \right)^{\alpha-1} - \delta_K \quad (5b)$$

### 2.3 Government

The government raises revenues by levying a tax on household income, and spends on transfer payments, a public consumption good, and investment in the economy’s stock of infrastructure. The government can also issue or purchase instantaneous one-period bonds (which are held by households). The government’s flow budget constraint can be written as:

$$B' = [1 + (1 - \tau)r]B + G_c + G_I + T - \tau[wL + rK] \quad (6)$$
where $B$ is the stock of public debt, $G_c$ is spending on the public consumption good, and $G_I$ is investment in public infrastructure. We assume that the government spends a constant fraction, $g_I$, of the economy’s final output on infrastructure investment, such that:

$$G_I = g_I Y, \ g_I \in (0, 1)$$

(7)

The stock of infrastructure evolves according to

$$K'_G = g_I Y + (1 - \delta_G) K_G$$

(8)

where $\delta_G$ is the depreciation rate of infrastructure.

### 2.4 Equilibrium

A stationary equilibrium in this economy is characterized by a value function, $v(a, \epsilon)$, agent-specific decision rules, $a'(a, \epsilon)$, $l(a, \epsilon)$, and $c(a, \epsilon)$, a time-invariant joint distribution of individual states, $F(a, \epsilon)$, factor prices, $w$ and $r$, government policy variables, $g_t$, $\tau$, $G_c$, and $T$, and the following aggregate conditions:

Labor market: $L = \sum_{\epsilon} \int_a \epsilon l(a, \epsilon) f(a, \epsilon) da$

(9)

Asset market: $A = \sum_{\epsilon} \int_a a'(a, \epsilon) f(a, \epsilon) da = K + B$

(10)

Goods market: $Y = \sum_{\epsilon} \int_a c(a, \epsilon) f(a, \epsilon) da + I + G_c + G_I$

(11)

where $f(a, \epsilon)$ is the density function associated with the distribution of individual states $F(a, \epsilon)$, $K$ and $B$ denote the economy-wide aggregate stocks of private capital and public
debt, respectively, and $I$ is aggregate private investment, given by:

$$I = \delta_K K$$

(12)

3 Calibration and Solution

The model is calibrated to be consistent with the long-run moments for the U.S. economy. To understand better the role played by public infrastructure, we compare our benchmark specification with infrastructure to a specification where infrastructure is absent. For the specification without infrastructure, the production function is given by:

$$Y = \Phi K^\alpha L^{1-\alpha} \text{, } \Phi > 0 \text{ and } \alpha \in (0, 1)$$

(13)

where $\Phi$ denotes an exogenously specified aggregate level of productivity for the production sector. In this specification, government spending entails only the public consumption good $G_c$ and transfers, $T$.

Table 1 provides a basic overview of our model’s calibration. Since we have two model specifications (with and without infrastructure), we begin by describing the parameter choices that are common to both. We set the parameter $\sigma$ in the utility function (1) to 1.5, which yields an intertemporal elasticity of substitution of about 0.67, within the range of estimated values reviewed by Guvenen (2006). The relative share of consumption in the utility functions, $\eta$, is set to 0.355 to match the aggregate share of time allocated to work in the United States, which is about 30 percent. The rate of time preference, $\beta$, is set to 0.95 in order to match a steady-state interest rate (or return on capital) of 3.5 percent, and the share of capital, $\alpha$, is set to its standard value of 0.3. The depreciation rate of private capital, $\delta_K$, is set to a standard value of 10 percent, while the depreciation rate for public infrastructure is set to 5 percent, to reflect the fact that infrastructure consists primarily of
structures, and depreciates at a lower rate than private capital. The income tax rate, $\tau$, in both specifications is set to approximately 0.4, to target total tax receipts as a share of GDP. The last common restriction between our specifications is the households’ individual borrowing constraint. Following Rohrs and Winter (2016), we assume that households can borrow up to 30 percent of output produced in the steady state equilibrium. As Rohrs and Winter (2016) report, this borrowing limit allows our model specifications to match the share of wealth held by the bottom quintile in the 2007 Survey of Consumer Finances. Both model specifications yield an equilibrium debt-to-GDP ratio of approximately 67 percent, which is consistent with the U.S. average from 1990 - 2014. The model specification with public infrastructure generates a capital output ratio of 2.7, which is also in line with its corresponding average in the U.S. data.

To ensure comparability across the two models, we make some adjustments for the fiscal variables across the two specifications. Specifically, in the model that includes public infrastructure, we set its corresponding output elasticity to 0.14, consistent with the average estimates from U.S. studies listed in Bom and Ligthart (2014) and with their meta-analysis regarding this elasticity. In the specification that does not include infrastructure, we calibrate the TFP parameter, $\Phi$, in (13) to ensure that both models yield the same productivity index (see (4)). Further, in the specification without infrastructure, we set the shares of government consumption, $G_c$, and transfers, $T$, to be 21.7 percent and 8.2 percent, respectively, consistent with Aiyagari and McGrattan (1998) and Trabandt and Uhlig (2011). In the model specification that includes infrastructure, we adjust the composition of the government’s budget by setting the share of public investment, $G_I$, in GDP to 2.4 percent.

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3 Our results are not sensitive to small variations in the depreciation rate for public infrastructure.
4 Source: NIPA Tables, Bureau of Economic Analysis.
5 According to the Penn World Tables (PWT 9.0), the average capital-output ratio for the United States was 2.96 for the period 1990 - 2014. The capital output ratio measures the share of private plus public capital in GDP. Consequently, the model specification without infrastructure generates a lower capital-output ratio in equilibrium.
following estimates from the Congressional Budget Office (Musick, 2010). Consequently, we reduce the share of government consumption in this model to 19.3 percent of GDP to ensure that the total share of government spending in GDP is identical across the two model specifications. The question we then ask is: what are the optimum shares of public debt in the two model specifications?

### 3.1 Income Shock Process

We adopt the shock process presented in Rohrs and Winter (2016). Specifically, the shock’s values and transition matrix are given by:

\[
\begin{align*}
    s &= [0.055, 0.551, 1.195, 7.351] \\
    \pi &= \begin{pmatrix}
        0.940 & 0.040 & 0.020 & 0.000 \\
        0.034 & 0.816 & 0.150 & 0.000 \\
        0.001 & 0.080 & 0.908 & 0.012 \\
        0.100 & 0.015 & 0.060 & 0.825
    \end{pmatrix}
\end{align*}
\]

This shock process was estimated following Castaneda et al. (2003) and was shown in Rohrs and Winter (2016) to yield model estimates of income and wealth distributions that align well with the U.S. data. Table 2 shows the distribution of net financial assets from the 2007 SCF: the top 20 percent of households hold more than 90 percent of net financial assets, while the bottom 80 percent’s share is about 10 percent, indicating a highly unequal distribution of wealth in the U.S. economy. Table 2 also shows the results of our baseline model with infrastructure and demonstrates that we retain a very good fit to the distribution of wealth and income in the US data. As can be seen, our model specification does a good job in

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\[\text{Data from 2007 Survey of Consumer Finances is used to compute a measure of net financial assets excluding longer term investments such as property, vehicles, business ownership, residential mortgages, and automobile loans. We choose to not reestimate the shock process as the values reported in Rohrs and Winter (2016) allow our model to match the data sufficiently well.}\]
matching the wealth and income distributions found in the data, especially for the top two and bottom quintiles.\footnote{Our specification without infrastructure also fits the U.S. wealth and income distributions well. Results available from authors upon request.}

## 3.2 Computational Methods

The results presented in the next several sections require the calculation and comparison of a number of stationary solutions and associated transition paths. We compute the stationary solution using standard methods. First, we make guesses on aggregate assets, $A$, and labor, $L$, and then use the aggregate equations described above to compute an internally consistent set of aggregates. Next, we discretize private assets, restricting their values to 250 unevenly spaced grid points on the interval $(-0.3 \times Y, 25)$, and we use value function iteration with golden section search and parabolic interpolation to solve for the agents’ decision rules. With the decision rules in hand, we compute the invariant density of individual states, $f(a, \epsilon)$, utilizing a finer grid of 500 points on the same interval. Lastly, new values of $A$ and $L$ can be recovered from the decision rules and densities and a new set of internally consistent aggregates can be recovered. This process is repeated until the change in $A$ and $L$ between iterations is sufficiently small. Once the stationary solution for the various policies have been recovered, we can compute the transition paths between the baseline policy, with a debt-to-GDP ratio of 67 percent, and the various alternative policies with different long run debt-to-GDP ratios. When computing the transition path between two stationary equilibria we make use of the methods outlined in Domeij and Heathcote (2004) and Heer and Maussner (2009). Specifically, we assume the economy starts at time $t = 1$ in the stationary equilibrium consistent with a debt-to-GDP ratio of 67 percent. At time $t = 2$ the government changes its debt-to-GDP policy triggering a transition to the new stationary equilibrium which converges.
to the long run solution over a period of several years.

### 3.3 Welfare

An important aspect of our analysis is the measurement of economic welfare from an underlying menu of government policies. We follow Aiyagari and McGrattan (1998), Floden (2001) and Rohrs and Winter (2016) and adopt the following utilitarian social welfare function:

\[
\Gamma = \int_a \int_\epsilon V(a, \epsilon) f(a, \epsilon) da d\epsilon
\]  

(14)

The welfare measure in (14) can be interpreted as the welfare level of the average individual in the economy. In reporting welfare changes in subsequent sections, we use a compensating variation measure, which quantifies the units of consumption that need to be transferred between two steady-states (say, generated by two different policies or shocks), such that the average individual is indifferent between these steady states. This leads to the following compensating variation measure for welfare changes:

\[
\Delta \Gamma = 1 - \frac{\left[ \int_a \int_\epsilon V_0(a, \epsilon) f_0(a, \epsilon) da d\epsilon \right]}{\left[ \int_a \int_\epsilon V_1(a, \epsilon) f_1(a, \epsilon) da d\epsilon \right]}^{\eta/1-\eta}
\]  

(15)

where the subscripts 0 and 1 refer to the baseline (pre-shock) and new (post-shock) steady states. As in Rohrs and Winter (2016), if $\Delta \Gamma > 0$, the average agent would prefer being at the new equilibrium without compensation for the change. On the other hand, if $\Delta \Gamma < 0$, then $\Delta \Gamma$ units of consumption is required to make the agent indifferent between the two steady states. This then becomes a measure of welfare loss across the two steady states.

While the compensating variation measure described in (15) provides the long run welfare costs of varying debt policies, this measure ignores the short run dynamics that occur as the

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Details regarding the computation the stationary equilibria and transition paths are available from the authors upon request.
economy transitions between steady states. Accounting for these transitional dynamics in our welfare measure is very important because adjustments to the debt level will impact variables differently depending on the time horizon considered. For example, in the long run reducing debt will reduce the tax rate in the economy as a lower debt level implies a lower debt servicing burden. However, in the short run, taxes must be increased significantly in order to pay down the debt. Differences such as these imply that the welfare effects of debt policies may be very different in the short and long run\textsuperscript{[9]} To this end, we also compute the welfare effects including transitional dynamics as:

\begin{equation}
\Delta \Gamma = 1 - \left[ \int_a \int_{\epsilon} \sum_{t=0}^{T} \beta^t U(c_0(a, \epsilon), l_0(a, \epsilon)) f_0(a, \epsilon) \text{d}a \text{d}\epsilon \right] \frac{1}{\eta (1 - \sigma)}
\end{equation}

where \( c_t(a, \epsilon), l_t(a, \epsilon), \) and \( f_{t-1}(a, \epsilon) \) denote the decision rules for consumption and leisure and the wealth density known at time \( t \) in the transition path while \( c_0(a, \epsilon), l_0(a, \epsilon), \) and \( f_0(a, \epsilon) \) denote the same values for the initial stationary equilibrium.

4 Steady-State Equilibrium

In this section, we characterize the steady-state equilibrium relationship between the level of public debt and the economy’s key aggregate and distributional variables. Figure 1 plots how the steady state level of these variables change as the share of public debt in GDP changes from its benchmark level, with the income tax rate adjusting to satisfy the government’s budget constraint.\textsuperscript{[10]} The red line indicates the equilibrium relationships for the model specification with infrastructure, while the black line represents the model without.

The productivity-enhancing role of public infrastructure raises both the marginal product

\textsuperscript{[9]} See Desbonett and Witzenbaum (2012) for further details on this issue.

\textsuperscript{[10]} We have also considered cases where the steady-state change in the level of public debt is accommodated by appropriate changes in government transfers and consumption. While we do not discuss those cases in the paper, the results are available on request from the authors.
of labor and capital, enabling the economy to produce more output. As the government reduces the steady-state ratio of public debt to GDP, it crowds in private capital which, in turn, feeds back into higher output. When infrastructure is present, this mechanism is amplified as reducing the steady-state ratio of public debt to GDP now crowds in both private capital and public infrastructure. This can be easily seen in Figure 1: the lower is public debt, the larger is the gap between the flow of aggregate output, the real wage rate, and the stock of private capital between the two model specifications. The higher flow of output also enables the agent to increase consumption at a faster rate in the model with public infrastructure, though diminishing returns set in at very low levels of public debt. The effect of public debt on productivity can be seen clearly in the first panel of Figure 1 which presents the coefficient on private factors in the production function for both model specifications.\footnote{In the baseline specification with infrastructure, productivity is $K^G$ and in the alternative specification productivity is given by $\Phi$, where $\Phi$ is fixed so that both specifications yield the same output level when the debt-to-GDP ratio equals 67\%.

As the debt level is reduced, $K^G$ increases allowing productivity in the baseline specification to rise. However, in the alternative specification in which infrastructure is absent, this productivity enhancing feature of debt reduction is ignored. This has implications for the optimal level of debt in the two model specifications, which we will explore further in the next section.

Figure 1 also plots the steady-state income and wealth Gini coefficients as a function of the share of public debt in GDP.\footnote{To compute the Gini coefficients we start by computing the wealth and income shares at the centile level, and we use these values to recover a discrete Lorenz curve. The gap between this discrete Lorenz curve and the 45 degree line is used to compute the Gini coefficients.\footnote{Since wealth may take on negative values, the gini coefficient may actually exceed 1.} Both model specifications yield very similar levels of long-run inequality for a given level of public debt, with the specification that includes public infrastructure generating slightly lower levels of wealth inequality.} Here, we find an inverse relationship for both distributional measures with respect to public debt: for both model specifications, as the steady-state share of public debt declines, income and wealth inequality increases.\footnote{Since wealth may take on negative values, the gini coefficient may actually exceed 1.}
inequality relative to the model without infrastructure. As the long-run stock of public debt declines, it tightens the borrowing constraint for households. For households that face a binding borrowing constraint, this implies less access to private capital. On the other hand, for wealthier households that do not face a borrowing constraint, this implies the opposite: the reduction in public debt crowds in private capital for their portfolios. Consequently, wealth (and income) inequality increases in equilibrium. We should also note here that since we do not model heterogeneity in the skill composition of the labor force, income inequality in our model is driven mainly by wealth inequality.

5 The Optimal Level of Public Debt

In this section, we ask the following question: what is the optimal quantity of public debt for the U.S. economy? More specifically, how does the presence of government investment in infrastructure affect the optimal quantity of public debt? There are two considerations here: first, given the government’s steady-state budget constraint, how do adjustments to the share of public debt in GDP affect long-run welfare? Second, how is the steady-state welfare comparison affected when one accounts for the transition path from one steady-state to another? To examine these issues, Figure 2 characterizes the optimal share of public debt for the two model specifications, namely with and without public infrastructure, under the following two scenarios: (i) from one stationary steady-state to another as the share of public debt is changed from its benchmark level (row 1), and (ii) from one stationary steady-state to another when the entire transition path between them is accounted for (row 2). Specifically, scenario (i) only accounts for the steady-state welfare change as the economy “switches” from the benchmark equilibrium level of debt, without taking into account the transition path. By contrast, scenario (ii) accounts for the intertemporal change in welfare as the economy’s aggregate variables such as output, consumption, labor, and capital adjust.
gradually over time when the benchmark level of public debt is altered. As before, we adjust the income tax rate to accommodate the change in the level of public debt. As in Figure 1, the red lines correspond to the specification with public infrastructure, while the black lines are for the model without infrastructure.

As can be seen from Figure 2, the optimum quantity of public debt depends critically on whether (i) infrastructure is included or excluded in the model specification, and (ii) the transitional path between the stationary equilibria is accounted for. The model without infrastructure is essentially similar to the one considered by Aiyagari and McGrattan (1998), modified by the income shock process used in Rohrs and Winter (2016). While Aiyagari and McGrattan (1998) document that the optimal level of public debt is about two-thirds of GDP, Rohrs and Winter (2016) show that modifying the income shock process to be consistent with data from the 2007 SCF leads to an optimal share of public debt that is negative. Starting with row 1 of Figure 2, where we plot the the welfare effects ignoring transitional dynamics, and for the specification without infrastructure, our results indicate that the government should run a net surplus of about 30 percent of GDP. This result is consistent with the findings of Rohrs and Winter (2016), underscoring the importance of the underlying income shock process for the calculation of optimal debt. However, the model specification with public infrastructure implies a very different level for the welfare-maximizing share of public debt: in this case, the optimal share of public debt is actually a much larger surplus of about 140 percent of GDP. This is an important result, suggesting that by not including infrastructure in the model specification, previous studies on this issue may have over-stated the optimal level of public debt by a significant amount. Further, we also note that the welfare profile implied by the model without public infrastructure is very flat around the optimum, indicating that reducing public debt from this level does not lead to significant long-run welfare gains for the economy. On the other hand, the welfare function for the model with infrastructure is relatively steeper around the optimum, indicating that the welfare loss
from deviating from this optimal level is non-trivial.

The intuition behind the different levels of optimal public debt across the two specifications can be explained as follows. When the government provides a productive public good like infrastructure, it acts as a complement to private factors in the production function, thereby raising the return to private capital and labor. Consequently, this reduces the precautionary savings motive for households, allowing them to sell their claims on the government. This causes households to be net debtors and the government to be a net creditor to the private sector at the optimum. Consequently, the government accumulates assets and runs a surplus, until diminishing returns to infrastructure set in, leading to the optimum. When infrastructure is absent from the model specification, the channel through which the government provision of public goods affects the household’s precautionary savings motive is absent and, hence, diminishing returns to capital set in much earlier (i.e., at a higher share of public debt in GDP).

The second row in Figure 2 characterizes the optimal level of public debt in the case where the transitional path between the stationary equilibria is accounted for when computing welfare effect. As mentioned above, as the level of public debt changes, the intertemporal welfare change depends on the economy’s transition path, which was not captured in the steady-state comparison plotted in row 1. In sharp contrast to the results obtained in row 1, we see now that once transitional dynamics are internalized, the optimal share of public debt is once again positive, as in Aiyagari and McGrattan (1998). However, whether public infrastructure is included in the model specification matters: for the model with public infrastructure, the optimal share of debt is now around 105% of GDP, while the exclusion of public infrastructure leads to a higher level of public debt, at around 125% of GDP. An interesting aspect of this result is its qualitative consistency with Aiyagari and McGrattan (1998), who found that the optimal share of debt was, in fact, close to its actual share in the 1990s, around two-thirds of GDP. The actual share of public debt in GDP for the
United States at the end of the first quarter of 2016 was 105.7% , while its average share between 2010-2015 was about 98%. These data are very consistent with the optimal debt level implied by our baseline model with public infrastructure. The key aspect of this result is that once we simultaneously (i) include public infrastructure in the model specification, (ii) calibrate the income shock process to match the earnings and wealth distributions in the United States, and (iii) account for the transition path between stationary states when computing welfare effects, the optimal level of public debt is once again positive, and close to its actual level in the data. Previous studies, by either ignoring one or more of these features, or considering them in isolation, may have overestimated the optimal level of public debt in the United States.

An important question that arises in this context is: why is the optimal level of public debt positive when transitional dynamics are accounted for, while it is negative when one assumes the economy can move instantly between stationary equilibra? The intuition lies in the fact that the steady state welfare comparison incorporates only the long-run effects of a change in the share of public debt, while ignoring the short run implications of changes in the composition of the government’s budget and key macroeconomic variables. For example, an increase in the share of public debt allows the government to reduce the income tax rate in the short run in order to satisfy the government’s budget constraint. This, in turn, raises the after-tax return on both capital and labor, thereby increasing both the flow of consumption and output. The increase in public debt also relaxes the borrowing constraint, which further increases welfare along the transition path, by allowing poorer households to smooth consumption over time. These short run welfare gains accumulate in transition, but must eventually trade-off against the long-run welfare losses, as the tax rate must be increased in the long-run to sustain the higher level of debt, and diminishing returns sets in for both public and private factors. However, the short-run welfare gains dominate the

14Source: The Federal Reserve Economic Database (FRED).
long-run losses, leading to an optimal level of debt that is positive in equilibrium. The presence of public infrastructure does, however, tend to reduce the optimal level of public debt, even when transitional dynamics are considered. This follows from the results presented in the previous section. When public infrastructure is included, the long run welfare costs associated with increasing public debt are amplified, as increasing the debt level crowds out both private capital and public infrastructure. Therefore, the inclusion of infrastructure leads to an optimal level of debt that is lower than that implied by the specification without infrastructure.

6 Transition Paths: Moving to the Optimal Debt Level

In order to better understand the short-run effects that occur during the transition between stationary states, we turn our attention to the transition paths taken by key aggregate variables in our model economy. Figure 3 presents these transition paths under a counterfactual policy experiment where the government instantaneously increases the debt-to-GDP ratio from its benchmark level of 67% to the optimal level of 105%.\(^{15}\) For clarity of exposition, we only plot the transition paths for the baseline model with infrastructure. Inspection of the bottom panel of Figure 3 shows the strong effect that the policy has on the income tax rate. Increasing debt leads to a sharp reduction in the tax rate in the short run, falling from approximately 40% to close to 5% during the periods immediately following the policy change. This reduction in taxes, coupled with the fact that an increase in debt relaxes the liquidity constraint for poor households, allows agents to increase consumption, savings (in the form of capital) and output on average. However, this tax cut is short lived, as the government has to raise the tax rate to sustain the higher level of public debt. Consequently,\(^{15}\)

\(^{15}\)We have also considered alternative policies where the government increases the debt-to-GDP ratio gradually over a period of 10 or 20 years. The results were consistent, and as such, we have decided to focus on the instantaneous policy change.
this reverses the process of capital accumulation and, in the long-run, reduces the equilibrium level of output, private capital, and consumption.

The welfare and distributional implications of the debt policy can be seen in the first row of Figure 3. The panel labeled *Instantaneous Welfare* plots the period-by-period welfare effects in consumption equivalent units while the panel labeled *Intertemporal Welfare* accumulates these effects. Inspection of these panels indicates that instantaneous welfare initially falls but then spikes to near 6% of consumption before decaying gradually to 0. This leads to an intertemporal welfare measure that starts negative but increases at a decreasing rate over the entire transition path. Lastly, in terms of inequality we see that the wealth Gini falls both in the short-run and in the long run, but reaches its minimum value during the periods immediately following the policy change.

7 Conclusions

In this paper, we have revisited an important policy issue that has recently received a lot of attention, namely the optimal share of public debt in an economy populated by heterogeneous agents. Previous studies, starting with the seminal work of Aiyagari and McGrattan (1998), have shown that this optimal share is positive, at least for the United States, possibly around two-thirds of GDP. More recent work has focused on the underlying income process that is used to calibrate these models to the data, showing that the Aiyagari-McGrattan result is indeed sensitive to the income process. One important issue this literature has neglected is the role of public investment in infrastructure in the context of heterogeneous agent models. Arguably, public infrastructure, by generating productivity benefits for private capital and labor, can help reduce the precautionary savings motive in these models where house-

\[16\text{At } t = 5, \text{ } \text{Instantaneous Welfare} \text{ presents the welfare effect of moving between } t = 4 \text{ and } t = 5 \text{ in the transition path while Intertemporal Welfare presents the entire welfare effect accumulated by } t = 5.\]

\[17\text{The initial reduction in welfare is simply an artifact of the fixed state variables at the time of the policy is changed.}\]
holds face idiosyncratic shocks and incomplete insurance markets. Another critical issue is the welfare consequences of a change in the stock of public debt along the transition path between steady states. Our results indicate that these considerations can have important consequences for the optimal quantity of public debt in the economy.

We introduce public infrastructure into a workhorse heterogeneous agent model that is calibrated to match the key aggregate and distributional moments of the United States. To understand better how our model relates to the previous literature, we compare our baseline specification to one without infrastructure. Our results indicate that when welfare effects are computed by comparing stationary equilibria, the optimal debt is a large public surplus, indicating that aggregate welfare maximization requires the government to be a net lender to the private sector. Moreover, we find that this optimal public surplus is significantly larger than those derived by previous studies that ignored public infrastructure. However, this result is completely reversed when the transitional dynamics between the stationary equilibria are internalized into the welfare calculations. In this case, the optimal debt is again positive, as in Aiyagari and McGrattan (1998). Interestingly, the model specification with public infrastructure implies an optimal share of public debt in GDP of about 105 percent—which is very close to the actual share of public debt in the United States since the Financial crisis of 2008-2009, and also qualitatively similar to the findings of Aiyagari and McGrattan (1998). These results are mainly driven by the differential trade-offs generated by a change in the composition of government spending and revenues for the household’s precautionary savings motive in the short run and the long run. From a distributional perspective, we find an inverse relationship between public debt reduction and wealth and income inequality, driven primarily by how the reduction in public debt affects the borrowing constraint for households for whom it is binding. Finally, we characterize the economy’s transitional adjustment to a counterfactual change in the level of public debt by the government.

In summary, our paper contributes to the existing literature on the optimal quantity of
public debt in two important ways. First, we show that ignoring the role of public investment in infrastructure may lead to a significant over-statement of the optimum quantity of debt. Second, we show that the characterization of the optimum is incomplete if one ignores the transitional adjustment path. Needless to say, there are several related issues that we do not yet consider, such as the internal composition of public spending between new investment and maintenance, a richer tax structure that differentiates between tax rates on capital and labor income, and the determination of the optimal level of infrastructure investment. These are important issues that we hope to address in future research.
### Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>Capital’s Income Share</td>
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<tr>
<td>$\beta$</td>
<td>0.95</td>
<td>Rental Rate of Approximately 3.5%</td>
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<tr>
<td>$\delta_K$</td>
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<td>Capital-Output Ratio</td>
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<tr>
<td>$\delta_G$</td>
<td>0.05</td>
<td>Capital-Output Ratio</td>
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<td>$\sigma$</td>
<td>1.50</td>
<td>Std. Arrow-Pratt CRRA</td>
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<tr>
<td>$\phi$</td>
<td>0.14</td>
<td>Elasticity of Y w.r.t Infrastructure</td>
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<tr>
<td>$g_I$</td>
<td>0.024</td>
<td>Infrastructure Investment-GDP Ratio</td>
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<tr>
<td>$\eta$</td>
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<td>Aggregate Labor in SS</td>
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<td>$\tau$</td>
<td>0.401</td>
<td>Tax Revenue - GDP Ratio</td>
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### Table 2: Wealth and Income Distributions

<table>
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<th>Wealth Distribution</th>
<th>Data</th>
<th>With $K_G$</th>
<th>Income Distribution</th>
<th>Data</th>
<th>With $K_G$</th>
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<td>91.02</td>
<td>Q5</td>
<td>61.39</td>
<td>58.68</td>
</tr>
</tbody>
</table>
Figure 1: Adjustment in Aggregate Variables

- **Productivity**: Depicts the adjustment in productivity with and without a knowledge gap (KG).
- **Output**: Shows the output adjustment with and without a knowledge gap.
- **Aggregate Capital**: Illustrates the change in aggregate capital.
- **Aggregate Consumption**: Demonstrates the adjustment in aggregate consumption.
- **Wages**: Reflects the wage adjustment.
- **Rental Rate**: Displays the rental rate adjustment.
- **Income Tax Rate**: Shows the change in income tax rate.
- **Income Gini**: Illustrates the income inequality.
- **Wealth Gini**: Depicts the wealth inequality.
Figure 2: Optimal Debt Profiles

Consumption Loss Ignoring Transitional Dynamics

Consumption Loss Including Transitional Dynamics
Figure 3: Transitions Paths: Moving to the Optimal Debt Level
References


Musick, N., 2010, ”Public spending on transportation and water infrastructure.” Congressional Budget Office, report.

