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Debt, Hedging, and Human Capital

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Abstract: This paper provides a theory of debt and hedging based on human capital. We distinguish human capital from physical capital in two ways: (1) human capital is inalienable and can exercise a one-sided option to leave the firm, and (2) human capital is not perfectly replaceable. We show that a firm may reach the first best solution while issuing debt or equity to outsiders provided that either the insiders receive a senior claim or that the firm hedges. We then show that, given asymmetric information concerning costs, the only viable solution has the firm issuing debt to outsiders and hedging.

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Key words: hedging, human capital, capital structure

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Debt, Hedging and Human Capital

A formal analysis regarding the relevancy of risk management by the corporation dates back to the work of Modigliani and Miller (1958). Their seminal result is that, absent frictions such as taxes, on balance sheet risk management in the form of leverage decisions make no difference in the value of the corporation. The extension of this result to off balance sheet arrangements, such as futures and swaps, is trivial once it is recognized that both transactions are just different forms of risk management. The intuition behind this famous result is, of course, that, absent frictions, investors can manage their own risk exposure in any way they please and therefore have no incentive to place a higher value on corporations who manage such risks at the corporate level.

Most of the subsequent literature regarding on balance sheet risk management (i.e., capital structure) and off balance sheet risk management has been developed using a theory of the firm that emphasizes the importance of physical assets in place or growth opportunities. However, human capital has supplanted physical assets as the primary determinant of firm value in a growing number of corporations, a point emphasized by Myers (1999) and Zingales (2000, pg. 1624). The theory they propose is based on the firm as a nexus of contracts between those parties that control assets and those that hold outside claims to the firm. This theory raises questions about traditional theories of the firm, including theories of hedging and capital structure that focus exclusively on the value of residual claimants. In particular, as Zingales has emphasized, attention needs to be paid to the role of human capital and the types of contracting needed to insure that first best outcomes can be obtained wherever possible.

The contracting issues that arise with regard to human capital are in some ways unique. Hart and Moore (1994) emphasize the inalienable nature of human capital in their development

of a theory of debt. Moreover, Qian (2003) has analyzed the capital structure decision when the firm employs workers with special skills and shows that firms with high levels of valuable human capital will hold less debt. In his model commitments to hi-tech workers acts as a substitute for leverage due to the better risk sharing properties of equity.

In this paper we extend the literature on capital structure and risk management by analyzing the role of hedging in a world where human capital is of first order importance. While general, we view the paper as particularly applicable to trading firms and financial institutions. Indeed, Qian (2003) has shown that these firms rank highest in his empirical measures of human capital intensity.

The model developed here is driven by two key assumptions. The first is that human capital is inalienable. Managers and workers cannot be forced to work and we assume that they are capable of generating some pecuniary benefits from outside opportunities. The second labor market assumption that we make is that managers and other employees outside the firm are not perfect substitutes for managers and workers currently in the firm. Assuming that we start in a world with the best manager/production technology mix, we assume that there is some cost, (in terms of lower output) that must be borne if managers and other skilled workers exercise their option to leave the firm. Of course, in equilibrium, the cost of the lost output within the firm must be paid for by the insiders that control the assets of the firm.

We analyze two situations. In the first there is symmetric information. In this case we contrast three alternative strategies. We first show that there is an overhang problem with simply issuing outside debt or equity. In this case the managers may opt for their outside option in some states even though it is socially optimal to stay with the firm because the realized profits are

insufficient to make the required payments to outside claimants and pay the managers enough to keep them from exercising their outside option.

In the second case we consider, the managers issue either some outside equity or junior debt. In addition, however, there is a senior claim (or salary) to cash flows held by the managers and other employees of the firm that at least equals the value of their outside option. We show that such an arrangement yields the first best solution. The firm could also issue outside debt or equity and hedge by transferring sufficient income from high profit states to low profit states so that the firm can honor its outside obligations and still pay insiders an amount at least equal to their outside obligation. This also yields the first best allocation. Therefore, we show that in a world of symmetric information simple explicit contracts such as salary can be used to bond managers to the firm. Hedging adds no additional value in this case and the risk management structure of the firm is a fairly simple one: senior claims to insiders and junior claims in the form of debt or outside equity to the outsiders.

In the third case that we consider there is asymmetric information regarding a second determinant of the firm's value (for simplicity, we focus on asymmetry about the firm's cost of production) where outside investors know the distribution of the cost of production but the managers know the realization. The price at which outsiders will purchase a claim is based on their knowledge distribution of possible values of the cost parameter. If this price depends on the distribution of the cost parameter however, it will almost surely differ from the fair price based on the managers' observation of the realized value. If outsiders underprice their claims, the managers will gladly accept the transfer. Even if outsiders overprice the claims, managers will nevertheless issue the claims if the project's expected profits exceed the wealth transfer to outside claimants.

However, if the firm has the option of issuing debt and making that debt riskless by hedging, then it can eliminate the difference between outside investors' and the managers' valuation of the debt. If the firm has access to this financing option, it will dominate the other options from the perspective of outside investors. Note, that hedging by itself is insufficient to resolve the asymmetry problem, as hedging cannot make the residual claim riskless in the presence of unhedgeable uncertainty about the firm's profitability. Therefore, there is a unique solution to the problem under asymmetric information and that solution involves hedging with outside debt. We note that such a contracting solution bonds the managers to the firm in such a way that they exercise their outside options to leave the firm only when it is socially desirable to do so. Thus, the risk management solution involves two explicit contracts. One is between insiders and outside debt holders while the second involves insiders and third parties that represent the offsetting futures positions.

Thus, unlike the Modigliani and Miller (1963) result, our results show that there is both an optimal capital structure and an optimal risk management strategy without the need to resort to tax based arguments. Even with symmetric information, capital structure decisions do influence firm value in the presence of inalienable human capital. The addition of even a small amount of information asymmetry is sufficient to make it optimal for the firm to have equity that is held by insiders and outside debt. Moreover, the firm should hedge its cash flows in a manner that minimizes the variability of total cash flows: that is cash flows from operations plus those from the hedge position. Outside equity leads to inefficient allocations and will only be chosen if hedging is not available.

In order to motivate our results consider an investment bank owned by its partners with an M&A section and a proprietary trading desk. Suppose that the bonuses in the M&A can be

influenced by total firm profitability. If the trading desk suffers a loss, say due to interest rate changes, bonuses in the M&A section may be jeopardized. In this case workers in both the M&A section and the trading desk may walk if they have viable alternative employment. Conversely, if the firm had hedged this risk then the loss on the trading position would have been offset by gains on say a futures contract thereby eliminating the incentive for workers to seek outside employment. If the bank needed to raise outside funding, investors in risky securities issued by the bank would demand compensation for the adverse selection risks associated with opacity about the value of the firm's existing proprietary assets and the future profitability of its trading and M&A operations. The lowest cost way to fund the firm would be for it to issue debt and make that debt riskless by hedging.

Our paper is related to a number of papers in the extant literature. Indeed, the last twenty years has brought forth a number of rationales for why firms should manage risk using off balance sheet contracts that are based on well known theories in corporate finance. The first involves managerial risk aversion, the convexity of the tax schedule or the existence of costly financial distress discussed by Stulz (1984) and Smith and Stulz (1985). One could view our model as extending Smith and Stulz (1985) by formally modeling financial distress costs in the form of the departure of valuable human capital rather than assuming the existence of such costs. Indeed, our results are consistent with the observation that when firms get into financial distress one often sees the top managers exit before formal bankruptcy is announced. Hedging, if possible, might have allowed the firm to avoid the financial distress brought on by the exit of valuable managers.[1]

Other rationales, while non-exclusive, fall into three categories; (a) alleviating the underinvestment problem first posed by Myers (1977), (b) dealing with asymmetric information

problems between insiders and outsiders (the Myers and Majluf (1984) "problem") and (c) agency cost problems like those discussed in Jensen and Meckling (1976).[2]

One paper that appears closely related to ours is that of Bessembinder (1991). Bessembinder argues that by hedging the firm can alleviate the debt overhang problem by shifting some of the distribution from default to non-default states in which the share of profits going to equity holders is larger. Thus, some projects in Myers (1977) setup that would not have been taken on absent hedging may now have a positive NPV to the firm's owners if the firm can commit to hedging in the futures market.

Although our paper is like Bessembinder's in that it considers the use of hedging to overcome a debt overhang problem, the focus of our paper is different and that leads to a difference in one of the models' key assumptions. Bessembinder's focus is on the scale of the investment by the firm ex ante, that is on whether and to what extent the firm's investment will fall short of its of its optimum value due to the debt overhang problem. Given this focus, the natural assumption is that uncertainty about the value of the hedgeable price is resolved after the investment has been made. In contrast, our focus is on the decision of managers and other insiders as to whether they will stay with the firm or exercise their outside option ex post. As this option may be exercised at any time, the natural assumption in our model is that insiders decide whether to exercise their outside option after the hedgeable price is observed.

The difference in assumptions has important implications for the results. The firm could eliminate the debt overhang problem in Bessembinder's model by issuing equity. In contrast, in our model a simple equity issue by itself does not solve the overhang problem even with symmetric information; moreover all of the equity issuance alternatives are dominated by the combination of debt and hedging with asymmetric information. Our model is thus best viewed

as a complement to Bessembinder. Bessembinder's model is the appropriate for analyzing the use of hedging to reduce debt overhang problem for firms whose biggest concern is avoiding Myer's under investment problem. In contrast, our model is the appropriate model for analyzing an overhang problem in payments to outside clients for firms primarily concerned about retaining human capital.

Another paper that has some similarities is Almazan, Suarez and Titman (2002) which focuses on the implications of information release for the terms under which the firm contracts with its employees. Some, but not all, workers in their model are willing to contract at less than their best outside wage if the workers perceive that the forgone wages will be more than offset by increased accumulation of human capital relative to their outside opportunity. However, this willingness to accept less than their best option depends on the perception that the firm is at the forefront of emerging technology, a perception which can be undercut if the firm is having financial difficulties.

The key difference in our papers is the assumption about the information set of key insiders. Almazan, Suarez and Titman (2002) assume that the key insiders (which they label the quick learners) understand some technology that is important to the firm but do not have information about the condition of the firm. The release of information changes the terms on which these employees will contract with the firm in an asymmetric way with bad information raising the cost of contracting more than the release of good information would reduce the costs. Almazan, Suarez and Titman (2002) cite innovative (or high tech) firms as examples of the types of firms to which their model is most applicable. In contrast, the role of losses in forcing the release of new information about the firm is irrelevant to our model; we assume the key insiders in our model have access to this information. As mentioned earlier, while our model may have

applications to a variety of settings, the model would be an especially good description of the senior staff at financial services firms, especially investment banks. Senior investment bankers are unlikely to work at a rate below their best outside opportunity in the belief that they will learn more about the technological state of the art. However, they are likely to leave if unhedged trading losses threaten to reduce or eliminate the annual bonuses which they would otherwise have earned.

Probably the most popularly cited work on rationales for hedging is provided by Froot, Scharfstein, and Stein (FSS) (1993) and Froot and Stein (1998). They revisit the asymmetric information model of Myers and Majluf (1984) and argue that costless hedging can prove to be value increasing if the firm faces a concave production technology and convex costs for raising outside capital.[3]

Our paper also differs from FSS. While they argue that outside financing in general is costly with asymmetric information, we emphasize the important role of human capital as an input to the production process and show that this leads to a unique risk management /capital structure configuration that involves hedging and the issuance of a particular type of outside claim-debt capital. Thus, our paper prescribes a unique solution to the financing problem that yields the first best allocation of resources.

The third type of rationale used for the value added that can be obtained from off balance sheet financing comes from agency type problems of Jensen and Meckling (1976). Holmstrom and Tirole (2000) and Leland (1998) study models whereby agency costs between equity holders and debt holders provide a rationale for carrying liquidity. For example, in Holmstrom and Tirole (2000) liquidity may be needed to keep a project going at some future date that could not be financed in the spot market due to agency costs. Leland (1998) on the other hand views hedging

as a way to increase debt capacity and thereby capture the additional tax shields associated with debt financing.

Our paper differs from Holmstrom and Tirole, and Leland in a number of important respects. It is different from Leland in the sense that we show that debt is an optimal part of the capital structure without the need to appeal to taxes or other market imperfections such as costly financial distress. The paper also differs from Holmstrom and Tirole in the sense that their assumed generic agency costs result in all solutions being second best. Moreover, our work differs from both of these papers in the sense that our contracting problems arise endogenously as a result of the particular nature of human capital.

The remainder of the paper is as follows. In section II we describe the model assumptions and setup. In section II(a) we look at the hedge/no hedge decision when there is no asymmetric information, while in section II(b) we model the problem as one where there is asymmetric information between managers and outside claimants. Section III contains some brief concluding remarks, some thoughts on further theoretical extensions and some possible empirical implications one might draw from the model.

I. Model Setup

We assume that all participants in the model are risk neutral. All of the equity is owned by senior managers and other highly skilled workers, which we will refer to simply as "the managers." The managers act as a single unit. The firm is incorporated so that the managers cannot walk away with cash flows that are due to outside claimants.

We assume that managers are not perfectly interchangeable. In particular we assume that initially each firm is run by the "best" management team associated with a given production process. Therefore, if these managers "exit" the firm there will be some loss of output. Without

loss of generality we assume that without the first best management, the firm ceases to operate, and corresponding cash flows from operations are zero.[4]

The model has two dates. At date zero all participants know the production and cost functions of all firms, along with the value of exit options to the managers. Furthermore, all market participants possess a distribution of the output price associated with production but the realization of this price is not known until date one. Managers observe the realization and decide whether to exercise their outside option. If the managers stay, the firm produces the optimal number of output units, q, and the product is sold at time 1. Finally, the gross profits from operation are then distributed.

In order to keep things simple, we assume that the marginal cost function for output is, $c(q) = \gamma q^2/2$. Production is assumed to be immediate so that profits are realized at date one. However, in order to have access to this production technology the firm must pay a fixed cost of K dollars that must be raised by issuing outside claims. We assume that the managers have zero wealth.

At date zero the firm must decide whether or not to hedge the price risk it faces at date one. Let the price R be distributed such that the range of R is a to b. The marginal distribution function is g(r) and the cumulative distribution function is given by G(r).

Since all market participants are risk neutral and the discount rate is assumed to be zero, the futures price, set at date zero, will equal the mean of the expected future spot price at date one. In particular, $r^f = E(R)$ is going to be a *result*, rather than an assumption, in our model, where r^f is the futures price set at date zero, R is the random (viewed from date zero) future spot price of the commodity and E() is the expectations operator.

The "unbiased" futures price result will hold so long as all futures markets participants know that there will be, with probability one, no default on these contracts. For this reason we compare only two hedging solutions. The first is the case of no hedging while the second is one where the firm hedges in such a way that, with probability one, there is sufficient cash flow to honor any commitments to outsiders.[5] Sticking to this comparison allows us to avoid the tricky problem of modeling default risk in the futures market, while still allowing us to derive our principal result: namely that there exist situations where hedging ex ante will increase firm value.[6]

We follow Bessembinder in assuming that the firm can credibly commit to entering into and maintaining a hedge. Bessembinder (1991) justifies this assumption on the ground that the firm may have reputational reasons for maintaining the hedge in a multiperiod model and that debt contracts may contain covenants requiring hedging. The extension of our model to a multiperiod setting would produce an even stronger reason for hedging. As Myers (1999) points out for firms that rely on human capital, "the assets go out to the parking lot every night" and need not return in the morning suggesting every day is a new period in the context of our model.

In order to complete the model we assume that the firm raises funds by issuing claims to outsiders. We first examine the implications of issuing debt claims and then of issuing equity claims. The outside claimants price their claims to break even in expectation. Assuming the risk free rate to be zero and assuming the managers have no initial capital, let us define the *marginal* profits from operations to be, for any realization of $R = r$, as

$$P = qr - \gamma q^2/2. \qquad (1)$$

Assuming the firm is in operation, the managers will maximize the marginal profits from operations at date one, i.e.,

$$q^*(r,\gamma) = r/\gamma. \tag{2}$$

Substituting (2) into (1) yields

$$P^*(r,\gamma) = r^2/2\gamma. \tag{3}$$

Assume that the managers can use their skills to generate optimal marginal profits (equation (3)) within the firm, assuming that they find it in their interest to keep the firm going. Alternatively, the managers can obtain U from outside employment.[7, 8] We assume that there is no direct cost to effort on the part of the managers, whether they work inside or outside the firm.

At date one the value of R is revealed as r, and the managers may choose to exert their effort either in the firm or outside of the firm. The social welfare maximizing outcome is that the managers stay with the firm if staying produces more wealth,

$$P^*(r,\gamma) = q^*(r,\gamma)r - \gamma(q^*(r,\gamma))^2/2 \geq U, \tag{4}$$

and otherwise leave to take the outside employment opportunity. Define r^c as the critical value of r such that the social welfare gains of staying exactly equal those of leaving:

$$r^c(\gamma,U) = \left(U + \gamma\left(\mathrm{q}^*(r,\gamma)\right)^2/2\right)/q^*(r,\gamma) = \sqrt{2\gamma U}\ . \tag{5}$$

If capital markets are efficient, there are no information asymmetries and the managers allocates their effort in a way that maximizes social welfare, their expected wealth is

$$V_0^{sw} = \int_{r^c}^{b} P^*\left(r,\gamma\right)dg(r) + \int_0^{r^c} U dg(r) - K\ . \tag{6}$$

This strategy produces the maximum marginal profits and pays outside investors only the opportunity cost of the resources they provide. Thus, the social welfare maximizing solution also maximizes the managers' expected wealth.

However, the managers are not necessarily entitled to receive P^* but rather must pay $C(r)$ to the outside claimants that supplied the initial fixed capital K. Thus, the managers' actual decision rule is chooses effort, e, at time one to:

$$\underset{e}{\text{Max}} \left((P^*(r,\gamma) - C(r)), U\right) . \qquad (7)$$

Thus, the actual decision will deviate from the managers' wealth maximizing rule whenever $P^* \geq U$ but $P^* - C(r)) < U$. The next section follows the basic model in analyzing a variety of financing alternatives given symmetric information. The following section considers the results given some small asymmetry.

II. Financing options under information symmetry

Whether the manger can obtain his wealth maximizing solution under information symmetry depends on how the payoffs to the outside claimants are structured. This section begins by considering the outcomes if the managers issue debt and then proceeds to analyze the case where the managers issue equity.

A. Debt Issuance

The simplest option for the managers would be to have the firm issue senior debt with no hedging. In this case the debt holders have first claim on any cash flow up to the face value of their debt.

Let $F^s(h)$ denote the face value that must be promised on a senior debt claim issued to the outside claimants in order for them to supply K in capital and still break even in expectation, knowing that there may exist states of the world where the firm is not in operation (i.e., the managers have left the firm). The term h denotes the futures position taken by the firm; $h > 0$ is a short position while $h < 0$ is a long position. Without loss of generality we assume that one unit of h equals one unit of q.

The problem with straight debt having first claim on the firm's cash flow is that it creates a debt overhang problem that causes the managers' choice of effort to deviate from the value maximizing effort decision. Substituting $F^s(0)$ into the managers' decision rule we get that

$$\underset{e}{\text{Max}}\ ((P^* - F^s(0)), U). \tag{8}$$

Using (4) and (3) we get the managers stay with the firm if

$$R \geq r^* = ((2\gamma(F^s(0) + U))^{1/2}), \tag{9}$$

where F(0) solves

$$F^S(0)(1- G(r^*)) = K, \tag{10}$$

and G(r) is the distribution function of r. Stated differently, the managers will inefficiently leave the firm whenever $P^* \geq U > P^* - F^s(0))$. Given an issue of senior debt and no hedging, the value of the managers' position at date zero is then given by

$$\begin{aligned} V^d(0) &= \int_{r^*}^{b} (P^*(r,\gamma) - F^s(0))dg(r) + \int_0^{r^*} U dg(r) \\ &= \int_{r^*}^{b} P^*(r,\gamma)\, dg(r) - K + \int_0^{r^*} U dg(r). \end{aligned} \tag{11}$$

This value is strictly less than the social welfare maximizing value because the managers abandon the firm and exercise their outside option when R takes a value of r^* or less rather than the social welfare maximizing rule of exercising their option at r^c since $r^* > r^c$.

As an alternative to issuing senior debt to outside investors, the firm could issue senior debt to the managers and junior debt to the outside investors. The senior debt would carry a face value at least equal to the managers' outside option, $F^s(0) = U$.[9] The face value of the junior debt would have to satisfy the condition that:

$$\int_{r^*}^{b} F^j(0)dg(r) + \int_{r^c}^{r^*} (P^*(r,\gamma) - F^s(0))dg(r) = K\ , \tag{12}$$

or

$$F^j(0) = \left(K - \int_{r^c}^{r^*} (P^*(r,\gamma) - F^s(0)) dg(r) \right) / \int_{r^*}^{b} dg(r). \tag{13}$$

The expected wealth of the managers is

$$\begin{aligned} V^{dj}(0) = & \int_{r^*}^{b} (P^*(r,\gamma) - F^j(0)) dg(r) \\ & + \int_{r^c}^{r^*} (P^*(r,\gamma) - (P^*(r,\gamma) - F^s(0))) dg(r) + \int_0^{r^c} U dg(r). \end{aligned} \tag{14}$$

Rearranging terms yields,

$$V^{dj}(0) = \int_{r^c}^{b} P^*(r,\gamma) dg(r) + \int_0^{r^c} U dg(r) - K \tag{15}$$

which is the same as the expression for the managers' maximum expected wealth, (6).

Alternatively, the managers could solve the problem by entering into a forward agreement. In order to simplify the analysis we assume that the hedge is riskless, this in turn requires that the firm be sufficiently profitable so that it can enter into a riskless hedge.[10]

Given h^* as the amount of hedging required to make the debt riskless,

$$h^* = K / (r^f - r^c). \tag{16}$$

We make the following two assumptions:

(Assumption 1) $\boldsymbol{q(r^c,\gamma) > h^* > 0.}$

(Assumption 2) $\boldsymbol{r^f - r^c > 0.}$

Proposition 1: **Both of these assumptions will hold if**

$$\boldsymbol{\gamma \leq (r^f)^2 (2U/(K+2U)^2).} \tag{17}$$

Proof: Substitute into the value of $q(r^c,\gamma)$ and h^* for r^c from equation (5) rearranging yields assumption 1. This proves that $q(r^c,\gamma)$ is greater than h^*. To prove r^f greater than r^c multiply both sides of the equation by $2U$ this yields

$$2U\gamma \leq (r^f)^2((2U)^2/(K+2U)^2), \tag{18}$$

Finally, take the square root of both sides of the equation and use equation (5). This shows that $r^f > r^c$ since K is positive, this shows that h^* must be strictly greater than zero. QED.

Given these assumptions, the managers may obtain first best by having the firm hedge h^*.[11]

Proposition 2: The managers will follow their welfare maximizing decision rule of staying with the firm if $P^* \geq U$ if the firm hedges by, h^*, the amount exactly required to make the debt riskless:

$$h^*(r^f - r^c) = K \qquad \text{or} \qquad h^* = K/(r^f - r^c).$$

Proof. See Appendix.

B. Equity Issuance

The issuance of equity rather than debt avoids the debt overhang problem. However, in order to sell equity to investors, the firm must give those investors the right to a share in the firm's profits. The outside equityholder's share absent hedging is S^e (0) with the managers receiving the remaining share of the profits, (1- S^e(0)).

The managers optimally allocate effort according to

$$\max_e \left(\left((1-S^e)P^*(r,\gamma)\right)\right), U). \tag{19}$$

This implies that the managers will stay with the firm if and only if

$$R \geq r^{**} = ((2\gamma(U/(1-S^e))^{1/2}), \tag{20}$$

where S^e solves

$$\int_0^{r^{**}} S^e P^*(r,\gamma) dg(r) = K\,. \tag{21}$$

The resulting value function for the managers is

$$V^e = \int_{r^{**}}^{b} S^e P^*(r,\gamma) dg(r) + \int_0^{r^{**}} U dg(r). \quad (22)$$

Substituting in the solution for $S^e(0)$ from (21)

$$V^e = \int_{r^{**}}^{b} P^*(r,\gamma) dg(r) + \int_0^{r^{**}} U dg(r) - K. \quad (23)$$

Which is less than the wealth maximizing value because r^{**} is greater than r^c. Thus, the "debt overhang" is not simply a problem with an outside obligation that takes priority over the managers' claim. An obligation that that has equal priority with the managers' payments may create an "equity overhang" that also reduces the managers' payments and induces them to inefficiently exercise their outside option.

Given that the equity overhang problem arises from the outside claims that have equal priority with the managers' claim, one potential way to prevent the problem would be to give the managers a debt claim with a face value of at least U. If the debt takes a value exactly equal to U, the firm would first redeem the debt as long as $P^* \geq U$, and would distribute dividends whenever $P^* > U$. If the firm earned less than U then the managers would exercise their outside option, the firm would have no profits and hence would neither redeem the debt nor pay dividends.

Outside investors' required claim on the firm's profits, S^e if the firm gave the managers a debt contract with a face value of U and sold an equity claim on any profits must solve:

$$S^e(0) \int_{r^c}^{b} (P^*(r,\gamma) - U) dg(r) = K, \quad (24)$$

or

$$S^e(0) = K / \int_{r^c}^{b} (P^*(r,\gamma) - U) dg(r). \quad (25)$$

The resulting expected wealth of the managers if the firm sells equity and gives them a debt contract, V^{ed}, is

$$
\begin{aligned}
V^{ed} &= (1-S^e(0))\int_{r^c}^{b}(P*(r,\gamma)-U)dg(r)+\int_{r^c}^{b}Udg(r)+\int_{a}^{r^c}Udg(r) \\
&= \int_{r^c}^{b}P*(r,\gamma)dg(r)-S^e(0)\int_{r^c}^{b}(P(r,\gamma)*-U)dg(r)+\int_{a}^{r^c}Udg(r).
\end{aligned} \tag{26}
$$

Substituting the value of S^e yields

$$
V^{ed} = \int_{r^c}^{b}P*(r,\gamma)dg(r)+\int_{a}^{r^c}Udg(r)-K\,. \tag{27}
$$

This equals the maximum wealth possible for the managers. The result of giving the managers a debt claim is analogous to renegotiating debt issued to outside investors. In both cases, the overhang problem is solved by giving the managers' claim to U priority over payments to outside investors.

Analogous to the debt case, the overhang problem may also be solved by transferring income from the high cash flow states to the low cash flow states so that the firm has sufficient income to both: (a) pay the outside investors their face value (for debt) or dividends (for equity) and (b) pay the managers an amount at least equal to their outside option U. The mechanism for transferring income is a hedge written on the source of uncertainty, r.

If the firm enters into a hedge such that the managers will not exercise their option unless $r < r^c$, then outside investors will demand that their share of the profits satisfy:

$$
S^e(h^*)\int_{r^c}^{b}(P*(r,\gamma)+(r^f-r)h^*)dg(r)+S^e(h^*)\int_{a}^{r^c}(r^f-r)h*dg(r)=K \tag{28}
$$

or

$$
S^e(h^*)=K/\left(\int_{r^c}^{b}(P*(r,\gamma)+(r^f-r)h^*)dg(r)+\int_{a}^{r^c}(r^f-r)h*dg(r)\right). \tag{29}
$$

The minimum size of the hedge, h^*, needed to retain the managers unless $r < r^c$ is the amount just sufficient so that the managers' share of the profits equals the value of their outside option:

$$
(1-S^e(h^*))[((r^c)^2/2\gamma)+(r^f-r^c)h^*]=U, \tag{30}
$$

solving for h^* yields

$$h^* = [U - ((r^c)^2/2\gamma)\ (1 - S^{e^*})]/\ (1 - S^{e^*}(h^*)(r^f - r^c), \tag{31}$$

substituting in using the definition of U, $U = (r^c)^2/2\gamma$

$$h^* = (S^e(h^*)\ U)/\ (1 - S^e(h^*))(r^f - r^c). \tag{32}$$

Given S^{e^*}, the value of the firm to the managers if the firm issues stock and hedges, V^{eh}, is

$$\begin{aligned} V^{eh} &= (1 - S^e(h^*))\int_{r^c}^{b} (P^*(r,\gamma) + (r^f - r)h^*)dg(r) \\ &+ \int_a^{r^c} (U + (1 - S^e(h^*))(r^f - r)h^*)dg(r). \end{aligned} \tag{33}$$

Substituting in the definition of S^{e^*} and rearranging terms yields:

$$V^{eh} = \int_{r^c}^{b} P^*(r,\gamma)dg(r) + \int_a^{b} (r^f - r)h^* dg(r) + \int_a^{r^c} Udg(r) - K\ . \tag{34}$$

Finally given that the expected value of the hedge in equilibrium is zero, the expected value of the firm to the managers is:

$$V^{eh} = \int_{r^c}^{b} P^*(r,\gamma)dg(r) + \int_a^{r^c} Udg(r) - K\ . \tag{35}$$

This is the maximum possible value for the managers' equity.

Thus, both debt and equity can create an "overhang" problem when the managers have inalienable capital and cannot be compelled to work. The overhang of a required payment to outside investors alters the managers' effort decision rule by reducing their share of the profits. As a result, there is a positive probability with simple debt and equity issues that the managers may leave the firm even though their value inside the firm exceeds their value outside the firm.

Given no information asymmetry, there are two strategies for both debt and equity issuance that result in the first best outcome for the managers despite the potential overhang problem. One strategy is to give the managers first claim on the firm's cash flows by giving them a debt issue that is senior to the claims sold to outside investors. If the amount of this

senior claim is at least equal to the value of the managers' outside option then they will stay with the firm so long as its marginal profits at least equal their outside option. The other strategy is to enter into a hedge agreement that will provide sufficient funds to make the required payment to outside creditors and a pay the managers at least the value of their outside option.

III. Financing options under information asymmetry

So far the analysis has assumed that the firm has only one source of risk and that information about the distribution of that risk is symmetric, both the managers and outsiders have the same information. However, a more common situation is that there is more than one source of risk and that the managers know more about at least one of the risks than the outsiders. Outsiders and managers may still be able to contract in the presences of asymmetric information but managers will have different valuations of the claims than outsiders. This section considers whether such differences in valuation arise and what impact the differences would have on security issuance. [12]

The focus of the analysis is on the four financing strategies that produced the maximum possible wealth for managers. The asymmetric information is about the value of the cost parameter Γ distributed such that the range of Γ is c to d.[13] Outside investors are assumed to know that Γ is drawn from some marginal distribution $l(\gamma)$ and the cumulative distribution is $L(\gamma)$. Insiders are assumed to know the realized value of Γ, γ.

The first two subsections examine the four financing strategies, first the two debt strategies and then the two equity strategies in the second subsection. These subsections consider the extent to which changes in γ would change insider's valuation of the outsider's claim and show how outsiders would solve for their claim. The third subsection considers the outcome when the insiders can chose among the four strategies.

A. Debt

The implication of information asymmetry about one aspect of the firm's profitability depends on whether the value of the claim is sensitive to the firm's profitability. If the claim is not sensitive to changes in the firm's profitability then both insiders and outsiders will assign the same value to any given claim and the information asymmetry is not relevant to the financing decision.

The value of a given junior debt claim, $F^j(0)$ given insider's knowledge of γ is:

$$\mathrm{E}(F^j(0) \mid \gamma) = \int_{r^*}^{b} F^j(0) dg(r) + \int_{r^c}^{r^*} \left(P^*\left(r, \gamma\right) - F^s(0) \right) dg(r) . \tag{36}$$

The sensitivity of this valuation to changes in γ may be obtained by differentiating with respect to γ:

$$\begin{aligned} \frac{\partial \mathrm{E}(F^j(0) \mid \gamma)}{\partial \gamma} = & \frac{\partial F^j(0)}{\partial \gamma}\left(1 - G\left(r^*\right)\right) - F^j(0)\frac{\partial r^*}{\partial \gamma} g(r^*) \\ & + (P^*(r^*, \gamma) - U)\frac{\partial r^*}{\partial \gamma} g(r^*) \\ & - \left(P\left(r^c, \gamma\right) - F^s\left(0\right)\right) g(r^c)\frac{\partial r^c}{\partial \gamma} \\ & + \int_{r^c}^{r^*} \frac{\partial P^*\left(r^*, \gamma\right)}{\partial \gamma} dg(r) \\ & = 0. \end{aligned} \tag{37}$$

The second two terms cancel due to smooth pasting and the fourth term is zero by the definition of $F^s(0)$ which is U. The term $\partial P^*/\partial\gamma$ is negative therefore the term $\partial F^j(0)/\partial\gamma$ must be positive. This result is intuitive; an increase in the cost of production reduces expected profits and thereby increases the demanded face value of the debt.

Although insiders' valuation of the junior claim depends on the realization of γ, the best that outsiders can do is to demand a face value on their claim that provides the same expected value as their initial investment K.

$$\int_c^d \left(\int_{r^*}^b F^j(0) dg(r) + \int_{r^c}^{r^*} (P^*(r,\gamma) - F^s(0)) dg(r) \right) dl(\gamma) = K . \qquad (38)$$

Insiders will be able to evaluate the value of $F^j(0)$ given their knowledge of γ, and given this knowledge will view the debt contract as either transferring wealth from themselves to outside investors or from outside investors to themselves.[14] However, if the only strategy available for financing the firm is to issue junior debt to outsiders and retain a senior claim then the firm will issue the debt provided the project is sufficiently profitable to offset any wealth transfer to outsiders.

The value of the debt with hedging is

$$\int_a^b F^j(h^*) dg(r) = F^j(h^*) = K . \qquad (39)$$

This valuation in which the outsiders are paid the same amount in every state is clearly independent of the realization γ. Thus, insiders and outsiders will agree on the value of the claim $F^j(h^*)$.

B. Equity

Outside equity may be accompanied by inside debt or by hedging. The insider's valuation if the firm issues debt to insiders with a face value of U:

$$S^e(0 \mid \gamma) = K / \int_{r^c}^b \left(P^*(r,\gamma) - U \right) dg(r) . \qquad (40)$$

In order to simplify the presentation, focus on $\partial \left(\int_{r^c}^b \left(P^*(r,\gamma) - U \right) dg(r) \right) \Big/ \partial\gamma$

$$\frac{\partial \left(\int_{r^c}^b \left(P^*(r,\gamma) - U \right) dg(r) \right)}{\partial \gamma} = \int_{r^c}^b \frac{\partial P^*(r,\gamma)}{\partial \gamma} dg(r) - \left(P^*(r^c,\gamma) - U \right) g(r^c) \frac{\partial r^c}{\partial \gamma} . \qquad (41)$$

Which is less than zero because $\partial P^*/\partial\gamma$ is negative and the second term is zero by from the definition of $P^*(r^c,\gamma)$. That is the dominator of (40) is decreasing in γ which implies that insiders estimate of the fair share of the firm that outsiders should receive is increasing in γ. Stated differently, an increase in costs reduces the firm's profitability and implies that outsiders will need a larger claim on the firm's residual cash flows to recover their initial investment K. Accordingly, in return for supplying the firm with K, outsiders will demand that, $S^e(0)$, their ownership interest satisfy:

$$S^e(0) = K / \left(\int_c^d \left(\int_{r^c}^b \left(P^*(r,\gamma) - U \right) dg(r) \right) dl(\gamma) \right). \qquad (42)$$

As was the case with the combination of junior and senior debt, insiders can compare the value of $S^e(0)$ demanded by outside investors with the fair value and determine whether the pricing of the outside equity will transfer wealth from insiders to outsiders or outsiders to insiders. Nevertheless, if the only financing strategy of issuing equity to outsiders and a debt claim with a face value of U to insiders is feasible and the firm is sufficiently profitable so that expected profits exceed any wealth transfers to outsiders.

The analysis of the strategy of issuing outside equity and hedging is similar, with the hedge cash flows replacing the payments on the junior debt.

$$S^e(h^*) = K / \int_{r^c}^b \left(P^*(r,\gamma) + \left(r^f - r \right) h^* \right) dg(r) . \qquad (43)$$

Differentiating the denominator of (43) with respect to γ results in an expression similar to that in equation (41), which has a value less than zero. Thus, the share of the firm demanded by outsiders when the firm hedges is similar to that in equation (42), substituting in $(r^f - r)h^*$ for U. Also as is true for the other equity financing strategy, the issuance of outside equity and hedging

is feasible if it is the only alternative and the firm is sufficiently profitable so that the expected profits exceed any wealth transfers to outsiders.[15]

C. The managers have access to all four financing options

The above analysis shows that each of the four financing strategies that are viable if considered individually and the firm is sufficiently profitable to offset any wealth transfer to creditors. Three of the four strategies required with positive probability that insiders issue claims to outsiders that are, given the insider's knowledge of γ, going to result in a wealth transfer to outside investors. However, no such wealth transfers need occur if the firm issues debt to outsiders and hedges its price risk. If insiders have access to an option that does not require wealth transfers, other strategies that involve wealth transfers will not be viable.

As an example of why the combination of debt and hedging would dominate the other alternatives, suppose the firm's other alternative is an equity issue to outsiders combined with hedging. The managers will first evaluate the equity issue to determine whether they receive a wealth transfer or will be transferring wealth to outside investors (or equivalently whether outsiders' demand a share that is smaller or larger than they would demand if they know γ). If the managers would be the beneficiary of the wealth transfer they would want the firm to issue equity and hedge. Alternatively, if outsiders would be the beneficiary then the managers would want the firm to issue debt and hedge. However, outside investors would recognize that they face something like a "winner's curse" problem with purchasing equity, that they only time they could buy equity is when it provided a wealth transfer to the firm's managers. In response, outside investors would refuse to purchase the equity and the firm's only financing strategy would be to issue debt and hedge its price risk.

III. Summary, Extensions and Empirical Implications

In this paper we derive the possible value added of hedging for firms in an equilibrium model with labor market frictions. Specifically, we assume that human capital is inalienable, managers have valuable exit options and that managers are not perfectly replaceable. In the case of symmetric information we show that a fixed salary along with outside debt or equity can yield the same first best allocation that can be obtained by hedging without a fixed salary. Conversely, we show that, in equilibrium, firms will find it value enhancing to hedge and issue outside debt if there is asymmetric information between managers and outsiders. This is the unique solution to the problem. Issuing outside equity and hedging will not solve this problem since it is not possible to make residual claims riskless, which is needed in order to achieve the first best allocations.

The model has a number of empirical implications. The first is that firms with a large amount of valuable human capital will find it advantageous to hedge. Thus, we should find that firms in the financial services industry, with little fixed capital, are more likely to hedge than other firms. Moreover, corporations with valuable R&D employees should find it more beneficial to hedge than other firms. More generally, if growth opportunities are more human capital intensive than assets in place, we should find that firms with more growth opportunities should hedge more. (See for example, Gay and Nam (1998).

Our results are also consistent with the idea that, to the extent that managers at large firms have more valuable outside exit options, large firms hedge relatively more than smaller firms. This same idea implies that managers with higher compensation should hedge more to the extent that compensation is the shadow price of outside options. Finally, we note that firms that are more profitable should hedge less and have less debt since cash flows are already close to

riskless if they can use internal capital. Thus, our theory suggests a reason why profitable firms both hedge less and hold less debt, a fact that Myers (2001) argues is not explainable by either the tradeoff theory of capital structure or the pecking order theory. Finally, it is the case that our theory implies that firms with higher debt levels should hedge more (see, e.g., Smith and Lin (2003)).

Our empirical predictions are consistent with much of the empirical literature on risk management at the corporate level. While consistent with existing empirical work, we note that it is possible to distinguish our theory from those in the existing literature by developing measures of human capital intensity similar to those in Qian (2003) and running cross sectional regressions using hedging and debt as the dependent variables and human capital intensity and control variables as regressors.

From a theoretical point of view we see a number of extensions to the model. The first is to introduce costly hedging. In a model with costly hedging, making the cash flows to outsiders riskless may not be the optimal solution. However, even in such a model there should be a benefit to debt relative to equity to the extent that hedging is able to reduce the risk of debt claims in a way that is not possible with outside equity.

The model could also be extended to incorporate multiple sources of risk (e.g., liquidity shocks and price risks) in a fashion similar to the model of Holmstrom and Tirole (2000). This would allow us to be able to separately value the benefits of liquidity (or financial slack) vs. the benefits that arise from hedging a particular price risk. In the model as it stands, all of the cash flow risk that the firm faces comes from variation in the output price.

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Endnotes

[1] Even if the managers are to some extent tied to the firm through no-compete clauses, these are insufficient to keep the manager from engaging in some outside activity, even if it is outside the industry. In our model, any positive value of the outside exit option will bring about states of financial distress if simple debt or equity is issued to outsiders.

[2] Stulz (1996) reviews these theories and argues that many firms risk management involves "taking a view" on market prices or interest rates. Faulkender (2005) provides some evidence for this view of risk management. Also see Triki (2005) for a review of the empirical literature on risk management and hedging.

[3] Recently Hogh et. al. (2003) have revisited the Froot and Stein model and showed that the complete hedging result developed in that paper is not in general correct.

[4] Our qualitative conclusions will not change as long as the loss of value associated with the management change is strictly positive. The assumption that we use here simply keeps the analytics much simpler.

[5] We have used a futures contract for the hedging solution because it is particularly tractable. With somewhat more complications we could have assumed that the firm uses options or other non-linear claims such as insurance to hedge the risk. All that matters is that the hedge makes the cash flows riskless to outside claimants. However, one would have to assume that the cost of the option or insurance contract is included with K when the firm raises outside capital.

[6] We also assume that the parties on the other side of the futures contract have sufficient capital so that there is no default on their part.

[7] In order to obtain the socially first best solutions later on in the paper we assume that this outside option is unique to the manager. That is, no other party can obtain the same level of U for the same effort. This assumption has no implications for the managers' maximization of their own wealth. However, altering this assumption could imply that maximizing the managers' wealth is not the same as maximizing social welfare.

[8] We assume that the realization of the output price and the managers outside exit option are independent. However, we could assume that there was some positive correlation between the two as long as the manager's outside opportunities have positive value at r^c.

[9] A similar result would hold if either: (1) the managers are given a salary at least equal to U that takes priority over the outside debt, or (2) the firm issued senior debt that was subject to renegotiation in which the managers would hold the power as they have the outside option. The analysis of renegotiation is identical to the case of the managers receiving senior debt if: (a) bargaining only occurs when the managers' share of the operating profits is less than U, and (b) the solution to the bargaining game is assumed to be that the managers receive only the value of their outside option. U. The conclusion would also be the same if the manager received more than U in some states renegotiation, albeit the details of the analysis would change somewhat. Moreover, we note that creditors may find renegotiating with the managers dominated by other strategies in a multiperiod setting.

[10] Even with hedging, the firm's marginal profits are quadratic in R given that the firm's scale depends on r and that the firm's cost function is quadratic in q.

[11] Unlike the case of managerial risk aversion, having the managers hedge on their own account would not result in the managers following the first best effort rule. The problem is that there is no link between payments received under the hedge and the managers' decision to remain with the firm. The managers could retain the payments from the hedge and exercise their outside option, thereby defaulting on the outside debt. Moreover, doing so would leave the managers with more wealth than remaining with the firm and paying off the outside debt.

[12] We also considered asymmetry in information about the value of the insider's option for employment outside the firm. However, this asymmetry need not result in any disagreement about security valuation if the outsiders are given a junior claim that just equals K in expected value terms and the insiders receive a senior claim on all of the

rest of the cash flow. In this case, a junior debt claim issued by the firm to outsiders would have payment structure that looked more like an equity claim than a debt claim, but it would solve the problem of asymmetry about U without creating any disagreement about the value of the cash flows to the junior claim.

[13] Asymmetric information about the distribution of R would produce similar results albeit at the cost of more complex mathematical expressions.

[14] Ignoring the measure zero possibility that that γ takes the one value where insiders would have the same value as outside claimants.

[15] We note that with equity and hedging the form of the mispricing may in fact alter the hedge put on by the manager. In this case, equity and hedging would not only result in wealth transfers but may also result in the managers deviating from the socially optimal effort decision.

Appendix

Proposition 2. The managers will follow their welfare maximizing decision rule of staying with the firm if $P^* \geq U$ if the managers hedges by, h^*, the amount exactly required to make the debt riskless:

$$\boldsymbol{h^* (r^f - r^c) = K} \qquad \textbf{or} \qquad \boldsymbol{h^* = K / (r^f - r^c).}$$

Proof. Consider three cases:

A) $\boldsymbol{r < r^c}$

The firm pays off the fixed obligation with the $h^* (r - r^c)$, which it can do under Assumption 2. The managers retain the remaining $h^* (r^f - r)$ and takes their outside employment option.

B) $\boldsymbol{r = r^c}$

The cash flow to the managers, C_0^{dh}, at time 0 if the managers hedge h^* is

$$C_0^{dh} = q^* r^c - \gamma(q^*)^2/2 + h^* (r^f - r^c) - K \qquad \text{(A1)}$$

The last two terms of equation (A1) drop out given the definition of h^* as the managers use the proceeds from the hedge to repay the creditors. Substituting in from equation (5) for the value of r^c yields

$$C_0^{dh} = q^* ((U + \gamma(q^*)^2/2) / q^*) - \gamma(q^*)^2/2 = U \qquad \text{(A2)}$$

Thus, the value of remaining with the firm is exactly equal in value to the managers' outside option and they have no incentive to exercise that option.

C) $r > r^c$

The value of the cash flows to the manager if the firm hedges is

$$\begin{aligned} C_0^{dh} &= q^*r - \gamma(q^*)^2/2 + h^*(r^f - r) - K \\ &= q^*r - \gamma(q^*)^2/2 + h^*(r^f - r^c) + h^*(r^c - r) - K \\ &= q^* r - \gamma(q^*)^2/2 + h^*(r^c - r) \\ &= q^* r^c + (q^* - h^*)(r - r^c) - \gamma(q^*)^2/2 \end{aligned} \qquad \text{(A3)}$$

substituting in the definition of *U yields*

$$C_0^{dh} = U + (q(r^c,\gamma) - h^*)(r - r^c) \qquad \text{(A4)}$$

which takes a value equal to or greater than U provided that $(q(r^c,\gamma) - h^*) \geq 0$. But by the assumption at r^c we have that $q(r^c,\gamma)$ is greater than h^*.

Give the above derivations, we can derive the sum of the managers' value in the two states with measure greater than zero:

$$
\begin{aligned}
V^{dh} &= \int_a^{r^c} \left(U + h*\left(r^f - r\right)\right) dg(r) + \int_{r^c}^{b} \left(P* + h*\left(r^f - r\right)\right) dg(r) - K \\
&= \int_a^{r^c} U dg(r) + \int_{r^c}^{b} P* dg(r) + \int_a^{b} h*\left(r^f - r\right) dg(r) - K \qquad \text{(A5)} \\
&= \int_a^{r^c} U dg(r) + \int_{r^c}^{b} P* dg(r) - K
\end{aligned}
$$

Which is the maximum possible wealth for the managers.

Q.E.D.