Idea Flows, Economic Growth, and Trade

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• Economists have long believed that international trade serves as a vehicle for the diffusion of technology—of ideas

• Much evidence of important growth effects of openness, trade


• Next slides, based on Maddison data, 1960 - 2000
INCOME LEVELS AND GROWTH RATES, 112 COUNTRIES

INCOME LEVELS AND GROWTH RATES, 112 COUNTRIES

Annual Growth Rate, 1960-2000

Income per capita, 1990 $, 1960
INCOME LEVELS AND GROWTH RATES, 39 OPEN ECONOMIES

Parameter Values

\[ \theta = 0.67 \]

\[ \mu = 0.017 \]
• What we do not have is good understanding of how this trade/diffusion effect works

• What is the process that links trade policy to diffusion, growth?

• What are the key parameters of this process?

• What evidence do we have on their magnitudes?

• Seek a framework that can help make progress on these questions
• We develop an endogenous growth model with many countries that explicitly connects trade and trade policy to sustained growth rates and transition dynamics.

• Model is built on work of Eaton, Kortum in two ways:
  
  

• View technology as distribution of productivity-related knowledge held by heterogeneous, individual people, firms, countries
• Construct a model of \( n \) country world, engaged in continuously balanced trade

• Many goods, many people in each: details later

• State variables are \( F_1, F_2, \ldots, F_n \): right cdfs of “cost” in \( \mathbb{R}_+ \)

\[
F_i(z) = \Pr\{\text{randomly chosen good has cost } \geq z \text{ if produced in } i\}
\]

• Densities \( f_i = -\partial F_i(z, t)/\partial x \)

(In E/K, A/L \( F_i \)'s are Weibull RVs. Not so here.)
• Constant trade costs matrix \( K = [\kappa_{ij}] : \kappa_{ij} = \text{units of goods arriving in } i \text{ per unit shipped from } j \)

• Populations \( L = (L_1, ..., L_n) \)

• Given \( K, L \) and technology profile \( F \), can solve for static trade equilibrium, including wage rates \( w = (w_1, ..., w_n) \)

• Theory also gives us equilibrium distributions of the costs of producers in \( j \) who sell in country \( i \), all \( i, j \).

• From these, can calculate right cdfs \( G_1, G_2, ..., G_n \) where

\[
G_i(z, t) = \Pr\{\text{seller active in } i \text{ at } t \text{ has cost } \geq z\}
\]
• We will use these to motivate a law of motion for $F_1, F_2, \ldots, F_n$ of the form

$$\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(G_i(z, t))$$ (*)

• Trade theory tells us how $F, K$ determine $G$; gives autonomous system

• Simulatable model of world trade, economic growth

• Law of motion (*) is main new idea of this paper
PLAN OF TALK

1 Technology Diffusion, Closed Economy

2 Technology Diffusion, $n$-Country World

3 Numerical Illustrations

4 Conclusions
1 Diffusion in Closed Economy

- Consumers have identical preferences over $[0, 1]$ continuum of goods
  \[ C = \left[ \int_0^1 c(s)^{1-1/\eta} ds \right]^{\eta/(\eta-1)} \]

- Good $s$ is produced with a linear, labor-only technology
  \[ y(s) = \frac{l(s)}{z(s)} \]

- $l(s)$ is labor input—1 unit per person—and $z(s)$ is cost (labor requirement)
• Exploit symmetry of utility function

• Re-label goods by their costs \( z \) and write time \( t \) utility as

\[
C(t) = \left[ \int_{\mathbb{R}_+} c(z)^{1-1/\eta} f(z, t) \, dz \right]^{\eta/(\eta-1)}
\]

• Here \( f(\cdot, t) \) is the density of costs.

• Use \( F(z, t) \) for the \textbf{right} cdf of cost, so density is \( f(z, t) = -\partial F(z, t) / \partial z \)
• In competitive equilibrium, price of good $z$ is $p(z) = wz$

• Ideal price index for the economy at date $t$ is

\[
p(t) = \left[ \int_{\mathbb{R}_+} p(z)^{1-1/\eta} f(z, t) dz \right]^{\eta/(\eta-1)}
\]

• Real per capita GDP $y(t)$ is real wage $w/p(t)$:

\[
y(t) = \left[ \int_{\mathbb{R}_+} z^{1-1/\eta} f(z, t) dz \right]^{-\eta/(\eta-1)}
\]

• Now need to describe evolution of $f, F$
• Model technological diffusion as process of search involving *technology managers*

• One manager per good, each identified with current cost $z$

• Technology management requires no time, earns no private return

• Manager of good $z$ operates the CRS, zero profit production described earlier

• Or others imitate him and do so: Who cares?
• Each manager $z$ meets others at given rate $\alpha$ per unit of time

• Each meeting a random draw from the population $f, F$ of managers of all goods

• When he meets another with cost $z' < z$ he adopts $z'$ for his own good

• Motivate law of motion for $F$ as

$$F(z, t + \Delta) = \Pr\{\text{my cost} > z \text{ at } t + \Delta\}$$
$$= \Pr\{\text{my cost} > z \text{ at } t\} \times \Pr\{\text{no lower draw in } (t, t + \Delta)\}$$
$$= F(z, t)F(z, t)^\alpha\Delta.$$ 

• Continuous draws, not Poisson arrivals
• Let $\Delta \to 0$ to obtain:

$$\frac{1}{F(z, t)} \frac{\partial F(z, t)}{\partial t} = \alpha \log(F(z, t))$$  \hspace{1cm} (DE)

• Write out general solution:

$$\log(F(z, t)) = \log(F(z, 0)) e^{\alpha t}$$

where $F(z, 0)$ is any given initial distribution (right cdf)

• Clear that model implies GDP growth of some kind.

• Easy to compute. How to interpret results, characterize possibilities?
• For empirical reasons, interest is in sustained growth of economies that either grow at a constant rate or will do so asymptotically.

• Central construct is balanced growth path (BGP): right cdf \( \Phi(z) \) (density \( \phi = -\Phi'(z) \)) and a growth rate \( \nu > 0 \) such that \( F(z, t) = \Phi(e^{\nu t} z) \) and

\[
\log(\Phi(e^{\nu t} z)) = \log(\Phi(z))e^{\alpha t}
\]

• On BGP

\[
f(z, t) = -\frac{\partial F(z, t)}{\partial z} = \phi(e^{\nu t} z)e^{\nu t}
\]
Then real GDP path is

\[ y(t) = \left[ \int_{\mathbb{R}^+} z^{1-1/\eta} \phi(e^{vt} z) e^{vt} dz \right]^{-\eta/(\eta-1)} \]

\[ = e^{vt} \left[ \int_{\mathbb{R}^+} x^{1-1/\eta} \phi(x) dx \right]^{-\eta/(\eta-1)} \]

We show that BGP takes Weibull form

\[ \Phi(z) = \exp(-\lambda z^\theta) \]

for some pair \( \lambda, \theta > 0 \) and \( \nu = \alpha \theta \)

Note that Weibull RV is just exponential RV raised to power \( \theta \)
• Also show that if initial distribution \( F(z, 0) \) satisfies

\[
\lim_{z \to 0} \frac{f(z, 0)z}{1 - F(z, 0)} = \frac{1}{\theta}
\]

for some \( \theta > 0 \) and

\[
\lim_{z \to 0} \frac{\log [F(z^\theta, 0)]}{z} = -\lambda
\]

for some \( \lambda > 0 \) then

\[
\lim_{t \to \infty} \log [F(e^{-\alpha \theta t}z, t)] = -\lambda z^{1/\theta} \quad \text{for all} \quad z > 0
\]
- Parameter $\theta$ measures mass near $z = 0$

- Costs are headed to zero so long run behavior determined by “left tail”

- High $\theta$ value means more low cost ideas waiting to be discovered

- Inverse of cost is productivity so high $\theta$ means thick “right tail” of productivity distribution, high growth rate

- Power Law
2 Diffusion in World Economy

- An $n$ country world. Populations $L_1, \ldots, L_n$

- Each country in autarky exactly as discussed

- Now open all of them to Eaton-Kortum trade in all goods

- Rename each good by its cost profile $z = (z_1, \ldots, z_n)$

\[
C_i(t) = \left[ \int_{\mathbb{R}_+^n} c_i(z)^{1-1/\eta} f(z, t) dz \right]^{\eta/(\eta-1)}
\]

where $f(z, t) = \prod_{i=1}^n f_i(z, t)$ is joint density
Given fixed trade costs $K$, technologies $F = (F_1, ..., F_n)$, solve for balanced growth equilibrium wages $w = (w_1, ..., w_n)$


Turn to dynamics. Technology managers native to $i$ now draw ideas from all managers, foreign and domestic, whose goods are currently being sold in $i$.

Searching managers include all managers in $i$, good and bad.

They meet managers from all $j$, but only those good enough to sell goods in $i$. 
• Want to replace autarky law of motion

\[
\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(F_i(z, t))
\]

with

\[
\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(G_i(z, t))
\] (*)

where

\[
G_i(z, t) = \Pr\{\text{seller active in } i \text{ at } t \text{ and has cost } \geq z\}
\]

• Theory tells us who these sellers are, given \(K, F\), and \(w\)
• Distributions $G_i$ stochastically dominate $F_i$

• Statement of familiar static gains from trade

• Also key to dynamic gains: $G_i$ provides people in $i$ a better intellectual environment than $F_i$ does

• Formula is

$$G_i(z, t) = \sum_{j=1}^{n} \int_{z}^{\infty} f(z_j, t) \prod_{k \neq j} F_k \left[ \frac{w_j(t) \kappa_{ik}}{w_k(t) \kappa_{ij} z_j} \right] dz_j$$
Can show that if (i) some $F_i$ consistent with sustained growth under autarchy, and (ii) trade is possible between any pair, then for all $i$

$$\lim_{z \to 0} \frac{zg_i(x)}{1 - G_i(x)} = \frac{1}{\theta} \quad \text{where} \quad \theta = \max_i \theta_i$$

That is $G_i's$ inherit common, fattest left tail from $F_i's$

Can also show that all countries have common BGP growth rate

$$\nu = \theta \sum_{i=1}^{n} \alpha_i \quad \text{where} \quad \theta = \max_i \theta_i$$

Scale economy? Yes. And note that trade costs not in the formula

Does not take much trade for the really good ideas to get around: think of Marco Polo and pasta
3 Numerical Illustrations

- Begin with world of \( n \) identical countries; symmetric trade cost \( \kappa \)

- Already know a lot from general theory:
  
  - Relative wages identical: set \( w = 1 \), all countries
  
  - common BGP growth rate: \( \nu = \theta \sum_{i=1}^{n} \alpha_i \), where \( \theta = \max_i \theta_i \)

  - \( \nu \) independent of \( \kappa \) value
Like to know more:

- what do the distributions $F$ and $G$ look like on a BGP?

- how do trade volumes and GDP levels vary with trade costs $\kappa$ and substitution elasticities $\eta$?

Consider world of identical economies, $n = 50$, $\theta = 0.2$, $\alpha = 0.002$, $\nu = 0.02$
• Next figure describes the distributions $F$ and $G$

• Have plotted distributions of productivities, $1/z$, rather than costs $z$

• x-axis on both panels is BGP productivity relative to mean of 1 for $\kappa = 1$ (costless trade) world

• Top panel shows productivity densities of each country ($F$) at different $\kappa$ levels. Bottom panel shows densities of sellers’ productivities ($G$) at different $\kappa$ levels

• Both are Frechet distributions in right tail: note common tails on each panel
Potential Producers from a Country

Sellers to a Country

productivity, $z^{-\theta} / \text{costless trade average}$
• Next figure also describes the symmetric world economy: the effects of changes in trade costs on real incomes and trade shares

• The x-axis shows trade costs, varying from autarky for all \((\kappa = 0)\) to costless trade \((\kappa = 1)\).

• Top panel plots per capita gdp levels, relative to gdp with costless trade

• Bottom panel plots trade volumes, relative to the costless trade case
• Solid blue lines show impact, static trade effects

• Other three curves on top panel are real gdp levels along the BGP

• Levels are shown for three values of $\eta$: elasticity of substitution

• Three curves on bottom panel are trade share levels along the BGP for three $\eta$ values
GDP / costless trade

Per-capita Income

impact
long-run, $\eta=2$
long-run, $\eta=4$
long-run, $\eta=5$

1−trade cost, $\kappa$

Trade Share

Import/GDP

1−trade cost, $\kappa$
• Next figures describe world with $n$ countries:
  
  – $n - 1$ identical (as above), common trade costs $\kappa = .75$

  – one small, open, larger trade cost $\kappa_n$ applied to all imports

• First figure shows BGP income levels and trade shares—relative to costless trade benchmark—for different trade cost levels of deviant
GDP / Costless trade

Per–capita Income

Import/GDP

Trade Share

1−trade cost, $\kappa_n$

$n−1$ symmetric

$n^{th}$ asymmetric
• Last figure shows time series of income and trade shares of the deviant country

• Deviant begins with higher trade costs, poorer economy

• At \( t = 0 \), deviant adopts trade cost \( \kappa = .75 \) of other countries

• Immediate jump in income, trade share shown: static trade effect

• Slow convergence to common BGP also shown
Per-capita Income, \( n^{th} \) Asymmetric Country

\[
\kappa_n(0) = 0.50 \\
\kappa_n(0) = 0.30 \\
\kappa_n(0) = 0.05
\]

Volume of Trade, \( n^{th} \) Asymmetric Country

Import/GDP

Years (since trade liberalization)
3 Conclusions

- Basic model general enough to support realistic calibration, policy simulations: see e.g. Alvarez/Lucas (2007)

- Our immediate goal here more modest: to understand the operating characteristics of a new, combined model of trade and growth

- General structure shares features of von Neumann (1937) or Parente/Prescott (1994): long run growth rate common to all; different policies induce different income levels

- Model makes operational distinction between static effects of trade policies and dynamic effects via trade related technology diffusion