The Risky Capital of Emerging Markets

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Lucas Paradox Revisited

Lucas Paradox: Why don’t returns to capital equalize?

Our answer: Capital in poorer countries is riskier.

⇒ Investor demands higher average returns there.

We show: differences in risk quantitatively account for differences in returns.
What we do...

First pass: workhorse CCAPM... works qualitatively; not quantitatively (paper)

- Given $\text{cov}(r_j, \Delta c_{US})$ from data, need $\gamma \approx 900$ to fit observed return diffs.

  $\Rightarrow$ Lucas Paradox: just another asset pricing puzzle!

Towards a resolution:

Key Building Blocks

1. Aguiar and Gopinath (2007): shocks to trend growth key in poor/emerging market BC’s +

2. Bansal and Yaron (2004): implications of these shocks for asset prices/returns, i.e., compensation for “long-run risks”
What we find...

Investments in poor countries are more exposed to global long-run risk
⇒ US investor demands higher average returns.

1. Risk accounts for 60-70% of return diff. between US & set of poorest countries

2. Results robust to different levels of disaggregation; grouping of countries

3. LRR key; SRR implies (tiny) *negative* risk premia in poor countries
Measuring Returns to Capital for US Investor

Environment: \( J \) regions;
2 sectors: \( K \) is freely traded—\( P_I, C \) is not (builds on Hsieh and Klenow, 2007);

Representative US agent consumes, considers buying \( K \) in US, investing it in \( j \).
Motivation: Poor countries import capital goods from rich.
(see Eaton and Kortum, 2001, Burstein et al., 2011, Mutreja et al., 2012)

Return from potential investment in \( j \) for US investor:

\[
R_t^j = \alpha \frac{P_{Y,j,t} Y_{j,t}}{K_{j,t}} \frac{1}{P_{I,t}} + (1 - \delta_{j,t+1}) \frac{P_{I,t+1}}{P_{I,t}}
\]

\( D_{j,t} \) in \( C_{US,t} \)
\( \Delta P \)
dividend yield, \( D/P \)
capital gains

\( P_I, P_{Y,j} \) are prices of \( K, Y \) relative to US price of \( C \) (Gomme et al., 2011).
Returns Revisited: “Bundles” of countries

Assume common $\alpha = 0.3$. 

144 countries bundled according to mean output per worker over period.

Data:

- PWT 8.0, 1950-2009: $K_{j,t}, \delta_{j,t}, P_{Y,US,2005} Y_{j,t}$—adjust by $\frac{P_{Y,US,t}}{P_{C,US,t}}$
- BEA 1950-2009:
  - $P_{I,US}$—price index of equipment + structures
  - $P_{Y,US}$—price index of output
  - $P_{C,US}$—price index of non-durables + services
Model

Endowment economy in spirit of Bansal and Yaron (2004):

- EZ US investor consumes, gets dividends from inv'ts in US + abroad
  1. Two sources of risk:
    - Small but persistent global component in $\Delta c^{(*)}$ and $\Delta d^{(*)}$ ($*=$foreign)
    - Small but persistent *-specific component; orthogonal to global component
  2. US investor prices exposure to global component
  3. Key challenge in calibration: disentangle global from *-specific shocks
Representative US-based investor endowed with $c$, $d$, $d^*$

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \eta_{t+1} \\
 x_{t+1} &= \rho x_t + e_{t+1} \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}
\end{align*}
\]

\[\eta_{t+1} \sim N(0, \sigma_\eta), \quad e_{t+1} \sim N(0, \sigma_e), \quad \mu_{t+1} \sim N(0, \sigma_\mu)\]

US variables:

- $\mu_c(\mu_d)$: unconditional mean growth rate of consumption (dividend)
- $\eta$: transitory shock to consumption growth
- $\phi$: exposure of dividend growth to trend shock in consumption growth
- $\pi$: exposure of dividend growth to transitory shock in consumption growth
- $\mu$: transitory shock to dividend growth
Model With Global and Local Shocks: Foreign Processes

\[ \Delta c^*_{t+1} = \mu_c^* + \xi^* x_t + x^*_t + \pi_c^* \eta_{t+1} + \eta^*_{t+1} \]
\[ x^*_{t+1} = \rho^* x^*_t + e^*_{t+1} \]
\[ \Delta d^*_{t+1} = \mu_d^* + \tilde{\phi}^* (\xi^* x_t + x^*_t) + \pi^* \eta_{t+1} + \pi_d^* \mu_{t+1} + \pi_{cd}^* \eta^*_{t+1} + \mu^*_{t+1} \]

\[ \eta^*_{t+1} \sim N(0, \sigma^* \eta) \, , \, e^*_{t+1} \sim N(0, \sigma^* e) \, , \, \mu^*_{t+1} \sim N(0, \tilde{\sigma} \mu^* ) \]

- \( \mu_c^*, (\mu_d^*) \): unconditional mean growth rate of consumption (dividend)
- \( \xi^* \): exposure of cons. growth to trend shock in US cons. growth
- \( \pi_c^* \): exposure of cons. growth to transitory shock in US cons. growth
- \( \eta^* \): transitory shock to consumption growth
- \( \tilde{\phi}^* \): exposure of dividend growth to local trend shock
- \( \pi^* \): exposure of dividend growth to transitory shock in US cons. growth
- \( \pi_d^* \): exposure of dividend growth to transitory shock in US dividend growth
- \( \pi_{cd}^* \): exposure of dividend growth to transitory shock in cons. growth
- \( \mu^* \): transitory shock to dividend growth
Model: US Investor’s Preferences

Representative US-based investor’s preferences

\[ V_t = \left[ (1 - \beta) C_t^{\psi - 1} + \beta \nu_t (V_{t+1})^{\psi - 1} \right]^{\psi - 1} \]

\[ \nu_t (V_{t+1}) = \left[ E_t \left( V_{t+1}^{1 - \gamma} \right) \right]^{1 - \gamma} \]

\begin{itemize}
  \item \( \nu_t (V_{t+1}) \) is certainty equivalent function
  \item \( \psi \) is IES; \( \gamma \geq 0 \) is risk aversion coefficient; CRRA special case iff \( \gamma = 1/\psi \)
\end{itemize}

Risk premia for US and foreign assets:

\[ \mathbb{E} [R_t^e] = \left( \phi - \frac{1}{\psi} \right) \left( \gamma - \frac{1}{\psi} \right) \frac{\kappa_{m,1}}{1 - \kappa_{m,1} \rho} \frac{\kappa_1}{1 - \kappa_1 \rho} \sigma_e^2 + \gamma \pi \sigma_{\eta}^2 \]

long-run risk

\[ \mathbb{E} [R_t^{e*}] = \left( \phi^* - \frac{1}{\psi} \right) \left( \gamma - \frac{1}{\psi} \right) \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho} \frac{\kappa_1}{1 - \kappa_1 \rho} \sigma_e^2 + \gamma \pi^* \sigma_{\eta}^2 \]

long-run risk

short-run risk

where \( \phi^* \equiv \tilde{\phi}^* \xi^* \) and \( \kappa \)’s are endogenous objects.
Model: Calibration of US Parameters

\[
\Delta c_{t+1} = \mu_c + x_t + \eta_{t+1}
\]

\[
x_{t+1} = \rho x_t + e_{t+1}
\]

\[
\Delta d_{t+1} = \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}
\]

\[\gamma = 10, \psi = 1.5, \beta = 0.99; \rho = 0.93 \text{ (Ferson, 2013; Bansal, Kiku, Yaron, 2012)}\]

\[
\mathbb{E}[\Delta c_t] = \mu_c
\]

\[
\text{cov}(\Delta c_t, \Delta c_{t+1}) = \rho \frac{\sigma_e^2}{1 - \rho^2}
\]

\[
\text{var}(\Delta c_t) = \frac{\sigma_e^2}{1 - \rho^2} + \sigma_\eta^2
\]

\[
\mathbb{E}[\Delta d_t] = \mu_d
\]

\[
\sqrt{\frac{\text{cov}(\Delta d_{t+1}, \Delta d_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}} = \phi
\]

\[
\text{cov}(\Delta d_t, \Delta c_t) = \phi \frac{\sigma_e^2}{1 - \rho^2} + \pi \sigma_\eta^2
\]

\[
\text{var}(\Delta d_t) = \phi^2 \frac{\sigma_e^2}{1 - \rho^2} + \pi^2 \sigma_\eta^2 + \sigma_\mu^2
\]
Rewrite the foreign dividend process:

\[
\Delta d_{t+1}^* = \mu_d^* + \phi_* x_t + \tilde{\phi}^* x_t^* + \pi_* \eta_{t+1} + \pi_d^* \mu_{t+1} + \pi_{cd}^* \eta_{t+1} + \mu_{t+1}^*
\]

Iterative procedure to identify \( \phi_* \) from:

\[
\text{cov} (r_{m,t}^*, r_{m,t}) = \frac{1}{\psi^2} \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \pi \sigma_{\eta}^2 + \pi_d^* \sigma_{\mu}^2 + \frac{\kappa_{m,1}}{1 - \kappa_{m,1} \rho} \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho} \left( 1 - \frac{1}{\psi} \right) \left( \phi^* - \frac{1}{\psi} \right) \sigma_e^2
\]

+ moment conditions for remaining parameters.

For intuition, consider following moment:

\[
\text{cov} (\Delta d_{t}^*, \Delta d_t) = \phi \phi^* \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \pi \sigma_{\eta}^2 + \pi_d^* \sigma_{\mu}^2
\]

- Both dividend growth and returns comove due to common transitory and persistent shocks, but return comovement is more sensitive to latter.
- Persistent shock \( \Rightarrow \Delta \text{ asset prices due to change in future growth prospect \( \Rightarrow \text{cov} (r_{m,t}^*, r_{m,t}) \text{ relative to cov} (\Delta d_{t}^*, \Delta d_t) \text{ is larger.} \)
### Data: Moments

#### US Moments

<table>
<thead>
<tr>
<th>Consumption</th>
<th>$E[\Delta c_t]$</th>
<th>$\text{cov}(\Delta c_{t+1}, \Delta c_t)$</th>
<th>$\text{std}(\Delta c_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.019</td>
<td>0.00024</td>
<td>0.022</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Dividends</th>
<th>$E[\Delta d_t]$</th>
<th>$\sqrt{\frac{\text{cov}(\Delta d_{t+1}, \Delta d_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}}$</th>
<th>$\text{cov}(\Delta d_t, \Delta c_t)$</th>
<th>$\text{std}(\Delta d_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.006$</td>
<td>2.19</td>
<td>0.00018</td>
<td>0.026</td>
</tr>
</tbody>
</table>

#### Foreign Moments

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$E[\Delta d_t^*]$</th>
<th>$\text{cov}(\Delta d_{t+1}^<em>, \Delta d_t^</em>)$</th>
<th>$\text{cov}(\Delta d_t^*, \Delta c_t)$</th>
<th>$\text{std}(\Delta d_t^*)$</th>
<th>$\text{cov}(\Delta d_t^*, \Delta d_t)$</th>
<th>$\text{cov}(r_t^*, r_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.017$</td>
<td>$0.00122$</td>
<td>$0.00011$</td>
<td>$0.083$</td>
<td>$0.00032$</td>
<td>$0.00109$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.015$</td>
<td>$0.00156$</td>
<td>$0.00011$</td>
<td>$0.074$</td>
<td>$0.00033$</td>
<td>$0.00100$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.011$</td>
<td>$0.00075$</td>
<td>$0.00015$</td>
<td>$0.063$</td>
<td>$0.00040$</td>
<td>$0.00085$</td>
</tr>
</tbody>
</table>

- **Portfolio moment computed as mean of country-specific moments**
- $\uparrow \text{cov}(\Delta d_t^*, \Delta d_t) + \downarrow \text{cov}(r^*, r)$ wrt income $\Rightarrow \uparrow \pi^{\star}_d + \downarrow \phi^{\star}

**Question**: What are the implications for returns to capital?
## Predicted VS Actual Returns: 3, 5, 10 Bundles

<table>
<thead>
<tr>
<th>3 Portfolios</th>
<th>5 Portfolios</th>
<th>10 Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{r}$</td>
<td>$r$</td>
<td>$\hat{r}$</td>
</tr>
<tr>
<td>1</td>
<td>10.55</td>
<td>13.02</td>
</tr>
<tr>
<td>2</td>
<td>9.30</td>
<td>11.07</td>
</tr>
<tr>
<td>3</td>
<td>7.18</td>
<td>8.05</td>
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<tr>
<td>US</td>
<td>5.94</td>
<td>6.02</td>
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<tr>
<td>5</td>
<td>6.28</td>
<td>6.75</td>
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<tr>
<td>US</td>
<td>5.94</td>
<td>6.02</td>
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<tr>
<td>9</td>
<td>6.60</td>
<td>7.63</td>
</tr>
<tr>
<td>10</td>
<td>6.04</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Average: 8.24 9.54 8.52 9.92 8.72 10.26

Spread: 1-US 4.61 7.00 5.08 8.36 6.44 10.38

Percent of actual corr($\hat{r}, r$): 66 61 62 1.00 0.92 0.91

Model delivers 61-66% of spread.
• 96 countries in PWT in or prior to 1961: corr(\(\hat{r}, r\)) is 0.61

• predicted semi-elasticity of returns wrt income is 55% of actual:
  -0.023 (model) vs. -0.013 (data)
## Decomposition: Long VS Short Run Risk

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Actual $r$</th>
<th>Predicted $\hat{r}$</th>
<th>$\hat{r}^f$</th>
<th>$\hat{r}_{sr}^e$</th>
<th>$\hat{r}_{lr}^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.02</td>
<td>10.55 = 1.29 + -0.28 + 9.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11.07</td>
<td>9.30 = 1.29 + -0.18 + 8.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8.05</td>
<td>7.18 = 1.29 + 0.08 + 5.82</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>6.02</td>
<td>5.94 = 1.29 + 0.22 + 4.44</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- High returns mainly driven by long-run risk
- Short-run risk compensation in low/middle-income countries
Conclusion

Can differences in risk quantitatively account for differences in returns?

Yes!

Risk (measured properly) accounts for 60-70% of difference in returns between poorest countries and US.

Key implication: Despite large return differentials, observed capital allocation is not so distant from that predicted by theory.

Future work should investigate sources of differences in long-run risk.
Model: Calibration of Foreign Parameters

Remaining moments on dividends similar to those for US.

Let \( \rho^* = \rho, \ \sigma_{\mu^*}^2 = \pi_{cd}^* \sigma_{\eta^*}^2 + \tilde{\sigma}_{\mu^*}^2\).

Guess \( \phi^* \), then use \( \text{cov}(\Delta d^*_t, \Delta d_t) \) and following moment conditions:

\[
\mathbb{E}[\Delta d_t^*] = \mu^*_d \\
\text{cov}(\Delta d_t^*, \Delta c_t) = \phi^* \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \sigma_{\eta}^2 \\
\text{cov}(\Delta d_{t+1}^*, \Delta d_t^*) = \left(\tilde{\phi}^* \sigma_{e^*}\right)^2 \frac{\rho^*}{1 - \rho^*2} + \left(\phi^* \sigma_{e}\right)^2 \frac{\rho}{1 - \rho^2} \\
\text{var}(\Delta d_t^*) = \left(\phi^* \right)^2 \frac{\sigma_e^2}{1 - \rho^2} + \left(\tilde{\phi}^* \sigma_{e^*}\right)^2 \frac{1}{1 - \rho^2} + \pi^* \sigma_{\eta}^2 + \pi_d^* \sigma_{\mu}^2 + \sigma_{\mu^*}^2
\]

Verify \( \phi^* \) using \( \text{cov}\left(r_{m,t}^*, r_{m,t}\right) \) above.

Notable difference:
\( \text{cov}(\Delta c_{t+1}^*, \Delta c_t^*) \) not needed b/c \( \tilde{\phi}^* \) and \( \sigma_{e^*} \) need not be separately identified.
Alternative moment for $\phi^*$

$$
\phi_{\text{biased}} = \sqrt{\frac{\text{cov}(\Delta d^*_{t+1}, \Delta d^*_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}} = \phi^* \sqrt{1 + \frac{1}{\xi^2} \frac{\sigma^2_{e^*}}{\sigma^2_{e^*}}}
$$

- larger country-specific trend shock $\sigma^2_{e^*}$ $\Rightarrow$ larger bias
- $\sigma^2_{e^*} = 0$ yields nearly identical results in our calibration
- $\pi_{d^*} = 0$ reverses returns; it ignores comovement due to transitory shocks

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Actual</th>
<th>Benchmark</th>
<th>$\sigma_{e^*} = 0$</th>
<th>$\pi_{d^*} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.02</td>
<td>10.55</td>
<td>11.06</td>
<td>10.56</td>
</tr>
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<td>2</td>
<td>11.07</td>
<td>9.30</td>
<td>12.05</td>
<td>9.33</td>
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<tr>
<td>3</td>
<td>8.05</td>
<td>7.18</td>
<td>9.59</td>
<td>7.19</td>
</tr>
</tbody>
</table>

Portfolio, Actual, Benchmark, Autocovariance, Baseline, $\pi_{d^*} = 0$
### Calibrated Model: Parameters

<table>
<thead>
<tr>
<th>Preferences:</th>
<th>$\gamma = 10$</th>
<th>$\psi = 1.5$</th>
<th>$\beta = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption:</td>
<td>$\rho = 0.93$</td>
<td>$\mu_c = 0.019$</td>
<td>$\sigma_e = 0.006$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\mu_d$</th>
<th>$\phi$</th>
<th>$\pi$</th>
<th>$\pi_d$</th>
<th>$\sigma_\mu$</th>
<th>$\tilde{\phi}^* \sigma_e^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.017</td>
<td>5.14</td>
<td>-1.24</td>
<td>-0.16</td>
<td>0.074</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>-0.015</td>
<td>4.23</td>
<td>-0.81</td>
<td>-0.00</td>
<td>0.061</td>
<td>0.011</td>
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<tr>
<td>3</td>
<td>-0.011</td>
<td>2.87</td>
<td>0.34</td>
<td>0.27</td>
<td>0.056</td>
<td>0.008</td>
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<tr>
<td>US</td>
<td>-0.006</td>
<td>2.19</td>
<td>0.98</td>
<td>-</td>
<td>0.020</td>
<td>-</td>
</tr>
</tbody>
</table>
With CRRA ($\gamma$) preferences:

$$\mathbb{E}[r^e_{jt}] \approx \gamma \text{cov}(r_{jt}, \Delta c_t)$$

- $r^e_{jt} \equiv r_{jt} - r_f$ is excess (net) return on portfolio j over 3-month t-bill at t
- $\Delta c_t$ is US (ND+S) consumption growth during 1950-2009

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$r^e$</th>
<th>$\text{cov}(r, \Delta c)$</th>
<th>$\gamma$ =</th>
<th>$\hat{r}^e$</th>
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<tbody>
<tr>
<td>1</td>
<td>11.80</td>
<td>0.00016</td>
<td>0.16</td>
<td>14.06</td>
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<td>2</td>
<td>9.85</td>
<td>0.00009</td>
<td>0.09</td>
<td>7.93</td>
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<td>3</td>
<td>6.83</td>
<td>0.00004</td>
<td>0.04</td>
<td>3.92</td>
</tr>
<tr>
<td>US</td>
<td>4.80</td>
<td>0.00004</td>
<td>0.04</td>
<td>3.23</td>
</tr>
<tr>
<td>Spread: 1-US</td>
<td>7.00</td>
<td>0.00012</td>
<td>0.12</td>
<td>10.83</td>
</tr>
</tbody>
</table>

$\gamma$ falls as granularity increases; at country level it is very high, $\approx 500$. 
# Predicted VS Actual Returns—Annually Rebalanced Portfolios

<table>
<thead>
<tr>
<th></th>
<th>3 Portfolios</th>
<th>5 Portfolios</th>
<th>10 Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{r}$</td>
<td>$r$</td>
<td>$\hat{r}$</td>
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<tr>
<td>1</td>
<td>10.56</td>
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<td>US</td>
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<td>5.94</td>
</tr>
<tr>
<td>Average:</td>
<td>8.07</td>
<td>8.98</td>
<td>8.31</td>
</tr>
<tr>
<td>Spread: 1-US</td>
<td>4.61</td>
<td>6.08</td>
<td>5.09</td>
</tr>
<tr>
<td>Percent of actual</td>
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<td></td>
<td>73</td>
</tr>
<tr>
<td>corr($\hat{r}, r$)</td>
<td>1.00</td>
<td></td>
<td>0.96</td>
</tr>
</tbody>
</table>
Key asset-pricing equation for any asset:

$$E_t[M_{t+1} R_{t+1}] = 1,$$

$M_{t+1}$ is US investor’s SDF.

In logs,

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{ct+1}$$

where $\theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}}$

and $r_{ct+1}$ is return on asset that pays consumption as dividend.
Solving the Model

Standard approach; approximations to make exercise computationally feasible

Note that $z_t = \log \left( \frac{P_t}{D_t} \right)$ is enough to characterize returns

- For each asset $k = \text{home}, \ast, \text{and asset that pays off aggregate consumption}$

To solve:

- Conjecture: $z^k_t = A^k_0 + A^k_1 x_t$

- Approximate log returns: $r^k_{m,t+1} = \kappa^k_0 + \kappa^k_1 z^k_{t+1} + \Delta d^k_{t+1} - z^k_t$
  where $\kappa$'s depend on $\bar{z}^k$

- Restrictions from Euler equation gives excess return (as in text) and rf rate

- Solve numerically for $\bar{z}^k = A^k_0 (\bar{z}^k)$ and for $A^k_1 (\bar{z}^k)$
Alternative Measurement Approaches

1. Caselli and Feyrer (2007): country-specific $P_I$, $P_C$, $P_Y$ from PWT.
2. Country-time specific $\alpha$’s + non-reproducible capital adjustment from WDI

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Baseline</th>
<th>Country prices</th>
<th>Country $\alpha$’s</th>
<th>Country prices &amp; $\alpha$’s</th>
<th>Baseline</th>
<th>Country prices</th>
<th>Country $\alpha$’s</th>
<th>Country prices &amp; $\alpha$’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.02***</td>
<td>12.00***</td>
<td>13.22***</td>
<td>13.63***</td>
<td>5.38**</td>
<td>3.32</td>
<td>8.24**</td>
<td>7.27</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(0.66)</td>
<td>(0.76)</td>
<td>(1.05)</td>
<td>(.78)</td>
<td>(2.81)</td>
<td>(1.25)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>2</td>
<td>11.07***</td>
<td>10.53***</td>
<td>13.15***</td>
<td>13.23***</td>
<td>5.21</td>
<td>5.89</td>
<td>8.12*</td>
<td>7.51</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.62)</td>
<td>(0.75)</td>
<td>(0.68)</td>
<td>(0.99)</td>
<td>(1.73)</td>
<td>(1.29)</td>
<td>(1.42)</td>
</tr>
<tr>
<td>3</td>
<td>8.05***</td>
<td>9.36**</td>
<td>9.17***</td>
<td>11.39***</td>
<td>3.91</td>
<td>10.03*</td>
<td>6.47</td>
<td>14.09***</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.33)</td>
<td>(0.49)</td>
<td>(0.44)</td>
<td>(0.85)</td>
<td>(1.48)</td>
<td>(1.29)</td>
<td>(1.22)</td>
</tr>
<tr>
<td>US</td>
<td>6.02</td>
<td>8.22</td>
<td>6.20</td>
<td>9.39</td>
<td>3.64</td>
<td>7.23</td>
<td>5.52</td>
<td>9.55</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.31)</td>
<td>(0.39)</td>
<td>(0.50)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: Table reports the returns to capital across portfolios under a number of measurement approaches. Baseline uses US prices from BEA. Country prices uses country-specific $P_Y$, $P_I$, $P_C$ from PWT. Country $\alpha$’s uses country-year $\alpha$ from PWT and subtracts from $\alpha$ the share of payments to non-reproducible capital from WDI, dropping the countries that have negative $\alpha$ for at least one year. Country prices and $\alpha$’s uses country prices and country-year $\alpha$ as described above. Baseline and Country prices cover years from 1950 to 2008. Country $\alpha$’s and Country prices and $\alpha$’s cover years from 1970 to 2008. The portfolios include only countries for which data are available. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.
Countries with open capital accounts obey link btwn GDP and returns.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Measure of Openness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chinn, Ito</td>
</tr>
<tr>
<td>1</td>
<td>10.38***</td>
</tr>
<tr>
<td></td>
<td>(0.66)</td>
</tr>
<tr>
<td>2</td>
<td>8.74***</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
</tr>
<tr>
<td>3</td>
<td>5.61</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
</tr>
<tr>
<td>US</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
</tr>
</tbody>
</table>

Notes: Table reports the returns to capital across portfolios for economies that are characterized as open according to three indices: Chinn/Ito, Quinn, and Grilli/Milesi-Ferretti, respectively. Chinn/Ito and Quinn openness cutoff is median value in sample. Grilli/Milesi-Ferretti openness indicator is unity. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.
Supplementary Results

Note:

- $A$'s depend on US parameters only
- $\sigma^2, \tilde{\phi}^2, \sigma^2$ and calibrated parameters are sufficient to compute $\kappa_{m,1}^*$

\[
A_{m,0}^* = \frac{\theta \log \beta - \gamma \mu + (\theta - 1) (\kappa_0 + A_0 (\kappa_1 - 1)) + \mu^* + \kappa_{m,0}^* + \frac{1}{2} \left( \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho^*} \right)^2 (\tilde{\phi}^* \sigma^*_e)^2}{1 - \kappa_{m,1}^*} + \frac{1}{2} (\pi^* - \gamma)^2 \sigma^2_{\eta} + \frac{1}{2} \left( (\theta - 1) \kappa_1 A_1 + \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho^*} (\phi^* - \frac{1}{\psi}) \right)^2 \sigma^2_e + \frac{1}{2} \pi^2 \sigma^2_{d \mu} + \frac{1}{2} \sigma^2_{\mu^*}}{1 - \kappa_{m,1}^*}
\]