Remittances, Migration, and the Small Open Economy
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Abstract: In the recent past we have observed an increasing movement of people from developing countries to more developed ones, and at the same time we have observed a significant increase in remittances flowing back to those labor exporting countries. A limited participation model is developed to account for these labor flows from a small open economy and the remittance’s flows into this small open economy, endogenizing the allocation of labor between these two markets from the small open economy’s perspective. The static macroeconomic impact of various levels of migrant assimilation to the developed country and subsistence requirements of the migrant population are examined. Monetary and technology shocks are then introduced to uncover the dynamic impact on the main aggregates of a small open economy.

Keywords: Migration; Remittances; Latin America; Limited participation model.

JEL Classification: O15; F24; N96

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1.- Introduction

We are witnessing an increasing interest in the impact of immigrant remittances on the recipient economy. One major reason is the significant increase in officially reported remittances flowing into developing countries. During the past ten years reported remittances have increased from $5 billion U.S. dollars to $69.2 billion U.S. dollars in 2008 (Inter-American Development Bank, 2009). This striking increase in reported remittances has been attributed to lower fees charged by money transfer institutions, to improved data collection on international flows, and to the renewed increase in migration to developed countries of the late 1990s and early 2000s. Preliminary data for 2009 suggests that this trend has slowed, with remittances flows to Latin America actually falling by almost 10%. This drop in remittances has been attributed to the global economic slowdown, but perhaps also suggests an end of the increase in reported remittances, an end to the “formalization” of previously unreported flows as lower fees led remitters to turn to money transfer institutions and hence allowed data collectors to count a higher proportion of remittance flows.

Recent economic research has examined the impact of remittances on the standard of living of receiving households and has found that remittances contribute to higher levels of consumption, health, and education (Keely and Tran (1989), Leon-Ledesma and Piracha (2004), De Hass (2006), Cox-Edwards and Ureta (2003)) as well as financial penetration and entrepreneurial promotion (Giuliano and Ruiz-Arranz (2009)). At the macro level researchers have found that remittances have a positive effect on economic activity, consumption, investment, and leisure (Durand et. Al (1996), Widgren and Martin (2002), Heilman (2006), Jansen, Vacaflorles, and Naufal (2009)) but can pose a threat on inflation, government policy, and the real exchange rate (Chami et. al. (2006), Amuedo-Dorantes and Pozo (2004), Acosta et. al. (2009)).

While these studies have contributed to our understanding of the effect of migrant workers’ remittances on the labor of exporting households and countries, they have concentrated on the monetary flows entering the developing countries and omitted the labor outflows that make possible these monetary flows. Most of the literature treats the migration process as a household decision, where the family decides to send abroad a portion of their labor force with the expectation that it will receive economic compensation after the migrant settles abroad. From a macroeconomic perspective one can envision a country allowing/facilitating a portion of its labor force to emigrate with the expectation that in the near future these workers living abroad will send remittances back home. In either case the home country will experience a decline in the number of workers available for production at home – and possibly in the quality of the remaining workers if one accounts for the fact that migrants may be more entrepreneurial
and more willing to take risks. This reduction in the home labor force reduces the level of output produced domestically. On the other hand, remittances might lower the domestic nominal interest rate, increase investment and capital, provide commercial connections, facilitate future human capital development, and even promote financial deepening, all of which can lead to higher levels of output in the future.

Of course the overall effect becomes dependent on the magnitude of these various impacts. Explicitly taking account of these flows would allow us to determine the full extent of the migration/remittances impact of the labor exporting countries. Further, the conventional wisdom in the remittances literature suggests that increases in remittances give rise to an increase in leisure time in the recipient country, as recipients enjoy additional income and respond by increasing consumption and leisure (i.e. Acosta et. al. (2009), Jansen, Vacaflores, and Naufal (2009)). This result is less clear if one considers that remittances are not just a gift from relatives but are in fact a household decision in their labor allocation. In this situation remittances require labor effort and this may lead to a different impact on household decisions regarding leisure.

In this paper we build a stochastic limited participation model to study the impact of migration and remittances on a remittances-receiving small open economy. Our model has a cash-in-advance constraint and costly adjustment of cash holdings in order to better capture exchange rate dynamics. We have endogenous migration and remittances decision by households in the small open economy, and we examine the impact of remittances on the steady state as well as on the dynamics of the main macroeconomic aggregates. The main contribution of this paper is to provide a model to examine the impact of migration and remittances on the main macroeconomic aggregates of a small open economy when migration and remittances decisions are endogenous.

The paper has the following organization. In Section 2 we present a brief literature review on migration and remittances. In Section 3 develops a theoretical model that endogenizes the migration decision and Section 4 discusses the results. Section 5 provides a robustness check and Section 6 summarizes and concludes.

2.- Literature Review

There have been many advances in the remittances literature at micro level. Using survey data researchers have uncovered motives for emigrants to send money to friends and relatives staying behind, concentrating on self-interest and altruism (Lucas and Stark (1985)). This survey data has also allowed researchers to uncover the uses of those remittances by the receiving households and its impact of the
local communities, especially in terms of consumption, education, health care, access to finances, and entrepreneurial growth. The magnitude of remittances has also been investigated, and recent data from the U.S. indicates that immigrants send to their relatives back home an average of $700 dollars per quarter, representing approximately 15% of recipient household income (SOURCE??).

The magnitude of remittances in some countries such as Honduras and Jordan reach levels above 10 percent of GDP, leading researchers to examine the impact of remittances on macroeconomic aggregates as well. However, the volume of macroeconomic research on remittances is much less than the volume of microeconomic research, and is skewed toward more recent papers.

With regards to the welfare of recipients, initially it was found that when emigration takes place the welfare of the remaining residents falls, or at best remains unchanged, as emigration reduces productivity (see Rivera-Batiz (1982)). However, Djajic (1998) suggests the possibility of the opposite impact. If foreign capital exists and if both migrant and remaining populations are identical, if the pattern of implicit trade of migrants and remaining population is the same, or if the emigrating population sends remittances that improves the terms of trade of the remaining population or are used for capital accumulation, then productivity might rise with out-migration. Recently, Michael (2003) found that marginal emigration benefits remaining workers when there is no capital mobility or non-traded goods, but an increase in capital mobility could dampen this positive effect or even reverse it. These last two findings are for permanent exogenous migration.

Endogenous migration is found to reduce domestic production and increase aggregate remittances, but its beneficial welfare implications on the remaining population is dependent on remittances being greater than the productivity of the migrant (McCormick and Wahba (2000)). The higher demand for non-traded goods that arises from the higher levels of remittances leads to a Dutch disease type of exchange rate appreciation, which has been extensively documented by now (Chami et. al. (2006), Amuedo-Dorantes and Pozo (2004), Acosta et. al. (2009)).

Results showing that an increase in remittances leads to a decline in work effort/supply are becoming common too. Acosta et al. (2009) show that remittances leads to a decline in labor supply, irrespective of the motivation for remitting. They also reports that remittances lead to an increase in the demand for non-tradable goods, creating the Dutch disease phenomenon as prices rise in this sector and appreciate the domestic currency. Jansen, Vacaflores, and Naufal (2009) also show that a remittances shock leads to an increase in leisure time (and consumption), and that the increase in leisure leads to an initial drop in domestic output even if capital is increasing. They also show that the end use of remittances and the cyclicality of these flows only affect the magnitude of the output responses, not the direction.
One caveat regarding the previous literature on the macroeconomic impact of remittances is that it has treated the migration decision as exogenous. This has important implications for the model results. In particular, a common finding is that increases in remittances reduce labor hours and hence output, as recipient households respond to the increased income flow by ‘purchasing’ more leisure as well as more consumption. With the introduction of endogenous migration and remittances, the household in the small open economy will be now deciding how to allocate its labor across countries. The remittance flows are not modeled as a gift from abroad but as a part of the household’s labor-leisure and labor allocation choice. As such the wealth impact on hours is attenuated, as is the output response. At the same time, the emigration of labor has other impacts on the economy in equilibrium, including reducing domestic output relative to a non-migration model as the domestic labor force is reduced.

In a somewhat similar approach, Mandelman and Zlate (2008) build a two-country DSGE model to account for migration from the labor-receiving country’s perspective, and find that emigration from the developing country increases with the expected stream of future wage gains by working in the foreign country, but such increases are reduced by increasing migration sunk costs. They also find that the introduction of capital flows (i.e., existence of bonds) alleviates the incentive for labor to migrate. While they endogenize the decision to migrate, they don’t include monetary issues and are silent about assimilation patterns in the destination country or the value that families place in being together.

Endogenizing the decision to supply labor between two different markets brings equilibrium difficulties, and thus requires the introduction of “migration brakes” to preserve labor distributions that reflect current patterns. Related work in the area has introduced migration frictions through a fixed migration cost, countercyclical taxes on migrating labor (Mandelman and Zlate (2008)), a requirement to work in a less remunerative sector for a period when returning to the source country (McCormick and Wahba (2000), or a requirement to go one period without work in the destination country.

The main frictions used in our model are given by the following three requirements. First we introduce an adjustment cost on money balances that reduces the reallocation of money balances between cash for consumption and deposits in the financial intermediary. This adjustment cost on money cash allows us to replicate the observed persistent liquidity effect. Then we introduce a cost of migration, in terms of time for the migrant population, to slow down the reallocation of labor between the two economies and to reflect migration costs. This cost of migration is incurred only on the portion of the population migrating abroad. We also introduce a term to account for the value that the household receives from family unity, the value that a household places in having the family together in one place.
We also explore on the effects of varying levels of assimilation on immigration and consequently remittances. Stark and Hyll (2008) show that migrants who due to altruism send remittances to their families back home exert higher effort to assimilate to the host country’s culture. They concentrate on the immigrant interpersonal comparisons that give rise to relative deprivation which affects the utility of the immigrant negatively. The more common understanding of assimilation suggests that the higher the rate of assimilation to the host country’s culture the weaker the family ties with the family left behind, and consequently the lower the level of remittances sent by the emigrant. The literature has shown that the longer the immigrant stays in the host country the more that he assimilates, and the increasingly smaller amounts of remittances being send back home (i.e. Funkhouser (1995)).

In the rest of this paper we develop and analyze a theoretical model in which households are allowed to supply their labor in the domestic economy and abroad, and that a fraction of foreign earnings from labor are sent back to the households in a small open economy. Households react as optimizing agents, and remittances are modeled as occurring in foreign currency. Our small open economy focus allows us to rationalize our (implicit) assumption that remittances do not impact the remitting economy. Our model generates the expected effects of remittances on optimizing agents, and our goal is to study the quantitative and qualitative dynamic responses that lead to the steady state results or that occur in response to shocks to the money supply and technology.

3.- Theoretical Model

We use a small open economy framework, with perfect competition in the goods market, such that domestic and foreign firms produce an identical good whose price in domestic currency (e.g. pesos) is given by \( P_t \). The law of one price holds, such that purchasing power parity is given by:

\[
P_t = s_t P^* \tag{1}
\]

where \( s_t \) is the nominal exchange rate – e.g. pesos per dollar – and \( P^* \) is the foreign price level (e.g. dollars).

The domestic population \( N_t \) is split between the domestic population living at home \( N_t^H \) and the domestic population living abroad, \( N_t^M \). We assume that the domestic population living at home, \( N_t^H \), grows at an exogenous constant rate \( f \) due to net births, except that migration abroad \( (J_t) \) reduces the domestic population living at home. That is,
\[ N_i^{H} = N_i^{H}(1 + f) - J_i. \]  

(2)

The domestic population living abroad, \( N_i^{M} \), also grows at the exogenous constant rate \( f \) due to net births, but the domestic population living abroad loses its domestic identity at the rate \( \chi > f \), so the net effect is for the domestic population living abroad to on balance shrink over time at the rate \( \chi - f \), except for migration \( J_i \). That is,

\[ N_i^{M} = N_i^{M}(1 + f - \chi) + J_i \]  

(3)

Without continued migration, the domestic population living abroad eventually disappears (is absorbed) into the foreign population.

Ours is a representative household model. The domestic economy has a fixed number of households, HH. The number of persons in a household is \((N_i / HH)\), so household size grows at the rate of population growth. The household consumes at home, but it provides labor both at home and abroad. The household allocates a fraction of its workers to the home labor market, \( N_i^{H} / N_i \), and the remaining fraction of its workers to the foreign labor market, \( N_i^{M} / N_i \). The household optimally allocates its workforce based on relative wages and on various costs of migration.

Each of the \((N_i / HH)\) individuals in the household have an allocation of one unit of time. Households decide what fraction of its workers will live at home and abroad, and also the number of hours worked for workers in each locale. The time available to the representative individual within a household is \( n_i \) (normalized to unity), divided between the fraction of time spent at home, \( n_i^{H} \), and the fraction of time spent abroad, \( n_i^{M} \). Thus we have:

\[ 1 = n_i = N_i / N_i; n_i = N_i^{H} / N_i + N_i^{M} / N_i = n_i^{H} + n_i^{M} \]  

(4)

\[ n_i^{H} = \frac{N_i^{H}}{N_i} = \frac{(N_i - N_i^{M})(1 + f) - J_i}{(1 + f)N_i - \chi N_i^{M}} = \frac{(1 + f)(1 - n_i^{M}) - J_i(1 + f - \chi n_i^{M})}{(1 + f) - \chi n_i^{M}} \]  

(5)

\[ n_i^{M} = \frac{N_i^{M}}{N_i} = \frac{N_i^{M}(1 + f - \chi) + J_i}{(1 + f)N_i - \chi N_i^{M}} = \frac{(1 + f - \chi)n_i^{M} + J_i(1 + f - \chi n_i^{M})}{(1 + f) - \chi n_i^{M}} \]  

(6)
The allocation of worker time at home and abroad depends on the migration decision, the rate at which the household transfers member time between home and abroad. The migration rate is labeled \( j_t = J_t / N_t \).

Household utility depends on the number of people in a household multiplied by the utility of the representative individual. The household migration decision impacts per-person utility but also changes the size of the household over time. That is, population growth is partly endogenous, because out-migration leads, ceteris paribus, to lower home population in the future, and, again ceteris paribus, to lower total utility. We can pose the household's optimization problem as a problem for the representative individual within a household. The representative agent is the household, and the agent’s objective is to choose a path for consumption, time allocation at home and abroad, and asset holdings to maximize:

\[
\sum_{t=0}^{\infty} \beta^t \frac{N_t}{HH} U(c_t, l_t^H, l_t^M, (n_t^H - n_t^M)^2)
\]

where \( c \) is individual consumption and \( l_t^H \) and \( l_t^M \) are the individual leisure hours at home and abroad, respectively. The last term in the utility function captures the disutility of splitting the family – we assume families prefer not to be split up. We constrain the amount of people living at home to be the larger portion of the distribution, \( n_t^H \geq 1/2 \). From here on all relationships will be denoted in representative agent terms.

We specify the following per-period individual utility function as:

\[
U_t = \frac{(c_t)^{1-\gamma_H-\gamma_M} (l_t^H)^{\gamma_H} (l_t^M)^{\gamma_M})^{1-\sigma}}{1-\sigma} + \Phi \frac{(n_t^H - n_t^M)^2}{2}
\]

Given the distribution of time at home and abroad, it is straightforward to specify individual leisure as \( l_t^H = n_t^H - h_t^H - \Omega_t \) and \( l_t^M = n_t^M - h_t^M - \Omega_t^* \), where \( h_t^H \) and \( h_t^M \) are hours worked in each location, \( \Omega \) is time spent adjusting money balances, and \( \Omega^* \) is the migration cost incurred by the migrant portion of the population. We assume that this adjustment cost of money holdings is borne by household members living at home, where the household is ‘headquartered.’ The adjustment cost for migration reduces leisure of household members living abroad. These two adjustment costs are specified as:
\[
\Omega_i = \frac{\varepsilon}{2} \left( \frac{M_i^c}{M_i^c} - g \right)^2 \quad \Omega_i^* = \frac{\psi}{2} (j_t - j)^2
\]

The cash-in-advance (CIA) constraint takes the usual form:

\[P_i c_t \leq M_i^c \quad (9)\]

where \( M_i^c \) denotes cash brought forward from period \( t - 1 \).

The representative agent can hold foreign assets that yield a risk-free exogenous nominal interest rate \( i^* \). In each period the individual buys foreign assets \( B_{t+1} \) denominated in the foreign currency, so the nominal exchange rate becomes a key variable in the portfolio decision.

The individual budget constraint is given by:

\[
\begin{align*}
\frac{N_{t+1}}{N_t} M_i^c + \frac{N_{t+1}}{N_t} M_i^b + s_t \frac{N_{t+1}}{N_t} B_{t+1} + P_i c_t & \leq M_i^c + P_i w_t h_t^H + s_t (1 - \varphi) P^* w^* h_t^M + (1 + i_t) M_i^b \\
& + s_t (1 + i^*_t) B_t + D^f_t + D^b_t
\end{align*}
\]

At time \( t \) the individual determines consumption \( c_t \), the distribution of money for the next period between the amount deposited in banks, \( M_i^b \), and the amount kept as cash, \( M_i^c \), the foreign asset position for the next period \( B_{t+1} \), the migration level \( j_t \), and the amount of time spent working at home and abroad, \( h_t^H \) and \( h_t^M \). In particular, the allocation of workers at home and abroad is largely predetermined at time \( t \), so the hours-worked decision determines labor income at time \( t \). The migration decision at time \( t \) will of course contribute to a different allocation of workers at home and abroad in future periods.

The representative agent’s income is determined by the real wage \( w_t \) received by the fraction of household workers working domestically, as well as the income received from the fraction of workers working abroad. The household also receives at the end of the period the profits (or dividends) from both the firm and the bank, \( D^f_t \) and \( D^b_t \), as well as interest on deposits and on foreign bonds.

Note that the income received from the fraction of the domestic population working abroad is the remittances, funds that are sent to the home country. These funds are denominated in the foreign
currency (e.g. dollars) and are a fraction of the total wages earned by the domestic population working abroad. We think of $\varphi$ as the fraction of income earned abroad that is required to cover living costs incurred when working abroad. Thus remittances in this specification, in the foreign currency, are endogenous by construction, and are specified as:

$$\mathcal{R}_t = (1-\varphi)P^*w^*h_t^M$$  \hspace{1cm} (11)$$

The household’s maximization problem can be represented by the value function

$$V(M_t^c, M_t^b, B_t, n_{t-1}^H, n_{t-1}^M) = \max_{\{c_t, b_t^H, b_t^M, M_{t+1}, M_{t+1}^b, B_{t+1}\}} \left\{ \frac{N_t}{HH}U(c_t, l_t^H, l_t^M, \frac{(n_t^H - n_{t-1}^M)^2}{2}) + \beta E_t V(M_{t+1}^c, M_{t+1}^b, B_{t+1}, n_{t+1}^H, n_{t+1}^M) \right\}$$

subject to the cash-in-advance constraint (9) and the budget constraint (10). Letting $\lambda_t$ denote the Lagrangean multiplier associated with the budget constraint, the first order conditions are given by:

$$P_t\lambda_t = \beta E_t \left[ \frac{P_{t+1}\lambda_{t+1}}{\pi_{t+1}} s_{t+1} N_t}{s_t N_{t+1}} (1+i_{t+1}^*) \right]$$  \hspace{1cm} (12)$$

$$P_t\lambda_t = \beta E_t \left[ \frac{P_{t+1}\lambda_{t+1}}{\pi_{t+1}} N_t}{N_{t+1}} (1+i_{t+1}) \right]$$  \hspace{1cm} (13)$$

$$\frac{\Gamma_t}{\pi_t} = \gamma_t^M (n_t^H - h_t^H - \Omega_t) \gamma_t^H (n_t^M - h_t^M - \Omega_t)$$  \hspace{1cm} (14)$$

$$2\frac{\Phi}{\gamma_t^H} (n_t^H - n_t^M) \Lambda_t \omega_t c_t^{(\gamma_t^H(1-\gamma^H\gamma^M))} l_t^{(\gamma_t^M(\gamma_t^H(1-\gamma^H\gamma^M))} (l_t^M)^{\gamma_t^M(\gamma_t^H(1-\gamma^H\gamma^M))} + \Lambda_t \omega_t (1-\gamma_t^M l_t^H) (1-\gamma_t^H l_t^M) = \hspace{1cm} (15)$$

$$\beta E_t \left[ -2\Phi(n_t^H - n_t) \Lambda_{t+1} w_{t+1}^{(1+f)(1+f-\chi)} \right] c_{t+1}^{(\gamma_t^H(1-\gamma^H\gamma^M))} l_{t+1}^{(\gamma_t^M(\gamma_t^H(1-\gamma^H\gamma^M))} (l_{t+1}^M)^{\gamma_t^M(\gamma_t^H(1-\gamma^H\gamma^M))} \right]$$

$$+ \Lambda_{t+1} w_{t+1} \gamma_{t+1}^M l_{t+1}^{(1+f)(1+f-\chi)} (1-f-\chi) (1-f-\chi)^2 - \Lambda_{t+1} w_{t+1} \gamma_{t+1}^M l_{t+1}^{(1+f)(1+f-\chi)} (1-f-\chi) (1-f-\chi)^2$$

$$P_t \lambda_t w_t \frac{\zeta}{M_t^c} + P_t \lambda_t \frac{N_{t+1}}{N_t} = \beta E_t \left[ P_{t+1} \lambda_{t+1} w_{t+1} \frac{\zeta M_{t+1}^c}{M_{t+1}} + \beta E_t \left[ P_{t+1} \lambda_{t+1} w_{t+1} \frac{\zeta M_{t+1}^c}{M_{t+1}} \right] \right]$$  \hspace{1cm} (16)$$

$$+ \beta E_t \left[ P_{t+1} \lambda_{t+1} w_{t+1} \frac{(1-\gamma^H - \gamma^M)(n_{t+1}^H - h_{t+1}^H - \Omega_{t+1})}{\gamma_t^H c_{t+1} \pi_{t+1}} \right]$$
Here \( \frac{N_{t+1}}{N_t} = 1 + f - \chi^M_t \) is the adjustment term for population growth. For notational ease we also define \( \Delta M^c_t = \frac{M^c_{t+1} N_{t+1}}{M^c_t N_t} \).

Equation (12) requires equality of the current marginal cost of buying foreign assets (in terms of wealth) with the gains in the following period from holding such assets today and Equation (13) requires equality between the costs and benefits of bank deposits, both adjusted for population growth. These are the usual asset pricing equations. Equation (14) requires equality between the marginal disutility of working at home and the marginal disutility of working abroad, adjusted by the ratio of remittances in domestic currency to the domestic real wage times inflation, while Equation (15) requires equality between the cost and benefit of migrating. Equation (16) equates the costs and benefits related to the choice made at time \( t \) of money holdings available for consumption in the following period. It is clear that if the adjustment cost is zero (\( \xi = 0 \)) then equation (16) will just equate the household’s cost of holding money in the current period to the marginal utility of consumption in the following period, properly discounted. However, when adjustment costs exist (\( \xi \neq 0 \)), the household will compare the cost of changing money holdings (cash) today to the benefits accrued in the next period with respect to the purchasing power of money holdings and the in-advance time saved rearranging the household portfolio.

The production technology of the firm is given by the following Cobb-Douglas function

\[
Y_t = e^{\xi} K^\alpha_t h^{1-\alpha}_t
\]

where \( \alpha \in [0,1] \) and \( K \) is the usual physical capital.

Nominal profits of the firm are given by

\[
D^f_t = P_t Y_t - P_t w_t h^M_t - P_t (1 + i_t) I_t - P_t \Theta_t
\]

with investment evolving according to the law of motion of the stock of physical capital,

\[
I_t = K_{t+1} \frac{N_{t+1}}{N_t} - (1 - \delta) K_t
\]

with \( \delta \) being the (constant) depreciation rate, and \( \Theta \) being the adjustment cost of capital in per representative agent terms. This last one is given by
\[
\Theta_t = \frac{\nu}{2} \left( K_{t+1} \frac{N_{t+1}}{N_t} - K_t \right)^2
\]  

(20)

Here the firm borrows at the beginning of time \( t \) to finance capital for period \( t+1 \), and the loan is repaid at the end of period \( t \) out of profits earned during period \( t \). The value function of the firm is then

\[
V(K_t) = \max \left\{ D_t^f + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \right] V(K_{t+1}) \right\}
\]

(21)

The first order necessary conditions for the household’s choice of labor and capital take the form:

\[
w_t = (1 - \alpha) \frac{Y_t}{h_t^m}
\]

(22)

\[(1 + i_t) \frac{N_{t+1}}{N_t} + \nu \left( K_{t+1} \frac{N_{t+1}}{N_t} - K_t \right) \frac{N_{t+1}}{N_t} = \]

\[
\beta E_t \left[ \frac{P_{t+1} \lambda_{t+1}}{P_t \lambda_t} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) + \nu \left( K_{t+2} \frac{N_{t+2}}{N_{t+1}} - K_{t+1} \right) \right) \right]
\]

(23)

Equation (22) indicates that the cost of hiring an additional worker should equal that worker’s marginal productivity, and equation (23) requires equality between the cost and benefit of the marginal investment.

The money stock evolves according to

\[
M_{t+1} \frac{N_{t+1}}{N_t} = M_t + X_t
\]

(24)

where the Central Bank’s money injection is defined as

\[
X_t = (g_t - 1)M_t
\]

(25)
and \( g_t \) represents the monetary growth factor\(^3\). Equation (24) indicates that money growth in the economy depends on the existing stock of money \( M_t \) and the monetary injection implemented by the central bank \( X_t \). The timing here is that \( M_t \) is the beginning-of-period \( t \) money stock.

At the beginning of the period, the financial intermediary or ‘bank’ receives deposits from the representative agent, \( M^b_t \), and also the monetary injection from the Central Bank, \( X_t \), as deposits. These funds are then available for lending to the firm to pay for the firm’s investment in physical capital. At the end of the period, the firm repays its loans, and the bank returns deposits to the household along with the appropriate interest payment.

To make this clearer, the bank’s nominal asset balance is given by

\[
P_t I_t = M^b_t + X_t
\]

Here \( P_t I_t \) are the loans made to firms to finance investment, and the right hand side lists the sources of funds including bank deposits and monetary injections from the central bank.

Bank profits per period are equal to the interest on loans minus interest paid on deposits. Note that the monetary injection directly into banks is a subsidy to the bank in that there is no interest expense incurred by the bank on those funds. Note also that we have equality between the loan rate and the deposit rate. Absent monetary injections, the bank earns zero economic profits, so:

\[
D^b_t = (1 + i_t) P_t I_t - (1 + i_t) M^b_t
\]

Putting both expressions together, profits of the intermediary depend only on the money injection provided by the monetary authority

\[
D^b_t = (1 + i_t) X_t
\]

It is worth to note that there is an uncovered interest rate parity condition (UIP) from combining equations (12) and (13):

\[
\pi_e + \lambda_{t+1} (1 + i_{t+1}) \frac{N_t}{N_{t+1}} = \pi_e (1 + \pi_{t+1}) \frac{e_{t+1}}{e_t} \frac{(1 + i^*_{t+1}) N_t}{N_{t+1}}
\]

\(^3\) Note that we specify monetary policy being determined by money growth instead of an interest-rate policy to better reflect the more common monetary policy of Latin America.
Here $\pi$ is the net inflation rate at time $t+1$. Since we are modeling a small open economy with international assets freely traded, the no-arbitrage condition leads to UIP.

The household can hold any quantity of foreign assets, subject only to its budget constraint. From equation (10) and market equilibrium we infer that foreign asset holdings evolve according to

$$s_t B_t \frac{N_{t+1}}{N_t} - s_t(1+i_t^*)B_t = P_t(Y_t - C_t - I_t) - \frac{U}{2}(K_{t+1} \frac{N_{t+1}}{N_t} - K_t)^2 + s_t R_t$$  \hspace{1cm} (30)

Equation (30) relates domestic production and absorption to an economy’s foreign asset position, giving the balance of payments equilibrium. If a country’s production is greater than its absorption, that country has a balance of trade surplus and a negative capital account, so its foreign asset holdings will increase when there are no remittances flowing into the country. Of course, the actual equilibrium impact of remittances on future bond holdings depends on its impact on output, consumption, and investment.

We also introduce the interest rate differential on bond holdings as

$$i_t^* = i_t^W - \tau s_{t-1} B_t$$  \hspace{1cm} (31)

where the interest in bonds is determined by the world interest rate and the net real foreign asset position, with $\tau$ calibrating the asset position. This assumption leads to a lower bond rate as the country’s net asset position improves. That is, the more foreign bonds held (valued in local currency), the lower is the interest rate on those bonds. The reason for this assumption is to avoid an instability problem with non-stationary behavior on bonds. (See the discussion in Karame, Patureau, and Sopraseuth (2008), Killman (2002), and Ghironi (2001)).

The shocks are given by the standard specifications: The monetary growth factor $g_t$ is specified as:

$$\log(g_{t+1}) = (1 - \rho_g) \log(\overline{g}) + \rho_g \log(g_t) + \varepsilon_{g,t+1}$$  \hspace{1cm} (32)

We specify the technology shock to the production function in the usual way,

$$\log(z_{t+1}) = (1 - \rho_z) \log(\overline{z}) + \rho_z \log(z_t) + \varepsilon_{z,t+1}$$  \hspace{1cm} (33)

Here $\varepsilon_{g,t+1}$ and $\varepsilon_{z,t+1}$ are white noise innovations with variance $\sigma_g^2$ and $\sigma_z^2$, respectively.
Steady State Equilibrium:

The calibration for the small open economy uses quarterly data and is based on Jansen, Vacaflores, and Naufal (2009) using a sample of Latin American countries: Bolivia, Brazil, Colombia, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Panama, and Peru. Table 1 lists the values we assign to the basic parameters. The capital share, $\alpha$, is set to 0.4. The subjective discount factor $\beta$ is set at 0.988, implying a real interest rate equal to 1.2% per quarter. The depreciation rate on capital is set to 2% per quarter. The long run gross inflation factor is given by $\Pi$, and is based on the average inflation factor of the countries in our sample. We set the average gross money growth rate parameter, $g$, to 1.038, or 3.8% per quarter. The parameters of the money process, $\rho_g$ and $\sigma_g$, are obtained from regressions. Finally, we calibrated the technology shock, persistence and variance, to standard levels.

Population growth, $f$, is assumed to be 1% per quarter, and the elasticity of leisure for domestic labor working in the domestic market, $\gamma^H$, is set at 0.67, while the elasticity of leisure for domestic labor working abroad, $\gamma^M$, is set to 0.08. Thus leisure taken by workers staying home contributes substantially more to household utility than leisure taken by workers sent abroad. (Note that the sum of these elasticities is 0.75, similar to the labor elasticity of Jansen, Vacaflores and Naufal (2009). For calibration purposes, time spent working is assumed to be 20% of total time, representing around 33 hours per week. The parameter $\sigma$ is set to the standard value of 1.5.

To facilitate calibration we specify foreign real wages as a proportion of domestic real wages, $w^* = \phi w$, and assume that foreign wages are 4 times larger than the domestic wages. (For instance, Mexico, a relatively high income state among the countries we considered, has a per capita income in 2009 of about $14,000, while the U.S. has a per capita income of about $47,000, a ratio of 3.3.) We also assume that the proportion of the foreign wage used for subsistence, $\phi$, is 73.5% of foreign income, so the migrant population sends 26.5% of their income back to their home country household in the form of remittances. (Our steady state calibrations have the home country wage at about $2.70 or about $380 per month at 33 hours per week, and the foreign wage is about $10.80 or $1,520 per month at 33 hours per week. Then 25% of this is $380 per month. We have documented evidence of remittances average about $300 per month, so our assumptions and steady state values appear to be within reason.)
We assume the existence of positive adjustment costs to allow for the liquidity effect, and consider the case of a small but positive adjustment cost parameter, $\xi = 10$. This positive adjustment costs represent lost time rearranging money cash balances of almost 2 minutes per week. Finally, we also allow for small but positive capital adjustment cost, $\nu$, migration costs, $\psi$, and value for family unity, $\Phi$.

The equations are written to describe a stationary system and are presented in the appendix. Nominal variables are made stationary by dividing them by the lagged domestic price level. The main variables are:

$$m_t = M_t / P_{t-1}; m^b_t = M^b_t / P^b_{t-1}; \pi_t = P_t / P_{t-1}; b_t = s_t B_t / P_{t-1}; \Gamma_t = s_t \Re_t / P_{t-1}$$

**Steady State Equilibrium**

We outline the calculation of steady state equilibrium values for the remaining variables in this section. Obviously adjustment costs and migration costs disappear in the steady state, and steady state values do not need time subscripts. In long-run equilibrium we assume the domestic gross inflation rate is given by the gross money growth rate adjusted for population growth, $\Pi = g / (1 + f - \chi n^M)$. This also leads to our steady state value for our definition of changes in money cash to be $\Delta M^c = g$.

We look at a steady state in which the domestic and foreign inflation levels are the same, so purchasing power parity implies that the nominal exchange rate is constant. Consequently the uncovered interest rate parity condition implies that the steady state domestic interest rate and the steady state interest rate on foreign bonds are equal ($i = i^*$).

---

4 Note that this assumption sets the steady-state nominal exchange rate to be constant, allowing a different steady-state foreign inflation rate will make the steady-state exchange rate grow at a constant rate.
The calibration of our model allows us to examine steady state differences under various assumptions of key parameters. Here we describe the main differences of steady state equilibrium for permanent changes in the assimilation rate of the migrant population to the host culture and permanent changes in the cost of living, or subsistence, for those emigrants working abroad.

The steady state values presented in Table 2 examine the behavior of the economy under three alternative assumptions about the rate at which the emigrant labor assimilates in the foreign culture, thus loosening ties with the migrant-supplying household and, in our model, leaving the household. This parameter is meant to represent the empirical evidence that as time goes by the immigrant assimilates with the new culture and his relationship with the culture of origin diminishes, leading to the migrant separating from the original household. We examine a 1%, 2%, and 5% quarterly assimilation rate. For comparison, the rate of population growth is 1% per quarter, and thus the baseline assimilation rate of 2% per quarter means that the stock of the migrant population shrinks through time if migration flows are zero.

A key finding is that as the assimilation rate increases the household will find it desirable to send more labor abroad to sustain the utility-maximizing distribution of labor between the domestic and foreign markets. In fact, the increase in emigration is large enough to expand the allocation of workers living abroad, raising the percent of household time allocated abroad from 0.0995 to 0.1 to 0.1016 as the assimilation rate increases from 1 to 2 to 5 percent. This higher emigration necessarily leads to a decline in the time allocated to the domestic market, and consequently leads to an increase in real wages for those workers remaining at home. Because we assume that in the steady state domestic workers spend 20 percent of their time working, this increased allocation of labor abroad leads to a marginal decline in hours worked in the domestic economy. The result also shows a slight increase in steady state investment as the assimilation rate increases. This is needed to support the 4.19% increase in capital as the assimilation rate increases from 1% to 2%, and the 14.18% increase in capital as the assimilation rate increases from 2% to 5%. The increase in capital outweighs the slight decline in domestic work hours and leads to an increase in steady state output. There is an increase consumption by 1.63% as the assimilation rate increases from 1% to 2% and by an additional 5.15% as the assimilation rate increases from 2% to 5%.

\footnote{This assumption may seem too rigid but is needed in order to determine the steady state allocation of labor between the two markets. We can determine the relative working time at home and abroad, but not the absolute level of each variable. Here we chose to fix domestic working time and make working time abroad endogenous. Alternatively we could fix working time abroad and have domestic working time be endogenous.}
The results also show an increase in migrant labor hours, which together with the slight increase in the stock of emigrants in the foreign economy leads to an increase in remittances of 4.5% as the assimilation rate increases from 1% to 2% and of an additional 14.67% as the assimilation rate increases from 2% to 5%. Somewhat counter-intuitively, the increase in assimilation rate results in higher utility, as the increased consumption is large enough to outweigh the declines in household leisure and the decline in family unity. Note however that this is per person utility, not household utility. The increased assimilation rate will lower the population growth rate in the home country as there is an increased outflow of workers, and this will lower overall household utility because households are smaller, ceteris paribus.

Table 2 – Steady State Values According to Rate of Assimilation Abroad

<table>
<thead>
<tr>
<th></th>
<th>1% per quarter</th>
<th>2% per quarter</th>
<th>5% per quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest Rate</td>
<td>0.0506</td>
<td>0.0506</td>
<td>0.0506</td>
</tr>
<tr>
<td>Investment</td>
<td>0.2121</td>
<td>0.2133</td>
<td>0.2168</td>
</tr>
<tr>
<td>Capital</td>
<td>7.3113</td>
<td>7.6177</td>
<td>8.6978</td>
</tr>
<tr>
<td>Output</td>
<td>0.7923</td>
<td>0.8052</td>
<td>0.8481</td>
</tr>
<tr>
<td>Domestic Time</td>
<td>0.9005</td>
<td>0.9</td>
<td>0.8984</td>
</tr>
<tr>
<td>D. Hours of Work</td>
<td>0.1801</td>
<td>0.18</td>
<td>0.1797</td>
</tr>
<tr>
<td>Migrant Time</td>
<td>0.0995</td>
<td>0.1</td>
<td>0.1016</td>
</tr>
<tr>
<td>M. Hours of Work</td>
<td>0.0203</td>
<td>0.0209</td>
<td>0.0226</td>
</tr>
<tr>
<td>Real Wages</td>
<td>2.6396</td>
<td>2.6839</td>
<td>2.8321</td>
</tr>
<tr>
<td>Migration</td>
<td>8.88E-04</td>
<td>0.0018</td>
<td>0.0045</td>
</tr>
<tr>
<td>Remittances</td>
<td>0.06</td>
<td>0.0627</td>
<td>0.0719</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.6754</td>
<td>0.6863</td>
<td>0.7229</td>
</tr>
<tr>
<td>Real Money Balances</td>
<td>0.8795</td>
<td>0.8925</td>
<td>0.9351</td>
</tr>
<tr>
<td>Real Money Cash</td>
<td>0.6948</td>
<td>0.7067</td>
<td>0.7467</td>
</tr>
<tr>
<td>Real Money Deposits</td>
<td>0.1847</td>
<td>0.1857</td>
<td>0.1884</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.0287</td>
<td>1.0298</td>
<td>1.0329</td>
</tr>
<tr>
<td>Bonds</td>
<td>3.0027</td>
<td>2.7418</td>
<td>1.7996</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>-0.0951</td>
<td>-0.0944</td>
<td>-0.0916</td>
</tr>
<tr>
<td>Utility</td>
<td>-2.5739</td>
<td>-2.5693</td>
<td>-2.5544</td>
</tr>
<tr>
<td>Family Unity</td>
<td>0.0209</td>
<td>0.0208</td>
<td>0.0207</td>
</tr>
</tbody>
</table>

Rate of assimilation abroad or rate at which the migrant population loses its national identity.

These results can be explained by considering a textbook closed economy Solow growth model. The increase in the assimilation rate leads to a decrease in the growth rate of the population of the small open economy, and as in the Solow growth model this leads to an increase in the capital stock per domestic worker. Here it also leads to an increase in the overall capital stock. In the textbook closed
economy Solow growth model this also leads to an increase in savings (and investment), and here we observe an increase in investment by 0.57% as the assimilation rate increases from 1% to 2% and by 1.64 percent as the assimilation rate increases from 2% to 5%. However, our model is of an open economy, so saving is actually equal to investment plus net exports, and this increases as the capital stock increases – by 1.62% as the assimilation rate increases from 1 to 2 percent and by 5.30% as the assimilation rate goes from 2 to 5 percent. In our model the increase in the capital stock and in saving (investment plus net exports) also increases, so there is saving to finance the required increase in spending on the capital stock. Interestingly, the ratio of savings to output in our model remains almost constant at 14.76% as the assimilation rate increases. The components of the saving rate between investment and net exports changes, however, as the assimilation rate increases.

We also analyze the steady state behavior of the economy under alternative assumptions about the percentage of the wages earned abroad that need to be devoted to fulfill subsistence requirements in the foreign economy. The steady state values of these variables are presented in Table 3 under three alternative assumptions about the percentage devoted to subsistence requirements, with such requirements being 73.6%, 73.5%, and 73.4% of foreign wages. This lower subsistence requirement in the foreign country is translated in an increase in the percentage of foreign wages (remittances) being sent to the household in the home country.

The representative household reacts to this decline in subsistence requirements by reducing the allocation of labor in the foreign market, with emigration falling by 10% as subsistence requirements fall from 73.6% to 73.5% and by an additional 11% as subsistence requirements fall from 73.5% to 73.4%. With higher potential remittances, and because of the utility cost of family disunity, households send fewer migrants to work abroad. This reduction in migration flows raise the domestic allocation of labor – and work hours – by almost 1.3% as subsistence requirements declines from 73.6% to 73.5% of foreign wages and by 1.27% as subsistence requirements decline from 73.5% to 73.4% of foreign wages. This increase in domestic labor, combined with the increase in investment and in physical capital, leads to an increase in output per capita of 0.91% as subsistence requirements falls from 73.6% to 73.5% and by almost 0.88% when subsistence requirements fall from 73.5% to 73.49%. Real wages decline as labor becomes more abundant, but since hours worked is calibrated to represent 20 percent of available time they also rise with the domestic population. Meanwhile consumption per capita is increased by similar rates than those of output, in part from the higher remunerations but also for the reduction in inflation.

This reduction in the subsistence requirements in the foreign economy allows the representative household to trade foreign leisure for domestic leisure. Here hours worked abroad falls by 37% as
subsistence requirements decline from 73.6% to 73.5% and by almost 58% when subsistence requirements fall from 73.5% to 73.49%. This leads to a consequent drop in the amount of remittances being sent to the home country. It also leads to a decline in the real wage at home, as the higher level of work hours at home represents an increase in labor supply. The net result of the lower subsistence requirement is higher levels of steady state consumption, leisure, and family unity, and consequently higher levels of utility.

<table>
<thead>
<tr>
<th>Table 3 – Steady State Values for Different Subsistence Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>73.6% of foreign wages</strong></td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>Capital</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Domestic Time</td>
</tr>
<tr>
<td>D. Hours of Work</td>
</tr>
<tr>
<td>Migrant Time</td>
</tr>
<tr>
<td>M. Hours of Work</td>
</tr>
<tr>
<td>Real Wages</td>
</tr>
<tr>
<td>Migration</td>
</tr>
<tr>
<td>Remittances</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Real Money Balances</td>
</tr>
<tr>
<td>Real Money Cash</td>
</tr>
<tr>
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</tr>
<tr>
<td>Inflation</td>
</tr>
<tr>
<td>Bonds</td>
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<tr>
<td>Trade Balance</td>
</tr>
<tr>
<td>Utility</td>
</tr>
<tr>
<td>Family Unity</td>
</tr>
</tbody>
</table>

_Holding the fact that foreign wages are 4.1 larger than domestic wages and that the assimilation rate is 2 percent per quarter._

In our model households maximize per capita welfare, but we might also want to consider the welfare of the population that remains in the small open economy. It is of particular interest to note that while investment, physical capital, output, and consumption increase in per capita term, if one adjusts these measures in terms of the population remaining in the home country, we find that they all decline. Thus if we speak strictly in terms of the population remaining behind, the reallocation of labor may not be beneficial to workers remaining at home. Simple calculations show that investment per remaining
workers drops by 0.1% (0.14%) as the subsistence requirement falls from 73.6% to 73.5% (73.5% to 73.4%). Meanwhile physical capital per remaining workers drops by 1% (0.95%) as the subsistence requirement falls from 73.6% to 73.5% (73.5% to 73.4%), and both output and consumption per remaining workers falls by 0.4% (0.37%) as the subsistence requirement falls from 73.6% to 73.5% (73.5% to 73.4%). With more workers now allocated to the home country, the higher level of investment is not enough to outweigh the increase in the domestic labor force, leading to reductions in capital and output in per domestic worker terms.

With respect to the textbook closed economy Solow growth model, the reduction in subsistence requirements leads to a decline in migration flows, which enhances the growth rate of the population because migrating workers are assimilated into the foreign country at a slower rate. Thus they remain part of the originating household for a longer period. This causes a reduction in the capital stock per domestic worker, although we see in the table that the overall capital stock increases because domestic workers increase. The reduction in subsistence requirements also leads to an increase in saving in the small open economy large enough to increase the capital stock and the level of output, similar to the case of the closed economy Solow model.

4.- Dynamic Responses

We examine the dynamics of the main macroeconomic aggregates of our small open economy, namely the nominal interest rate, output, consumption, the nominal exchange rate, migration flows, and remittances following expansionary monetary and technology shocks, to then examine the overall effect on the welfare of the receiving economy, measured by their utility and the trade balance. We present results under the baseline calibration of Table 1, with a small but positive adjustment cost of about 3 minutes per week ($\xi = 10$).

4.1 Monetary Shock

We first analyze the behavior of the economy to a positive 1% shock to the rate of money growth in our baseline calibration through its impact on the main macroeconomic aggregates. The introduction of a positive monetary shock lowers the interest rate slightly on impact (by 50 base points). The monetary shock raises inflation momentarily, which reduces the value of real money balances and induces households to increase their holdings of money cash the next period to satisfy a given level of consumption. However, since the monetary expansion goes through the financial intermediary and households cannot withdraw their deposits within the period without incurring adjustment costs from rearranging real money balances, this creates an excess supply of funds that outweighs the inflationary pressure and that lowers the nominal interest rate. This is the typical liquidity effect, and it has a persistent
effect on the interest rate that can be observed below in the top-left panel of Figure 2. The magnitude of the drop and its persistence is determined by the adjustment costs on real money balances. It is only in the following period that the household will start to reduce its money deposits ($M_{t+1}^b$) to satisfy consumption, and thus exert an upward pressure on the interest rate.

The instantaneous fall in the nominal interest rate reduces the return on domestic savings, and since households cannot immediately reallocate their funds towards the foreign asset it leads to an instantaneous depreciation of the nominal exchange rate on impact on the order of 3.5 percent. The overshooting of the nominal exchange rate shown in the bottom-left panel of Figure 2 is due to uncovered interest rate parity, which requires the interest rate differential to be equal to the expected rate of appreciation in the following periods, leading to the subsequent appreciation of the exchange rate until it returns to its steady state as the liquidity effect dissipates.

As it is typically found in the literature, an expansionary monetary shock generates a positive wealth effect, which is allocated to increases in leisure in the first period because of the cash-in-advance constraint and the adjustment cost of money holdings. In our model which allows for the reallocation of labor between the home and foreign markets, the household is able to smooth consumption by a combination of reallocation of labor and changes in work effort. Since the rise in inflation coupled with the cash in advance constraint reduces consumption levels on impact, and the increased leisure from the positive wealth effect reduces near term wage income, the household chooses to allocate more labor abroad to reduce the fall in income at home. This rise in emigration is shown in the bottom-center of Figure 1, with emigration flows increasing by almost 4 percent on impact. As the inflation dissipates in the second period and economic activity starts to improve, the subsequent dynamics show that the household reallocates labor towards the domestic market. The household reduces its time devoted to the domestic economy by almost 0.01% (and his work effort by 0.02%), which is matched by an increase in time allocated abroad by almost 0.07%.
Since capital is fixed for the period, this slight reallocation of labor away from the domestic economy is compounded by the lower work effort due to the wealth effect and leads to a slight drop in output on impact, which is shown in the top-center of Figure 1. The household reacts by reducing its effort on impact, but it also places more labor abroad to ameliorate the negative effect on consumption. However, from the second period onwards we observe an improvement of investment per worker, with its consequent effect on capital, as the interest rate remains below its steady state level due to the liquidity effect, and we see an increase in work effort due to the real wage remaining above its steady state levels. Output returns to its original steady state level one and a half quarters after the monetary shock and peaks after 12 quarters before starting to decline. It is difficult to see in Figure 2, but output eventually returns to the initial steady state values.

The consumption dynamics following the monetary injection are mainly generated by inflationary pressures during the period of the shock. Given that the consumption level is determined by the cash-in-advance constraint, and since the amount on money-cash available for consumption is predetermined, inflation generated by the larger money supply reduces consumption instantaneously. The consumption dynamics from the second period onwards arises from the rearrangement between money-cash and money-deposits. Since agents anticipate inflation, and in order to preserve their consumption in the future, households increase their future amount of nominal money-cash the period of the shock \( M_{t+1}^c \). Because it is costly to change the ratio \( \frac{M_{t+1}^c}{M_t^c} \) when there are positive adjustment costs, this ratio is adjusted...
smoothly and thus induces persistence in the adjustment of consumption. This is shown in the top-right panel in Figure 2.

The dynamics of real remittances are solely influenced by the amount of work abroad, since the exchange rate fully incorporates the inflationary fluctuation in our flexible exchange rate model. As shown above in the bottom-right panel in Figure 2, remittances first increase by almost 0.4% due to the increased work participation in the foreign market – hours worked abroad increase by a similar 0.4% – to then decline as work effort abroad is smoothly reduced to 1.8% below the original steady state. Remittances return eventually, and slowly, to the initial steady state.

4.2 Technology Shock

We now analyze the behavior of the economy to a positive 1% technological shock in our baseline calibration through its impact on the main macroeconomic aggregates. The technology shock leads to a greater marginal product of capital, which exerts an upward pressure on investment large enough to offset the downward pressure coming from the higher nominal interest rate, and thus results in a 1 percent increase in investment per capita on impact. The positive technological shock has a positive effect on the retention of domestic labor on impact, and through its upward pressure on the real wage labor participation is also enhanced. The higher wages induce the representative agent to increase its hours worked by almost 2.4% on impact. Since capital is fixed the period of the shock, this higher work effort leads to an increase in output of about 2.4%, which is shown in the top-center panel in Figure 3. It is only after one period that investment starts to taper off, leading to an expansion of capital levels above steady state. It is also at this period that we observed a reversal of the return of workers from abroad together with the decline of worked hours in the domestic economy, which is large enough to outweigh the improvement in capital to lead to the continuous decline in output towards its initial steady state.

The drop in inflation produced by the technology shock and produces a positive wealth effect even as we allow for labor dynamics between the two markets. The technological shock leads to a drop of inflation of about 1.6% on impact. Since real money-cash is determined the previous period, this positive wealth effect leads to a rise in consumption of around 1.6%, shown in the top-right panel of Figure 3, before it monotonically returns to the steady state level through its adjustment of money cash balances.
The greater marginal productivity of capital from the technology shock dominates the one-period drop in inflation to put an upward pressure on the nominal interest rate, increasing the nominal interest rate by almost 8% on impact – approximately 40 basis points – shown in the top-left panel of Figure 3. The subsequent weaker demand for loans is smaller than the prolonged increase in deposits, and thus exerts a downward pressure on the nominal interest rate that pushes it down towards its initial steady state level. Of course, this dynamics are determined by the adjustment of money-cash balances, and continue until investment, inflation, and money-deposits returns to their initial steady state levels.

The dynamics of the nominal exchange rate on impact are dominated by the drop in inflation and the spike in the interest rate, appreciating the domestic currency on impact by a similar 1.6%, as shown in the bottom-left panel in Figure 3. However, its subsequent dynamics are given by the faster return of the domestic interest rate relative to the return of the interest on foreign bonds, thus leading to a continuous depreciation due to uncovered interest rate parity, which requires the interest rate differential to be equal to the expected rate of appreciation.

The technological shock has an attractive effect on labor towards the domestic economy, with the higher wages allowing the household to reduce its migration flows to the foreign market by almost 5% on impact, as shown above in the bottom-center panel of Figure 3. However, since the shock also leads to an appreciation of the domestic currency, work effort abroad is also increased on impact to compensate for the declining labor allocation abroad – note that the labor flows continue to be below the replacement steady state level for the remaining time period.

For the case of real remittances, the results show that its dynamics are only influenced by the amount of work effort abroad, since the internal inflationary pressures are fully incorporated in the
nominal exchange rate. Remittances first increase by almost 2% due to the time allocated to the foreign market, but they then start to decline as the workers allocated to the foreign market starts to trade off some work effort for the continuous deprecation of the exchange rate that arises from the second period onwards. This dynamics are shown in the bottom-right panel of Figure 3.

It is important to point out that our small open economy preserves the main dynamics observed in the data, like the liquidity effect, the response of work effort of the domestic workers, the behavior of output, etc. It is able to do this while also providing insight into the reallocation of labor between the two markets.

5.- Conclusion

Our limited participation model for a small open economy with migration and remittances is able to capture the behavior of the main macroeconomic aggregates to monetary and technology shocks, in accord with empirical evidence. The introduction of adjustment costs on money holdings allows for a persistent liquidity effect, and overshooting of the nominal exchange rate, in response to monetary innovations.

With respect to permanent changes assimilation rates and subsistence requirements, our model shows that as the assimilation rate increases the household will send more labor abroad to sustain the optimal distribution of labor. In fact, the increase in emigration is large enough to expand the allocation of workers living abroad and reduces the time allocated to the domestic market. This marginal decline in hours worked in the domestic economy is, however, outweighed by the increase in investment and capital to lead to the increase in steady state output. The higher wages compensate the decline in work effort to lead to higher remuneration that outweighs the rising inflation to increase consumption. The increase in emigrants, and labor hours, leads to an increase in remittances. These dynamics leads to a large enough increase in household consumption that outweighs the declines in household leisure and family unity, resulting in an increase in utility. These results are remarkably in agreement with the textbook closed economy Solow growth model, with the decrease in the growth rate of the population of the small open economy and the increase in savings (investment plus net exports) increasing the capital stock.

The decline in subsistence requirements results in a reduction of the allocation of labor to the foreign market, in part for the greater potential remittances from fewer workers. This increase in domestic labor – and work hours – combines with the increase in investment and in physical capital to increase output. The higher domestic remunerations (although lower remittances) and lower inflation produce an increase in consumption. This reduction in the subsistence requirements in the foreign economy allows
the representative household to trade foreign leisure for domestic leisure, but leads to higher levels of consumption, leisure, and family unity, and consequently in higher levels of utility too. However, even if our model maximizes the per capita welfare of the whole population, it is of particular interest to note that if one adjusts these measures in terms of the population remaining in the home country, we actually find that investment, capital, output, and consumption fall, and consequently such reallocation of labor is not that beneficial to the workers remaining at home. With respect to the textbook Solow growth model, the reduction in subsistence requirements leads to a decline in migration flows and thus enhances the growth rate of the population but also increases the savings of the small open economy by a large enough amount to lead to an increase in the capital stock and output, similar to those of the closed economy case.

With respect to the dynamic impacts, the introduction of a positive monetary shock lowers the interest rate slightly on impact. This is the typical liquidity effect, and the magnitude of the drop in the interest rate is determined by the cost of money adjustments. Here we observe an increase in migration flows that also leads to a decline in hours worked, which leads to the initial drop in output since capital is fixed for that period. The behavior of consumption is negatively affected by the inflation generated by the larger money supply – from the CIA constraint – and the household increases its holdings of foreign bonds, leading to an instantaneous slight overshooting depreciation of the nominal exchange rate on impact. Remittances increase for a period as the work participation in the foreign economy is initially increased, but then it is reduced below steady state levels as work effort abroad is reduced to take advantage of the appreciating currency.

In the other hand the technology shock leads to a greater marginal product of capital that enough to offset the downward pressure on investment that comes from the higher nominal interest rate and thus results in an increase in investment per person on impact. Labor is also reallocated towards the domestic market, with combined increase in domestic work hours from the higher wage is reflected in a rise in output. Inflation presents the typical initial drop and leads to the improvement in consumption. The dynamics of the nominal exchange rate on impact are dominated by the drop in inflation, but its subsequent dynamics are given by the greater fall in the interest on foreign bonds relative to the fall in the nominal interest rate, increasing the relative return on domestic savings and thus inducing households to hold less foreign assets – from uncovered interest rate parity. Real remittances first increase due to the increased work participation in the foreign market, but then returns to below steady state levels as the domestic currency depreciates.
References


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Appendix (not for publication)

A.1. System of Equations in real terms

(8) \( \pi_i = \frac{s_i - \pi_i^*}{s_{t+1}} \)

(6) \( \pi_i C_i = m_i^c \)

(2) \( \Lambda_i = \beta \frac{E}{\pi_i} \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} (1 + i_{t+1}) \frac{1}{(1 + f - \chi n_i^M)} \right] \)

(4) \( \Gamma_i = \frac{\gamma^M (n_i^M - h_i^M - \Omega_i)}{\gamma^H (n_i^M - h_i^M - \Omega_i)} h_i^M w_i \)

(1) \( \Lambda_i = \beta \frac{E}{\pi_i} \left[ (1 + i_t^*) \frac{s_{t+1}}{s_t} \frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{1}{(1 + f - \chi n_i^M)} \right] \)

(5) \( 2 \Phi \frac{\gamma^H (n_i^M - n_i^H)}{\gamma^H} \Lambda_i w_i e^{(\sigma-1)(1-\gamma^H - \gamma^M)} (l_i^M) (\gamma^H (\sigma-1)+1) (l_i^M) \gamma^M (\sigma-1) + \Lambda_i w_i (1 - \frac{\gamma^M l_i^H}{\gamma^H l_i^M} (1 - \psi(j_t - j))) = \)

\[ \beta E \left[ -2 \Phi (n_i^H - n_i^M) \frac{\Lambda_{t+1} w_{t+1}}{\gamma^H} \frac{(1 + f)(1 + f - \chi)}{(1 + f - \chi n_i^M)^2} \right] \]

(3) \( \Lambda_i (1 + f - \chi n_i^M) w_i \frac{\xi}{m_i^c} \pi_i (\Delta M_i^c - g) + \Lambda_i (1 + f - \chi n_i^M) = \)

\[ \beta E \left[ \Lambda_{t+1} w_{t+1} \frac{\xi}{m_i^c} \Delta M_i^c (\Delta M_i^c - g) + \beta E \left[ \Lambda_{t+1} w_{t+1} \frac{(1 - \gamma^H - \gamma^M)(n_i^H - h_i^H - \Omega_i)}{\gamma^H c_{t+1} \pi_{t+1}} \right] \right] \]

(13) \( Y_t = e^\alpha K_i^{H^1-\alpha} \)

(14) \( I_t = (1 + f - \chi n_i^M) K_{t+1} - (1 - \delta) K_i \)

(15) \( w_t = (1 - \alpha) \frac{Y_t}{h_i^H} \)

(16) \( (1 + i_t) \frac{N_{t+1}}{N_t} + v \left( K_{t+1} \frac{N_{t+1}}{N_t} - K_i \right) \frac{N_{t+1}}{N_t} = \)
\[ \beta E_t \left[ \frac{P_{t+1} \lambda_{t+1}}{P_t \lambda_t} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) + \nu \left( K_{t+2} \frac{N_{t+2}}{N_{t+1}} - K_{t+1} \right) \right) \right] \]

(11) \( (1 + f - \chi n_t^M) m_{t+1} = g_t \frac{m_t}{\pi_t} \)

(10) \( \pi_t I_t = m_t^b + (g_t - 1) m_t \)

(12) \( \Gamma_t = (1 - \varphi) h_t^M w^* \)

(9) \( (1 + f - \chi n_t^M) b_{t+1} - \frac{s_{t-1}}{s_{t+1}} (1 + i_t^*) \frac{b_t}{\pi_t} = Y_t - c_t - I_t - \frac{\nu}{2} \left( K_{t+1} \frac{N_{t+1}}{N_t} - K_t \right)^2 + \Gamma_t \)

(21) \( \Delta M_t^c = \frac{m_{t+1} \pi_t}{m_t^c} (1 + f - \chi n_t^M) \)

(7) \( m_t = m_t^b + m_t^c \)

(17) \( 1 = n_t^H + n_t^M \)

(18) \( n_t^H = \frac{(1 + f)^* (1 - n_{t-1}^M) - j_t (1 + f - \chi n_{t-1}^M)}{(1 + f) - \chi n_{t-1}^M} \)

(19) \( n_t^M = \frac{(1 + f - \chi)n_{t-1}^M + j_t (1 + f - \chi n_{t-1}^M)}{(1 + f) - \chi n_{t-1}^M} \)

(20) \( i_t^* = i_t^w - \varphi b_t \)

(22) \( \log(g_{t+1}) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_t) + \epsilon_{g_{t+1}} \)

(23) \( \log(z_{t+1}) = (1 - \rho_z) \log(\bar{z}) + \rho_z \log(z_t) + \epsilon_{z_{t+1}} \)
A.2. The log-linearized system of equations is given by

\begin{align*}
(8) & \quad 0 = -\hat{\pi}_t + \hat{s}_t - \hat{s}_{t-1} \\
(6) & \quad 0 = \hat{\pi}_t + \hat{C}_t - \hat{m}^c_v \\
(2) & \quad 0 = E \left[ -\Lambda_t + \frac{i}{1+i} \hat{i}_{t+1} + \Lambda_{t+1} - \hat{\pi}_{t+1} + \frac{2\pi n^M M^t}{1 + f - \chi n^M \hat{M}_t} \right] \\
(4) & \quad 0 = \hat{w}_t - \hat{\Gamma}_t - \frac{n^M M^t}{n^M - h^M} \hat{n}_t^M + \left( 1 + \frac{h^M M^t}{n^M - h^M} \right) \hat{\nu}_t^M + \frac{n^H M^t}{n^H - h^H} \hat{n}_t^H - \frac{h^H}{n^H - h^H} \hat{\nu}_t^H \\
(1) & \quad 0 = E \left[ -\Lambda_t + \Lambda_{t+1} + \hat{s}_{t+1} - \hat{s}_t - \hat{\pi}_{t+1} + \frac{2\pi n^M M^t}{1 + f - \chi n^M \hat{M}_t} + \frac{i^*}{1+i^*} \hat{i}_{t+1} \right] \\
(3) & \quad 0 = E \left[ - (1 + f - \chi n^M) \Lambda_t + S \Lambda_{t+1} - S \hat{\pi}_{t+1} + S \hat{w}_{t+1} - S \hat{c}_{t+1} + S \frac{n^H M^t}{n^H - h^H} \hat{n}_t^H - S \frac{h^H}{n^H - h^H} \hat{\nu}_t^H \\
& \quad + \chi n^M \hat{n}_t^M + \beta \pi^2 w \frac{c}{m^c} \Delta M_{t+1} - \pi w \frac{c}{m^c} \Delta M^c \left( 1 + f - \chi n^M \right) \hat{M}_t \right] \\
\text{where} \quad S = \frac{\beta w (1 - \gamma^H - \gamma^M) \hat{M}_t}{\gamma^H \hat{C}_t} \\
(5) & \quad 0 = E \left[ A44 \Lambda_{t+1} + A45 \hat{w}_{t+1} + A46 \hat{n}^H_{t+1} + A47 \hat{n}^M_{t+1} - A48 \hat{c}_{t+1} + A49 \hat{h}^H_{t+1} + A50 \hat{h}^M_{t+1} - A51 \hat{j}_t \right. \\
& \quad - A52 \Lambda_t - A53 \hat{w}_t - A54 \hat{c}_t - A55 \hat{h}^H_t + A56 \hat{h}^M_t - A57 \hat{n}^H_t + A58 \hat{n}^M_t \right] \\
\text{Where} A44-A58 \text{ are parameters.} \\
(13) & \quad 0 = -\hat{Y}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{h}^H_t + \hat{z}_t \\
(14) & \quad 0 = \frac{I}{K} \hat{J}_t + \chi n^M \hat{n}^H_t - (1 + f - \chi n^M) \hat{K}_{t+1} + (1 - \delta) \hat{K}_t \\
(15) & \quad 0 = -\hat{w}_t + \hat{\gamma}_t - \hat{h}^H_t \\
(7) & \quad 0 = -(m) \hat{m}_t + (m^b) \hat{m}^b_t + (m^c) \hat{m}^c_t
\end{align*}
(10) \[ 0 = -\mathbf{\hat{\pi}}_t - \mathbf{\hat{I}}_t + \frac{m^b}{I_\pi} \mathbf{\hat{m}}^b_t + \frac{m}{I_\pi} (g-1) \mathbf{\hat{m}}_t + \frac{mg}{I_\pi} \mathbf{\hat{g}}_t, \]

(16) \[
0 = E_t \left[ -\frac{\beta \nu \chi n^M}{1 + f - \chi n^M} \mathbf{\hat{n}}^M_{t+1} + \beta \nu K \mathbf{\hat{K}}_{t+2} + \nu K \mathbf{\hat{K}}_t + \frac{\beta (1-\delta) i}{1 + f - \chi n^M} \mathbf{\hat{I}}_{t+1} + \frac{\alpha \beta Y}{K (1 + f - \chi n^M)} \mathbf{\hat{Y}}_{t+1} \right. \\
- \left. \left( \frac{\beta}{1 + f - \chi n^M} + \nu K (1 + f - \chi n^M) + \frac{\alpha \beta Y}{K (1 + f - \chi n^M)} \right) \mathbf{\hat{K}}_{t+1} + \left( \frac{(1+i) \chi n^M}{1 + f - \chi n^M} + \nu K \chi n^M \right) \mathbf{\hat{n}}^M_t - (i) \mathbf{\hat{i}}_t \right. \\
+ \frac{\beta}{1 + f - \chi n^M} \left( \frac{\alpha Y}{K} + (1-\delta)(1+i) + \nu (K(1 + f - \chi n^M) - K) \right) \mathbf{\hat{\lambda}}_{t+1} \\
- \frac{\beta}{1 + f - \chi n^M} \left( \frac{\alpha Y}{K} + (1-\delta)(1+i) + \nu (K(1 + f - \chi n^M) - K) \right) \mathbf{\hat{\lambda}}_t \right]

(11) \[ 0 = -(1 + f - \chi n^M) \mathbf{\hat{m}}^M_{t+1} + \frac{g}{\pi} \mathbf{\hat{m}}_t - \frac{g}{\pi} \mathbf{\hat{\pi}}_t + \frac{g}{\pi} \mathbf{\hat{g}}_t + \chi n^M \mathbf{\hat{n}}^M_t \]

(9) \[
0 = -(1 + f - \chi n^M) \mathbf{\hat{h}}^M_{t+1} + \frac{(1+i)^*}{\pi} \mathbf{\hat{s}}_t - \frac{(1+i)^*}{\pi} \mathbf{\hat{s}}_{t+1} + \frac{(1+i)^*}{\pi} \mathbf{\hat{b}}_t - \frac{(1+i)^*}{\pi b} \mathbf{\hat{b}}_t + \frac{Y}{b} \mathbf{\hat{Y}}_t + \frac{C}{b} \mathbf{\hat{C}}_t \\
- \frac{I}{b} \mathbf{\hat{I}}_t + \frac{\Gamma}{b} \mathbf{\hat{\Gamma}}_t - (\chi n^M \left( 1 + \frac{\nu K}{b} (K(1 + f - \chi n^M) - K) \right)) \mathbf{\hat{n}}^M_t + \frac{i^*}{\pi} \mathbf{\hat{i}}^* \\
- \left( \frac{\nu K}{b} (K(1 + f - \chi n^M) - K)(1 + f - \chi n^M) \right) \mathbf{\hat{K}}_{t+1} + \frac{\nu K}{b} (K(1 + f - \chi n^M) - K) \mathbf{\hat{K}}_t \]

(21) \[ 0 = -\Delta M^c_t \mathbf{\hat{\hat{m}}^M_{t+1}} + \mathbf{\hat{\hat{\pi}}}_t - \mathbf{\hat{\hat{m}}}_t - \frac{\chi n^M}{1 + f - \chi n^M} \mathbf{\hat{\hat{n}}^M_t} \]

(12) \[ 0 = \mathbf{\hat{\hat{h}}^M_t} - \mathbf{\hat{\hat{\pi}}}_t \]

(17) \[ 0 = n^M \mathbf{\hat{n}}^M_t + n^M \mathbf{\hat{\hat{n}}^M_t} \]

(18) \[ 0 = [\chi n^M - (1 + f) j] \mathbf{\hat{j}}_t + [\chi n^M - (1 + f) n^M + \chi n^M n^M] \mathbf{\hat{n}}^M_{t+1} + [\chi n^M n^M - (1 + f)n^M] \mathbf{\hat{n}}^M_t \]

(19) \[ 0 = [-\chi n^M + (1 + f) j] \mathbf{\hat{j}}_t - [\chi n^M - (1 + f - \chi) n^M - \chi(n^M)^2] \mathbf{\hat{n}}^M_{t-1} + [\chi(n^M)^2 - (1 + f)n^M] \mathbf{\hat{n}}^M_t \]

(20) \[ 0 = \mathbf{\hat{i}}^* + \frac{\nu b}{i^*} \mathbf{\hat{i}}^*_t \]
(22) \[ \hat{g}_{t+1} = \rho_g \hat{g}_t + \epsilon_{g_{t+1}} \]

(23) \[ \hat{z}_{t+1} = \rho_z \hat{z}_t + \epsilon_{z_{t+1}} \]

A.3. Solving

The system is given by 23 equations with 23 variables. The endogenous state variables are given by \{ \hat{m}_t, \hat{h}_t, \hat{K}_t, \hat{m}_t^c, \hat{n}_t^M, \hat{\lambda}_t, \hat{i}_t, \hat{\mu}_t^M, \Delta \hat{M}_t \}, the other endogenous variables of the system are \{ \hat{\pi}_t, \hat{m}_t^b, \hat{C}_t, \hat{i}_t^*, \hat{\theta}_t, \hat{n}_t^H, \hat{\mu}_t^H, \hat{\gamma}_t^*, \hat{\Gamma}_t^*, \hat{\Gamma}_t \}, and the exogenous state variable are \{ \hat{g}_t, \hat{z}_t \}. 