Churning, firm inter-connectivity, and labor market fluctuations

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Abstract

In the US, during downturns of economic activity, firms change their profit rankings more often. Motivated by this fact, this paper studies the effect of firms’ transitions in profit distributions, or churning, on the business cycle. Specifically, I develop and estimate a modified Diamond-Mortenson-Pissarides search and matching model in which an increase in churning leads to a contraction of the labor market and a decline in output. The key feature of the model is firm inter-connectivity. Churning affects the aggregate economy by increasing firms’ chances of cooperating with partners that they are reluctant to work with, which induces firms and their potential partners to reduce inter-firm cooperation. A reduction of inter-firm cooperation depresses other economic activities, such as recruiting. The main prediction of the model is that an increase in the churning of an industry causes a recession within industry and in its linked industries, which is consistent with the evidence I document in the paper. The model’s key mechanism, furthermore, is supported by microeconomic evidence.

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1 Introduction

In the US, periods of rising unemployment are also periods of accelerated movement of firms’ rankings across the profit distribution. That is, during downturns of economic activity, there is an increase in the churning of firms’ rankings in the profit distribution. Moreover, a higher churning of an industry is usually accompanied by an slower employment growth within the industry and in its linked industries. Motivated by these facts, I study the effect of churning on the labor market through the lens of a Diamond-Mortenson-Pissarides (DMP hereafter) model. Based on my model, I argue that variations in churning is a significant factor explaining the cyclical labor market fluctuations, and firm inter-connectivity across industries serves as a crucial propagation mechanism.

The main idea of the paper is that firms collaborate with other firms. In choosing partners, firms care about the quality of their partners. The churning of firms’ rankings increases the chance that they will match with partners who they are reluctant to work with. When the termination of a match is costly, both the firms and their potential partners would optimally avoid inter-firm cooperation which depresses other economic activities such as recruiting of workers.

In section 3, I implement the idea by supplementing the canonical DMP model with firm inter-connectivity. I assume that all firms prefer to cooperate with high-ranking partners; hence, in equilibrium, firms only initiate partnership with similarly ranked partners. I show that churning increases unemployment if and only if the production function is supermodular; that is, firms have a comparative advantage in cooperating with a similarly ranked partner. The intuition is that when the production function is supermodular, cooperation between similarly ranked firms maximizes their aggregate profit. Churning generates mismatch between differently ranked firms, which decreases firms’ expected profits and reduces their incentive to create jobs. The main prediction of the model is that an increase in churning of one industry would cause an aggregate recession.

In section 4, I test my model’s key mechanism—particularly the supermodular production function—by conducting inference on the model’s testable microeconomic implications. The challenge of the test is that firms initiate partnership with similarly ranked partners as long as the production function is monotone, therefore supermodularity cannot be directly identified from firms’ choice of partner. To overcome the challenge, I examine the cross-industry variation in the choice of inter-firm cooperation contract. While supermodularity does not affect firms’ choice of partner, it affects firms’ choice of cooperation contract. When a firm is uncertain about its future type or its partner’s future type, and if mismatches are inefficient, the firm does
not want to make the commitment by choosing a long-term contract. Specifically, my model predicts that with supermodularity, fewer firms choose long-term contracts in industries with higher churning. I test my model using vertical integration and sourcing to proxy for long- and short-term contract. I find that a higher churning of an industry and its linked industries is associated with a lower vertical integration, which supports the main mechanism of my model.

The question of the extent to which churning contributes to the business cycle, however, remains to be seen. In section 5, I answer this question by embedding the simple model into a real business cycle (RBC) model disturbed by shocks to churning and several other shocks that have been commonly studied in dynamic stochastic general equilibrium (DSGE) models. After estimating the model using Bayesian method, I find that shocks to churning emerge as the major source of persistent joint movements in unemployment and other macro variables: they account for 27 percent of variation in unemployment and 32 percent of variation in aggregate output.

As a byproduct, my paper provides a theory of endogenous total factor productivity (TFP) and speaks to the Shimer puzzle (Shimer (2005)). According to my estimates, the production function is supermodular, which implies that mismatched firms are, on average, less productive. Churning of an industry raises the share of mismatched firms in the existing matches both within the industry and in its linked industries, leading to a gradual decline in aggregate TFP. This decline in aggregate TFP, however, is mild compared with the surge in unemployment.

My paper is related to the literature on uncertainty shock. Uncertainty shock, as defined by Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), characterizes the exogenous change in volatility of idiosyncratic productivity. In many settings, uncertainty shocks induce fluctuations in the rate of churning by varying firms’ transition probability in the productivity distribution. Most research in this literature emphasizes that firms concern with the uncertainty of their business condition due to the irreversibility and the non-convex adjustment cost of capital and labor input. The innovation of my paper is to argue that firms also concern with the uncertainty about the business condition of their partner due to the irreversibility of partnership and the opportunity cost of mismatch. Moreover, as shown by Schaal (2012), the standard DMP model predicts an increase in the job creation in response to a higher uncertainty, which is inconsistent with data. In contrast, my model implies that a high uncertainty generates a large decline in the job creation. Lastly, while existing models imply that uncertainty shocks at the industry level lead to economic downturn within the industry; my paper has a new and attractive implication that industry uncertainty shocks cause economic downturn not only within the industry but also in linked industries, which is consistent with the evidence in the data.¹

¹While no existing work studies the interaction between industry linkage and uncertainty shocks, Alessandria,
2 Empirical evidence

In this section, I present the empirical evidence that motivates my research. I first construct measures of churning on both industry level and aggregate level. Then I use the measures to show that churning is associated with the condition of the labor market at both aggregate level and industry level; Moreover, I show that churning of an industry is also associated with the condition of the labor markets of its linked industries. Lastly, I discuss the difference between churning and uncertainty shock.

2.1 Measuring churning of firms’ profit rankings

I use compustat fundamentals annual from 1960 to 2013. Compustat fundamentals annual is a data set of listed companies which contains more than 370,000 observations and covers 112 3-

Choi, Kaboski, and Midrigan (2015a) show that in a two-country trade model, uncertainty shocks to one country induce negative comovement of the two countries via the non-convex adjustment cost channel.
digit NAICS industries. For each period and within each industry I rank firms by profit, which is measured by Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA). Then I categorize firms into two types, high \( H \) and low \( L \), based on ranking; a firm is \( H \) type if its profit is above median, \( L \) type if below median.

A rotation occurs when a firm changes its type in consecutive periods, either from \( H \) to \( L \) or vice versa. Industry rotation rate is the fraction of firms that have changed type. Specifically, industry rotation rate for industry \( i \) in period \( t \) is defined as

\[
\text{Rot}_{i,t} = \frac{\# \text{rotation}_{i,t}}{\# \text{firm}_{i,t}}
\]

Industry rotation rate \( \text{Rot}_{i,t} \) measures the churning of firms’ rankings in industry \( i \) in period \( t \). High industry rotation rate implies that a firm is more likely to change its ranking within the industry profit distribution.

To measure churning of the aggregate economy, I construct the aggregate rotation rate by aggregating the industry rotation rates weighted by industry value-added. Specifically, aggregate rotation rate \( \text{Rot}_t \) is defined as

\[
\text{Rot}_t = \sum_i \text{Rot}_{i,t} \cdot \frac{\text{Value added}_{i,t}}{\text{GDP}_t}
\]

Aggregate rotation rate \( \text{Rot}_t \) measures churning of firms’ within-industry ranking for the entire economy. Similar to the industry rotation rate, a higher aggregate rotation rate indicates a higher churning in the economy. In this paper, I use the term aggregate rotation rate and rotation rate interchangeably.

It is worth noting that there exist several alternative ways to measure churning, such as

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2 Ideally, firms should be ranked by their “true abilities” to promote the inter-firm match’s joint pay-off, which is difficult to measure. Yet it is still reasonable to conjecture that profits are positively correlated with the “true abilities”. Several studies, such as Rhodes-Kropf and Robinson (2008), found that high-earning firms are more likely to match with high-earning firms in M&A activities, which supports my conjecture and justifies profit as a legitimate proxy.

3 For every two years, I keep the panel balanced. The following is an example to illustrate the measurement. Industry A has four firms, A1—A4; each firm’s type depends on ranking of yearly profits. Rotations occur in two of four firms in year 2; firms A1 and A3 switch their types, yielding a 0.5 rotation rate in year 2.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Year 1 profit (type)</th>
<th>Year 2 profit (type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>12 ( (H) )</td>
<td>11 ( (L) )</td>
</tr>
<tr>
<td>A2</td>
<td>10 ( (H) )</td>
<td>15 ( (H) )</td>
</tr>
<tr>
<td>A3</td>
<td>8 ( (L) )</td>
<td>10 ( (L) )</td>
</tr>
<tr>
<td>A4</td>
<td>9 ( (L) )</td>
<td>12 ( (H) )</td>
</tr>
</tbody>
</table>

\[ \Rightarrow \text{Rot}_{A,2} = 0.5 \]
the autocorrelation of firms’ ranking. I choose rotation rate because it is the exact empirical counterpart of churning in the model described in the next section. Rotation rates are used as observable in the estimation of the model.

Figure 1 plots the rotation rate and the national unemployment rate from 1960 to 2013. Rotation rate closely comoves with unemployment rate over the business cycles and is strongly countercyclical. For example, in the great recession, rotation rate hits more than 9 percent compared with the pre-recession rate of 5.5 percent.\textsuperscript{4}

Some may think that the cyclicality of rotation rate and its comovement with unemployment rate is mainly driven by firms’ small variation of profit around the median point, which would undermine the economic importance of my measurement. To address this concern, I refine the categorization of firms into four quartiles. By definition, rotation is the switching of rankings from 1st and 2nd quartiles to 3rd and 4th, or vice versa. The switchings between 2nd and 3rd quartile might contain small variation of profit around median point. Hence I ignore switchings between 2nd and 3rd quartiles and focus only on the ones between non-adjacent quartiles, which I denote as large rotations. By construction, large rotation is a subset of rotation and only includes very sizable changes in rankings.

I define large rotation rates in the same way as rotation rates:

\[
\text{Rot}^{\text{large}}_{i,t} = \frac{\# \text{large rotation}_{i,t}}{\# \text{firm}_{i,t}}
\]

\[
\text{Rot}_{t}^{\text{large}} = \sum_i \text{Rot}^{\text{large}}_{i,t} \cdot \frac{\text{Value added}_{i,t}}{\text{GDP}_t}
\]

where \(\text{Rot}^{\text{large}}_{i,t}\) is large rotation rate for industry \(i\) in period \(t\). Similarly, \(\text{Rot}_{t}^{\text{large}}\) is aggregate large rotation rate, which is the weighted sum of large industry rotation rates.

Table 1 reports a set of results of regressions of the national unemployment rate on measures of churning. In addition to the benchmark measure of churning when firms are ranked by profit, I also rank firms by profit margin, which is defined as the ratio of profit to sales. For each method of ranking, I use both rotation rate and large rotation rate as explanatory variables. As shown in Table 1, coefficient \(\beta\) is significant positive in all cases. Therefore, measures of churning of the economy are positively correlated with unemployment rate.

It is important to clarify whether rotation rate’s comovement with unemployment rate is mostly driven by a few industries, while other industries have fairly flat rotation rates. If that’s the case, rotation rate reflects only some industry phenomena and might not be important for

\textsuperscript{4}This pattern is consistent with result found in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) who found churning to be negatively correlated with consumption growth. They used manufacturing census data and ranked firms by total factor productivities (TFP).
Table 1: Measures of churning are positively correlated with unemployment rate

\[ \text{Unemployment}_t = \alpha + \beta \cdot \text{Rot}_t + \epsilon_t \]

| Independent variable | Coefficient | \( P > |t| \) | \( \text{Adj R} - \text{squared} \) | # of years |
|----------------------|-------------|----------------|-----------------|-----------|
| \( \text{Rot}^{\text{Profit}} \) | 0.50*** (0.18) | 0.01 | 0.11 | 54 |
| \( \text{Large Rot}^{\text{Profit}} \) | 0.42*** (0.15) | 0.01 | 0.11 | 54 |
| \( \text{Rot}^{\text{Profit margin}} \) | 0.54*** (0.09) | 0.00 | 0.40 | 54 |
| \( \text{Large Rot}^{\text{Profit margin}} \) | 0.47*** (0.11) | 0.00 | 0.24 | 54 |

\( \text{Unemployment}_t \) = civilian unemployment rate \( t \), BLS, 1960-2013
\( \text{Rot}_t \) = aggregate rotation rate at \( t \), Compustat
\( \text{Large Rot}_t \) = switching rate between non-adjacent quartiles
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

macroeconomics. To address this concern, I conduct industry panel regressions to show that measures of churning at the industry level are negatively associated with the industry employment growth. Specifically, I specify:

\[ \text{Employment growth}_i,t = X_i + \gamma_t + \beta \cdot \text{Rot}_i,t + \epsilon_{i,t} \]

where \( X_i \) is industry fixed effect and \( \gamma_t \) is year effect. \( \text{Rot}_i,t \) is industry rotation rate or industry large rotation rate. Firms are ranked either by profit or profit margin. I use industry employment growth rate as the the dependent variable because it is well known that unemployment is not well measured at the industry level.

Table 2 reports the results of the industry panel regression. In all cases, an increase in measures of churning within an industry is associated with a decline in the industry employment growth.

As there is strong inter-industry linkage in the U.S. economy in terms of flow of intermediate input and output across industries, it is interesting to exam if churning of an industry

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5Examples of studies of inter-industry input-output linkage in the real business cycle models can be found in Long Jr and Plosser (1983); Dupor (1999); Horvath (2000). A more recent literature on the production network, such as Atalay, Hortacsu, Roberts, and Syverson (2011); Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), also documents a strong inter-industry linkage in the US economy.
Table 2: Measures of industry churning are negatively associated with industry employment growth

\[
Employment\ growth_{i,t} = X_i + \gamma_t + \beta \cdot Rot_{i,t} + \epsilon_{i,t}
\]

| Independent variable | Coefficient | \( P > |t| \) | \( R - squared \) | \( Number\ of\ observations \) |
|----------------------|-------------|----------------|-----------------|-----------------------------|
| \( Rot^{Profit} \)   | -0.16***    | 0.00           | 0.34            | 15 \times 41               |
| \( Large\ Rot^{Profit} \) | -0.21***    | 0.00           | 0.34            | 15 \times 41               |
| \( Rot^{Profit\ margin} \) | -0.11***    | 0.00           | 0.34            | 15 \times 41               |
| \( Large\ Rot^{Profit\ margin} \) | -0.17***    | 0.00           | 0.29            | 15 \times 41               |

\( Employment\ growth_{i,t} \) = industry employment growth in year \( t \), NIPA, 1999-2013  
\( X_i \) = industry and fixed effect  
\( \gamma_t \) = year effect  
\( Rot_{i,t} \) = industry rotation rate in year \( t \), Compustat  
I include private non-farm industries that are in both Compustat and NIPA and have more than 8 firms from 1999 to 2013  
SE is in the parentheses.  
1, 2, 3 asterisks denote significance at the ten, five, one percent level.
is also associated with the condition of its linked industries’ labor market. To examine it, I conduct industry panel regressions of industry employment growth on not only the industry’s own measures of churning, but also its linked industries’ measures of churning. Specifically, I specify:

\[ \text{Employment growth}_{it} = X_i + \gamma_t + \beta_1 \cdot \text{Rot}_{it} + \beta_2 \sum_j w_{ij} \text{Rot}_{jt} + \epsilon_{it} \]

where \( X_i \) is industry fixed effect and \( \gamma_t \) is year effect. \( \text{Rot}_{it} \) is industry rotation rate for industry \( i \) in year \( t \). \( \sum_j w_{ij} \text{Rot}_{jt} \) is the weighted average of linked industries’ rotation rates.

Table 3 reports the results of the above regressions. In all cases, an increase in measures of churning within an industry and an increase in its linked industry are both associated with a decline in the industry employment growth.

To summarize the empirical findings of this subsection, measures of churning are significantly associated with the condition of the labor market at both aggregate level and industry level. Moreover, measures of churning of an industry is significantly associated with the condition of labor markets of its linked industries. Given these findings, it is reasonable to conjecture that variations in churning contribute to the labor market fluctuations, and inter-firm linkage across industries serve as a propagation mechanism.

2.2 Churning and uncertainty shock

Churning is closely related to uncertainty shock. Following Bloom (2009); Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), uncertainty shock is defined as exogenous change in volatility of idiosyncratic productivity. In many settings, an increase in uncertainty leads to higher churning.\(^6\) There are two main reasons why I distinguish churning from uncertainty in this paper.

First, churning is not equivalent to uncertainty. In fact, there are many cases in which uncertainty preserves firms’ initial rankings and therefore has no effect on churning.\(^7\) In the uncertainty shock literature, most empirical work measure uncertainty as cross-firm dispersion which characterizes the shape of the entire distribution and can be independent to individual

\(^6\)This is often true if the productivity process is parameterized as an auto-regressive process. For example, if idiosyncratic productivity follows

\[ z_{i,t+1} = \rho z_{i,t} + \sigma_t \epsilon_{i,t+1}, \epsilon_{i,t+1} \sim N(0, 1) \]

and if there is positive shock to \( \sigma_t \), firms’ rankings in productivity distribution changes more often, which leads to a higher churning.

\(^7\)One example is: \( z_{1,t+1} = \exp(|\sigma_{1,t} \epsilon_{1,t+1}|) \) and \( z_{2,t+1} = \exp(-|\sigma_{2,t} \epsilon_{2,t+1}|) \). Uncertainty shock does not change the two firms’ rankings since we always have \( z_{2,t+1} < z_{1,t+1} \).
Table 3: Measures of industry churning and linked industries’ churning are negatively correlated with industry employment growth

\[ Employment\ growth_{i,t} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \sum_j w_{i,j} Rot_{j,t} + \epsilon_{i,t} \]

| Measures of churning | \( \beta_1 \) | \( P > |t| \) | \( \beta_2 \) | \( P > |t| \) | Number of observations |
|----------------------|--------------|-------------|--------------|-------------|----------------------|
|                      | Coefficient | \( P > |t| \) | Coefficient | \( P > |t| \) | \( \text{year} \times \text{industry} \) |
| \( Rot^{Profit} \)    | \(-0.17^{***}\) | 0.01        | \(-0.24^{***}\) | 0.01        | 15 \( \times \) 41 |
| (0.04)                |             |             | (0.09)       |             |                      |
| \( Large\ Rot^{Profit} \) | \(-0.20^{***}\) | 0.00        | \(-0.27^{***}\) | 0.01        | 15 \( \times \) 41 |
| (0.06)                |             |             | (0.10)       |             |                      |
| \( Rot^{Profit\ margin} \) | \(-0.09^{***}\) | 0.01        | \(-0.17^{***}\) | 0.01        | 15 \( \times \) 41 |
| (0.03)                |             |             | (0.07)       |             |                      |
| \( Large\ Rot^{Profit\ margin} \) | \(-0.16^{***}\) | 0.01        | \(-0.15^{*}\)  | 0.10        | 15 \( \times \) 41 |
| (0.06)                |             |             | (0.09)       |             |                      |

\( Employment\ growth_{i,t} = \) industry employment growth, NIPA, 1999-2013
\( X_i = \) industry fixed effect
\( \gamma_t = \) year effect
\( Rot_{i,t} = \) industry rotation rate at \( t \), Compustat
\( w_{i,j} = \) share of intermediate goods from industry \( i \) to industry \( j \), BEA 2007 IO tables
I include private non-farm industries that are in both Compustat and NIPA and have more than 8 firms in Compustat from 1999 to 2013
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.
firms’ rankings.\textsuperscript{8}

Second, churning has straightforward interpretations that are distinct from those of uncertainty shock. Uncertainty is usually motivated as dispersion of economic outlook or as an occurrence of unexpected critical event.\textsuperscript{9} In contrast, churning is likely to be caused by the introduction of new technologies which are then adopted by firms who are not industry leaders, or by a shift of consumer preference on different products,\textsuperscript{10} or firms’ heterogeneous exposures to aggregate shocks,\textsuperscript{11} which are economic disturbances that are common in the economy. In this paper, I study the effect of churning on the economy, which also provides a new transmission and amplification mechanism for the above mentioned economic disturbances.

3 A simple model

Motivated by the preceding empirical findings, in this section I demonstrate the idea of this paper through a simple DMP model modified with firm inter-connectivity. I introduce the baseline environment of the model in sections 3.1 to 3.3. Then I derive the key analytical results in section 3.4 to show that churning increases unemployment and decreases measured TFP if and only if (1) there is firm inter-connectivity, and (2) the production function is supermodular.

3.1 Baseline environment

Time is discrete. The economy has two distinct sets of firms indexed by $A$ and $B$, so two labor markets. New production opportunities, corresponding to job vacancies ($v_i$) are created at cost $\chi$. Each set of firms is associated with a labor market that is populated by a measure one of risk-neutral individuals who can be either employed in set $i$ ($e_i$) or unemployed and searching for a job ($u_i$). As will be shown, in this model, the two sets always comove exactly and workers are indifferent between the two labor markets.\textsuperscript{12} Hence, for simplicity, I assume workers cannot move across labor markets.

At the beginning of each period, unemployed workers and job vacancies are matched in the two frictional labor markets. Matching probability depends on the ratio of the number

\textsuperscript{8}One exception is Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), who used churning of firms’ rankings in TFP as evidence of uncertainty shock.
\textsuperscript{9}See Bloom (2009)
\textsuperscript{10}For example, consumers previously prefer goods A to goods B, but suddenly flip the ordering. The producers of the two goods would switch ranking of output and profit.
\textsuperscript{11}For example, a persistent increase in the oil price can possibly induce a change in the relative profitability of car markers who specialize in SUV and ones specialize in vehicles with lower fuel consumption.
\textsuperscript{12}The quantitative model in section (4) would describe the situation when the two sets do not comove and labor can move across labor markets.
of vacancies to the number of unemployed workers. If unmatched, the firms exit the model and the workers remain unemployed. If matched, the new firms\textsuperscript{13} draw a type from $H$ (high productivity) or $L$ (low productivity)\textsuperscript{14} randomly with probability 50 percent and 50 percent. Then the firm is instantaneously assigned to a partner of the same type from the other set; That is, an $H$ type firm is assigned to an $H$ type partner while an $L$ type is assigned to an $L$ type partner.\textsuperscript{15,16} The positive assortative assignment is the unique Nash equilibrium in this economy.

Production takes place directly after the assignment of partner. A firm’s productivity depends on a firm’s own type and its partner’s type. I denote a firm’s productivity as $z_{ijk}^i$ if it is in set $i$ with type $j$ cooperating with a $k$ type partner. By assumption, a new firm’s productivity is either $z_{i}^{HH}$ or $z_{i}^{LL}$.

Firm and worker split output with fixed share $\tau$ and $1 - \tau$. As a firm’s output is simply the productivity, its profit is $\tau \cdot z_{ij}^{ik}$ and worker’s wage is $\tau \cdot z_{ij}^{ik}$. For simplicity I assume there is no unemployment insurance or disutility of working, so workers always prefer working to unemployment.\textsuperscript{17}

The most important setting of the model is, after production of goods, type of each firm from set $A$ switches with a Markov switching process, which is described by a Markov switching matrix $\Pi_t$ with each row adds up to one. Here $\Pi_t$ has a time subscript, meaning rotation rates are time-varying: in period $t$ a type $j$ firm switches to type $k$ with probability $\rho_{jk}^t$. The rotation processes are i.i.d. across firms.\textsuperscript{18}

\[
\Pi_t = \begin{bmatrix}
\rho_{HH}^t & \rho_{HL}^t \\
\rho_{LH}^t & \rho_{LL}^t
\end{bmatrix}
\]

As the empirical counterpart of $H$ type and $L$ type is the above the median and below the median in the productivity distribution, the measure of $H$ type firms should always equal to the measure of $L$ type firms, therefore $\Pi_t$ is restricted to be symmetric with $\rho_{LH}^t = \rho_{HL}^t$.

My analysis focuses on shock to the rotation rates. For simplicity, I ignore rotation of type for firms in set $B$; that is, their types are permanent. However, their productivities can change

\textsuperscript{13} New firms are newly created firm–worker matches.

\textsuperscript{14} Here firms are ranked by productivity. In the appendix E, I show the case in which firms are not ranked by productivity, but by consumer’s preference on different products. The two settings have similar results.

\textsuperscript{15} For simplicity, here I assume firms are assigned to their partner without searching friction. In the quantitative model, I will add searching friction to the inter-firm matching process.

\textsuperscript{16} In appendix F, I show that the main results of this section is not altered if one assumes that firms draw types before matching with worker.

\textsuperscript{17} The setting of wage and unemployment benefit deviates from canonical DMP model. The quantitative model in section 3 considers Nash bargaining and positive unemployment benefit. The assumption of wage setting does not affect the main result.

\textsuperscript{18} $\rho_{jk}^t$ is also the fraction of firm with type $j$ in set $A$ that rotates to type $k$, and off-diagonal entries corresponds to the empirical rotation rate I constructed in the previous section.
due to the switching of their partner’s type.

Following the canonical DMP model, worker-firm matches are destroyed exogenously at the end of each period with fixed rate $\delta$. Upon destruction of the match, workers become unemployed in the same set since I assume that they cannot move across labor markets.

In sum, events unfold as follows: At the beginning of the period, the aggregate productivity and rotation rate of set $A$ are observed. Firms post vacancy to match with unemployed workers. If successfully matched, firms randomly draw their type, either H or L, then are assigned to a partner with the same type. If not matched, the firm exits the model. The production takes place right after the assignment. Then the firms in set $A$ rotate their type randomly with type rotation rate. At the end of each period, firms are destroyed and disappear from the model exogenously with a fixed rate $\delta$, in which case the worker becomes unemployed.

### 3.2 Tightness ratios

In this subsection, I introduce some key notations of the frictional labor market. A firm can post a vacancy in either set. The cost of posting a vacancy is $\chi$. There is free entry into vacancy posting on the part of firms.

Matching function $m(\mu_{i,t}, v_{i,t})$ determines how many matches are formed given the number of vacancies $v_{i,t}$ and the number of unemployed workers $u_{i,t}$.

Labor market tightness ratio $\theta_{i,t}$ is the ratio of the number of vacancies to the number of unemployed workers in set $i$:

$$\theta_{i,t} = \frac{v_{i,t}}{u_{i,t}}, \quad i \in \{A, B\}$$

Tightness ratios are an equilibrium object, they are taken parametrically by both firms and workers.

Job finding rate $\mu_{i,t}$ is the probability for an unemployed worker to match with a vacancy. Vacancy filling rate $q_{i,t}$ is the probability for a vacancy matching with an unemployed worker. The matching function is assumed to be homogeneous of degree one, hence job finding rate and vacancy filling rate are functions of the market tightness ratio:

$$\mu_{i,t} = \frac{m(u_{i,t}, v_{i,t})}{u_{i,t}} = \mu(\theta_{i,t}) \quad i \in \{A, B\}$$

$$q_{i,t} = \frac{m(u_{i,t}, v_{i,t})}{v_{i,t}} = q(\theta_{i,t})$$
Unemployment rates are determined by the joint force of job creation and job destruction. In each period, unemployed workers flow out of unemployment with the job finding rate $\mu_{i,t}$. At the same time, jobs are destroyed endogenously and workers become unemployed with the job destruction rate $\delta$. Flow motion of unemployment is

$$u_{i,t+1} = u_{i,t} - \mu_{i,t} \cdot u_{i,t} + \delta \cdot e_{i,t}$$

$i \in \{A, B\}$ (1)

$u_{i,t} (e_{i,t})$ is measure of unemployment (employment) with $u_{i,t} = 1 - e_{i,t}$, because measure of workforce of each set is 1 and workers can be either working or unemployed searching for a job.

Since the job destruction rate is assumed to be fixed, fluctuation of unemployment is solely governed by the tightness ratio $\theta_{i,t}$. A lower tightness ratio halts job creation and increases unemployment. The simple model aims to find the conditions under which increases in rotation rate $\rho_{H}^{H,L}$ cause an increase in unemployment $u_{A,t}$ and $u_{B,t}$ by reducing tightness ratios $\theta_{A,t}$ and $\theta_{B,t}$.

### 3.3 Firm’s value function

In this subsection, I demonstrate firms’ value functions.

As utility function is linear in this simple model, the firm’s value function is simply present value of profit. In set $A$, for a firm with type $j$ and working with a partner with type $k$, its value function is

$$J_{A,t}^{jk} = \tau \cdot z_{A,t}^{jk} + \beta \left(1 - \delta\right) E_{t} \left(\rho_{t}^{jH} J_{A,t+1}^{H} + \rho_{t}^{jL} J_{A,t+1}^{L}\right)$$

$j, k \in \{H, L\}$ (2)

Similar to the notation of idiosyncratic productivity, the superscript $jk$ indicates that the firm is type $j$ and it cooperates with another firm of type $k$; subscript indicates set and time period.

This value is composed of the contemporary profit, $\tau$ fraction of output $z_{A,t}^{jk}$, plus the expected discounted value from the next period on. In the next period, with probability $\rho_{t}^{jH}$ and complementary probability $\rho_{t}^{jL}$, the firm becomes $H$ type and $L$ type and therefore has value of $J_{A,t+1}^{H}$ and $J_{A,t+1}^{L}$.

Value of firms in set $B$ is described by a similar equation:

$$J_{B,t}^{jk} = \tau \cdot z_{B,t}^{jk} + \beta \left(1 - \delta\right) E_{t} \left(\rho_{t}^{jH} J_{B,t+1}^{H} + \rho_{t}^{jL} J_{B,t+1}^{L}\right)$$

(3)
The firm receives contemporary profit $\tau \cdot z^j_{\tau k}$, and in the next period, although by assumption its own type is not subject to rotation and will always be $j$, the type of its partner in set $A$ rotates from $k$ to $H$ and $L$ with probability $\rho_{kH_t}$ and $\rho_{kL_t}$.

In each set, there are a large number of firms that can potentially post vacancies as long as they pay the cost $\chi$. The value of posting a vacancy in set $i$ is

$$V_{i,t} = -\chi + f(\theta_{i,t}) \left( \frac{J_{i,HH}^{i,t}}{2} + \frac{J_{i,LL}^{i,t}}{2} \right) + (1 - f(\theta_{i,t})) \max_i E_t (V_{A,t+1}, E_t (V_{B,t+1}), 0)$$

where $f(\theta_{i,t})$ denotes a firm’s vacancy filling rate. If successfully matched with a worker, the firm draws $H$ or $L$ type with 50 percent and 50 percent probability and assigned to a partner with the same type. The maximization of the expectation term implies that firms who fail to match with a worker can choose to post a vacancy in either market or to be inactive in the following period.

Some useful notions

Before characterizing the equilibrium of the labor market, let me introduce some useful notions and results.

**Definition 1.** (Strict monotonicity)

1. Production function is strictly monotone if

$$z_{i,H}^j > z_{i,L}^j \quad z_{k,H}^i > z_{k,L}^i \quad i \in \{A, B\}$$

2. Value function is strictly monotone if

$$J_{i,H}^j > J_{i,L}^j \quad J_{k,H}^i > J_{k,L}^i \quad i \in \{A, B\}$$

Strict monotonicity holds if a firm’s productivity and value are strictly increasing in both the firm’s own type and its partner’s type.

**Definition 2.** (Supermodularity)
1. Production function is supermodular if

\[ z_i^{HH} - z_i^{LH} > Z_i^{HL} - z_i^{LL} \quad i \in \{A, B\} \]

2. Value function is supermodular if

\[ J_i^{HH} - J_i^{LH} > J_i^{HL} - J_i^{LL} \quad i \in \{A, B\} \]

The production function is supermodular when marginal production is increasing in partner’s type. Similarly, the value function is supermodular when marginal value is increasing in partner’s type. Another way to interpret supermodularity is that positive assortative matching yields higher aggregate productivity or value than cross matching between different type of firms.

**Lemma 1.**

1. Value function is strictly monotone if production function is strictly monotone.

2. Value function is supermodular if and only if production function is supermodular.

Proof of Lemma 1 can be found in appendix C. The first line of Lemma 1 shows the sufficient condition under which firms are better off as a H type firm or matching with H type partner. The second line shows that it’s more efficient for firms to match with firms of same type if and only if the production function is supermodular; cross matching between different types induces efficiency loss.

**Proposition 1.** Positive assortative assignment is the unique Nash equilibrium if idiosyncratic productivity is strictly monotone.

Proposition 1 illustrates that the positive assortative assignment can be endogenized given firms are more productive matching with H type than with L type. The intuition is that, as there is no search friction between firms, H type firms only match with H type firms while L type firms are only left to match with L type firms.

### 3.4 Free entry condition and the equilibrium of the labor market

The equilibrium of the model is determined by free entry condition. In this subsection, I first derive the free entry condition. Then based on the free entry condition, I illustrate the effect of rotation rate on the equilibrium of the labor market.
3.4.1 Free entry condition

The equilibrium level of tightness ratio $\theta_{i,t}$ is determined by the following free entry conditions

$$V_{i,t} = 0, \quad i \in \{A, B\}$$

By plugging the free entry condition into equation 4, we get

$$\chi = f(\theta_{i,t}) \cdot \left( \frac{J_{i,t}^{HH}}{2} + \frac{J_{i,t}^{LL}}{2} \right) \quad i \in \{A, B\} \quad (5)$$

The LHS of equation 5 is vacancy cost. The RHS is the expected payoff of the vacancy. Firms would post vacancies up to the point that its expected payoff exactly compensates for the cost. Because the vacancy filling rate $f(\theta_{i,t})$ is a decreasing function of $\theta_{i,t}$, a smaller expected matching value $\frac{1}{2} \left( J_{i,t}^{HH} + J_{i,t}^{LL} \right)$ would induce a lower tightness ratio $\theta_{i,t}$. The intuition is that when the expected matching value is small, fewer firms want to post or maintain vacancies which then leads to a lower tightness ratio.

The case with no firm inter-connectivity

When there is no firm inter-connectivity as in the canonical DMP model, a firm’s productivity and value depends only on its own type

$$J_{A,t}^j = \tau \cdot \zeta_{A,t}^j + \beta (1 - \delta) E_t \left( \rho_t^{jH} J_{A,t+1}^{jH} + \rho_t^{jL} J_{A,t+1}^{jL} \right) \quad j, k \in \{H, L\} \quad (6)$$

$$J_{B,t}^j = \tau \cdot \zeta_{B,t}^j + \beta (1 - \delta) E_t \left( J_{B,t+1}^j \right) \quad (7)$$

The free entry condition is

$$\chi = f(\theta_{i,t}) \cdot \left( \frac{J_{i,t}^H}{2} + \frac{J_{i,t}^L}{2} \right) \quad i \in \{A, B\} \quad (8)$$

Equations 6 and 8 are identical to the value function and free entry condition of the canonical DMP model with two productivity states. In other words, the canonical DMP model is nested in the benchmark model with firm inter-connectivity by restricting firms’ productivity to be unaffected by type of partners; that is, $\zeta_{i,t}^{HH} = \zeta_{i,t}^{HL} = \zeta_{i,t}^{LH} = \zeta_{i,t}^{LL}$.

Take vacancy filling rate to the LHS of free entry condition equation 8 and plug equation 6
into the RHS, we get

\[
\frac{\chi}{f(\theta_{A,t})} = \frac{1}{2} \left[ \tau \cdot z_{A,t}^H + \beta (1 - \delta) E_t (\rho_t^{HH} J_{A,t+1}^H + \rho_t^{HL} J_{A,t+1}^L) \right] \\
+ \frac{1}{2} \left[ \tau \cdot z_{A,t}^L + \beta (1 - \delta) E_t (\rho_t^{LH} J_{A,t+1}^H + \rho_t^{LL} J_{A,t+1}^L) \right]
\]

As the Markov switching matrix \( \Pi_{A,t} \) is symmetric, that is, \( \rho_t^{HH} + \rho_t^{HL} = 1, \rho_t^{HL} + \rho_t^{LL} = 1 \), which is a restriction that ensures the measure of \( H \)-type and \( L \)-type firms are always same. The above equation can be further derived as

\[
\frac{\chi}{f(\theta_{A,t})} = \frac{1}{2} \left[ \tau \cdot (z_{A,t}^H + z_{A,t}^L) + \beta (1 - \delta) E_t (J_{A,t+1}^H + J_{A,t+1}^L) \right]
\]

As shown in equation 9, rotation rate becomes irrelevant in the free entry condition set \( A \), hence does not affect the equilibrium level of set \( A \)'s tightness ratio. The intuition is that with a higher \( \rho^{HL} \), that is, \( H \)-type firms are more likely to become \( L \)-type; at the same time \( L \)-type firms are equally likely to become \( H \)-type, as \( \rho^{LH} \) also increases. Adding them together, the expected value of vacancy does not change hence rotation rate has no aggregate effect.

Because set \( B \) does not have rotation, it’s free entry condition is simply

\[
\frac{\chi}{f(\theta_{B,t})} = \frac{1}{2} \left[ \tau \cdot (z_{B,t}^H + z_{B,t}^L) + \beta (1 - \delta) E_t (J_{B,t+1}^H + J_{B,t+1}^L) \right]
\]

In set \( B \), the equilibrium level of tightness ratio \( \theta_{B,t} \) is also not affected by rotation rate.

I summarize the above results in the following proposition.

**Proposition 2.** When there’s no firm inter-connectivity, rotation rate has no effect on the equilibrium level of tightness ratio in either labor market.

Proposition 2 shows that firm inter-connectivity is a necessary condition for rotation rate to affect the labor market. Therefore in the subsequent part, I focus on the case with firm inter-connectivity.

**The case with firm inter-connectivity**

When there’s firm inter-connectivity, rotation rate enters into matching value and can potentially affect the equilibrium level of the tightness ratio. To illustrate this, take vacancy filling rate to the \( LHS \) of free entry condition equation 5 and plug equations 2 and 3 into \( RHS \), free entry
condition of set $A$ then becomes

$$\frac{\chi}{f(\theta_{A,t})} = \frac{1}{2} \left[ \tau \cdot z_{A,t}^{HH} + \beta (1 - \delta) E_t \left( \rho_t^{HH} J_{A,t+1}^{HH} + \rho_t^{HL} J_{A,t+1}^{HL} \right) \right]$$

$$+ \frac{1}{2} \left[ \tau \cdot z_{A,t}^{LL} + \beta (1 - \delta) E_t \left( \rho_t^{LL} J_{A,t+1}^{LL} + \rho_t^{HL} J_{A,t+1}^{HL} \right) \right]$$

Since the Markov switching matrix is symmetric, that is $\rho_t^{HH} = 1 - \rho_t^{HL}$, $\rho_t^{LL} = 1 - \rho_t^{LH}$ and $\rho_t^{HL} = \rho_t^{LH}$, the above equation is equivalent to

$$\frac{\chi}{f(\theta_{A,t})} = \frac{1}{2} \left[ \tau \cdot z_{A,t}^{HH} + \beta (1 - \delta) E_t \left( J_{A,t+1}^{HH} \right) \right]$$

$$+ \frac{1}{2} \left[ \tau \cdot z_{A,t}^{LL} + \beta (1 - \delta) E_t \left( J_{A,t+1}^{LL} \right) \right]$$

$$- \frac{1}{2} \beta (1 - \delta) \rho_t^{HL} E_t \left( \frac{J_{A,t+1}^{HH} + J_{A,t+1}^{LL} - J_{A,t+1}^{HL} - J_{A,t+1}^{LH}}{mismatch \ loss} \right)$$

As equation 11 is stochastic, I will use temporary shock to rotation rate in period $t$ as an illustration, and defer the discussion of persistent shock to the next subsection. Suppose in period $t$, there is a temporary increase in the rotation rate $\rho_t^{HL}$. It is easy to see that the first two rows of the RHS of equation 11 are not affected as the shock is temporary and values should return to the steady state in period $t + 1$. The shock to $\rho_t^{HL}$ would affects the last row only. Specifically, with supermodularity, or equivalently $(J_{A,t+1}^{HH} + J_{A,t+1}^{LL} - J_{A,t+1}^{HL} - J_{A,t+1}^{LH})$ is positive, a temporary increase in $\rho_t^{HH}$ would increase expected efficiency loss, which then translates into a decline in expected matching value. As a result, fewer firms post vacancies, driving tightness ratio to a lower level.

In addition to the change in the expected matching value, a higher rotation rate increases the share of mismatched firms in the next period. When production function is supermodular, mismatched firms, on average, are less productive. This composition effect decreases the averaged TFP in set $A$ in period $t + 1$. 

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For firm in set $B$, its free entry condition has a similar form

\[
\frac{\chi_f}{f(\theta_{B,t})} = \frac{1}{2} \left( \tau \cdot z_{B,t}^{HH} + \beta (1 - \delta) E_t \left( J_{B,t+1}^{HH} \right) \right) \\
+ \frac{1}{2} \left( \tau \cdot z_{B,t}^{LL} + \beta (1 - \delta) E_t \left( J_{B,t+1}^{LL} \right) \right) \\
- \frac{1}{2} \beta (1 - \delta) \rho_t^{HL} E_t \left( \frac{J_{B,t+1}^{HH} + J_{B,t+1}^{LL} - J_{B,t+1}^{HL} - J_{B,t+1}^{LH}}{\text{mismatch loss}} \right)
\] (12)

Although firms in set $B$ do not have rotation, they are faced with the risk of mismatch. When firms in set $B$ has supermodular production and value functions, a temporary increase in $\rho_t^{HH}$ would lower the equilibrium level of tightness ratio in period $t$ and averaged TFP in period $t+1$.

### 3.5 The main result: rotation rate and labor market equilibrium

In the previous subsection, I show that an temporary increase in the rotation rate reduces labor market tightness ratio and therefore increases unemployment if two conditions hold: (1) there is firm inter-connectivity; and (2) efficiency loss caused by mismatch $E_t \left( J_{B,t+1}^{HH} + J_{B,t+1}^{LL} - J_{B,t+1}^{HL} - J_{B,t+1}^{LH} \right)$ is positive. In this subsection, I generalize the temporary shock to a persistent shock case and present the main results.

**Proposition 3.** If production function is strictly monotone, and there is firm inter-connectivity, and the persistence of the rotation shock is bounded below $|\psi| < 1 - \rho^{HL} - \rho^{LH}$, a persistent increase in the rotation rate of one set

1. decreases the equilibrium tightness ratio of both sets
2. decreases the average productivity of both sets in the next period

if and only if the production function is supermodular.

Proposition 3 is the key result of section 3. The proof of proposition 3 can be found in appendix C. The first part formally shows the necessary and sufficient condition under which an increase in rotation rate reduces job creation and increases unemployment. The intuition is similar with the temporary shock case illustrated in the previous subsection: positive shock to the rotation rate of one set of firms increases the probability of mismatch for these firms and their partners. With supermodularity mismatch risk negatively affects firms’ average expected value, which will reduce job creation and increase unemployment.

The second part of proposition 3 shows that the model generates endogenous change in TFP. It works in two channels. First, higher rotation rate makes more existing firms become
mismatched. In the model, with the supermodular production function, mismatched firms have lower productivity on average and a larger fraction of mismatched firms leads to a lower average productivity of the economy. Second, faced with a higher mismatch risk, firms post fewer vacancies, which results in decreased creation of new firms. By construction, there’s no mismatch in new firms, hence they have higher average productivity. As a result, the drop in the inflow of new firms also contributes to a decline in average productivity.

4 Endogenous choice of inter-firm cooperation contract and testable microeconomic implication

In this section, I test the empirical relevance of my model using testable microeconomic implication that my model is not designed to match with. The main mechanism of my model is that when there’s firm inter-connectivity and the production function is supermodular, churning induces mismatches which reduce output and job creation. While churning and firm inter-connectivity are observed in the economy, supermodularity cannot be directly observed. Although the idea of supermodularity of inter-firm matches has been hypothesized by many studies,\(^{19}\) there is no direct microeconomic evidence on it. In this section, I test the assumption of supermodularity using the model’s implication on firms’ endogenous choice of inter-firm cooperation contract.

The plan of this section is as follows: I first demonstrate the main idea of the test. Then I extend the simple model with endogenous choice of cooperation contract, which allows me to derive the model’s testable microeconomic implication and implement the test. Last, I present the result of the test.

4.1 Intuition of the test

Supermodularity depicts the efficiency of the allocation of different types of firms, which is not directly observed. Firms would always initiate match with the same type partner as long as the production function is monotone, that is, every firm prefer to match with a \(H\) type partner. Hence one cannot identify supermodularity from firms’ choice of partner.

To overcome this challenge, I consider a slight extension to the simple model. Specifically, I allow firms to choose the duration of the cooperation contract before matching with partners. And I show that although supermodularity does not affect firms’ choice of partner, it does affect firms’ choice of cooperation contract. The intuition is that, when a firm is uncertain about its

\(^{19}\)For example, Becker (1973); Kremer (1993); Rhodes-Kropf and Robinson (2008); Shimer and Smith (2000)
own future type or its partner’s future type, and then mismatch is inefficient, the firm does not want to make the commitment by choosing a long duration contract. In the empirical analysis, I use vertical integration and sourcing to proxy long- and short-duration contracts.\textsuperscript{20}

In particular, I show that supermodularity predicts two cross-industry patterns: first, fewer firms choose vertical integration (long-duration cooperation) in industries with higher churning; second, fewer firms choose vertical integration when their partners are in industries with higher churning. The intuition is as follows: while vertical integration helps firms to avoid transaction costs,\textsuperscript{21} it prevents firms from dissolving mismatches and re-allocating to new cooperation relationships.

Remember that in any mismatch, the high type always wants to leave, and the low type always wants to stay. Supermodularity means that on average, keeping a mismatch is an inefficient thing to do, because the opportunity cost to the high type is higher than the benefit to the low type, and sourcing is an option to hedge against the mismatch risk. When a industry or its linked industries has higher churning, more firms choose sourcing and fewer choose vertical integration.

4.2 Extend the simple model

In this subsection, I extend the simple model by letting firms choose the type of cooperation contract either vertical integration (VI) or sourcing (SC). If a firm chooses vertical integration, it is permanently matched with its partner.\textsuperscript{22} In contrast, if a firm chooses sourcing, it can sever mismatched partnership with probability $\nu_i$ then match with a new partner. Parameter $\nu_i$ is a number between zero and one depending on maturity of sourcing contract industry in industry $i$.\textsuperscript{23} This rematch option comes with a price: each period, firms with sourcing contracts pay a transaction cost $\eta \sim iid \mathcal{N}(\tilde{\eta}_i, \tilde{\sigma}_i^2)$, where $\eta$ is firm fixed effect; $\tilde{\eta}_i$ and $\tilde{\sigma}_i$ are mean and dispersion of transaction cost within industry $i$.

As I do not have measurement of each industry’s transaction costs or on their sourcing contracts’ maturities, in the empirical analysis I treat $\nu_i$, $\tilde{\eta}_i$ and $\tilde{\sigma}_i$ as constant across industries and I ignore subscript $i$ in the rest of this section.

\textsuperscript{20}Vertical integration means a firm internalizes the cooperation relationship, which is practically permanent cooperation; Sourcing is a temporary cooperation arrangement, the duration of which depends on maturity of the contract.

\textsuperscript{21}Common transaction costs include bargaining cost, contracting cost and hold-up risk, which has been discussed extensively in Transaction Cost Economics (TCE) literature pioneered by Coase (1937); Williamson (1979, 1981)

\textsuperscript{22}The simple model studied in the previous section, in which all firms use vertical integration, is nested in the extended model.

\textsuperscript{23}For example, if a contract’s maturity is 2 years, $\nu$ is 1/8 in a quarterly model.
For simplicity, as in the simple model there are two industries $i$ and $j$. Only industry $j$ has rotation, which is governed by a Markov switching matrix
\[
\begin{bmatrix}
\rho_{j}^{HH} & \rho_{j}^{HL} \\
\rho_{j}^{LH} & \rho_{j}^{LL}
\end{bmatrix},
\]
while industry $i$ does not have rotation.

I assume that the draw of transaction cost $\eta$ and the choice of cooperation contract takes place directly after a firm matches with a worker. Hence events unfold as follows: at the beginning of the period, homogeneous firms post vacancies to match with unemployed workers. If successfully matched, a firm draws its transaction cost $\eta$ from $\mathcal{N}(\bar{\eta}, \sigma^2)$, then chooses between vertical integration and sourcing. Then firms randomly draw their type, either H or L, and match with a same type partner. In each period, existing firms in industry $j$ rotate their type randomly. At the end of each period, firms are destroyed and exit the model exogenously with fixed rate $\delta$.

**Value function and free entry condition**

In industry $i$, value functions for firms choosing sourcing as cooperation contract are:

\[
J_{i,SC}^{HH}(\eta) = \tau \cdot (z_{i}^{HH} - \eta) + \beta (1 - \delta) E_t \left[ \rho_{j}^{HH} J_{i,SC}^{HH}(\eta) + (1 - \nu) \rho_{j}^{HL} J_{i,SC}^{HL}(\eta) + \nu \rho_{j}^{HL} J_{i,SC}^{HH}(\eta) \right]
\]

\[
J_{i,SC}^{LL}(\eta) = \tau \cdot (z_{i}^{LL} - \eta) + \beta (1 - \delta) E_t \left[ \rho_{j}^{LL} J_{i,SC}^{LL}(\eta) + (1 - \nu) \rho_{j}^{LH} J_{i,SC}^{LH}(\eta) + \nu \rho_{j}^{LH} J_{i,SC}^{LL}(\eta) \right]
\]

The time subscripts are omitted in the above equations. The value of $H$ type firms is composed of contemporary profit and continuation value in the next period; with rotation rate $\rho_{j}^{HL}$, the firm’s partner becomes $L$ type, and with probability $\nu$, the firm rematches with another $H$ type partner. The subscript $SC$ denotes that the firm uses sourcing as cooperation contract. The value of $L$ type firms is similar. With rotation rate $\rho_{j}^{LH}$, its partner becomes $H$ type and mismatch occurs; the $H$ type firm would terminate the partnership and the $L$ type firm rematches with an $L$ type partner.

The value functions for firms with vertical integration are same as those in the original simple model. A firm that chooses vertical integration is exempt from transaction cost $\eta$, but does not have a rematch option when mismatch occurs.

\[
J_{i,VI}^{HH} = \tau \cdot z_{i}^{HH} + \beta (1 - \delta) E_t \left( \rho_{j}^{HH} J_{i,VI}^{HH} + \rho_{j}^{HL} J_{i,VI}^{HL} \right)
\]

\[
J_{i,VI}^{LL} = \tau \cdot z_{i}^{LL} + \beta (1 - \delta) E_t \left( \rho_{j}^{LL} J_{i,VI}^{LL} + \rho_{j}^{LH} J_{i,VI}^{LH} \right)
\]
The value of posting a vacancy in industry $i$ is

$$V_i = -\chi + f(\theta_i) \int \max \left\{ \frac{J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)}{2}, \frac{J_{i,VI}^{HH} + J_{i,VI}^{LL}}{2} \right\} dF(\eta)$$

$$+ (1 - f(\theta_i)) \max_i \{ E_i(V_i), 0 \}$$

(17)

where $F(\eta)$ is cumulative distribution function (CDF) of the transaction cost distribution.

After matching with a worker with vacancy filling rate $f(\theta_i)$, the firm chooses between vertical integration and sourcing. If a firm chooses sourcing, the firm is subject to the transaction cost $\eta$ and the expected value is $\frac{1}{2} \left( J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta) \right)$. If the firm chooses vertical integration, the expected value is $\frac{1}{2} \left( J_{i,VI}^{HH} + J_{i,VI}^{LL} \right)$. Firms choose the cooperation contract by comparing the above two terms.

The free entry condition in industry $i$ is

$$V_i = 0$$

From equation 17, we get

$$\frac{\chi}{f(\theta_i)} = \int \max \left\{ \frac{J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)}{2}, \frac{J_{i,VI}^{HH} + J_{i,VI}^{LL}}{2} \right\} dF(\eta)$$

(18)

The choice of contract

The focus of this section is firms’ choice of contract and how it relates to rotation rate. I first show that firms’ choice of contract can be categorized by a threshold $\eta^*_i$: firms with transaction cost $\eta$ beyond $\eta^*_i$ would choose vertical integration; ones with $\eta$ below $\eta^*_i$ would choose sourcing.

I conjecture then verify that in industry $i$, there exists a threshold $\eta^*_j$ that makes a firm indifferent between sourcing and vertical integration. According to the free entry condition equation 18, $\eta^*_j$ is determined by the following equation:

$$\frac{1}{2} \left( J_{i,VI}^{HH} + J_{i,VI}^{LL} \right) = \frac{1}{2} \left[ J_{i,SC}^{HH}(\eta^*_j) + J_{i,SC}^{LL}(\eta^*_j) \right]$$

One can show that $\eta^*_i$ is determined by

$$\tau \cdot \eta^*_i = \beta (1 - \delta) g (\rho_{j,HL}, \nu) \cdot (z_{j,HL} - z_{j,LL} - z_{i,HL} - z_{i,LL})$$

(19)
\[
\frac{\partial g(\rho_{HL}^j, \nu)}{\partial \rho_{HL}^j} > 0 \text{ and } \frac{\partial g(\rho_{HL}^j, \nu)}{\partial \nu} > 0
\]

The LHS of equation 19 is the cut-off firm’s transaction cost that it can save with vertical integration. The RHS is the hedgable part of mismatch risk it can hedge with sourcing, which depends on rematch probability, industry \( j \)’s rotation rate and its mismatch loss. Function \( g \) is defined in appendix; it is increasing in \( \rho_{HL}^j \) and decreasing in \( \nu \). The intuition of equation 19 is that the cut-off firm should find transaction cost exactly equal to the hedgable part of mismatch risk.

I denote \( \tilde{z}_i = z_{iHH}^i + z_{iLL}^i - z_{iHL}^i - z_{iLH}^i \) as industry \( i \)’s productivity loss induced by mismatch.

**Proposition 4.** The threshold level of transaction cost of industry \( i \) exists and is uniquely determined by

\[
\eta_i^* = \frac{\beta (1 - \delta) g(\rho_{HL}^j, \nu) \cdot \tilde{z}_i}{\tau}
\]

In proposition 5, I show that a firm’s choice of contract is categorized by the threshold.

**Proposition 5.** In industry \( i \), a firm would choose vertical integration if \( \eta > \eta_i^* \), sourcing if \( \eta < \eta_i^* \). The share of firms that choose vertical integration is \( 1 - F(\eta_i^*) \).

The intuition of the proposition is that firms with \( \eta \) higher than \( \eta_i^* \) would choose vertical integration because the transaction cost is too high and it wants to avoid the expensive transaction cost by internalizing the inter-firm cooperation; in contrast, firms with \( \eta \) below \( \eta_i^* \) would choose sourcing.

**Supermodularity, rotation rate, and vertical integration**

In the following part, I discuss the relationship between rotation rate \( \rho_{HL}^j \) and threshold \( \eta_i^* \) and hence the share of firms choosing vertical integration \( 1 - F(\eta_i^*) \). I interrogate how the relationship is affected by supermodular production function. In particular, I show that under some exclusion assumption, rotation rate \( \rho_{HL}^j \) is positively correlated with share of firms choosing vertical integration \( 1 - F(\eta_i^*) \) across industries if and only if the production function is supermodular.

Specifically, I denote \( \bar{z} = \frac{\sum_{i=1}^N \tilde{z}_i}{N} \) as the mean of the \( N \) industries’ productivity loss induced by mismatch.

Our goal is to test if \( \bar{Z} \) is positive; that is, whether supermodularity holds on average. We
can rewrite equation ?? as
\[
\eta_i^* \cdot \tau = \beta (1 - \delta) g (\rho_{iH}^L, \nu) \left[ \bar{z} + (\bar{z}_i - \bar{z}) \right]
\]
\[
\frac{\eta_i^* - \bar{\eta}}{\bar{\sigma}} = -\frac{\bar{\eta}}{\bar{\sigma}} + \frac{\beta (1 - \delta)}{\bar{\sigma} \tau} \cdot \bar{z} \cdot g (\rho_{jH}^L, \nu) + \frac{\beta (1 - \delta)}{\bar{\sigma} \tau} g (\rho_{jH}^L, \nu) (\bar{z}_i - \bar{z}) \quad (20)
\]

The above equations show that the threshold of industry \(i\) depends on rotation rate \(\rho_{iH}^L\), and an interactive term between rotation rate \(\rho_{jH}^L\) and productivity loss that deviates from average level \(\bar{z}_i - \bar{z}\).

Using the same method, I show that higher rotation rate of industry \(j\) increases mismatch risk for firms in industry \(j\). In the model, for simplicity, I assume that there is no rotation in industry \(i\). If we allow for rotation in both industry \(i\) and \(j\), there is a symmetric result: higher rotation rate of industry \(i\) increases mismatch risk for firms in industry \(i\). Similar to equation 20, we have the following equation:
\[
\frac{\eta_i^* - \bar{\eta}}{\bar{\sigma}} = -\frac{\bar{\eta}}{\bar{\sigma}} + \frac{\beta (1 - \delta)}{\bar{\sigma} \tau} \cdot \bar{z} \cdot g (\rho_{jH}^L, \nu) + \frac{\beta (1 - \delta)}{\bar{\sigma} \tau} g (\rho_{jH}^L, \nu) (\bar{z}_i - \bar{z}) \quad (21)
\]

As the last terms of equations are unobserved, I need to make the following exclusive assumption and treat the last terms as an error term in the empirical analysis.\(^{24}\)

**Assumption 1.** The cross industry variation in the supermodularity of production function is orthogonal to the cross industry variation in the rotation rate.

\[
E (\bar{z}_i - \bar{z} \mid \rho_j, \rho_i) = 0, \text{ for any } i \text{ and } j
\]

I present the the key result of this section in Proposition 6

**Proposition 6.** With the exclusive assumption 1, the share of firms choosing vertical integration \(1 - F (\eta_i^*)\) is positively correlated with rotation rates \(\rho_{iH}^L\) and \(\rho_{jH}^L\) if and only if the production function of industry \(j\) is supermodular.

Proposition 6 is a testable result given that both rotation rate and vertical integration can be measured in the data.

\(^{24}\)The instrumental variable (IV) approach can hardly exempt me from making this exclusion assumption, because any IV that is correlated with \(\rho_{jH}^L\) also correlates with \(\rho_{jH}^L (z_j - \bar{z})\) unless \(E (z_j - \bar{z} \mid \rho_j) = 0\). There’s little prior reason to defy this assumption: \(J_j - J\) depends on production function, while \(\rho_j\) should be related to technology progress or preference shift; they seem to driven by different factors. In the following empirical analysis, I impose that mismatch loss is weakly exogenous to rotation rate and transaction cost.
4.3 Empirical analysis

In the empirical study, I focus on backward vertical integration and sourcing of 3-digit NAICS industries. Data is described in Appendix C.25

I use proposition 6 to test supermodularity. I combine equation 20 and equation 21, which yield the following specification26

\[ VI_i = \alpha + \beta_1 \text{Rot}_i + \beta_2 \sum_j w_{i,j} \text{Rot}_j + \beta_3 \text{MSRF}_i + \epsilon_i \]

The dependent variable \( VI_i \) is the share of firms choosing vertical integration in industry \( i \). It corresponds to \( 1 - F(\eta^*_i) \) in the extended model I described in section 5.2. I proxy it with the ratio of industry value added to industry sales.27

The second term on the \( RHS \) \( \text{Rot}_i \) is the mean of industry \( i \)'s rotation rate, constructed in section 2.

The third term \( \sum_j w_{i,j} \text{Rot}_j \) is the weighted average of supplier industries’ rotation rates. Weight \( w_{i,j} \) is the share of intermediate goods industry \( i \) purchases from industry \( j \). In the model, for simplicity I have only two industries. In the data, there are many industries and each one is matched with multiple industries. My model implies that the choice of contract depends on rotation rate of the industry as well as the rotation rates of its linked industries.

\( \text{MSRF}_i \) is market share of the representative firm in industry \( i \), measured by share of top 50 firms in industry sales constructed by Census Bureau. \( \text{MSRF}_i \) proxies for industry \( i \)'s concentration, which is not explicitly modeled in my model. I include it for two reasons: First, it has been found to be important in the vertical integration literature—including it as explanatory variable avoids biased results. Second, there might be concern that rotation rate has information on industry’s competition intensity and, therefore the interpretation of my result can be blurred. As a common proxy for competition intensity, \( \text{MSRF}_i \) can help to address this potential concern.

The goal of the empirical analysis is to test whether \( \beta_1 \) and \( \beta_2 \) are negative as predicted by my model. The null hypothesis is \( \beta_1 = \beta_2 = 0 \).

As in the data, mean and standard deviation of industry rotation rate are correlated. I also consider a generalized method of moments (GMM) specification in which I use the standard deviation of industry rotation rate as an instrumental variable to increase estimation efficiency.

25 Backward vertical integration means that downstream producer buys up the upstream supplier.
26 I use a cross sectional regression instead of panel regression as I don’t have a panel of measure of vertical integration.
27 I use this approximation because I do not have firm level input-output data. See Acemoglu, Griffith, Aghion, and Zilibotti (2010); Atalay, Hortaçu, and Syverson (2014) for examples of measuring vertical integration using firm level data.
The empirical results are reported in Table 4. According to the results, industry rotation rate and supplier industries’ rotation rates both have a significant negative effect on vertical integration, which suggests that production function is supermodular.

Table 4: Measures of churning are negatively correlated with vertical integration across industries

\[ VI_i = \alpha + \beta_1 Rot_i + \beta_2 \sum_j w_{i,j} Rot_j + \beta_3 MSRF_i + \epsilon_i \]

<table>
<thead>
<tr>
<th>Specification</th>
<th>OLS Profit</th>
<th>GMM</th>
<th>OLS Profit Margin</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rot_i</td>
<td>-1.34*</td>
<td>-2.22***</td>
<td>-0.84</td>
<td>-1.02**</td>
</tr>
<tr>
<td></td>
<td>(0.75)</td>
<td>(0.78)</td>
<td>(0.52)</td>
<td>(0.51)</td>
</tr>
<tr>
<td>[ \sum_{j} w_{i,j} Rot_j ]</td>
<td>-3.62***</td>
<td>-3.03***</td>
<td>-1.81***</td>
<td>-1.50***</td>
</tr>
<tr>
<td></td>
<td>(0.65)</td>
<td>(0.73)</td>
<td>(0.45)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>MSRF_i</td>
<td>-0.16***</td>
<td>-0.16***</td>
<td>-0.15***</td>
<td>-0.13***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R^2</td>
<td>0.44</td>
<td>0.42</td>
<td>0.38</td>
<td>0.37</td>
</tr>
<tr>
<td>J-stat</td>
<td>5.66**</td>
<td>5.66**</td>
<td>5.66**</td>
<td>5.66**</td>
</tr>
<tr>
<td>Number of industries</td>
<td>57</td>
<td>57</td>
<td>57</td>
<td>57</td>
</tr>
</tbody>
</table>

Standard deviation of rotation rate is treated as IV.

J-stat is over-identification test statistics.

SE is in the parentheses.

1, 2, 3 asterisks denote significance at the ten, five, one percent level.

\( VI_j = \text{degree of vertical integration}, \frac{\text{Value added}_i}{\text{Sales}_j}, \text{BEA 2007 IO tables} \)

\( Rot_j = \text{mean of rotation rate}, \text{Compustat} \)

\( w_{i,j} = \text{share of intermediate goods from industry } j, \text{BEA 2007 IO tables} \)

\( MSRF_j = \text{share of top 50 firms in industry sales, Census Bureau} \)

Column (1) shows the estimation result of OLS using rotation rates based on ranking of profit as explanatory variables. \( \beta_1 \) and \( \beta_2 \) are negative and significant at 10% and 1% levels. If I include standard deviation of the rotation rate as an instrumental variable and adding an additional moment condition, as shown in column (2), \( \beta_1 \) and \( \beta_2 \) are still negative and both significant at the 1% level.

When I compute the industry rotation rates by ranking firms by profit margin, I get similar results. Column (3) shows the OLS result: both \( \beta_1 \) and \( \beta_2 \) are estimated to be negative. While \( \beta_2 \) is still significant at the 1% level, \( \beta_1 \) is not significant at the 10% level due to a large standard error. By adding an additional moment condition, as shown in column (4), \( \beta_1 \) and \( \beta_2 \) become
significant at 5% and 1% level.

To sum up, Table 4 shows that, when an industry has a higher rotation rate or its linked industries have higher rotation rates, the industry would have less vertical integration. The intuition is as follows: a higher churning in an industry or in its linked industries are associated with greater mismatch risk; firms use sourcing to hedge against mismatch risk and avoid vertical integration as the inter-firm cooperation relationship. This supports the assumption of supermodularity and the main mechanism of the model.

5 A quantitative model

In this section, I embed the model studied in section 4 into a general equilibrium, real business cycle (RBC) model. The RBC model includes a number of features that are absent from the DMP framework, notably accumulation of capital and concavity of utility and production functions, which are critical in quantitative macroeconomic analysis. The primary purpose of this analysis is to evaluate the quantitative contribution of rotation shock to the business cycle, comparing with the common shocks that have been intensively studied in the literature. The second purpose is to show robustness of the cyclical properties of labor market tightness ratio and unemployment to the inclusion of these neoclassical features. The last purpose is to ensure that the model is able to replicate the stylized cyclical behavior of macro aggregates, such as output, consumption and investment. In section 4.1, I describe the model and derive the free entry condition, which determines the equilibrium of the labor market. Then in section 5.2, I provide a set of analytical results characterizing some key properties of the model. In section 5.3 I estimate the model with Bayesian method and use numerical analysis to evaluate the importance of rotation shock.

5.1 Description of the quantitative model

I modify Andolfatto (1996) to include firm inter-connectivity and endogenous choice of inter-firm cooperation contract. With the model studied in section 4, this model considers several additional features: There is capital, and both utility and production functions are concave; Firms match with partners in frictional inter-firm matching market; Labor can move across labor markets.

The economy is populated by a continuum of households. There are two distinct labor
markets or sets of firms: $A$ and $B$. Production of final goods requires two intermediate inputs. Each set specializes in one intermediate goods. Firms from each set cooperate with partners from the other set: they produce final goods with intermediate goods then sell final goods in a competitive market. Representative household makes two decisions. First, it optimally allocate income to consumption and investment. Second, it sends its unemployed members to search for job vacancies in either labor market.

At the beginning of each period, firms post job vacancies to match with unemployed workers in either labor market. The matching process is frictional and the search is random. If successfully matched, the firm becomes a single firm since it does not have a partner yet. A new single firm randomly draws a transaction cost $\eta \sim N(\bar{\eta}, \sigma^2)$. Observing the transaction cost, the firm chooses its cooperation contract to be either vertical integration (VI) or sourcing (SC). Then the firm randomly draws a type—either $H$ or $L$ with probability 50 percent and 50 percent. I assume that single firms can produce final goods, but with low productivity.\footnote{One interpretation is single firm needs to produce both intermediate goods, one of which is not their comparative advantage. The other interpretation is that without inter-firm cooperation, single firm can only sell intermediate goods to a commodity market which is very competitive and yields a very low price.}

To have an inter-firm cooperation, single firms search for partners in frictional inter-firm matching market. As inter-firm search incurs no cost, every firm chooses to enter the inter-firm matching market. Search is directed, hence single firms are then divided into submarkets depending on their own types, their target partner’s type, and the choice of cooperation contract. For example, the $H$ type firms in set $A$ who wants to vertically integrate with an $L$ type partner would be located into the same submarket with the $L$ type firms in set $B$ who want to vertically integrate with an $H$ type partner.

In theory, there can be eight submarkets: $(H - H) - VI$, $(H - L) - VI$, $(L - H) - VI$, $(L - L) - VI$, $(H - H) - SC$, $(H - L) - SC$, $(L - H) - SC$, $(L - L) - SC$. For example, the $(H - L) - VI$ sub-market accommodates the $H$ type firms in set $A$ who are looking for an $L$ type partner and the $L$ type firms in set $B$ who are looking for an $H$ type partner, both sides choose vertical integration as the cooperation contract. In the model, I show that under certain conditions, firms only search for same type partner; that is, there are only four submarkets operating in the Nash equilibrium: $(H - H) - VI$, $(L - L) - VI$, $(H - H) - SC$, and $(L - L) - SC$ markets. The matching technology is similar with the one in the labor market—the probability of finding partner depends on number of single firms from each set in each sub-market. If successfully matched, the firm becomes a cooperative firm since it has a partner to cooperate with. If choosing SC, a firm can choose to sever its cooperation with probability $\nu$, but needs to pay transaction cost $\eta$ in every period. If choosing VI, the transaction cost is waived, yet the firm cannot separate and rematch.
Production takes place after the matching processes. In each set there are twelve categories of firms: $H-VI$, $L-VI$, $H-SC$, $L-SC$, $HH-VI$, $HL-VI$, $LL-VI$, $HH-SC$, $HL-SC$, $LH-SC$, $LL-SC$; the first four are single firms and the remains are cooperative firms; each category is specified by firm’s type and the cooperation contract. Production is organized by representative firms; they organize firms into production departments indexed by categories. They then rent capital from households and optimally allocate it to each production department.

After production, all firms rotate type according to a Markov switching process. Rotation rates are governed by the time varying and symmetric Markov switching matrices $\Pi_{A,t} = \begin{bmatrix} \rho_{HH}^{A,t} & \rho_{HL}^{A,t} \\ \rho_{LH}^{A,t} & \rho_{LL}^{A,t} \end{bmatrix}$ and $\Pi_{B,t} = \begin{bmatrix} \rho_{HH}^{B,t} & \rho_{HL}^{B,t} \\ \rho_{LH}^{B,t} & \rho_{LL}^{B,t} \end{bmatrix}$. For each firm-to-firm match, the two Markov switching processes are independent with each other. At the end of each period, firms are destroyed with fixed rate $\delta$.

5.1.1 Notations of the labor market and the inter-firm matching market

The notations of the labor market are similar to those used in the simple model.

There are twelve types of firms, or employments. The measure of firms in set $i$ with type $j$ choosing cooperation contract $l$ is denoted by $n^l_{i,j}$.

As in the simple model, tightness ratio $\theta_{i,t}$ is the ratio of the number of vacancies posted by firms in set $i$ to the number of workers looking for these jobs:

$$\theta_{i,t} = \frac{v_{i,t}}{u_{i,t}}$$

Job finding rate $\mu_i(\theta_{i,t})$ measures the probability of an unemployed worker matching with a vacancy. The vacancy filling rate $f_i(\theta_{i,t})$ is the probability for a vacancy matching with an unemployed worker.

Aggregate unemployment is determined by job creation and job destruction in the two sets.

$$u_{t+1} = u_t - \left(\mu_{A,t}u_{A,t} + \mu_{B,t}u_{B,t}\right) + \delta (1 - u_t)$$

with

$$u_t = u_{A,t} + u_{B,t}$$

In the above equation, $u_t$ is aggregate unemployment. Notice that unlike the simple model, in this model unemployed workers can freely move across labor markets. Thus $u_{A,t}$ and $u_{B,t}$ are

---

29 As in DMP models, the production unit is firm-employment match, hence I use firm and employment interchangeably.
endogenously determined by states of the economy.

Similar to the labor market tightness ratio, the tightness ratios of inter-firm matching market is the ratio of the number of single firms in two sets. In the Nash equilibrium, firms only match with firms of same type. Thus there are two endogenously segmented markets. In sum, the model has two sets and two submarkets which give rise to four tightness ratios.

\[
\tilde{\theta}_{A,j,k}^{i} = \frac{n_{A,j,k}^i}{n_{B,j,k}^i}, \quad j,k \in \{H,L\}, \ l \in \{VI,SC\}
\]

\[
\tilde{\theta}_{B,j,k}^{i} = \frac{n_{B,j,k}^i}{n_{A,j,k}^i}
\]

where \(n_{i,l,t}^j\) is the measure of single firms of type \(j\) in set \(i\) and plan to use inter-firm cooperation contract \(l\). I use tilde to indicate the inter-firm tightness ratios.

Inter-firm matching rate is the probability of a single firm matching with a partner.

\[
p_{A,j,k}^{i} = \frac{\tilde{M}(n_{A,j,k}^i,n_{B,j,k}^i)}{n_{A,j,k}^i}, \quad j,k \in \{H,L\}, \ l \in \{VI,SC\}
\]

\[
p_{B,j,k}^{i} = \frac{\tilde{M}(n_{B,j,k}^i,n_{A,j,k}^i)}{n_{B,j,k}^i}
\]

where \(\tilde{M}(n_{A,j,k}^i,n_{B,j,k}^i)\) is matching functions for inter-firm matching which is assumed to be homogeneous of degree one. As the result, set \(i\)’s cooperation matching rate \(p_{i,j,k}^{i}\) is an increasing function of inter-firm tightness ratio \(\tilde{\theta}_{i,j,k}^{i}\).

Furthermore, I impose both the worker-firm matching function and the inter-firm matching function \(\tilde{M}\) to be Cobb-Douglas, that is,

\[
M(u_{A,j,k},v_{A,j,k}) = \phi_A(u_{A,j,k})^{1-\alpha_1}(v_{A,j,k})^{\alpha_1}
\]

\[
M(u_{B,j,k},v_{B,j,k}) = \phi_B(u_{B,j,k})^{1-\alpha_1}(v_{B,j,k})^{\alpha_1}
\]

and

\[
\tilde{M}(n_{A,j,k}^i,n_{B,j,k}^i) = \psi(n_{A,j,k}^i)^{1-\alpha_2}(n_{B,j,k}^i)^{\alpha_2}, \ j,k \in \{H,L\}, \ l \in \{VI,SC\}
\]

where I restrict the labor market matching elasticity to be same in the two labor markets. Moreover, the inter-firm matching functions are identical across submarkets.
5.1.2 Household

The representative household derives utility from consumption, while working incurs disutility $\xi_n$. A household’s problem is:

$$\max E_t \left\{ \sum \beta^t \xi_{c,t} \left[ \log (C_t - \Psi_c \bar{C}_{t-1}) - \xi_{n,t} \sum_i \sum_j \sum_l n_{i,j,l}^t \right] \right\}$$

(22)

with

$$i \in \{ A, B \}$$

$$j \in \{ H, L, HH, HL, LH, LL \}$$

$$l \in \{ VI, SC \}$$

where $\Psi_c$ is an external habit parameter, $C_t$ denotes consumption, $\bar{C}_{t-1}$ is the economy’s average consumption in period $t - 1$. As defined in the previous subsection, $n_{i,j,l}^t$ is the measure of employment in production department indexed by type $j$ in set $i$, and has (or is willing to have) cooperation contract $l$. There are two exogenous preference shocks: shock to discount rate $\xi_{c,t}$ which changes household’s evaluation of future utility and shock to disutility of labor $\xi_{n,t}$.

The household’s problem is subject to three classes of flow motions:

1. The transition rules of unemployment and employment

$$u_{t+1} = u_t - (\mu_{A,t} u_{A,t} + \mu_{B,t} u_{B,t}) + \delta (1 - u_t)$$

(23)

$$\begin{bmatrix}
  n_{A,VI,t+1}^H \\
  \vdots \\
  n_{A,SC,t+1}^L \\
\end{bmatrix}
= \Phi \left( \hat{\theta}_{A,t}, \Pi_t, \Omega_t \right) \begin{bmatrix}
  n_{A,VI,t}^H \\
  \vdots \\
  n_{A,SC,t}^L \\
\end{bmatrix}
+ \Xi \left( \theta_{A,t}, \Omega_t \right) u_{A,t}$$

(24)

$$\begin{bmatrix}
  n_{B,VI,t+1}^H \\
  \vdots \\
  n_{B,SC,t+1}^L \\
\end{bmatrix}
= \Phi \left( \hat{\theta}_{B,t}, \Pi_t, \Omega_t \right) \begin{bmatrix}
  n_{B,VI,t}^H \\
  \vdots \\
  n_{B,SC,t}^L \\
\end{bmatrix}
+ \Xi \left( \theta_{B,t}, \Omega_t \right) u_{B,t}$$

(25)

Equation 23 is the flow motion of unemployment. As unemployed worker can freely move across labor markets, $u_{A,t}$ and $u_{B,t}$ are optimally chosen by the representative household given the state of the economy.

Equations 24 and 25 are flow motions for each type of employment. The detail of equations
24 and 25 are explained in appendix A. In the equation, Φ and Ξ are transition matrices whose elements are determined by vectors of cooperation tightness ratios \( \hat{\theta}_{A,t} = [\hat{\theta}^H_{A,V_1,t}, \hat{\theta}^L_{A,V_1,t}, \hat{\theta}^H_{A,SC,t}, \hat{\theta}^L_{A,SC,t}] \), \( \hat{\theta}_{B,t} = [\hat{\theta}^H_{B,V_1,t}, \hat{\theta}^L_{B,V_1,t}, \hat{\theta}^H_{B,SC,t}, \hat{\theta}^L_{B,SC,t}] \), Markov switching matrices \( \Pi_t = [\Pi_{A,t}, \Pi_{B,t}] \), and labor market tightness ratios \( \theta_{A,t} \) and \( \theta_{B,t} \), and lastly a vector of exogenous states \( \Omega_t \).

(2) The budget constraint

\[
C_t + T_t + I_t = \int \sum_i \sum_j \sum_l [w_{i,j,t}(\eta) \cdot n_{i,j,t}(\eta)] d\eta + z \cdot \sum_i u_{i,t} + r_{k,t}K_t + D_t
\]

(26)

The representative household’s income is composed of wage income \( \sum_i \sum_j \sum_l [w_{i,j,t}(\eta) \cdot n_{i,j,t}(\eta)] d\eta \), the unemployment insurance \( z \cdot u_{i,t} \), capital rent \( r_{k,t}K_t \), and dividend \( D_t \). Household allocates income to consumption \( C_t \), investment \( I_t \), and receives lump-sum tax \( T_t \).

(3) The representative household invests to accumulate capital

\[
K_{t+1} = (1 - d)K_t + \left[ 1 - S \left( \xi_{t,t} \left[ \frac{I_t}{I_{t-1}} \right] \right) \right] I_t
\]

(27)

\( I_{t-1} \) is investment in the previous period, and function \( S \) models investment adjustment cost which equals zero on steady state. Investment is subject to shock to marginal efficiency of investment (MEI) \( \xi_{t,t} \) which affects the efficiency of the transformation of investment to productive capital.

5.1.3 Representative firm

The representative firm has twelve production departments indexed by type \( j \) and cooperation contract \( l \). The firm’s endogenous state variables consists of measures of current employment of each production department. Representative firms make four decisions: (1) rent capital from household then optimally allocate to each department, (2) post vacancy to match with worker,\(^{30}\) (3) send single firms to search for partners in the inter-firm matching market. For the third decision, I impose that in the Nash equilibrium, single firms only search for same type partners.

\(^{30}\)Noticing that firm-worker match type is revealed only after the match is formed, firms post one type of vacancy only.
In set \( i \), the representative firm’s value function takes the following form:

\[
J_i\left(\left\{n_{A,l}^i(\eta)\right\}, \left\{n_{B,l}^i(\eta)\right\}, \Omega\right) = \max_{\nu_i, \{k_i^j\}} \left\{ \sum_j z_i^j \left[ \sum_l \left[ \left( k_{i,l}^j(\eta) \right)^{\alpha} \left( n_{i,l}^j(\eta) \right)^{1-\alpha} \right] \right] \right\} \tag{28}
- \sum_j \int_{w_i} w_{i,SCI}^j(\eta) \cdot n_{i,SCI}^j(\eta) + r_k k_{i,SCI}^j(\eta) + \eta \right] d\eta \right] \right\} \tag{Sourcing}
- \sum_j \int_{w_i} w_{i,VII}^j(\eta) \cdot n_{i,VII}^j(\eta) + r_k k_{i,VII}^j(\eta) \right] d\eta \right] \right\} \tag{Vertical integration}
- \nu_i \cdot \chi + \beta E \left[ \frac{\lambda'}{\lambda} J_i\left(\left\{n_{A,l}^i(\eta)\right\}, \left\{n_{B,l}^i(\eta)\right\}, \Omega'\right) \right] \]

with

\[
i \in \{A, B\} \\
j \in \{H, L, HH, HL, LH, LL\} \\
l \in \{VI, SCI\}

subject to the transition rules of employment described by equations 24 and 25 and also the constraint from the matching technology in the labor market

\[
\nu_i = u_i \cdot \theta_i \tag{29}
\]

Firm’s production function is Cobb-Douglas. TFP is determined by two components: (1) the firm’s idiosyncratic productivity \( z_i^j \) which depends on type \( j \); (2) aggregate TFP \( x \).\(^{31}\) Firms need to pay the cost on labor, capital, job vacancy. In addition, for firms who use SC as the inter-firm cooperation contract, they need to pay transaction cost \( \eta \). I use \( \Xi \) to denote the set of firms who use SC. Naturally, \( \Xi^{C} \) contains the firms using VI, who do not pay the transaction cost. Wage \( w_{i,l}^j(\eta) \) is determined by Nash bargaining, which will be described in the next subsection. Labor market tightness ratio \( \theta_i \) and capital rental rate \( r_k \) are equilibrium outcomes and are taken by firms as given. It is worthnoting that in search and matching models, employment is pre-determined, which is similar to capital stock in most models; and vacancies are similar to investment. Firms adjust future employment by posting vacancies in the labor market, while

\(^{31}\)Aggregate TFP contains a trend, hence vacancy posting cost \( \chi \) and unemployment insurance \( z \) also grows with the trend of aggregate TFP in order to keep the tightness ratio and unemployment stationary.
they cannot adjust contemporary employment.

5.1.4 Nash Bargaining and wage determination

Following the DMP search and matching model, wage is determined by Nash bargaining. In each period firms and workers set wages to solve the following Nash bargaining problem:

\[
    w_{ij}(\eta) = \arg \max_w \left( \frac{\frac{\partial V}{\partial n_{ij}(\eta)} - \frac{\partial V}{\partial u_i}}{\lambda} \right)^{1-\tau} \left( \frac{\partial J_i}{\partial n_{ij}(\eta)} \right)^\tau
\]

In the above equation, time subscripts are omitted. \(\tau\) is a parameter of bargaining share that measures the firm’s bargaining power. \(\frac{\partial V}{\partial n_{ij}(\eta)}\) and \(\frac{\partial V}{\partial u_i}\) are the representative household’s marginal value of employment and unemployment respectively. \(\frac{\partial J_i}{\partial n_{ij}(\eta)}\) is firm’s marginal value of employment. The detail of these variables can be found in appendix B. As the result of the Nash bargaining, the total surplus of each match is divided by the firm and worker with shares \(\tau\) and \(1 - \tau\).

It can be shown that the wage has the following close form solution

\[
    w_{i,SC}(\eta) = \tau \left( z + \frac{\xi_n}{\lambda} \right) + (1 - \tau) \left[ xz_i^j \left( \frac{k_{i,SC}(\eta)}{n_{i,SC}(\eta)} \right)^{\alpha} - \eta + \chi \theta_i \right]
\]

and

\[
    w_{i,VI} = \tau \left( z + \frac{\xi_n}{\lambda} \right) + (1 - \tau) \left[ xz_i^j \left( \frac{k_{i,VI}(\eta)}{n_{i,VI}(\eta)} \right)^{\alpha} + \chi \theta_i \right]
\]

Wage is the weighted average of three components: (1) the worker’s marginal product of labor \((MPL)\) \(xz_i^j \left( \frac{k_{i}(\eta)}{n_{i}(\eta)} \right)^{\alpha}\); (2) the flow value of unemployment, or the opportunity cost of employment, measured by unemployment insurance \(z\) plus marginal rate of substitution \((MRS)\) of consumption for leisure, which is disutility of labor \(\xi_n\) adjusted by marginal utility; and (3) \(\chi \theta_i\), the competition pressure from the other firms who are actively hiring workers. In the case of SC, there is a transaction cost component in the wage. It deviates from the models with a competitive labor market in which wage always equals \(MPL\) and \(MRS\).

Following the canonical DMP search and matching models, I define the total surplus of a firm-worker match as

\[
    TS_{ij}(\eta) = \lambda \frac{\partial J_i}{\partial n_{ij}(\eta)} + \frac{\partial V}{\partial n_{ij}(\eta)} - \frac{\partial V}{\partial u_i}
\]
And with Nash Bargaining we have

\[
\lambda \frac{\partial J_i}{\partial n_{i,l}^j(\eta)} = \tau T S_{{i,l}}^j(\eta)
\]

\[
\frac{\partial V}{\partial n_{i,l}^j(\eta)} - \frac{\partial V}{\partial u_i} = (1 - \tau) T S_{{i,l}}^j(\eta)
\]

5.1.5 The choice of contract and the free entry condition

In this subsection, I show the determination of cooperation contract and the free entry condition.

A firm’s first order condition with regard to the measure of vacancies \( v_i \) gives the free entry condition of the model

\[
\chi_f(\theta_i) = \mathbb{E} \left\{ \frac{\lambda}{\hat{\lambda}} \max \left[ \frac{\partial J_i}{\partial n_{i,SC}^H(\eta)} + \frac{\partial J_i}{\partial n_{i,SC}^L(\eta)} + \frac{\partial J_i}{\partial n_{i,VI}^H} + \frac{\partial J_i}{\partial n_{i,VI}^L} }{2}, \frac{\partial J_i}{\partial n_{i,SC}^H(\eta)} + \frac{\partial J_i}{\partial n_{i,SC}^L(\eta)} + \frac{\partial J_i}{\partial n_{i,VI}^H} + \frac{\partial J_i}{\partial n_{i,VI}^L} } } \right\}
\]

(32)

where \( J_{i,n_{i,l}} \) is a firm’s marginal value of employment with type \( j \), cooperation contract \( l \), in set \( i \).

A single firm with transaction cost \( \eta \) would choose VI if and only if

\[
\frac{\partial J_i}{\partial n_{i,SC}^H(\eta)} + \frac{\partial J_i}{\partial n_{i,SC}^L(\eta)} < \frac{\partial J_i}{\partial n_{i,VI}^H} + \frac{\partial J_i}{\partial n_{i,VI}^L}
\]

or equivalently

\[
T S_{i,SC}^H(\eta) + T S_{i,SC}^L(\eta) < T S_{i,VI}^H + T S_{i,VI}^L
\]

We can categorize firms’ choice of contract with the following proposition

**Proposition 7.** In industry \( i \) and in period \( t \), a single firm would choose vertical integration if \( \eta \geq \eta_i^* \), sourcing if \( \eta < \eta_i^* \). The threshold is determined by

\[
\eta_i^* = \beta (1 - \delta) (\rho_{A,t} + \rho_{B,t}) E_t [\tilde{T} S_{i,VI,t+1} - (1 - \nu) \tilde{T} S_{i,SC,t+1}(\eta_i^*)]
\]

with

\[
\tilde{T} S_{i,SC} \triangleq T S_{i,SC}^{HH}(\eta) + T S_{i,SC}^{LL}(\eta) - T S_{i,SC}^{HL}(\eta) - T S_{i,SC}^{LH}(\eta)
\]

\[
\tilde{T} S_{i,VI} \triangleq T S_{i,VI}^{HH} + T S_{i,VI}^{LL} - T S_{i,VI}^{LH} - T S_{i,VI}^{HL}
\]

In the above proposition, as I proved in the appendix, both \( \tilde{T} S_{i,SC} \) and \( \tilde{T} S_{i,VI} \) are independent
with \( \eta \). With proposition 7, we can rewrite free entry condition equation 32 as

\[
\chi = \tau \beta f(\theta_{i,t}) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ \int_{-\infty}^{\eta^*_t} \frac{T^{S_{i,SC,t+1}}(\eta_t) + T^{S_{i,VL,t+1}}(\eta_t)}{2} dF(\eta) \right] + \left( 1 - F(\eta^*_t) \right) \Delta \hat{T} S_{i,t+1} \right\}
\]

(33)

In the appendix C, I prove the following result, which substantially simplifies the computation of the model.

**Proposition 8.** The free entry condition of set \( i \) is

\[
\chi = \tau \beta f(\theta_{i,t}) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ \int_{-\infty}^{\eta^*_t} \frac{T^{S_{i,VL,t+1}}(\eta_t) + T^{S_{i,VL,t+1}}(\eta_t)}{2} + F(\eta^*_t) \Delta \hat{T} S_{i,t+1} \right] \right\}
\]

with

\[
\Delta \hat{T} S_{i,t} = -\hat{\eta}_t + \beta (1 - \delta) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ \Delta \hat{T} S_{i,t+1} - (\rho_{A,t} + \rho_{B,t}) (1 - \nu) \hat{T} S_{i,SC,t+1} + (\rho_{A,t} + \rho_{B,t}) \hat{T} S_{i,VL,t+1} \right] \right\}
\]

\[
\hat{\eta}_t \triangleq \int_{-\infty}^{\eta^*_t} \eta dF(\eta) \frac{F(\eta^*_t)}{F(\eta^*_t)}
\]

**5.1.6 Economy resource constraint and shocks**

I close the model by presenting the economic resource constraint and the processes of shocks.

The economy can allocate output—the aggregate of all departments’ output net of transaction cost—to consumption, investment and vacancies. Thus the economy resource constraint is:

\[
C + I + \chi \sum_i v_i = x \sum_i \sum_j \sum_l z_{i,j}^l \left( k_{i,t}^l(\eta) \right)^\alpha \left( n_{i,t}^l(\eta) \right)^{1-\alpha} - \sum_i \sum_j \left( \int_{-\infty}^{\eta} \eta d\eta \right)
\]

I include exogenous shocks to six variables in the model. Besides shocks to the two rotation rates \( \rho_{A,t}^{HH} \) and \( \rho_{B,t}^{HH} \), I consider shock to aggregate TFP, shock to investment adjustment cost \( \xi_I \), shock to inter-temporal preference \( \xi_C \) and shock to labor disutility \( \xi_L \).

\[
\log(x_t) = \mu^* + \rho_A \log(x_{t-1}) + \sigma_x \epsilon_{x,t}, \quad \epsilon_{x,t} \sim N(0,1)
\]
The shocks are governed by $AR(1)$ processes that are independent to each other:

$$
\log(\xi_{w,t}) = \rho \log(\xi_{w,t-1}) + \sigma_w \epsilon_{w,t}, \quad \epsilon_{w,t} \sim N(0,1)
$$

$$
w = x, I, C, L
$$

The values of all shock variables are observable to economic agents at beginning of each period. To save space, I stack them into a vector of states

$$
\Omega = [x, \xi_I, \Pi_A, \Pi_B, \xi_C, \xi_L]
$$

$\Pi_A$ and $\Pi_B$ are symmetric Markov switching matrices pinned down by rotation rates $\rho_A^{HL}$ and $\rho_B^{HL}$

### 5.2 The sufficient condition for PAM

In the models studied in the previous sections, without inter-firm search friction, PAM is always the Nash equilibrium as long as the production function is monotone in partner’s type. In this model with inter-firm search friction, however, monotonicity is not sufficient for PAM: there exists a possibility that a large amount of of $L$ type single firms are willing to chase for just a few $H$ type single firms, while the few $H$ type firm might be happy to match with those $L$ type if the matching probability is high enough. In proposition 9, I show the sufficient condition for PAM to be the Nash equilibrium.

**Proposition 9.** The positive assortative matching is Nash equilibrium if

$$
\left(\frac{T_{S_{A,l}}^{HH} - T_{S_{A,l}}^{H}}{T_{S_{A,l}}^{HL} - T_{S_{A,l}}^{H}}\right)^{\frac{1}{2}} \times \left(\frac{T_{S_{B,l}}^{H} - T_{S_{B,l}}^{L}}{T_{S_{B,l}}^{HL} - T_{S_{B,l}}^{L}}\right)^{\frac{1}{2}} > 1
$$

and

$$
\left(\frac{T_{S_{A,l}}^{HH} - T_{S_{A,l}}^{H}}{T_{S_{A,l}}^{HL} - T_{S_{A,l}}^{H}}\right)^{\frac{1}{2}} \times \left(\frac{T_{S_{B,l}}^{H} - T_{S_{B,l}}^{L}}{T_{S_{B,l}}^{HL} - T_{S_{B,l}}^{L}}\right)^{\frac{1}{2}} > 1
$$

The proof of proposition 9 can be found in appendix C. The intuition of the proof is to find the condition under which the $H$ type single firms have no incentive to search for $L$ type partner. Specifically, I first impose positive assortative matching so that inter-firm matching

\footnote{In another version, I specified the TFP process as a random walk process with drift as in Fernández-Villaverde and Rubio-Ramírez (2007) and gets similar result in the variance decomposition.}

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market is endogenously segmented to two sub-markets. Then I identify the sufficient condition under which no firm would like to deviate from the Nash equilibrium with positive assortative matching.

When solving and estimating the model, I take the same strategy as in the proof of proposition 9: I first impose positive assortative matching to be the Nash equilibrium, then verify that the inequality conditions depicted in proposition 9 hold.

5.3 Estimation of the quantitative model

In this subsection, I estimate the model described above and quantitatively assess the effect of rotation shock. I first describe the data and estimation strategy, then report the estimation result. With the estimation result, I conduct impulse response analysis to study the effect of rotation shock on the labor market variables and other key macro variables. Then I study the role of firm inter-connectivity and supermodularity with two counterfactual experiments, one is to simulate a model with no firm inter-connectivity (canonical DMP model), the other one has firm inter-connectivity but with no supermodularity. As shocks to rotation rate induces endogenous movement in the measured TFP, I compare the shocks to rotation rate and TFP shocks and comment on how my model speaks to the Shimer’s puzzle (Shimer (2005)). Lastly, I use variance decomposition to evaluate the importance of rotation shock relative to other shocks.

5.3.1 Data and methodology

Data

The sample period is 1969Q1 to 2013Q4. The estimation uses seven variables: rotation rates of sets $A$ and $B$, real interest rate, aggregate unemployment rate, aggregate job opening rate, growth rates of real investment and real consumption per capita, growth rate of per hour real wage. Data sources are described in Appendix D. Keeping the same notation as in the description of the model above, and writing $\Delta$ to indicate first differences, the full vector of observables is

$$\left[ \rho_{A,t}^{HL}, \rho_{B,t}^{HL}, R_t, u_t, v_t, \Delta \log (I_t), \Delta \log (C_t), \Delta \log (w_t) \right]$$

The model counterparts of the observables follow directly from the description of the model.
Methodology

I estimate the model using Bayesian method. I first solve and re-scale the model then log-linearize the non-linear model around a deterministic steady state and write the linearized equilibrium conditions in a state-space form. The resulting linear rational expectations model can then be solved by methods such as in Sims (2002). Define a vector of model variables $X_t$, and a data vector of observable variables $Z_t$. The state-space representation of the model can then be written as

$$
\begin{align*}
X_t &= \Gamma X_{t-1} + \Psi \epsilon_t \\
Z_t &= \Phi X_t + \eta_t
\end{align*}
$$

where $\Gamma$ and $\Psi$ are coefficient matrices, the elements of which are typically non-linear functions of the structural parameters, and $\Phi$ is a selection matrix that maps the model variables to the observables. The innovations of the shocks are collected by $\epsilon_t$. $\eta_t$ is vector of observation errors.

The use of eight series of observables requires the inclusion of at least eight independent sources of variation. I consider six independent shocks: aggregate TFP shock, two set-specific rotation shocks, marginal efficiency of investment (MEI) shock, discount rate shock, and labor disutility shock. As the number of observable is larger than the number of shocks, the model cannot be identified. So I also include observation errors for three observable, $\rho_{HL}^{A,i}, \rho_{HL}^{B,i}$ and $\nu_t$, as additional disturbances. In the estimation, I made two restrictions. First, the production is monotone in firm’s type, that is, $z_{i}^{Hj} > z_{i}^{Lj}, z_{i}^{jH} > z_{i}^{jL}, i = A, B, j = H, L$. Second, the total surplus is positive for any firm-worker match, so that no firm-worker match wants a separation.

In the implementation of the Bayesian estimation procedure, I use the Kalman-filter to evaluate the likelihood function of the observable. I then combine the likelihood function with the prior distribution of the model’s parameters to obtain the posterior distribution. I then evaluate the posterior distribution using the random-walk Metropolis-Hasting algorithm. Further details on the computational procedure can be found in An and Schorfheide (2007).

5.3.2 Calibration and prior

Calibration

Firstly, I calibrate some parameters based on the typical values used in calibration studies. Inter-firm matching is assumed to be symmetric; that is, given the two sectors have the same measure
Table 5: Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Calibration</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm-firm match elasticity</td>
<td>$\alpha_2$</td>
<td>0.5</td>
<td>Inter-sector Symm.</td>
</tr>
<tr>
<td>Max TFP</td>
<td>$z_{HH}$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Ratio of productivities</td>
<td>$z_{LL}/z_{HH}$</td>
<td>0.44</td>
<td>Syverson (2004)</td>
</tr>
<tr>
<td>Vacancy cost</td>
<td>$\chi$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Unemployment insurance</td>
<td>$z$</td>
<td>0.25</td>
<td>Department of Labor</td>
</tr>
<tr>
<td>Exog separation rate</td>
<td>$\delta_1$</td>
<td>0.05</td>
<td>Shimer (2012)</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.99</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>0.34</td>
<td>Capital income share in the US</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
<td>10% annual depreciation rate</td>
</tr>
</tbody>
</table>

of single firms, the inter-firm matching probability is same for the two sectors. Therefore inter-firm matching elasticity $\alpha_0$ is set to 0.5. I normalize the maximum TFP $z_{HH}$ as 1. As the empirical rotation rate is low on steady state (5% annually), the majority of firms fall into the $HH$ and $LL$ category. Hence I set $z_{LL}$ to 0.44, which targets to inter-quartile ratio of productivity for manufacturing firms in U.S., as documented in Syverson (2011).\footnote{It also implies that the inter-quartile ratio of the wage distribution is 0.62 at the posterior mode.} Vacancy posting cost $\chi$ is normalized as 1. Following Shimer (2012), I set the exogenous separation rate $\delta_1$ as 0.05. Discount rate $\beta$ is set to 0.99, as in standard models. Capital share $\alpha$ and depreciation rate of capital $\delta$ are set to 0.34 and 0.025. The calibrations are summarized in Table 5.3.2.

Priors for the common parameters of DSGE models

The other parameters are estimated using Bayesian method. For parameters that are present in common DSGE studies, I keep priors close to previous work (e.g. Justiniano, Primiceri, and Tambalotti (2011)). In particular, for investment adjustment cost, I choose Gamma distributions with a mean of 4 and a standard deviation of 2. Prior for habit persistence is a distribution with a mean 0.5 and standard deviation 0.1. Prior for trend productivity growth rate is a Normal distribution with mean 0.004 and standard deviation 0.001.
I follow the literature in choosing rather diffuse priors for the structural shock processes. The priors for the autocorrelation parameters are Beta distributions with mean 0.5 and standard deviation 0.1. The priors for the standard deviations of shocks are Inverse Gamma distributions: priors for standard deviations of shocks have mean 0.01 and standard deviation 0.002.

Following Ilut and Schneider (2014), the priors for the standard deviation of observables are set in the following way: for an observable $W$ with unconditional standard deviation $\sigma_W$, the prior for the standard deviation of measurement error on $W$ is an Inverse Gamma distribution with mean $0.1\sigma_W$ and standard deviation $0.4\sigma_W$. Therefore, at the prior mean, measurement error would explain 1% fluctuation in $W$; at one standard deviation, it would explain 16% fluctuations in $W$. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^H_A$</td>
<td>$z^H_A / z^{HH}_A$, $z^L_A / z^{LL}_A$</td>
<td>beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma^H_B$</td>
<td>$z^H_B / z^{HH}_B$, $z^L_B / z^{LL}_B$</td>
<td>beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma^{HL}_A$, $\gamma^{HL}_B$</td>
<td>$z^H_A / z^{HH}_A$, $z^L_A / z^{HH}_A$, $z^H_B / z^{HH}_B$, $z^L_B / z^{HH}_B$</td>
<td>beta</td>
<td>0.6</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Separation probability for SC</td>
<td>beta</td>
<td>0.25</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>Mean of transaction cost</td>
<td>normal</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Std. of transaction cost</td>
<td>inv gamma</td>
<td>2.0</td>
</tr>
<tr>
<td>$\phi_A$</td>
<td>A’s matching efficiency</td>
<td>beta</td>
<td>0.7</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>B’s matching efficiency</td>
<td>beta</td>
<td>0.7</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inter-firm ME</td>
<td>beta</td>
<td>0.7</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Bargaining share of firm</td>
<td>beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\xi_l / \lambda$</td>
<td>Mean of labor disutility</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\psi_c$</td>
<td>Habit persistence</td>
<td>beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Investment adj. cost</td>
<td>normal</td>
<td>4</td>
</tr>
<tr>
<td>$100 \times \mu^*$</td>
<td>Trend TFP growth</td>
<td>normal</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Table 7: Estimation result II

<table>
<thead>
<tr>
<th>Para Description</th>
<th>Prior Type</th>
<th>Mean</th>
<th>Std</th>
<th>Mode</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 × ( r_\text{ot}_A ) Mean of Rot in A</td>
<td>beta</td>
<td>1</td>
<td>0.1</td>
<td>1.24</td>
<td>[1.12,1.42]</td>
</tr>
<tr>
<td>100 × ( r_\text{ot}_B ) Mean of Rot in B</td>
<td>beta</td>
<td>1</td>
<td>0.1</td>
<td>1.32</td>
<td>[1.29,1.52]</td>
</tr>
<tr>
<td>100 × ( \sigma_{\text{rot}_A} ) Std of shock to Rot in A</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>0.40</td>
<td>[0.37,0.43]</td>
</tr>
<tr>
<td>100 × ( \sigma_{\text{rot}_B} ) Std of shock to Rot in B</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>0.38</td>
<td>[0.35,0.40]</td>
</tr>
<tr>
<td>100 × ( \sigma_z ) Std of aggregate TFP</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>0.71</td>
<td>[0.68,0.75]</td>
</tr>
<tr>
<td>100 × ( \sigma_{\xi_c} ) Std of discount rate shock</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>1.97</td>
<td>[1.81,2.14]</td>
</tr>
<tr>
<td>100 × ( \sigma_{\xi_i} ) Std of investment shock</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>0.88</td>
<td>[0.78,0.98]</td>
</tr>
<tr>
<td>100 × ( \sigma_{\xi_l} ) Std of disutility shock</td>
<td>inv gamma</td>
<td>1</td>
<td>0.2</td>
<td>39.12</td>
<td>[31.28,46.53]</td>
</tr>
<tr>
<td>( \rho_{\text{Rot}_A} ) Pers of shock to Rot in A</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.92</td>
<td>[0.90,0.94]</td>
</tr>
<tr>
<td>( \rho_{\text{Rot}_B} ) Pers of shock to Rot in B</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.91</td>
<td>[0.89,0.92]</td>
</tr>
<tr>
<td>( \rho_z ) Pers of aggregate TFP</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.95</td>
<td>[0.94,0.97]</td>
</tr>
<tr>
<td>( \rho_{\xi_c} ) Pers of discount rate shock</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.75</td>
<td>[0.72,0.76]</td>
</tr>
<tr>
<td>( \rho_{\xi_i} ) Pers of investment shock</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.37</td>
<td>[0.41,0.45]</td>
</tr>
<tr>
<td>( \rho_{\xi_l} ) Pers of disutility shock</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.94</td>
<td>[0.94,0.95]</td>
</tr>
</tbody>
</table>

Priors for the productivities

Since I have calibrated \( z_{HH}^i \) and \( z_{LL}^i \), it remains to estimate \( z_{HL}^i \), \( z_{LH}^i \), \( z_{H}^i \) and \( z_{L}^i \). I set their priors as Beta distributions. For single firms, I set the prior mean of \( z_{H}^i \) and \( z_{L}^i \) to 0.5 and 0.2, 50 percent of \( z_{HH}^i \) and \( z_{LL}^i \). That is to say, at the prior mean, finding a cooperation can double productivity. Priors standard deviation of productivities are set to 0.1.

The prior mean of \( z_{HH}^i \) and \( z_{LL}^i \) is set to be 0.6, which means that cooperating with an L type partner can improve H type’s productivity by 20%. I impose that in any inter-firm match partners split output evenly, hence \( z_{A}^{HL} = z_{B}^{HL} \) and \( z_{A}^{LH} = z_{B}^{LH} \). \(^{34}\)

\(^{34}\) As revealed in proposition ??, it is \((z_{HH}^i)^{1 / \alpha} + (z_{LL}^i)^{1 / \alpha} - (z_{HL}^i)^{1 / \alpha} - (z_{LH}^i)^{1 / \alpha}\) that can be identified. Given that \( z_{HH}^i \) and \( z_{LL}^i \) are calibrated, I can identify \((z_{HL}^i)^{1 / \alpha} + (z_{LH}^i)^{1 / \alpha}\). However, I cannot identify \( z_{HL}^i \) and \( z_{LH}^i \) separately, hence I impose them to be same.
Priors for the parameters of the labor market and the cooperation contract

For most labor market parameters, I choose fairly dispersed priors and set the prior mean based on calibration studies. I set prior for labor disutility $\xi_n$ to Beta distribution with mean 0.2 and standard deviation 0.1, combining with unemployment insurance implies that unemployment benefit is 45% of average wage when the model is computed at the prior mean.

I set the prior for labor market matching efficiency to Beta distribution with a mean of 0.7 and a standard deviation of 0.1, so that steady-state unemployment rate is 5% at the prior mean. I don’t have information to target the range of inter-firm matching efficiency, so I set its prior to have the same mean as labor market matching efficiency but with higher dispersion. For the bargaining share of firm $\tau$, calibration studies use a wide range of values centering around 0.5. Therefore, I set a beta prior around 0.5. For the labor market matching elasticity $\alpha_1$, instead of estimating it, I impose $\alpha_1 = \tau$ to satisfy the Hosios condition (Hosios (1990)), which ensures that the labor market matching is efficient.

For the parameters related to the inter-firm cooperation contract, I set the prior for the separation probability of sourcing contract to a Beta distribution with mean 0.25, which implies that the average duration of sourcing contract is one year. The prior for the mean of the transaction cost distribution is Normal distributed with mean of 0 and a standard deviation of 0.2.

5.3.3 Parameter estimates

Tables 6 and 7 compare the posterior distributions to the priors. The posterior estimates of the parameters that are unrelated to labor market and productivities are in line with previous estimations of DSGE models. Therefore I do not comment on them.

Parameters of the labor market and the cooperation contract

Labor market matching efficiencies $\phi_A$ and $\phi_B$ have posterior mode of 0.73 and 0.75, which are close to the prior mean and imply a job finding rate of 0.56. Remember that the prior mean is set to match the steady state unemployment rate; the estimates of labor market matching efficiency well capture the empirical unemployment rate. Inter-firm matching efficiency $\psi$ has a posterior mode of 0.9. Given that the inter-firm tightness ratio is one in steady state, the steady state inter-firm matching rate is 0.9 at the posterior mode; that is, on average, a firm spends 1.1 quarters to find a partner.

---

35Labor disutility needs to be adjusted by marginal utility to be comparable with monetary variables such as wage and unemployment insurance.
At its posterior mode, a firm’s bargaining share is 0.76. This is similar with the result of Lubik (2009), who estimated firm’s bargaining share to be 0.97. As I impose the Hosio condition to hold, the labor market matching elasticity is also 0.76. Labor disutility has a posterior mode of 0.11. Combining unemployment insurance with the adjusted labor disutility, the implied flow value of unemployment is about 37% of the average wage on steady state.

Both the bargaining weight and unemployment benefit of my estimation are in line with the conventional calibration of DMP models, such as in Shimer (2005). It is well known that in canonical DMP model which is driven only by exogenous TFP shocks, an extremely high flow value of unemployment (higher than 0.9 of mean wage) and low firm bargaining share (lower than 0.1) are often needed to generate realistic volatility of unemployment. In contrast, my model fits the data without help from high flow value of unemployment and low firm bargaining share. The main reason is that, in my model, the fluctuation of unemployment is mostly driven by rotation shocks and labor disutility shocks. As will be shown in the subsequent subsections, without the help of high flow value of unemployment and low firm bargaining share, rotation shocks and labor disutility shocks are still able to generate volatile unemployment while keeping productivity variation mild.

The separation probability of sourcing contract has a posterior mode of 0.12, which is much smaller than the prior mean and indicates that the data suggests a large friction preventing mismatched firms from reallocating to more efficient partnerships. In the posterior mode, the transaction cost distribution has a mean of -0.13 and standard deviation of 1.01, which implies the ratio of industry value added to industry sales to be 0.45, which is close to 0.53 as measured in the BEA 2007 input-output table.

**Productivities**

Productivities of single firms in set $A$ $z_A^H$ and $z_A^L$ are estimated to be 0.46 and 0.50. This means that firms can double their productivity when matching with a same type partner. For mismatched cooperative firms in set $A$, their productivities $z_A^{HL}$ and $z_A^{LH}$ are 0.43. These results imply that an $H$ type firm would find it more productive to work alone than to cooperate with an $L$ type partner; an $L$ type firm would barely gain any productivity by working with an $H$ type partner, in contrast with working with an $L$ type partner. Productivities of firms in set $B$ have very similar estimates.

With the above results, we can verify that the conditions in propositions 9 are satisfied at the posterior mode, so that in the Nash equilibrium, single firms initiate match with same type partners. Specifically, I confirm that $H$ type single firms prefers to match with $H$ type partners in both sets, which guarantees that no firm would deviate from the Nash equilibrium in which
firms only search for same type partners.

Moreover, I verify that the value function, or equivalently $MPL$, is supermodular in the steady state at the posterior mode. This ensures that an increase in rotation rate would lead to a rise in unemployment, which I will confirm using impulse response analysis.

5.3.4 Impulse response analysis

In order to understand the effect of rotation shock on the economy, I conduct impulse response analysis of rotation shock.

Benchmark impulse response

Consider the reaction of the economy to a positive rotation shock to set $A$; that is, a persistent increase in the probability of switching type for firms in set $A$. The lines in Figure 2 are the impulse responses, in percentage deviations from the steady state, to a one standard deviation increase in rotation rate.

Figure 2: Impulse response to 1 std. of rotation shock in set A

The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set $A$. The shock increases rotation rates $\rho_{AH}$. As the Markov switching matrix is symmetric, it implies an increase in $\rho_{AL}^{HH}$, and decline in $\rho_{AH}^{HH}$ and $\rho_{AL}^{LL}$. A higher rotation rate generates a labor market slack in which unemployment increases and vacancies drop. This is because higher
rotation rate increases the probability of mismatch between different types of firms. Notice that value function and $MPL$ are supermodular at posterior mode, hence the risk of mismatch negatively affects firms’ expected values which reduces their incentive to create jobs.

Positive rotation shock also leads to a gradual decline in measured TFP. Decline in average productivity comes from a composition effect. A higher churning increases the share of mismatched firms. Since they have lower production efficiency on average, their rising share leads to a decline in average productivity.

The increase in churning also leads to a decline in macro aggregates such as consumption, investment and output. Three channels generate the aggregate recession. First, increase in churning hurts aggregate production efficiency by misallocating firms to inefficient partnerships, which causes a decline in aggregate output. Second, similar to the response of measured TFP, rotation shock leads to a decline in $MPK$, which makes households hold back investment. This then translates into a gradual decline in capital stock. Lastly, a weakening labor market reduces employment, which contributes to decreasing output.

**Comparing rotation shock to TFP shock**

In the model, rotation shock gives rise to endogenous change in TFP. What drives the cyclical variation in the measured aggregate TFP is still one of the major open questions in macroeconomics (Rebelo (2005)). My paper provides a novel and empirically plausible mechanism: a higher rotation rate of one set of firms increases the share of mismatched firms in both sets of firms, which induces an endogenous U-shape decline in the measured aggregate TFP. The natural question is: is there any quantitative difference between rotation shock and exogenous aggregate TFP shock in terms of their impact on the labor market variables and other macro aggregates? If so, can rotation shock speak to the Shimer puzzle (Shimer (2005))? As pointed out by Shimer (2005), canonical DMP search and matching model, which is propagated by exogenous TFP shocks, has two shortcomings: (1) it under-predicts the volatility of the labor market variables, such as unemployment and job openings, relative to TFP,\(^{36}\) (2) it over-predicts the correlation between the labor market variables and TFP. Exogenous TFP shock induces a perfect correlation between labor market variables and measured TFP; while they are only mildly correlated in the data.\(^{37}\)

To answer the above questions, I simulate a model with exogenous aggregate TFP shock

\(^{36}\)There is a vast literature on this, see Hall (2005); Hornstein, Krusell, and Violante (2005); Mortensen and Nagypal (2007); Hall and Milgrom (2008); Costain and Reiter (2008); Hagedorn and Manovskii (2008); Fujita and Ramey (2009); Gertler and Trigari (2009); Pissarides (2009) for example.

only, then compare with the impulse response to rotation shock in Figure 3. Blue lines are impulse response to positive rotation shock, while the black dashed lines correspond to the TFP shock. To make the two cases comparable, I engineer the path of TFP shock so that the measured TFP is identical in the two cases.

Figure 3: Compare rotation shock and TFP shock

The panels plot the percent deviation from the steady state induced by either one standard deviation shock to the rotation rate or a series of shocks to aggregate TFP whose path is set to make the measured TFP is identical as the rotation shock case.

The responses of labor market variables are very different in the two cases. First, shock to rotation rate generates a much stronger effect in the labor market than shock to aggregate TFP. Moreover, shock to aggregate TFP generates perfect comovement between the labor market variables and the measured TFP, which is inconsistent with data (Gervais, Jaimovich, Siu, and Yedid-Levi (2015)). In contrast, rotation rate generates a sudden drop in job openings and tightness ratio and a gradual U-shape decline in the measured TFP, which breaks its perfect correlation with the labor market variables. Therefore, shock to rotation rate significantly improves the performance of DMP model.

**Simulation without firm inter-connectivity and supermodularity**

In the simple model, as shown in Proposition 3, both firm inter-connectivity and supermodularity are necessary for a positive rotation shock to increase unemployment rate. To understand
Figure 4: Impulse responses to rotation shock in counterfactual analysis

The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set $A$.

The role of firm inter-connectivity and supermodularity in the quantitative model, I simulate two counterfactual models, one canonical DMP model with no firm inter-connectivity, the other one with firm inter-connectivity yet without supermodularity. Then I compare the effect of rotation shock on these two models to the benchmark case. In Figure 4, red lines are impulse response functions in the model with no firm inter-connectivity; black dashed lines are for model without supermodularity; blue lines correspond to the benchmark case with both firm inter-connectivity and supermodularity.

When firm inter-connectivity is removed, my model becomes identical to the canonical DMP model and rotation shock does not have any aggregate effect. In this case, churning only shifts the dispersion between the marginal values of different types. However, the mean of marginal values, which determines the aggregate job creation, is not affected.

When there is firm inter-connectivity, but MPL is not supermodular, that is \( \frac{z_{i}^{HH}}{1-\alpha} - \frac{z_{i}^{HL}}{1-\alpha} = \frac{z_{i}^{LH}}{1-\alpha} - \frac{z_{i}^{LL}}{1-\alpha} \), positive rotation shock does not increase unemployment. In fact, according to the impulse response function, higher churning results in higher job creation and lower unemployment. This is a general equilibrium effect. The intuition is as follows: while MPL is not supermodular, productivity is still supermodular; that is, we still have \( z_{i}^{HH} - z_{i}^{HL} > \)
Therefore a higher churning leads to diminished output and a decline in consumption growth. As a result, discount factor $\beta^{uc}$ rises and firms are more willing to allocate resource to the activities generating future profit, including the recruiting of worker. While this general equilibrium effect also appears in the benchmark model, it is dominated by the mismatch risk effect.

### 5.3.5 Variance decomposition

To evaluate the contribution of rotation shock to the business cycle, I conduct variance decomposition at business cycle frequency and report the results in Table 8. Each row reports the shares of fluctuations explained by the shocks at the business cycle frequency; that is, the share of variances explained at frequencies between 6 and 32 quarters, computed by a bandpass filter as in Stock and Watson (1999).

Table 8: Variance decomposition at the business cycle frequency

<table>
<thead>
<tr>
<th>Variance decomposition</th>
<th>$TFP$</th>
<th>$DR$</th>
<th>$DL$</th>
<th>$MEI$</th>
<th>$Rot_A$</th>
<th>$Rot_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>0.03</td>
<td>0.02</td>
<td>0.67</td>
<td>0.01</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Tightness ratio</td>
<td>0.02</td>
<td>0.02</td>
<td>0.68</td>
<td>0.01</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Output</td>
<td>0.62</td>
<td>0.00</td>
<td>0.06</td>
<td>0.00</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td>Measured TFP</td>
<td>0.48</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>Investment</td>
<td>0.05</td>
<td>0.16</td>
<td>0.04</td>
<td>0.45</td>
<td>0.25</td>
<td>0.22</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.25</td>
<td>0.08</td>
<td>0.03</td>
<td>0.37</td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>

$TFP$ is aggregate total factor productivity. $DR$ is discount rate. $DL$ is disutility of labor. $MEI$ is marginal efficiency of investment. $Rot_i$ is rotation rate for set $i$.

Rotation shocks account for a significant fraction of variation of all macro variables listed in the table. In particular, rotation shocks contribute to 27 percent of labor market fluctuations and 32 percent of fluctuation of aggregate output. There are two main reasons for the data to assign such a big role to rotation shocks. First, as displayed in the impulse response functions,

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38I thank Cosmin Ilut for generously sharing his matlab code for conducting the variance decomposition at the business cycle frequency.
rotation shocks have strong effect on both labor market variables as well as macro aggregates without relying on extreme values of parameters. The second reason is that rotation shock is the only type of shock considered in the model that can account for the joint movement of all observable variables, hence it is preferred by the data.

Shocks to TFP have a very minor effect on unemployment rate and tightness ratio due to lack of amplification of productivity shock in search and matching models. However, they still account for large fractions of the variances of macro aggregates such as output and measured TFP. This is consistent with the main findings of the business cycle literature.

Shocks to discount rate, or preference shocks, contribute to a large fraction of investment and consumption fluctuations. It also explains a moderate fraction of labor market fluctuations. While it is well known that higher discount rate shifts resources from consumption to investment, its effect on labor market variables is less well known. Remember that in search and matching models, hiring is an inter-temporal action similar to investment—firms increase future employment by posting vacancies at today’s cost. Thus shock to discount rate has a direct impact on firms’ incentive to post vacancies by changing their evaluation of future cash flow. This effect has been emphasized by Hall (2014). In a recent paper by Albertini and Poirier (2014), they estimated a search and matching model with shocks to TFP and discount rate and found that the shock to the discount rate predominantly account for fluctuation of unemployment. In the estimation, however, they did not include real interest rate as observable, hence the discount rate shock is not disciplined. In contrast, I include real interest rate as observable and the data assigns much less role to discount rate compared to the finding by Albertini and Poirier (2014).

Shocks to marginal efficiency of investment, or shocks to investment adjustment cost, have a large effect on consumption and investment, yet has little contribution to the other variables’ fluctuations.

Shocks to labor disutility have a very large effect on labor market variables. In many previous studies, fluctuation of labor disutility has been found to prominently account for variation in labor input. A positive shock to the labor disutility increases the option value of unemployment, which makes the total surplus of potential worker-firm matches shrink and hence reduces firm’s incentive to create jobs.
6 Conclusions

This paper is motivated by two empirical facts. The first is that during downturns of economic activity, there is an increase in churning of firms’ rankings in the profit distribution. Second, a higher churning of an industry is usually accompanied by an economic downturn within the industry and in its linked industries. Based on the two facts, this paper studies the effect of churning on the business cycle emphasizing the role of firm inter-connectivity.

Specifically, I construct and estimate a Diamond-Mortenson-Pissarides search and matching model featured by firm inter-connectivity and supermodular production function; that is, firms have comparative advantage in cooperating with same type partners. The main prediction of the model is that an increase in churning of an industry causes a recession within its own industry and in its linked industries. According to the estimation, variations in churning is one of the major sources of persistent and joint movements in unemployment and other macro variables.

Lastly, I study the model’s implication on the cross industry variation in the vertical integration. The model implies that when firms have comparative advantage in cooperating with same type partners; in an industry with higher churning, or whose linked industries have higher churning, firms are less likely to choose vertical integration. This implication is consistent with the evidence in the data, which supports the main mechanism of the model, particularly the supermodular production function.

References


Appendix

A Transition rule of firms (employments)

The transition rule of firms (or equivalently, employment) of each department indexed by type in sector A is governed by the following equations.

**Firms with vertical integration (VI)**

- **H type single firm**
  
  \[ n'_{A,VI} = (1 - \delta) \left[ (1 - p^H_{A,VI})(1 - \rho^H_{A,VI})n^H_{A,VI} + (1 - p^L_{A,VI})p^L_{A,VI}n^L_{A,VI} \right] + \frac{1}{2} (1 - F(\eta^*)) \mu_A u_A \]  
  
  (A.1)

  where \( p^H_A \) and \( p^L_A \) are cooperation matching rate with \( p^H_{A,VI} = \frac{\bar{M}(n^H_{A,VI}, n^H_{B,VI})}{n^H_{A,VI}} \) and \( p^L_{A,VI} = \frac{\bar{M}(n^L_{A,VI}, n^L_{B,VI})}{n^L_{A,VI}} \)

- **L type single firm**
  
  \[ n'_{A,VI} = (1 - \delta) \left[ (1 - p^L_{A,VI})(1 - \rho^L_{A,VI})n^L_{A,VI} + (1 - p^H_{A,VI})p^H_{A,VI}n^H_{A,VI} \right] + \frac{1}{2} (1 - F(\eta^*)) \mu_A u_A \]  
  
  (A.2)

- **H type cooperative firm matched with H type partner**
  
  \[ n'_{A,VI} = (1 - \delta) \left[ p^H_{A,VI}n^H_{A,VI} + \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho^H_{A} \rho^{qH}_{B} n^{lq}_{A,VI} \right] \]  
  
  (A.3)

- **H type cooperative firm matched with L type partner**
  
  \[ n'_{A,VI} = (1 - \delta) \left[ \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho^{lH}_{A} \rho^{qL}_{B} n^{lq}_{A,VI} \right] \]  
  
  (A.4)

- **L type cooperative firm matched with H type partner**
  
  \[ n'_{A,VI} = (1 - \delta) \left[ \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho^{lL}_{A} \rho^{qH}_{B} n^{lq}_{A,VI} \right] \]  
  
  (A.5)
• L type cooperative firm matched with L type partner

\[ n_{A,VI}^{LL} = (1 - \delta) \left[ p_{A,VI}^{L} n_{A,VI}^{L} + \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_{A}^{HL} \rho_{B}^{qH} n_{A,VI}^{lq} \right] \quad (A.6) \]

The first two equations are flow motions for single firms. Take H type as an example. Existing H type single firms may flow out in three conditions: first, with probability \( \delta \) an H type single firm can be destructed exogenously; Second, with probability \( p_{A}^{H} \) it is matched with another firm; Third, with probability \( \rho_{A}^{HL} \) the firm rotate to a L type single firm. At the same time, there are two kinds of inflow into H type: L type single firm becomes H type with probability \( \rho_{A}^{LH} \); unemployed workers becomes H type employment (firm) with job finding rate \( \mu_{A} \). \( (1 - F(\eta^{*})) \) fraction of the new created firms choose to do vertical integration, and half of them draw an H type.

The last four equations are flow motion for cooperative firms. Take the HH type as an example. With probability \( \delta \) existing firms are exogenously destructed. And with probability \( \rho_{A}^{HL} \rho_{B}^{gL} \) an lq type firm becomes HH. Lastly, H type single firms match to another H type single firm and becomes HH with probability \( p_{A}^{H} \).

**Firms with sourcing (SC)**

• H type single firm

\[ n_{A,SC}^{H} = (1 - \delta) \left[ (1 - p_{A,SC}^{H})(1 - \rho_{A}^{HL})n_{A,SC}^{H} + (1 - p_{A,VI}^{L}) \rho_{A}^{HL} n_{A,SC}^{L} \right] \quad (A.7) \]

\[ + \frac{1}{2} F(\eta^{*}) \mu_{A} u_{A} + v (1 - \delta) \left[ \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_{A}^{HL} \rho_{B}^{qH} n_{A,SC}^{lq} \right] \]

where \( p_{A}^{H} \) and \( p_{L}^{L} \) are cooperation matching rate with \( p_{A,SC}^{H} = \frac{\mathcal{M}(n_{A,SC}^{H}, n_{B,SC}^{H})}{n_{A,SC}^{H}} \) and \( p_{A,SC}^{L} = \frac{\mathcal{M}(n_{A,SC}^{L}, n_{B,SC}^{L})}{n_{A,SC}^{L}} \).

• L type single firm

\[ n_{A,SC}^{L} = (1 - \delta) \left[ (1 - p_{A,SC}^{L})(1 - \rho_{A}^{LH})n_{A,SC}^{L} + (1 - p_{A,SC}^{H}) \rho_{A}^{LH} n_{A,SC}^{H} \right] \quad (A.8) \]

\[ + \frac{1}{2} F(\eta^{*}) \mu_{A} u_{A} + v (1 - \delta) \left[ \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_{A}^{HL} \rho_{B}^{qH} n_{A,SC}^{lq} \right] \]
• H type cooperative firm matched with H type partner

\[ n_{H,H'}^{H,\text{VI}} = (1 - \delta) \left( \rho_{A,SC}^H n_{H,SC}^H + \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_A^{lH} \rho_B^{qH} n_{A,SC}^{lq} \right) \]  

(A.9)

• H type cooperative firm matched with L type partner

\[ n_{H,L'}^{H,\text{VI}} = (1 - \nu) (1 - \delta) \left( \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_A^{lH} \rho_B^{qL} n_{A,SC}^{lq} \right) \]  

(A.10)

• L type cooperative firm matched with H type partner

\[ n_{L,H'}^{L,\text{VI}} = (1 - \nu) (1 - \delta) \left( \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_A^{lL} \rho_B^{qH} n_{A,SC}^{lq} \right) \]  

(A.11)

• L type cooperative firm matched with L type partner

\[ n_{L,L'}^{L,\text{VI}} = (1 - \delta) \left( \rho_{A,SC}^L n_{L,SC}^L + \sum_{l=\{H,L\}} \sum_{q=\{H,L\}} \rho_A^{lL} \rho_B^{qL} n_{A,SC}^{lq} \right) \]  

(A.12)

The flow motion equations for SC firms are similar with the VI case, except that mismatched firms have \( \nu \) probability to sever the partnership and becomes a single firm.

Putting the equations to matrix form, I can stack the transition rule in matrix form.

For firms choosing VI

\[
\begin{bmatrix}
  n_{H,H'}^{H,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
  n_{H,H'}^{H,\text{VI}} \\
  n_{H,L'}^{H,\text{VI}} \\
  n_{L,H'}^{L,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
  n_{H,H'}^{H,\text{VI}} \\
  n_{H,L'}^{H,\text{VI}} \\
  n_{L,H'}^{L,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
\end{bmatrix}
= \Phi_{\text{VI}} \times
\begin{bmatrix}
  n_{H,H'}^{H,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
  n_{H,H'}^{H,\text{VI}} \\
  n_{H,L'}^{H,\text{VI}} \\
  n_{L,H'}^{L,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
  n_{H,H'}^{H,\text{VI}} \\
  n_{H,L'}^{H,\text{VI}} \\
  n_{L,H'}^{L,\text{VI}} \\
  n_{L,L'}^{L,\text{VI}} \\
\end{bmatrix}
+ \Xi_{\text{VI}} \times u_A
\]  

(A.13)
where

\[
\Phi_{VI} = \begin{bmatrix}
(1 - p_{A,VI}^H)(1 - \rho_{A}^{HL}) & (1 - p_{A,VI}^L) & \rho_{A}^{LH} & 0 & 0 & 0 & 0 \\
(1 - p_{A,VI}^H) & \rho_{A}^{LH} & 0 & 0 & 0 & 0 & 0 \\
p_{A,VI}^H & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \ast \Pi_T^A \times \Pi_T^B
\]

(A.14)

\[
\Xi_{VI} = \begin{bmatrix}
\frac{1}{2} (1 - F(\eta^*)) \mu(\theta_A) \\
\frac{1}{2} (1 - F(\eta^*)) \mu(\theta_A) \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(A.15)

In the \( \Phi \) matrix, \( \ast \) is Kronecker product and \( \Pi_A \) and \( \Pi_B \) are Markov switching matrices that governs rotation of types.

Similarly, for firms choosing SC

\[
\begin{bmatrix}
n_{A,SC}^{H'} \\
n_{A,SC}^{L'} \\
n_{A,SC}^{HH'} \\
n_{A,SC}^{HL'} \\
n_{A,SC}^{LH'} \\
n_{A,SC}^{LL'} \\
\end{bmatrix} = \Phi_{SC} \times \begin{bmatrix}
n_{A,SC}^H \\
n_{A,SC}^L \\
n_{A,SC}^{HH} \\
n_{A,SC}^{HL} \\
n_{A,SC}^{LH} \\
n_{A,SC}^{LL} \\
\end{bmatrix} + \Xi_{SC} \times u_A
\]

(A.16)

where

\[
\Xi_{SC} = \begin{bmatrix}
\frac{1}{2} F(\eta^*) \mu(\theta_A) \\
\frac{1}{2} F(\eta^*) \mu(\theta_A) \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(A.17)
, and

$$
\Phi_{SC} = \begin{bmatrix}
(1 - p_{A,SC}^H)(1 - p_{A,SC}^L) & (1 - p_{A,SC}^L)(1 - p_{A,SC}^H)

(1 - p_{A,SC}^H) & 0

0 & 0

0 & p_{A,SC}^L

\end{bmatrix}
$$

where \((\Pi_A^T \otimes \Pi_B^T)_{(k,:)}\) denotes the \(kth\) row of \((\Pi_A^T \otimes \Pi_B^T)\) matrix.

Combine the VI and SC firms, the flow motion of employment is then described by

$$
\begin{bmatrix}
n_{H,A,VI}' \\
\vdots \\
n_{L,L,SC}^{LL}' \\
\end{bmatrix}
_{12 \times 1}

= \Phi
_{12 \times 12}
\begin{bmatrix}
n_{H,A,VI}' \\
\vdots \\
n_{L,L,SC}^{LL}' \\
\end{bmatrix}
_{12 \times 1}

+ \Xi
_{12 \times 1}
\times u_A
$$

where

$$
\Phi = \begin{bmatrix}
\Phi_{SC} \\
\Phi_{VI}
\end{bmatrix}
$$

and

$$
\Xi = \begin{bmatrix}
\Xi_{SC} \\
\Xi_{VI}
\end{bmatrix}
$$

B Non-linear system of equations

Household’s F.O.C.s

- Marginal utility of consumption

$$
\xi_{c,t} (C_t - \Psi_c \bar{C}_{t-1})^{-1} = \lambda_t
$$

where \(\lambda\) is the Lagrange multiplier on the household’s budget constraint. In equilibrium \(C = \bar{C}\)

- F.O.C of investment

$$
\lambda_t = q_t \left[ (1 - S_t) - S_t' \xi_{t+1} \frac{I_{t+1}}{I_{t-1}} \right] + \beta E \left[ q_{t+1} S_{t+1}' \xi_{t+1} \frac{I_{t+1}^2}{I_t^2} \right]
$$
where \( q \) is Tobin’s \( q \).

- F.O.C of capital stock

\[
\lambda_t r_{k,t} = -q_t (1 - d) + \frac{q_{t-1}}{\beta}
\]  
(B.3)

**Household’s marginal value of employment and unemployment**

- Household’s marginal value of employment for each type of employee

\[
\begin{bmatrix}
V'_{n_i^{H,VI}} \\
\vdots \\
V'_{n_i^{L,SC}}
\end{bmatrix}
= -\xi_n + \lambda
\begin{bmatrix}
w_{i,VI}^H \\
\vdots \\
w_{i,SC}^L
\end{bmatrix} + \beta \Phi^T E
\begin{bmatrix}
V'_{n_i^{H,VI}} \\
\vdots \\
V'_{n_i^{L,SC}}
\end{bmatrix}
\]  
(B.4)

where \( \Phi \) is defined in the last section and \( T \) denotes transpose of matrix.

- Equilibrium condition for perfect labor mobility and household’s marginal value of unemployment.

\[
V_{u_A} = V_{u_B} = V_u
\]  
(B.5)

where

\[
V_{u_A} = z \cdot \lambda + \beta (1 - \delta - \mu (\theta_A)) E \begin{bmatrix} V'_{u_A} \\ 1 \times 12 \end{bmatrix} + \beta \Xi^T E \begin{bmatrix}
V'_{n_i^{H,VI}} \\
\vdots \\
V'_{n_i^{L,SC}}
\end{bmatrix}
\]

\[
V_{u_B} = z \cdot \lambda + \beta (1 - \delta - \mu (\theta_B)) E \begin{bmatrix} V'_{u_A} \\ 1 \times 12 \end{bmatrix} + \beta \Xi^T E \begin{bmatrix}
V'_{n_i^{H,VI}} \\
\vdots \\
V'_{n_i^{L,SC}}
\end{bmatrix}
\]

With assumption of perfect labor mobility, the marginal value of unemployment should equalize across two sectors in any state of economy.
Firm’s FOCs

- Firm’s F.O.C. of capital input.

\[ xz_i^H \left( \frac{k_{i,VI}^H}{xn_{i,VI}} \right) \alpha^{-1} = \cdots = xz_i^{LL} \left( \frac{k_{i,SC}^{LL}}{xn_{i,SC}} \right) \alpha^{-1} = r_K \tag{B.6} \]

Firm allocates capital to production departments to the point that \( mpk \) of each department equal to rental rate of capital.

- Firm’s F.O.C. of vacancy posting.

\[ \chi = \beta \int \left\{ \begin{array}{c} E \left[ \lambda^' \frac{\partial f}{\partial n_{i,VI}^H(\eta)} \right] \\ \vdots \\ E \left[ \lambda^' \frac{\partial f}{\partial n_{i,SC}^{LL}(\eta)} \right] \end{array} \right\} dF(\eta) \tag{B.7} \]

Firm chooses the tightness ratio so that its expected marginal value equals to vacancy posting cost.

Firm’s envelope conditions

- Firm, number of employee in each department

\[ \lambda \left[ \begin{array}{c} \frac{\partial J_i}{\partial n_{i,VI}^H(\eta)} \\ \vdots \\ \frac{\partial J_i}{\partial n_{i,SC}^{LL}(\eta)} \end{array} \right] = \lambda \left[ \begin{array}{c} m_{i,VI}^H \\ \vdots \\ m_{i,SC}^{LL} \end{array} \right] - \lambda \left[ \begin{array}{c} w_{i,VI}^H \\ \vdots \\ w_{i,SC}^{LL} \end{array} \right] + \beta \Phi^T (\bar{\theta}, \Pi) E \left( \lambda^' \left[ \begin{array}{c} \frac{\partial V'}{\partial n_{i,VI}^H(\eta)} \\ \vdots \\ \frac{\partial V'}{\partial n_{i,SC}^{LL}(\eta)} \end{array} \right] \right) \tag{B.8} \]

Total surplus

Remember that the definition of total surplus \( TS \) as

\[ TS_{i,j}^j(\eta) = \lambda \frac{\partial J_i}{\partial n_{i,j}^j(\eta)} + \frac{\partial V}{\partial n_{i,j}^j(\eta)} - \frac{\partial V}{\partial u_i} \]
And with Nash Bargaining we have

\[
\frac{\lambda}{\partial n_{i,l}^j(\eta)} \frac{\partial J_i}{\partial n_{i,l}^j(\eta)} = \tau T S_{i,l}^j(\eta) \tag{B.9}
\]

\[
\frac{\partial V}{\partial n_{i,l}^j(\eta)} - \frac{\partial V}{\partial u_i} = (1 - \tau) T S_{i,l}^j(\eta) \tag{B.10}
\]

By combining household’s F.O.C., firm’s F.O.C. and envelope conditions together one can get the following total surplus representation

\[
\begin{bmatrix}
TS_{i,l}^H(\eta) \\
TS_{i,l}^L(\eta)
\end{bmatrix} = \lambda \left( \begin{bmatrix}
mp_{ii}^{HH} \\
mp_{ii}^{HL} \\
mp_{ii}^{LH} \\
mp_{ii}^{LL}
\end{bmatrix} - z - \frac{(1 - \delta)(1 - \tau) \chi \theta_i}{\tau} \right) - \xi_n \tag{B.11}
\]

\[
+ \beta (1 - \delta) E \left[ \begin{bmatrix}
\rho_i^{HH} T S_{i,l}^{HH}(\eta) + (1 - p_i^H) \left( \rho_i^{HH} T S_{i,l}^{H}(\eta) + \rho_i^{HL} T S_{i,l}^L(\eta) \right) \\
\rho_i^{LL} T S_{i,l}^{LL} + (1 - p_i^L) \left( \rho_i^{LH} T S_{i,l}^H(\eta) + \rho_i^{LL} T S_{i,l}^L(\eta) \right)
\end{bmatrix} \right]
\]

\[
\begin{bmatrix}
TS_{i,l}^{HH}(\eta) \\
TS_{i,l}^{HL}(\eta) \\
TS_{i,l}^{LH}(\eta) \\
TS_{i,l}^{LL}(\eta)
\end{bmatrix} = \lambda \left( \begin{bmatrix}
mp_{ii}^{HH} \\
mp_{ii}^{HL} \\
mp_{ii}^{LH} \\
mp_{ii}^{LL}
\end{bmatrix} - z - \frac{(1 - \delta)(1 - \tau) \chi \theta_i}{\tau} \right) - \xi_n \tag{B.12}
\]

\[
+ \beta (1 - \delta) \left( \Pi_A^T \otimes \Pi_B^T \right) E \left[ \begin{bmatrix}
TS_{i,l}^{HH'}(\eta) \\
TS_{i,l}^{HL'}(\eta) \\
TS_{i,l}^{LH'}(\eta) \\
TS_{i,l}^{LL'}(\eta)
\end{bmatrix} \right]
\]

And the free entry condition of the labor market \(i\) becomes

\[
\lambda \chi = \beta \tau \int \left\{ \mathbf{z}_{\delta x}^T E \begin{bmatrix}
TS_{i,Vl}^H(\eta) \\
\vdots \\
TS_{i,SC}^{LL}(\eta)
\end{bmatrix} \right\} dF(\eta) \tag{B.13}
\]

C Proof of lemmas and propositions

C.1 Proof of results of section 3 and 4

In this subsection, I will use the following notations

\[
\mathbf{f}_i = J_i^{HH} + J_i^{LL} - J_i^{HL} - J_i^{LH}
\]

66
We immediately get that \( \tilde{z} = z_{i}^{HH} + z_{i}^{LL} - z_{i}^{HL} - z_{i}^{LH} \)
\( \bar{J}_i = \frac{j_i^{HH} + j_i^{LL}}{2} \)
\( \tilde{z}_i = \frac{j_i^{HH} + j_i^{LL}}{2} \)
\( \bar{J}_{i,l}(\eta) = J_{i,l}^{HH}(\eta) + J_{i,l}^{LL}(\eta) - J_{i,l}^{HL}(\eta) - J_{i,l}^{LH}(\eta) \)
\( i \in \{A, B\}, \ l \in \{VI, SC\} \)

As the two sets of firms are symmetric, to save space I will drop the subscript \( i \) in some cases.

**Lemma 1**

1. Value function is strictly monotone if idiosyncratic productivity is strictly monotone.
2. Value function is supermodular if and only if production function is supermodular.

**Proof.** (1) Use firms’ value functions, we get

\[
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix}
= \tau \cdot \begin{bmatrix}
z_{i}^{HH} - z_{i}^{HL} \\
z_{i}^{HL} - z_{i}^{LL}
\end{bmatrix} + \beta (1 - \delta) \begin{bmatrix}
\rho_{i}^{HH} & \rho_{i}^{HL} \\
\rho_{i}^{HL} & \rho_{i}^{LL}
\end{bmatrix}
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
z_{i}^{HH} - z_{i}^{HL} \\
z_{i}^{HL} - z_{i}^{LL}
\end{bmatrix} = \begin{bmatrix}
1 - \beta (1 - \delta) \rho_{i}^{HH} & -\beta (1 - \delta) \rho_{i}^{HL} \\
-\beta (1 - \delta) \rho_{i}^{HL} & 1 - \beta (1 - \delta) \rho_{i}^{LL}
\end{bmatrix}
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix} = \begin{bmatrix}
1 - \beta (1 - \delta) \rho_{i}^{HH} & -\beta (1 - \delta) \rho_{i}^{HL} \\
-\beta (1 - \delta) \rho_{i}^{HL} & 1 - \beta (1 - \delta) \rho_{i}^{LL}
\end{bmatrix}
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix}
\]

We immediately get that

\[
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix} > \begin{bmatrix}
0 \\
0
\end{bmatrix}
\text{if} \begin{bmatrix}
z_{i}^{HH} - z_{i}^{HL} \\
z_{i}^{HL} - z_{i}^{LL}
\end{bmatrix} > \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

With the same method we can prove that

\[
\begin{bmatrix}
J_{i}^{HH} - J_{i}^{HL} \\
J_{i}^{HL} - J_{i}^{LL}
\end{bmatrix} > \begin{bmatrix}
0 \\
0
\end{bmatrix}
\text{if} \begin{bmatrix}
z_{i}^{HH} - z_{i}^{HL} \\
z_{i}^{HL} - z_{i}^{LL}
\end{bmatrix} > \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(2) It is to show that \( \bar{J} > 0 \) if and only if \( \tilde{z} > 0 \). Use the value functions of firms equation 2 and equation refE2, we get

\[
\tilde{J}_{t} = \tau \cdot x_{t} \cdot z_{t}^{HH} + \beta (1 - \delta) E_{t} \left( \rho_{t}^{HH} J_{t+1}^{HH} + \rho_{t}^{HL} J_{t+1}^{HL} \right) - \left[ \tau \cdot x_{t} \cdot z_{t}^{HL} + \beta (1 - \delta) E_{t} \left( \rho_{t}^{HH} J_{t+1}^{HL} + \rho_{t}^{HL} J_{t+1}^{LH} \right) \right] + \tau \cdot x_{t} \cdot z_{t}^{LL} + \beta (1 - \delta) E_{t} \left( \rho_{t}^{LL} J_{t+1}^{HL} + \rho_{t}^{LH} J_{t+1}^{LH} \right) - \left[ \tau \cdot x_{t} \cdot z_{t}^{LL} + \beta (1 - \delta) E_{t} \left( \rho_{t}^{LL} J_{t+1}^{HH} + \rho_{t}^{LH} J_{t+1}^{LH} \right) \right]
\]
Rearranging the terms, we get

\[ \tilde{J}_t = \tau \cdot x_t (\tilde{z}_t^{HH} + \tilde{z}_t^{LL} - \tilde{z}_t^{HL} - \tilde{z}_t^{LH}) + \beta (1 - \delta) (\rho_t^{HH} - \rho_t^{LH}) E_t (J_{t+1}^{HH}) + \beta (1 - \delta) (\rho_t^{LL} - \rho_t^{HL}) E_t (J_{t+1}^{LL}) + \beta (1 - \delta) (\rho_t^{LH} - \rho_t^{HH}) E_t (J_{t+1}^{LH}) + \beta (1 - \delta) (\rho_t^{HL} - \rho_t^{LL}) E_t (J_{t+1}^{HL}) \]

Notice that the Markov switching matrix is symmetric, we have

\[ \rho_t^{HH} - \rho_t^{LH} = \rho_t^{LL} - \rho_t^{HL}, \quad 1 - 2\rho_t^{HL} = 1 - 2\rho_t^{HH} \]

\[ \rho_t^{LH} - \rho_t^{HH} = -(1 - 2\rho_t^{HL}), \quad \rho_t^{HL} - \rho_t^{LL} = -(1 - 2\rho_t^{HL}) \]

Thus we have

\[ \tilde{J}_t = \tau \cdot x_t \tilde{z}_t + \beta (1 - 2\rho_t^{HL}) (1 - \delta) E_t (\tilde{J}_{t+1}) \quad (C.1) \]

On steady state, we have

\[ \bar{J} = \frac{\tau \cdot x \cdot \bar{z}}{1 - \beta (1 - 2\rho^{HL})(1 - \delta)} \]

Since \(1 - \beta (1 - 2\rho^{HL})(1 - \delta) > 0\), \(\bar{J} > 0\) if and only if \(\bar{z} > 0\).

**Proposition 3**

If production function is strictly monotone, and there is firm inter-connectivity, a persistent increase in the rotation rate of one set, with persistence bounded below \(|\psi| < 1 - \rho^{HL} - \rho^{LH}\), state would lead to

1. a decrease in the equilibrium tightness ratio of both sets
2. a decrease in the average productivity of both sets in the next period

if and only if the production function is supermodular.

**Proof.** I will show when \(\bar{z} > 0\), \(\theta\) is decreasing with \(\rho^{HL}\). Notice that \(\theta\) is increasing in \(\bar{J}\) according to free entry condition, we only need to show \(\bar{J}\) is decreasing with \(\rho^{HL}\)
Fixing aggregate productivity \( x \) to one, we have the following equations

\[
\bar{J}_t = \bar{z} + \beta (1 - \delta) \left[ E_t (\bar{J}_{t+1}) - \frac{\rho_t^{HL} E_t (\bar{J}_{t+1})}{2} \right] \quad (C.2)
\]

\[
\check{J}_t = \check{z} + \beta (1 - \delta) (1 - 2 \rho_t^{HL}) E_t (\check{J}_{t+1}) \quad (C.3)
\]

\[
\rho_t^{HL} = \psi \rho_{t-1}^{HL} + \varepsilon_t \quad (C.4)
\]

Denote \( F_{t,t+s} = \frac{1}{2} E_t \left( \rho_{t,s}^{HL} \check{J}_{t,s+1} \right) \), the first equation becomes

\[
\check{J}_t = \frac{\bar{z}}{1 - \beta (1 - \delta)} - \sum_{s=0}^{\infty} F_{t,t+s}
\]

It suffices to show that \( \frac{dF_{t,t+s}}{d\varepsilon_t} > 0 \) for \( t > 0 \quad s \geq 0 \). Here I prove the inequality for \( s = 0 \), the cases for \( s > 0 \) are similar.

\[
\frac{dF_{t,t}}{d\varepsilon_t} = \frac{d \left[ \rho_t^{HL} E_t (\check{J}_{t+1}) \right]}{d\rho_t^{HL}} = E_t (\check{J}_{t+1}) + \rho_t^{HL} E_t (\check{J}_{t+1}) \frac{d \rho_t^{HL}}{d\rho_t^{HL}}
\]

According to equation (C.3), we have

\[
E_t (\check{J}_{t+1}) = \bar{z} \cdot \sum_{s=0}^{\infty} E_t \left[ \beta^s (1 - \delta)^s \prod_{k=0}^{s} (1 - 2 \rho_t^{HL}) \right]
\]

Therefore

\[
\frac{E_t (\check{J}_{t+1})}{d\rho_t^{HL}} = \bar{z} \cdot \sum_{s=0}^{\infty} E_t \left\{ \beta^s (1 - \delta)^s \prod_{k=0}^{s} (1 - 2 \rho_t^{HL}) \right\} \cdot \sum_{k=0}^{s} \left( \frac{2}{1 - 2 \rho_t^{HL}} \cdot \frac{d \rho_t^{HL}}{d\rho_t^{HL}} \right) \\
> \bar{z} \cdot \sum_{s=0}^{\infty} E_t \left\{ \beta^s (1 - \delta)^s \prod_{k=0}^{s} (1 - 2 \rho_t^{HL}) \right\} \cdot \sum_{k=0}^{s} \left( -2 \cdot \frac{1 - \psi^k}{1 - \psi} \right) \\
> -2 \cdot \bar{z} \cdot \sum_{s=0}^{\infty} E_t \left[ \beta^s (1 - \delta)^s \prod_{k=0}^{s} (1 - 2 \rho_t^{HL}) \right] \frac{1}{1 - \psi} \bar{z} \cdot \sum_{s=0}^{\infty} E_t (\check{J}_{t+1}) \\
= -\frac{2}{1 - \psi} \cdot E_t (\check{J}_{t+1})
\]
When \( \frac{1-\psi}{\rho^m} > 2 \), we immediately get

\[
\frac{dF_{t,i}}{dE_t} > \left( 1 - \frac{2\rho^{HL}}{1-\psi} \right) E_t (\tilde{J}_{t+1}) > 0
\]

\[ \square \]

**Proposition 4**

The threshold level of transaction cost of industry \( i \) exists and is uniquely determined by

\[
\tau \cdot \eta^*_i = \beta (1-\delta) g (\rho^{HL}_j, \nu) \cdot (z^{HH}_i + z^{LL}_i - z^{HL}_i - z^{LH}_i)
\]

with \( \frac{\partial g (\rho^{HL}, \nu)}{\partial \rho^{HL}} > 0 \) and \( \frac{\partial g (\rho^{HL}, \nu)}{\partial \nu} > 0 \)

**Proof.** The threshold level of transaction cost is determined by

\[
J^{HH}_{i,V1} + J^{LL}_{i,V1} = J^{HH}_{i,SC}(\eta^*_i) + J^{LL}_{i,SC}(\eta^*_i) \tag{C.5}
\]

The LHS of equation C.5 can be derived as

\[
J^{HH}_{i,V1} + J^{LL}_{i,V1} = \tau (z^{HH}_i + z^{LL}_i) + \beta (1-\delta) E_t \left[ J^{HH}_{i,V1} + J^{LL}_{i,V1} \right] - \rho_t \tilde{J}_{i,V1} \tag{C.6}
\]

The RHS of equation C.5 can be derived as

\[
J^{HH}_{i,SC}(\eta^*_i) + J^{LL}_{i,SC}(\eta^*_i) = \tau (z^{HH}_i + z^{LL}_i - 2\eta^*_i) + \beta (1-\delta) E_t \left[ J^{HH}_{i,SC}(\eta^*_i) + J^{LL}_{i,SC}(\eta^*_i) \right] - \rho_t (1-\nu) \tilde{J}_{i,SC}(\eta^*_i) \tag{C.7}
\]

Subtract equation C.6 from C.7, we get

\[
2\tau \eta^*_i = \beta (1-\delta) \rho_t [ \tilde{J}_{i,V1} - (1-\nu) \tilde{J}_{i,SC}(\eta^*_i) ] \tag{C.8}
\]

I first present a useful lemma.

**Lemma.** Mismatch loss for firms choosing sourcing is constant within industry, that is

\[
\tilde{J}_{i,SC}(\eta^*_i) = \tilde{J}_{i,SC}
\]

for \( \eta \sim \mathcal{N}(\tilde{\eta}_i, \sigma_i^2) \)
Proof. By plugging value functions into the above equation and using the method of deriving equation C.1, we get

\[ \tilde{J}_{SC,t}(\eta) = \tau \tilde{z}_t + \beta (1-\delta) \left[ 1 - \left( \rho_t^{HL} + \rho_t^{LH} \right) (1-v) \right] E_t \left( \tilde{J}_{SC,t}(\eta) \right) \]

As shown in the above equation, mismatch loss \( \tilde{J}_{S,t}(\eta) \) is independent of contraction cost \( \eta \), and I will drop the \( \eta \) term in the following proof.

With the above lemma, we immediately get that in the steady state,

\[ \tilde{J}_{SC} = \frac{\tau \tilde{z}}{1-\beta (1-\delta) \left[ 1 - 2\rho (1-v) \right]} \] (C.9)

, where \( \rho \triangleq \rho^{HL} = \rho^{LH} \).

For firms who choose vertical integration, the mismatch loss is same as the simple model, and we have

\[ \tilde{J}_{VI} = \frac{\tau \tilde{z}}{1-\beta (1-\delta) \left[ 1 - 2\rho \right]} \] (C.10)

Plug equations C.9 and C.10 into equation C.8, we get

\[ \eta^*_i = \frac{\beta (1-\delta) \tilde{z}}{2} \left[ \frac{\rho}{1-\beta (1-\delta) \left[ 1 - 2\rho \right]} - \frac{(1-v) \rho}{1-\beta (1-\delta) \left[ 1 - 2\rho (1-v) \right]} \right] \]

Denote

\[ g(\rho, v) = \frac{\rho \nu}{2 \left[ 1 - \beta (1-\delta) \left[ 1 - 2\rho (1-v) \right] \right] \cdot \left[ 1 - \beta (1-\delta) \left[ 1 - 2\rho (1-v) \right] \right]} \]

Therefore,

\[ \eta^*_i = \beta (1-\delta) \tilde{z} \cdot g(\rho, v) \]

With tedious algebra, we can show that

\[ \frac{\partial g(\rho, v)}{\partial \rho} = \frac{\nu \left[ 1 - \beta (1-\delta) + \beta^2 (1-\delta)^2 (1-4(1-v)\rho^2) \right]}{\left[ 1 - \beta (1-2\rho) \right]^2 \cdot \left[ 1 - \beta (1-\delta) \left[ 1 - 2\rho (1-v) \right] \right]^2} \]

and

\[ \frac{\partial g(\rho, v)}{\partial v} = \frac{\rho \left[ 1 - \beta (1-\delta) (1-4\rho) + \beta^2 (1-\delta)^2 (1-4\rho + 4\rho^2) \right]}{\left[ 1 - \beta (1-2\rho) \right]^2 \cdot \left[ 1 - \beta (1-\delta) \left[ 1 - 2\rho (1-v) \right] \right]^2} \]
In the data, $\rho$ is never larger than 0.2, hence we have
\[ \frac{\partial g(\rho, \nu)}{\partial \rho} > 0 \]
and
\[ \frac{\partial g(\rho, \nu)}{\partial \nu} > 0 \]

**Proposition 5**

In industry $i$, a firm would choose vertical integration if $\eta > \eta^*_i$, sourcing if $\eta < \eta^*_i$. The share of firms that choose vertical integration is $1 - F(\eta^*_i)$.

**Proof.** From a firm’ value functions, it can be shown that
\[ \eta \succ \eta^* = \frac{v\beta (1 - \delta) \rho^{HL} \cdot J_i}{\tau} \]
\[ \Leftrightarrow \frac{1}{2} (J_{i,VI}^{HH} + J_{i,VI}^{LL}) \geq \frac{1}{2} [J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)] \]

**C.2 Proof for results in section 5**

In this subsection, I will use the following notations
\[ T\tilde{S}_{i,l}(\eta) = TS_{i,l}^{HH}(\eta) + TS_{i,l}^{LL}(\eta) - TS_{i,l}^{HL}(\eta) - TS_{i,l}^{LL}(\eta) \]
\[ \tilde{z}_i = (z_{i,HH})^{1/a} - (z_{i,HL})^{1/a} - (z_{i,LH})^{1/a} + (z_{i,LL})^{1/a} \]
\[ \overline{z}_i = (z_{i,HH})^{1/a} + (z_{i,LL})^{1/a} \]
\[ i \in \{A, B\}, l \in \{VI, SC\} \]

As sets $A$ and $B$ are symmetric, to save space I will drop the subscript $i$ in some cases.

**Proposition 7**

In industry $i$ and in period $t$, a single firm would choose vertical integration if $\eta \geq \eta^*_i$, sourcing if $\eta < \eta^*_i$. The threshold is determined by
\[ \eta^*_i = \beta (1 - \delta) (\rho_{A,i} + \rho_{B,i}) E_t [\tilde{T}\tilde{S}_{i,VI,t+1} - (1 - \nu) \tilde{T}\tilde{S}_{i,SC,t+1}(\eta^*_i)] \]
Proof. For firms choosing VI, it is easy to show that

$$\frac{TS_{VI,t}^{HH} + TS_{VI,t}^{LL}}{2} = \bar{z}_t + \frac{1}{2} \beta (1 - \delta) E_t \left[ TS_{VI,t}^{HH} + TS_{VI,t}^{LL} \right] - \rho_t \tilde{T}S_{VI,t+1}$$

Also, it can be shown that

$$\tilde{T}S_{VI,t} = \bar{z}_t + \beta (1 - \delta) \left[ 1 - 2 (\rho_{A,t} + \rho_{B,t}) \right] E_t (\tilde{T}S_{VI,t+1})$$

For firms choosing SC

$$\frac{TS_{SC,t}^{HH} (\eta) + TS_{SC,t}^{LL} (\eta)}{2} = (\bar{z}_t - \eta_t) + \frac{1}{2} \beta (1 - \delta) E_t \left[ TS_{SC,t+1}^{HH} (\eta) + TS_{SC,t+1}^{LL} (\eta) \right] - (\rho_{A,t} + \rho_{B,t}) (1 - \nu) \tilde{T}S_{SC,t+1}$$

It can be shown that

$$\tilde{T}S_{SC,t} = \bar{z}_t + \beta (1 - \delta) \left[ 1 - 2 (\rho_{A,t} + \rho_{B,t}) (1 - \nu) \right] E_t (\tilde{T}S_{SC,t+1})$$

The threshold is determined by

$$TS_{SC,t}^{H} (\eta) + TS_{SC,t}^{L} (\eta) = TS_{VI,t}^{HH} + TS_{VI,t}^{LL}$$

or equivalently

$$p_{SC,t}^{HH} E_t \left( TS_{SC,t+1}^{HH} \right) + (1 - p_{SC,t}^{HH}) E_t \left( TS_{SC,t+1}^{HH} \right) + p_{SC,t}^{LL} E_t \left( TS_{SC,t+1}^{LL} \right) + (1 - p_{SC,t}^{LL}) E_t \left( TS_{SC,t+1}^{LL} \right)$$

In PAM equilibrium, $p_{SC,t}^{HH} = p_{SC,t}^{LL} = p_{VI,t}^{HH} = p_{VI,t}^{LL}$

To have analytical solution, I make the assumption that

$$E_t \left( TS_{I,t+1}^{j} (\eta) \right) \approx TS_{I,t}^{j} (\eta)$$

Equation C.11 can be approximated by

$$TS_{SC,t}^{HH} (\eta) + TS_{SC,t}^{LL} (\eta) = TS_{VI,t}^{HH} + TS_{VI,t}^{LL}$$

By solving equation C.12 with the results above, the threshold of transaction cost is deter-
mined by

\[ \eta_i^* = \beta (1 - \delta) E_t \left[ (\rho_{A,t} + \rho_{B,t}) \bar{T}_{SV,tl+1} - (1 - \nu) (\rho_{A,t} + \rho_{B,t}) \bar{T}_{SC,t+1} \right] \]

\[ \square \]

**Proposition**

The free entry condition of set \( i \) is

\[ \lambda_t \chi = \beta f(\Theta_{t,l}) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{\partial J_i}{\partial n_{i,VI,t+1}} + \frac{\partial J_i}{\partial n_{i,VI,t+1}} \right] + F(\eta_i^*) \Delta \hat{H}_{i,t+1} \right\} \]

with

\[ \Delta \hat{H}_{i,t} = -\tau \hat{h}_t + \beta (1 - \delta) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [\Delta \hat{H}_{i,t+1} - (\rho_{A,t} + \rho_{B,t}) (1 - \nu) \hat{H}_{i,SC,t+1} + (\rho_{A,t} + \rho_{B,t}) \hat{H}_{i,VI,t+1}] \right\} \]

\[ \hat{h}_t \triangleq \int_{-\infty}^{\eta_i^*} \eta dF(\eta) \]

**Proposition 9**

The positive assortative matching is Nash equilibrium if

\[ \left( \frac{\partial J_A}{\partial n_{AI}^H(\eta)} - \frac{\partial J_A}{\partial n_{AI}^H(\eta)} \right) \frac{1}{\alpha_2} \times \left( \frac{\partial J_B}{\partial n_{BI}^H(\eta)} - \frac{\partial J_B}{\partial n_{BI}^H(\eta)} \right) \frac{1}{\alpha_2} > 1 \]

and

\[ \left( \frac{\partial J_B}{\partial n_{BI}^H(\eta)} - \frac{\partial J_B}{\partial n_{BI}^H(\eta)} \right) \frac{1}{\alpha_2} \times \left( \frac{\partial J_A}{\partial n_{AI}^H(\eta)} - \frac{\partial J_A}{\partial n_{AI}^H(\eta)} \right) \frac{1}{\alpha_2} > 1 \]

**Proof.** I only need to show that no \( H \) type firms want to search for \( L \) type partners. I will show this for the firms in set \( A \). Firms in set \( B \) have the same result.

For the \( H \) type single firm in set \( A \) choosing cooperation contract \( l \), the expected gain of searching for \( H \) type partner is

\[ \tilde{M} \left( n_{A,l}^H, n_{B,l}^H \right) \left( \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} \right) \]

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where $\tilde{M}(n_{A,l}^H, n_{B,l}^H)$ is the probability of matching with a $H$ type partner; $\left(\frac{\partial J_A}{\partial n_{A,l}^H(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^H(\eta)}\right)$ is the marginal benefit conditional on matching with a $H$ type partner.

Similarly, for the $L$ type single firm in set $B$, the expected gain of searching for $L$ type partner is

$$\tilde{M} \left( \frac{n_{A,l}^L}{n_{B,l}^L} \right) \left( \frac{\partial J_A}{\partial n_{A,l}^L(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^L(\eta)} \right)$$

The Nash equilibrium is that firms search for same type partner only. Now assuming that an infinitesimal measure $\Delta_A^H$ of $H$ firms in set $A$ declare that they are searching for $L$ type partner in set $B$. Knowing this, a certain measure of $L$ type firms in set $B$ will attempt to match with those $H$ type firms; the measure is pinned down by the condition under which they are indifferent between matching with $H$ and $L$. Specifically, the measure of $L$ type firms in set $B$ who commit deviating, $\Delta_B^L$, is determined by

$$\tilde{M} \left( \frac{\Delta_A^H, \Delta_B^L}{\Delta_A^H} \right) \left( \frac{\partial J_B}{\partial n_{B,l}^L(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^L(\eta)} \right) = \tilde{M} \left( \frac{n_{A,l}^L, n_{B,l}^L}{n_{B,l}^L} \right) \left( \frac{\partial J_B}{\partial n_{B,l}^L(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^L(\eta)} \right)$$

(C.13)

Given the measure of $L$ type firms in set $B$ attempting to match with $H$ type firms in set $A$, the $H$ type firms in set $A$ who search for $L$ type partner expect to gain

$$\tilde{M} \left( \frac{\Delta_A^H, \Delta_B^L}{\Delta_A^H} \right) \left( \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} \right)$$

Now I want to show that the above expected gain is less than the expected gain of searching for $H$ type partner; that is,

$$\tilde{M} \left( \frac{\Delta_A^H, \Delta_B^L}{\Delta_A^H} \right) \left( \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} \right) < \tilde{M} \left( \frac{n_{A,l}^H, n_{B,l}^H}{n_{A,l}^H} \right) \left( \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^H(\eta)} \right)$$

(C.14)

As $n_{A,l}^H = n_{A,l}^L = n_{B,l}^H = n_{B,l}^L$ in PAM equilibrium. It is easy to show that $\tilde{M}(n_{A,l}^H, n_{B,l}^H) = \tilde{M}(n_{A,l}^L, n_{B,l}^L)$...
\[
\frac{\bar{m}(n_{i,j}^H, a_{i,j}^H)}{n_{i,j}^H} = \psi, \text{ and denote } \theta_{\text{dev}} = \frac{\dot{\bar{m}}_B}{\bar{m}_A}, \text{ we can rewrite equation C.13 and inequation C.14 as}
\]

\[
\psi(\theta_{\text{dev}})^{\alpha_2 - 1} \left( \frac{\partial J_B}{\partial n_{i,j}^H(\eta^H)} - \frac{\partial J_B}{\partial n_{i,j}^L(\eta^L)} \right) = \psi \left( \frac{\partial J_B}{\partial n_{i,j}^L(\eta)} - \frac{\partial J_B}{\partial n_{i,j}^L(\eta)} \right)
\]

\[
\psi(\theta_{\text{dev}})^{\alpha_2} \left( \frac{\partial J_A}{\partial n_{i,j}^H(\eta^H)} - \frac{\partial J_A}{\partial n_{i,j}^H(\eta^H)} \right) < \psi \left( \frac{\partial J_A}{\partial n_{i,j}^H(\eta)} - \frac{\partial J_A}{\partial n_{i,j}^H(\eta)} \right)
\]

which is equivalent with

\[
\theta_{\text{dev}} = \left( \frac{\partial J_B}{\partial n_{i,j}^L(\eta)} - \frac{\partial J_B}{\partial n_{i,j}^L(\eta)} \right)^{\frac{1}{\alpha_2 - 1}}
\]

\[
\theta_{\text{dev}} < \left( \frac{\partial J_A}{\partial n_{i,j}^H(\eta^H)} - \frac{\partial J_A}{\partial n_{i,j}^H(\eta^H)} \right)^{\frac{1}{\alpha_2}}
\]

We immediately see that the inequation of proposition implies the above condition.

\[\square\]

**D Data**

**Data for section 2**

I obtain annual firm-level balance sheet information from Compustat for North America. My base Compustat sample covers the period 1960-2013 for 112 3-digit industries, and consists of 31069 publicly-traded firms who have NAICS code. I define profit as Earnings before Interest, Taxes and Amortization (Ebita). Profit margin is defined as the ratio of Ebita to sales. Sales growth is the growth rate of sales. For each two consecutive years, I keep the panel balanced by disregarding delist and enlist of firm in the two years.\(^{39}\)

In the balanced panel for each two consecutive years, each firm-year observation is categorized into H type or L type according to its position in the profit/profit margin distribution of 3-digit NAICS industry the firm belongs

\(^{39}\) Most of enlist and delist do not carry information on turbulence of firms’ rankings. While delist due to bankruptcy and insolvency may represent a permanent rotation that is not included in my measurement, it comprise less than 3 percents of delist, which is quantitatively insignificant for my analysis.
to: a firm is H type if its annual profit is above median; L type if below median.\footnote{As Ebita is available only in the annual dataset, in each year, I rank firms by their profit and profit margin using annual series of Ebida and sales, then compute the annual rotation rates.} A rotation occurs when a firm switches its type in consecutive years. Rotation rate is the ratio of number of rotation to the number of firms.

I obtain industry value added series constructed by BEA, which are used for the weights to construct the aggregate rotation rate. Yearly industry value added is available only for 1-digit industries (sector), while value added for 2-digit industries are only available every five years in Input-Output (I-O) Accounts Data. I impute value added of 2 digit industries by treating their shares of value added within each 1-digit industry as constant overtime. In particular, value added of industry $i$ in sector $j$ is

$$\text{Value added}_{i,j,t} = \text{Value added}_{j,t} \times S_{i,j}$$

where $S_{i,j}$ is the share of value added of industry $i$ in sector $j$ computed from 2007 Input-Output (I-O) table constructed by BEA.

In the regression analysis shown in tables 1 and 2, I use annual series of aggregate unemployment rate from 1970 to 2013, obtained from Current Employment Statistics (CES). I construct industry employment growth from the annual series of full-time equivalent employees by industry from 1998 to 2013\footnote{Starting from 1998, BEA changed the classification of industries, hence I only use the series after that.}, obtained from National Income and Product Accounts (NIPAs) constructed by BEA. In the industry panel regression, I use the private and non-farm 3 digit NAICS industries that (1) show up in both Compustat and NIPA, and (2) have more than 8 firms in Compustat over the sample periods.

Data for section 5

For the estimation, I use quarterly observations on eight data series from 1969 Q1 to 2013 Q4: aggregate unemployment, aggregate job openings rate, growth rate of real consumption per capita, growth rate of real investment per capita, growth rate of real per-hour wage, real interest rate, and rotation rate for the two sectors.

The benchmark model have two sets/sectors. The real economy, however, contains much more sectors, even according to coarsest categorization. To conduct estimation, I need to construct empirical counterpart of the two sector model. I categorize the two-digit NAICS industries into two categories: industries that are more production related, including Mining and
logging, Construction, Manufacturing, and industries that are more productive service related, including Trade transportation and utilities, Information, Financial activities, Professional and business services. The former category consists 19 percent of non-government employment while the latter consists 48 percent. Four industries which are inappropriate to fit into either category are dropped: Education and health services, Leisure and hospitality, Other services, and Government. I construct the two annual series of sectoral rotation rates by averaging rotation rates at the 3 digit NAICS industry level weighted by their valued added. Then I interpolate them into quarterly series.

For the other observable variables, I use Quarterly aggregate unemployment rate is from Current Employment Statistics (CES). Quarterly job openings are obtained from HWOL as described by Barnichon (2010). I do not use sectoral unemployment rate and job openings because it is nontrivial to identify the sector the unemployed workers belong to. Real interest rate series are constructed by NY Fed. The other series of macro aggregates are extracted from National Income and Product Accounts (NIPAs).

E Churning of rankings in consumer’s preference

In the main part of the model, I assume that firms are ranked by TFP. However, the mechanism is not restricted by churning of rankings in TFP. When firms are ranked by consumer’s preference on them and churning is induced by a shift in consumer’s preference, one can get the similar result.

In this subsection, I briefly describe the case in which churning is caused by a shift in consumer’s preference.

Same as the benchmark model, households derives utility from consumption and disutility from working. What’s different here is that now households’ consumption basket comprises of continuum of non-durable goods which are not perfectly substitutable to each other:

\[ C_t = \left( \int \psi(j) \cdot q(j)^{\rho} \, dj \right)^{\frac{1}{\beta}} \]  \hspace{1cm} (E.1)

where \( \psi(j) \) is households’ preference on goods \( j \).
In each period, representative households solve the following static problem

\[
\max \left( \int \psi(j) \cdot q(j)^\rho \, dj \right)
\]

\[s.t. \quad \int p(j) \cdot q(j) \, dj = C
\]

where \(p(j)\) is goods \(j\)’s price and \(C\) is nominal consumption.

Define \(\sigma = \frac{1}{1-\rho}\) and \(P = \left( \int \psi(j)^\sigma \cdot q(j)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}}\), it’s easy to derive the demand curve from the first order condition of the above problem.

\[
\frac{q(j)}{c} = \left( \frac{p(j) / \psi(j)}{P} \right)^{-\sigma}
\]

(E.2)

where \(c\) is real consumption with \(c = C/P\). Notice that equation E.2 is similar to the classic result of Dixit and Stiglitz (1977) except that the price here is adjusted by preference: given the same level of sales, goods with higher \(\psi\) charges a higher price because households gets higher preference from it. In the subsequent part, I will show how churning of \(\psi\) can play a similar role of churning of productivity.

Same as benchmark model, assume there are two sets of firms, each set has two type of firms: H and L. Firms can match with firms from the other set. Their product’s preference \(\psi\) not only depends on their own type, but also on their cooperation partner’s type. So within each set there are six production departments: \(j\) can be \(H, L, HH, HH, HH, HH\), that is within in each set there are six levels of preference \(\psi(H), \psi(L), \psi(HH), \psi(HL), \psi(LH), \psi(LL)\)

Firms’ value function is similar to that of benchmark model, except here firms have same TFPs but have different preference \(\psi\). In particular, type \(j\) firm’s sales is

\[
\frac{p(j) q(j)}{P} = \left( \frac{q(j)}{c} \right)^{-\frac{1}{\sigma}} \psi(j) q(j)
\]

(E.3)

Plug the production function \(q(j) = z k(j)^\alpha n(j)^{1-\alpha}\) into the above equation and take first order conditions, it can be solved that

\[
MPL(j) = \left( \psi(j) q(j)^{-\sigma} c^{\alpha + \sigma} z \alpha^\alpha \right)^{\frac{1}{1-\alpha}}
\]

(E.4)

It can be seen that \(\psi(j)\) affects \(MPL(j)\) in the same way as TFP and proposition 5 applies to this case.
F Alternative assumptions of ordering of events

In the simple model, I assume that the matching process proceeds in the following steps:

1. Homogeneous firms pay fixed cost.
2. Homogeneous firms and homogeneous workers are matched in the labor market.
3. Homogeneous firms draw types from $H$ and $L$ hence become heterogeneous.
4. Heterogeneous firms match with same type partners from the other set.

I denote the above ordering of events as $T_1$. Some might ask whether the main results of section 3, particularly proposition 3, depend on the assumption that firms draw types after matching with worker. In this subsection, I show that the main results are robust to change of ordering of events.

Consider the following ordering of events:

1. Homogeneous firms pay fixed cost.
2. Homogeneous firms draw types from $H$ and $L$ hence become heterogeneous.
3. Heterogeneous firms and homogeneous workers are matched in the labor market.
4. Heterogeneous firms match with same type partners from the other set.

I denote the above ordering of events as $T_2$. Different with $T_1$, $T_2$ assumes that firms match with worker after drawing types.

As production takes place after the matching process, value function does not depend on the ordering of events in the matching process. Take set $A$ as example, value of firms is described by equation 2 in section 3

$$J_{A,t}^{jk} = \tau \cdot x_t \cdot z_{A,t}^{jk} + \beta (1 - \delta) E_t \left( \rho_t^{jH} J_{A,t+1}^{Hk} + \rho_t^{jL} J_{A,t+1}^{Lk} \right)$$

$j, k \in \{H, L\}$

In step 2 of $T_2$, denote the value of drawing type $H$ and $L$ as $\tilde{J}_A^{H,t}$ and $\tilde{J}_A^{L,t}$. We immediately get

$$\tilde{J}_A^{H,t} = f(\theta_{A,t}) J_{A,t}^{HH}$$
$$\tilde{J}_A^{L,t} = f(\theta_{A,t}) J_{A,t}^{LL}$$

(F.1)

(F.2)
The free entry condition of set $A$, which takes place in step 1 of $T_2$, is

$$\chi = \frac{J_{A,t}^H}{2} + \frac{J_{A,t}^L}{2} \quad (F.3)$$

Plug equation 2, equation F.1 and equation F.2 into equation F.3, the free entry condition of set $A$ becomes

$$\chi = \frac{f(\theta_{A,t}) \left[ \tau \cdot x_t \cdot z_{A,t}^{HH} + \beta \left( 1 - \delta \right) E_t \left( \rho_t^{HH} J_{A,t+1}^{HH} + \rho_t^{HL} J_{A,t+1}^{HL} \right) \right]}{2} \cdot \frac{f(\theta_{A,t}) \left[ \tau \cdot x_t \cdot z_{A,t}^{LL} + \beta \left( 1 - \delta \right) E_t \left( \rho_t^{LH} J_{A,t+1}^{HL} + \rho_t^{LL} J_{A,t+1}^{LL} \right) \right]}{2} \quad (F.4)$$

It is easy to see that equation F.4, which is the free entry condition under $T_2$, is identical with the free entry condition under $T_1$. Therefore, the models under orderings of events $T_1$ and $T_2$ are equivalent, and should yield same results.
Figure 6: Rotation rate, ranked by profit margin

Figure 7: Large rotation rate, ranked by profit margin